Chapter 4 Exercise Solutions

Several exercises in this chapter differ from those in the 4th edition. An "*" following the exercise number indicates that the description has changed. New exercises are denoted with an "☺". A second exercise number in parentheses indicates that the exercise number has changed.

4-1.
"Chance" or "common" causes of variability represent the inherent, natural variability of a process - its background noise. Variation resulting from "assignable" or "special" causes represents generally large, unsatisfactory disturbances to the usual process performance. Assignable cause variation can usually be traced, perhaps to a change in material, equipment, or operator method.

A Shewhart control chart can be used to monitor a process and to identify occurrences of assignable causes. There is a high probability that an assignable cause has occurred when a plot point is outside the chart's control limits. By promptly identifying these occurrences and acting to permanently remove their causes from the process, we can reduce process variability in the long run.

4-2.
The control chart is mathematically equivalent to a series of statistical hypothesis tests. If a plot point is within control limits, say for the average $\overline{X}$, the null hypothesis that the mean is some value is not rejected. However, if the plot point is outside the control limits, then the hypothesis that the process mean is at some level is rejected. A control chart shows, graphically, the results of many sequential hypothesis tests.

NOTE TO INSTRUCTOR FROM THE AUTHOR (D.C. Montgomery):
There has been some debate as to whether a control chart is really equivalent to hypothesis testing. Deming (see *Out of the Crisis*, MIT Center for Advanced Engineering Study, Cambridge, MA, pp. 369) writes that:

> "Some books teach that use of a control chart is test of hypothesis: the process is in control, or it is not. Such errors may derail self-study".

Deming also warns against using statistical theory to study control chart behavior (false-alarm probability, OC-curves, average run lengths, and normal curve probabilities. Wheeler (see "Shewhart's Charts: Myths, Facts, and Competitors", *ASQC Quality Congress Transactions* (1992), Milwaukee, WI, pp. 533–538) also shares some of these concerns:

> "While one may mathematically model the control chart, and while such a model may be useful in comparing different statistical procedures on a theoretical basis, these models do not justify any procedure in practice, and their exact probabilities, risks, and power curves do not actually apply in practice."

4-2 continued

On the other hand, Shewhart, the inventor of the control chart, did not share these views in total. From Shewhart (*Statistical Method from the Viewpoint of Quality Control* (1939), U.S. Department of Agriculture Graduate School, Washington DC, p. 40, 46):

> "As a background for the development of the operation of statistical control, the formal mathematical theory of testing a statistical hypothesis is of outstanding importance, but it would seem that we must continually keep in mind the fundamental difference between the formal theory of testing a statistical hypothesis and the empirical theory of testing a hypothesis employed in the operation of statistical control. In the latter, one must also test the hypothesis that the sample of data was obtained under conditions that may be considered random. …
> The mathematical theory of distribution characterizing the formal and mathematical concept of a state of statistical control constitutes an unlimited storehouse of helpful suggestions from which practical criteria of control must be chosen, and the general theory of testing statistical hypotheses must serve as a background to guide the choice of methods of making a running quality report that will give the maximum service as time goes on."

Thus Shewhart does not discount the role of hypothesis testing and other aspects of statistical theory. However, as we have noted in the text, the purposes of the control chart are more general than those of hypothesis tests. The real value of a control chart is monitoring stability over time. Also, from Shewhart's 1939 book, (p. 36):

> "The control limits as most often used in my own work have been set so that after a state of statistical control has been reached, one will look for assignable causes when they are not present not more than approximately three times in 1000 samples, when the distribution of the statistic used in the criterion is normal."

Clearly, Shewhart understood the value of statistical theory in assessing control chart performance.

My view is that the proper application of statistical theory to control charts can provide useful information about how the charts will perform. This, in turn, will guide decisions about what methods to use in practice. If you are going to apply a control chart procedure to a process with unknown characteristics, it is prudent to know how it will work in a more idealized setting. In general, before recommending a procedure for use in practice, it should be demonstrated that there is some underlying model for which it performs well. The study by Champ and Woodall (1987), cited in the text, that shows the ARL performance of various sensitizing rules for control charts is a good example. This is the basis of the recommendation against the routine use of these rules to enhance the ability of the Shewhart chart to detect small process shifts.

