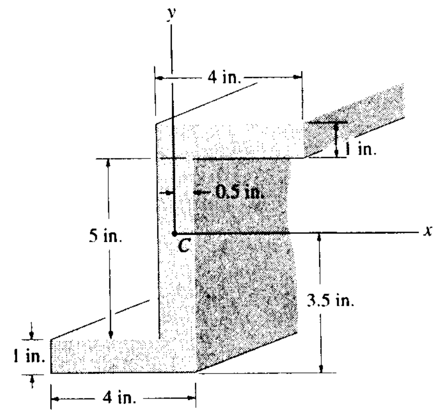
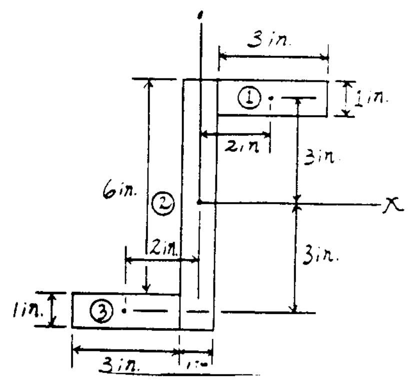
10-69. Determine the product of inertia of the cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.



Product of Inertia : The area for each segment, its centroid and product of inertia with respect to x and y axes are tabulated below.

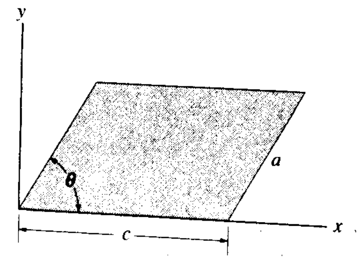


Segment	A_i (in²)	$(d_x)_i$ (in.)	$(d_y)_i$ (in.)	$(I_{xy})_i$ (in⁴)
1	3(1)	2	3	18.0
2	7(1)	0	0	0
3	3(1)	−2	−3	18.0

Thus,

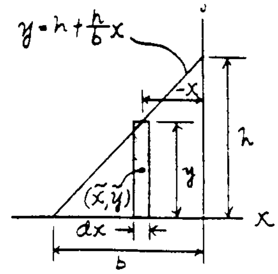
$$I_{xy} = \Sigma(I_{xy})_i = 36.0 \text{ in}^4 \qquad \textbf{Ans}$$

10-70. Determine the product of inertia of the parallelogram with respect to the x and y axes.



Product of Inertia of the Triangle : The product of inertia with respect to x and y axes can be determined by integration. The area of the differential element parallel to y axis is $dA = y\,dx = \left(h + \frac{h}{b}x\right)dx$ [Fig. (a)] . The coordinates of the centroid for this element are $\tilde{x} = -x$, $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(h + \frac{h}{b}x\right)$. Then the product of inertia for this element is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\,\tilde{x}\tilde{y}$$
$$= 0 + \left[\left(h + \frac{h}{b}x\right)dx\right](-x)\left[\frac{1}{2}\left(h + \frac{h}{b}x\right)\right]$$
$$= -\frac{1}{2}\left(h^2 x + \frac{h^2}{b^2}x^3 + \frac{2h^2}{b}x^2\right)dx$$



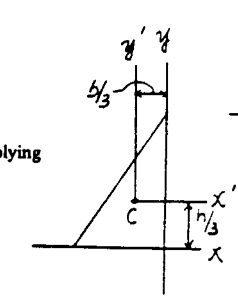
(a)

Performing the integration, we have

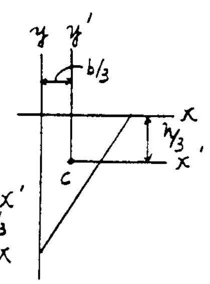
$$I_{xy} = \int dI_{xy} = -\frac{1}{2}\int_{-b}^{0}\left(h^2 x + \frac{h^2}{b^2}x^3 + \frac{2h^2}{b}x^2\right)dx = -\frac{b^2h^2}{24}$$

The product of inertia with respect to centroidal axes, x' and y', can be determined by applying Eq. 10 – 8 [Fig. (b) or (c)] .

$$I_{xy} = \bar{I}_{x'y'} + A d_x d_y$$
$$-\frac{b^2h^2}{24} = \bar{I}_{x'y'} + \frac{1}{2}bh\left(-\frac{b}{3}\right)\left(\frac{h}{3}\right)$$
$$\bar{I}_{x'y'} = \frac{b^2h^2}{72}$$



(b)

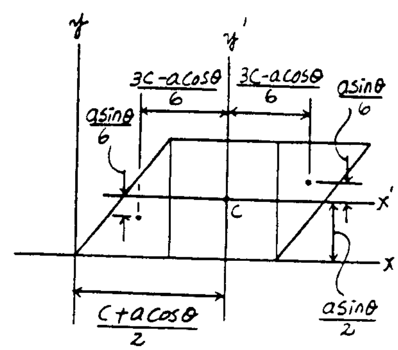


(c)

Here, $b = a\cos\theta$ and $h = a\sin\theta$. Then, $\bar{I}_{x'y'} = \dfrac{a^2b^2\sin^2\theta\cos^2\theta}{72}$.

Product of inertia of the parallelogram [Fig. (d)] with respect to centroidal x' and y' axes, is

$$\bar{I}_{x'y'} = 2\left[\frac{a^4\cos^2\theta\sin^2\theta}{72} + \frac{1}{2}(a\sin\theta)(a\cos\theta)\left(\frac{3c - a\cos\theta}{6}\right)\left(\frac{a\sin\theta}{6}\right)\right]$$
$$= \frac{a^3 c\sin^2\theta\cos\theta}{12}$$

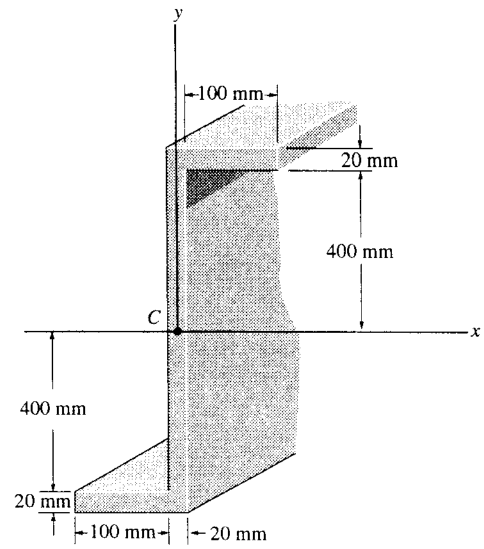


(d)

The product of inertia of the parallelogram [Fig. (d)] about x and y axes is

$$I_{xy} = \bar{I}_{x'y'} + A d_x d_y$$
$$= \frac{a^3 c\sin^2\theta\cos\theta}{12} + (a\sin\theta)(c)\left(\frac{c + a\cos\theta}{2}\right)\left(\frac{a\sin\theta}{2}\right)$$
$$= \frac{a^2 c\sin^2\theta}{12}(4a\cos\theta + 3c)$$ **Ans**

10-71. Determine the product of inertia of the cross sectional area with respect to the x and y axes.

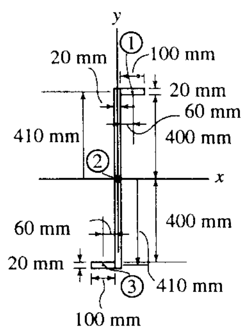


Product of Inertia: The area for each segment, its centroid and product of inertia with respect to x and y axes are tabulated below.

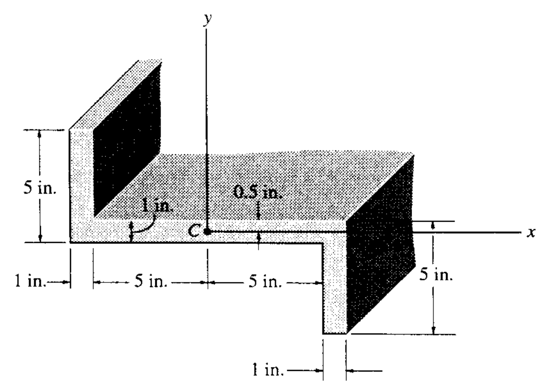
Segment	A_i (mm^2)	$(d_x)_i$ (mm)	$(d_y)_i$ (mm)	$(I_{xy})_i$ (mm^4)
1	100(20)	60	410	49.2(10^6)
2	840(20)	0	0	0
3	100(20)	−60	−410	49.2(10^6)

Thus,

$I_{xy} = \Sigma(I_{xy})_i = 98.4(10^6)\text{mm}^4$ **Ans**



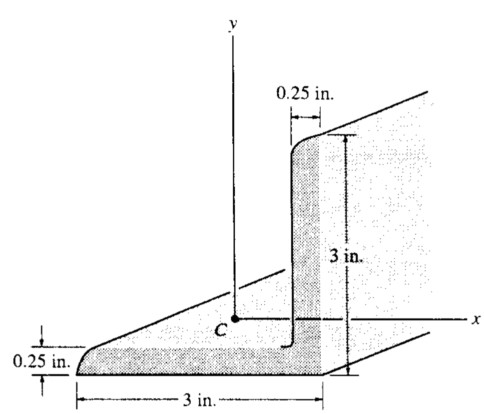
***10-72.** Determine the product of inertia of the beam's cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.



$I_{xy} = 5(1)(5.5)(-2) + 5(1)(-5.5)(2)$

$= -110 \text{ in}^4$ **Ans**

10-73. Determine the product of inertia for the angle with respect to the x and y axes passing through the centroid C. Assume all corners to be square.



Centroid:

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0.125(0.25)(3) + 1.625(0.25)(2.75)}{0.25(3) + 0.25(2.75)} = 0.8424 \text{ in}$$

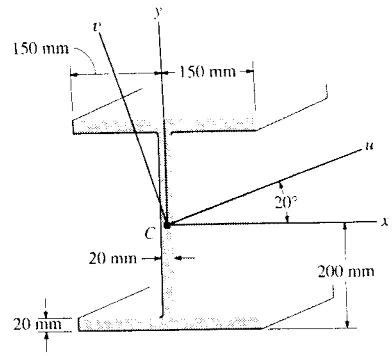
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{1.5(0.25)(3) + 0.125(0.25)(2.75)}{0.25(3) + 0.25(2.75)} = 0.8424 \text{ in}$$

Product of inertia about x and y axes:

$I_{xy} = 0.25(3)(0.7174)(0.6576) + 0.25(2.75)(-0.7826)(-0.7174)$

$= 0.740 \text{ in}^4$ **Ans**

10-74. Determine the product of inertia for the beam's cross-sectional area with respect to the u and v axes.



Moments of inertia I_x and I_y.

$$I_x = \frac{1}{12}(300)(400)^3 - \frac{1}{12}(280)(360)^3 = 511.36(10)^6 \text{ mm}^4$$

$$I_y = 2\left[\frac{1}{12}(20)(300)^3\right] + \frac{1}{12}(360)(20)^3 = 90.24(10)^6 \text{ mm}^4$$

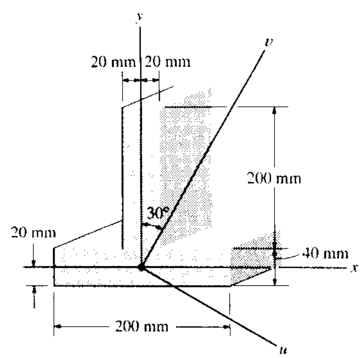
The section is symmetric about both x and y axes; therefore $I_{xy} = 0$.

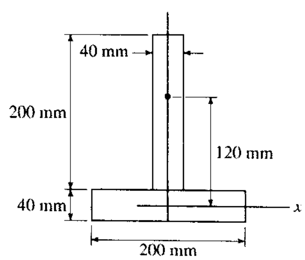
$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \left(\frac{511.36 - 90.24}{2}\sin 40^\circ + 0\cos 40^\circ\right)10^6$$

$$= 135(10)^6 \text{ mm}^4$$ **Ans**

10-75. Determine the moments of inertia I_u and I_v of the cross-sectional area.





Moment and Product of Inertia about x and y Axes: Since the shaded area is symmetrical about the y axis, $I_{xy} = 0$.

$$I_x = \frac{1}{12}(40)(200^3) + 40(200)(120^2) + \frac{1}{12}(200)(40^3)$$

$$= 142.93(10^6) \text{ mm}^4$$

$$I_y = \frac{1}{12}(200)(40^3) + \frac{1}{12}(40)(200^3) = 27.73(10^6) \text{ mm}^4$$

Moment of Inertia about the Inclined u and v Axes: Applying Eq. 10-9 with $\theta = -30°$, we have

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \left(\frac{142.93 + 27.73}{2} + \frac{142.93 - 27.73}{2}\cos(-60^\circ) - 0[\sin(-60^\circ)]\right)(10^6)$$

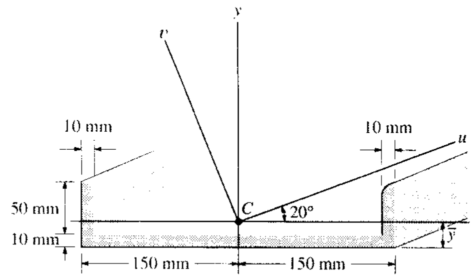
$$= 114(10^6) \text{ mm}^4$$ **Ans**

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \left(\frac{142.93 + 27.73}{2} - \frac{142.93 - 27.73}{2}\cos(-60^\circ) - 0[\sin(-60^\circ)]\right)(10^6)$$

$$= 56.5(10^6) \text{ mm}^4$$ **Ans**

***10-76.** Determine the distance $\bar{y}$ to the centroid of the area and then calculate the moments of inertia I_u and I_v of the channel's cross-sectional area. The u and v axes have their origin at the centroid C. For the calculation, assume all corners to be square.



$$\bar{y} = \frac{300(10)(5) + 2[(50)(10)(35)]}{300(10) + 2(50)(10)} = 12.5 \text{ mm} \qquad \textbf{Ans}$$

$$I_x = \left[\frac{1}{12}(300)(10)^3 + 300(10)(12.5 - 5)^2\right]$$

$$+ 2\left[\frac{1}{12}(10)(50)^3 + 10(50)(35 - 12.5)^2\right]$$

$$= 0.9083(10^6) \text{ mm}^4$$

$$I_y = \frac{1}{12}(10)(300)^3 + 2\left[\frac{1}{12}(50)(10)^3 + 50(10)(150 - 5)^2\right]$$

$$= 43.53(10^6) \text{ mm}^4$$

$I_{xy} = 0$ (By symmetry)

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{0.9083(10^6) + 43.53(10^6)}{2} + \frac{0.9083(10^6) - 43.53(10^6)}{2}\cos 40^\circ - 0$$

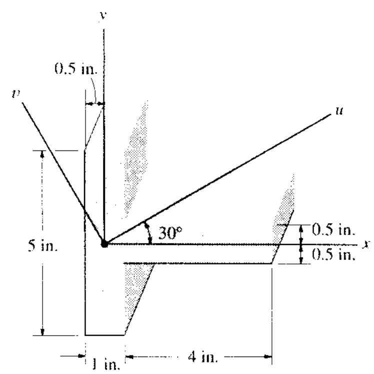
$$= 5.89(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \frac{0.9083(10^6) + 43.53(10^6)}{2} - \frac{0.9083(10^6) - 43.53(10^6)}{2}\cos 40^\circ + 0$$

$$= 38.5(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

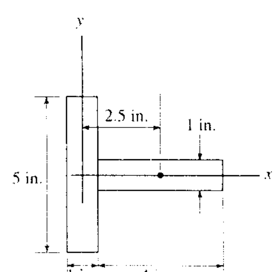
***10-77.** Determine the moments of inertia of the shaded area with respect to the u and v axes.



Moment and Product of Inertia about x and y Axes: Since the shaded area is symmetrical about the x axis, $I_{xy} = 0$.

$$I_x = \frac{1}{12}(1)(5^3) + \frac{1}{12}(4)(1^3) = 10.75 \text{ in}^4$$

$$I_y = \frac{1}{12}(1)(4^3) + 1(4)(2.5^2) + \frac{1}{12}(5)(1^3) = 30.75 \text{ in}^4$$



Moment of Inertia about the Inclined u and v Axes: Applying Eq. 10-9 with $\theta = 30°$, we have

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{10.75 + 30.75}{2} + \frac{10.75 - 30.75}{2}\cos 60° - 0(\sin 60°)$$

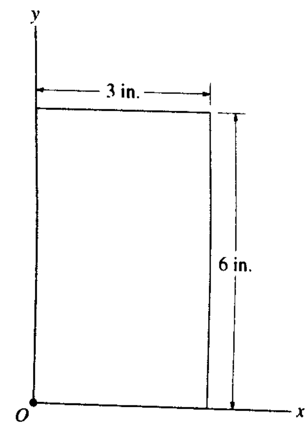
$$= 15.75 \text{ in}^4 \qquad \textbf{Ans}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \frac{10.75 + 30.75}{2} - \frac{10.75 - 30.75}{2}\cos 60° + 0(\sin 60°)$$

$$= 25.75 \text{ in}^4 \qquad \textbf{Ans}$$

10-78. Determine the directions of the principal axes with origin located at point O, and the principal moments of inertia for the rectangular area about these axes.



$$I_x = \frac{1}{12}(3)(6)^3 + (3)(6)(3)^2 = 216\text{ in}^4$$

$$I_y = \frac{1}{12}(6)(3)^3 + (3)(6)(1.5)^2 = 54\text{ in}^4$$

$$I_{xy} = \bar{x}\bar{y}A = (1.5)(3)(3)(6) = 81\text{ in}^4$$

$$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y} = \frac{-2(81)}{216-54} = -1$$

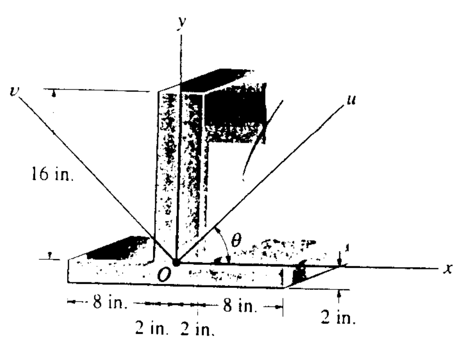
$\theta = -22.5°$ **Ans**

$$I_{\substack{max\\min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = \frac{216+54}{2} \pm \sqrt{\left(\frac{216-54}{2}\right)^2 + (81)^2}$$

$I_{max} = 250\text{ in}^4$ **Ans**

$I_{min} = 20.4\text{ in}^4$ **Ans**

10-79. Determine the moments of inertia I_u, I_v and the product of inertia I_{uv} of the beam's cross-sectional area. Take $\theta = 45°$.



$$I_x = \frac{1}{12}(20)(2)^3 + 20(2)(1)^2 + \frac{1}{12}(4)(16)^3 + 4(16)(8)^2$$

$$= 5.515(10^3)\text{ in}^4$$

$$I_y = \frac{1}{12}(2)(20)^3 + \frac{1}{12}(16)(4)^3$$

$$= 1.419(10^3)\text{ in}^4$$

$$I_{xy} = 0$$

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{5.515 + 1.419}{2}(10^3) + \frac{5.515 - 1.419}{2}(10^3)\cos 90° - 0$$

$= 3.47(10^3)\text{ in}^4$ **Ans**

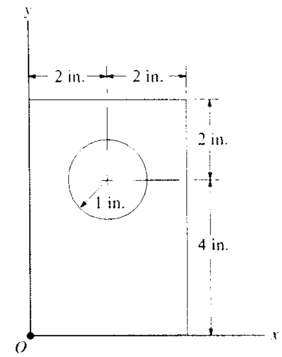
$I_v = 3.47(10^3)\text{ in}^4$ **Ans**

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \frac{5.515 - 1.419}{2}(10^3)\sin 90° + 0$$

$= 2.05(10^3)\text{ in}^4$ **Ans**

***10-80.** Determine the directions of the principal axes with origin located at point O, and the principal moments of inertia of the area about these axes.



$$I_x = \left[\frac{1}{12}(4)(6)^3 + (4)(6)(3)^2\right] - \left[\frac{1}{4}\pi(1)^4 + \pi(1)^2(4)^2\right]$$

$$= 236.95 \text{ in}^4$$

$$I_y = \left[\frac{1}{12}(6)(4)^3 + (4)(6)(2)^2\right] - \left[\frac{1}{4}\pi(1)^4 + \pi(1)^2(2)^2\right]$$

$$= 114.65 \text{ in}^4$$

$$I_{xy} = [0 + (4)(6)(2)(3)] - [0 + \pi(1)(2)(4)] = 118.87 \text{ in}^4$$

$$\tan 2\theta_P = \frac{-I_{xy}}{\frac{I_x - I_y}{2}} = \frac{-118.87}{\frac{(236.95 - 114.65)}{2}}$$

$$\theta_P = -31.388°; \quad 58.612°$$

Thus,

$\theta_{P1} = -31.4°; \quad \theta_{P2} = 58.6°$ **Ans**

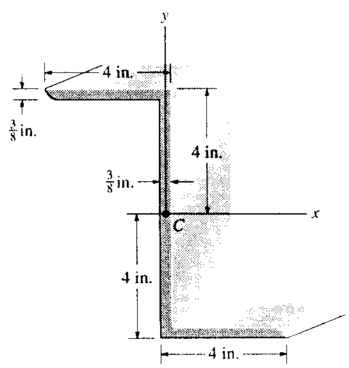
$$I_{\substack{max \\ min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{236.95 + 114.65}{2} \pm \sqrt{\left(\frac{236.95 - 114.65}{2}\right)^2 + (118.87)^2}$$

