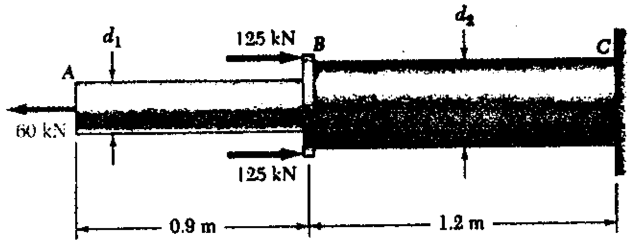
CHAPTER 1

PROBLEM 1.1

1.1 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that $d_1 = 30$ mm and $d_2 = 50$ mm, find the average normal stress in the mid section of (a) rod AB, (b) rod BC.



SOLUTION

(a) rod AB Force: $P = 60 \times 10^3$ N tension

Area: $A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30 \times 10^{-3})^2 = 706.86 \times 10^{-6}\ m^2$

Normal stress: $\sigma_{AB} = \frac{P}{A} = \frac{60 \times 10^3}{706.86 \times 10^{-6}} = 84.88 \times 10^6$ Pa

$\sigma_{AB} = 84.9$ MPa ◄

(b) rod BC

Force: $P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3$ N

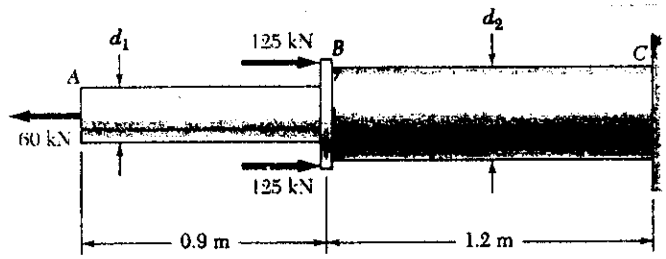
Area: $A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(50 \times 10^{-3})^2 = 1.9635 \times 10^{-3}\ m^2$

Normal stress: $\sigma_{BC} = \frac{P}{A} = \frac{-190 \times 10^3}{1.9635 \times 10^{-3}} = -96.77 \times 10^6$ Pa

$\sigma_{BC} = -96.8$ MPa ◄

PROBLEM 1.2

1.2 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that the average normal stress must not exceed 150 MPa in either rod, determine the smallest allowable values of the diameters d_1 and d_2.



SOLUTION

rod AB

Force: $P = 60 \times 10^3$ N   Stress: $\sigma_{AB} = 150 \times 10^6$ Pa

Area: $A = \frac{\pi}{4} d_1^2$

$\sigma_{AB} = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma_{AB}}$

$\frac{\pi}{4} d_1^2 = \frac{P}{\sigma_{AB}}$

$d_1^2 = \frac{4P}{\pi \sigma_{AB}} = \frac{(4)(60 \times 10^3)}{\pi (150 \times 10^6)} = 509.3 \times 10^{-6}\ m^2$

$d_1 = 22.56 \times 10^{-3}$ m $d_1 = 22.6$ mm ◄

rod BC

Force $P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3$ N

Stress: $\sigma_{BC} = -150 \times 10^6$ Pa   Area: $A = \frac{\pi}{4} d_2^2$

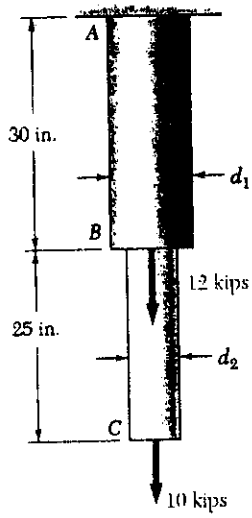
$\sigma_{BC} = \frac{P}{A} = \frac{4P}{\pi d_2^2}$

$d_2^2 = \frac{4P}{\pi \sigma_{BC}} = \frac{(4)(-190 \times 10^3)}{\pi (-150 \times 10^6)} = 1.6128 \times 10^{-3}\ m^2$

$d_2 = 40.16 \times 10^{-3}$ m $d_2 = 40.2$ mm ◄

PROBLEM 1.3

1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that $d_1 = 1.25$ in. and $d_2 = 0.75$ in., find the normal stress at the midpoint of (*a*) rod *AB*, (*b*) rod *BC*.



SOLUTION

(a) rod AB

$$P = 12 + 10 = 22 \text{ kips}$$

$$A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(1.25)^2 = 1.2272 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{22}{1.2272} = 17.93 \text{ ksi}$$

(b) rod BC

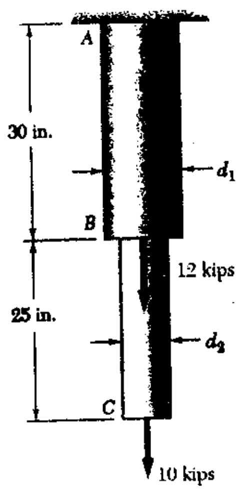
$$P = 10 \text{ kips}$$

$$A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(0.75)^2 = 0.4418 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{10}{0.4418} = 22.6 \text{ ksi}$$

PROBLEM 1.4

1.4 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Knowing that the normal stress must not exceed 25 ksi in either rod, determine the smallest allowable values of the diameters d_1 and d_2.



SOLUTION

rod AB:

$$P = 12 + 10 = 22 \text{ kips}$$

$$\sigma_{AB} = 25 \text{ ksi} \qquad A_{AB} = \frac{\pi}{4} d_1^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{4P}{\pi d_1^2}$$

$$d_1^2 = \frac{4P}{\pi \sigma_{AB}} = \frac{(4)(22)}{\pi (25)} = 1.1205 \text{ in}^2$$

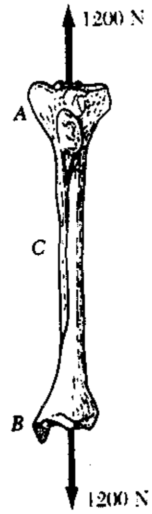
$$d_1 = 1.059 \text{ in}$$

rod BC: $P = 10$ kips $\quad \sigma_{BC} = 25$ ksi $\quad A_{BC} = \frac{\pi}{4} d_2^2$

$$d_2^2 = \frac{4P}{\pi \sigma_{BC}} = \frac{(4)(10)}{\pi (25)} = 0.5093 \text{ in}^2 \qquad d_2 = 0.714 \text{ in.}$$

PROBLEM 1.5

1.5 A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.



SOLUTION

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $A = \frac{\pi}{4}(d_1^2 - d_2^2)$

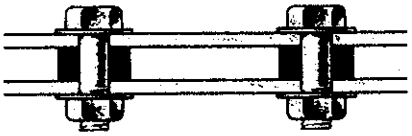
$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25\times10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80\times10^6}$$

$$= 222.9\times10^{-6}\ m^2$$

$d_1 = 14.93\times10^{-3}$ m $\qquad d_1 = 14.93$ mm

PROBLEM 1.6



1.6 Two steel plates are to be held together by means of ¼-in.-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 30 ksi in the bolts and 18 ksi in the spacers, determine the outer diameter of the spacers which yields the most economical and safe design.

SOLUTION

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time the spacer pushes that plate upward with a compressive force P_s. In order to maintain equilibrium

$$P_b = P_s$$

For the bolt $\quad \sigma_b = \frac{P_b}{A_b} = \frac{4P_b}{\pi d_b^2} \quad$ or $\quad P_b = \frac{\pi}{4}\sigma_b d_b^2$

For the spacer $\quad \sigma_s = \frac{P_s}{A_s} = \frac{4P_s}{\pi(d_s^2 - d_b^2)} \quad$ or $\quad P_s = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

Equating P_b and P_s

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$$

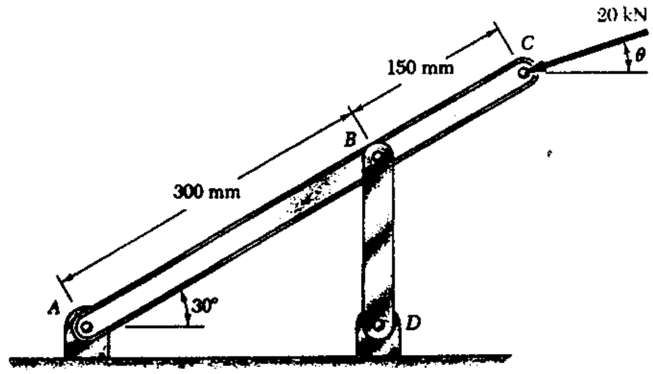
$$d_s^2 = d_b^2 + \frac{\sigma_b}{\sigma_s}d_b^2 = \left(1 + \frac{\sigma_b}{\sigma_s}\right)d_b^2$$

$$d_s = \left(1 + \frac{30}{18}\right)\left(\frac{1}{4}\right)^2 = 0.16667 \text{ in}^2$$

$$d_s = 0.408 \text{ in.}$$

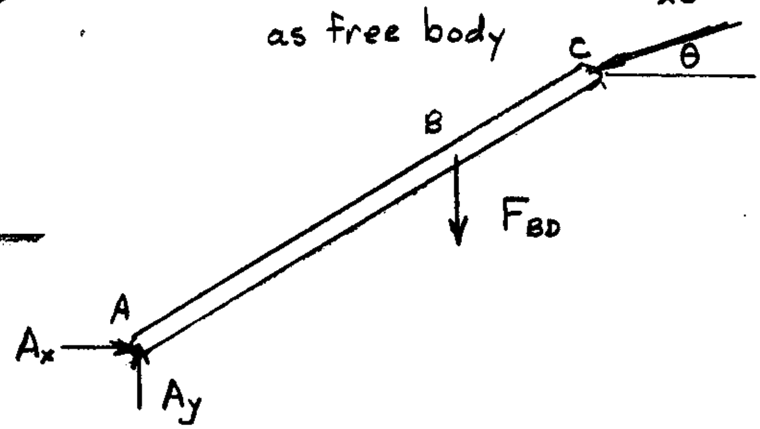
PROBLEM 1.7

1.7 Link *BD* consists of a single bar 30 mm wide and 12 mm thick. Knowing that each pin has a 10-mm diameter, determine the maximum value of the average normal stress in link *BD* if (*a*) $\theta = 0$, (*b*) $\theta = 90$



SOLUTION

Use bar ABC as free body



$\Sigma M_A = 0$

(a) $\theta = 0°$ $\quad (0.450 \sin 30°)(20\times10^3) - (0.300 \cos 30°) F_{BD} = 0$

$F_{BD} = 17.32 \times 10^3$ N

(b) $\theta = 90°$ $\quad (0.450 \cos 30°)(20\times10^3) - (0.300 \cos 30°) F_B = 0$

$F_{BD} = -30 \times 10^3$ N

Areas

(a) tension loading $\quad A = (0.030 - 0.010)(0.012) = 240 \times 10^{-6}$ m²

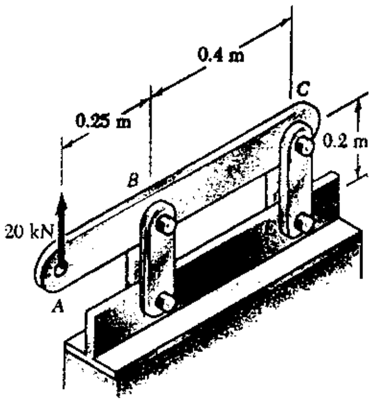
(b) compression $\quad A = (0.030)(0.012) = 360 \times 10^{-6}$ m²

Stresses

(a) $\sigma = \dfrac{F_{BC}}{A} = \dfrac{17.32\times10^3}{240\times10^{-6}} = 72.2\times10^6$ $\quad$ 72.2 MPa ◄

(b) $\sigma = \dfrac{F_{BC}}{A} = \dfrac{-30\times10^3}{360\times10^{-6}} = -83.3\times10^6$ $\quad$ −83.3 MPa ◄

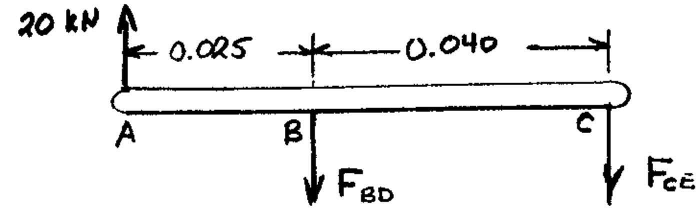
PROBLEM 1.8



1.8 Each of the four vertical links has an 8 × 36- mm uniform rectangular cross section and each of the four pins has a 16- mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.

SOLUTION

Use bar ABC as a free body.



$\Sigma M_C = 0 \qquad (0.040)F_{BD} - (0.025 + 0.040)(20\times10^3) = 0$

$F_{BD} = 32.5 \times 10^3$ N $\qquad$ Link BD is in tension

$\Sigma M_B = 0 \qquad -(0.040)F_{CE} - (0.025)(20\times10^2) = 0$

$F_{CE} = -12.5\times10^3$ N $\qquad$ Link CE is in compression.

Net area of one link for tension = $(0.008)(0.036 - 0.016)$

$= 160\times10^{-6}$ m². For two parallel links $A_{net} = 320\times10^{-6}$ m²

Tensile stress in link BD

(a) $\quad \sigma_{BD} = \dfrac{F_{BD}}{A_{net}} = \dfrac{32.5\times10^3}{320\times10^{-6}} = 101.56\times10^6$ or 101.6 MPa ◄

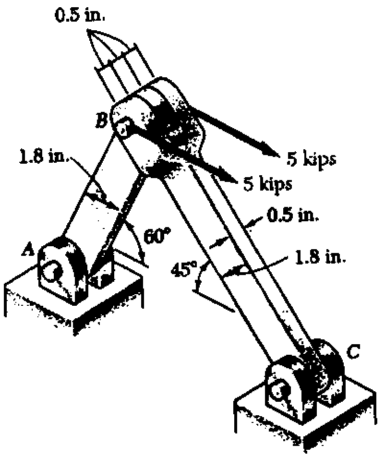
Area for one link in compression = $(0.008)(0.036)$

$= 288\times10^{-6}$ m². For two parallel links $A = 576\times10^{-6}$ m²

(b) $\quad \sigma_{CE} = \dfrac{F_{CE}}{A} = \dfrac{-12.5\times10^3}{576\times10^{-6}} = -21.70\times10^{6}$ or -21.7 MPa ◄

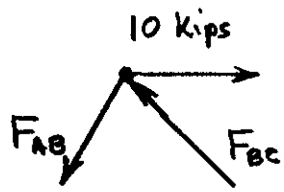
PROBLEM 1.9

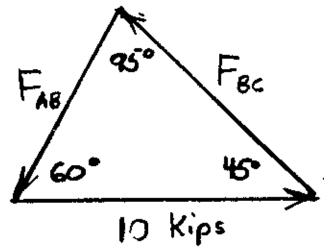
1.9 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link AB, (*b*) in link BC.



SOLUTION

Use joint B as free body.





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{10}{\sin 95°}$$

$F_{AB} = 7.3205$ kips $\qquad F_{BC} = 8.9658$ kips.

Link AB is a tension member

Minimum section at pin $A_{net} = (1.8 - 0.8)(0.5) = 0.5 \text{ in}^2$

(a) Stress in AB $\quad \sigma_{AB} = \dfrac{F_{AB}}{A_{net}} = \dfrac{7.3205}{0.5} = 14.64$ ksi

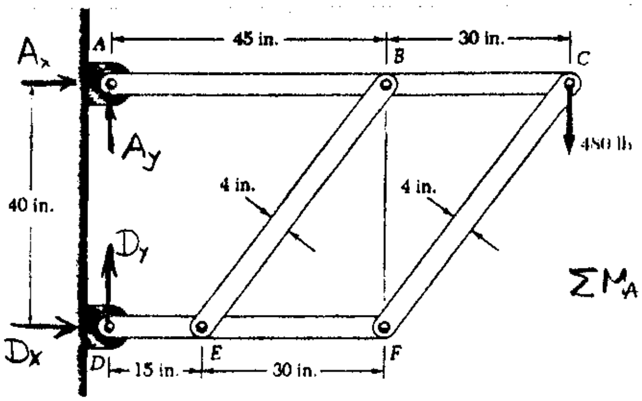
Link BC is a compression member

Cross sectional area is $A = (1.8)(0.5) = 0.9 \text{ in}^2$

(b) Stress in BC $\quad \sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{-8.9658}{0.9} = -9.96$ ksi

PROBLEM 1.10

1.10 The frame shown consists of *four* wooden members, *ABC*, *DEF*, *BE*, and *CF*. Knowing that each member has a 2 × 4-in. rectangular cross section and that each pin has a ½ - in. diameter, determine the maximum value of the average normal stress (*a*) in member *BE*, (*b*) in member *CF*.



SOLUTION

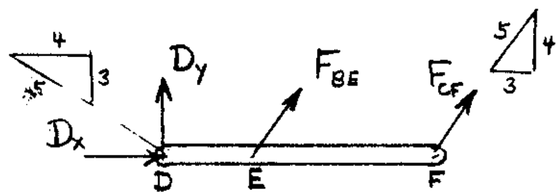
Add support reactions to figure as shown.

Using entire frame as free body

$$\Sigma M_A = 0 \qquad 40\,D_x - (45+30)(480) = 0$$

$$D_x = 900 \text{ lb.}$$

Use member DEF as free body



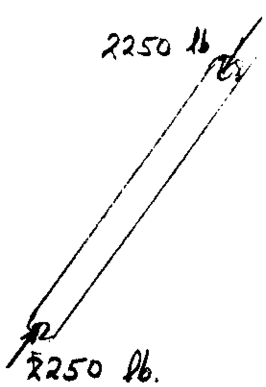
$$\Sigma F \nwarrow = 0$$

$$\tfrac{3}{5}D_y - \tfrac{4}{5}D_x = 0$$

$$D_y = \tfrac{4}{3}D_x = 1200 \text{ lb.}$$

$$\Sigma M_F = 0 \qquad -(30)(\tfrac{4}{5}F_{BE}) - (30+15)D_y = 0 \qquad F_{BE} = -2250 \text{ lb.}$$

$$\Sigma M_E = 0 \qquad (30)(\tfrac{4}{5}F_{CE}) - (15)D_y = 0 \qquad F_{CF} = 750 \text{ lb.}$$



Stress in compression member BE

Area $\quad A = 2\text{ in} \times 4\text{ in} = 8\text{ in}^2$

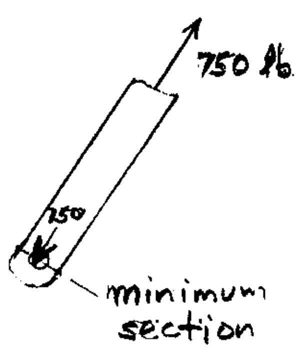
(a) $\sigma_{BE} = \dfrac{F_{BE}}{A} = \dfrac{-2250}{8} = -281$ psi ◄

Stress in tension member CF

Minimum section area occurs at pin.

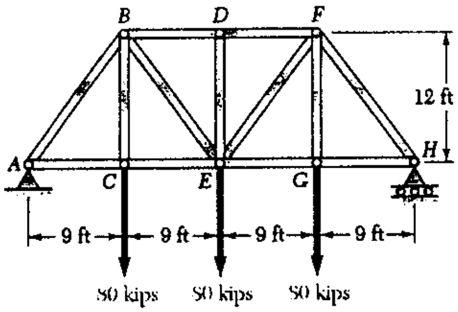
$A_{min} = (2)(4.0-0.5) = 7.0\text{ in}^2$

(b) $\sigma_{CF} = \dfrac{F_{CF}}{A_{min}} = \dfrac{750}{7.0} = 107.1$ psi ◄



PROBLEM 1.11

1.11 For the Pratt bridge truss and loading shown, determine the average normal stress in member *BE*, knowing that the cross-sectional area of that member is 5.87 in^2.



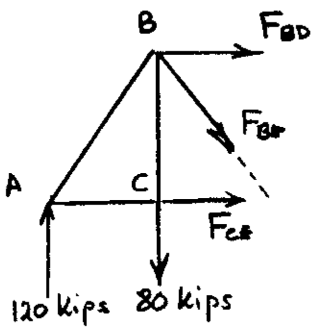
SOLUTION

Use entire truss as free body

$\Sigma M_H = 0$

$(9)(80) + (18)(80) + (27)(80) - 36\,A_y = 0$

$A_y = 120$ kips



Use portion of truss to the left of a section cutting members BD, BE, and CE.