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4-3.
Relative to the control chart, the type I error represents the probability of concluding the process is out of control when it isn't, meaning a plot point is outside the control limits when in fact the process is still in control. In process operation, high frequencies of false alarms could lead could to excessive investigation costs, unnecessary process adjustment (and increased variability), and lack of credibility for SPC methods.

The type II error represents the probability of concluding the process is in control, when actually it is not; this results from a plot point within the control limits even though the process mean has shifted out of control. The effect on process operations of failing to detect an out-of-control shift would be an increase in non-conforming product and associated costs.

4-4.
The statement that a process is in a state of statistical control means that assignable or special causes of variation have been removed; characteristic parameters like the mean, standard deviation, and probability distribution are constant; and process behavior is predictable. One implication is that any improvement in process capability (i.e., in terms of non-conforming product) will require a change in material, equipment, method, etc.

4-5.
No. The fact that a process operates in a state of statistical control does not mean that nearly all product meets specifications. It simply means that process behavior (mean and variation) is statistically predictable. We may very well predict that, say, 50% of the product will not meet specification limits! *Capability* is the term, which refers to the ability to meet product specifications, and a process must be in control in order to calculate capability.

4-6.
The logic behind the use of 3-sigma limits on Shewhart control charts is that they give good results in practice. Narrower limits will result in more investigations for assignable causes, and perhaps more false alarms. Wider limits will result in fewer investigations, but perhaps fewer process shifts will be promptly identified.

Sometimes probability limits are used - particularly when the underlying distribution of the plotted statistic is known. If the underlying distribution is unknown, care should be exercised in selecting the width of the control limits. Historically, however, 3-sigma limits have been very successful in practice.

4-7.
Warning limits on control charts are limits that are inside the control limits. When warning limits are used, control limits are referred to as action limits. Warning limits, say at 2-sigma, can be used to increase chart sensitivity and to signal process changes more quickly than the 3-sigma action limits. The Western Electric rule, which addresses this type of shift is to consider a process to be out of control if 2 of 3 plot points are between 2 sigma and 3 sigma of the chart centerline.

4-8.
The concept of a rational subgroup is used to maximize the chance for detecting variation between subgroups. Subgroup samples can be structured to identify process shifts. If it is expected that a process will shift and stay at the new level until a corrective action, then sampling consecutive (or nearly) units maximizes the variability between subgroups and minimizes the variability within a subgroup. This maximizes the probability of detecting a shift.

4-9.
I would want assignable causes to occur between subgroups and would prefer to select samples as close to consecutive as possible. In most SPC applications, process changes will not be self-correcting, but will require action to return the process to its usual performance level. The probability of detecting a change (and therefore initiating a corrective action) will be maximized by taking observations in a sample as close together as possible.

4-10.
This sampling strategy will very likely underestimate the size of the true process variability. Similar raw materials and operating conditions will tend to make any five-piece sample alike, while variability caused by changes in batches or equipment may remain undetected. An out-of-control signal on the R chart will be interpreted to be the result of differences between cavities. Because true process variability will be underestimated, there will likely be more false alarms on the $\bar{x}$ chart than there should be.

4-11.
(a)
No.

(b)
The problem is that the process may shift to an out-of-control state and back to an in-control state in less than one-half hour. Each subgroup should be a random sample of all parts produced in the last 2½ hours.

4-12.
No. The problem is that with a slow, prolonged trend upwards, the sample average will tend to be the value of the 3rd sample --- the highs and lows will average out. Assume that the trend must last 2½ hours in order for a shift of detectable size to occur. Then a better sampling scheme would be to simply select 5 consecutive parts every 2½ hours.

4-13.
No. If time order of the data is not preserved, it will be impossible to separate the presence of assignable causes from underlying process variability.
4-14.
An operating characteristic curve for a control chart illustrates the tradeoffs between sample size *n* and the process shift that is to be detected. Generally, larger sample sizes are needed to increase the probability of detecting small changes to the process. If a large shift is to be detected, then smaller sample sizes can be used.

