

## Chapter 10

# Fundamentals of Metal Casting

### QUALITATIVE PROBLEMS

**10.15 Describe the stages involved in the contraction of metals during casting.**

The stages involved in the contraction of metals during casting are outlined in Section 10.5.2 on p. 274. The student is encouraged to elaborate further regarding these stages, also providing some data from the technical literature for purposes of comparison.

**10.16 Explain the reasons why heat transfer and fluid flow are important in metal casting.**

Heat transfer and fluid flow have a direct effect on the formation or suppression of defects in metal casting. Defects such as porosity (due to either shrinkage or gas), hot tears, and misruns (i.e., when the molten metal in a casting freezes before the mold is completely filled, shutting off that portion of the mold) are all controlled by these factors. Furthermore, the grain structure (hence properties such as strength and toughness of a metal casting) is dependent upon the rate and direction of heat transfer.

**10.17 We know that pouring metal at a high rate into a mold has certain disadvantages. Are there any disadvantages to pouring it very slowly?**

If a metal is poured too slowly it may solidify while it is still in the gating system or before completely filling the mold cavities. This will result in an incomplete or partial casting. This situation can be overcome by using a mold with a lower thermal conductivity or a preheated mold, but these lead to reduced mold life and longer cycle times.

**10.18 Describe the events depicted in Fig. 10.5.**

Due to a greater freezing range (see Eq. (10.1) on p. 264), gray cast iron forms an extensive dendritic structure and requires a considerable amount of time to solidify (Fig. 10.5a on p. 265). Steel, on the other hand, has a shorter freezing range and, thus, has a less extensive

dendritic structure. As the carbon in the steel is increased, however, there is a greater tendency to form dendrites and hence the time to solidify increases. The effect of a chill mold is to greatly decrease the time for the metal to solidify, and this limit dendrite formation.

**10.19 Would you be concerned about the fact that parts of internal chills are left within the casting? What materials do you think chills should be made of and why?**

The fact that a part of the chill remains within the casting should be considered in the design of the part. The following factors should be taken into consideration:

- (a) Any gas entrained in the molten metal when it contacts the chill may not readily escape; the chill could be a location where gas bubbles are in locally high concentration, and this can be a stress concentration.
- (b) The chill may not fuse with the casting, developing regions of weakness.
- (c) The material from which the chill is made should be compatible with the metal being cast, i.e., it should have approximately the same composition of the metal being poured.

If these factors are understood and provided for, the fact that a piece of the chill remains within the casting should generally not be a significant concern.

**10.20 What practical demonstrations can you offer to indicate the relationship of the solidification time to the volume and surface area?**

By the student. As an example, consider the following: If a swimming pool is filled with water and an equal volume of water is spread on a road were both subjected to a temperature below freezing, it is easy to see which would freeze first. Students should give other examples to illustrate this situation.

**10.21 Explain why you may want to subject a casting to various heat treatments.**

Heat treatments (described in Chapter 4) such as quenching and tempering, among others, are carried out to optimize the grain structure of metal castings, thereby controlling and enhancing mechanical properties. Heat treating can control microporosity, which is a main reason that castings are weak in tension.

**10.22 Why does porosity have detrimental effects on the mechanical properties of castings? Would physical properties (such as thermal and electrical conductivity) also be affected by porosity? Explain.**

Pores are, in effect, internal discontinuities that are prone to propagate under external stresses. Thus, the toughness of a material, for example, will decrease as a result of porosity. Furthermore, the presence of pores in a metal part under tension requires that the material around the pores support a greater load than if no pores were present; thus the strength and elastic modulus are also lowered. Considering thermal and electrical conductivity, porosity decreases both the thermal and electrical conductivity because of the presence of a vacuum or air.

**10.23 A spoked handwheel is to be cast in gray iron. In order to prevent hot tearing of the spokes, would you insulate the spokes or chill them? Explain.**

Referring to Table 10.1 on p. 275, we first note that gray iron undergoes an expansion up to 2.5% on solidification. Although this fact may suggest that hot tearing cannot occur (see Fig.

10.12 on p. 276), we should also consider contraction of the spokes during cooling. Since the hot tearing tendency will be reduced as the strength increases, it would be advisable to chill the spokes to develop this strength.

**10.24 Which of the following considerations is/are important for a riser to function properly? Must it: (a) have a surface area larger than the part being cast, (b) be kept open to atmospheric pressure, and/or (c) solidify first? Why?**

Both (a) and (c) would result in a situation contrary to a riser's purpose, that is, if a riser solidifies first, it cannot feed the mold cavity to avoid shrinkage in the part. Concerning (b), when the molten metal enters the mold cavity, the air which was in the mold has to be forced out. If a riser is not open to the atmosphere, either the gas will become dissolved into the metal (due to the increased pressure and depending on solubility), or sufficient pressure will build up which may crack the mold. Thus, a riser should be kept open to atmospheric pressure in order for it to function properly.