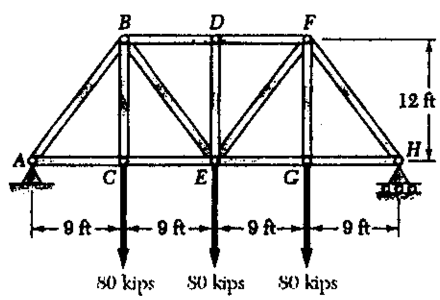
$+\uparrow\ \Sigma F_y = 0$

$120 - 80 - \frac{12}{15} F_{BE} = 0 \qquad \therefore\ F_{BE} = 50$ kips

$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{50}{5.87} = 8.52$ ksi

PROBLEM 1.12

1.12 Knowing that the average normal stress in member *CE* of the Pratt bridge truss shown must not exceed 21 ksi for the given loading, determine the cross-sectional area of that member which will yield the most economical and safe design. Assume that both ends of the member will be adequately reinforced.



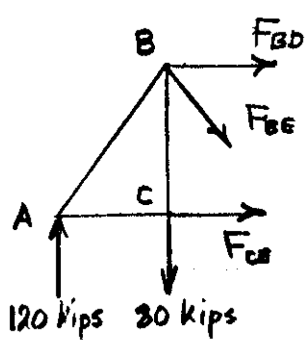
SOLUTION

Use entire truss as free body

$\Sigma M_H = 0$

$(9)(80) + (18)(80) + (27)(80) - 36\,A_y = 0$

$A_y = 120$ Kips



Use portion of truss to the left of a section cutting members BD, BE, and CE

$\Sigma M_B = 0$

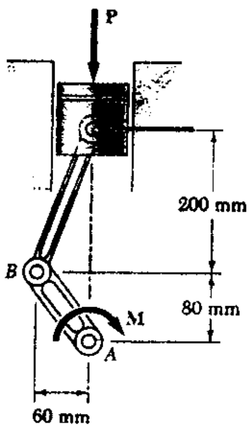
$12\,F_{CE} - (9)(120) = 0 \qquad \therefore \qquad F_{CE} = 90$ kips

$\sigma_{CE} = \frac{F_{CE}}{A_{CE}}$

$A_{CE} = \frac{F_{CE}}{\sigma_{CE}} = \frac{90}{21} = 4.29\ \text{in}^2$

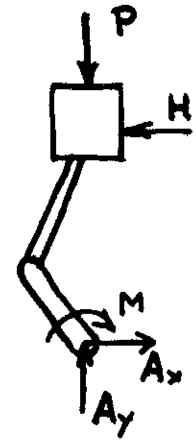
PROBLEM 1.13

1.13 A couple **M** of magnitude 1500 N · m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm² uniform cross section.



SOLUTION

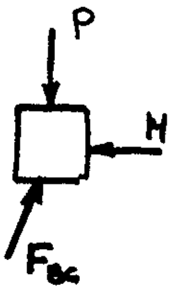
Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y.

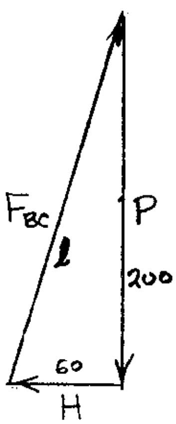


$\circlearrowleft \Sigma M_A = 0$

$(0.280\ \text{m})H - 1500\ \text{N·m} = 0$

$H = 5.3571 \times 10^3\ \text{N}$





Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BC} by proportions.

$$l = \sqrt{200^2 + 60^2} = 208.81\ \text{mm}$$

$$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86 \times 10^3\ \text{N}$$

(a) $P = 17.86\ \text{kN}$ ◄

$$\frac{F_{BC}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{BC} = 18.643 \times 10^3\ \text{N}$$

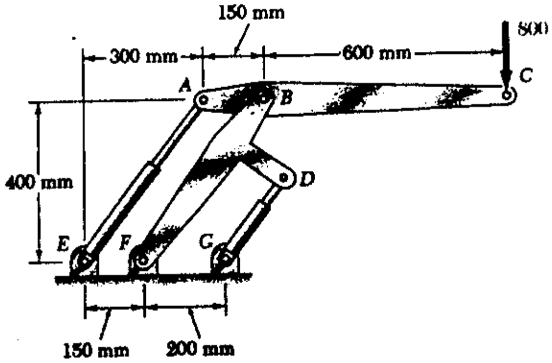
Rod BC is a compression member. Its area is $450\ \text{mm}^2 = 450 \times 10^{-6}\ \text{m}^2$

Stress $\quad \sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{-18.643 \times 10^3}{450 \times 10^{-6}} = -41.4 \times 10^6\ \text{Pa}$

(b) $\sigma_{BC} = -41.4\ \text{MPa}$ ◄

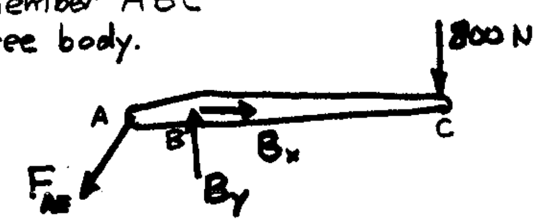
PROBLEM 1.14

1.14 Two hydraulic cylinders are used to control the position of the robotic arm *ABC*. Knowing that the control rods attached at *A* and *D* each have a 20-mm diameter and happen to be parallel in the position shown, determine the average normal stress in (*a*) member *AE*, (*b*) member *DG*.



SOLUTION

Use member ABC as free body.

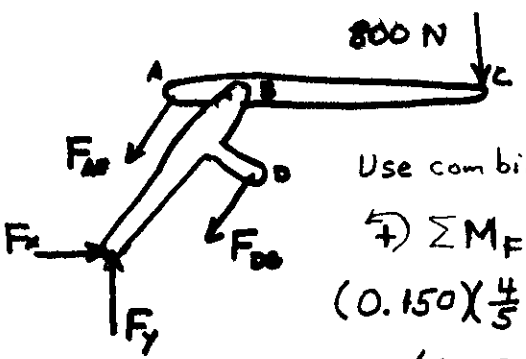


$\curvearrowleft + \; \Sigma M_B = 0 \qquad (0.150)\frac{4}{5}F_{AE} - (0.600)(800) = 0 \qquad F_{AE} = 4 \times 10^3$ N

Area of rod in member AE is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20\times10^{-3})^2 = 314.16\times10^{-6}$ m^2

Stress in rod AE: $\sigma_{AE} = \frac{F_{AE}}{A} = \frac{4\times10^3}{314.16\times10^{-6}} = 12.73\times10^6$ Pa

(a) $\sigma_{AE} = 12.73$ MPa ◄



Use combined members ABC and BFD as free body.

$\curvearrowleft + \; \Sigma M_F = 0$

$(0.150)(\frac{4}{5}F_{AE}) - (0.200)(\frac{4}{5}F_{DG})$

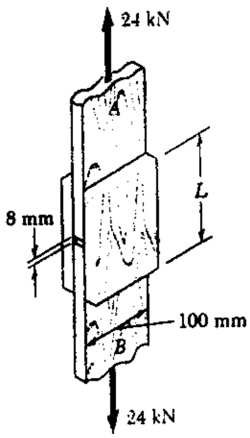
$- (1.050 - 0.350)(800) = 0 \qquad F_{DG} = -1500$ N

Area in rod DG is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20\times10^{-3})^2 = 314.16\times10^{-6}$ m^2

Stress in rod DG: $\sigma_{DG} = \frac{F_{DG}}{A} = \frac{-1500}{3.1416\times10^{-6}} = -4.77\times10^6$ Pa

(b) $\sigma_{DG} = -4.77$ MPa ◄

PROBLEM 1.15



1.15 The wooden members A and B are to be joined by plywood splice plates which will be fully glued on the surfaces in contact. As part of the design of the joint and knowing that the clearance between the ends of the members is to be 8 mm, determine **the smallest allowable length L if the average shearing stress in the glue is not to exceed 800 kPa.**

SOLUTION

There are four separate areas of glue. Each area must transmit half of the 24 kN load.

Therefore $F = 12 \text{ kN} = 12 \times 10^3 \text{ N}$

Shearing stress in glue $\tau = 800 \times 10^3 \text{ Pa}$

$$\tau = \frac{F}{A} \quad \therefore \quad A = \frac{F}{\tau} = \frac{12 \times 10^3}{800 \times 10^3} = 15 \times 10^{-3} \text{ m}^2$$

Let l = length of glue area and w = width = 100 mm = 0.1 m

$$A = lw \quad \therefore \quad l = \frac{A}{w} = \frac{15 \times 10^{-3}}{0.1} = 150 \times 10^{-3} \text{ m} = 150 \text{ mm}$$

$$L = 2l + \text{gap} = (2)(150) + 8 = 308 \text{ mm}$$

PROBLEM 1.16

1.16 Determine the diameter of the largest circular hole which can be punched into a sheet of polystyrene 6-mm thick, knowing that the force exerted by the punch is 45 kN and that a 55-MPa average shearing stress is required to cause the material to fail.

SOLUTION

$A = \pi d t$ for cylindrical failure surface

Shearing stress $\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau}$

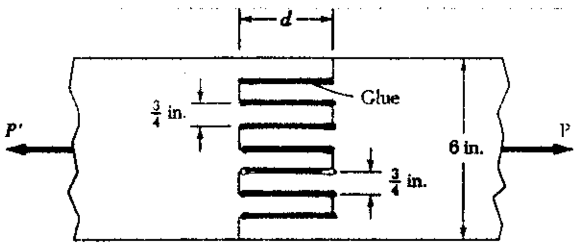
Equating A's $\pi d t = \frac{P}{\tau}$

Solving for d: $d = \frac{P}{\pi t \tau} = \frac{45 \times 10^3}{\pi (0.006)(55 \times 10^6)} = 43.4 \times 10^{-3} \text{ m}$

$$d = 43.4 \text{ mm}$$

PROBLEM 1.17

1.17 Two wooden planks, each 7/8 - in. thick and 6 in. wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 120 psi, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude P = 1200 lb.



SOLUTION

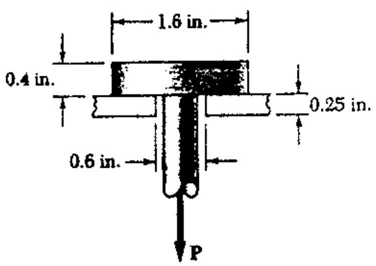
Seven surfaces carry the total load P = 1200 lb.

Area $A = (7)(\frac{7}{8})\, d = \frac{49}{8} d$

$\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau} \qquad \frac{49}{8} d = \frac{1200}{120} \qquad d = 1.633 \text{ in}$ ◄

PROBLEM 1.18

1.18 A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 0.6-in.-diameter hole has been drilled. Knowing that the shearing stress must not exceed 18 ksi in the steel rod and 10 ksi in the aluminum plate, determine the largest load **P** which may be applied to the rod.



SOLUTION

For steel $\quad A_1 = \pi d t = \pi(0.6)(0.4)$
$= 0.7540 \text{ in}^2$

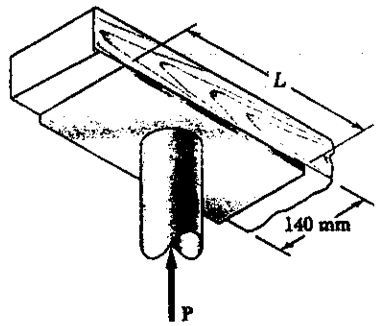
$\tau_1 = \frac{P}{A_1} \quad \therefore \quad P = A_1 \tau_1 = (0.7540)(18)$
$= 13.57$ kips

For aluminum $\quad A_2 = \pi d t = \pi(1.6)(0.25) = 1.2566 \text{ in}^2$

$\tau_2 = \frac{P}{A_2} \quad \therefore \quad P = A_2 \tau_2 = (1.2566)(10) = 12.57$ kips

Limiting value of P is the smaller value $\therefore \quad P = 12.57$ kips ◄

PROBLEM 1.19



1.19 The axial force in the column supporting the timber beam shown is $P = 75$ kN. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 3.0 MPa.

SOLUTION

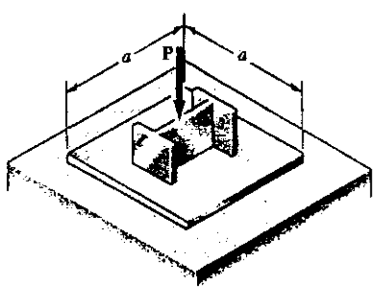
$$\sigma_b = \frac{P}{A} = \frac{P}{Lw}$$

Solving for L: $L = \frac{P}{\sigma_b w} = \frac{75 \times 10^3}{(3.0 \times 10^6)(0.140)}$

$= 178.6 \times 10^{-3}$ m

$L = 178.6$ mm

PROBLEM 1.20



1.20 An axial load **P** is supported by a short W250 × 67 column of cross-sectional area $A = 8580 \text{ mm}^2$ and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 150 MPa and that the bearing stress on the concrete foundation must not exceed 12.5 MPa, determine the side a of the plate which will provide the most economical and safe design.

SOLUTION

Area of column: $A = 8580 \text{ mm}^2 = 8580 \times 10^{-6} \text{ m}^2$

Normal stress in column: $\sigma = 150 \times 10^6$ Pa

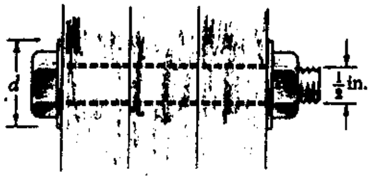
$\sigma = \frac{P}{A}$ ∴ $P = A\sigma = (8580 \times 10^{-6})(150 \times 10^6)$

$= 1.287 \times 10^6$ N

Bearing plate: $\sigma_b = \frac{P}{A_b}$ and $A_b = a^2$ for square plate.

$A_b = a^2 = \frac{P}{\sigma_b} = \frac{1.287 \times 10^6}{12.5 \times 10^6} = 321 \times 10^{-3}$ m or 321 mm

PROBLEM 1.21



1.21 Three wooden planks are fastened together by a series of bolts to form a column. The diameter of each bolt is ½ in. and the inner diameter of each washer is ⅝ in., which is slightly larger than the diameter of the holes in the planks. Determine the smallest allowable outer diameter d of the washers, knowing that the average normal stress in the bolts is 5 ksi and that the bearing stress between the washers and the planks must not exceed 1.2 ksi.

SOLUTION

Bolt: $A_{bolt} = \frac{\pi}{4} d_b^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$

$\sigma_b = \frac{P}{A}$ $\therefore$ Tensile force in bolt $P = \sigma_b A = (5)(0.19635) = 0.98175$ kips

Washer: inside diameter $= d_i = \frac{5}{8}$ in, outside diameter $= d_o$

Bearing area $A_w = \frac{\pi}{4}(d_o^2 - d_i^2)$ and $A_w = \frac{P}{\sigma_b}$.

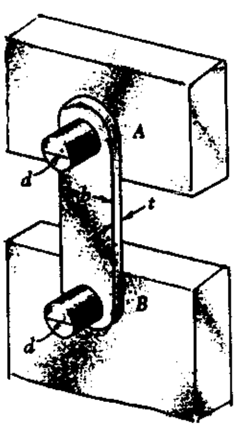
Equating $\frac{\pi}{4}(d_o^2 - d_i^2) = \frac{P}{\sigma_b}$

$$d_o^2 = d_i^2 + \frac{4P}{\pi\sigma_b} = \left(\frac{5}{8}\right)^2 + \frac{(4)(0.98175)}{\pi(12.5)} = 1.4323 \text{ in}^2$$

$$d_o = 1.197 \text{ in}$$

PROBLEM 1.22

1.22 Link AB, of width $b = 2$ in. and thickness $t = \frac{1}{4}$ in., is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is - 20 ksi and that the average shearing stress in each of the two pins is 12 ksi, determine (a) the diameter d of the pins, (b) the average bearing stress in the link.



SOLUTION

Rod AB is in compression.

$A = bt$ where $b = 2$ in and $t = \frac{1}{4}$ in

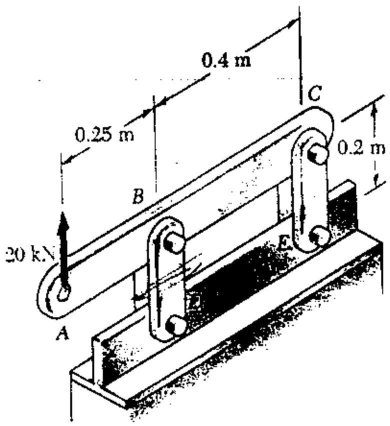
$P = -\sigma A = -(-20)(2)(\frac{1}{4}) = 10$ kips

Pin: $\tau_p = \frac{P}{A_p}$ and $A_p = \frac{\pi}{4} d^2$

(a) $d = \sqrt{\frac{4A_p}{\pi}} = \sqrt{\frac{4P}{\pi\tau_p}} = \sqrt{\frac{(4)(10)}{\pi(12)}} = 1.030$ in

(b) $\sigma_b = \frac{P}{dt} = \frac{10}{(1.030)(0.25)} = 38.8$ ksi

PROBLEM 1.23

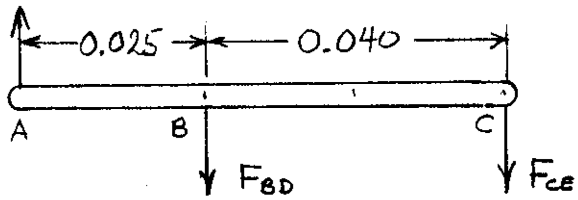


1.8 Each of the four vertical links has an 8 × 36- mm uniform rectangular cross section and each of the four pins has a 16- mm diameter.

1.23 For the assembly and loading of Prob. 1.8, determine (*a*) the average shearing stress in the pin at *B*, (*b*) the average bearing stress at *B* in link *BD*, (*c*) the average bearing stress at *B* in member *ABC*, knowing that this member has a 10 × 50-mm uniform rectangular cross section.

SOLUTION

Use bar ABC as a free body



$\Sigma M_C = 0 \qquad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$

$F_{BD} = 32.5 \times 10^3 \text{ N}$

(a) Shear pin at B $\qquad \tau = \dfrac{F_{BD}}{2A}$ for double shear

where $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

$\tau = \dfrac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \qquad 80.8 \text{ MPa}$ ◀

(b) Bearing link BD $\qquad A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$

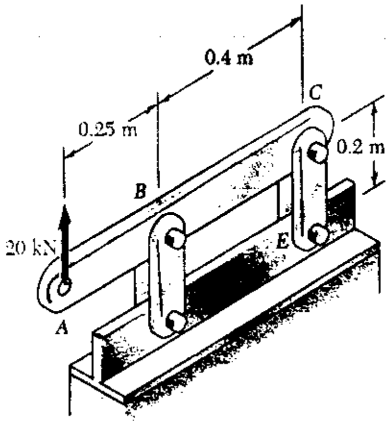
$\sigma_b = \dfrac{\frac{1}{2}F_{BD}}{A} = \dfrac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \qquad 127.0 \text{ MPa}$ ◀

(c) Bearing in ABC at B

$A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$

$\sigma_b = \dfrac{F_{BD}}{A} = \dfrac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \qquad 203 \text{ MPa}$ ◀

PROBLEM 1.24

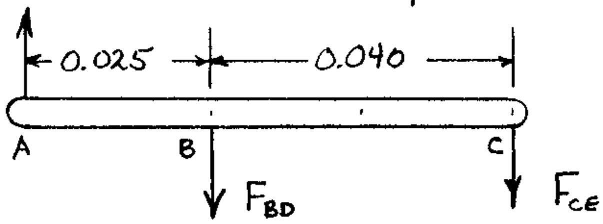


1.8 Each of the four vertical links has an 8 × 36- mm uniform rectangular cross section and each of the four pins has a 16- mm diameter.

1.24 For the assembly and loading of Prob. 1.8, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in link *CE*, (*c*) the average bearing stress at *C* in member *ABC*, knowing that this member has a 10 × 50-mm uniform rectangular cross section.