4-15.
The costs of sampling, excessive defective units, and searches for assignable causes impact selection of the control chart parameters of sample size *n*, sampling frequency *h*, and control limit width. The larger *n* and *h*, the larger will be the cost of sampling. This sampling cost must be weighed against the cost of producing non-conforming product.

4-16.
Type I and II error probabilities contain information on statistical performance; an ARL results from their selection. ARL is more meaningful in the sense of the operations information that is conveyed and could be considered a measure of the process performance of the sampling plan.

4-17.
Evidence of runs, trends or cycles? NO. There are no runs of 5 points or cycles. So, we can say that the plot point pattern appears to be random.

4-18.
Evidence of runs, trends or cycles? YES, there is one "low - high - low - high" pattern (Samples 13 – 17), which might be part of a cycle. So, we can say that the pattern does not appear random.

4-19.
Evidence of runs, trends or cycles? YES, there is a "low - high - low - high - low" wave (all samples), which might be a cycle. So, we can say that the pattern does not appear random.

4-20.
Three points exceed the 2-sigma warning limits - points #3, 11, and 20.

4-21.
Check:

- Any point outside the 3-sigma control limits? NO.
- 2 of 3 beyond 2 sigma of centerline? NO.
- 4 of 5 at 1 sigma or beyond of centerline? YES. Points #17, 18, 19, and 20 are outside the lower 1-sigma area.
- 8 consecutive points on one side of centerline? NO.

One out-of-control criteria is satisfied.

4-22.
Four points exceed the 2-sigma warning limits - points #6, 12, 16, and 18.

4-23.
Check:

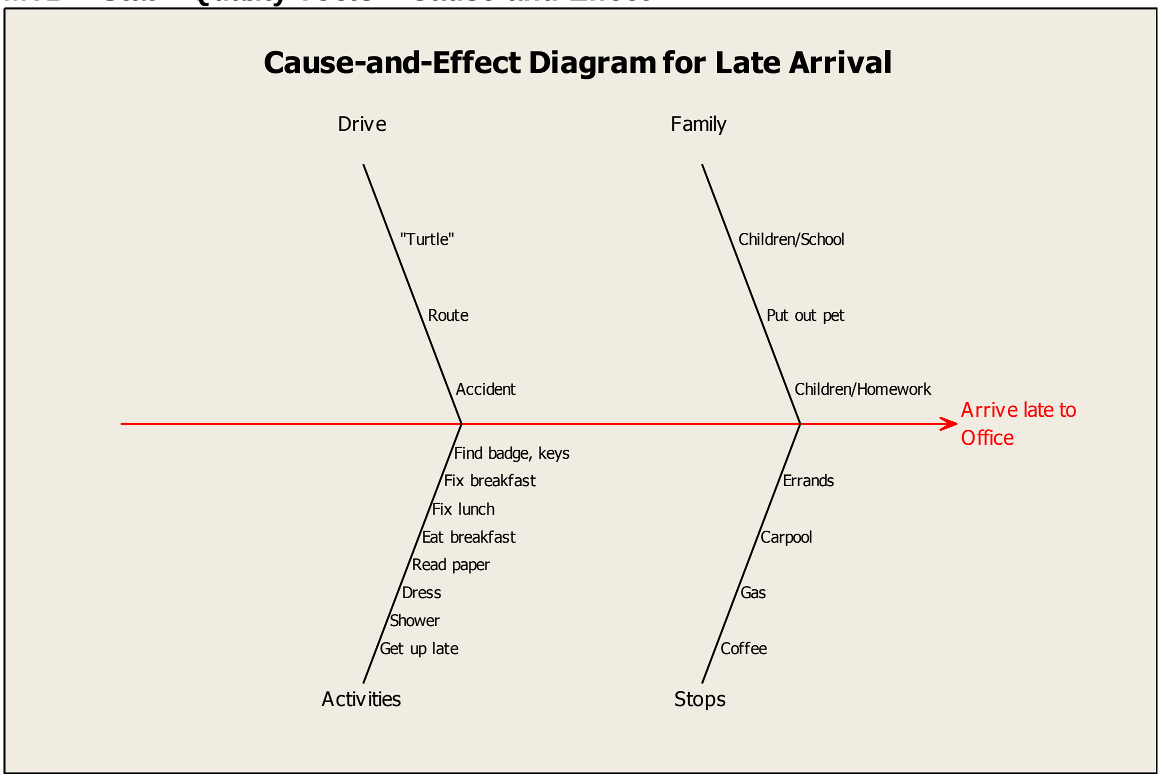
- Any point outside the 3-sigma control limits? NO. (Point #12 is within the lower 3-sigma control limit.)
- 2 of 3 beyond 2 sigma of centerline? YES, points #16, 17, and 18.
- 4 of 5 at 1 sigma or beyond of centerline? YES, points #5, 6, 7, 8, and 9.
- 8 consecutive points on one side of centerline? NO.