**10.25 Explain why the constant  $C$  in Eq. (10.7) depends on mold material, metal properties, and temperature.**

The constant  $C$  in Eq. (10.7) on p. 272 takes into account various factors such as the conductivity of the mold material and external temperature. For example, zircon sand (zirconium silicate) has a higher thermal conductivity than basic silica sand; as a result, a part cast in a zircon mold of equal volume and surface area to that of a part cast in silica will require less time to solidify.

**10.26 Are external chills as effective as internal chills? Explain.**

The answer depends on the location of the chills in the mold (see Fig. 10.14 on p. 278). That is, if a surface needs to be chilled (say, for example, to directionally solidify a casting), then an external chill is as effective as an internal chill. Often, however, chilling is required at some depth beneath the surface of a casting. For this condition an internal chill would be more effective.

**10.27 Explain why gray cast iron undergoes expansion rather than contraction during solidification, as shown in Table 10.1.**

As gray cast iron solidifies, a period of graphitization occurs during the final stages. This causes an expansion that counteracts the shrinkage of the metal during solidification, and results in an overall expansion.

**10.28 Referring to Fig. 10.11, explain why internal corners (such as A) develop a thinner skin than external corners (such as B) during solidification.**

We note in Fig. 10.11 on p. 273 that the internal corner A has a larger volume of material near to its surface area than does the external corner B. This situation can be visualized even better by assuming that the angles at A and B are less than  $90^\circ$ . Consequently, a point at a certain distance inward from corner A will remain at a higher temperature than a point at the same distance inward from corner B. Therefore, during the same time period, corner A will develop a thinner skin than will corner B. This can also be explained by considering heat flow directions, and noting that the volume in a corner will solidify most quickly because

heat can be quickly extracted from this area; at an internal corner such as at A, the mold material cannot conduct as much heat as quickly.

**10.29 Note the shape of the two risers in Fig. 10.8, and discuss your observations with respect to Eq. (10.7).**

This is an open-ended problem, and a number of observations can be made. The side riser (at left in Fig. 10.8 on p. 268) has a greater volume than the top riser shown on the right. As a result and referring to Eq. (10.7) on p. 272, we would expect the side riser to require a longer solidification time than the top riser. This is, as one would expect, because the metal closest to the point of entry, i.e., sprue and runner, will be the hottest. The metal near the side riser should remain liquid longer than that near the top riser, thus requiring a larger riser because that portion of the casting is intended to be the last to solidify.

**10.30 Is there any difference in the tendency for shrinkage void formation for metals with short and long freezing ranges, respectively? Explain.**

Consider an alloy poured into a mold, where the exterior solidified and a solidification front progresses towards the casting center. In an alloy with a large freezing range, the presence of a large mushy zone is more likely to occur and, thus, the formation of microporosity. However, in an alloy with a short freezing range, the formation of gross shrinkage voids is more likely to occur near the center of the casting. The total porosity is the same in this case. With proper gating and riser design, the porosity for the short freezing range alloy is easier to eliminate or control.

**10.31 What is the influence of the cross-sectional area of the spiral channel in Fig. 10.9 on fluidity test results? What is the effect of sprue height? If this test is run with the test setup heated to elevated temperatures, would the test results be more useful? Explain.**

Referring to Fig. 10.9 on p. 272, we can make the following observations:

- (a) The greater the cross-sectional area of the spiral channel, the further the metal will flow in the mold. Consider Eq. (10.7) on p. 272, which describes the solidification time.
- (b) An increase in sprue height would increase the velocity of the metal that enters the spiral, thus allowing the metal to flow further into the spiral than for a lower sprue height.
- (c) Tests can be, and are, conducted with the test setup used at elevated temperatures, showing the effect of a preheated mold on the fluidity of the molten metal. Such a test is especially useful for the investment or die-casting processes described in Sections 11.2.6 on p. 300 and 11.3.5 on p. 306, respectively.