$I_{max} = 309 \text{ in}^4$ **Ans**

$I_{min} = 42.1 \text{ in}^4$ **Ans**

10-81. Determine the principal moments of inertia of the beam's cross-sectional area about the principal axes that have their origin located at the centroid C. Use the equations developed in Section 10-7. For the calculation, assume all corners to be square.



$$I_x = 2\left[\frac{1}{12}(4)\left(\frac{3}{8}\right)^3 + 4\left(\frac{3}{8}\right)\left(4 - \frac{3}{16}\right)^2\right] + \frac{1}{12}\left(\frac{3}{8}\right)\left(8 - \frac{6}{8}\right)^3$$

$$= 55.55 \text{ in}^4$$

$$I_y = 2\left[\frac{1}{12}\left(\frac{3}{8}\right)\left(4 - \frac{3}{8}\right)^3 + \frac{3}{8}\left(4 - \frac{3}{8}\right)\left\{\left(\frac{4 - \frac{3}{8}}{2}\right) + \frac{3}{16}\right\}^2\right]$$

$$+ \frac{1}{12}(8)\left(\frac{3}{8}\right)^3$$

$$= 13.89 \text{ in}^4$$

$$I_{xy} = \Sigma \bar{x}\bar{y}A$$

$$= -2[(1.813 + 0.1875)(3.813)(3.625)(0.375)] + 0$$

$$= -20.73 \text{ in}^4$$

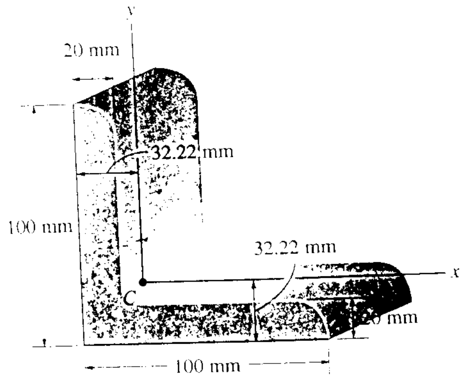
$$I_{max/min} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{55.55 + 13.89}{2} \pm \sqrt{\left(\frac{55.55 - 13.89}{2}\right)^2 + (-20.73)^2}$$

$I_{max} = 64.1 \text{ in}^4$ **Ans**

$I_{min} = 5.33 \text{ in}^4$ **Ans**

10-82. Determine the principal moments of inertia for the angle's cross-sectional area with respect to a set of principal axes that have their origin located at the centroid C. Use the equation developed in Section 10-7. For the calculation, assume all corners to be square.



$$I_x = \left[\frac{1}{12}(20)(100)^3 + 100(20)(50-32.22)^2\right] + \left[\frac{1}{12}(80)(20)^3 + 80(20)(32.22-10)^2\right]$$

$$= 3.142(10^6)\ \text{mm}^4$$

$$I_y = \left[\frac{1}{12}(100)(20)^3 + 100(20)(32.22-10)^2\right] + \left[\frac{1}{12}(20)(80)^3 + 80(20)(60-32.22)^2\right]$$

$$= 3.142\,(10^6)\ \text{mm}^4$$

$$I_{xy} = \Sigma \bar{x}\bar{y}A$$

$$= -(32.22-10)(50-32.22)(100)(20) - (60-32.22)(32.22-10)(80)(20)$$

$$= -1.778(10^6)\ \text{mm}^4$$

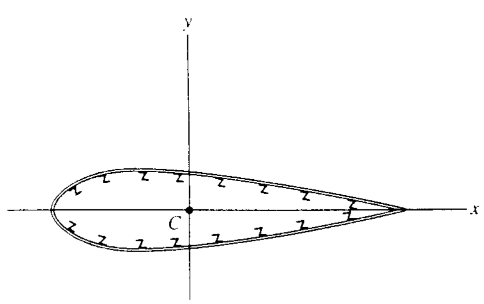
$$I_{max/min} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$= 3.142(10^6) \pm \sqrt{0 + \{(-1.778)(10^6)\}^2}$$

$I_{max} = 4.92(10^6)\ \text{mm}^4$ **Ans**

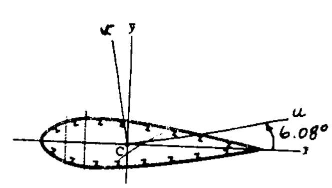
$I_{min} = 1.36(10^6)\ \text{mm}^4$ **Ans**

10-83. The area of the cross section of an airplane wing has the following properties about the x and y axes passing through the centroid C: $\bar{I}_x = 450\ \text{in}^4$, $\bar{I}_y = 1730\ \text{in}^4$, $\bar{I}_{xy} = 138\ \text{in}^4$. Determine the orientation of the principal axes and the principal moments of inertia.



$$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y} = \frac{-2(138)}{450-1730}$$

$\theta = 6.08°$ **Ans**



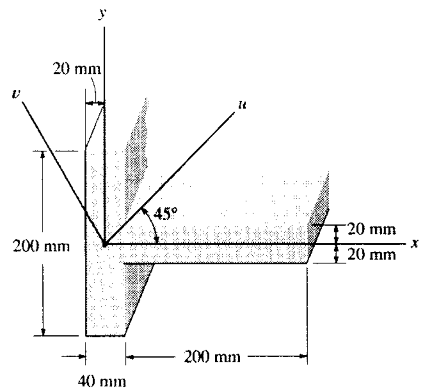
$$I_{max/min} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{450 + 1730}{2} \pm \sqrt{\left(\frac{450-1730}{2}\right)^2 + 138^2}$$

$I_{max} = 1.74(10^3)\ \text{in}^4$ **Ans**

$I_{min} = 435\ \text{in}^4$ **Ans**

***10-84.** Determine the moments of inertia I_u and I_v of the shaded area.



Moment and Product of Inertia about x and y Axes: Since the shaded area is symmetrical about the x axis, $I_{xy} = 0$.

$$I_x = \frac{1}{12}(200)(40^3) + \frac{1}{12}(40)(200^3) = 27.73(10^6)\ \text{mm}^4$$

$$I_y = \frac{1}{12}(40)(200^3) + 40(200)(120^2) + \frac{1}{12}(200)(40^3)$$

$$= 142.93(10^6)\ \text{mm}^4$$

Moment of Inertia about the Inclined u and v Axes: Applying Eq. 10-9 with $\theta = 45°$, we have

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

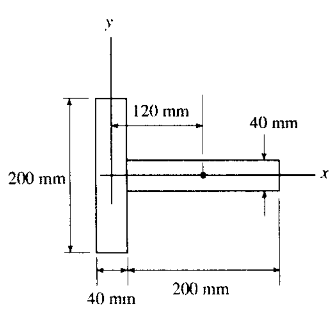
$$= \left(\frac{27.73 + 142.93}{2} + \frac{27.73 - 142.93}{2}\cos 90° - 0(\sin 90°)\right)(10^6)$$

$$= 85.3(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

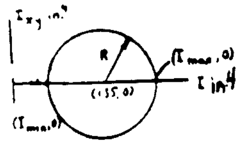
$$= \left(\frac{27.73 + 142.93}{2} - \frac{27.73 - 142.93}{2}\cos 90° - 0(\sin 90°)\right)(10^6)$$

$$= 85.3(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$



10-85. Solve Prob. 10-78 using Mohr's circle.

See solution to Prob. 10-78.



$I_x = 216\ \text{in}^4$

$I_y = 54\ \text{in}^4$

$I_{xy} = 81\ \text{in}^4$

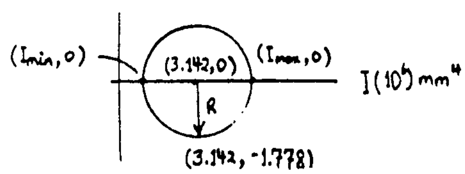
Center of circle : $\dfrac{I_x + I_y}{2} = 135$

$R = \sqrt{(216 - 135)^2 + (81)^2} = 114.55$

$I_{max} = 135 + 114.55 = 250\ \text{in}^4$ **Ans**

$I_{min} = 135 - 114.55 = 20.4\ \text{in}^4$ **Ans**

10-86. Solve Prob. 10-81 using Mohr's circle.



See prob. 10 – 81.

$I_x = 55.55\ \text{in}^4$

$I_y = 13.89\ \text{in}^4$

$I_{xy} = -20.73\ \text{in}^4$

Center of circle;

$\dfrac{I_x + I_y}{2} = 34.72\ \text{in}^4$

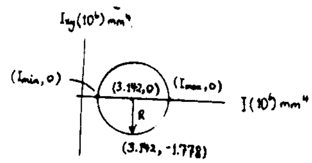
$R = \sqrt{(55.55 - 34.72)^2 + (-20.73)^2} = 29.39\ \text{in}^4$

$I_{max} = 34.72 + 29.39 = 64.1\ \text{in}^4$ **Ans**

$I_{min} = 34.72 - 29.39 = 5.33\ \text{in}^4$ **Ans**

10-87. Solve Prob. 10-82 using Mohr's circle.

See Prob. 10 – 82.



$I_x = 3.142(10^6)\ \text{mm}^4$

$I_y = 3.142(10^6)\ \text{mm}^4$

$I_{xy} = -1.778(10^6)\ \text{mm}^4$

Center of circle;

$$\frac{I_x + I_y}{2} = 3.142(10^6)\ \text{mm}^4$$

$$R = \sqrt{(3.142 - 3.142)^2 + (-1.778)^2}(10^6) = 1.778(10^6)\ \text{mm}^4$$

$I_{max} = 3.142(10^6) + 1.778(10^6) = 4.92(10^6)\ \text{mm}^4$ **Ans**

$I_{min} = 3.142(10^6) - 1.778(10^6) = 1.36(10^6)\ \text{mm}^4$ **Ans**

***10-88.** Solve Prob. 10-80 using Mohr's circle.

See solution to Prob. 10 - 80.

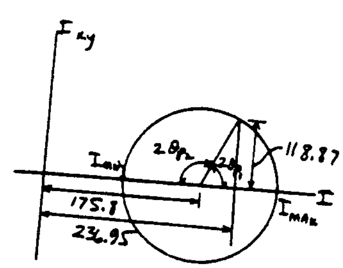
$I_x = 236.95\ \text{in}^4$

$I_y = 114.65\ \text{in}^4$

$I_{xy} = 118.87\ \text{in}^4$

$$\frac{I_x + I_y}{2} = \frac{236.95 + 114.65}{2} = 175.8\ \text{in}^4$$

$$R = \sqrt{(236.95 - 175.8)^2 + (118.87)^2} = 133.68\ \text{in}^4$$



$I_{max} = (175.8 + 133.68) = 309\ \text{in}^4$ **Ans**

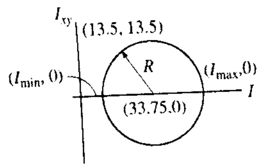
$I_{min} = (175.8 - 133.68) = 42.1\ \text{in}^4$ **Ans**

$$2\theta_{p_1} = \tan^{-1}\left(\frac{118.87}{(236.95 - 175.8)}\right) = 62.78°$$

$\theta_{p_1} = -31.4°$ **Ans**

$\theta_{p_2} = 90° - 31.4° = 58.6°$ **Ans**

10-89. Solve Prob. 10-83 using Mohr's circle.



From Prob. 10-83,

$\bar{I}_{\bar{x}} = 450 \text{ in}^4$, $\bar{I}_{\bar{y}} = 1730 \text{ in}^4$, $\bar{I}_{\bar{x}\bar{y}} = 138 \text{ in}^4$

Center of circle

$$\frac{\bar{I}_{\bar{x}} + \bar{I}_{\bar{y}}}{2} = \frac{450 + 1730}{2} = 1090 \text{ in}^4$$

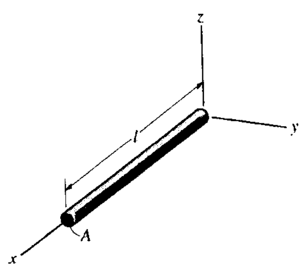
Radius $R = \sqrt{(-640)^2 + (138)^2}$

$R = 654.71(-640)^2 + (138)^2$

$I_{max} = 1090 + 654.71 = 1744.7 = 1.74(10^3) \text{ in}^4$ **Ans**

$I_{min} = 1090 - 654.71 = 435 \text{ in}^4$ **Ans**

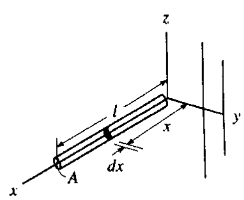
***10-90.** Determine the moment of inertia I_y for the slender rod. The rod's density ρ and cross-sectional area A are constant. Express the result in terms of the rod's total mass m.



$$I_y = \int_M x^2 dm$$

$$= \int_0^l x^2(\rho A\, dx)$$

$$= \frac{1}{3}\rho A l^3$$

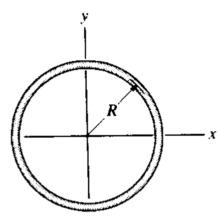


$m = \rho A l$

Thus,

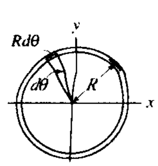
$$I_y = \frac{1}{3} m l^2 \quad \textbf{Ans}$$

10-91. Determine the moment of inertia of the thin ring about the z axis. The ring has a mass m.



$$I_z = \int_0^{2\pi} \rho A(R\, d\theta)R^2 = 2\pi \rho A R^3$$

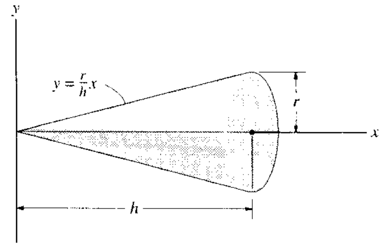
$$m = \int_0^{2\pi} \rho A R\, d\theta = 2\pi \rho A R$$



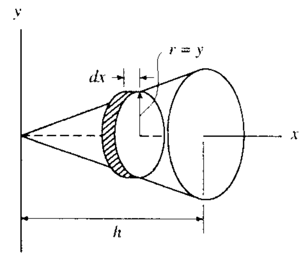
Thus,

$I_z = mR^2$ **Ans**

***10-92.** Determine the moment of inertia I_x of the right circular cone and express the result in terms of the total mass m of the cone. The cone has a constant density ρ.



Differential Disk Element: The mass of the differential disk element is $dm = \rho dV = \rho\pi y^2\,dx = \rho\pi\left(\frac{r^2}{h^2}x^2\right)dx$. The mass moment of inertia of this element is $dI_x = \frac{1}{2}dmy^2 = \frac{1}{2}\left[\rho\pi\left(\frac{r^2}{h^2}x^2\right)dx\right]\left(\frac{r^2}{h^2}x^2\right) = \frac{\rho\pi r^4}{2h^4}x^4\,dx$.



Total Mass: Performing the integration, we have

$$m = \int_m dm = \int_0^h \rho\pi\left(\frac{r^2}{h^2}x^2\right)dx = \frac{\rho\pi r^2}{h^2}\left(\frac{x^3}{3}\right)\Big|_0^h = \frac{1}{3}\rho\pi r^2 h$$

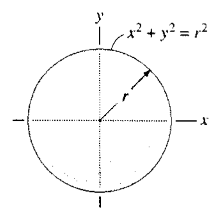
Mass Moment of Inertia: Performing the integration, we have

$$I_x = \int dI_x = \int_0^h \frac{\rho\pi r^4}{2h^4}x^4\,dx = \frac{\rho\pi r^4}{2h^4}\left(\frac{x^5}{5}\right)\Big|_0^h = \frac{1}{10}\rho\pi r^4 h$$

The mass moment of inertia expressed in terms of the total mass is

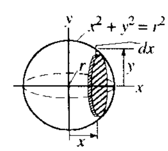
$$I_x = \frac{3}{10}\left(\frac{1}{3}\rho\pi r^2 h\right)r^2 = \frac{3}{10}mr^2 \qquad \textbf{Ans}$$

10-93. Determine the moment of inertia I_x of the sphere and express the result in terms of the total mass m of the sphere. The sphere has a constant density ρ.



$$d\,I_x = \frac{y^2\,dm}{2}$$

$$dm = \rho\,dV = \rho(\pi y^2\,dx) = \rho\pi(r^2 - x^2)\,dx$$



$$d\,I_x = \frac{1}{2}\rho\pi(r^2 - x^2)^2\,dx$$

$$I_x = \int_{-r}^{r} \frac{1}{2}\rho\pi(r^2 - x^2)^2\,dx$$

$$= \frac{8}{15}\pi\rho r^5$$

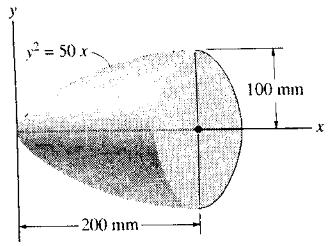
$$m = \int_{-r}^{r} \rho\pi(r^2 - x^2)\,dx$$

$$= \frac{4}{3}\rho\pi r^3$$

Thus,

$$I_x = \frac{2}{5}mr^2 \quad \textbf{Ans}$$

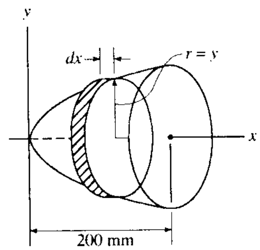
10-94. Determine the radius of gyration k_x of the paraboloid. The density of the material is $\rho = 5\ \text{Mg/m}^3$.



Differential Disk Element: The mass of the differential disk element is $dm = \rho dV = \rho\pi y^2\,dx = \rho\pi(50x)\,dx$. The mass moment of inertia of this element is $dI_x = \frac{1}{2}\,dmy^2 = \frac{1}{2}[\rho\pi(50x)\,dx](50x) = \frac{\rho\pi}{2}(2500x^2)\,dx$.

Total Mass: Performing the integration, we have

$$m = \int_m dm = \int_0^{200\text{ mm}} \rho\pi(50x)\,dx = \rho\pi(25x^2)\Big|_0^{200\text{ mm}} = 1(10^6)\rho\pi$$



Mass Moment of Inertia: Performing the integration, we have

$$I_x = \int dI_x = \int_0^{200\text{ mm}} \frac{\rho\pi}{2}(2500x^2)\,dx$$

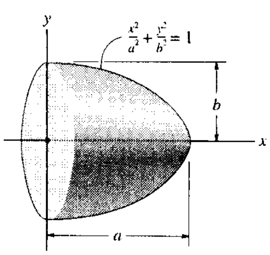
$$= \frac{\rho\pi}{2}\left(\frac{2500x^3}{3}\right)\Bigg|_0^{200\text{ mm}}$$

$$= 3.333(10^9)\rho\pi$$

The radius of gyration is

$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{3.333(10^9)\rho\pi}{1(10^6)\rho\pi}} = 57.7\text{ mm} \quad \textbf{Ans}$$

10-95. Determine the moment of inertia of the semiellipsoid with respect to the x axis and express the result in terms of the mass m of the semiellipsoid. The material has a constant density ρ.

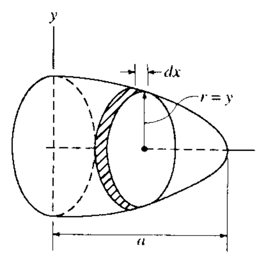


Differential Disk Element: Here, $y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$. The mass of the differential disk element is $dm = \rho dV = \rho\pi y^2\,dx = \rho\pi b^2\left(1 - \frac{x^2}{a^2}\right)dx$. The mass moment of inertia of this element is $dI_x = \frac{1}{2}\,dmy^2 = \frac{1}{2}\left[\rho\pi b^2\left(1 - \frac{x^2}{a^2}\right)dx\right]\left[b^2\left(1 - \frac{x^2}{a^2}\right)\right] = \frac{\rho\pi b^4}{2}\left(\frac{x^4}{a^4} - \frac{2x^2}{a^2} + 1\right)dx$.