Two out-of-control criteria are satisfied.

4-24.
The pattern in Figure (a) matches the control chart in Figure (2).
The pattern in Figure (b) matches the control chart in Figure (4).
The pattern in Figure (c) matches the control chart in Figure (5).
The pattern in Figure (d) matches the control chart in Figure (1).
The pattern in Figure (e) matches the control chart in Figure (3).

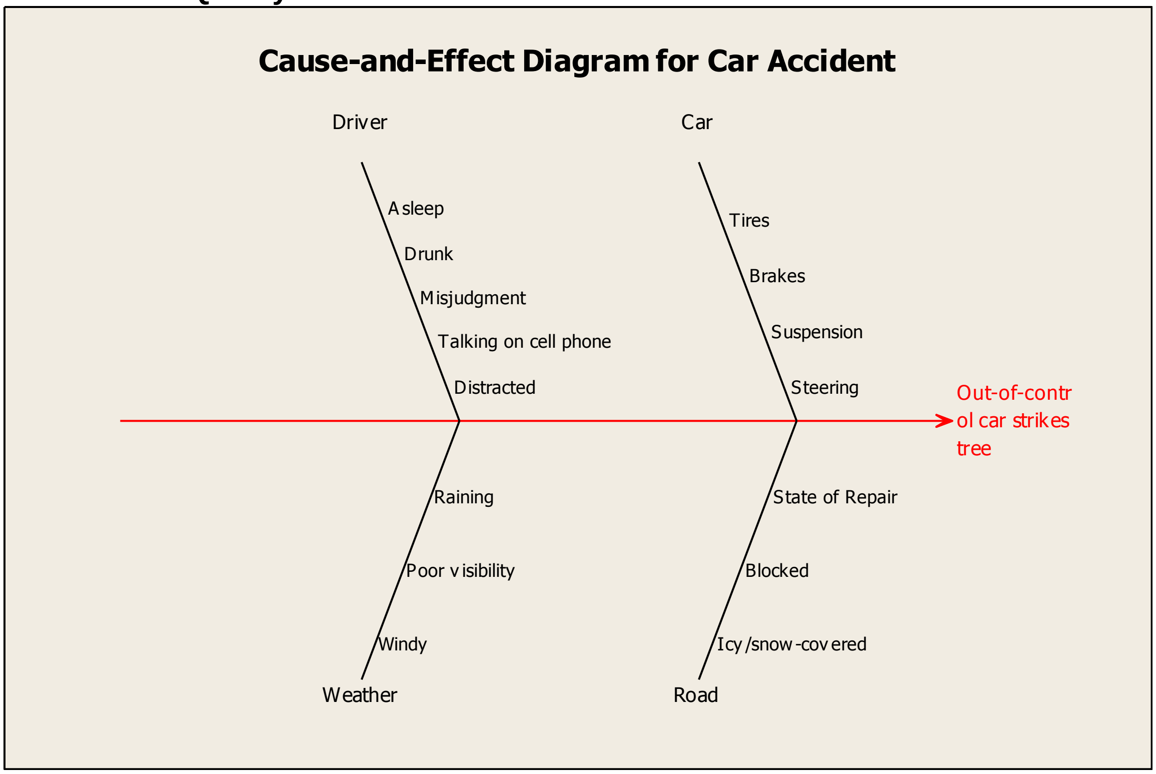
4-25 (4-30).
Many possible solutions.