**10.32 It has long been observed by foundrymen and ingot casters that low pouring temperatures (i.e., low superheat) promote formation of equiaxed grains over columnar grains. Also, equiaxed grains become finer as the pouring temperature decreases. Explain these phenomena.**

Equiaxed grains develop in castings near the mold wall where rapid cooling and solidification takes place by heat transfer through the relatively cool mold. With low pouring temperatures, cooling to the solidification temperature is faster because of the lower heat capacity of the

molten metal. With a high pouring temperature, cooling to the solidification temperature is slower. The mold still dissipates the heat but the metal being poured remains molten for a longer period of time, thus producing columnar grains in the direction of heat conduction. As the pouring temperature is decreased, equiaxed grains become finer because the energy required to heat the mold is a larger fraction of the heat in the molten metal. Thus there is more rapid initial cooling as the mold temperature is increased.

**10.33 What would you expect to occur (in casting metal alloys) if the mold was agitated aggressively (vibrated) after the molten metal had been in the mold for a sufficient amount of time to form a skin?**

By the student. Several effects can occur. The most obvious is that any dendrites which may exist in the slushy phase will be broken up by the agitation. Also, agitation will aid in more rapid cooling, because of the increased contribution of convection and also because the mold/casting interface will have a lower thermal resistance.

**10.34 If you examine a typical ice cube, you will see pockets and cracks in the cube. However, some ice cubes are tubular in shape and do not have noticeable air pockets or cracks in their structure. Explain these phenomena.**

Note that this is not universally true; ice expands when it solidifies, and needs to be cooled sufficiently before stresses develop that crack the ice. The reason for this is that the ice cube first begins to solidify at its outside surfaces; the interior then contracts as it begins to cool. Since there is no riser or an equivalent means, the ice cube develops microcracks in the interior. The effect is actually less than for metals because water has a minimum specific volume at  $-4^{\circ}\text{C}$ , whereas most metals shrink further while undergoing phase changes during solidification. Tubular ice pieces are formed by the exposure of water to copper tubes that have a refrigerant pumped through them. Thus, they solidify from the inside outward. The pockets are gases that are soluble in the water but have lower solubility in ice.

**10.35 How can you tell whether cavities in a casting are due to shrinkage or entrained air bubbles?**

The simplest method is observing them under a microscope. Air bubbles will have sufficient surface tension while the metal is liquid to form a spherical cavity, whereas shrinkage pores will be far more jagged because they are formed by localized fracture of the solidified metal. There are other tests that can be performed as well, for example, shrinkage cavities will theoretically be under a vacuum, whereas an air bubble will be filled with gas. Therefore, casting can be performed in the presence of a gas that can be traced, such as argon or helium. The casting can be remelted in vacuum and outgassing of argon or helium can be measured.

**10.36 Describe the drawbacks to having a riser that is (a) too large and (b) too small.**

A riser that is too large wastes material, adds to the solidification time and will require additional finishing operations. Also, a large riser can adversely affect the solidification pattern and lead to voids or cold shuts in the casting. In addition, large risers may be difficult to locate in the sand mold. On the other hand, a riser that is too small may not provide sufficient molten metal to compensate for solidification shrinkage in the casting. Also, it may solidify prematurely, so that it fails to serve as a riser; it may not influence the solidification front;

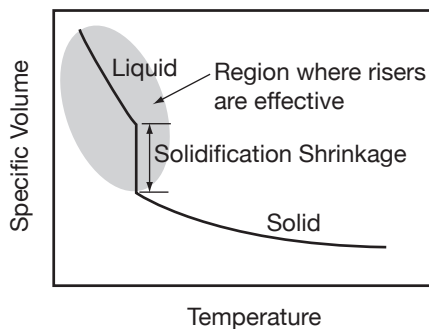
and it may require higher preheat levels, leading to more shrinkage pores and hence lower casting quality.

**10.37 What are the benefits and drawbacks to having a pouring temperature that is much higher than the metal's melting temperature? What are the advantages and disadvantages in having the pouring temperature remain close to the melting temperature?**

If the pouring temperature is much higher than that of the mold temperature, there is less danger that the metal will solidify in the mold and it is likely that even intricate molds can be fully filled. This situation makes runners, gates, wells, etc., easier to design because their cross-sections are less critical for complete mold filling. The main drawback is that there is an increased likelihood of shrinkage pores, cold shuts, and other defects associated with shrinkage, and an increased likelihood of entrained air since the viscosity will be lower at the higher pouring temperature. If the pouring temperature is close to the melting temperature, there will be less likelihood of shrinkage pores and entrained air. However, there is the danger of the molten metal solidifying in a runner before the mold cavity is completely filled; this may be overcome with higher injection pressures but clearly have a cost implication.

## QUANTITATIVE PROBLEMS

**10.38 Sketch a graph of specific volume versus temperature for a metal that shrinks as it cools from the liquid state to room temperature. On the graph, mark the area where shrinkage is compensated for by risers.**



The graph for specific volume versus temperature is shown to the left, including compensation for shrinkage. The risers can compensate for shrinkage from the superheat temperature to solidification temperature, and also, if properly designed, for shrinkage associated with solidification.