SOLUTION

Use bar ABC as a free body



$\sum M_B = 0 \quad -(0.040)F_{CE} - (0.025)(20\times 10^3) = 0 \qquad F_{CE} = -12.5\times 10^3$

(a) Shear in pin at C $\qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.016)^2 = 201.06\times 10^{-6}\ m^2$

Double shear $\quad \tau = \frac{F_{CE}}{2A} = \frac{12.5\times 10^3}{(2)(201.06\times 10^{-6})} = 31.1\times 10^{-6}$

31.1 MPa ◄

(b) Bearing in link CE at C

$A = dt = (0.016)(0.008) = 128\times 10^{-6}\ m^2$

$\sigma_b = \frac{\frac{1}{2}F_{CE}}{A} = \frac{(0.5)(12.5\times 10^3)}{128\times 10^{-6}} = 48.8\times 10^6 \qquad$ 48.8 MPa ◄

(c) Bearing in ABC at C

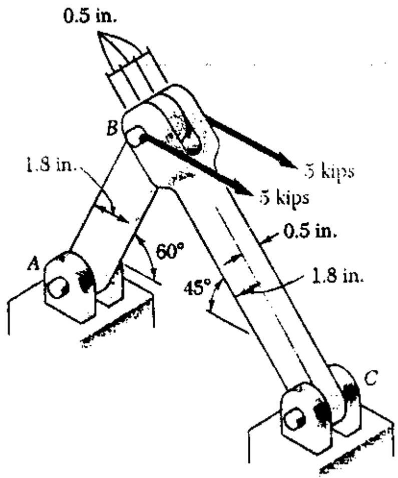
$A = dt = (0.016)(0.010) = 160\times 10^{-6}\ m^2$

$\sigma_b = \frac{F_{CE}}{A} = \frac{12.5\times 10^3}{160\times 10^{-6}} = 78.1\times 10^6 \qquad$ 78.1 MPa ◄

PROBLEM 1.25

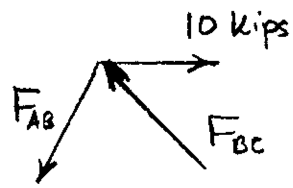
1.9 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link AB, (*b*) in link BC.

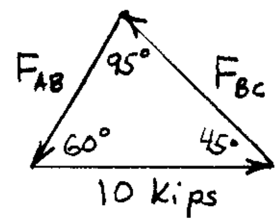
1.25 For the assembly and loading of Prob. 1.9, determine (*a*) the average shearing stress in the pin at A, (*b*) the average bearing stress at A in member AB.



SOLUTION

Use joint B as free body.





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ} \qquad F_{AB} = 7.3205 \text{ kips}$$

(a) Shearing stress in pin at A $\qquad \tau = \dfrac{F_{AB}}{2A_p}$

where $A_p = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.8)^2 = 0.5026 \text{ in}^2$

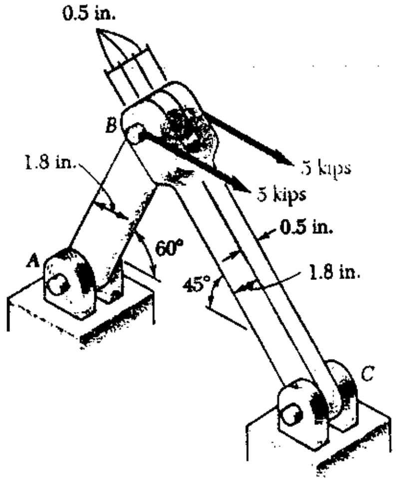
$$\tau = \frac{7.3205}{(2)(0.5026)} = 7.28 \qquad\qquad 7.28 \text{ ksi}$$

(b) Bearing stress at A in member AB

$$A_b = t\,d = (0.5)(0.8) = 0.4 \text{ in}^2$$

$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{7.3205}{0.4} = 18.30 \qquad\qquad 18.30 \text{ ksi}$$

PROBLEM 1.26

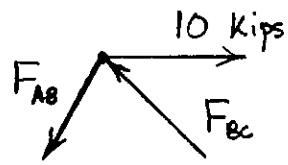


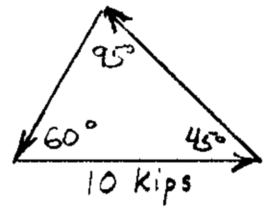
1.9 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link AB, (*b*) in link BC.

1.26 For the assembly and loading of Prob. 1.9, determine (*a*) the average shearing stress in the pin at C, (*b*) the average bearing stress at C in member BC, (*c*) the average bearing stress at B in member BC.

SOLUTION

Use joint B as free body





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ} \qquad F_{BC} = 8.9658 \text{ kips}$$

(a) Shearing stress in pin at C $\qquad \tau = \dfrac{F_{BC}}{2A_p}$

$$A_p = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.8)^2 = 0.5026 \text{ in}^2$$

$$\tau = \frac{8.9658}{(2)(0.5026)} = 8.92 \qquad 8.92 \text{ ksi}$$

(b) Bearing stress at C in member BC $\qquad \sigma_b = \dfrac{F_{BC}}{A}$

$$A = td = (0.5)(0.8) = 0.4 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.4} = 22.4 \qquad 22.4 \text{ ksi}$$

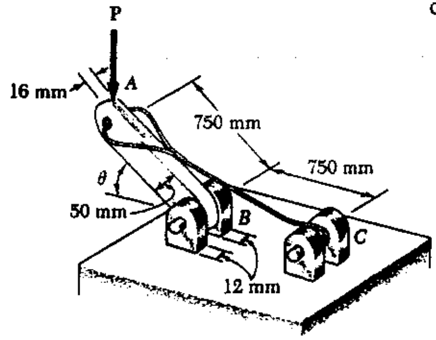
(c) Bearing stress at B in member BC $\qquad \sigma_b = \dfrac{F_{BC}}{A}$

$$A = 2td = 2(0.5)(0.8) = 0.8 \text{ in}^2$$

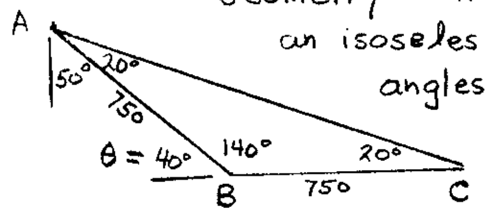
$$\sigma_b = \frac{8.9658}{0.8} = 11.21 \qquad 11.21 \text{ ksi}$$

PROBLEM 1.27

1.27 Knowing that $\theta = 40°$ and $P = 9$ kN, determine (*a*) the smallest allowable diameter of the pin at B if the average shearing stress in the pin is not to exceed 120 MPa, (*b*) the corresponding average bearing stress in member AB at B, (*c*) the corresponding average bearing stress in each of the support brackets at B.

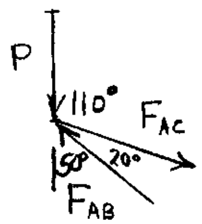


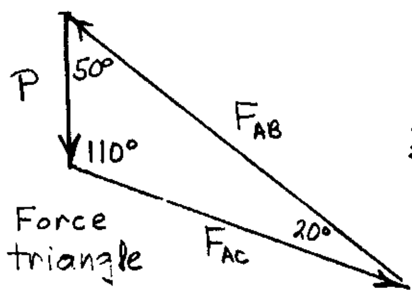
SOLUTION



Geometry: Triangle ABC is an isoseles triangle with angles shown here.

Use joint A as a free body.





Force triangle

Law of Sines applied to force triangle

$$\frac{P}{\sin 20°} = \frac{F_{AB}}{\sin 110°} = \frac{F_{AC}}{\sin 50°}$$

$$F_{AB} = \frac{P \sin 110°}{\sin 20°} = \frac{(9)\sin 110°}{\sin 20°} = 24.73 \text{ kN}$$

(a) Allowable pin diameter.

$$\tau = \frac{F_{AB}}{2A_P} = \frac{F_{AB}}{2\frac{\pi}{4}d^2} = \frac{2F_{AB}}{\pi d^2} \quad \text{where } F_{AB} = 24.73 \times 10^3 \text{ N}$$

$$d^2 = \frac{2F_{AB}}{\pi \tau} = \frac{(2)(24.73\times 10^3)}{\pi(120\times 10^6)} = 131.18 \times 10^{-6} \text{ m}^2$$

$$d = 11.45 \times 10^{-3} \text{ m} \qquad 11.45 \text{ mm} ◄$$

(b) Bearing stress in AB at A.

$$A_b = t d = (0.016)(11.45\times 10^{-3}) = 183.26 \times 10^{-6} \text{ m}^2$$

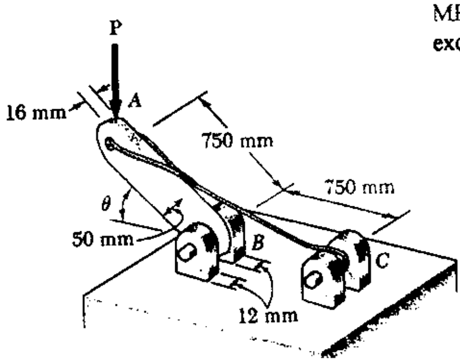
$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{24.73\times 10^3}{183.26\times 10^{-6}} = 134.9 \times 10^6 \qquad 134.9 \text{ MPa} ◄$$

(c) Bearing stress in support brackets at B

$$A = t d = (0.012)(11.45\times 10^{-3}) = 137.4 \times 10^{-6} \text{ m}^2$$

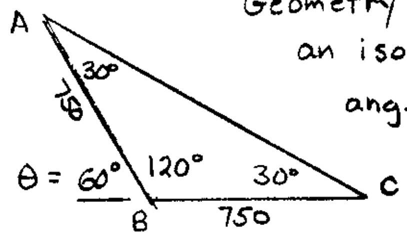
$$\sigma_b = \frac{\frac{1}{2}F_{AB}}{A} = \frac{(0.5)(24.73\times 10^3)}{137.4\times 10^{-6}} = 90.0 \times 10^6 \qquad 90.0 \text{ MPa} ◄$$

PROBLEM 1.28



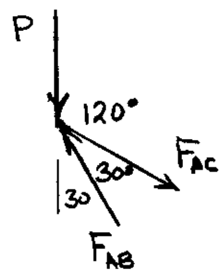
1.28 Determine the largest load **P** which may be applied at A when $\theta = 60$, knowing that the average shearing stress in the 10-mm-diameter pin at B must not exceed 120 MPa and that the average bearing stress in member AB and in the bracket at B must not exceed 90 MPa..

SOLUTION

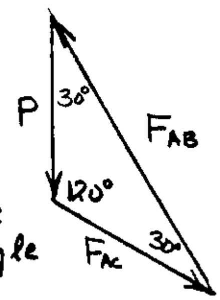


Geometry: Triangle ABC is an isoseles triangle with angles shown here

Use joint A as free body



Force triangle



Law of sines applied to force triangle

$$\frac{P}{\sin 30^\circ} = \frac{F_{AB}}{\sin 120^\circ} = \frac{F_{AC}}{\sin 30^\circ}$$

$$P = \frac{F_{AB}\sin 30^\circ}{\sin 120^\circ} = 0.57735\, F_{AB}$$

$$P = \frac{F_{AC}\sin 30^\circ}{\sin 30^\circ} = F_{AC}$$

If shearing stress in pin at B is critical

$$A_p = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54\times10^{-6}\ \text{m}^2$$

$$F_{AB} = 2A\tau = (2)(78.54\times10^{-6})(120\times10^{6}) = 18.850\times10^{3}\ \text{N}$$

IF bearing stress in member AB at bracket at A is critical

$$A_b = td = (0.016)(0.010) = 160\times10^{-6}\ \text{m}^2$$

$$F_{AB} = A_b\sigma_b = (160\times10^{-6})(90\times10^{6}) = 14.40\times10^{3}\ \text{N}$$

If bearing stress in the bracket at B is critical

$$A_b = 2td = (2)(0.012)(0.010) = 240\times10^{-6}\ \text{m}^2$$

$$F_{AB} = A_b\sigma_b = (240\times10^{-6})(90\times10^{6}) = 21.6\times10^{3}\ \text{N}$$

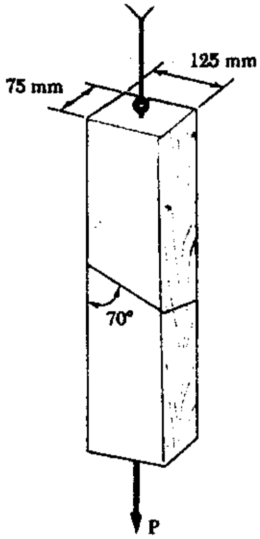
Allowable F_{AB} is the smallest, i.e. 14.40×10^{3} N

Then, from Statics $P_{allow} = (0.57735)(14.40\times10^{3})$

$= 8.31\times10^{3}$ N　　　8.31 kN ◄

PROBLEM 1.29

1.29 The 6- kN load **P** is supported by two wooden members of 75 × 125- mm uniform rectangular cross section which are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.



SOLUTION

$P = 6 \times 10^3$ N $\qquad \theta = 90° - 70° = 20°$

$A_0 = (0.075)(0.125) = 9.375 \times 10^{-3}$ m^2

$$\sigma = \frac{P}{A_0}\cos^2\theta = \frac{(6 \times 10^3)\cos^2 20°}{9.375 \times 10^{-3}} = 565 \times 10^3$$

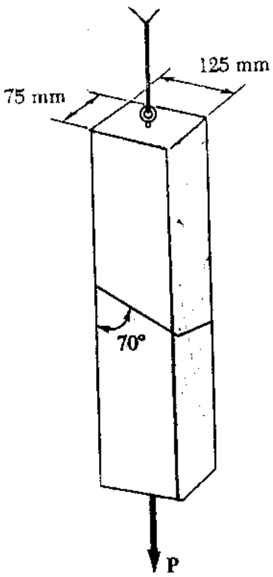
$\sigma = 565$ kPa ◄

$$\tau = \frac{P}{2A_0}\sin 2\theta = \frac{(6 \times 10^3)\sin 40°}{(2)(9.375 \times 10^{-3})} = 206 \times 10^3$$

$\tau = 206$ kPa ◄

PROBLEM 1.30

1.30 Two wooden members of 75 × 125- mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 500 kPa, determine (*a*) the largest load **P** which can be safely supported, (*b*) the corresponding shearing stress in the splice.



SOLUTION

$A_0 = (0.075)(0.125) = 9.375 \times 10^{-3}$ m^2

$\theta = 90° - 70° = 20° \qquad \sigma = 500 \times 10^3$ Pa

$$\sigma = \frac{P}{A_0}\cos^2\theta$$

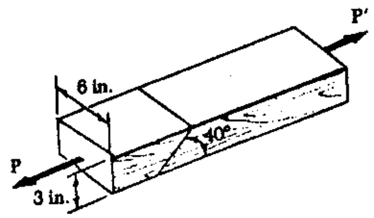
$$P = \frac{A_0\sigma}{\cos^2\theta} = \frac{(9.375 \times 10^{-3})(500 \times 10^3)}{\cos^2 20°} = 5.3085 \times 10^3$$

(a) $\qquad P = 5.31$ kN ◄

$$\tau = \frac{P \sin 2\theta}{2A_0} = \frac{(5.3085 \times 10^3)\sin 40°}{(2)(9.375 \times 10^{-3})} = 181.99 \times 10^3$$

(b) $\qquad \tau = 182.0$ kPa ◄

PROBLEM 1.31



1.31 Two wooden members of 3 × 6- in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 90 psi, determine (*a*) the largest load **P** which can be safely applied, (*b*) the corresponding tensile stress in the splice.

SOLUTION

$\theta = 90° - 40° = 50°$

$A_o = (3)(6) = 18 \text{ in}^2$

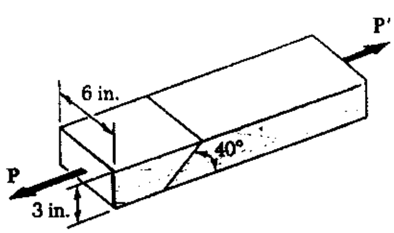
$\tau = \frac{P}{2A} \sin 2\theta$

$P = \frac{2A\tau}{\sin 2\theta} = \frac{(2)(18)(90)}{\sin 100°} = 3290$

(a) $P = 3290 \text{ lb.}$

(b) $\sigma = \frac{P \cos^2\theta}{A_o} = \frac{3290 \cos^2 50°}{18} = 75.5$ $\quad \sigma = 75.5 \text{ psi}$

PROBLEM 1.32



1.32 Two wooden members of 3 × 6- in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 2400$ lb, determine the normal and shearing stresses in the glued splice.

SOLUTION

$\theta = 90° - 40° = 50°$ $\quad P = 2400 \text{ lb.}$

$A_o = (3)(6) = 18 \text{ in}^2$

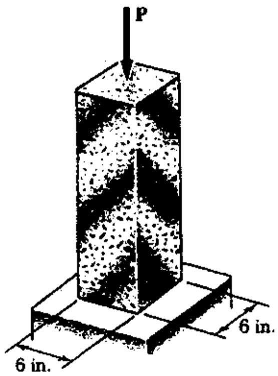
$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{(2400)\cos^2 50°}{18} = 55.1$

$\sigma = 55.1 \text{ psi}$

$\tau = \frac{P}{2A}\sin 2\theta = \frac{(2400)\sin 100°}{(2)(18)} = 65.7$

$\tau = 65.7 \text{ psi}$

PROBLEM 1.33



1.33 A centric load **P** is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 2.5 ksi, determine (*a*) the magnitude of **P**, (*b*) the orientation of the surface on which the maximum shearing stress occurs, (*c*) the normal stress exerted on that surface, (*d*) the maximum value of the normal stress in the block.

SOLUTION

$A_o = (6)(6) = 36 \text{ in}^2 \qquad \tau_{max} = 2.5 \text{ ksi}$

$\theta = 45°$ for plane of τ_{max}

(a) $\tau_{max} = \dfrac{|P|}{2A_o} \quad \therefore \quad |P| = 2A_o \tau_{max} = (2)(36)(2.5)$

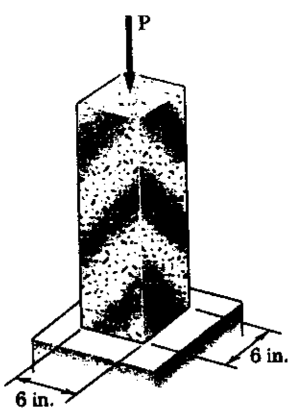
$= 180$ Kips

(b) $\sin 2\theta = 1 \qquad 2\theta = 90° \qquad \theta = 45°$

(c) $\sigma_{45} = \dfrac{P}{A_o}\cos^2 45° = \dfrac{P}{2A_o} = \dfrac{-180}{(2)(36)} = -2.5 \text{ ksi}$

(d) $\sigma_{max} = \dfrac{P}{A_o} = \dfrac{-180}{36} = -5 \text{ ksi}$

PROBLEM 1.34



1.34 A 240- kip load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.

SOLUTION

$A_o = (6)(6) = 36 \text{ in}^2$

$\sigma = \dfrac{P}{A_o}\cos^2\theta = \dfrac{-240}{36}\cos^2\theta = -6.67\cos^2\theta$

(a) max tensile stress $= 0$ at $\theta = 90°$

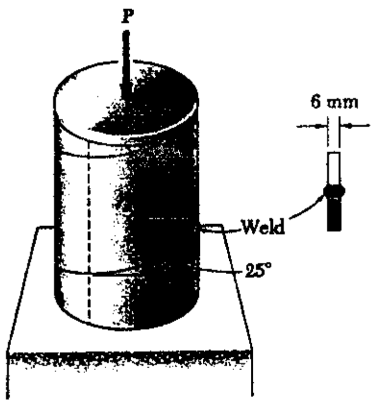
max. compressive stress $= 6.67$ ksi

at $\theta = 0°$

(b) $\tau_{max} = \dfrac{P}{2A_o} = \dfrac{240}{(2)(36)} = 3.33 \text{ ksi}$

at $\theta = 45°$

PROBLEM 1.35



1.35 A steel pipe of 300- mm outer diameter is fabricated from 6- mm- thick plate by welding along a helix which forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that a 250- kN axial force **P** is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

SOLUTION

$d_o = 0.300 \text{ m} \qquad r_o = \frac{1}{2} d_o = 0.150 \text{ m}$

$r_i = r_o - t = 0.150 - 0.006 = 0.144 \text{ m}$

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.150^2 - 0.144^2)$

$= 5.54 \times 10^{-3} \text{ m}^2$

$\theta = 25°$

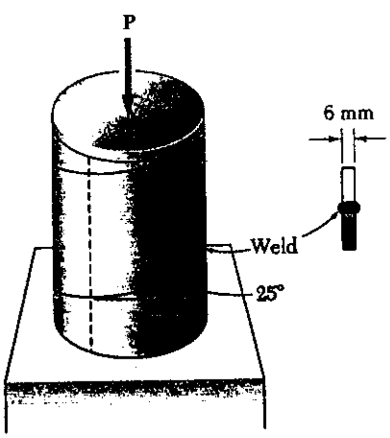
$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{-250 \times 10^3 \cos^2 25°}{5.54 \times 10^{-3}}$

$= -37.1 \times 10^6 \qquad \sigma = -37.1 \text{ MPa}$

$\tau = \frac{P}{2A_o}\sin 2\theta = \frac{-250 \times 10^3 \sin 50°}{(2)(5.54 \times 10^{-3})}$

$= -17.28 \times 10^6 \qquad \tau = 17.28 \text{ MPa}$

PROBLEM 1.36



1.36 A steel pipe of 300- mm outer diameter is fabricated from 6- mm- thick plate by welding along a helix which forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in directions respectively normal and tangential to the weld are $\sigma = 50$ MPa and $\tau = 30$ MPa, determine the magnitude P of the largest axial force that can be applied to the pipe.