Total Mass: Performing the integration, we have

$$m = \int_m dm = \int_0^a \rho\pi b^2\left(1 - \frac{x^2}{a^2}\right)dx = \rho\pi b^2\left(x - \frac{x^3}{3a^2}\right)\Bigg|_0^a$$

$$= \frac{2}{3}\rho\pi ab^2$$



Mass Moment of Inertia: Performing the integration, we have

$$I_x = \int dI_x = \int_0^a \frac{\rho\pi b^4}{2}\left(\frac{x^4}{a^4} - \frac{2x^2}{a^2} + 1\right)dx$$

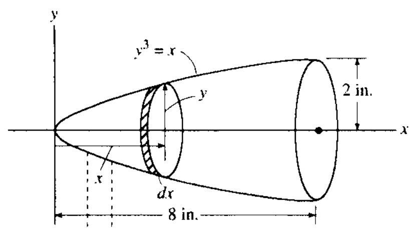
$$= \frac{\rho\pi b^4}{2}\left(\frac{x^5}{5a^4} - \frac{2x^3}{3a^2} + x\right)\Bigg|_0^a$$

$$= \frac{4}{15}\rho\pi ab^4$$

The mass moment of inertia expressed in terms of the total mass is

$$I_x = \frac{2}{5}\left(\frac{2}{3}\rho\pi ab^2\right)b^2 = \frac{2}{5}mb^2 \quad \textbf{Ans}$$

***10-96.** Determine the radius of gyration k_x. The specific weight of the material is $\gamma = 380\ \text{lb/ft}^3$.



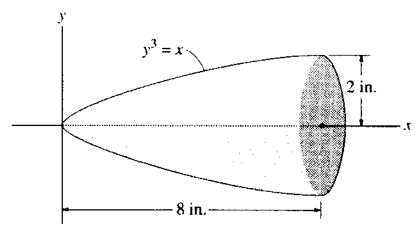
$$dm = \rho\, dV = \rho\pi y^2\, dx$$

$$d\,I_x = \frac{1}{2}(dm)y^2 = \frac{1}{2}\pi\rho y^4\, dx$$

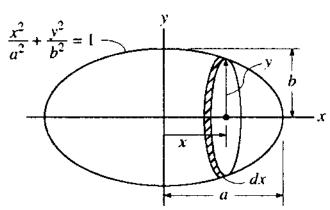
$$I_x = \int_0^8 \frac{1}{2}\pi\rho x^{4/3}\, dx = 86.17\rho$$

$$m = \int_0^8 \pi\rho x^{2/3}\, dx = 60.32\rho$$

$$k_x = \sqrt{\frac{I_x}{m}} = \sqrt{\frac{86.17\rho}{60.32\rho}} = 1.20\text{ in.} \quad \textbf{Ans}$$



10-97. Determine the moment of inertia of the ellipsoid with respect to the x axis and express the result in terms of the mass m of the ellipsoid. The material has a constant density ρ.



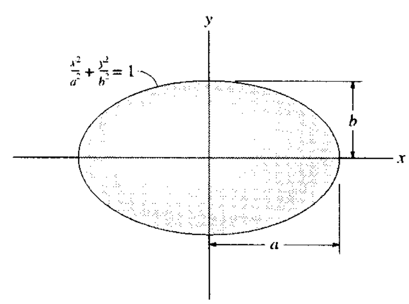
$$d\,I_x = \frac{y^2 dm}{2}$$

$$m = \int_V \rho\, dV = \int_{-a}^{a} \rho\pi b^2\left(1 - \frac{x^2}{a^2}\right) dx = \frac{4}{3}\pi\rho a b^2$$

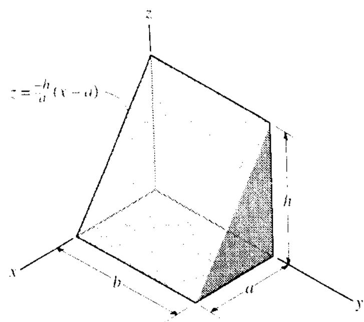
$$I_x = \int_{-a}^{a} \frac{1}{2}\rho\pi b^4\left(1 - \frac{x^2}{a^2}\right)^2 dx = \frac{8}{15}\pi\rho a b^4$$

Thus,

$$I_x = \frac{2}{5}mb^2 \quad \textbf{Ans}$$



10-98. Determine the moment of inertia of the homogenous triangular prism with respect to the y axis. Express the result in terms of the mass m of the prism. *Hint*: For integration, use thin plate elements parallel to the x-y plane having a thickness of dz.



Differential Thin Plate Element: Here, $x = a\left(1 - \frac{z}{h}\right)$. The mass of the differential thin plate element is $dm = \rho dV = \rho bx dz = \rho ab \left(1 - \frac{z}{h}\right) dz$. The mass moment of inertia of this element about y axis is

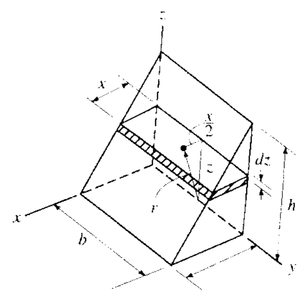
$$dI_y = dI_G + dmr^2$$

$$= \frac{1}{12} dmx^2 + dm\left(\frac{x^2}{4} + z^2\right)$$

$$= \frac{1}{3} x^2\,dm + z^2\,dm$$

$$= \left[\frac{a^2}{3}\left(1 - \frac{z}{h}\right)^2 + z^2\right]\left[\rho ab\left(1 - \frac{z}{h}\right)dz\right]$$

$$= \frac{\rho ab}{3}\left(a^2 + \frac{3a^2}{h^2}z^2 - \frac{3a^2}{h}z - \frac{a^2}{h^3}z^3 + 3z^2 - \frac{3z^3}{h}\right)dz$$



Total Mass: Performing the integration, we have

$$m = \int_m dm = \int_0^h \rho ab\left(1 - \frac{z}{h}\right)dz = \rho a b\left(z - \frac{z^2}{2h}\right)\Bigg|_0^h = \frac{1}{2}\rho abh$$

Mass Moment of Inertia: Performing the integration, we have

$$I_y = \int dI_y = \int_0^h \frac{\rho ab}{3}\left(a^2 + \frac{3a^2}{h^2}z^2 - \frac{3a^2}{h}z - \frac{a^2}{h^3}z^3 + 3z^2 - \frac{3z^3}{h}\right)dz$$

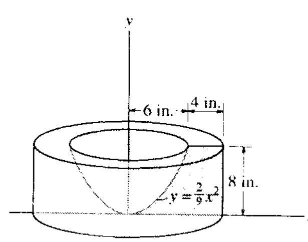
$$= \frac{\rho ab}{3}\left(a^2 z + \frac{a^2}{h^2}z^3 - \frac{3a^2}{2h}z^2 - \frac{a^2}{4h^3}z^4 + z^3 - \frac{3z^4}{4h}\right)\Bigg|_0^h$$

$$= \frac{\rho abh}{12}(a^2 + h^2)$$

The mass moment of inertia expressed in terms of the total mass is

$$I_y = \frac{1}{6}\left(\frac{\rho abh}{2}\right)(a^2 + h^2) = \frac{m}{6}(a^2 + h^2) \quad \textbf{Ans}$$

10-99. The concrete shape is formed by rotating the shaded area about the y axis. Determine the moment of inertia I_y. The specific weight of concrete is $\gamma = 150\ \text{lb/ft}^3$.



$$d\,I_y = \frac{1}{2}(dm)(10)^2 - \frac{1}{2}(dm)x^2$$

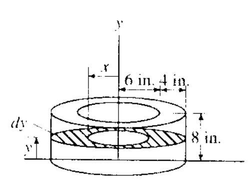
$$= \frac{1}{2}\{\pi\rho(10)^2\,dy\}(10)^2 - \frac{1}{2}\pi\rho x^2\,dy\,x^2$$

$$I_y = \frac{1}{2}\pi\rho\left[\int_0^8 (10)^4\,dy - \int_0^8 \left(\frac{9}{2}\right)^2 y^2\,dy\right]$$

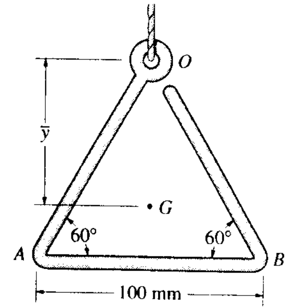
$$= \frac{\frac{1}{2}\pi(150)}{32.2(12)^3}\left[(10)^4(8) - \left(\frac{9}{2}\right)^2\left(\frac{1}{3}\right)(8)^3\right]$$

$$= 324.1\ \text{slug}\cdot\text{in}^2$$

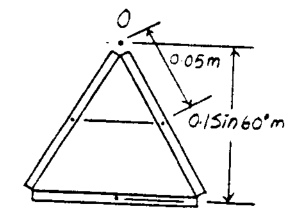
$$I_y = 2.25\ \text{slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$



***10-100.** Determine the moment of inertia of the wire triangle about an axis perpendicular to the page and passing through point O. Also, locate the mass center G and determine the moment of inertia about an axis perpendicular to the page and passing through point G. The wire has a mass of 0.3 kg/m. Neglect the size of the ring at O.



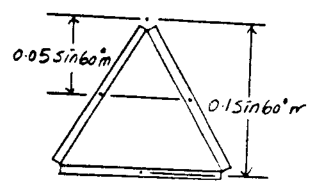
Mass Moment of Inertia About an Axis Through Point O : The mass for each wire segment is $m_i = 0.3(0.1) = 0.03$ kg. The mass moment of inertia of each segment about an axis passing through the center of mass can be determined using $(I_G)_i = \frac{1}{12}ml^2$. Applying Eq. 10 – 16, we have



$$I_O = \Sigma(I_G)_i + m_i d_i^2$$
$$= 2\left[\frac{1}{12}(0.03)\left(0.1^2\right) + 0.03\left(0.05^2\right)\right]$$
$$+ \frac{1}{12}(0.03)\left(0.1^2\right) + 0.03(0.1\sin 60°)^2$$
$$= 0.450\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

Location of Centroid :

$$\bar{y} = \frac{\Sigma \tilde{y}m}{\Sigma m} = \frac{2[0.05\sin 60°(0.03)] + 0.1\sin 60°(0.03)}{3(0.03)}$$
$$= 0.05774\ \text{m} = 57.7\ \text{mm} \qquad \textbf{Ans}$$

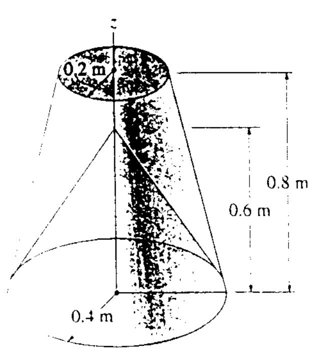


Mass Moment of Inertia About an Axis Through Point G : Using the result $I_O = 0.450\left(10^{-3}\right)$ kg · m² and $d = \bar{y} = 0.05774$ m and applying Eq. 10 – 16, we have

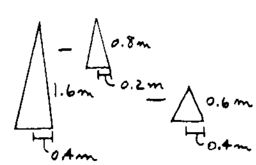
$$I_O = I_G + md^2$$
$$0.450\left(10^{-3}\right) = I_G + 3(0.03)\left(0.05774^2\right)$$

$$I_G = 0.150\left(10^{-3}\right)\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

10-101. Determine the moment of inertia I_z of the frustum of the cone which has a conical depression. The material has a density of 200 kg/m³.

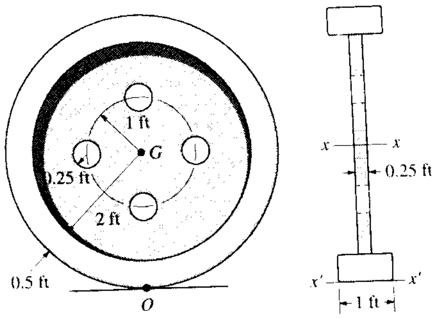


$$I_z = \frac{3}{10}\left[\frac{1}{3}\pi(0.4)^2(1.6)(200)\right](0.4)^2$$
$$- \frac{3}{10}\left[\frac{1}{3}\pi(0.2)^2(0.8)(200)\right](0.2)^2$$
$$- \frac{3}{10}\left[\frac{1}{3}\pi(0.4)^2(0.6)(200)\right](0.4)^2$$



$$I_z = 1.53\ \text{kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

10-102. Determine the moment of inertia of the wheel about the x axis that passes through the center of mass G. The material has a specific weight of $\gamma = 90\ \text{lb/ft}^3$.



Mass Moment of Inertia About an Axis Through Point G: The mass moment of inertia of each disk about an axis passing through the center of mass can be determine using $(I_G)_i = \frac{1}{2}mr^2$. Applying Eq. 10-16, we have

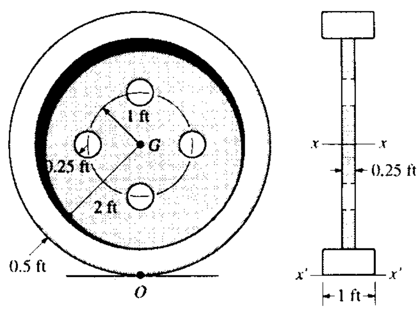
$$I_G = \Sigma(I_G)_i + m_i d_i^2$$

$$= \frac{1}{2}\left[\frac{\pi(2.5^2)(1)(90)}{32.2}\right](2.5^2) - \frac{1}{2}\left[\frac{\pi(2^2)(0.75)(90)}{32.2}\right](2^2)$$

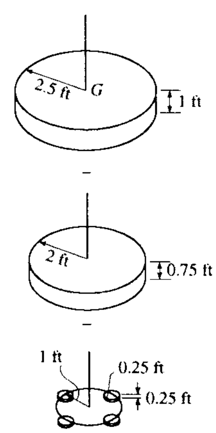
$$-4\left\{\frac{1}{2}\left[\frac{\pi(0.25^2)(0.25)(90)}{32.2}\right](0.25^2) + \left[\frac{\pi(0.25^2)(0.25)(90)}{32.2}\right](1^2)\right\}$$

$$= 118\ \text{slug}\cdot\text{ft}^2$$ **Ans**

10-103. Determine the moment of inertia of the wheel about the x' axis that passes through point O. The material has a specific weight of $\gamma = 90$ lb/ft^3.



Mass Moment of Inertia About an Axis Through Point G: The mass moment of inertia of each disk about an axis passing through the center of mass can be determine using $(I_G)_i = \frac{1}{2}mr^2$. Applying Eq. 10-16, we have



$$I_G = \Sigma(I_G)_i + m_i d_i^2$$

$$= \frac{1}{2}\left[\frac{\pi(2.5^2)(1)(90)}{32.2}\right](2.5^2) - \frac{1}{2}\left[\frac{\pi(2^2)(0.75)(90)}{32.2}\right](2^2)$$

$$-4\left\{\frac{1}{2}\left[\frac{\pi(0.25^2)(0.25)(90)}{32.2}\right](0.25^2) + \left[\frac{\pi(0.25^2)(0.25)(90)}{32.2}\right](1^2)\right\}$$

$$= 118.25 \text{ slug}\cdot\text{ft}^2$$

Mass Moment of Inertia About an Axis Through Point O: The mass of the wheel is

$$m = \frac{\pi(2.5^2)(1)(90)}{32.2} - \frac{\pi(2^2)(0.75)(90)}{32.2} - 4\left[\frac{\pi(0.25^2)(0.25)(90)}{32.2}\right]$$

$$= 27.989 \text{ slug}$$

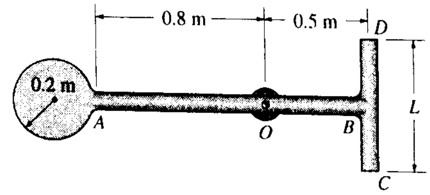
Using the result $I_G = 118.25$ slug · ft^2 and applying Eq. 10-16, we have

$$I_O = I_G + md^2$$

$$= 118.25 + 27.989(2.5^2)$$

$$= 293 \text{ slug}\cdot\text{ft}^2$$ **Ans**

***10-104.** The pendulum consists of a disk having a mass of 6 kg and slender rods AB and DC which have a mass of 2 kg/m. Determine the length L of DC so that the center of the mass is at the bearing O. What is the moment of inertia of the assembly about an axis perpendicular to the page and passing through point O?

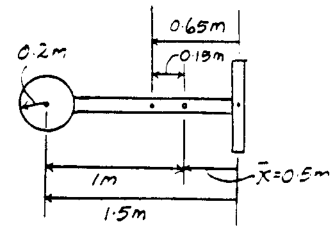


***Location of Centroid* :** This problem requires $\bar{x} = 0.5$ m.

$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m}$$

$$0.5 = \frac{1.5(6) + 0.65[1.3(2)] + 0[L(2)]}{6 + 1.3(2) + L(2)}$$

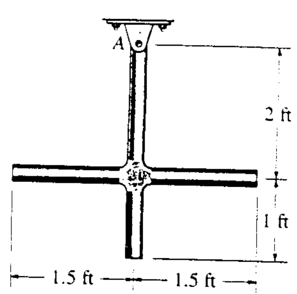
$$L = 6.39 \text{ m} \qquad \textbf{Ans}$$



***Mass Moment of Inertia About an Axis Through Point O* :** The mass moment of inertia of each rod segment and disk about an axis passing through the center of mass can be determine using $(I_G)_i = \frac{1}{12}ml^2$ and $(I_G)_i = \frac{1}{2}mr^2$. Applying Eq. 10 – 16, we have

$$\begin{aligned} I_O &= \Sigma(I_G)_i + m_i d_i^2 \\ &= \frac{1}{12}[1.3(2)](1.3^2) + [1.3(2)](0.15^2) \\ &\quad + \frac{1}{12}[6.39(2)](6.39^2) + [6.39(2)](0.5^2) \\ &\quad + \frac{1}{2}(6)(0.2^2) + 6(1^2) \\ &= 53.2 \text{ kg}\cdot\text{m}^2 \end{aligned} \qquad \textbf{Ans}$$

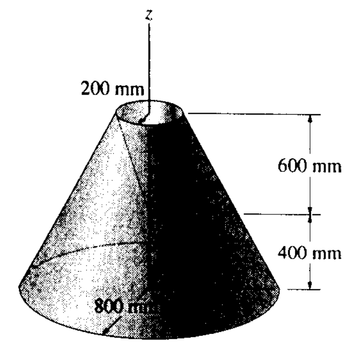
10-105. The slender rods have a weight of 3 lb/ft. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through point A.



$$I = \frac{1}{3}(3(\frac{3}{32.2}))(3)^2 + \frac{1}{12}(3(\frac{3}{32.2}))(3)^2 + (3(\frac{3}{32.2}))(2)^2$$

$$= 2.17 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

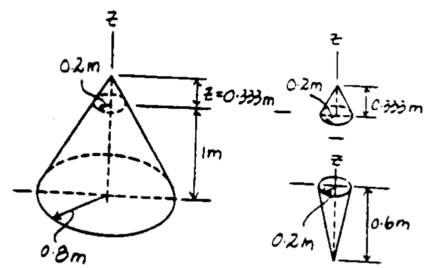
10-106. Determine the moment of inertia I_z of the frustrum of the cone which has a conical depression. The material has a density of 200 kg/m³.



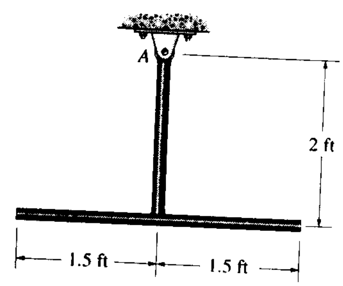
Mass Moment of Inertia About z Axis **: From similar triangles,** $\frac{z}{0.2} = \frac{z+1}{0.8}$, $z = 0.333$ m . The mass moment of inertia of each cone about z axis can be determine using $I_z = \frac{3}{10}mr^2$.

$$I_z = \Sigma(I_z)_i = \frac{3}{10}\left[\frac{\pi}{3}(0.8^2)(1.333)(200)\right](0.8^2) - \frac{3}{10}\left[\frac{\pi}{3}(0.2^2)(0.333)(200)\right](0.2^2) - \frac{3}{10}\left[\frac{\pi}{3}(0.2^2)(0.6)(200)\right](0.2^2)$$

$$= 34.2 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

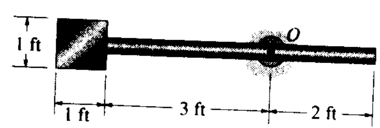


10-107. The slender rods have a weight of 3 lb/ft. Determine the moment of inertia of the assembly about an axis perpendicular to the page and passing through point A



$$I_A = \frac{1}{3}\left[\frac{3(2)}{32.2}\right](2)^2 + \frac{1}{12}\left[\frac{3(3)}{32.2}\right](3)^2 + \left[\frac{3(3)}{32.2}\right](2)^2 = 1.58 \text{ slug}\cdot\text{ft}^2 \qquad \textbf{Ans}$$

***10-108.** The pendulum consists of a plate having a weight of 12 lb and a slender rod having a weight of 4 lb. Determine the radius of gyration of the pendulum about an axis perpendicular to the page and passing through point O



$$I_O = \Sigma I_G + md^2$$

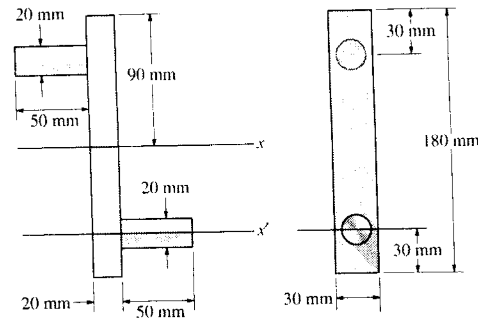
$$= \frac{1}{12}\left(\frac{4}{32.2}\right)(5)^2 + \left(\frac{4}{32.2}\right)(0.5)^2 + \frac{1}{12}\left(\frac{12}{32.2}\right)(1^2+1^2) + \left(\frac{12}{32.2}\right)(3.5)^2$$

$$= 4.917 \text{ slug}\cdot\text{ft}^2$$

$$m = \left(\frac{4}{32.2}\right) + \left(\frac{12}{32.2}\right) = 0.4969 \text{ slug}$$

$$k_O = \sqrt{\frac{I_O}{m}} = \sqrt{\frac{4.917}{0.4969}} = 3.15 \text{ ft} \qquad \textbf{Ans}$$

10-109. Determine the moment of inertia of the overhung crank about the x axis. The material is steel having a density of $\rho = 7.85\ \text{Mg/m}^3$.