**10.39 A round casting is 0.2 m (7.9 in.) in diameter and 0.5 m (19.7 in.) in length. Another casting of the same metal is elliptical in cross-section with a major to minor axis ratio of 2 and has the same length and cross-sectional area as the round casting. Both pieces are cast under the same conditions. What is the difference in the solidification times of the two castings?**

For the same length and cross-sectional area (thus the same volume), and same casting conditions, the same  $C$  value should occur. The surface area and volume of the round casting

is

$$A_{\text{round}} = 2\pi rl + 2\pi r^2 = 0.377 \text{ m}^2$$

$$V_{\text{round}} = 2\pi r^2 h = 0.031 \text{ in}^2$$

Since the cross-sectional area of the ellipse is the same as that for the cylinder, and it has a major and minor diameter of  $a$  and  $b$ , respectively, where  $a = 2b$ , then

$$\pi ab = \pi r^2 \quad \rightarrow \quad 2b^2 = r^2 \quad \rightarrow \quad b = 0.071 \text{ m}$$

So that  $a = 0.14 \text{ m}$ . The surface area of the ellipse-based part is (see a basic geometry text for the area equation derivations):

$$A_{\text{ellipse}} = 2\pi ab + 2\pi\sqrt{a^2 + b^2}h = 0.556 \text{ m}^2$$

The volume is still  $0.031 \text{ in}^2$ . According to Eq. (10.7) on page 250, we thus have

$$\frac{T_{\text{round}}}{T_{\text{ellipse}}} = \frac{(V/A_{\text{round}})^2}{(V/A_{\text{ellipse}})^2} = \left( \frac{A_{\text{ellipse}}}{A_{\text{round}}} \right)^2 = 2.17$$

**10.40 A 100-mm (4-in.) thick square plate and a right circular cylinder with a radius of 100 mm (4 in.) and a height of 50 mm each have the same volume. If each is to be cast using a cylindrical riser, will each part require the same size riser to ensure proper feeding? Explain.**

First note that it is important for the riser to solidify after the casting has solidified. A casting that solidifies rapidly would be expected to require a smaller riser than one that solidifies over a longer period of time. Let's now calculate the relative solidification times. For the cylindrical part, we have

$$V = \pi r^2 h = \pi(4)^2(2) = 100.5 \text{ in}^3$$

and

$$A = 2\pi r^2 + 2\pi r h = 2\pi(4)^2 + 2\pi(4)(2) = 150.8 \text{ in}^2$$

Thus

$$t_{\text{cylinder}} = C(100.5/150.8)^2 = 0.444C$$

For a square plate with sides  $L$  and height  $h$ , we have

$$V = 100.5 = L^2 h = L^2(4), \text{ or } L = 5.0 \text{ in.}$$

and

$$A = 2L^2 + 4Lh = 130 \text{ in}^2$$

Thus

$$t_{\text{plate}} = C(100.5/130)^2 = 0.59C$$

Therefore, the cylindrical casting will take less time to solidify and hence will require a smaller riser.

- 10.41** Assume that the top of a round sprue has a diameter of 3 in. (75 mm) and is at a height of 8 in. (200 mm) from the runner. Based on Eq. (10.5), plot the profile of the sprue diameter as a function of its height. Assume that the sprue wall has a diameter of 0.25 in. (6 mm) at the bottom.

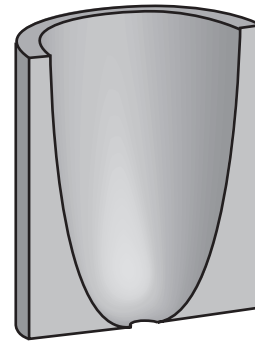
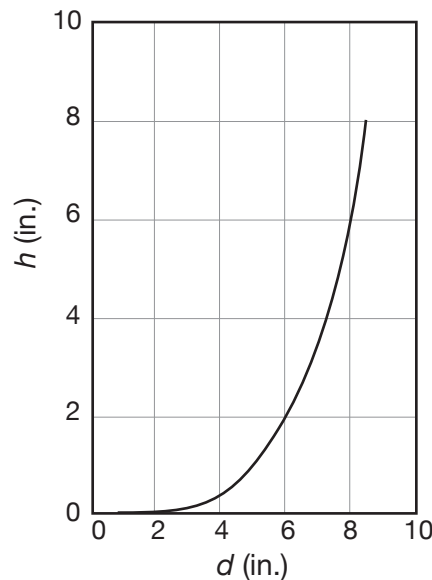
From Eq. (10.5) and substituting for the area, it can be shown that

$$\frac{d_1^2}{d^2} = \sqrt{\frac{h}{h_1}} \rightarrow d = \sqrt{d_1^2 \sqrt{\frac{h_1}{h}}} \rightarrow d = Ch^{-0.25}$$