SOLUTION

$d_o = 0.300 \text{ m} \qquad r_o = \frac{1}{2} d_o = 0.150 \text{ m}$

$r_i = r_o - t = 0.150 - 0.006 = 0.144 \text{ m}$

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.150^2 - 0.144^2)$

$= 5.54 \times 10^{-3} \text{ m}^2$

$\theta = 25°$

Based on $|\sigma| = 50$ MPa: $\quad \sigma = \frac{P}{A_o}\cos^2\theta$

$P = \frac{A_o \sigma}{\cos^2\theta} = \frac{(5.54 \times 10^{-3})(50 \times 10^6)}{\cos^2 25°} = 337 \times 10^3$

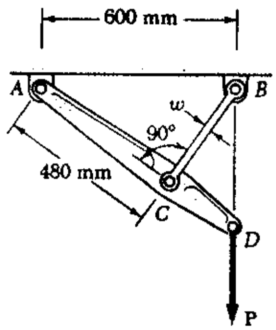
Based on $|\tau| = 30$ MPa $\quad \tau = \frac{P}{2A_o}\sin 2\theta$

$P = \frac{2A_o \tau}{\sin 2\theta} = \frac{(2)(5.54 \times 10^{-3})(30 \times 10^6)}{\sin 50°} = 434 \times 10^3$

Smaller value is the allowable value of P $\therefore$ $\quad P = 337$ kN

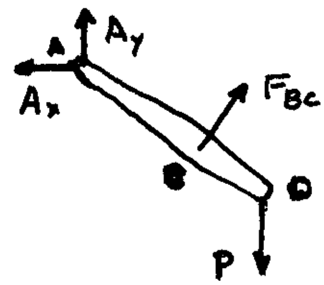
PROBLEM 1.37

1.37 Link BC is 6 mm thick, has a width w = 25 mm, and is made of a steel with a 480- MPa ultimate strength in tension. What was the safety factor used if the structure shown was designed to support a 16-kN load **P** ?



SOLUTION

Use bar ACD as a free body and note that member BD is a two-force member



$\Sigma M_A = 0$

$(480) F_{BC} - (600) P = 0$

$$F_{BC} = \frac{600}{480} P = \frac{(600)(16\times10^3)}{480} = 20\times10^3 \text{ N}$$

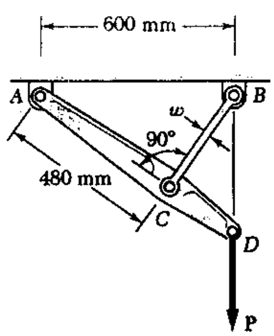
Ultimate load for member BC $\quad F_U = \sigma_U A$

$$F_U = (480\times10^6)(0.006)(0.025) = 72\times10^3 \text{ N}$$

Factor of safety $\quad F.S. = \frac{F_U}{F_{BC}} = \frac{72\times10^3}{20\times10^3} = 3.60$ ◄

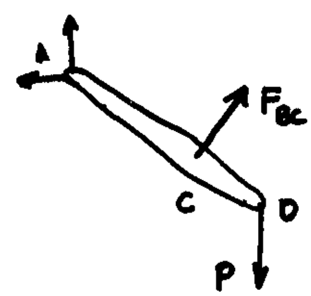
PROBLEM 1.38

1.38 Link BC is 6 mm thick and is made of a steel with a 450- MPa ultimate strength in tension. What should be its width w if the structure shown is being designed to support a 20-kN load **P** with a factor of safety of 3 ?



SOLUTION

Use bar ACD as a free body and note that member BC is a two-force member.



$\Sigma M_A = 0$

$480 F_{BC} - 600 P = 0$

$$F_{BC} = \frac{600 P}{480} = \frac{(600)(20\times10^3)}{480} = 25\times10^3 \text{ N}$$

For a factor of safety F.S. = 3, the ultimate load of member BC

$$F_U = (F.S.)(F_{BC}) = (3)(25\times10^3) = 75\times10^3 \text{ N}$$

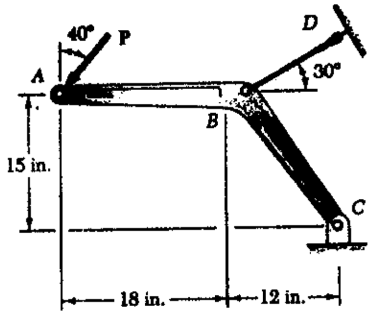
But $F_U = \sigma_U A \quad \therefore \quad A = \frac{F_U}{\sigma_U} = \frac{75\times10^3}{450\times10^6} = 166.67\times10^{-6} \text{ m}^2$

For a rectangular section $\quad A = wt \quad$ or $\quad w = \frac{A}{t} = \frac{166.67\times10^{-6}}{0.006}$

$w = 27.8\times10^{-3}$ m or 27.8 mm ◄

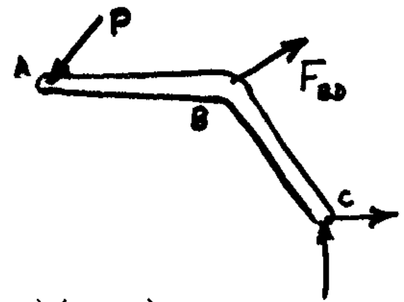
PROBLEM 1.39

1.39 Member ABC, which is supported by a pin and bracket at C and a cable BD, was designed to support the 4-kip load **P** as shown. Knowing that the ultimate load for cable BD is 25 kips, determine the factor of safety with respect to cable failure.



SOLUTION

Use member ABC as a free body and note that member BD is a two-force member.



$$\Sigma M_c = 0$$

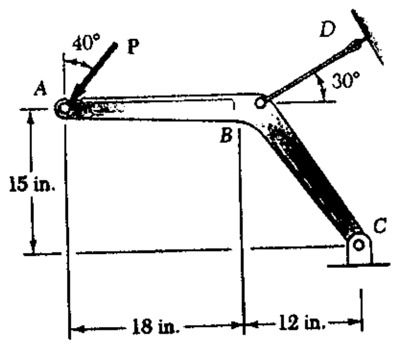
$$(P\cos 40°)(30\text{ in}) + (P\sin 40°)(15\text{ in.}) - (F_{BD}\cos 30°)(15\text{ in}) - (F_{BD}\sin 30°)(12\text{ in}) = 0$$

$$F_{BD} = \frac{32.623}{18.990}P = \frac{(32.623)(4)}{18.990} = 6.8715\text{ kips}$$

Factor of safety for cable BD $\quad F.S. = \frac{F_{ult}}{F_{BD}} = \frac{25}{6.8715} = 3.64$

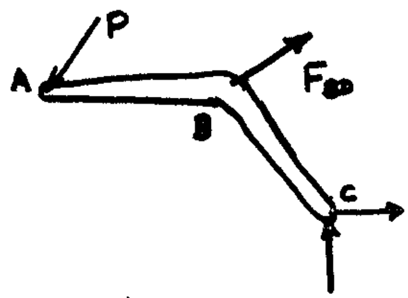
PROBLEM 1.40

1.40 Knowing that the ultimate load for cable BD is 25 kips and that a factor of safety of 3.2 with respect to cable failure is required, determine the magnitude of the largest force **P** which can be safely applied as shown to member ABC.



SOLUTION

Use member ABC as a free body and note that member BD is a two-force member.



$$\Sigma M_c = 0$$

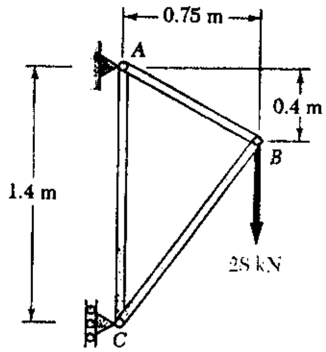
$$(P\cos 40°)(30\text{ in.}) + (P\sin 40°)(15\text{ in}) - (F_{BD}\cos 30°)(15\text{ in}) - (F_{BD}\sin 30°)(12\text{ in}) = 0$$

$$P = \frac{18.990}{32.623}F_{BD} = 0.58216\, F_{BD}$$

Allowable load for member BD is $F_{BD} = \frac{F_{ult}}{F.S.} = \frac{25}{3.2} = 7.8125\text{ kips}$

Allowable load $P = (0.58216)(7.8125) = 4.55\text{ kips}$

PROBLEM 1.41



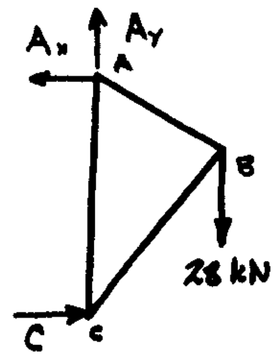
1.41 Members *AB* and *AC* of the truss shown consist of bars of square cross section made of the same alloy. It is known that a 20- mm- square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If bar *AB* has a 15- mm- square cross section, determine (*a*) the factor of safety for bar *AB*, (*b*) the dimensions of the cross section of bar *AC* if it is to have the same factor of safety as bar *AB*.

SOLUTION

Length of member AB

$$\ell_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

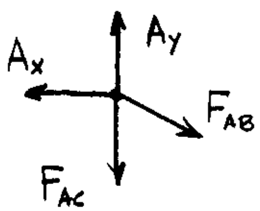
Use entire truss as a free body



$\circlearrowleft \Sigma M_C = 0 \quad 1.4 A_x - (0.75)(28) = 0 \quad A_x = 15 \text{ kN}$

$+\uparrow \Sigma F_y = 0 \quad A_y - 28 = 0 \quad A_y = 28 \text{ kN}$

Use joint A as free body



$\xrightarrow{+} \Sigma F_x = 0 \quad \frac{0.75}{0.85} F_{AB} - A_x = 0$

$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$

$+\uparrow \Sigma F_y = 0 \quad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$

$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$

For the test bar $\quad A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \quad P_U = 120 \times 10^3 \text{ N}$

For the material $\quad \sigma_U = \frac{P_U}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For bar AB $\quad F.S. = \frac{F_U}{F_{AB}} = \frac{\sigma_U A}{F_{AB}} = \frac{(300 \times 10^6)(0.015)^2}{17 \times 10^3}$

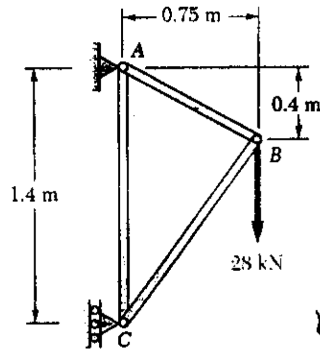
$= 3.97$ ◄

(b) For bar AC $\quad F.S. = \frac{F_U}{F_{AC}} = \frac{\sigma_U A}{F_{AC}} = \frac{\sigma_U a^2}{F_{AC}}$

$$a^2 = \frac{(F.S.) F_{AC}}{\sigma_U} = \frac{(3.97)(20 \times 10^3)}{300 \times 10^6} = 264.7 \times 10^{-6} \text{ m}^2$$

$a = 16.27 \times 10^{-3}$ m $\qquad$ 16.27 mm ◄

PROBLEM 1.42



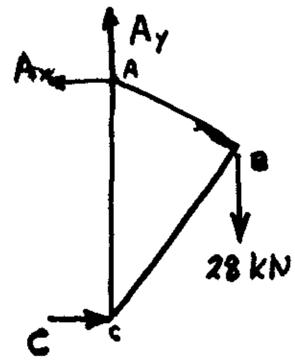
1.42 Members *AB* and *AC* of the truss shown consist of bars of square cross section made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If a factor of safety of 3.2 is to be achieved for both bars, determine the required dimensions of the cross section of (*a*) bar *AB*, (*b*) bar *AC*.

SOLUTION

Length of member AB

$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

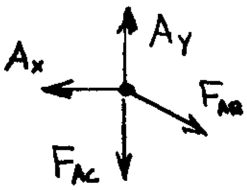
Use entire truss as a free body



$\circlearrowleft \Sigma M_c = 0 \qquad 1.4 A_x - (0.75)(28) = 0 \qquad A_x = 15 \text{ kN}$

$+\uparrow \Sigma F_y = 0 \qquad A_y - 28 = 0 \qquad A_y = 28 \text{ kN}$

Use joint A as free body



$\xrightarrow{+} \Sigma F_x = 0 \qquad \frac{0.75}{0.85} F_{AB} - A_x = 0$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$+\uparrow \Sigma F_y = 0 \qquad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar $\quad A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \qquad P_u = 120 \times 10^3 \text{ N}$

For the material $\quad \sigma_u = \frac{P_u}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For member AB $\quad \text{F.S.} = \frac{P_u}{F_{AB}} = \frac{\sigma_u A}{F_{AB}} = \frac{\sigma_u a^2}{F_{AB}}$

$$a^2 = \frac{(\text{F.S.}) F_{AB}}{\sigma_u} = \frac{(3.2)(17 \times 10^3)}{300 \times 10^6} = 181.33 \times 10^{-6} \text{ m}^2$$

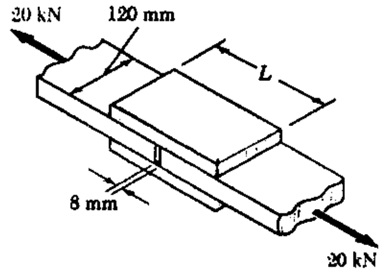
$a = 13.47 \times 10^{-3}$ m $\qquad$ 13.47 mm ◀

(b) For member AC $\quad \text{F.S.} = \frac{P_u}{F_{AC}} = \frac{\sigma_u A}{F_{AC}} = \frac{\sigma_u b^2}{F_{AC}}$

$$b^2 = \frac{(\text{F.S.}) F_{AC}}{\sigma_u} = \frac{(3.2)(20 \times 10^3)}{300 \times 10^6} = 213.33 \times 10^{-6} \text{ m}^2$$

$b = 14.61 \times 10^{-3}$ m $\qquad$ 14.61 mm ◀

PROBLEM 1.43



1.43 The two wooden members shown, which support a 20-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.8 MPa and the clearance between the members is 8 mm. Determine the factor of safety, knowing that the length of each splice is $L = 200$ mm.

SOLUTION

There are 4 separate areas of glue. Each glue area must transmit 10 kN of shear load.

$$P = 10\times10^3\ N$$

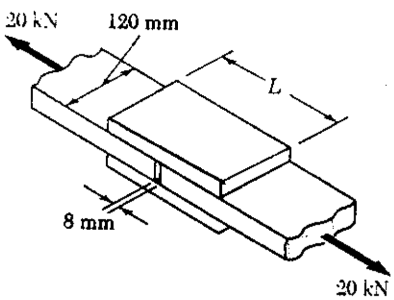
Length of splice $L = 2\ell + c$ where ℓ = length of glue and c = clearance. $\ell = \frac{1}{2}(L - c) = \frac{1}{2}(0.200 - 0.008) = 0.096$ m.

Area of glue $A = \ell w = (0.096)(0.120) = 11.52\times10^{-3}\ m^2$

Ultimate load $P_U = \tau_U A = (2.8\times10^6)(11.52\times10^{-3}) = 32.256\times10^3$ N

Factor of safety $F.S. = \dfrac{P_U}{P} = \dfrac{32.256\times10^3}{10\times10^3} = 3.23$ ◀

PROBLEM 1.44



1.43 The two wooden members shown, which support a 20-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.8 MPa and the clearance between the members is 8 mm.

1.44 For the joint and loading of Prob. 1.43, determine the required length L of each splice if a factor of safety of 3.5 is to be achieved.

SOLUTION

There are 4 separate areas of glue. Each glue area must transmit 10 kN of shear load.

$$P = 10\times10^3\ N$$

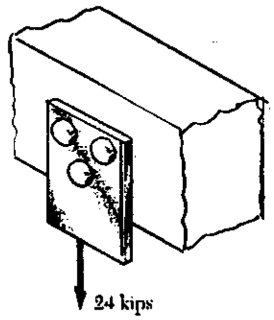
Required ultimate load $P_U = (F.S.)(P) = (3.5)(10\times10^3) = 35\times10^3$ N

Required length ℓ of each glue area

$$P_U = \tau_U A = \tau_U \ell w \qquad \ell = \frac{P_U}{\tau_U w} = \frac{35\times10^3}{(2.8\times10^6)(0.120)} = 104.17\times10^{-3}\ m$$

Length of splice $L = 2\ell + c = (2)(104.17\times10^{-3}) + 0.008$
$= 216.3\times10^{-3}$ m 216 mm ◀

PROBLEM 1.45



1.45 Three $\frac{3}{4}$ - in.-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 24-kip load and that the ultimate shearing stress for the steel used is 52 ksi, determine the factor of safety for this design.

SOLUTION

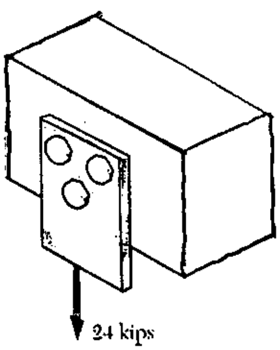
For each bolt $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$

$P_U = A\tau_U = (0.4418)(52) = 22.97 \text{ kips}$

Per bolt $P = \frac{24}{3} = 8 \text{ kips}$

$\text{F.S.} = \frac{P_U}{P} = \frac{22.97}{8} = 2.87$

PROBLEM 1.46



1.46 Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 24-kip load, that the ultimate shearing stress for the steel used is 52 ksi, and that a factor of safety of 3.37 is desired, determine the required diameter of the bolts.

SOLUTION

For each bolt $P = \frac{24}{3} = 8 \text{ kips}$

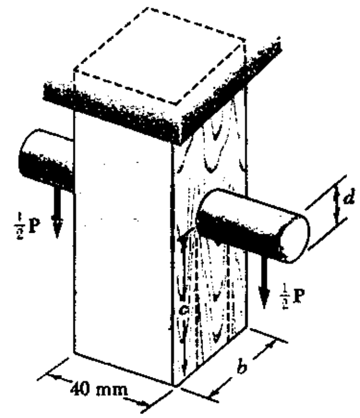
Required $P_U = (\text{F.S.})P = (3.37)(8) = 26.96 \text{ kips}$

$\tau_U = \frac{P_U}{A} \therefore A = \frac{P_U}{\tau_U} = \frac{26.96}{52} = 0.51846 \text{ in}^2$

$A = \frac{\pi}{4}d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.51846)}{\pi}}$

$= 0.8125 \text{ in.}$

PROBLEM 1.47



1.47 A load **P** is supported as shown by a steel pin which has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 150 MPa in shear. Knowing that the diameter of the pin is d = 16 mm and that the magnitude of the load is P = 20 kN, determine (*a*) the factor of safety for the pin, (*b*) the required values of b and c if the factor of safety for the wooden member is to be the same as that found in part *a* for the pin.