Let m = mass of one handle.

$m = \rho(\pi r^2 h)$

$= (7.85 \times 10^3)\pi(0.010)^2(0.050)$

$= 0.1233$ kg

Let M = mass of bar.

$M = \rho(abc)$

$= (7.85 \times 10^3)(0.03)(0.18)(0.02)$

$= 0.8478$ kg

For the assembly,

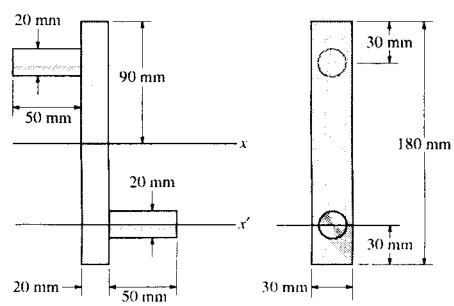
$$I_x = 2\left(\frac{1}{2}mr^2 + md^2\right) + \frac{1}{12}M(a^2 + b^2)$$

$$= 2\left[\frac{1}{2}(0.1233)(0.010)^2 + (0.1233)(0.060)^2\right]$$

$$+ \frac{1}{12}(0.8478)[(0.030)^2 + (0.18)^2]$$

$= 3.25 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ **Ans**

10-110. Determine the moment of inertia of the overhung crank about the x' axis. The material is steel having a density of $\rho = 7.85\ \text{Mg/m}^3$.



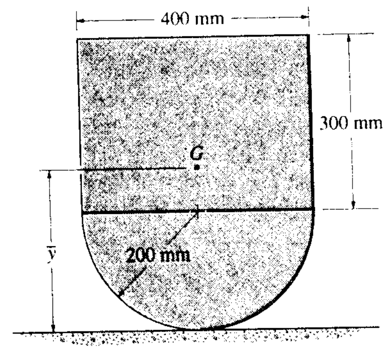
From 10-109, $m = 0.1233$ kg, $M = 0.8478$ kg, and $\bar{I}_x = 3.25 \times 10^{-3}\text{kg}\cdot\text{m}^{-2}$.

$I_{x'} = I_x + (2m + M)d^2$

$= 3.25 \times 10^{-3} + [2(0.1233) + 0.8478](0.060)^2$

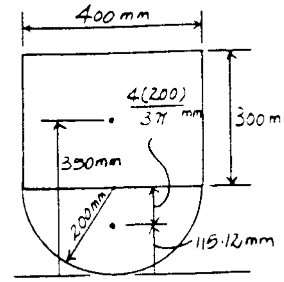
$= 7.20 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ **Ans**

10-111. Determine the location of $\bar{y}$ of the center of mass G of the assembly and then calculate the moment of inertia about an axis perpendicular to the page and passing through G. The block has a mass of 3 kg and the mass of the semicylinder is 5 kg.



Location of Centroid :

$$\bar{y} = \frac{\Sigma \tilde{y} m}{\Sigma m} = \frac{350(3) + 115.12(5)}{3+5} = 203.20 \text{ mm} = 203 \text{ mm} \qquad \textbf{Ans}$$

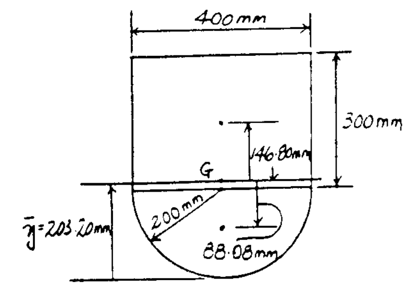


Mass Moment of Inertia About an Axis Through Point G : The mass moment of inertia of a rectangular block and a semicylinder about an axis passing through the center of mass perpendicular to the page can be determine using

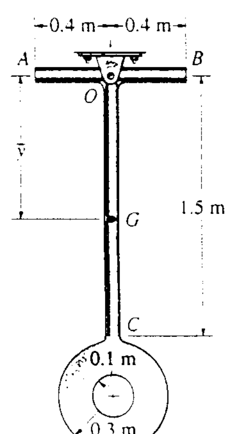
$$(I_z)_G = \frac{1}{12}m\left(a^2+b^2\right) \text{ and } (I_z)_G = \frac{1}{2}mr^2 - m\left(\frac{4r}{3\pi}\right)^2 = 0.3199mr^2$$

respectively. Applying Eq. 10 – 16, we have

$$\begin{aligned} I_G &= \Sigma (I_z)_{G_i} + m_i d_i^2 \\ &= \left[\frac{1}{12}(3)\left(0.3^2+0.4^2\right) + 3\left(0.1468^2\right)\right] \\ &\quad + \left[0.3199(5)\left(0.2^2\right) + 5\left(0.08808^2\right)\right] \\ &= 0.230 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans} \end{aligned}$$



***10-112.** The pendulum consists of two slender rods AB and OC which have a mass of 3 kg/m. The thin plate has a mass of 12 kg/m^2. Determine the location $\bar{y}$ of the center of mass G of the pendulum, then calculate the moment of inertia of the pendulum about an axis perpendicular to the page and passing through G.



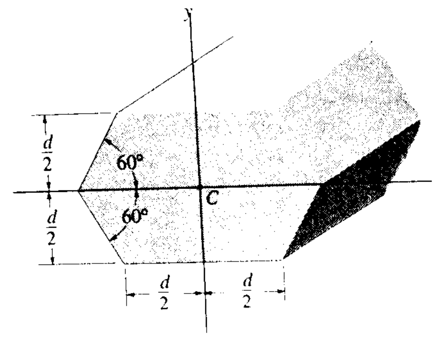
$$\bar{y} = \frac{1.5(3)(0.75) + \pi(0.3)^2(12)(1.8) - \pi(0.1)^2(12)(1.8)}{1.5(3) + \pi(0.3)^2(12) - \pi(0.1)^2(12) + 0.8(3)}$$

$$= 0.8878 \text{ m} = 0.888 \text{ m} \qquad \textbf{Ans}$$

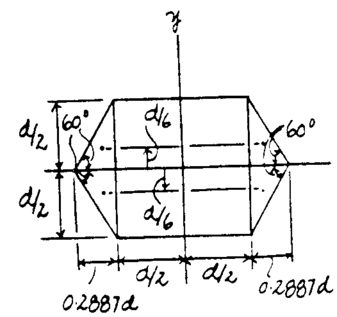
$$\begin{aligned} I_G = &\frac{1}{12}(0.8)(3)(0.8)^2 + 0.8(3)(0.8878)^2 \\ &+ \frac{1}{12}(1.5)(3)(1.5)^2 + 1.5(3)(0.75 - 0.8878)^2 \\ &+ \frac{1}{2}[\pi(0.3)^2(12)(0.3)^2 + [\pi(0.3)^2(12)](1.8-0.8878)^2 \\ &- \frac{1}{2}[\pi(0.1)^2(12)(0.1)^2 - [\pi(0.1)^2(12)](1.8-0.8878)^2 \end{aligned}$$

$$I_G = 5.61 \text{ kg}\cdot\text{m}^2 \qquad \textbf{Ans}$$

10-113. Determine the moment of inertia of the beam's cross-sectional area about the x axis which passes through the centroid C.

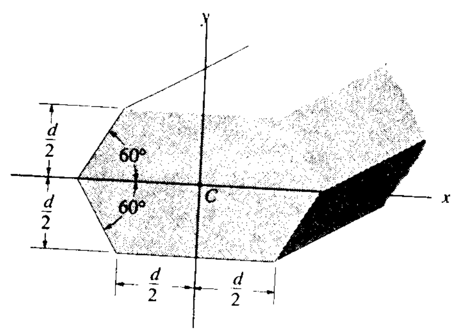


Moment of Inertia : The moment of inertia about the x axis for the composite beam's cross section can be determined using the parallel − axis theorem $I_x = \Sigma(\bar{I}_x + Ad_y^2)_i$.

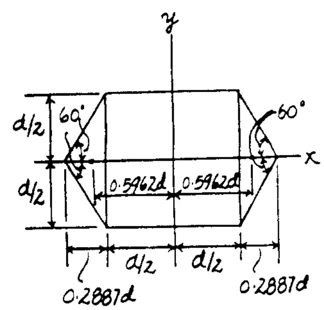


$$I_x = \left[\frac{1}{12}(d)(d^3)+0\right] + 4\left[\frac{1}{36}(0.2887d)\left(\frac{d}{2}\right)^3 + \frac{1}{2}(0.2887d)\left(\frac{d}{2}\right)\left(\frac{d}{6}\right)^2\right]$$
$$= 0.0954d^4 \qquad \textbf{Ans}$$

10-114. Determine the moment of inertia of the beam's cross-sectional area about the y axis which passes through the centroid C.

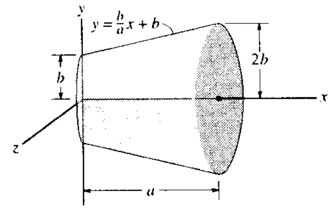


Moment of Inertia : The moment of inertia about yaxis for the composite beam's cross section can be determined using the parallel − axis theorem $I_y = \Sigma(\bar{I}_y + Ad_x^2)_i$.



$$I_y = \left[\frac{1}{12}(d)(d^3)+0\right] + 2\left[\frac{1}{36}(d)(0.2887d)^3 + \frac{1}{2}(d)(0.2887d)(0.5962d)^2\right]$$
$$= 0.187d^4 \qquad \textbf{Ans}$$

10-115. Determine the moment of inertia I_x of the body and express the result in terms of the total mass m of the body. The density is constant.



$$dm = \rho dV = \rho\pi y^2\,dx = \rho\pi\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx$$

$$dI_x = \tfrac{1}{2}dm\,y^2 = \tfrac{1}{2}\rho\pi y^4\,dx$$

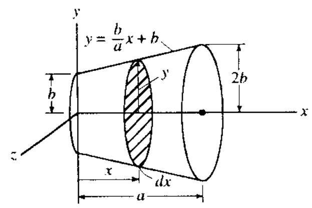
$$dI_x = \frac{1}{2}\rho\pi\left(\frac{b^4}{a^4}x^4 + \frac{4b^4}{a^3}x^3 + \frac{6b^4}{a^2}x^2 + \frac{4b^4}{a}x + b^4\right)dx$$

$$I_x = \int dI_x = \tfrac{1}{2}\rho\pi\int_0^a\left(\frac{b^4}{a^4}x^4 + \frac{4b^4}{a^3}x^3 + \frac{6b^4}{a^2}x^2 + \frac{4b^4}{a}x + b^4\right)dx$$

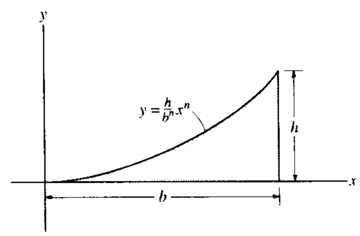
$$= \frac{31}{10}\rho\pi a b^4$$

$$m = \int_m dm = \rho\pi\int_0^a\left(\frac{b^2}{a^2}x^2 + \frac{2b^2}{a}x + b^2\right)dx = \frac{7}{3}\rho\pi a b^2$$

$$I_x = \frac{93}{70}mb^2$$ **Ans**



***10-116.** Determine the moments of inertia I_x and I_y of the shaded area.



$$I_x = \int d\,I_x$$

$$= \int_0^b \frac{1}{3}y^3\,dx = \int_0^b \frac{h^3}{3\,b^{3n}}x^{3n}\,dx$$

$$= \frac{h^3}{(3n+1)3b^{3n}}b^{3n+1}$$

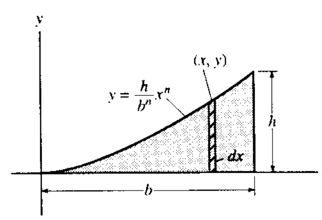
$$= \frac{1}{3(3n+1)}bh^3$$ **Ans**

$$I_y = \int x^2 dA$$

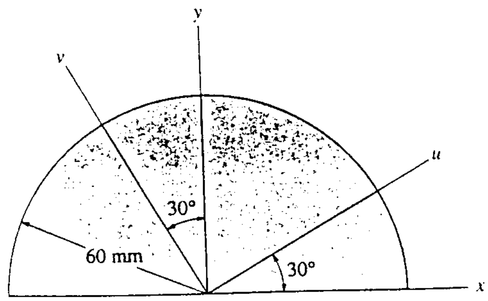
$$= \int_0^b \frac{h}{b^n}x^{n+2}\,dx$$

$$= \frac{h}{b^n(n+3)}b^{n+3}$$

$$= \frac{1}{n+3}b^3 h$$ **Ans**



10-117. Determine the moments of inertia I_u and I_v and the product of inertia I_{uv} for the semicircular area.



$$I_x = I_y = \frac{1}{8}\pi(60)^4 = 5\,089\,380.1 \text{ mm}^4$$

$I_{xy} = 0$ (Due to symmetry)

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{5\,089\,380.1 + 5\,089\,380.1}{2} + 0 - 0$$

$I_u = 5.09(10^6)\ \text{mm}^4$ **Ans**

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \frac{5\,089\,380.1 + 5\,089\,380.1}{2} - 0 + 0$$

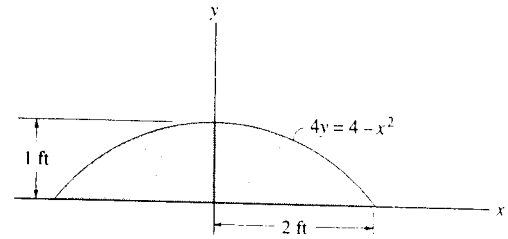
$I_v = 5.09(10^6)\ \text{mm}^4$ **Ans**

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

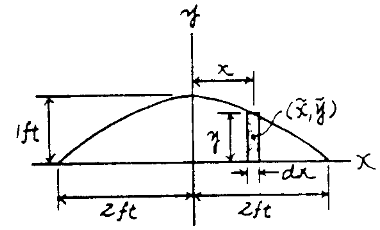
$= 0 + 0$

$I_{uv} = 0$ **Ans**

*10-118. Determine the moment of inertia of the shaded area about the y axis.



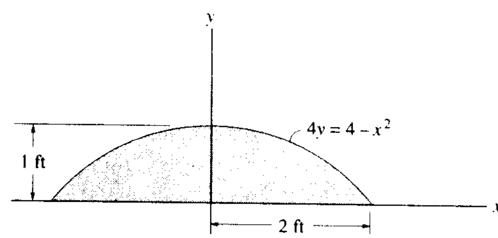
Differential Element : Here, $y = \frac{1}{4}(4-x^2)$. The area of the differential element parallel to the y axis is $dA = y\,dx = \frac{1}{4}(4-x^2)\,dx$.



Moment of Inertia : Applying Eq. 10-1 and performing the integration, we have

$$
\begin{aligned}
I_y &= \int_A x^2 dA = \frac{1}{4}\int_{-2\text{ft}}^{2\text{ft}} x^2(4-x^2)\,dx \\
&= \frac{1}{4}\left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]_{-2\text{ft}}^{2\text{ft}} \\
&= 2.13\ \text{ft}^4 \qquad \textbf{Ans}
\end{aligned}
$$

10-119. Determine the moment of inertia of the shaded area about the x axis.



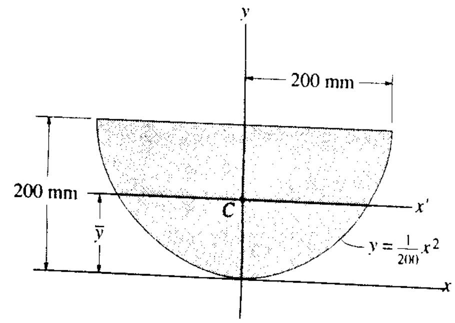
Differential Element : Here, $y = \frac{1}{4}(4-x^2)$. The area of the differential element parallel to the y axis is $dA = y\,dx$. The moment of inertia of this differential element about the x axis is

$$
\begin{aligned}
dI_x &= d\bar{I}_{x'} + dA\,\tilde{y}^2 \\
&= \frac{1}{12}(dx)y^3 + y\,dx\left(\frac{y}{2}\right)^2 \\
&= \frac{1}{3}\left[\frac{1}{4}(4-x^2)\right]^3 dx \\
&= \frac{1}{192}(-x^6 + 12x^4 - 48x^2 + 64)\,dx
\end{aligned}
$$

Moment of inertia : Performing the integration, we have

$$
\begin{aligned}
I_x &= \int dI_x = \frac{1}{192}\int_{-2\text{ft}}^{2\text{ft}} (-x^6 + 12x^4 - 48x^2 + 64)\,dx \\
&= \frac{1}{192}\left(-\frac{1}{7}x^7 + \frac{12}{5}x^5 - 16x^3 + 64x\right)\Bigg|_{-2\text{ft}}^{2\text{ft}} \\
&= 0.610\ \text{ft}^4 \qquad \textbf{Ans}
\end{aligned}
$$

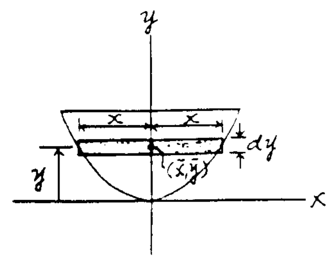
***10-120.** Determine the moment of inertia of the area about the x axis. Then, using the parallel-axis theorem, find the moment of inertia about the x' axis that passes through the centroid C of the area. $\bar{y} = 120$ mm.



Differential Element : Here, $x = \sqrt{200}y^{\frac{1}{2}}$. The area of the differential element parallel to the x axis is $dA = 2x dy = 2\sqrt{200}y^{\frac{1}{2}}dy$.

Moment of Inertia : Applying Eq. 10 – 1 and performing the integration, we have

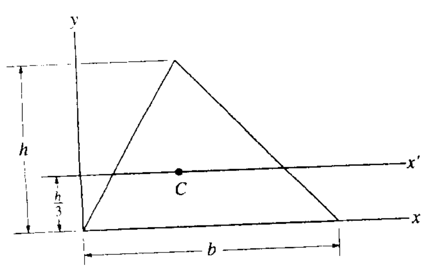
$$I_x = \int_A y^2 dA = \int_0^{200\text{mm}} y^2\left(2\sqrt{200}y^{\frac{1}{2}}dy\right)$$
$$= 2\sqrt{200}\left(\frac{2}{7}y^{\frac{7}{2}}\right)\Big|_0^{200\text{mm}}$$
$$= 914.29(10^6)\ \text{mm}^4 = 914(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$



The moment of inertia about the x' axis can be determined using the parallel - axis theorem. The area is $A = \int_A dA = \int_0^{200\text{mm}} 2\sqrt{200}y^{\frac{1}{2}}dy = 53.33(10^3)\ \text{mm}^2$

$$I_x = \bar{I}_{x'} + Ad_y^2$$
$$914.29(10^6) = \bar{I}_{x'} + 53.33(10^3)(120^2)$$
$$\bar{I}_{x'} = 146(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

10-121. Determine the moment of inertia of the triangular area about (a) the x axis, and (b) the centroidal x' axis.



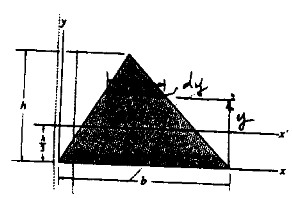
$$\frac{s}{h-y} = \frac{b}{h}$$
$$s = \frac{b}{h}(h-y)$$

(a) $dA = s\,dy = [\frac{b}{h}(h-y)]\,dy$

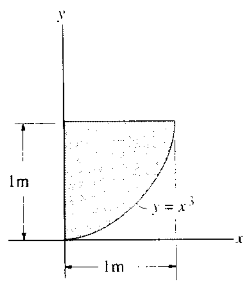
$$I_x = \int y^2\,dA$$
$$= \int_0^h y^2[\frac{b}{h}(h-y)]dy$$
$$= \frac{bh^3}{12} \qquad \textbf{Ans}$$

(b) $I_x = \bar{I}_{x'} + A\,d^2$

$$\frac{bh^3}{12} = \bar{I}_{x'} + \frac{1}{2}bh(\frac{h}{3})^2$$
$$\bar{I}_{x'} = \frac{bh^3}{36} \qquad \textbf{Ans}$$



10-122. Determine the product of inertia of the shaded area with respect to the x and y axes.



Differential Element: Here, $x = y^{\frac{1}{3}}$. The area of the differential element parallel to the x axis is $dA = x\,dy = y^{\frac{1}{3}}dy$. The coordinates of the centroid for this element are $\tilde{x} = \frac{x}{2} = \frac{1}{2}y^{\frac{1}{3}}$, $\tilde{y} = y$. Then the product of inertia for this element i.