The difficulty is that the reference location for height measurements is not known. The boundary conditions are that at  $h = h_o$ ,  $d = 0.25$  (where  $h_o$  is the height at bottom of the sprue from the reference location) and at  $h = h_o + 8$  in.,  $d = 3$  in. The first boundary condition yields  $0.25 = C(h_o)^{-0.25}$ ;  $h_o = 0.25C^{-4}$ . The second boundary condition yields

$$3 = C(h_o + 8)^{-0.25} = C(0.25C^{-4} + 8)^{-0.25}$$

This equation is solved numerically as  $C = 5.04$ , so that  $h_o = 0.000386$ . These values are substituted into the expression above to get the plot shown below. Note that  $h_o$  is the location of the bottom of the sprue and that the sprue is axisymmetric. The sprue shape based on this curve is shown to the right. Note that normally a pouring basin would be included in the design, and the sprue would either be tapped to the side or would sit on top of a well.



- 10.42** Pure aluminum is poured into a sand mold. The metal level in the pouring basin is 8 in. above the metal level in the mold, and the runner is circular with a 0.5-in. diameter. What is the velocity and rate of flow of the metal into the mold? Is the flow turbulent or laminar?

Equation (10.3) on page 268 gives the metal flow; assuming the pressure does not change appreciably in the channel and that there is no friction in the sprue, the flow is

$$h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g}$$



Where the subscript 1 indicates the top of the sprue and 2 the bottom. If we assume that the velocity at the top of the sprue is very low (as would occur with the normal case of a pouring basin on top of the sprue with a large cross-sectional area), then  $v_1 = 0$ . Therefore, the velocity at the bottom of the sprue is

$$v_2^2 = 2g(h_1 - h_2) \rightarrow v_2 = \sqrt{2g\Delta h} = \sqrt{2(32.2 \text{ ft/s}^2)(12 \text{ in/ft})(8 \text{ in})} = 78.6 \text{ in./s}$$

If the opening is 0.5-in. in diameter, the flow rate is  $Q = v_2 A = 15.4 \text{ in}^3/\text{s}$ . Pure aluminum has a density of  $2700 \text{ kg/m}^3$  (Table 3.1 on p. 103) and a viscosity that is comparable to water at room temperature ( $0.15 \times 10^{-6} \text{ lb-s/in}^2$ ). The Reynolds number, from Eq. (10.6) on p. 270, is then

$$\text{Re} = \frac{vD\rho}{\eta} = \frac{(78.6 \text{ in./s})(0.5 \text{ in.})(2700 \text{ kg/m}^3)}{0.15 \times 10^{-6} \text{ lb-sec/in}^2} = 68,000$$

**10.43 A cylinder with a diameter of 1 in. and height of 3 in. solidifies in three minutes in a sand casting operation. What is the solidification time if the cylinder height is doubled? What is the time if the diameter is doubled?**

The surface area of the cylinder is given by

$$A = 2\left(\frac{\pi}{4}d^2\right) + \pi dh = \frac{\pi}{2}(1 \text{ in.})^2 + \pi(1)(3) = 3.5\pi \text{ in}^2$$

and the volume is

$$V = \frac{\pi}{4}d^2h = \frac{\pi}{4}(1)^2(3) = 0.75\pi \text{ in.}^3$$

From Eq. (10.7), we can evaluate the constant  $C$  as:

$$\text{Solidification time} = C\left(\frac{V}{A}\right)^2 \rightarrow 3 \text{ min} = C\left(\frac{0.75\pi \text{ in.}^3}{3.5\pi \text{ in}^2}\right)^2$$

or  $C = 65.33 \text{ min/in}^2$ . If the height is doubled, so that  $h = 6 \text{ in.}$ , then

$$A = 2\left(\frac{\pi}{4}d^2\right) + \pi dh = \frac{\pi}{2}(1 \text{ in.})^2 + \pi(1)(6) = 6.5\pi \text{ in}^2$$

$$V = \frac{\pi}{4}d^2h = \frac{\pi}{4}(1)^2(6) = 1.5\pi \text{ in}^3$$

so that from Chvorinov's rule, we have

$$\text{Solidification time} = C\left(\frac{V}{A}\right)^2 = (65.33 \text{ min/in}^2)\left(\frac{1.5\pi \text{ in}^3}{6.5\pi \text{ in}^2}\right)^2 = 3.48 \text{ min}$$