SOLUTION

$$P = 20 \text{ kN} = 20 \times 10^3 \text{ N}$$

(a) Pin: $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

Double shear $\quad \tau = \frac{P}{2A} \qquad \tau_U = \frac{P_U}{2A}$

$$P_U = 2A\tau_U = (2)(201.16 \times 10^{-6})(150 \times 10^6)$$

$$= 60.319 \times 10^3 \text{ N}$$

$$\text{F.S.} = \frac{P_U}{P} = \frac{60.319 \times 10^3}{20 \times 10^3} = 3.02$$

(b) Tension in wood $\quad P_U = 60.319 \times 10^3$ N for same F.S.

$$\sigma_U = \frac{P_U}{A} = \frac{P_U}{w(b-d)} \quad \text{where } w = 40 \text{ mm} = 0.040 \text{ m}$$

$$b = d + \frac{P_U}{w\sigma_U} = 0.016 + \frac{60.319 \times 10^3}{(0.040)(60 \times 10^6)} = 41.1 \times 10^{-3} \text{ m}$$

$$b = 41.1 \text{ mm}$$

Shear in wood $\quad P_U = 60.319 \times 10^3$ N for same F.S.

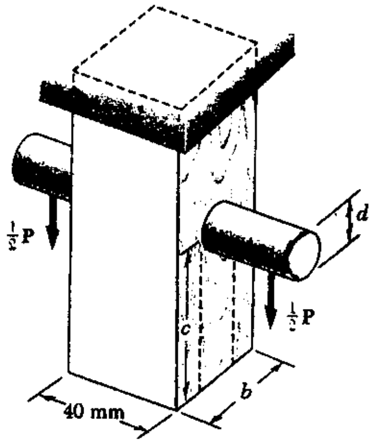
Double shear; each area is $A = wc$

$$\tau_U = \frac{P_U}{2A} = \frac{P_U}{2wc}$$

$$c = \frac{P_U}{2w\tau_U} = \frac{60.319 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 100.5 \times 10^{-3} \text{ m}$$

$$c = 100.5 \text{ mm}$$

PROBLEM 1.48



1.47 A load **P** is supported as shown by a steel pin which has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 150 MPa in shear.

1.48 For the support of Prob. 1.47, knowing that $b = 40$ mm, $c = 55$ mm and $d = 12$ mm, determine the allowable load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

Based on double shear in pin

$$P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U$$

$$= \frac{\pi}{4}(2)(0.012)^2(150\times10^6) = 33.93\times10^3\ N$$

Based on tension in wood

$$P_U = A\sigma_U = w(b-d)\sigma_U$$

$$= (0.040)(0.040-0.012)(60\times10^6)$$

$$= 67.2\times10^3\ N$$

Based on double shear in the wood

$$P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times10^6)$$

$$= 33.0\times10^3\ N$$

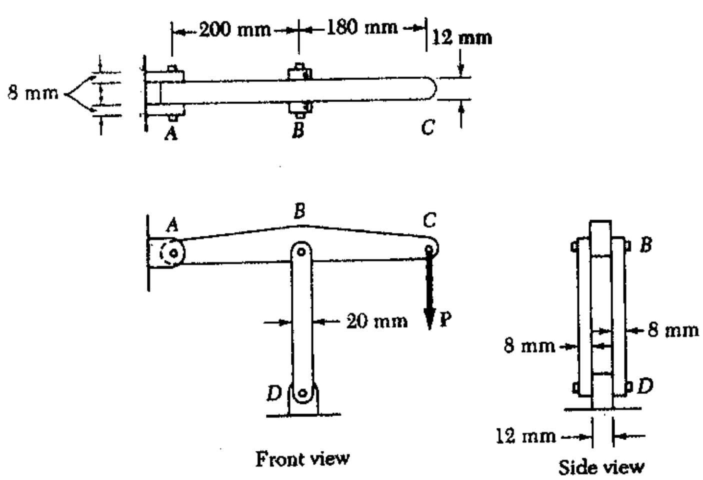
Use smallest $\quad P_U = 33.0\times10^3\ N$

Allowable $\quad P = \frac{P_U}{F.S.} = \frac{33.0\times10^3}{3.2} = 10.31\times10^3\ N$

10.31 kN

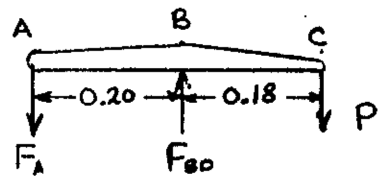
PROBLEM 1.49

1.49 In the structure shown, an 8- mm- diameter pin is used at A, and 12- mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.



SOLUTION

Statics: Use ABC as free body.



$\Sigma M_B = 0 \qquad 0.20\,F_A - 0.18\,P = 0$

$P = \frac{10}{9} F_A$

$\Sigma M_A = 0 \qquad 0.20\,F_{BD} - 0.38\,P = 0$

$P = \frac{10}{19} F_{BD}$

Based on double shear in pin A

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.008)^2 = 50.266 \times 10^{-6}\ m^2$

$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(50.266\times10^{-6})}{3.0} = 3.351\times10^3\ N$

$P = \frac{10}{9} F_A = 3.72\times10^3\ N$

Based on double shear in pins at B and D

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10\times10^{-6}\ m^2$

$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(113.10\times10^{-6})}{3.0} = 7.54\times10^3\ N$

$P = \frac{10}{19} F_{BD} = 3.97\times10^3\ N$

Based on compression in links BD

For one link $\quad A = (0.020)(0.008) = 160\times10^{-6}\ m^2$

$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250\times10^6)(160\times10^{-6})}{3.0} = 26.7\times10^3\ N$

$P = \frac{10}{19} F_{BD} = 14.04\times10^3\ N$

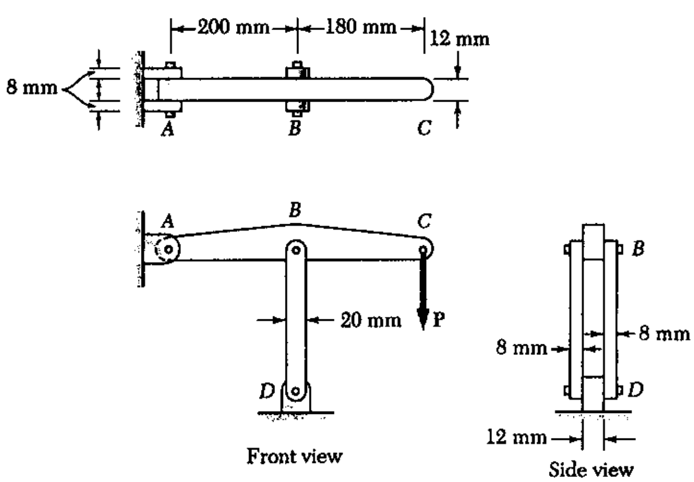
Allowable value of P is smallest $\therefore \quad P = 3.72\times10^3\ N$

3.72 kN

PROBLEM 1.50

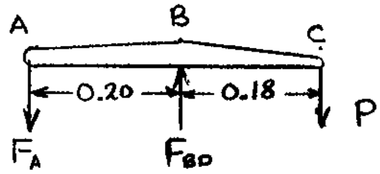
1.49 In the structure shown, an 8- mm- diameter pin is used at *A*, and 12- mm-diameter pins are used at *B* and *D*. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining *B* and *D*, determine the allowable load **P** if an overall factor of safety of 3.0 is desired

1.50 In an alternative design for the structure of Prob. 1.49, a pin of 10-mm-diameter is to be used at *A*. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.



SOLUTION

Statics: Use ABC as free body.



$\Sigma M_B = 0 \qquad 0.20\, F_A - 0.18\, P = 0$

$P = \frac{10}{9} F_A$

$\Sigma M_A = 0 \qquad 0.20\, F_{BD} - 0.38\, P = 0$

$P = \frac{10}{19} F_{BD}$

Based on double shear in pin A

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\ m^2$

$F_A = \frac{2 \tau_U A}{F.S.} = \frac{(2)(100\times10^6)(78.54\times10^{-6})}{3.0} = 5.236 \times 10^3\ N$

$P = \frac{10}{9} F_A = 5.82 \times 10^3\ N$

Based on double shear in pins at B and D

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ m^2$

$F_{BD} = \frac{2 \tau_U A}{F.S.} = \frac{(2)(100\times10^6)(113.10\times10^{-6})}{3.0} = 7.54 \times 10^3\ N$

$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3\ N$

Based on compression in links BD

For one link $\quad A = (0.020)(0.008) = 160 \times 10^{-6}\ m^2$

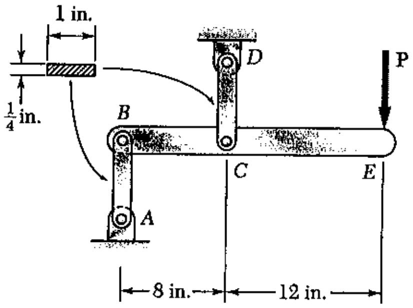
$F_{BD} = \frac{2 \sigma_U A}{F.S.} = \frac{(2)(250\times10^6)(160\times10^{-6})}{3.0} = 26.7 \times 10^3\ N$

$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3\ N$

Allowable value of P is smallest $\therefore \quad P = 3.97 \times 10^3\ N$

3.97 kN

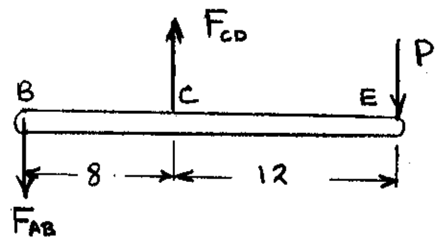
PROBLEM 1.51



1.51 Each of the steel links AB and CD is connected to a support and to member BCE by $\frac{1}{2}$ - in.- diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 24 ksi for the steel used in the pins and that the ultimate normal stress is 60 ksi for the steel used in the links, determine the allowable load **P** if an overall factor of safety of 3.2 is desired. (Note that the links are not reinforced around the pin holes.)

SOLUTION

Use BCE as free body



$\Sigma M_B = 0$

$8F_{CD} - 20P = 0$

$P = \frac{2}{5} F_{CD}$

$\Sigma M_C = 0 \qquad 8F_{AB} - 12P = 0 \qquad P = \frac{2}{3} F_{AB}$

Both links have the same area and same pin diameter ; hence, being of the same material, they will have the same ultimate load.

Based on pin in single shear

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$

$F_U = \tau_U A = (24)(0.19635) = 4.7124$ kips

Based on tension in link

$A = (b - d)t = (1 - \frac{1}{2})(\frac{1}{4}) = 0.125 \text{ in}^2$

$F_U = \sigma_U A = (60)(0.125) = 7.50$ Kips

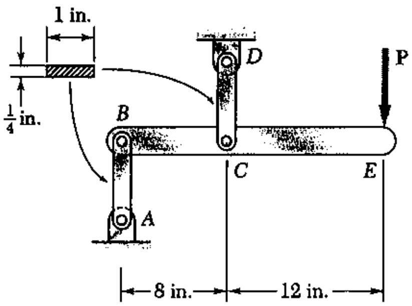
Ultimate load for link is smallest: $\quad F_U = 4.7124$ kips.

Allowable load for link $\quad F = \frac{F_U}{F.S.} = \frac{4.7124}{3.2} = 1.4726$ kips

Allowable load for structure $\quad P = \frac{2}{5} F = 0.589$ Kips

$F = 589$ lb

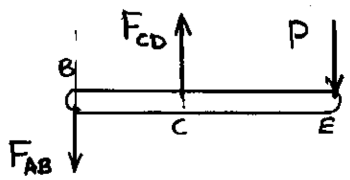
PROBLEM 1.52



1.51 Each of the steel links AB and CD is connected to a support and to member BCE by $\frac{1}{2}$ - in.- diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 24 ksi for the steel used in the pins and that the ultimate normal stress is 60 ksi for the steel used in the links, determine the allowable load **P** if an overall factor of safety of 3.2 is desired. (Note that the links are not reinforced around the pin holes.

1.52 An alternative design is being considered to support member BCE of Prob. 1.51 in which link CD will be replaced by two links, each of $\frac{1}{8} \times$ 1-in. cross section, causing the pins at C and D to be in double shear. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION



Use member BCE as free body

$\Sigma M_B = 0 \qquad 8F_{CD} - 20P = 0 \qquad P = \frac{2}{5}F_{CD}$

$\Sigma M_C = 0 \qquad 8F_{AB} - 12P = 0 \qquad P = \frac{2}{3}F_{AB}$

Based on pin A in single shear

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(\frac{1}{2})^2 = 0.19635 \text{ in}^2$

$F_U = \tau_U A = (24)(0.19635) = 4.7124$ kips

Based on tension in link AB

$A = (b-d)t = (1-\frac{1}{2})(\frac{1}{4}) = 0.125 \text{ in}^2$

$F_U = \sigma_U A = (60)(0.125) = 7.50$ kips

Ultimate load for link AB is smallest, i.e. $F_U = 4.7124$ kips

Corresponding ultimate load for structure: $P_U = \frac{2}{3}F_U = 3.1416$ kips

Based on pins at C and D in double shear

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(\frac{1}{2})^2 = 0.19635 \text{ in}^2$

$F_U = 2\tau_U A = (2)(24)(0.19635) = 9.4248$ kips

Based on tension in links BC

$A = (b-d)t = (1-\frac{1}{2})(\frac{1}{8}) = 0.0625 \text{ in}^2$ (one link)

$F_U = 2\sigma_U A = (2)(60)(0.0625) = 7.50$ kips (total, both links)

Ultimate load for links BC is smallest, i.e. $F_U = 7.50$ kips

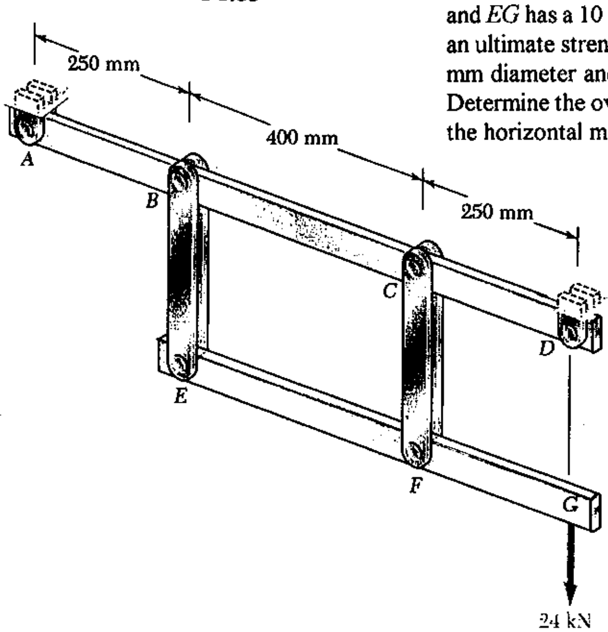
Corresponding ultimate load for structure $P_U = \frac{2}{5}F_U = 3.00$ kips.

Actual ulimate load is smallest, i.e. $P_U = 3.00$ kips

Allowable load for structure $P = \frac{P_U}{F.S.} = \frac{3.00}{3.2} = 0.938$ kip

$P = 938$ lb.

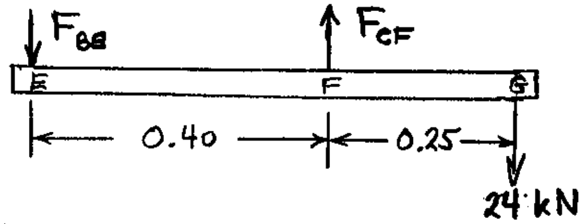
PROBLEM 1.53



1.53 Each of the two vertical links CF connecting the two horizontal members AD and EG has a 10×40- mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at C and F has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links CF and the pins connecting them to the horizontal members.

SOLUTION

Use member EFG as free body.



$\circlearrowleft \Sigma M_E = 0$

$0.40\, F_{CF} - (0.65)(24 \times 10^3) = 0$

$F_{CF} = 39 \times 10^3$ N

Based on tension in links CF

$A = (b - d)\,t = (0.040 - 0.02)(0.010) = 200 \times 10^{-6}\ m^2$ (one link)

$F_U = 2\sigma_U A = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160.0 \times 10^3$ N

Based on double shear in pins

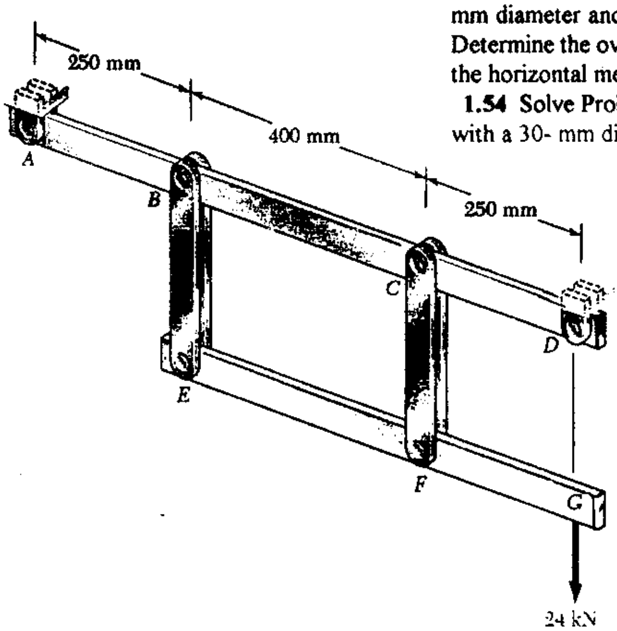
$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\ m^2$

$F_U = 2\tau_U A = (2)(150 \times 10^6)(314.16 \times 10^{-6}) = 94.248 \times 10^3$ N

Actual F_U is smaller value, i.e. $F_U = 94.248 \times 10^3$ N

Factor of safety $F.S. = \frac{F_U}{F_{CF}} = \frac{94.248 \times 10^3}{39 \times 10^3} = 2.42$ ◄

PROBLEM 1.54

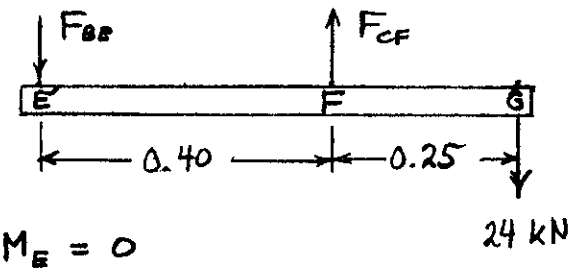


1.53 Each of the two vertical links *CF* connecting the two horizontal members *AD* and *EG* has a 10 × 40- mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at *C* and *F* has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links *CF* and the pins connecting them to the horizontal members.