MTB > Stat > Quality Tools > Cause-and-Effect



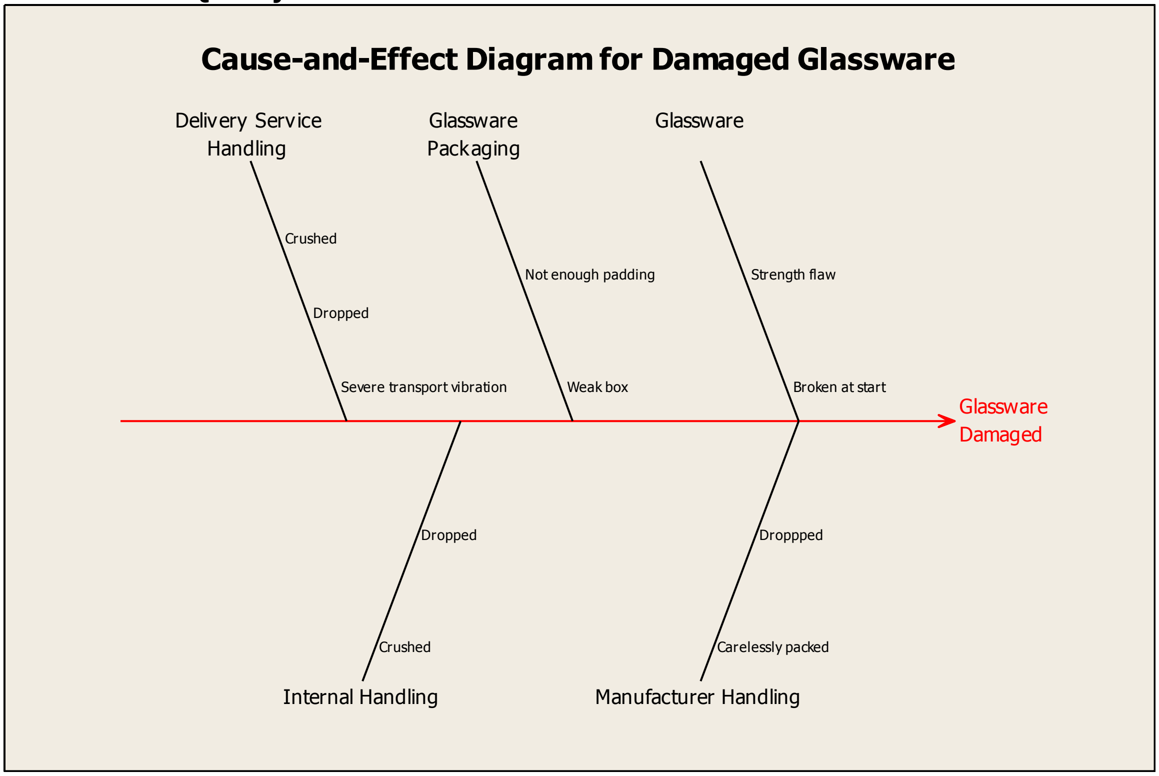
4-26 (4-31).
Many possible solutions.