which represents an increase of about 28 seconds. If the diameter is doubled to 6 in., then

$$A = 2\left(\frac{\pi}{4}d^2\right) + \pi dh = \frac{\pi}{2}(2 \text{ in.})^2 + \pi(2)(3) = 8\pi \text{ in}^2$$

$$V = \frac{\pi}{4}d^2h = \frac{\pi}{4}(2)^2(3) = 3\pi \text{ in}^3$$

and

$$\text{Solidification time} = C\left(\frac{V}{A}\right)^2 = (65.33 \text{ min/in}^2)\left(\frac{3\pi \text{ in}^3}{8\pi \text{ in}^2}\right)^2 = 9.19 \text{ min}$$

Thus, the solidification time increases by more than six minutes.

- 10.44** The volume flow rate of metal into a mold is  $0.01 \text{ m}^3/\text{min}$ . The top of the sprue has a diameter of 20 mm, and its length is 200 mm. What diameter should be specified at the bottom of the sprue to prevent aspiration? What is the resultant velocity and Reynolds number at the bottom of the sprue if the metal being cast is aluminum with a viscosity of  $0.004 \text{ N}\cdot\text{s}/\text{m}^2$ ?

Note that the metal volume flow rate is  $Q = 0.01 \text{ m}^3/\text{min} = 1.667 \times 10^{-4} \text{ m}^3/\text{s}$ . Again, let's use the subscripts 1 for the top and 2 for the bottom of the sprue. Since  $d_1 = 20 \text{ mm} = 0.02 \text{ m}$ ,

$$A_1 = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.02 \text{ m})^2 = 3.14 \times 10^{-4} \text{ m}^2$$

Therefore,

$$v_1 = \frac{Q}{A_1} = \frac{1.667 \times 10^{-4} \text{ m}^3/\text{s}}{3.14 \times 10^{-4} \text{ m}^2} = 0.531 \text{ m/s}$$

Assuming no frictional losses and recognizing that the pressure at the top and bottom of the sprue is atmospheric, Eq. (10.3) gives

$$h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g} \quad \rightarrow \quad 0.2 \text{ m} + \frac{(0.531 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 0 + \frac{v_2^2}{2(9.81 \text{ m/s}^2)}$$

or  $v_2 = 1.45 \text{ m/s}$ . To prevent aspiration, the sprue opening should be the same as that required by flow continuity, or

$$Q = A_2 v_2 = 1.667 \times 10^{-4} \text{ m}^3/\text{s} = A_2(1.45 \text{ m/s}) \quad \rightarrow \quad A_2 = 1.150 \times 10^{-4} \text{ m}^2$$

hence  $d = 12 \text{ mm}$ . To calculate the Reynolds number, we first note from Table 3.1 that the density of aluminum is  $2700 \text{ kg}/\text{m}^3$ . The density for molten aluminum will of course be lower, but not significantly so, so this value is sufficient for this problem. From Eq. (10.6),

$$\text{Re} = \frac{vD\rho}{\eta} = \frac{(1.45 \text{ m/s})(0.012 \text{ m})(2700 \text{ kg}/\text{m}^3)}{0.004 \text{ N}\cdot\text{s}/\text{m}^2} = 11,745$$

As discussed on p. 270, this is typical and is a mixture of laminar and turbulent flow.

- 10.45** A rectangular mold with dimensions  $100 \text{ mm} \times 200 \text{ mm} \times 400 \text{ mm}$  is filled with aluminum with no superheat. Determine the final dimensions of the part as it cools to room temperature. Repeat the analysis for gray cast iron.

Note that the initial volume of the box is  $(0.100)(0.200)(0.400) = 0.008 \text{ m}^3$ . From Table 10.1 on p. 275, the volumetric contraction for aluminum is 6.6%. Therefore, the box volume will be

$$V = (1 - 0.066)(0.008 \text{ m}^3) = 0.007472 \text{ m}^3$$

Assuming the box has the same aspect ratio as the mold (1:2:4), and that warpage can be ignored, then we can calculate the dimensions of the box after solidification as  $97.7 \text{ mm} \times 195.5 \text{ mm} \times 391 \text{ mm}$ . From Table 3.1 on p. 103, the melting point of aluminum is  $660^\circ\text{C}$ , with a coefficient of thermal expansion of  $23.6 \mu\text{m}/\text{m}^\circ\text{C}$ . Thus, the total strain in cooling from  $660^\circ\text{C}$  to room temperature ( $25^\circ\text{C}$ ) is

$$\epsilon = \alpha\Delta t = (23.6 \mu\text{m}/\text{m}^\circ\text{C})(660^\circ\text{C} - 25^\circ\text{C}) = 0.0150$$