1.54 Solve Prob. 1.53, assuming that the pins at *C* and *F* have been replaced by pins with a 30- mm diameter.

SOLUTION

Use member EFG as free body.



$$\circlearrowleft \Sigma M_E = 0$$

$$0.40\, F_{CF} - (0.65)(24 \times 10^3) = 0$$

$$F_{CF} = 39 \times 10^3 \text{ N}$$

Based on tension in links CF

$$A = (b - d)t = (0.040 - 0.030)(0.010) = 100 \times 10^{-6} \text{ m}^2 \quad (\text{one link})$$

$$F_U = 2\sigma_U A = (2)(400 \times 10^6)(100 \times 10^{-6}) = 80.0 \times 10^3 \text{ N}$$

Based on double shear in pins

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.030)^2 = 706.86 \times 10^{-6} \text{ m}^2$$

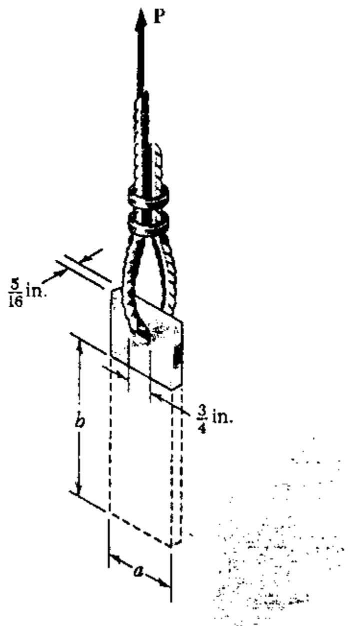
$$F_U = 2\tau_U A = (2)(150 \times 10^6)(706.86 \times 10^{-6}) = 212.06 \times 10^3 \text{ N}$$

Actual F_U is smaller value, i.e. $F_U = 80.0 \times 10^3$ N

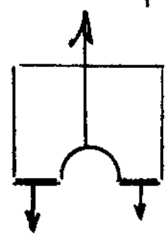
Factor of safety $\quad F.S. = \dfrac{F_U}{F_{CF}} = \dfrac{80.0 \times 10^3}{39 \times 10^3} = 2.05$

PROBLEM 1.55

1.55 A steel plate $\frac{5}{16}$ - in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ - in., the ultimate strength of the steel used is 36 ksi , and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when P = 2.5 kips, determine (a) the required width a of the plate, (b) the minimum depth b to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)



SOLUTION



Based on tension in plate

$$A = (a - d)\,t$$

$$P_U = \sigma_U A$$

$$F.S. = \frac{P_U}{P} = \frac{\sigma_U (a-d)\,t}{P}$$

Solving for b

$$a = d + \frac{(F.S.)P}{\sigma_U t} = \frac{3}{4} + \frac{(3.60)(2.5)}{(36)(\frac{5}{16})}$$

$$a = 1.550 \text{ in}$$

Based on shear between plate and concrete slab

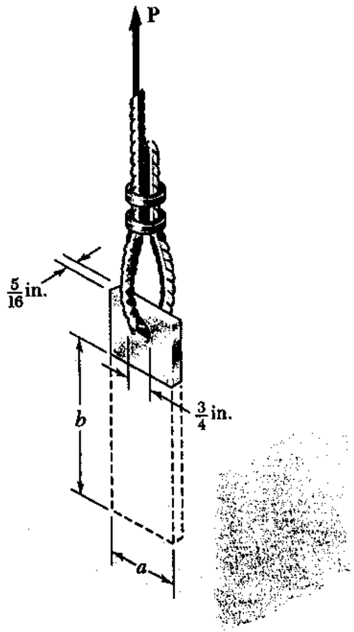
$$A = \text{perimeter} \times \text{depth} = 2\,(a + t)\,b \qquad \tau_U = 0.300 \text{ ksi}$$

$$P_U = \tau_U A = 2\tau_U (a + t)\,b \qquad F.S. = \frac{P_U}{P}$$

Solving for b $\quad b = \frac{(F.S.)P}{2(a+t)\tau_U} = \frac{(3.6)(2.5)}{(2)(1.550 + \frac{5}{16})(0.300)}$

$$b = 8.05 \text{ in}$$

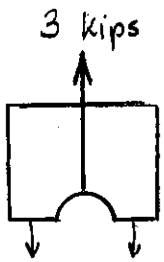
PROBLEM 1.56



1.55 A steel plate $\frac{5}{16}$ - in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$ - in., the ultimate strength of the steel used is 36 ksi , and the ultimate bonding stress between plate and concrete is 300 psi.

1.56 Determine the factor of safety for the cable anchor of Prob. 1.55 when $P = 3$ kips, knowing that $a = 2$ in. and $b = 7.5$ in.

SOLUTION



Based on tension in plate

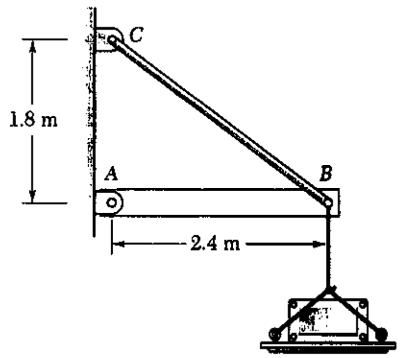
$$A = (a - d)t$$
$$= (2 - \tfrac{3}{4})(\tfrac{5}{16}) = 0.3906 \text{ in}^2$$
$$P_U = \sigma_U A$$
$$= (36)(0.3906) = 14.06 \text{ kips}$$
$$F.S. = \frac{P_U}{P} = \frac{14.06}{3} = 4.69$$

Based on shear between plate and concrete slab

$$A = \text{perimeter} \times \text{depth} = 2(a+t)b = 2(2+\tfrac{5}{16})(7.5)$$
$$A = 34.69 \text{ in}^2 \qquad \tau_U = 0.300 \text{ ksi}$$
$$P_U = \tau_U A = (0.300)(34.69) = 10.41 \text{ kips}$$
$$F.S. = \frac{P_U}{P} = \frac{10.41}{3} = 3.47$$

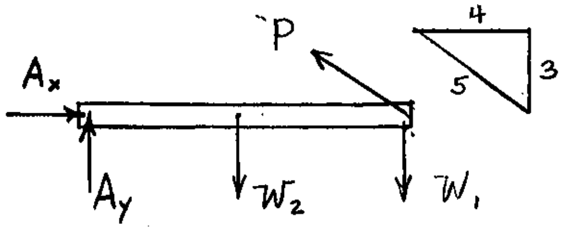
Actual factor of safety is the smaller value $\qquad F.S. = 3.47$

PROBLEM 1.57



***1.57** A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (*a*) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load which can be safely placed on the platform. (*b*) What is the corresponding conventional factor of safety for rod BC?

SOLUTION



$$+\circlearrowleft \Sigma M_A = 0 \qquad (2.4)\tfrac{3}{5}P - 2.4\,W_1 - 1.2\,W_2 \qquad \therefore\ P = \tfrac{5}{3}W_1 + \tfrac{5}{6}W_2$$

For dead loading $\quad W_1 = (40)(9.81) = 392.4 \text{ N}$

$$W_2 = (50)(9.81) = 490.5 \text{ N}$$

$$P_D = (\tfrac{5}{3})(392.4) + (\tfrac{5}{6})(490.5) = 1.0628 \times 10^3 \text{ N}$$

For live loading $\quad W_1 = mg \qquad W_2 = 0$

$$P_L = \tfrac{5}{3}mg \qquad \text{from which} \qquad m = \tfrac{3}{5}\frac{P_L}{g}$$

Design criterion

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

$$P_L = \frac{\phi P_U - \gamma_D P_D}{\gamma_L} = \frac{(0.90)(12\times10^3) - (1.25)(1.0628\times10^{-3})}{1.6}$$

$$= 5.920\times10^3 \text{ N}$$

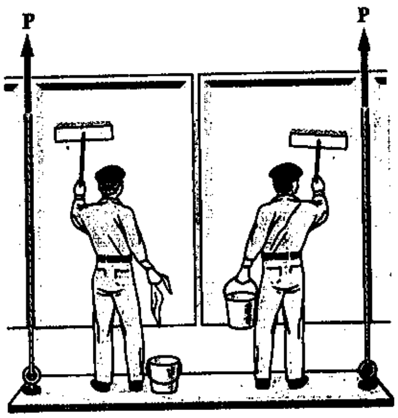
Allowable load $\quad m = \tfrac{3}{5}\,\dfrac{5.92\times10^3}{9.81} = 362 \text{ kg}$ ◄

Conventional factor safety

$$P = P_D + P_L = 1.0628\times10^3 + 5.920\times10^3 = 6.983\times10^3 \text{ N}$$

$$F.S. = \frac{P_U}{P} = \frac{12\times10^3}{6.983\times10^3} = 1.718$$ ◄

PROBLEM 1.58



***1.58** The Load and Resistance Factor Design method is to be used to select the two cables which will raise and lower a platform supporting two window washers. The platform weighs 160 lb and each of the window washers is assumed to weigh 195 lb with his equipment. Since these workers are free to move on the platform, 75% of their total weight and of the weight of their equipment will be used as the design live load of each cable.. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the conventional factor of safety for the selected cables?

SOLUTION

$$\gamma_D P_D + \gamma_L P_L = \varphi P_U$$

$$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\varphi}$$

$$= \frac{(1.2)(\frac{1}{2} \times 160) + (1.5)(\frac{3}{4} \times 2 \times 195)}{0.85}$$

$$= 629 \text{ lb.}$$

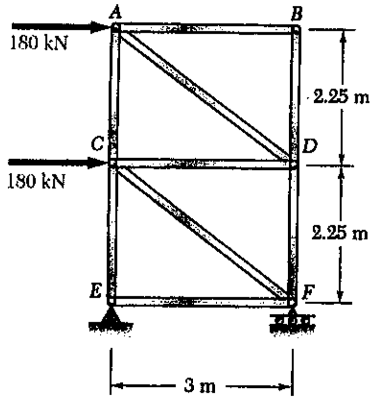
Conventional factor of safety

$$P = P_D + P_L = \tfrac{1}{2} \times 80 + 0.75 \times 2 \times 195 = 372.5 \text{ lb}$$

$$\text{F.S.} = \frac{P_U}{P} = \frac{629}{372.5} = 1.689$$

PROBLEM 1.59

1.59 For the truss and loading shown, determine the average normal stress in member *DF*, knowing that the cross-sectional area of that member is 2500 mm^2.

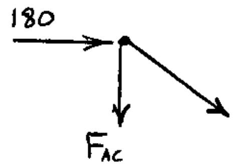


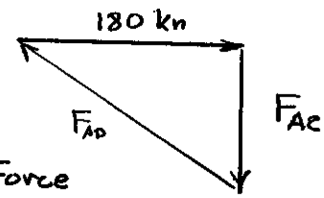
SOLUTION

Using method of joints to find member forces

Joint B : AB and BD are zero force members.

Joint A : $\ell_{AD} = \sqrt{3^2 + 2.25^2} = 3.75$ m





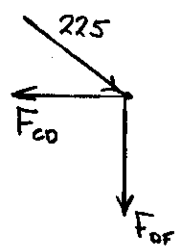
Force Trangle

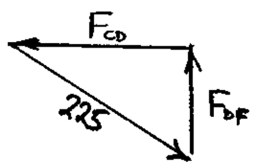
By similar triangles

$$\frac{F_{AD}}{3.75} = \frac{180}{3} \quad \therefore \quad F_{AD} = 225 \text{ lb.}$$

(compression)

Joint D





By similar triangles

$$\frac{F_{DE}}{2.25} = \frac{225}{3.75}$$

$$F_{DF} = 135 \text{ kN (comp)}$$

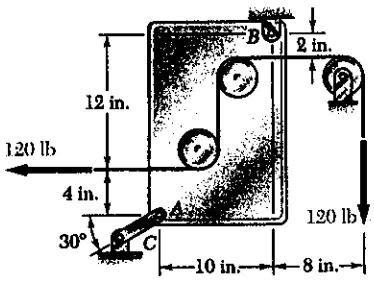
$$= 135 \times 10^3 \text{ N}$$

Area : $A_{DE} = 2500 \text{ mm}^2 = 2500 \times 10^{-6} \text{ m}^2$

Stress : $\sigma_{DF} = -\dfrac{135 \times 10^3}{2500 \times 10^{-6}} = -54 \times 10^6 \text{ Pa} = -54.0 \text{ MPa}$ ◄

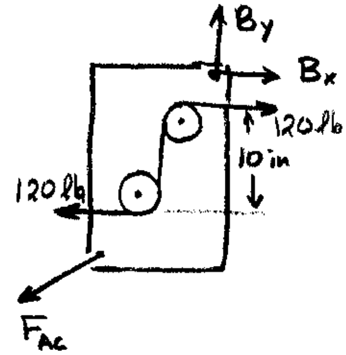
PROBLEM 1.60

1.60 Link AC has a uniform rectangular cross section $\frac{1}{8}$ in. thick and 1 in. wide. Determine the normal stress in the central portion of that link.



SOLUTION

Use the plate together with two pulleys as a free body. Note that the cable tension causes at 1200 lb-in clockwise couple to act on the body.



$$\circlearrowright \Sigma M_B = 0$$

$$-(12+4)(F_{AC}\cos 30^\circ) + (10)(F_{AC}\sin 30^\circ) - 1200 = 0$$

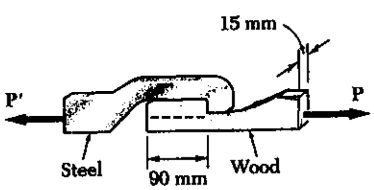
$$F_{AC} = -\frac{1200}{16\cos 30^\circ - 10\sin 30^\circ} = -135.50 \text{ lb.}$$

Area of link AC: $A = 1 \text{ in} \times \frac{1}{8} \text{ in} = 0.125 \text{ in}^2$

Stress in link AC: $\sigma_{AC} = \frac{F_{AC}}{A} = -\frac{135.50}{0.125} = 1084 \text{ psi} = 1.084 \text{ ksi}$ ◄

PROBLEM 1.61

1.61 When the force **P** reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.



Area being sheared

$A = 90 \text{ mm} \times 15 \text{ mm} = 1350 \text{ mm}^2 = 1350 \times 10^{-6} \text{ m}^2$

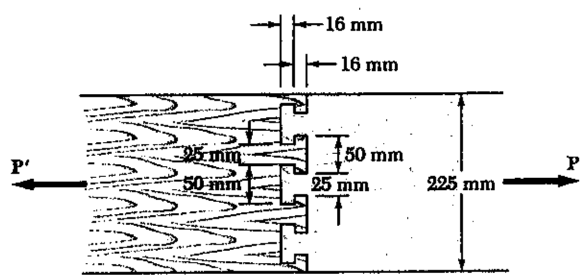
Force $P = 8 \times 10^3$ N

Shearing stress $\tau = \frac{P}{A} = \frac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^3 \text{ Pa} = 5.93 \text{ MPa}$ ◄

PROBLEM 1.62

1.62 Two wooden planks, each 12 mm thick and 225 mm wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 8 MPa, determine the magnitude P of the axial load which will cause the joint to fail.

SOLUTION



Six areas, each 16 mm × 12 mm are being sheared.

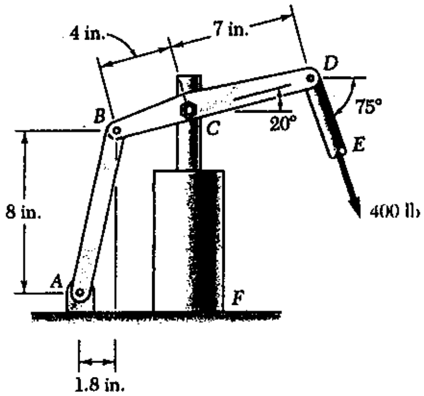
Area: $A = (6)(16)(12) = 1152 \text{ mm}^2$

$= 1152 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{P}{A}$$

$$P = \tau A = (8 \times 10^6)(1152 \times 10^{-6}) = 9.22 \times 10^3 \text{ N} = 9.22 \text{ kN}$$ ◄

PROBLEM 1.63

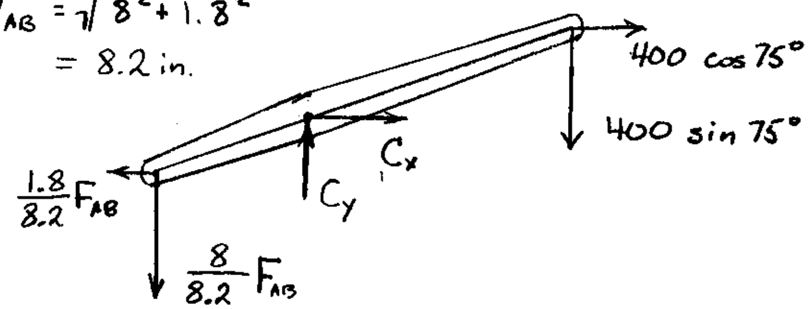


1.63 The hydraulic cylinder *CF*, which partially controls the position of rod *DE*, has been locked in the position shown. Member *BD* is $\frac{5}{8}$ in. thick and is connected to the vertical rod by a $\frac{3}{8}$ - in.- diameter bolt. Determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at *C* in member *BD*.

SOLUTION

Use member BCD as a free body, and note that AB is a two force member.

$$\ell_{AB} = \sqrt{8^2 + 1.8^2}$$
$$= 8.2 \text{ in.}$$



$$\circlearrowleft \Sigma M_C = 0 \qquad (4\cos 20°)\left(\tfrac{8}{8.2} F_{AB}\right) - (4\sin 20°)\left(\tfrac{1.8}{8.2} F_{AB}\right)$$
$$- (7\cos 20°)(400 \sin 75°) - (7\sin 20°)(400\cos 75°) = 0$$

$$3.36678\ F_{AB} - 2789.35 = 0 \qquad \therefore\ F_{AB} = 828.49 \text{ lb.}$$

$$\xrightarrow{+} \Sigma F_x = 0 \qquad -\tfrac{1.8}{8.2} F_{AB} + C_x + 400\cos 75° = 0$$

$$C_x = \frac{(1.8)(828.49)}{8.2} - 400\cos 75° = 78.34 \text{ lb.}$$

$$+\uparrow \Sigma F_y = 0 \qquad -\tfrac{8}{8.2} F_{AB} + C_y - 400\sin 75° = 0$$

$$C_y = \frac{(8)(828.49)}{8.2} + 400\sin 75° = 1194.65 \text{ lb.}$$

$$C = \sqrt{C_x^2 + C_y^2} = 1197.2 \text{ lb.}$$

(a) Shearing stress in the bolt: $\qquad P = 1197.2$ lb.

$$A = \tfrac{\pi}{4} d^2 = \tfrac{\pi}{4}\left(\tfrac{3}{8}\right)^2 = 0.11045 \text{ in}^2$$

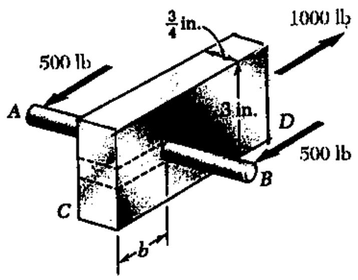
$$\tau = \frac{P}{A} = \frac{1197.2}{0.11045} = 10.84 \times 10^3 \text{ psi} = 10.84 \text{ ksi} \quad ◀$$

(b) Bearing stress at C in member BCD: $\qquad P = 1197.2$ lb.

$$A_b = dt = \left(\tfrac{3}{8}\right)\left(\tfrac{5}{8}\right) = 0.234375 \text{ in}^2$$

$$\sigma_b = \frac{P}{A_b} = \frac{1197.2}{0.234375} = 5.11 \times 10^3 \text{ psi} = 5.11 \text{ ksi} \quad ◀$$

PROBLEM 1.64



1.64 A $\frac{1}{2}$ - in.- diameter steel rod AB is fitted to a round hole near end C of the wooden member CD. For the loading shown, determine (a) the maximum average normal stress in the wood, (b) the distance b for which the average shearing stress is 90 psi on the surfaces indicated by the dashed lines, (c) the average bearing stress on the wood.

SOLUTION

(a) Maximum normal stress in the wood

$$A_{net} = \frac{3}{4}\left(3 - \frac{1}{2}\right) = 1.875 \text{ in}^2$$

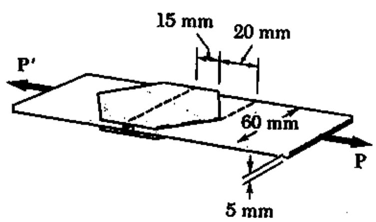
$$\sigma = \frac{P}{A_{net}} = \frac{1000}{1.875} = 533 \text{ psi}$$ ◀

(b) $$\tau = \frac{P}{A} = \frac{P}{2bt}$$

$$b = \frac{P}{2t\tau} = \frac{1000}{(2)(\frac{3}{4})(90)} = 7.41 \text{ in.}$$ ◀

(c) $$\sigma_b = \frac{P}{A_b} = \frac{P}{dt} = \frac{1000}{(\frac{1}{2})(\frac{3}{4})} = 2667 \text{ psi}$$ ◀

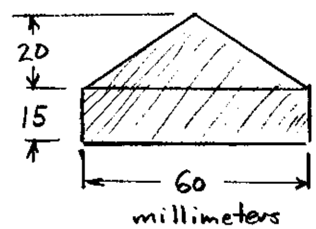
PROBLEM 1.65



1.65 Two plates, each 3-mm thick, are used to splice a plastic strip as shown. Knowing that the ultimate shearing stress of the bonding between the surfaces is 900 kPa, determine the factor of safety with respect to shear when P = 1500 N.

SOLUTION

Bond area: (See figure)

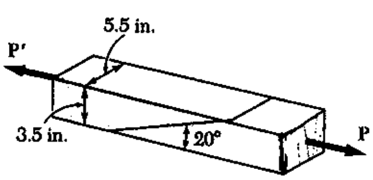


$$A = \frac{1}{2}(60)(20) + (15)(60)$$
$$= 1500 \text{ mm}^2 = 1500 \times 10^{-6} \text{ m}^2$$

$$P_U = 2A\tau_U = (2)(1500 \times 10^{-6})(900 \times 10^3) = 2700 \text{ N}$$

$$F.S. = \frac{P_U}{P} = \frac{2700}{1500} = 1.800$$ ◀

PROBLEM 1.66



1.66 Two wooden members of 3.5 × 5.5-in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 75 psi, determine the largest axial load **P** which can be safely applied.