MTB > Stat > Quality Tools > Cause-and-Effect



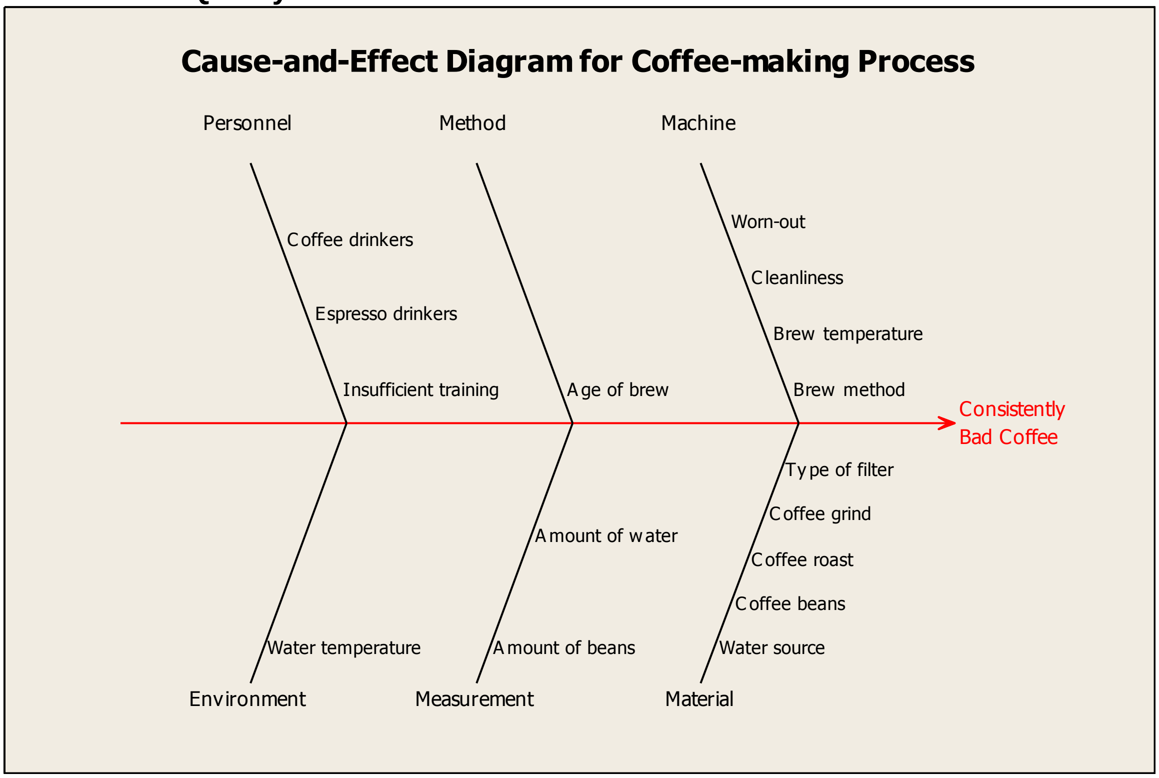
4-27 (4-32).
Many possible solutions.

MTB > Stat > Quality Tools > Cause-and-Effect

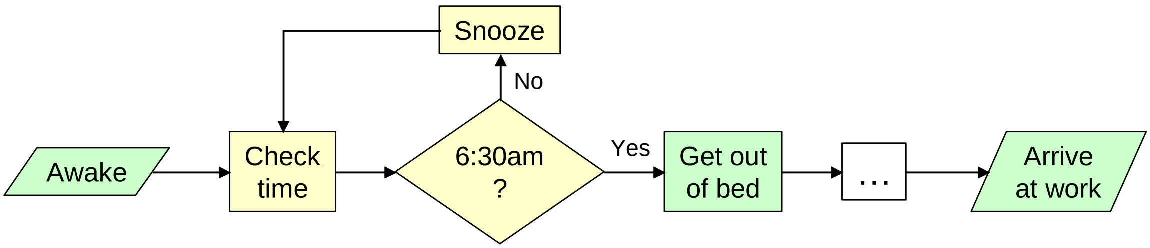


4-28☺.
Many possible solutions.