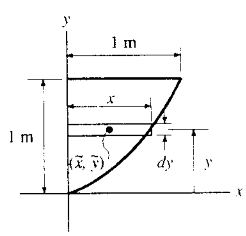
$$dI_{xy} = dI_{x'y'} + dA\,\tilde{x}\,\tilde{y}$$

$$= 0 + \left(y^{\frac{1}{3}}dy\right)\left(\frac{1}{2}y^{\frac{1}{3}}\right)(y)$$

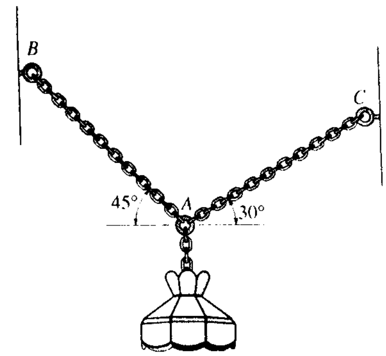
$$= \frac{1}{2}y^{\frac{5}{3}}dy$$

Product of Inertia: Performing the integration, we have

$$I_{xy} = \int dI_{xy} = \int_0^{1\,\text{m}} \frac{1}{2}y^{\frac{5}{3}}dy = \frac{3}{16}y^{\frac{8}{3}}\Big|_0^{1\,\text{m}} = 0.1875\ \text{m}^4 \quad \textbf{Ans}$$



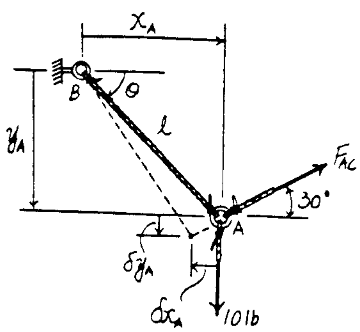
11-1. Use the method of virtual work to determine the tensions in cable AC. The lamp weighs 10 lb.



Free Body Diagram : The tension in cable AC can be determined by releasing cable AC. The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only $\mathbf{F}_{AC}$ and the weight of lamp (10 lb force) do work.

Virtual Displacements : Force $\mathbf{F}_{AC}$ and 10 lb force are located from the fixed point B using position coordinates y_A and x_A.

$$x_A = l\cos\theta \qquad \delta x_A = -l\sin\theta\,\delta\theta \qquad [1]$$
$$y_A = l\sin\theta \qquad \delta y_A = l\cos\theta\,\delta\theta \qquad [2]$$



Virtual - Work Equation : When y_A and x_A undergo positive virtual displacements δy_A and δx_A, the 10 lb force and horizontal component of $\mathbf{F}_{AC}$, $F_{AC}\cos 30°$ do positive work while the vertical component of $\mathbf{F}_{AC}$, $F_{AC}\sin 30°$ does negative work.

$$\delta U = 0; \qquad 10\delta y_A - F_{AC}\sin 30°\,\delta y_A + F_{AC}\cos 30°\,\delta x_A = 0 \qquad [3]$$

Substituting Eqs. [1] and [2] into [3] yields

$$(10\cos\theta - 0.5F_{AC}\cos\theta - 0.8660F_{AC}\sin\theta)\,l\delta\theta = 0$$

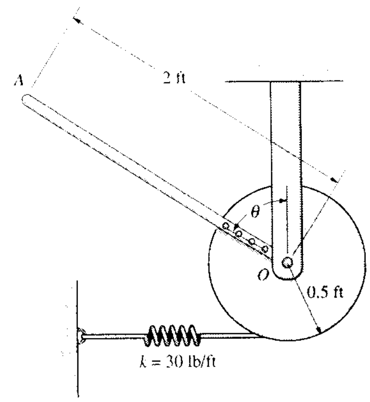
Since $l\delta\theta \neq 0$, then

$$F_{AC} = \frac{10\cos\theta}{0.5\cos\theta + 0.8660\sin\theta}$$

At the equilibrium position $\theta = 45°$,

$$F_{AC} = \frac{10\cos 45°}{0.5\cos 45° + 0.8660\sin 45°} = 7.32 \text{ lb} \qquad \textbf{Ans}$$

11-2. The uniform rod OA has a weight of 10 lb. When the rod is in vertical position, $\theta = 0°$, the spring is unstretched. Determine the angle θ for equilibrium if the end of the spring wraps around the periphery of the disk as the disk turns.



Free Body Diagram: The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force and the weight of rod (10 lb force) do work.

Virtual Displacements: The 10 lb force is located from the fixed point B using the position coordinate y_B, and the virtual displacement of point C is δx_C.

$y_B = 1\cos\theta \qquad \delta y_B = -\sin\theta\delta\theta$ [1]

$\delta x_C = 0.5\delta\theta$ [2]

Virtual—Work Equation: When points B and C undergo positive virtual displacements δy_B and δx_C, the 10 lb force and the spring force F_{sp}, do positive work.

$\delta U = 0; \quad 10\delta y_B + F_{sp}\delta x_C = 0$ [3]

Substituting Eqs. [1] and [2] into [3] yields

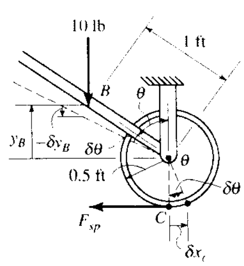
$(-10\sin\theta + 0.5F_{sp})\,\delta\theta = 0$ [4]

However, from the spring formula, $F_{sp} = kx = 30(0.5\theta) = 15\theta$. Substituting this value into Eq. [4] yields

$(-10\sin\theta + 7.5\theta)\,\delta\theta = 0$

Since $\delta\theta \neq 0$, then

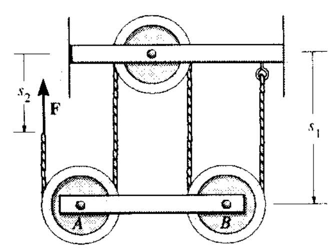
$-10\sin\theta + 7.5\theta = 0$



Solving by trial and error

$\theta = 0°$ and $\theta = 73.1°$ **Ans**

11-3. Determine the force **F** acting on the cord which is required to maintain equilibrium of the horizontal 10-kg bar AB. *Hint*: Express the total *constant vertical length l* of the cord in terms of position coordinates s_1 and s_2. The derivative of this equation yields a relationship between δ_1 and δ_2.



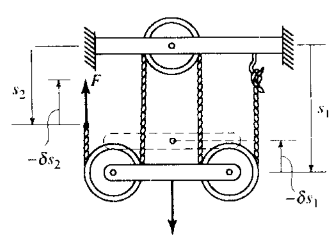
Free—Body Diagram: Only force **F** and the weight of link AB (98.1 N) do work.

Virtual Displacements: Force **F** and the weight of link AB (98.1 N) are located from the top of the fixed link using position coordinates s_2 and s_1. Since the cord has a constant length, l, then

$4s_1 - s_2 = l \qquad 4\delta s_1 - \delta s_2 = 0$ [1]

Virtual—Work Equation: When s_1 and s_2 undergo positive virtual displacements δs_1 and δs_2, the weight of link AB (98.1 N) and force **F** do positive work and negative work, respectively.

$\delta U = 0; \quad 98.1(-\delta s_1) - F(-\delta s_2) = 0$ [2]



Substituting into Eq. [2] into [1] yields

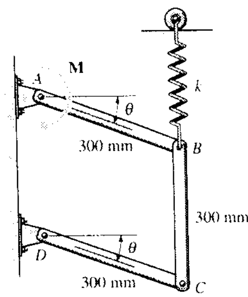
$(-98.1 + 4F)\,\delta s_1 = 0$

Since $\delta s_1 \neq 0$, then

$-98.1 + 4F = 0$

$F = 24.5$ N **Ans**

11-4. Each member of the pin-connected mechanism has a mass of 8 kg. If the spring is unstretched when $\theta = 0^\circ$, determine the angle θ for equilibrium. Set $k = 2500$ N/m and $M = 50$ N·m.



$y_1 = 0.15\sin\theta$

$y_2 = 0.3\sin\theta$

$\delta y_1 = 0.15\cos\theta\delta\theta$

$\delta y_2 = 0.3\cos\theta\delta\theta$

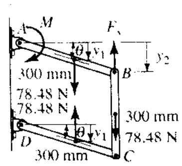
$\delta U = 0;\quad 2(78.48)\delta y_1 + 78.48\delta y_2 - F_2\delta y_2 + 50\delta\theta = 0$

$[2(78.48)(0.15\cos\theta) + 78.48(0.3\cos\theta) - F_2(0.3\cos\theta) + 50]\delta\theta = 0$

$47.088\cos\theta - F_2(0.3\cos\theta) + 50 = 0$

$F_2 = 2500(0.3\sin\theta) = 750\sin\theta$

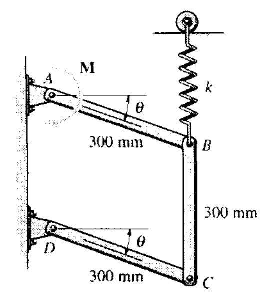
$47.088\cos\theta - 112.5\sin 2\theta + 50 = 0$



Solving, $\theta = 27.4^\circ$ **Ans**

or $\theta = 72.7^\circ$ **Ans**

11-5. Each member of the pin-connected mechanism has a mass of 8 kg. If the spring is unstretched when $\theta = 0^\circ$, determine the required stiffness k so that the mechanism is in equilibrium when $\theta = 30^\circ$. Set M = 0.



$y_1 = 0.15\sin\theta, \quad y_2 = 0.3\sin\theta$

$\delta y_1 = 0.15\cos\theta\delta\theta, \quad \delta y_2 = 0.3\cos\theta\delta\theta$

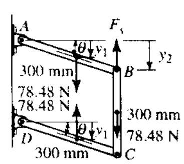
$\delta U = 0;\quad 2(78.48)\delta y_1 + 78.48\delta y_2 - F_2\delta y_2 = 0$

$[2(78.48)(0.15\cos\theta) + 78.48(0.3\cos\theta) - F_2(0.3\cos\theta)]\delta\theta = 0$

$\theta = 30^\circ;\quad F_2 = k(0.3\sin 30^\circ) = 0.15k$

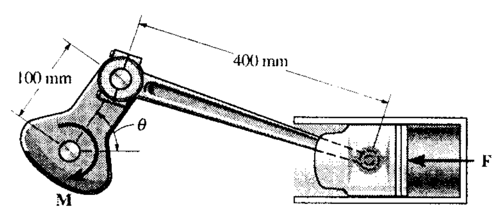
$2(78.48)(0.15\cos 30^\circ) + 78.48(0.3\cos 30^\circ)$

$\quad - 0.15k(0.3\cos 30^\circ) = 0$



$k = 1.05$ kN/m **Ans**

11-6. The crankshaft is subjected to a torque of $M =$ 50 N·m. Determine the horizontal compressive force F applied to the piston for equilibrium when $\theta = 60^\circ$.



$(0.4)^2 = (0.1)^2 + x^2 - 2(0.1)(x)(\cos\theta)$

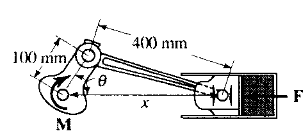
$0 = 0 + 2x\delta x + 0.2x\sin\theta\delta\theta - 0.2\cos\theta\delta x$

$\delta U = 0;\quad -50\delta\theta - F\delta x = 0$

For $\theta = 60^\circ$, $x = 0.4405$ m

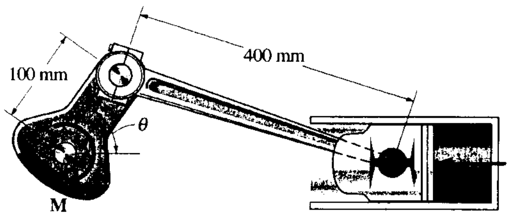
$\delta x = -0.09769\delta\theta$

$(-50 + 0.09769F)\delta\theta = 0$

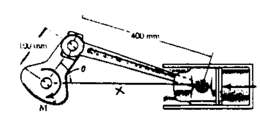


$F = 512$ N **Ans**

11-7. The crankshaft is subjected to a torque of $M = 50\ \mathrm{N \cdot m}$. Determine the horizontal compressive force F and plot the result of F (ordinate) versus θ (abscissa) for $0° \le \theta \le 90°$.



$$(0.4)^2 = (0.1)^2 + x^2 - 2(0.1)(x)(\cos\theta) \qquad (1)$$



$$0 = 0 + 2x\,\delta x + 0.2x\sin\theta\,\delta\theta - 0.2\cos\theta\,\delta x$$

$$\delta x = \left(\frac{0.2x\sin\theta}{0.2\cos\theta - 2x}\right)\delta\theta$$

$$\delta U = 0; \qquad -50\delta\theta - F\delta x = 0$$

$$-50\delta\theta - F\left(\frac{0.2x\sin\theta}{0.2\cos\theta - 2x}\right)\delta\theta = 0, \qquad \delta\theta \neq 0$$

$$F = \frac{50(2x - 0.2\cos\theta)}{0.2x\sin\theta}$$

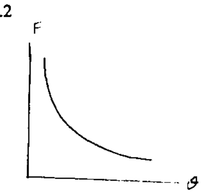
From Eq. (1)

$$x^2 - 0.2x\cos\theta - 0.15 = 0$$

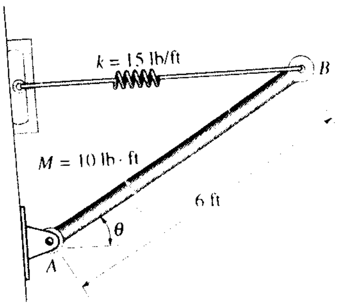
$$x = \frac{0.2\cos\theta \pm \sqrt{0.04\cos^2\theta + 0.6}}{2}, \qquad \text{since } \sqrt{0.04\cos^2\theta + 0.6} > 0.2$$

$$x = \frac{0.2\cos\theta + \sqrt{0.04\cos^2\theta + 0.6}}{2}$$

$$F = \frac{500\sqrt{0.04\cos^2\theta + 0.6}}{(0.2\cos\theta + \sqrt{0.04\cos^2\theta + 0.6})\sin\theta} \qquad \textbf{Ans}$$



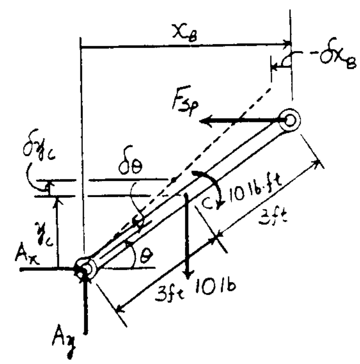
*11-8. Determine the force developed in the spring required to keep the 10 lb uniform rod AB in equilibrium when $\theta = 35°$.



Free - Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force F_{sp}, the weight of the rod (10 lb) and the 10 lb · ft couple moment do work.

Virtual Displacements : The spring force F_{sp} and the weight of the rod (10 lb) are located from the fixed point A using position coordinates x_B and x_C, respectively.

$$x_B = 6\cos\theta \qquad \delta x_B = -6\sin\theta\,\delta\theta \qquad [1]$$
$$y_C = 3\sin\theta \qquad \delta y_C = 3\cos\theta\,\delta\theta \qquad [2]$$



Virtual - Work Equation : When points B and C undergo positive virtual displacements δx_B and δy_C, the spring force F_{sp} and the weight of the rod (10 lb) do negative work. The 10 lb · ft couple moment does negative work when rod AB undergoes a positive virtual rotation $\delta\theta$.

$$\delta U = 0; \qquad -F_{sp}\,\delta x_B - 10\delta y_C - 10\delta\theta = 0 \qquad [3]$$

Substituting Eqs. [1] and [2] into [3] yields

$$(6F_{sp}\sin\theta - 30\cos\theta - 10)\,\delta\theta = 0 \qquad [4]$$

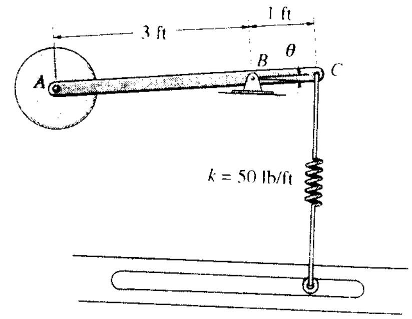
Since $\delta\theta \neq 0$, then

$$6F_{sp}\sin\theta - 30\cos\theta - 10 = 0$$
$$F_{sp} = \frac{30\cos\theta + 10}{6\sin\theta}$$

At the equilibrium position, $\theta = 35°$. Then

$$F_{sp} = \frac{30\cos 35° + 10}{6\sin 35°} = 10.0\text{ lb} \qquad \textbf{Ans}$$

11-9. Determine the angles θ for equilibrium of the 4-lb disk using using the principle of the virtual work. Neglect the weight of the rod. The spring is unstretched when $\theta = 0°$ and always remains in the vertical position due to the roller guide.

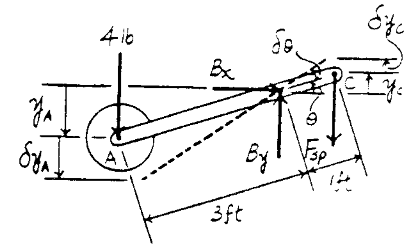


Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force F_{sp} and the weight of the disk (4 lb) do work.

Virtual Displacements : The spring force F_{sp} and the weight of the disk (4 lb) are located from the fixed point B using position coordinates y_C and y_A, respectively.

$$y_C = 1\sin\theta \qquad \delta y_C = \cos\theta\,\delta\theta \qquad [1]$$

$$y_A = 3\sin\theta \qquad \delta y_A = 3\cos\theta\,\delta\theta \qquad [2]$$



Virtual - Work Equation : When points C and A undergo positive virtual displacements δy_C and δy_A, the spring force F_{sp} does negative work while the weight of the disk (4 lb) do positive work.

$$\delta U = 0; \qquad 4\delta y_A - F_{sp}\,\delta y_C = 0 \qquad [3]$$

Substituting Eqs.[1] and [2] into [3] yields

$$(12 - F_{sp})\cos\theta\,\delta\theta = 0 \qquad [4]$$

However, from the spring formula, $F_{sp} = kx = 50(1\sin\theta) = 50\sin\theta$. Substituting this value into Eq.[4] yields

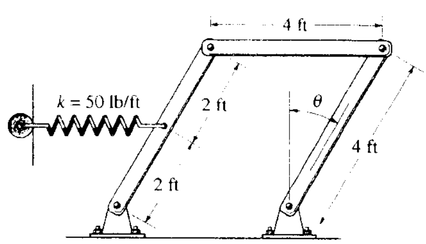
$$(12 - 50\sin\theta)\cos\theta\,\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

$$12 - 50\sin\theta = 0 \qquad \theta = 13.9° \qquad \textbf{Ans}$$

$$\cos\theta = 0 \qquad \theta = 90° \qquad \textbf{Ans}$$

11-10. If each of the three links of the mechanism has a weight of 20 lb, determine the angle θ for equilibrium of the spring, which, due to the roller guide, always remains horizontal and is unstretched when $\theta = 0°$.



$$x = 2\sin\theta, \qquad \delta x = 2\cos\theta\,\delta\theta$$

$$y_1 = 2\cos\theta, \qquad \delta y_1 = -2\sin\theta\,\delta\theta$$

$$y_2 = 4\cos\theta, \qquad \delta y_2 = -4\sin\theta\,\delta\theta$$

$$\Delta x = 2\sin\theta$$

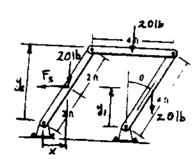
$$F_s = k\Delta x = 50(2\sin\theta) = 100\sin\theta$$

$$\delta U = 0; \qquad -20\delta y_2 - 2(20\delta y_1) - F_s\delta x = 0$$

$$[20(4\sin\theta) + 2(20)(2\sin\theta) - F_s(2\cos\theta)]\delta\theta = 0$$

$$[160\sin\theta - 200\sin\theta\cos\theta]\delta\theta = 0$$

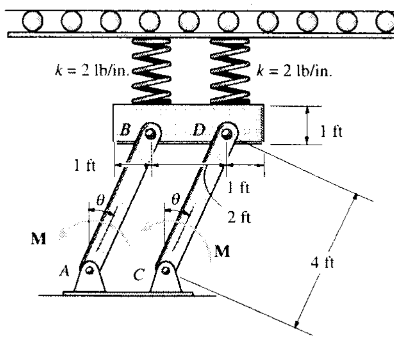
$$F_s = k(4\cos\theta - 4\cos 45°)$$



Hence, $\sin\theta = 0; \qquad \theta = 0° \qquad$ **Ans**

$$\cos\theta = \frac{160}{200}; \qquad \theta = 36.9° \qquad \textbf{Ans}$$

11-11. When $\theta = 20^\circ$, the 50-lb uniform block compresses the two vertical springs 4 in. If the uniform links AB and CD each weigh 10 lb, determine the magnitude of the applied couple moments **M** needed to maintain equilibrium when $\theta = 20^\circ$.



Free Body Diagram: The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive virtual displacement $\delta\theta$, only the spring forces F_{sp}, the weight of the block (50 lb), the weights of the links (10 lb) and the couple moment **M** do work.