SOLUTION

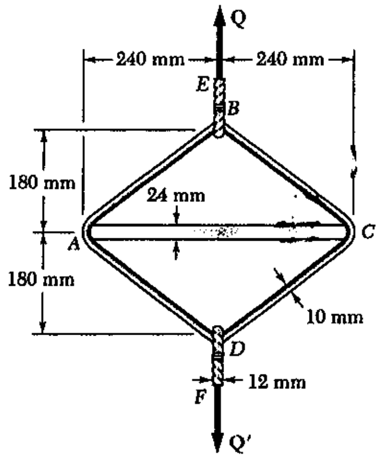
$$A_o = (3.5)(5.5) = 19.25 \text{ in}^2$$

$$\theta = 90° - 20° = 70°$$

$$\tau = \frac{P}{A_o}\sin\theta\cos\theta = \frac{P}{2A_o}\sin 2\theta$$

$$P = \frac{2A_o\tau}{\sin 2\theta} = \frac{(2)(19.25)(75)}{\sin 140°} = 4492 \text{ lb} = 4.49 \text{ kips}$$ ◀

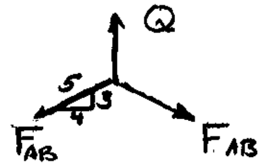
PROBLEM 1.67



1.67 A steel loop *ABCD* of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod *AC*. Cables *BE* and *DF*, each of 12-mm diameter, are used to apply the load **Q**. Knowing that the ultimate strength of the aluminum used for the rod is 260 MPa and that the ultimate strength of the steel used for the loop and the cables is 480 MPa, determine the largest load **Q** which can be applied if an overall factor of safety of 3 is desired.

SOLUTION

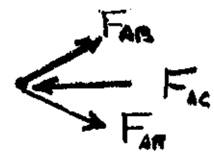
Using joint B as free body and considering symmetry



$$2 \cdot \frac{3}{5} F_{AB} - Q = 0$$

$$Q = \frac{6}{5} F_{AB}$$

Using joint A as a free body and considering symmetry



$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$

Based on strength of cable BE

$$Q_U = \sigma_U A = \sigma_U \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{ N}$$

Based on strength of steel loop

$$Q_U = \frac{6}{5} F_{AB,U} = \frac{6}{5} \sigma_U A = \frac{6}{5} \sigma_U \frac{\pi}{4} d^2$$

$$= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{ N}$$

Based on strength of rod AC

$$Q_U = \frac{3}{4} F_{AC,U} = \frac{3}{4} \sigma_U A = \frac{3}{4} \sigma_U \frac{\pi}{4} d^2$$

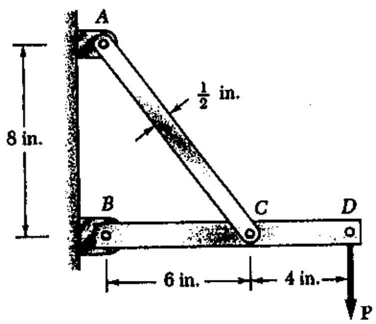
$$= \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{ N}$$

Actual ultimate load Q_U is the smallest $\therefore$ $Q_U = 45.24 \times 10^3$ N

Allowable load $Q = \frac{Q_U}{F.S.} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3$ N

$= 15.08$ kN ◀

PROBLEM 1.68

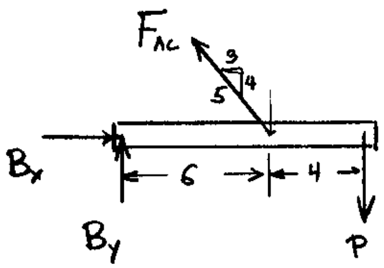


1.68 Link AC has a uniform $\frac{1}{4} \times \frac{1}{2}$- in. uniform rectangular cross section and is made of a steel with a 60-ksi ultimate normal stress. It is connected to a support at A and to member BCD at C by $\frac{3}{8}$-in.-diameter pins, while member BCD is connected to a support at B by a $\frac{5}{16}$-in.-diameter pin. All of the pins are in single shear and are made of a steel with a 25-ksi ultimate shearing stress. Knowing that an overall factor of safety of 3.25 is desired, determine the largest load **P** which can be safely applied at D. Note that link AC is not reinforced around the pin holes.

SOLUTION

$\circlearrowleft \Sigma M_B = 0 \qquad (6)(\frac{3}{5}F_{AC}) - 10\,P = 0$

$F_{AC} = 2.0833\,P \qquad P = 0.480\,F_{AC}$



$\xrightarrow{+} \Sigma F_x = 0 \qquad B_x - \frac{3}{5}F_{AC} = 0$

$B_x = \frac{3}{5}F_{AC} = (\frac{3}{5})(2.0833\,P) = 1.25\,P$

$+\uparrow \Sigma F_y = 0 \qquad B_y + \frac{4}{5}F_{AC} - P = 0$

$B_y = P - \frac{4}{5}(2.0833\,P) = -0.66667\,P$

$B = \sqrt{B_x^2 + B_y^2} = 1.41667\,P, \qquad P = 0.70588\,B$

Based on strength of link AC: $\quad \sigma_U = 60$ ksi

$A_{net} = (\frac{1}{4})(\frac{1}{2} - \frac{3}{8}) = 0.03125\ in^2; \quad F_{AC,U} = \sigma_U A_{net} = (60)(0.03125) = 1.875$ kip

$P_U = (0.480)(1.875) = 0.900$ kip.

Based on strength of pin at C: $\quad A_{pin} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(\frac{3}{8})^2 = 0.11045\ in^2$

$\tau_U = 25$ ksi $\qquad F_{AC,U} = \tau_U A_{pin} = (25)(0.11045) = 2.761$ kip.

$P_U = (0.480)(2.761) = 1.325$ kip.

Based on strength of pin at B: $\quad A_{pin} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(\frac{5}{16})^2 = 0.07670\ in^2$

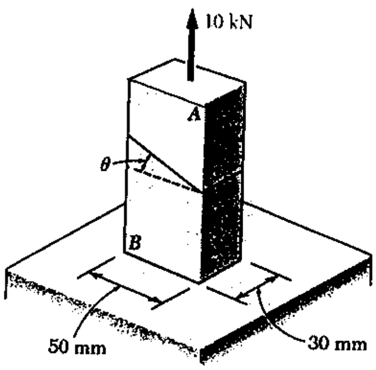
$B_U = \tau_U A_{pin} = (25)(0.07670) = 1.9175$ kip.

$P_U = (0.70588)(1.9175) = 1.3535$ kip

Actual P_U is the smallest $\therefore \quad P_U = 0.900$ kip.

Allowable value for P: $\quad P = \frac{P_U}{FS} = \frac{0.900}{3.25} = 0.277$ kip $= 277$ lb. ◄

PROBLEM 1.69



1.69 The two portions of member *AB* are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine the range of values of θ for which the factor of safety of the member is at least 3.0.

SOLUTION

$A_o = (0.030)(0.050) = 1.50 \times 10^{-3}\ \text{m}^2$

$P = 10 \times 10^3\ \text{N} \qquad P_U = (F.S)P = 30 \times 10^3\ \text{N}$

Based on tensile stress

$$\sigma_U = \frac{P_U}{A_o}\cos^2\theta$$

$$\cos^2\theta = \frac{\sigma_U A_o}{P_U} = \frac{(17 \times 10^6)(1.50 \times 10^{-3})}{30 \times 10^3} = 0.85$$

$\cos\theta = 0.92195 \qquad \theta = 22.79° \qquad \theta \geq 22.79°$

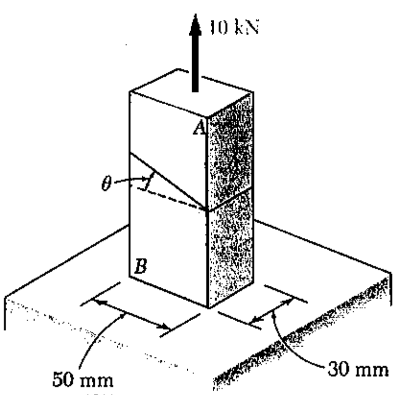
Based on shearing stress $\quad \tau_U = \frac{P_U}{A_o}\sin\theta\cos\theta = \frac{P_U}{2A_o}\sin 2\theta$

$$\sin 2\theta = \frac{2A_o\tau_U}{P_U} = \frac{(2)(1.50 \times 10^{-3})(9 \times 10^6)}{30 \times 10^3} = 0.900$$

$2\theta = 64.16° \qquad \theta = 32.08° \qquad \theta \leq 32.08°$

Hence $\quad 22.79° \leq \theta \leq 32.08°$ ◄

PROBLEM 1.70



1.70 The two portions of member *AB* are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine (*a*) the value of θ for which the factor of safety of the member is maximum, (*b*) the corresponding value of the factor of safety. (*Hint:* Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

SOLUTION

$A_o = (0.030)(0.050) = 1.50 \times 10^{-3}\ \text{m}^2$

At the optimum angle $\quad (F.S.)_\sigma = (F.S.)_\tau$

Normal stress: $\sigma = \frac{P}{A_o}\cos^2\theta \quad \therefore \quad P_{U,\sigma} = \frac{\sigma_U A_o}{\cos^2\theta}$

$$(F.S.)_\sigma = \frac{P_{U,\sigma}}{P} = \frac{\sigma_U A_o}{P\cos^2\theta}$$

Shearing stress: $\tau = \frac{P}{A_o}\sin\theta\cos\theta \quad \therefore \quad P_{U,\tau} = \frac{\tau_U A_o}{\sin\theta\cos\theta}$

$$(F.S.)_\tau = \frac{P_{U,\tau}}{P} = \frac{\tau_U A_o}{P\sin\theta\cos\theta}$$

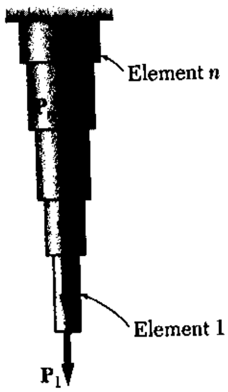
Equating: $\frac{\sigma_U A_o}{P\cos^2\theta} = \frac{\tau_U A_o}{P\sin\theta\cos\theta}$;

Solving: $\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{\tau_U}{\sigma_U} = \frac{9}{17} = 0.52941$; $\qquad \theta_{opt} = 27.9°$ ◄

Then $\quad P_U = \frac{\sigma_U A_o}{\cos^2\theta} = \frac{(17 \times 10^6)(1.50 \times 10^{-3})}{\cos^2 27.9°} = 32.65 \times 10^3$;

$$F.S. = \frac{P_U}{P} = \frac{32.65 \times 10^6}{10 \times 10^3} = 3.26$$ ◄

PROBLEM 1.C1



1.C1 A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by $\mathbf{P}_i$, with the magnitude P_i of this load being assumed positive if $\mathbf{P}_i$ is directed downward as shown and negative otherwise. (*a*) Write a computer program which can be used with either SI or U.S. customary units to determine the average stress in each element of the rod. (*b*) Use this program to solve Probs. 1.1 and 1.3.

SOLUTION

FORCE IN ELEMENT i:

It is the sum of the forces applied to that element and all lower ones:

$$F_i = \sum_{k=1}^{i} P_k$$

AVERAGE STRESS IN ELEMENT i:

Area $= A_i = \frac{1}{4}\pi d_i^2$ Ave stress $= \dfrac{F_i}{A_i}$

PROGRAM OUTPUTS

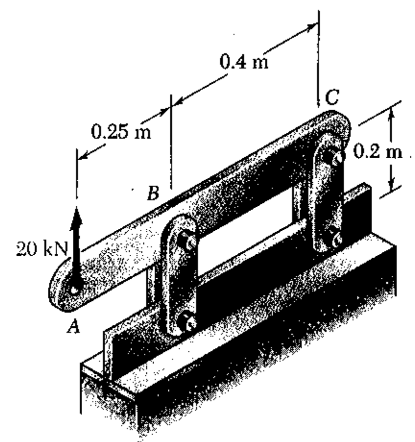
Problem 1.1

Element	Stress (MPa)
1	84.883
2	-96.766

Problem 1.3

Element	Stress (ksi)
1	22.635
2	17.927

PROBLEM 1.C2

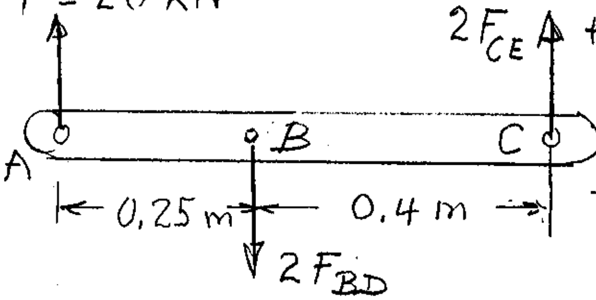


1.C2 A 20-kN force is applied as shown to the horizontal member ABC. Member ABC has a 10×50-mm uniform rectangular cross section and is supported by four vertical links, each of 8×36-mm uniform rectangular cross section. Each of the four pins at A, B, C, and D has the same diameter d and is in double shear. (*a*) Write a computer program to calculate for values of d from 10 to 30 mm, using 1-mm increments, (1) the maximum value of the average normal stress in the links connecting pins B and D, (2) the average normal stress in the links connecting pins C and E, (3) the average shearing stress in pin B, (4) the average shearing stress in pin C, (5) the average bearing stress at B in member ABC, (6) the average bearing stress at C in member ABC. (*b*) Check your program by comparing the values obtained for $d = 16$ mm with the answers given for Probs. 1.8, 1.23, and 1.24. (*c*) Use this program to find the permissible values of the diameter d of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve part *c*, assuming that the thickness of member ABC has been reduced from 10 to 8 mm.

SOLUTION

FORCES IN LINKS

$P = 20$ kN



F. B. DIAGRAM OF ABC:

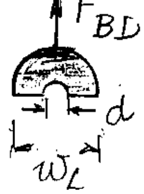
$+\circlearrowleft \Sigma M_C = 0: \; 2F_{BD}(BC) - P(AC) = 0$

$F_{BD} = P(AC)/2(BC)$ (TENSION)

$+\circlearrowleft \Sigma M_B = 0: \; 2F_{CE}(BC) - P(AB) = 0$

$F_{CE} = P(AB)/2(BC)$ (COMP.)

(1) LINK BD

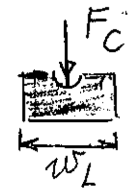


Thickness $= t_L$

$A_{BD} = t_L(w_L - d)$

$\sigma_{BD} = +F_{BD}/A_{BD}$

(2) LINK CE



Thickness $= t_L$

$A_{CE} = t_L w_L$

$\sigma_{CE} = -F_{CE}/A_{CE}$

(3) PIN B

$\tau_B = F_{BD}/(\pi d^2/4)$

(4) PIN C

$\tau_C = F_{CE}/(\pi d^2/4)$

(5) BEARING STRESS AT B

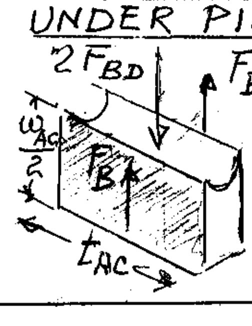
Thickness of member $AC = t_{AC}$

Sig Bear $B = F_{BD}/(d\, t_{AC})$

(6) BEARING STRESS AT C

Sig Bear $C = F_{CE}/(d\, t_{AC})$

SHEARING STRESS IN ABC UNDER PIN B



$F_B = \tau_{AC}\, t_{AC}\, (w_{AC}/2)$

$\Sigma F_y = 0: \; 2F_B = 2F_{BD}$

$\tau_{AC} = \dfrac{2F_{BD}}{t_{AC}\, w_{AC}}$

(CONTINUED)

PROBLEM 1.C2 CONTINUED **PROGRAM OUTPUTS**

INPUT DATA FOR PARTS (a), (b), (c): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 10 mm, WAC = 50 mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	~~206.90~~	79.58	~~325.00~~	125.00
11.00	81.25	-21.70	~~170.99~~	65.77	~~295.45~~	113.64
12.00	84.64	-21.70	~~143.68~~	55.26	~~270.83~~	104.17
13.00	88.32	-21.70	~~122.43~~	47.09	~~250.00~~	96.15
14.00	92.33	-21.70	~~105.56~~	40.60	~~232.14~~	89.29
15.00	96.73	-21.70	~~91.96~~	35.37	216.67	83.33
16.00	101.56	-21.70	80.82	31.08	203.12	78.13 ← (b)
17.00	106.91	-21.70	71.59	27.54	191.18	73.53
18.00	112.85	-21.70	63.86	24.56	180.56	69.44
19.00	119.49	-21.70	57.31	22.04	171.05	65.79
20.00	126.95	-21.70	51.73	19.89	162.50	62.50
21.00	135.42	-21.70	46.92	18.04	154.76	59.52
22.00	145.09	-21.70	42.75	16.44	147.73	56.82
23.00	~~156.25~~	-21.70	39.11	15.04	141.30	54.35
24.00	~~169.27~~	-21.70	35.92	13.82	135.42	52.08
25.00	~~184.66~~	-21.70	33.10	12.73	130.00	50.00
26.00	~~203.13~~	-21.70	30.61	11.77	125.00	48.08
27.00	~~225.69~~	-21.70	28.38	10.92	120.37	46.30
28.00	~~253.91~~	-21.70	26.39	10.15	116.07	44.64
29.00	~~290.18~~	-21.70	24.60	9.46	112.07	43.10
30.00	~~338.54~~	-21.70	22.99	8.84	108.33	41.67

(c) ANSWER : 16 mm ≤ d ≤ 22 mm ← (c)

CHECK: For d = 22 mm, Tau AC = 65 MPa < 90 MPa O.K.

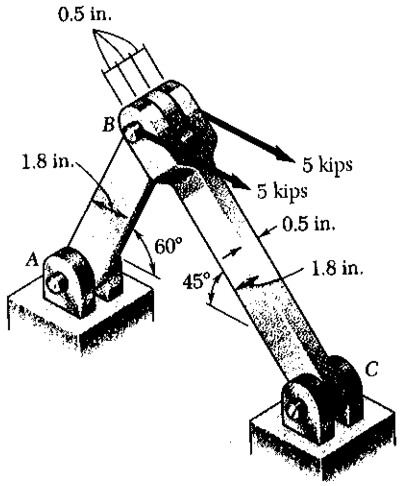
INPUT DATA FOR PART (d): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 8 mm, WAC = 50 mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.13	-21.70	~~206.90~~	79.58	~~406.25~~	156.25
11.00	81.25	-21.70	~~170.99~~	65.77	~~369.32~~	142.05
12.00	84.64	-21.70	~~143.68~~	55.26	~~338.54~~	130.21
13.00	88.32	-21.70	~~122.43~~	47.09	~~312.50~~	120.19
14.00	92.33	-21.70	~~105.56~~	40.60	~~290.18~~	111.61
15.00	96.73	-21.70	~~91.96~~	35.37	~~270.83~~	104.17
16.00	101.56	-21.70	80.82	31.08	~~253.91~~	97.66
17.00	106.91	-21.70	71.59	27.54	~~238.97~~	91.91
18.00	112.85	-21.70	63.86	24.56	225.69	86.81
19.00	119.49	-21.70	57.31	22.04	213.82	82.24
20.00	126.95	-21.70	51.73	19.89	203.12	78.13
21.00	135.42	-21.70	46.92	18.04	193.45	74.40
22.00	145.09	-21.70	42.75	16.44	184.66	71.02
23.00	~~156.25~~	-21.70	39.11	15.04	176.63	67.93
24.00	~~169.27~~	-21.70	35.92	13.82	169.27	65.10
25.00	~~184.66~~	-21.70	33.10	12.73	162.50	62.50
26.00	~~203.13~~	-21.70	30.61	11.77	156.25	60.10
27.00	~~225.69~~	-21.70	28.38	10.92	150.46	57.87
28.00	~~253.91~~	-21.70	26.39	10.15	145.09	55.80
29.00	~~290.18~~	-21.70	24.60	9.46	140.09	53.88
30.00	~~338.54~~	-21.70	22.99	8.84	135.42	52.08

(d) ANSWER : 18 mm ≤ d ≤ 22 mm ← (d)

CHECK: For d = 22 mm, Tau AC = 81.25 MPa < 90 MPa O.K.