So that the final box dimensions are  $96.2 \times 192.5 \times 385$  mm. For gray cast iron, the metal expands upon solidification. Assuming the mold will allow expansion, the volume after solidification is given by

$$V = (1.025)(0.008 \text{ m}^3) = 0.0082 \text{ m}^3$$

If the box has the same aspect ratio as the initial mold cavity, the dimensions after solidification are  $100.8 \times 201.7 \times 403.3$  mm. Using the data for iron in Table 3.1 on p. 103, the melting point is taken as  $1537^\circ\text{C}$  and the coefficient of thermal expansion is  $11.5 \mu\text{m}/\text{m}^\circ\text{C}$ . Therefore

$$\epsilon = \alpha\Delta t = (11.5\mu\text{m}/\text{m}^\circ\text{C})(1537^\circ\text{C} - 25^\circ\text{C}) = 0.0174$$

So that the final dimensions are  $99.0 \times 198.1 \times 396$  mm. Note that even though the cast iron needed to cool off from a higher initial temperature, the box of cast iron is much closer to the mold dimensions than the aluminum.

- 10.46** The constant  $C$  in Chvorinov's rule is given as  $3 \text{ s}/\text{mm}^2$ , and is used to produce a cylindrical casting with a diameter of 75 mm and height of 125 mm. Estimate the time for the casting to fully solidify. The mold can be broken safely when the solidified shell is at least 20 mm. Assuming the cylinder cools evenly, how much time must transpire after pouring the molten metal before the mold can be broken?

Note that for the cylinder

$$A = 2 \left( \frac{\pi}{4} d^2 \right) + \pi dh = 2 \left[ \frac{\pi}{4} (75)^2 \right] + \pi (75)(125) = 38,290 \text{ mm}^2$$

$$V = \frac{\pi}{4} d^2 h = \frac{\pi}{4} (75)^2 (125) = 5.522 \times 10^5 \text{ mm}^3$$

From Chvorinov's rule given by Eq. (10.7) on p. 272,

$$t = C \left( \frac{V}{A} \right)^2 = (3 \text{ s}/\text{mm}^2) \left( \frac{5.522 \times 10^5}{38,290} \right)^2 = 624 \text{ s}$$

or, just over ten seconds to solidify. The second part of the problem is far more difficult, and different answers can be obtained depending on the method of analysis. The solution is not as straightforward as it may seem initially. For example, one could say that the 20 mm wall is 53.3% of the thickness, so that  $0.533(624) = 333$  seconds is needed. However, this would be insufficient because an annular section at an outside radius has more material than one closer to the center. It is reasonable and conservative to consider the time required for the remaining cylinder to solidify. Using  $h = 85$  mm and  $d = 35$  mm, the solidification time is found to be 21.8 seconds. Therefore, one still has to wait 602 seconds before the mold can be broken.

- 10.47** Assume that you are an instructor covering the topics described in this chapter, and you are giving a quiz on the numerical aspects to test the understanding of the students. Prepare two quantitative problems and supply the answers.

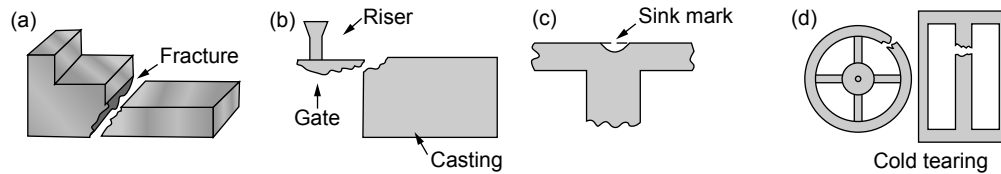
By the student. This is a good, open-ended question that requires considerable focus and understanding on the part of the students, and has, in the past, been found to be a very valuable homework problem.

## SYNTHESIS, DESIGN AND PROJECTS

**10.48** Can you devise fluidity tests other than that shown in Fig. 10.9? Explain the features of your test methods.

By the student. As a suggestion, tests could involve convergent sections, moving walls, or gravity assistance. Note that the tests should allow for a competition between pressure-driven flow and cooling of the molten metal along the path.

**10.49** Figure P10.49 indicates various defects and discontinuities in cast products. Review each one and offer solutions to avoid them.