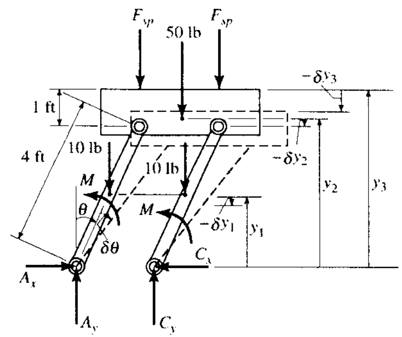
Virtual Displacements: The spring forces F_{sp}, the weight of the block (50 lb) and the weight of the links (10 lb) are located from the fixed point C using position coordinates y_3, y_2 and y_1 respectively.

$y_3 = 1 + 4\cos\theta \quad \delta y_3 = -4\sin\theta\delta\theta$ [1]

$y_2 = 0.5 + 4\cos\theta \quad \delta y_2 = -4\sin\theta\delta\theta$ [2]

$y_1 = 2\cos\theta \quad \delta y_1 = -2\sin\theta\delta\theta$ [3]

Virtual—Work Equation: When y_1, y_2 and y_3 undergo positive virtual displacements δy_1, δy_2 and δy_3, the spring forces F_{sp}, the weight of the block (50 lb) and the weights of the links (10 lb) do negative work. The couple moment **M** does negative work when the links undergo a positive virtual rotation $\delta\theta$.



$\delta U = 0; \quad -2F_{sp}\delta y_3 - 50\delta y_2 - 20\delta y_1 - 2M\delta\theta = 0$ [4]

Substituting Eqs. [1], [2] and [3] into [4] yields

$$(8F_{sp}\sin\theta + 240\sin\theta - 2M)\,\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

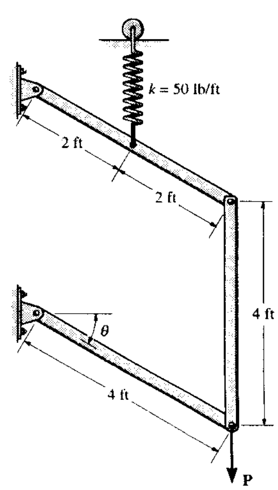
$8F_{sp}\sin\theta + 240\sin\theta - 2M = 0$

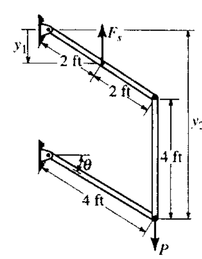
$M = \sin\theta(4F_{sp} + 120)$

At the equilibrium position $\theta = 20^\circ$, $F_{sp} = kx = 2(4) = 8$ lb.

$M = \sin 20^\circ[4(8) + 120] = 52.0$ lb · ft **Ans**

***11-12.** The spring is unstretched when $\theta = 0^\circ$. If $P = 8$ lb, determine the angle θ for equilibrium. Due to the roller guide, the spring always remains vertical. Neglect the weight of the links.





$y_1 = 2\sin\theta, \quad \delta y_1 = 2\cos\theta\,\delta\theta$

$y_2 = 4\sin\theta + 4, \quad \delta y_2 = 4\cos\theta\,\delta\theta$

$F_s = 50(2\sin\theta) = 100\sin\theta$

$\delta U = 0; \quad -F_s\,\delta y_1 + P\delta y_2 = 0$

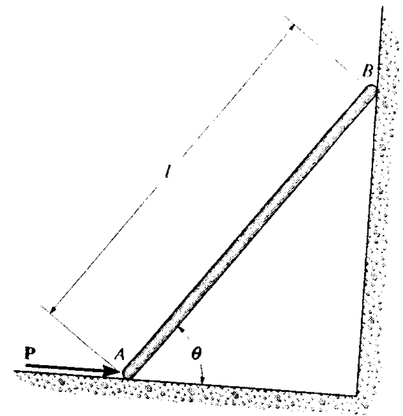
$-100\sin\theta(2\cos\theta\,\delta\theta) + 8(4\cos\theta\,\delta\theta) = 0$

Assume $\Theta < 90^\circ$, so $\cos\Theta \neq 0$.

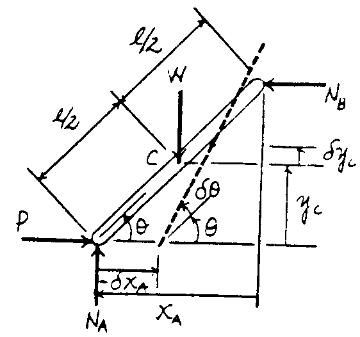
$200\sin\theta = 32$

$\theta = 9.21^\circ$ **Ans**

11-13. The thin rod of weight W rest against the smooth wall and floor. Determine the magnitude of force **P** needed to hold it in equilibrium for a given angle θ.



Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the weight of the rod W and force **P** do work.



Virtual Displacements : The weight of the rod W and force **P** are located from the fixed points A and B using position coordinates y_C and x_A, respectively

$$y_C = \frac{l}{2}\sin\theta \qquad \delta y_C = \frac{l}{2}\cos\theta\,\delta\theta \qquad [1]$$

$$x_A = l\cos\theta \qquad \delta x_A = -l\sin\theta\,\delta\theta \qquad [2]$$

Virtual - Work Equation : When points C and A undergo positive virtual displacements δy_C and δx_A, the weight of the rod W and force **F** do negative work.

$$\delta U = 0; \qquad -W\delta y_C - P\delta y_A = 0 \qquad [3]$$

Substituting Eqs. [1] and [2] into [3] yields

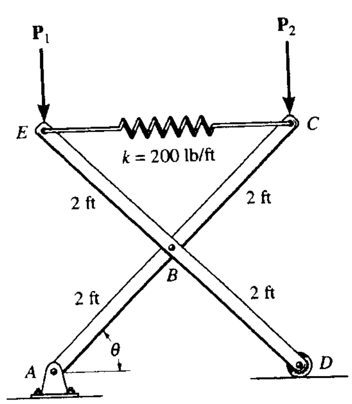
$$\left(Pl\sin\theta - \frac{Wl}{2}\cos\theta\right)\delta\theta = 0$$

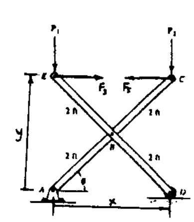
Since $\delta\theta \neq 0$, then

$$Pl\sin\theta - \frac{Wl}{2}\cos\theta = 0$$

$$P = \frac{W}{2}\cot\theta \qquad \textbf{Ans}$$

▪**11-14.** The 4-ft members of the mechanism are pin-connected at their centers. If vertical forces $P_1 = P_2 = 30$ lb act at C and E as shown, determine the angle θ for equilibrium. The spring is unstretched when $\theta = 45°$. Neglect the weight of the members.





$y = 4\sin\theta, \quad x = 4\cos\theta$

$\delta y = 4\cos\theta\ \delta\theta, \quad \delta x = -4\sin\theta\ \delta\theta$

$\delta U = 0; \quad -F_s\delta x - 30\delta y - 30\delta y = 0$

$[-F_s(-4\sin\theta) - 60(4\cos\theta)]\delta\theta = 0$

$F_s = 60\left(\frac{\cos\theta}{\sin\theta}\right)$

Since $F_s = k(4\cos\theta - 4\cos 45°) = 200(4\cos\theta - 4\cos 45°)$

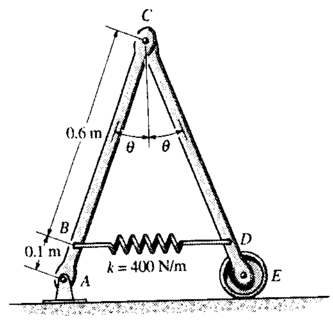
$60\cos\theta = 800(\cos\theta - \cos 45°)\sin\theta$

$\sin\theta - 0.707\tan\theta - 0.075 = 0$

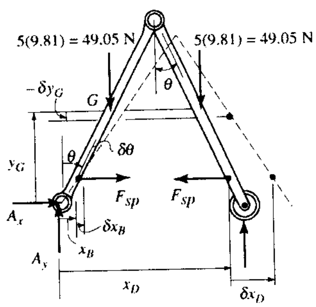
$\theta = 16.6°$ **Ans**

and $\theta = 35.8°$ Ans

11-15. The spring has an unstretched length of 0.3 m. Determine the angle θ for equilibrium if the uniform links each have a mass of 5 kg.



Free Body Diagram: The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force F_{sp} and the weights of the links (49.05 N) do work.



Virtual Displacements: The position of points B, D and G are measured from the fixed point A using position coordinates x_B, x_D and y_G, respectively.

$$x_B = 0.1\sin\theta \qquad \delta x_B = 0.1\cos\theta\delta\theta \qquad [1]$$

$$x_D = 2(0.7\sin\theta) - 0.1\sin\theta = 1.3\sin\theta \qquad \delta x_D = 1.3\cos\theta\delta\theta \qquad [2]$$

$$y_G = 0.35\cos\theta \qquad \delta y_G = -0.35\sin\theta\delta\theta \qquad [3]$$

Virtual—Work Equation: When points B, D and G undergo positive virtual displacements δx_B, δx_D and δy_G, the spring force F_{sp} that acts at point B does positive work while the spring force F_{sp} that acts at point D and the weight of link AC and CE (49.05 N) do negative work.

$$\delta U = 0; \quad 2(-49.05\delta y_G) + F_{sp}(\delta x_B - \delta x_D) = 0 \qquad [4]$$

Substituting Eqs. [1], [2] and [3] into [4] yields

$$(34.335\sin\theta - 1.2F_{sp}\cos\theta)\,\delta\theta = 0 \qquad [5]$$

However, from the spring formula, $F_{sp} = kx = 400[2(0.6\sin\theta) - 0.3] = 480\sin\theta - 120$. Substituting this value into Eq. [5] yields

$$(34.335\sin\theta - 576\sin\theta\cos\theta + 144\cos\theta)\,\delta\theta = 0$$

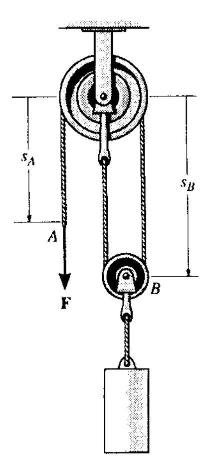
Since $\delta\theta \neq 0$, then

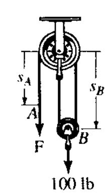
$$34.335\sin\theta - 576\sin\theta\cos\theta + 144\cos\theta = 0$$

$\theta = 15.5°$ **Ans**

and $\theta = 85.4°$ **Ans**

***11-16.** Determine the force F needed to lift the block having a weight of 100 lb. *Hint*: Note that the coordinates s_A and s_B can be related to the *constant* vertical length l of the cord.





$$l = s_A + 2s_B$$

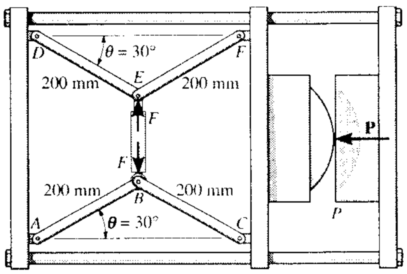
$$\delta s_A = -2\delta s_B$$

$$\delta U = 0; \quad W\delta s_B + F\delta s_A = 0$$

$$100\delta s_B + F(-2\delta s_B) = 0$$

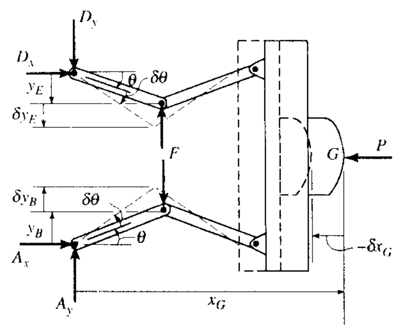
$F = 50$ lb **Ans**

11-17. The machine shown is used for forming metal plates. It consists of two toggles ABC and DEF, which are operated by hydraulic cylinder BE. The toggles push the moveable bar FC forward, pressing the plate p into the cavity. If the force which the plate exerts on the head is $P = 8$ kN, determine the force **F** in the hydraulic cylinder when $\theta = 30°$.



Free Body Diagram: The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the forces **F** and **P** do work.

Virtual Displacements: The force **F** acting on joints E and B and force **P** are located from the fixed points D and A using position coordinates y_E and y_B, respectively. The location for force **P** is measured from the fixed point A using position coordinate x_G.



$$y_E = 0.2\sin\theta \qquad \delta y_E = 0.2\cos\theta\delta\theta \qquad [1]$$

$$y_B = 0.2\sin\theta \qquad \delta y_B = 0.2\cos\theta\delta\theta \qquad [2]$$

$$x_G = 2(0.2\cos\theta) + l \qquad \delta x_G = -0.4\sin\theta\delta\theta \qquad [3]$$

Virtual—Work Equation: When points E, B and G undergo positive virtual displacements δy_E, δy_B and δx_G, force **F** and **P** do negative work.

$$\delta U = 0; \qquad -F\delta y_E - F\delta y_B - P\delta x_G = 0 \qquad [4]$$

Substituting Eqs. [1], [2] and [3] into [4] yields

$$(0.4P\sin\theta - 0.4F\cos\theta)\,\delta\theta = 0$$

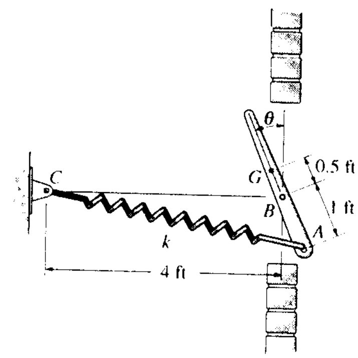
Since $\delta\theta \neq 0$, then

$$0.4P\sin\theta - 0.4F\cos\theta = 0 \qquad F = P\tan\theta$$

At equilibrium position $\theta = 30°$ set $P = 8$ kN, we have

$F = 8\tan 30° = 4.62$ kN **Ans**

11-18. The vent plate is supported at B by a pin. If it weighs 15 lb and has a center of gravity at G, determine the stiffness k of the spring so that the plate remains in equilibrium at $\theta = 30°$. The spring is unstretched when $\theta = 0°$.

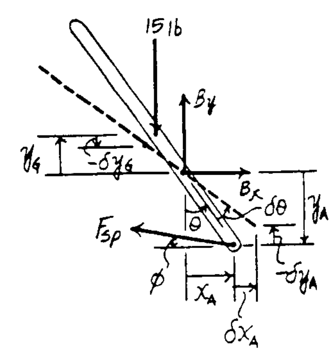


Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force F_{sp} and the weight of the vent plate (15 lb force) do work.

Virtual Displacements : The weight of the vent plate (15 lb force) is located from the fixed point B using the position coordinate y_G. The horizontal and vertical position of the spring force F_{sp} are measured from the fixed point B using the position coordinates x_A and y_A, respectively.

$$y_G = 0.5\cos\theta \qquad \delta y_G = -0.5\sin\theta\,\delta\theta \qquad [1]$$
$$y_A = 1\cos\theta \qquad \delta y_A = -\sin\theta\,\delta\theta \qquad [2]$$
$$x_A = 1\sin\theta \qquad \delta x_A = \cos\theta\,\delta\theta \qquad [3]$$

Virtual - Work Equation : When y_G, y_A and x_A undergo positive virtual displacements δy_G, δy_A and δx_A, the weight of the vent plate (15 lb force), horizontal component of F_{sp}, $F_{sp}\cos\phi$ and vertical component of F_{sp}, $F_{sp}\sin\phi$ do negative work.



$$\delta U = 0; \qquad -F_{sp}\cos\phi\,\delta x_A - F_{sp}\sin\phi\,\delta y_A - 15\delta y_G = 0 \qquad [4]$$

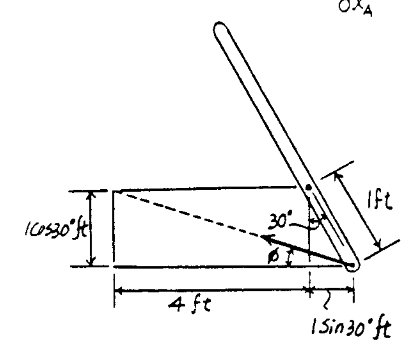
Substituting Eqs. [1], [2] and [3] into [4] yields

$$\left(-F_{sp}\cos\theta\cos\phi + F_{sp}\sin\theta\sin\phi + 7.5\sin\theta\right)\delta\theta = 0$$
$$\left(-F_{sp}\cos(\theta+\phi) + 7.5\sin\theta\right)\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

$$-F_{sp}\cos(\theta+\phi) + 7.5\sin\theta = 0$$
$$F_{sp} = \frac{7.5\sin\theta}{\cos(\theta+\phi)}$$

At equilibrium position $\theta = 30°$, the angle $\phi = \tan^{-1}\left(\frac{1\cos 30°}{4 + 1\sin 30°}\right) = 10.89°$.

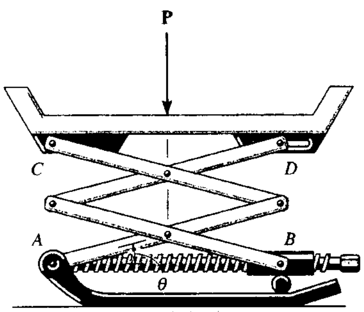


$$F_{sp} = \frac{7.5\sin 30°}{\cos(30° + 10.89°)} = 4.961 \text{ lb}$$

Spring Formula : From the geometry, the spring stretches $x = \sqrt{4^2 + 1^2 - 2(4)(1)\cos 120°} - \sqrt{4^2 + 1^2} = 0.4595$ ft.

$$F_{sp} = kx$$
$$4.961 = k(0.4595)$$
$$k = 10.8 \text{ lb/ft} \qquad \textbf{Ans}$$

11-19. The scissors jack supports a load **P**. Determine the axial force in the screw necessary for equilibrium when the jack is in the position θ. Each of the four links has a length L and is pin-connected at its center. Points B and D can move horizontally.



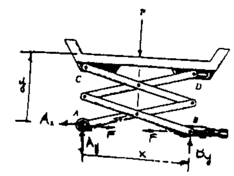
$x = L\cos\theta, \qquad \delta x = -L\sin\theta\,\delta\theta$

$y = 2L\sin\theta, \qquad \delta y = 2L\cos\theta\,\delta\theta$

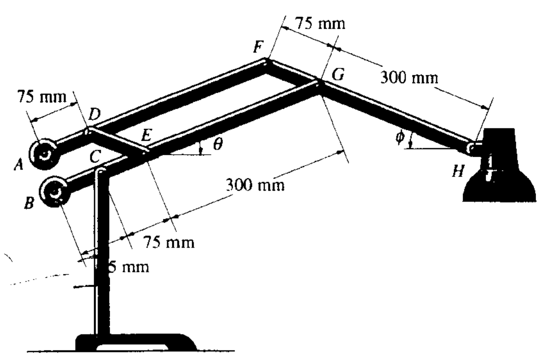
$\delta U = 0; \qquad -P\delta y - F\delta x = 0$

$-P(2L\cos\theta\,\delta\theta) - F(-L\sin\theta\,\delta\theta) = 0$

$F = 2P\cot\theta$ **Ans**



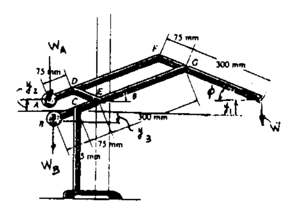
***11-20.** Determine the mass of A and B required to hold the 400-g desk lamp in balance for any angles θ and ϕ. Neglect the weight of the mechanism and the size of the lamp.



$y_1 = 300\sin\phi - 375\sin\theta$

$y_2 = 75\sin\theta + 75\sin\phi - 75\sin\theta = 75\sin\phi$

$y_3 = 75\sin\theta$



Displacement $\delta\theta$ (only)

$\delta y_1 = -375\cos\theta\,\delta\theta$

$\delta y_2 = 0$

$\delta y_3 = 75\cos\theta\,\delta\theta$

$\delta U = 0; \qquad W\delta y_1 - W_A\delta y_2 + W_B\delta y_3 = 0$

$W(-375\cos\theta\,\delta\theta) - 0 + W_B(75\cos\theta\,\delta\theta) = 0$

$$W_B = \frac{375}{75}W = \frac{375}{75}(0.4)(9.81) = 19.62\text{ N}$$

$$m_B = \frac{19.62}{9.81} = 2\text{ kg}$$ **Ans**

Displacement $\delta\phi$ (only)

$\delta y_1 = 300\cos\phi\,\delta\phi$

$\delta y_2 = 75\cos\phi\,\delta\phi$

$\delta y_3 = 0$

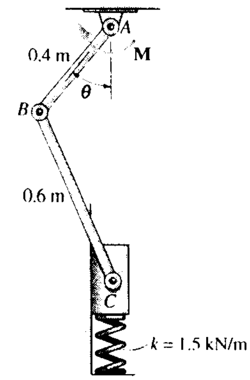
$\delta U = 0; \qquad W\delta y_1 - W_A\delta y_2 + W_B\delta y_3 = 0$

$W(300\cos\phi\,\delta\phi) - W_A(75\cos\phi\,\delta\phi) + 0 = 0$

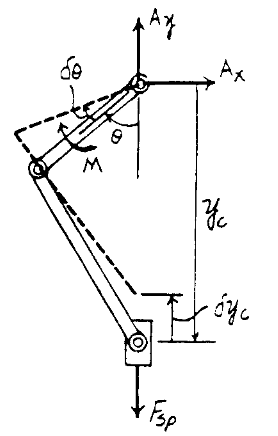
$$W_A = \frac{300}{75}W = \frac{300}{75}(0.4)(9.81) = 15.70\text{ N}$$

$$m_A = \frac{15.70}{9.81} = 1.60\text{ kg}$$ **Ans**

11-21. The piston C moves vertically between the two smooth walls. If the spring has a stiffness of $k = 1.5$ kN/m and is unstretched when $\theta = 0°$, determine the couple **M** that must be applied to link AB to hold the mechanism in equilibrium; $\theta = 30°$.



Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force F_{sp} and couple moment **M** do work.



Virtual Displacements : The spring force F_{sp} is located from the fixed point A using the position coordinate y_C. Using the law of cosines

$$0.6^2 = y_C^2 + 0.4^2 - 2(y_C)(0.4)\cos\theta \qquad [1]$$

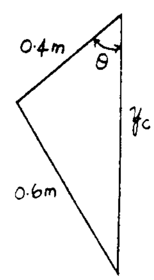
Differentiating the above expression, we have

$$0 = 2y_C\,\delta y_C - 0.8\,\delta y_C\cos\theta + 0.8y_C\sin\theta\,\delta\theta$$

$$\delta y_C = \frac{0.8y_C\sin\theta}{0.8\cos\theta - 2y_C}\delta\theta \qquad [2]$$

Virtual - Work Equation : When point C undergoes a positive virtual displacement δy_C, the spring force F_{sp} does positive work. The couple moment **M** does positive work when link AB undergoes a positive virtual rotation $\delta\theta$.

$$\delta U = 0; \qquad F_{sp}\,\delta y_C + M\delta\theta = 0 \qquad [3]$$



Substituting Eq.[1] into [2] yields

$$\left(\frac{0.8y_C\sin\theta}{0.8\cos\theta - 2y_C}F_{sp} + M\right)\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

$$\frac{0.8y_C\sin\theta}{0.8\cos\theta - 2y_C}F_{sp} + M = 0$$

$$M = -\frac{0.8y_C\sin\theta}{0.8\cos\theta - 2y_C}F_{sp} \qquad [4]$$

At the equilibrium position, $\theta = 30°$. Substituting into Eq.[1],

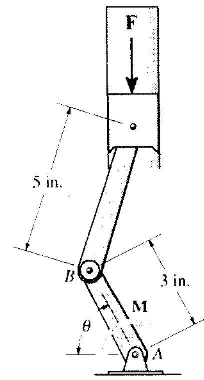
$$0.6^2 = y_C^2 + 0.4^2 - 2(y_C)(0.4)\cos 30°$$

$$y_C = 0.9121 \text{ m}$$

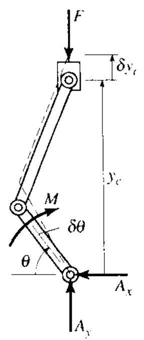
The spring stretches $x = 1 - 0.9121 = 0.08790$ m. Then the spring force is $F_{sp} = kx = 1500(0.08790) = 131.86$ N. Substituting the above results into Eq. [4], we have

$$M = -\left[\frac{0.8(0.9121)\sin 30°}{0.8\cos 30° - 2(0.9121)}\right]131.86 = 42.5 \text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

11-22. The crankshaft is subjected to a torque of $M = 50$ lb · ft. Determine the vertical compressive force **F** applied to the piston for equilibrium when $\theta = 60°$.



Free Body Diagram: The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the force **F** and couple moment **M** do work.



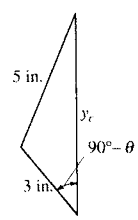
Virtual Displacements: Force **F** is located from the fixed point A using the positional coordinate y_C. Using the law of cosines.

$$5^2 = y_C^2 + 3^2 - 2(y_C)(3)\cos(90° - \theta) \qquad [1]$$

However, $\cos(90° - \theta) = \sin\theta$. Then Eq. [1] becomes $25 = y_C^2 + 9 - 6y_C\sin\theta$. Differentiating this expression, we have

$$0 = 2y_C\delta y_C - 6\delta y_C\sin\theta - 6y_C\cos\theta\delta\theta$$

$$\delta y_C = \frac{6y_C\cos\theta}{2y_C - 6\sin\theta}\delta\theta \qquad [2]$$



Virtual—Work Equation: When point C undergoes a positive virtual displacement δy_C, force **F** does negative work. The couple moment **M** does positive work when link AB undergoes a positive virtual rotation $\delta\theta$.

$$\delta U = 0; \quad -F\delta y_C + M\delta\theta = 0 \qquad [3]$$

Substituting Eq. [2] into [3] yields

$$\left(-\frac{6y_C\cos\theta}{2y_C - 6\sin\theta}F + M\right)\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

$$-\frac{6y_C\cos\theta}{2y_C - 6\sin\theta}F + M = 0$$

$$F = \frac{2y_C - 6\sin\theta}{6y_C\cos\theta}M \qquad [4]$$

At the equilibrium position, $\theta = 60°$. Substituting into Eq. [1], we have

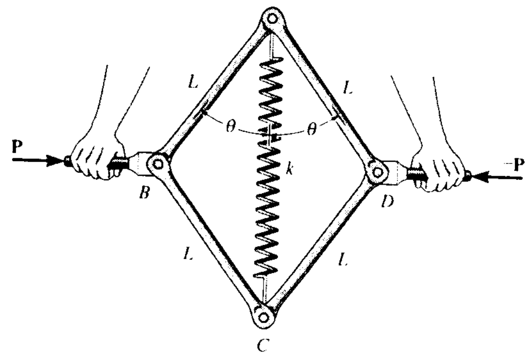
$$5^2 = y_C^2 + 3^2 - 2(y_C)(3)\cos 30°$$

$$y_C = 7.368 \text{ in.}$$

Substituting the above results into Eq. [4] and setting $M = 50$ lb · ft, we have

$$F = \left[\frac{2(7.368) - 6\sin 60°}{6(7.368)\cos 60°}\right]50(12 \text{ in./1 ft}) = 259 \text{ lb} \qquad \textbf{Ans}$$

11-23. The assembly is used for exercise. It consist of four pin-connected bars, each of length L, and a spring of stiffness k and unstretched length a ($<2L$). If horizontal forces **P** and $-$**P** are applied to the handles so that θ is slowly decreased, determine the angle θ at which the magnitude of **P** becomes a maximum.



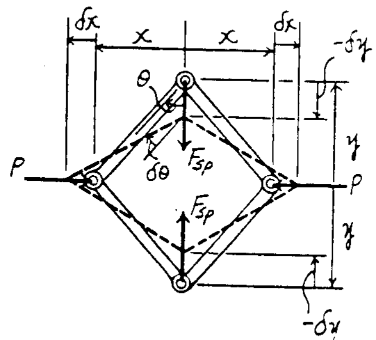
Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, the spring force F_{sp} and force **P** do work.

Virtual Displacements : The spring force F_{sp} and force **P** are located from the fixed point D and A using position coordinates y and x, respectively.

$$y = L\cos\theta \qquad \delta y = -L\sin\theta\,\delta\theta \qquad [1]$$
$$x = L\sin\theta \qquad \delta x = L\cos\theta\,\delta\theta \qquad [2]$$

Virtual - Work Equation : When points A, C, B and D undergo positive virtual displacement δy and δx, the spring force F_{sp} and force **P** do negative work.

$$\delta U = 0; \qquad -2F_{sp}\,\delta y - 2P\delta x = 0 \qquad [3]$$



Substituting Eqs. [1] and [2] into [3] yields

$$\left(2F_{sp}\sin\theta - 2P\cos\theta\right)L\delta\theta = 0 \qquad [4]$$

From the geometry, the spring stretches $x = 2L\cos\theta - a$. Then, the spring force $F_{sp} = kx = k(2L\cos\theta - a) = 2kL\cos\theta - ka$. Substituting this value into Eq. [4] yields

$$(4kL\sin\theta\cos\theta - 2ka\sin\theta - 2P\cos\theta)L\delta\theta = 0$$

Since $L\delta\theta \neq 0$, then

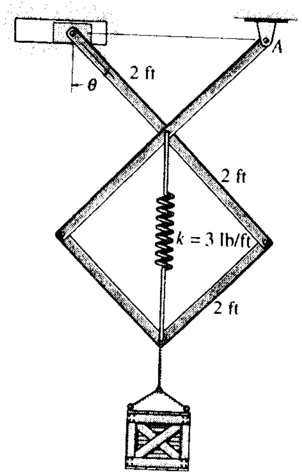
$$4kL\sin\theta\cos\theta - 2ka\sin\theta - 2P\cos\theta = 0$$
$$P = k(2L\sin\theta - a\tan\theta)$$

In order to obtain maximum P, $\dfrac{dP}{d\theta} = 0$.

$$\frac{dP}{d\theta} = k\left(2L\cos\theta - a\sec^2\theta\right) = 0$$

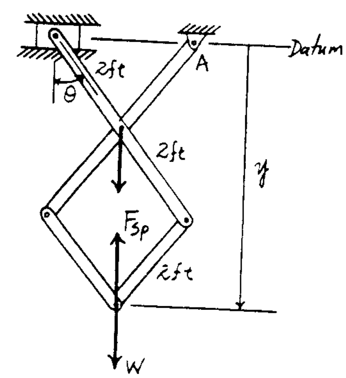
$$\theta = \cos^{-1}\left(\frac{a}{2L}\right)^{\frac{1}{3}} \qquad \textbf{Ans}$$

*11-24. Determine the weight W of the crate if the angle $\theta = 45°$. The springs are unstretched when $\theta = 60°$. Neglect the weights of the members.



Potential Function : The datum is established at point A. Since the center of gravity of the crate is below the datum, its potential energy is negative. Here, $y = (4\sin\theta + 2\sin\theta) = 6\sin\theta$ ft and the spring stretches $x = 2(2\sin\theta - 2\sin 30°) = (4\sin\theta - 2)$ ft.

$$\begin{aligned} V &= V_e + V_g \\ &= \frac{1}{2}kx^2 - Wy \\ &= \frac{1}{2}(3)(4\sin\theta - 2)^2 - W(6\sin\theta) \\ &= 24\sin^2\theta - 24\sin\theta - 6W\sin\theta + 6 \end{aligned}$$



Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$.

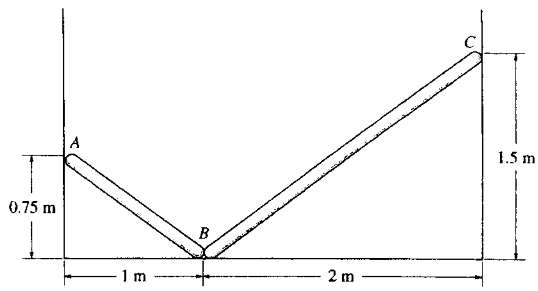
$$\frac{dV}{d\theta} = 48\sin\theta\cos\theta + 24\cos\theta + 6W\cos\theta = 0 \qquad [1]$$

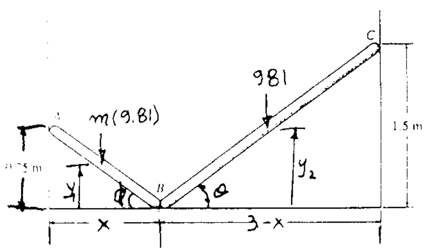
At equilibrium position, $\theta = 45°$. Substituting this value into Eq.[1], we have

$$48\sin 45°\cos 45° + 24\cos 45° - 6W\cos 45° = 0$$

$$W = 1.66 \text{ lb} \qquad \textbf{Ans}$$

11-25. Rods AB and BC have centers of mass located at their midpoints. If all contacting surfaces are smooth and BC has a mass of 100 kg, determine the appropriate mass of AB required for equilibrium.





$$x = 1.25\cos\phi; \qquad 3 - x = 2.5\cos\theta$$

$$3 - 1.25\cos\phi = 2.5\cos\theta$$

$$1.25\sin\phi\ \delta\phi = -2.5\sin\theta\ \delta\theta$$

$$1.25\left(\frac{0.75}{1.25}\right)\delta\phi = -2.5\left(\frac{1.5}{2.5}\right)\delta\theta$$

$$0.75\delta\phi = -1.5\delta\theta$$

$$\delta\phi = -\delta\theta$$

$$y_1 = \left(\frac{1.25}{2}\right)\sin\phi$$

$$y_2 = 1.25\sin\theta$$

$$\delta y_1 = 0.625\cos\phi\ \delta\phi$$

$$\delta y_2 = 1.25\cos\theta\ \delta\theta$$

$$\delta U = 0; \qquad -m(9.81)\delta y_1 - 981\delta y_2 = 0$$

$$-m(9.81)(0.625\cos\phi\ \delta\phi) - 981(1.25\cos\theta\ \delta\theta) = 0$$

$$-m(9.81)(0.625)\left(\frac{1}{1.25}\right)(-2\delta\theta) - 981(1.25)\left(\frac{2}{2.5}\right)\delta\theta = 0$$

$$[m(9.81) - 981]\delta\theta = 0$$

$m = 100$ kg **Ans**

11-26. If the potential function for a conservative one-degree-of-freedom system is $V = (8x^3 - 2x^2 - 10)$ J, where x is given in meters, determine the positions for equilibrium and investigate the stability at each of these positions.

$$V = 8x^3 - 2x^2 - 10$$

$$\frac{dV}{dx} = 24x^2 - 4x = 0$$

$$(24x - 4)x = 0$$

$x = 0$ and $x = 0.167$ m **Ans**

$$\frac{d^2V}{dx^2} = 48x - 4$$

$x = 0$, $\frac{d^2V}{dx^2} = -4 < 0$ **Unstable** **Ans**

$x = 0.167$ m, $\frac{d^2V}{dx^2} = 4 > 0$ **Stable** **Ans**

11-27. If the potential function for a conservative one-degree-of-freedom system is $V = (12 \sin 2\theta + 15 \cos \theta)$ J, where $0° < \theta < 180°$, determine the positions for equilibrium and investigate the stability at each of these positions.

$V = 12\sin 2\theta + 15\cos\theta$

$\frac{dV}{d\theta} = 0; \quad 24\cos 2\theta - 15\sin\theta = 0$

$24(1 - 2\sin^2\theta) - 15\sin\theta = 0$

$48\sin^2\theta + 15\sin\theta - 24 = 0$

Choosing the angle $0° < \theta < 180°$

$\theta = 34.6°$ **Ans**

and

$\theta = 145°$ **Ans**

$\frac{d^2V}{d\theta^2} = -48\sin 2\theta - 15\cos\theta$

$\theta = 34.6°, \quad \frac{d^2V}{d\theta^2} = -57.2 < 0$ Unstable **Ans**

$\theta = 145°, \quad \frac{d^2V}{d\theta^2} = 57.2 > 0$ Stable **Ans**

***11-28.** If the potential function for a conservative one-degree-of-freedom system is $V = (10 \cos 2\theta + 25 \sin \theta)$ J, where $0° < \theta < 180°$, determine the positions for equilibrium and investigate the stability at each of these positions.

$V = 10\cos 2\theta + 25\sin\theta$

For equilibrium :

$\frac{dV}{d\theta} = -20\sin 2\theta + 25\cos\theta = 0$

$(-40\sin\theta + 25)\cos\theta = 0$

$\theta = \sin^{-1}(\frac{25}{40}) = 38.7°$ and $141°$ **Ans**

and

$\theta = \cos^{-1} 0 = 90°$ **Ans**

Stability : $\frac{d^2V}{d\theta^2} = -40\cos 2\theta - 25\sin\theta$

$\theta = 38.7°, \quad \frac{d^2V}{d\theta^2} = -24.4 < 0,$ Unstable **Ans**

$\theta = 141°, \quad \frac{d^2V}{d\theta^2} = -24.4 < 0,$ Unstable **Ans**

$\theta = 90°, \quad \frac{d^2V}{d\theta^2} = 15 > 0,$ Stable **Ans**

11-29. If the potential function for a conservative two-degree-of-freedom system is $V = (9y^2 + 18x^2)$ J, where x and y are given in meters, determine the equilibrium position and investigate the stability at this position.

$$V = 9y^2 + 18x^2$$

$$\frac{\partial V}{\partial x} = 36x = 0; \quad x = 0$$

$$\frac{\partial V}{\partial y} = 18y = 0; \quad y = 0$$

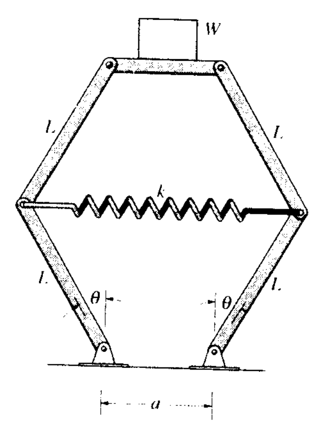
(0,0) is a position for equilibrium **Ans**

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 36 + 18 = 54 > 0$$

$$\left(\frac{\partial^2 V}{\partial x \partial y}\right)^2 - \left(\frac{\partial^2 V}{\partial x^2}\right)\left(\frac{\partial^2 V}{\partial y^2}\right) = 0 - 36(18) = -648 < 0$$

stable **Ans**

11-30. The spring of the scale has an unstretched length of a. Determine the angle θ for equilibrium when a weight W is supported on the platform. Neglect the weight of the members. What value W would be required to keep the scale in neutral equilibrium when $\theta = 0°$?



Potential Function : The datum is established at point A. Since the weight W is above the datum, its potential energy is positive. From the geometry, the spring stretches $x = 2L\sin\theta$ and $y = 2L\cos\theta$.

$$\begin{aligned} V &= V_e + V_g \\ &= \frac{1}{2}kx^2 + Wy \\ &= \frac{1}{2}(k)(2L\sin\theta)^2 + W(2L\cos\theta) \\ &= 2kL^2\sin^2\theta + 2WL\cos\theta \end{aligned}$$

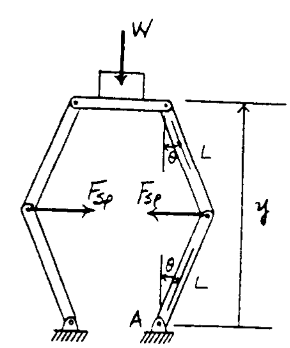
Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = 4kL^2\sin\theta\cos\theta - 2WL\sin\theta = 0$$

$$\frac{dV}{d\theta} = 2kL^2\sin 2\theta - 2WL\sin\theta = 0$$

Solving,

$$\theta = 0° \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{W}{2KL}\right) \qquad \textbf{Ans}$$



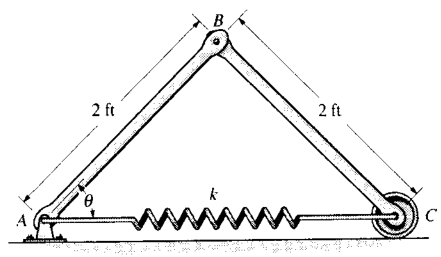
Stability : To have neutral equilibrium at $\theta = 0°$, $\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} = 0$.

$$\frac{d^2V}{d\theta^2} = 4kL^2\cos 2\theta - 2WL\cos\theta$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} = 4kL^2\cos 0° - 2WL\cos 0° = 0$$

$$W = 2kL \qquad \textbf{Ans}$$

11-31. The two bars each have a weight of 8 lb. Determine the required stiffness k of the spring so that the two bars are in equilibrium when $\theta = 30°$. The spring has an unstretched length of 1 ft.



$$V = 2(8)(1\sin\theta) + \frac{1}{2}k(4\cos\theta - 1)^2$$

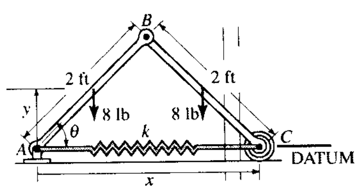
$$\frac{dV}{d\theta} = 16\cos\theta + k(4\cos\theta - 1)(-4\sin\theta)$$

$$\frac{dV}{d\theta} = 16\cos\theta - 4k(4\cos\theta - 1)\sin\theta$$

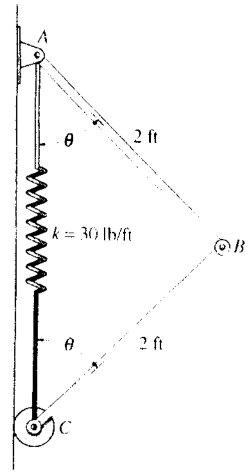
$$\theta = 30°, \quad \frac{dV}{d\theta} = 0$$

$$16\cos 30° - 4k(4\cos 30° - 1)\sin 30° = 0$$

$k = 2.81$ lb/ft **Ans**



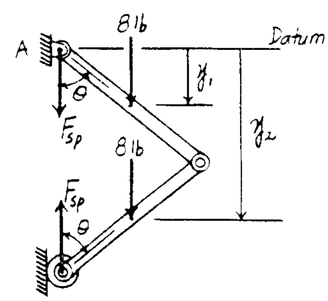
***11-32.** The two bars each have a weight of 8 lb. Determine the angle θ for the equilibrium and investigate the stability at the equilibrium position. The spring has an unstretched length of 1 ft.