PROBLEM 1.C3

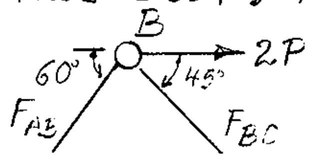


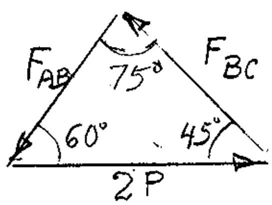
1.C3 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Each of the three pins at A, B, and C has the same diameter d and is in double shear. (a) Write a computer program to calculate for values of d from 0.50 to 1.50 in., using 0.05-in. increments, (1) the maximum value of the average normal stress in member AB, (2) the average normal stress in member BC, (3) the average shearing stress in pin A, (4) the average shearing stress in pin C, (5) the average bearing stress at A in member AB, (6) the average bearing stress at C in member BC, (7) the average bearing stress at B in member BC. (b) Check your program by comparing the values obtained for $d = 0.8$ in. with the answers given for Probs. 1.9, 1.25, and 1.26. (c) Use this program to find the permissible values of the diameter d of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 22 ksi, 13 ksi, and 36 ksi. (d) Solve part c, assuming that a new design is being investigated, in which the thickness and width of the two members are changed, respectively, from 0.5 to 0.3 in. and from 1.8 in. to 2.4 in.

SOLUTION

FORCES IN MEMBERS AB AND BC

FREE BODY: PIN B





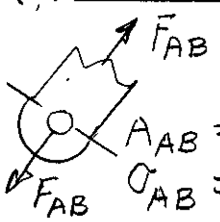
FROM FORCE TRIANGLE:

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{2P}{\sin 75°}$$

$$F_{AB} = 2P(\sin 45°/\sin 75°)$$

$$F_{BC} = 2P(\sin 60°/\sin 75°)$$

(1) MAX. AVE. STRESS IN AB



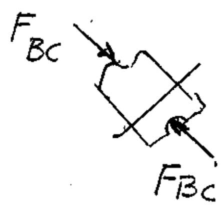
Width $= w$

Thickness $= t$

$A_{AB} = (w - d)t$

$\sigma_{AB} = F_{AB}/A_{AB}$

(2) AVE. STRESS IN BC



$A_{BC} = wt$

$\sigma_{BC} = F_{BC}/A_{BC}$

(3) PIN A

$\tau_A = (F_{AB}/2)/(\pi d^2/4)$

(4) PIN C

$\tau_C = (F_{BC}/2)/(\pi d^2/4)$

(5) BEARING STRESS AT A

Sig Bear A $= F_{AB}/dt$

(6) BEARING STRESS AT C

Sig Bear C $= F_{BC}/dt$

(7) BEARING STRESS AT B IN MEMBER BC

Sig Bear B $= F_{BC}/2dt$

(CONTINUED)

PROBLEM 1.C3 CONTINUED **PROGRAM OUTPUTS**

INPUT DATA FOR PARTS (a), (b), (c): P = 5 kips, w = 1.8 in., t = 0.5 in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	11.262	-9.962	18.642	22.831	29.282	35.863	17.932
0.550	11.713	-9.962	15.406	18.869	26.620	32.603	16.301
0.600	12.201	-9.962	12.945	15.855	24.402	29.886	14.943
0.650	12.731	-9.962	11.030	13.510	22.525	27.587	13.793
0.700	13.310	-9.962	9.511	11.649	20.916	25.616	12.808
0.750	13.944	-9.962	8.285	10.147	19.521	23.909	11.954
0.800	14.641	-9.962	7.282	8.918	18.301	22.414	11.207
0.850	15.412	-9.962	6.450	7.900	17.225	21.096	10.548
0.900	16.268	-9.962	5.754	7.047	16.268	19.924	9.962
0.950	17.225	-9.962	5.164	6.324	15.412	18.875	9.438
1.000	18.301	-9.962	4.660	5.708	14.641	17.932	8.966
1.050	19.521	-9.962	4.227	5.177	13.944	17.078	8.539
1.100	20.916	-9.962	3.852	4.717	13.310	16.301	8.151
1.150	22.525	-9.962	3.524	4.316	12.731	15.593	7.796
1.200	24.402	-9.962	3.236	3.964	12.201	14.943	7.471
1.250	26.620	-9.962	2.983	3.653	11.713	14.345	7.173
1.300	29.282	-9.962	2.758	3.377	11.262	13.793	6.897
1.350	32.536	-9.962	2.557	3.132	10.845	13.283	6.641
1.400	36.603	-9.962	2.378	2.912	10.458	12.808	6.404
1.450	41.831	-9.962	2.217	2.715	10.097	12.367	6.183
1.500	48.803	-9.962	2.071	2.537	9.761	11.954	5.977

(b)

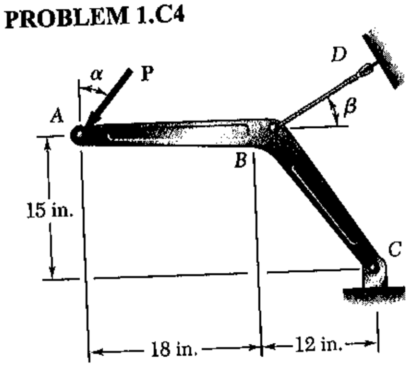
(c) ANSWER: 0.70 in. ≤ d ≤ 1.10 in. (c)

INPUT DATA FOR PART (d): P = 5 kips, w = 2.4 in., t = 0.3 in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	12.843	-12.452	18.642	22.831	48.803	59.772	29.886
0.550	13.190	-12.452	15.406	18.869	44.367	54.338	27.169
0.600	13.556	-12.452	12.945	15.855	40.669	49.810	24.905
0.650	13.944	-12.452	11.030	13.510	37.541	45.978	22.989
0.700	14.354	-12.452	9.511	11.649	34.860	42.694	21.347
0.750	14.789	-12.452	8.285	10.147	32.536	39.848	19.924
0.800	15.251	-12.452	7.282	8.918	30.502	37.357	18.679
0.850	15.743	-12.452	6.450	7.900	28.708	35.160	17.580
0.900	16.268	-12.452	5.754	7.047	27.113	33.206	16.603
0.950	16.829	-12.452	5.164	6.324	25.686	31.459	15.729
1.000	17.430	-12.452	4.660	5.708	24.402	29.886	14.943
1.050	18.075	-12.452	4.227	5.177	23.240	28.463	14.231
1.100	18.771	-12.452	3.852	4.717	22.183	27.169	13.584
1.150	19.521	-12.452	3.524	4.316	21.219	25.988	12.994
1.200	20.335	-12.452	3.236	3.964	20.335	24.905	12.452
1.250	21.219	-12.452	2.983	3.653	19.521	23.909	11.954
1.300	22.183	-12.452	2.758	3.377	18.771	22.989	11.495
1.350	23.240	-12.452	2.557	3.132	18.075	22.138	11.069
1.400	24.402	-12.452	2.378	2.912	17.430	21.347	10.674
1.450	25.686	-12.452	2.217	2.715	16.829	20.611	10.305
1.500	27.113	-12.452	2.071	2.537	16.268	19.924	9.962

(d) ANSWER: 0.85 in. ≤ d ≤ 1.25 in. (d)

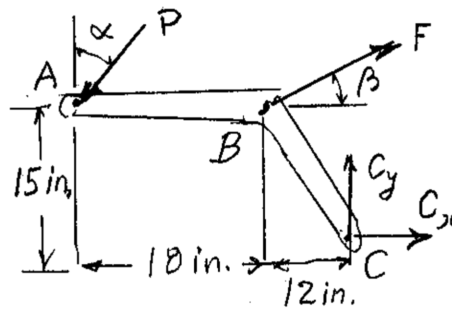
PROBLEM 1.C4



1.C4 A 4-kip force **P** forming an angle α with the vertical is applied as shown to member *ABC*, which is supported by a pin and bracket at *C* and by a cable *BD* forming an angle β with the horizontal. (*a*) Knowing that the ultimate load of the cable is 25 kips, write a computer program to construct a table of the values of the factor of safety of the cable for values of α and β from 0 to 45°, using increments in α and β corresponding to 0.1 increments in $\tan\alpha$ and $\tan\beta$. (*b*) Check that for any given value of α the maximum value of the factor of safety is obtained for $\beta = 38.66°$ and explain why. (*c*) Determine the smallest possible value of the factor of safety for $\beta = 38.66°$, as well as the corresponding value of α, and explain the result obtained.

SOLUTION

(a) DRAW F. B. DIAGRAM OF ABC:



$$+\circlearrowleft \sum M_C = 0: (P\sin\alpha)(15\text{ in.}) + (P\cos\alpha)(30\text{ in.}) - (F\cos\beta)(15\text{ in.}) - (F\sin\beta)(12\text{ in.}) = 0$$

$$F = P\,\frac{15\sin\alpha + 30\cos\alpha}{15\cos\beta + 12\sin\beta}$$

$$F.S. = F_{ult}/F$$

OUTPUT FOR $P = 4$ kips AND $F_{ult} = 20$ kips

VALUES OF FS

ALPHA \ BETA	0	5.71	11.31	16.70	21.80	26.56	30.96	34.99	38.66	41.99	45.00
0.000	3.125	3.358	3.555	3.712	3.830	3.913	3.966	3.994	4.002	3.995	3.977
5.711	2.991	3.214	3.402	3.552	3.666	3.745	3.796	3.823	3.830	3.824	3.807
11.310	2.897	3.113	3.295	3.441	3.551	3.628	3.677	3.703	3.710	3.704	3.687
16.699	2.837	3.049	3.227	3.370	3.477	3.553	3.600	3.626	3.633	3.627	3.611
21.801	2.805	3.014	3.190	3.331	3.438	3.512	3.560	3.585	3.592	3.586	3.570
26.565	2.795	3.004	3.179	3.320	3.426	3.500	3.547	3.572	3.579	3.573	3.558
30.964	2.803	3.013	3.189	3.330	3.436	3.510	3.558	3.583	3.590	3.584	3.568
34.992	2.826	3.036	3.214	3.356	3.463	3.538	3.586	3.611	3.619	3.612	3.596
38.660	2.859	3.072	3.252	3.395	3.503	3.579	3.628	3.653	3.661	3.655	3.638
41.987	2.899	3.116	3.298	3.444	3.554	3.631	3.680	3.706	3.713	3.707	3.690
45.000	2.946	3.166	3.351	3.499	3.611	3.689	3.739	3.765	3.773	3.767	3.750

↑(b)

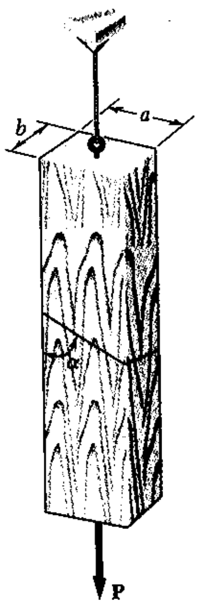
(b) When $\beta = 38.66°$, $\tan\beta = 0.8$ and cable BD is perpendicular to the lever arm BC.

(c) F.S. = 3.579 for $\alpha = 26.6°$; P is perpendicular to the lever arm AC

NOTE:

The value F.S. = 3.579 is the smallest of the values of F.S. corresponding to $\beta = 38.66°$ and the largest of those corresponding to $\alpha = 26.6°$. The point $\alpha = 26.6°$, $\beta = 38.66°$ is a "saddle point", or "minimax" of the function F.S.(α, β).

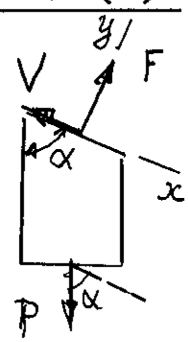
PROBLEM 1.C5



1.C5 A load **P** is supported as shown by two wooden members of uniform rectangular cross section which are joined by a simple glued scarf splice. (*a*) Denoting by σ_U and τ_U, respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of a, b, P, σ_U, and τ_U, expressed in either SI or U.S. customary units, and for values of α from 5 to 85° at 5° intervals, can be used to calculate (1) the normal stress in the joint, (2) the shearing stress in the joint, (3) the factor of safety relative to failure in tension, (4) the factor of safety relative to failure in shear, (5) the overall factor of safety for the glued joint. (*b*) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.32, knowing that $\sigma_U = 1.26$ MPa and $\tau_U = 1.50$ MPa for the glue used in Prob. 1.29, and that $\sigma_U = 150$ psi and $\tau_U = 214$ psi for the glue used in Prob. 1.32. (*c*) Verify in each of these two cases that the shearing stress is maximum for $\alpha = 45°$.

SOLUTION

(1) and (2) Draw the F.B. diagram of lower member:



$\searrow\!+ \sum F_x = 0: \quad -V + P\cos\alpha = 0 \qquad V = P\cos\alpha$

$+\!\nearrow \sum F_y = 0: \quad F - P\sin\alpha = 0 \qquad F = P\sin\alpha$

Area $= ab/\sin\alpha$

Normal stress:

$$\sigma = \frac{F}{\text{Area}} = (P/ab)\sin^2\alpha$$

Shearing stress: $\tau = \dfrac{V}{\text{Area}} = (P/ab)\sin\alpha\cos\alpha$

(3) F.S. for tension (normal stresses)

$FSN = \sigma_U/\sigma$

(4) F.S. for shear:

$FSS = \tau_U/\tau$

(5) OVERALL F.S.:

FS = The smaller of FSN and FSS.

(CONTINUED)

PROBLEM 1.C5 CONTINUED PROGRAM OUTPUTS

For Prob. 1.29: $P = 6$ kN
$a = 125$ mm, $b = 75$ mm, $\alpha = 70°$, $\sigma_U = 1.26$ MPa, $\tau_U = 1.50$ MPa,

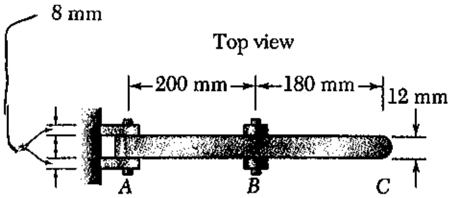
ALPHA	SIG(MPa)	TAU(MPa)	FSN	FSS	FS	
5.0000	0.0049	0.0556	259.1782	26.9942	26.9942	
10.0000	0.0193	0.1094	65.2905	13.7053	13.7053	
15.0000	0.0429	0.1600	29.3899	9.3750	9.3750	
[illegible].0000	0.0749	0.2057	16.8301	7.2925	7.2925	
25.0000	0.1143	0.2451	11.0229	6.1191	6.1191	
30.0000	0.1600	0.2771	7.8750	5.4127	5.4127	
35.0000	0.2106	0.3007	5.9842	4.9883	4.9883	
40.0000	0.2644	0.3151	4.7649	4.7598	4.7598	
45.0000	0.3200	0.3200	3.9375	4.6875	3.9375	(c)
50.0000	0.3756	0.3151	3.3549	4.7598	3.3549	
55.0000	0.4294	0.3007	2.9340	4.9883	2.9340	
60.0000	0.4800	0.2771	2.6250	5.4127	2.6250	
65.0000	0.5257	0.2451	2.3968	6.1191	2.3968	
70.0000	0.5651	0.2057	2.2296	7.2925	2.2296	(b)
75.0000	0.5971	0.1600	2.1101	9.3750	2.1101	
80.0000	0.6207	0.1094	2.0300	13.7053	2.0300	
85.0000	0.6351	0.0556	1.9838	26.9942	1.9838	

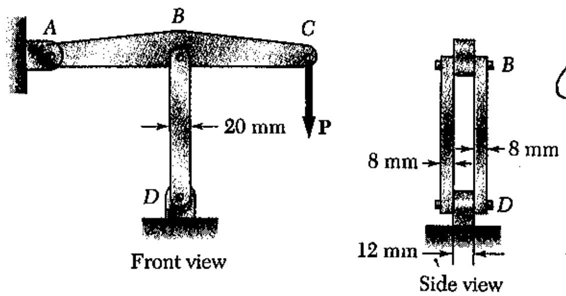
For Prob. 1.32: $P = 2400$ lb
$a = 6$ in., $b = 3$ in., $\alpha = 40°$, $\sigma_U = 150$ psi, $\tau_U = 214$ psi.

ALPHA	SIG(psi)	TAU(psi)	FSN	FSS	FS	
5.0000	1.0128	11.5765	148.1018	18.4857	18.4857	
10.0000	4.0205	22.8013	37.3089	9.3854	9.3854	
[illegible].0000	8.9316	33.3333	16.7942	6.4200	6.4200	
20.0000	15.5970	42.8525	9.6172	4.9939	4.9939	
25.0000	23.8142	51.0696	6.2988	4.1904	4.1904	
30.0000	33.3333	57.7350	4.5000	3.7066	3.7066	
35.0000	43.8653	62.6462	3.4196	3.4160	3.4160	
40.0000	55.0901	65.6538	2.7228	3.2595	2.7228	(b)
45.0000	66.6667	66.6667	2.2500	3.2100	2.2500	(c)
50.0000	78.2432	65.6538	1.9171	3.2595	1.9171	
55.0000	89.4680	62.6462	1.6766	3.4160	1.6766	
60.0000	100.0000	57.7350	1.5000	3.7066	1.5000	
65.0000	109.5192	51.0696	1.3696	4.1904	1.3696	
70.0000	117.7363	42.8525	1.2740	4.9939	1.2740	
75.0000	124.4017	33.3333	1.2058	6.4200	1.2058	
80.0000	129.3128	22.8013	1.1600	9.3854	1.1600	
85.0000	132.3205	11.5765	1.1336	18.4857	1.1336	

PROBLEM 1.C6

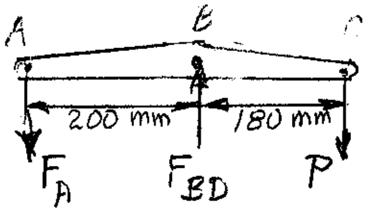
1.C6 Member *ABC* is supported by a pin and bracket at *A* and by two links which are pin-connected to the member at *B* and to a fixed support at *D*. (*a*) Write a computer program to calculate the allowable load P_{all} for any given values of (1) the diameter d_1 of the pin at *A*, (2) the common diameter d_2 of the pins at *B* and *D*, (3) the ultimate normal stress σ_U in each of the two links, (4) the ultimate shearing stress τ_U in each of the three pins, (5) the desired overall factor of safety *F.S.* Your program should also indicate which of the following three stresses is critical: the normal stress in the links, the shearing stress in the pin at *A*, or the shearing stress in the pins at *B* and *D*. (*b* and *c*) Check your program by using the data of Probs. 1.49 and 1.50, respectively, and comparing the answers obtained for P_{all} with those given in the text. (*d*) Use your program to determine the allowable load P_{all}, as well as which of the stresses is critical, when $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa for aluminum links, $\tau_U = 100$ MPa for steel pins, and $F.S. = 3.2$.





SOLUTION

(a) F.B. DIAGRAM OF ABC:



$\sum M_A = 0$: $P = \frac{200}{380} F_{BD}$

$\sum M_B = 0$: $P = \frac{200}{180} F_A$

(1) For given d_1 of pin A: $F_A = 2(\tau_U/FS)(\pi d_1^2/4)$, $P_1 = \frac{200}{180} F_A$

(2) For given d_2 of pins B and D: $F_{BD} = 2(\tau_U/FS)(\pi d_2^2/4)$, $P_2 = \frac{200}{380} F_{BD}$

(3) For ultimate stress in links BD: $F_{BD} = 2(\sigma_U/FS)(0.02)(0.008)$, $P_3 = \frac{200}{380} F_{BD}$

(4) For ult. shearing stress in pins: P_4 is the smaller of P_1 and P_2

(5) For desired overall F.S.: P_5 is the smaller of P_3 and P_4

If $P_3 < P_4$, stress is critical in links

If $P_4 < P_3$ and $P_1 < P_2$, stress is critical in pin A

If $P_4 < P_3$ and $P_2 < P_1$, stress is critical in pins B and D

PROGRAM OUTPUTS

(b) Prob. 1.49. DATA: $d_1 = 8$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.0$

$P_{all} = 3.72$ kN. Stress in pin A is critical ◀

(c) Prob. 1.50. DATA: $d_1 = 10$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.0$

$P_{all} = 5.97$ kN.; Stress in pins B and D is critical ◀

(d) DATA: $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.2$

$P_{all} = 5.79$ kN. Stress in links is critical ◀