MTB > Stat > Quality Tools > Cause-and-Effect

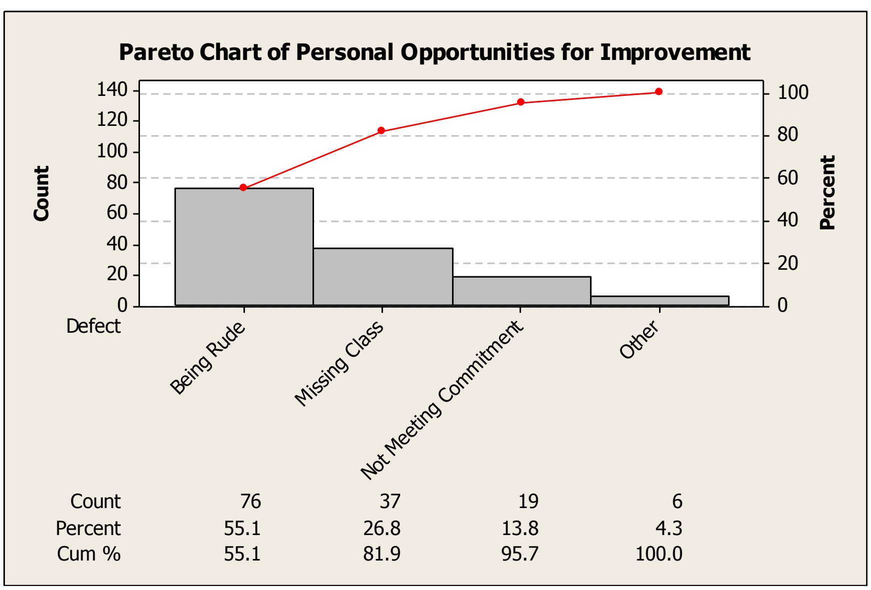


4-29☺.
Many possible solutions, beginning and end of process are shown below. Yellow is non-value-added activity; green is value-added activity.



4-31☺.
Example of a check sheet to collect data on personal opportunities for improvement. Many possible solutions, including defect categories and counts.

	Month/Day									
Defect	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**…**	**31**	**TOTAL**
Overeating	0	2	1	0	1	0	1	…	1	6
Being Rude	10	11	9	9	7	10	11	…	9	76
Not meeting commitments	4	2	2	2	1	0	1	…	7	19
Missing class	4	6	3	2	7	9	4	…	2	37
Etc.										
TOTAL	18	21	15	13	16	19	17		19	138



To reduce total count of defects, "Being Rude" represents the greatest opportunity to make an improvement. The next step would be to determine the causes of "Being Rude" and to work on eliminating those causes.

4-32☺.
$m = 5$

$$\alpha_1 = \Pr\{\text{at least 1 out-of-control}\} = \Pr\{\text{1 of 5 beyond}\} + \Pr\{\text{2 of 5 beyond}\} + \cdots + \Pr\{\text{5 of 5 beyond}\}$$

$$= 1 - \Pr\{\text{0 of 5 beyond}\} = 1 - \binom{5}{0}(0.0027)^0(1-0.0027)^5 = 1 - 0.9866 = 0.0134$$

MTB > Calc > Probability Distributions > Binomial, Cumulative Probability

Cumulative Distribution Function

```
Binomial with n = 5 and p = 0.0027
x  P( X <= x )
0     0.986573
```

$m = 10$

$$\alpha_1 = 1 - \Pr\{\text{0 of 10 beyond}\} = 1 - \binom{10}{0}(0.0027)^0(1-0.0027)^{10} = 1 - 0.9733 = 0.0267$$

Cumulative Distribution Function

```
Binomial with n = 10 and p = 0.0027
x  P( X <= x )
0     0.973326
```

$m = 20$

$$\alpha_1 = 1 - \Pr\{\text{0 of 20 beyond}\} = 1 - \binom{20}{0}(0.0027)^0(1-0.0027)^{20} = 0.0526$$

Cumulative Distribution Function

```
Binomial with n = 20 and p = 0.0027
x  P( X <= x )
0     0.947363
```

$m = 30$

$$\alpha_1 = 1 - \Pr\{\text{0 of 30 beyond}\} = 1 - \binom{30}{0}(0.0027)^0(1-0.0027)^{30} = 0.0779$$

Cumulative Distribution Function

```
Binomial with n = 30 and p = 0.0027
x  P( X <= x )
0     0.922093
```

$m = 50$

$$\alpha_1 = 1 - \Pr\{\text{0 of 50 beyond}\} = 1 - \binom{50}{0}(0.0027)^0(1-0.0027)^{50} = 0.1025$$

Cumulative Distribution Function

```
Binomial with n = 50 and p = 0.0027
x  P( X <= x )
0     0.873556
```

Although the probability that a single point plots beyond the control limits is 0.0027, as the number of samples increases (*m*), the probability that at least one of the points is beyond the limits also increases.

4-33☺.
When the process mean μ and variance σ^2 are unknown, they must be estimated by sample means $\bar{x}$ and standard deviations *s*. However, the points used to estimate these sample statistics are not independent—they do not reflect a random sample from a population. In fact, sampling frequencies are often designed to increase the likelihood of detecting a special or assignable cause. The lack of independence in the sample statistics will affect the estimates of the process population parameters.