**FIGURE P10.49**

By the student. Some examples are:

- (a) Notice that fracture occurred at one of the two steps in the casting, so that fracture is at stress riser. A better design would utilize a more gradual fillet radius.
- (b) Fracture at the gate indicates this runner section is too narrow and will solidified first; this gate should be larger.

**10.50** The fluidity test shown in Fig. 10.9 only illustrates the principle of this test. Design a setup for such a test, showing the type of materials and the equipment to be used. Explain the method by which you would determine the length of the solidified metal in the spiral passage.

By the student. This is an open-ended problem with a large number of potential solutions, automated or manual and made of different materials to accommodate different metals. Students should be encouraged to pursue their own creative solutions to this problem. Note that if the spiral pattern is known, an angular measurement can be converted to length.

**10.51** Utilizing the equipment and materials available in a typical kitchen, design an experiment to reproduce results similar to those shown in Fig. 10.11. Comment on your observations.

By the student. For example, a simple experiment can be performed with melted chocolate and a coffee cup. If a parting agent (oil) is first sprayed inside the cup and then molten chocolate is poured into the cup, after a short while the molten center portion can be poured out of the cup, leaving a solidified shell. This effect can be made more pronounced by using cups that have been chilled in a freezer. It is also interesting to investigate object shapes with steps as those shown in Fig. 10.11.

**10.52 One method of relieving stress concentrations in a part is to apply a small, uniform plastic deformation to it. Make a list of your concerns and recommendations if such an approach is suggested for a casting.**

The plastic deformations, if tensile or flexural, need to be applied with great care on castings. The main reason is that castings are typically weak in tension, due to micropores which act as stress risers. While the intent may be to relieve residual stresses, the result may be to fracture the part.

**10.53 If a casting of a given shape is to be doubled in volume, describe the effects on mold design, including the required change in the size of risers, runners, chokes, and sprues.**

By the student. This is an open ended problem, and students may interpret this problem differently in their answer. Some of the considerations are:

- It must be realized that if the volume is doubled, with no changes in any other mold features, the molten metal still has to flow through the same sprues and at the same velocity for twice as long as the initial design. This may be accomplished without any changes in sprue, runner, etc., size at all if the original design had large features to begin with or if a high superheat is prescribed.
- Otherwise, these features will have to be increased in size, but not by a factor of two - Eqs. (10.3)-(10.5) give the design guidelines.
- Students may discuss the need for chills, or the use of vacuum casting or alternative processes to prevent entrained gases or shrinkage pores in larger castings.

**10.54 Small amounts of slag often persist after skimming and are introduced into the molten-metal flow in casting. Recognizing that the slag is much less dense than the metal, design mold features that will remove small amounts of slag before the metal reaches the mold cavity.**

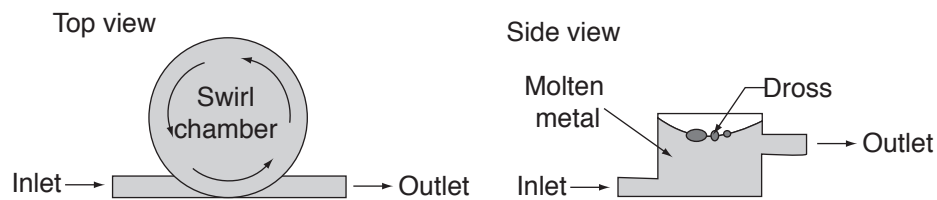
There are several trap designs in use in foundries. An excellent discussion of cross trap design is given in J. Campbell, *Castings*, 1991, Reed Educational Publishers, pp. 53-55. A conventional and effective cross trap is the following design:



The design is based on the principle that a trap at the end of a runner will capture the first material through the runner and keep it away from the gates. The design shown above is a wedge-type trap. Metal entering the runner contacts the wedge, and the leading front of the metal wave is chilled and attaches itself to the runner wall, and thus it is kept out of the mold cavity. The wedge must be designed to avoid reflected waves that would recirculate the dross or slag.

The following design is a swirl trap, which is based on the principle that the dross or slag is less dense than the metal. The metal enters the trap off of the center, inducing a swirl in the molten metal as the trap is filled with molten metal. Since it is much less dense than the

metal, the dross or slag remains in the center of the swirl trap. Since the metal is tapped from the outside periphery, dross or slag is excluded from entering the casting.



**10.55** Figure II.1 shows a variety of components in a typical automobile that are produced by casting. Think of other products, such as power tools and small appliances, and make a similar illustration as done in that figure.

By the student. Among many possible solutions, the following is an example showing cast parts. Students should be encouraged to develop their own designs, based on disassembly of a product, which can be a challenging project.