Potential Function : The datum is established at point A. Since the center of gravity of the bars are below the datum, their potential energy is negative. Here, $y_1 = 1\cos\theta$ ft, $y_2 = 2\cos\theta + 1\cos\theta = 3\cos\theta$ ft and the spring stretches $x = 2(2\cos\theta) - 1 = (4\cos\theta - 1)$ ft

$$\begin{aligned} V &= V_e + V_g \\ &= \frac{1}{2}kx^2 - \Sigma Wy \\ &= \frac{1}{2}(30)(4\cos\theta - 1)^2 - 8(1\cos\theta) - 8(3\cos\theta) \\ &= 240\cos^2\theta - 152\cos\theta + 15 \end{aligned}$$

Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$.



$$\frac{dV}{d\theta} = -480\sin\theta\cos\theta + 152\sin\theta = 0$$

$$\frac{dV}{d\theta} = -240\sin 2\theta + 152\sin\theta = 0$$

Solving,

$$\theta = 0^\circ \quad \text{or} \quad \theta = 71.54^\circ = 71.5^\circ \qquad \textbf{Ans}$$

Stability :

$$\frac{d^2V}{d\theta^2} = -480\cos 2\theta + 152\cos\theta$$

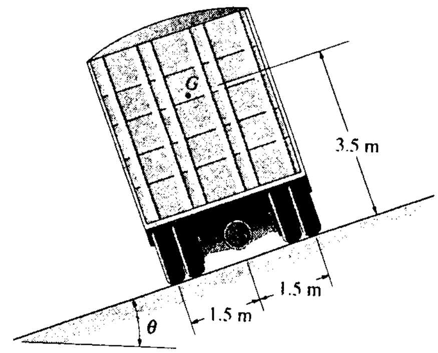
$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = -480\cos 0^\circ + 152\cos 0^\circ = -328 < 0$$

Thus, the system is in **unstable equilibrium** at $\theta = 0^\circ$ **Ans**

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=71.54^\circ} = -480\cos 143^\circ + 152\cos 71.54^\circ = 431.87 > 0$$

Thus, the system is in **stable equilibrium** at $\theta = 71.54^\circ$ **Ans**

11-33. The truck has a mass of 20 Mg and a mass center at G. Determine the steepest grade θ along which it can park without overturning and investigate the stability in this position.



Potential Function : The datum is established at point A. Since the center of gravity for the truck is above the datum, its potential energy is positive. Here, $y = (1.5\sin\theta + 3.5\cos\theta)$ m.

$$V = V_g = Wy = W(1.5\sin\theta + 3.5\cos\theta)$$

Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$

$$\frac{dV}{d\theta} = W(1.5\cos\theta - 3.5\sin\theta) = 0$$

Since $W \neq 0$,

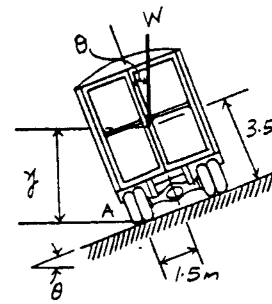
$$1.5\cos\theta - 3.5\sin\theta = 0$$
$$\theta = 23.20° = 23.2° \qquad \textbf{Ans}$$

Stability :

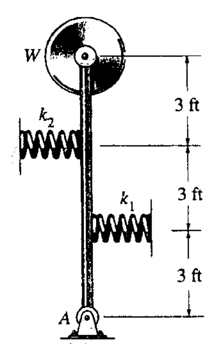
$$\frac{d^2V}{d\theta^2} = W(-1.5\sin\theta - 3.5\cos\theta)$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=23.20°} = W(-1.5\sin 23.20° - 3.5\cos 23.20°) = -3.81W < 0$$

Thus, the truck is in **unstable equilibrium** at $\theta = 23.2°$ **Ans**



11-34. The bar supports a weight of $W = 500$ lb at its end. If the springs are originally unstretched when the bar is vertical, determine the required stiffness $k_1 = k_2 = k$ of the springs so that the bar is in neutral equilibrium when it is vertical.



$$y = 9\cos\theta$$

$$x_1 = 3\sin\theta$$

$$x_2 = 6\sin\theta$$

$$V = 500(9\cos\theta) + \frac{1}{2}k(3\sin\theta)^2 + \frac{1}{2}k(6\sin\theta)^2$$

$$V = 4500\cos\theta + k(22.5\sin^2\theta)$$

$$\frac{dV}{d\theta} = -4500\sin\theta + k(22.5\sin 2\theta)$$

Require, $\frac{dV}{d\theta} = 0$; $\quad -4500\sin\theta + k(45\sin\theta\cos\theta) = 0$

$\sin\theta = 0$; $\quad \theta = 0°$

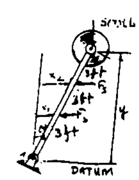
$$\frac{d^2V}{d\theta^2} = -4500\cos\theta + k(45\cos 2\theta)$$

Neutral equilibrium requires $\frac{d^2V}{d\theta^2} = 0$

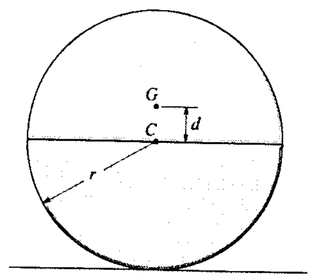
$$-4500\cos\theta + k(45\cos 2\theta) = 0$$

When $\theta = 0°$, $\quad -4500 + 45k = 0$

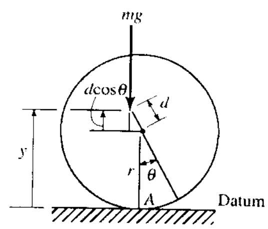
$k = 100$ lb/ft **Ans**



11-35. The cylinder is made of two materials such that it has a mass of m and a center of gravity at point G. Show that when G lies above the centroid C of the cylinder, the equilibrium is unstable.



Potential Function: The datum is established at point A. Since the center of gravity of the cylinder is above the datum, its potential energy is positive. Here, $y = r + d\cos\theta$.



$$V = V_g = W_y = mg(r + d\cos\theta)$$

Equilibrium Position: The system is in equilibrium if $\frac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = -mgd\sin\theta = 0$$

$\sin\theta = 0 \qquad \theta = 0°$.

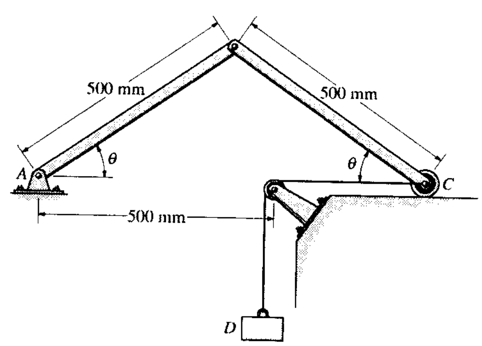
Stability:

$$\frac{d^2V}{d\theta^2} = -mgd\cos\theta$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} = -mgd\cos 0° = -mgd < 0$$

Thus, the cylinder is in unstable equilibrium at $\theta = 0°$ (*Q.E.D.*)

***11-36.** Determine the angle θ for equilibrium and investigate the stability at this position. The bars each have a mass of 3 kg and the suspended block D has a mass of 7 kg. Cord DC has a total length of 1 m.



$l = 500$ mm

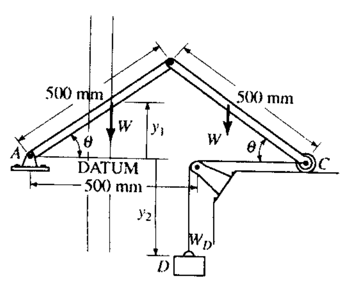
$y_1 = \frac{l}{2}\sin\theta$

$y_2 = l + 2l(1 - \cos\theta) = l(3 - 2\cos\theta)$

$V = 2Wy_1 - W_D y_2$

$= Wl\sin\theta - W_D l(3 - 2\cos\theta)$

$$\frac{dV}{d\theta} = l(W\cos\theta - 2W_D\sin\theta) = 0$$



$$\tan\theta = \frac{W}{2W_D} = \frac{3(9.81)}{14(9.81)} = 0.2143$$

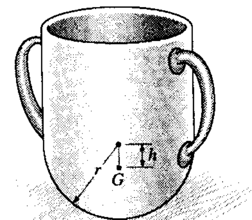
$\theta = 12.1°$ **Ans**

$$\frac{d^2V}{d\theta^2} = l(-W\sin\theta - 2W_D\cos\theta)$$

$\theta = 12.1°$, $\frac{d^2V}{d\theta^2} = 0.5[-3(9.81)\sin 12.1° - 14(9.81)\cos 12.1°]$

$= -70.2 < 0$ Unstable **Ans**

11-37. The cup has a hemispherical bottom and a mass m. Determine the position h of the center of mass G so that the cup is in neutral equilibrium.



Potential Function: The datum is established at point A. Since the center of gravity of the cup is above the datum, its potential energy is positive. Here, $y = r - h\cos\theta$.

$$V = V_g = Wy = mg(r - h\cos\theta)$$

Equilibrium Position: The system is in equilibrium if $\frac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = mgh\sin\theta = 0$$

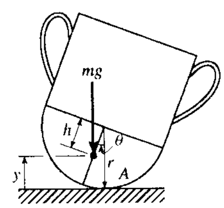
$$\sin\theta = 0 \quad \theta = 0^\circ.$$

Stability: To have neutral equilibrium at $\theta = 0^\circ$, $\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = 0$.

$$\frac{d^2V}{d\theta^2} = mgh\cos\theta$$

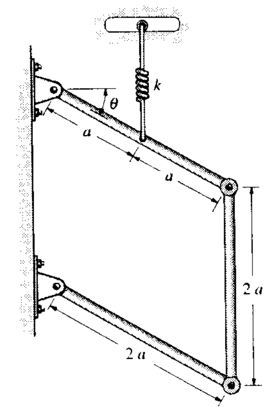
$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = mgh\cos 0^\circ = 0$$

$$h = 0 \qquad \textbf{Ans}$$



Note: Stable Equilibrium occurs if $h > 0\left(\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = mgh\cos 0^\circ > 0\right)$.

11-38. If each of the three links of the mechanism has a weight W, determine the angle θ for equilibrium. The spring, which always remains vertical, is unstretched when $\theta = 0^\circ$

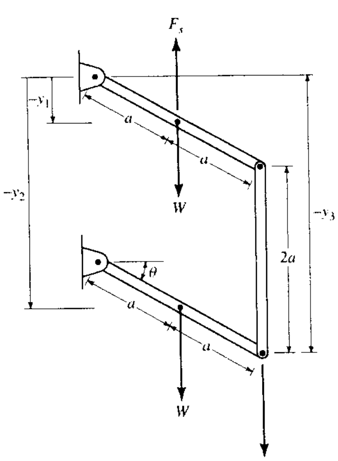


$y_1 = a\sin\theta \quad \delta y_1 = a\cos\theta\,\delta\theta$

$y_2 = 2a + a\sin\theta \quad \delta y_2 = a\cos\theta\,\delta\theta$

$y_3 = 2a + 2a\sin\theta \quad \delta y_3 = 2a\cos\theta\,\delta\theta$

$F_s = ka\sin\theta$



$\delta U = 0;\ (W - F_s)\delta y_1 + W\delta y_2 + W\delta y_3 = 0$

$(W - ka\sin\theta)a\cos\theta\,\delta\theta + Wa\cos\theta\,\delta\theta + W(2a)\cos\theta\,\delta\theta = 0$

Assume $\theta < 90^\circ$, so $\cos\theta \neq 0$.

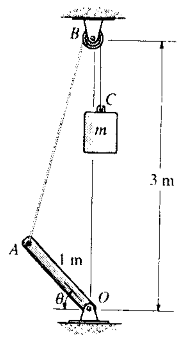
$4W - ka\sin\theta = 0$

$$\theta = \sin^{-1}\left(\frac{4W}{ka}\right) \qquad \textbf{Ans}$$

or

$$\theta = 90^\circ \qquad \textbf{Ans}$$

11-39. If the uniform rod OA has a mass of 12 kg, determine the mass m that will hold the rod in equilibrium when $\theta = 30^\circ$. Point C is coincident with B when OA is horizontal. Neglect the size of the pulley at B.



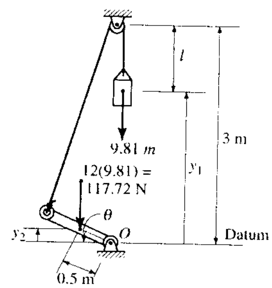
Geometry: Using the law of cosines,

$$l_{A'B} = \sqrt{1^2 + 3^2 - 2(1)(3)\cos(90^\circ - \theta)} = \sqrt{10 - 6\sin\theta}$$

$$l_{AB} = \sqrt{1^2 + 3^2} = \sqrt{10}\ \text{m}$$

$$l = l_{AB} - l_{A'B} = \sqrt{10} - \sqrt{10 - 6\sin\theta}$$

Potential Function: The datum is established at point O. Since the center of gravity of the rod and the block are above the datum, their potential energy is positive.



Here, $y_1 = 3 - l = [3 - (\sqrt{10} - \sqrt{10 - 6\sin\theta})]$ m and $y_2 = 0.5\sin\theta$ m.

$$V = V_g = W_1 y_1 + W_2 y_2$$

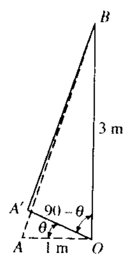
$$= 9.81\ \text{m}[3 - (\sqrt{10} - \sqrt{10 - 6\sin\theta})] + 117.72(0.5\sin\theta)$$

$$= 29.43\ \text{m} - 9.81\ \text{m}(\sqrt{10} - \sqrt{10 - 6\sin\theta}) + 58.86\sin\theta$$

Equilibrium Position: The system is in equilibrium if $\left.\frac{dV}{d\theta}\right|_{\theta = 30^\circ} = 0.$

$$\frac{dV}{d\theta} = -9.81\ \text{m}\left[-\frac{1}{2}(10 - 6\sin\theta)^{-\frac{1}{2}}(-6\cos\theta)\right] + 58.86\cos\theta$$

$$= -\frac{29.43\ \text{m}\cos\theta}{\sqrt{10 - 6\sin\theta}} + 58.86\cos\theta$$

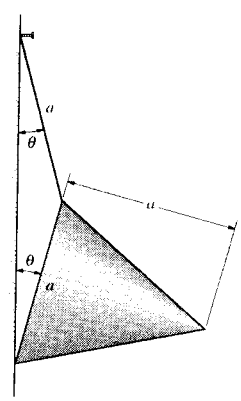


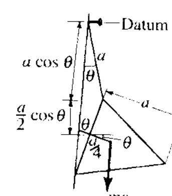
At $\theta = 30^\circ$,

$$\left.\frac{dV}{d\theta}\right|_{\theta = 30^\circ} = -\frac{29.43m\cos 30^\circ}{\sqrt{10 - 6\sin 30^\circ}} + 58.86\cos 30^\circ = 0$$

$$m = 5.29\ \text{kg}$$ **Ans**

***11-40.** The uniform right circular cone having a mass m is suspended from the cord as shown. Determine the angle θ at which it hangs from the wall for equilibrium. Is the cone in stable equilibrium?





$$V = -\left(\frac{3a}{2}\cos\theta + \frac{a}{4}\sin\theta\right)mg$$

$$\frac{dV}{d\theta} = -\left(-\frac{3a}{2}\sin\theta + \frac{a}{4}\cos\theta\right)mg = 0$$

$$3\sin\theta = 0.5\cos\theta$$

$$\tan\theta = 0.1667$$

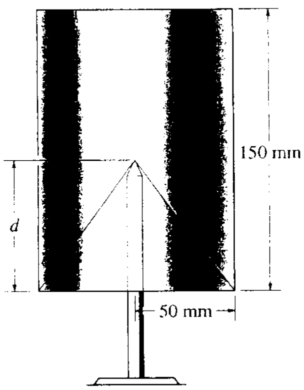
$$\theta = 9.46^\circ$$ **Ans**

$$\frac{d^2V}{d\theta^2} = -\left(-\frac{3a}{2}\cos\theta - \frac{a}{4}\sin\theta\right)mg$$

$\theta = 9.46^\circ$, $\frac{d^2V}{d\theta^2} = 1.52\,a\,mg > 0$

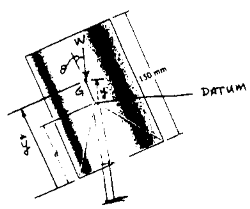
Stable **Ans**

11-41. The homogeneous cylinder has a conical cavity cut into its base as shown. Determine the depth d of the cavity so that the cylinder balances on the pivot and remains in neutral equilibrium.



$$\bar{y} = \frac{\Sigma \tilde{y} V}{\Sigma V} = \frac{75\pi(50)^2(150) - \frac{d}{4}(\frac{1}{3}\pi)(50)^2 d}{\pi(50)^2(150) - \frac{1}{3}\pi(50)^2 d}$$

$$\bar{y} = \frac{11250 - \frac{d^2}{12}}{150 - \frac{d}{3}}$$



$y = (\bar{y} - d)\cos\theta$

$V = (\bar{y} - d)\cos\theta(W)$

$$\frac{dV}{d\theta} = -W(\bar{y} - d)\sin\theta = 0$$

$\theta = 0°$ (equilibrium position)

$$\frac{d^2V}{d\theta^2} = -W(\bar{y} - d)\cos\theta = 0$$

At $\theta = 0°$, $\bar{y} = d$

$$11250 - \frac{d^2}{12} = 150d - \frac{d^2}{3}$$

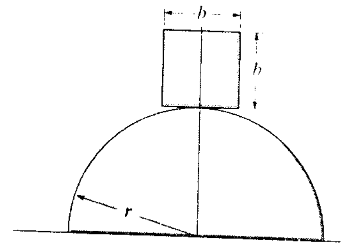
$0.25d^2 - 150d + 11250 = 0$

$d = 512.1$ mm > 150 mm (N.G!)

Also,

$d = 87.9$ mm **Ans**

11-42. A homogeneous block rests on top of the cylindrical surface. Derive the relationship between the radius of the cylinder, r, and the dimension of the block, b, for stable equilibrium. *Hint:* Establish the potential energy function for a small angle θ, i.e., approximate $\sin\theta \approx \theta$, and $\cos\theta \approx 1 - \theta^2/2$.



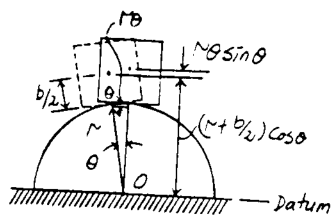
Potential Function : The datum is established at point O. Since the center of gravity for the block is above the datum, its potential energy is positive. Here, $y = \left(r + \frac{b}{2}\right)\cos\theta + r\theta\sin\theta$.

$$V = Wy = W\left[\left(r + \frac{b}{2}\right)\cos\theta + r\theta\sin\theta\right] \qquad [1]$$

For small angle θ, $\sin\theta = \theta$ and $\cos\theta = 1 - \frac{\theta^2}{2}$. Then Eq.[1] becomes

$$V = W\left[\left(r + \frac{b}{2}\right)\left(1 - \frac{\theta^2}{2}\right) + r\theta^2\right]$$

$$= W\left(\frac{r\theta^2}{2} - \frac{b\theta^2}{4} + r + \frac{b}{2}\right)$$



Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$

$$\frac{dV}{d\theta} = W\left(r - \frac{b}{2}\right)\theta = 0 \qquad \theta = 0°$$

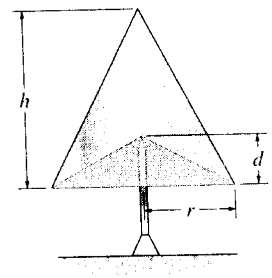
Stability : To have stable equilibrium, $\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} > 0$.

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0°} = W\left(r - \frac{b}{2}\right) > 0$$

$$\left(r - \frac{b}{2}\right) > 0$$

$$b < 2r \qquad \textbf{Ans}$$

11-43. The homogeneous cone has a conical cavity cut into it as shown. Determine the depth of d of the cavity in terms of h so that the cone balances on the pivot and remains in neutral equilibrium.

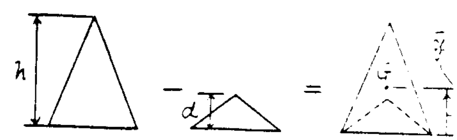


$$\bar{y} = \frac{\left(\frac{h}{4}\right)\left(\frac{1}{3}\pi r^2 h\right) - \left(\frac{d}{4}\right)\left(\frac{1}{3}\pi r^2 d\right)}{\frac{1}{3}\pi r^2 h - \frac{1}{3}\pi r^2 d} = \frac{h^2 - d^2}{4(h-d)} = \frac{1}{4}(h+d) \qquad [1]$$

Potential Function : The datum is established at point A. Since the center of gravity of the cone is above the datum, its potential energy is positive. Here,

$$y = (\bar{y} - d)\cos\theta = \left[\frac{1}{4}(h+d) - d\right]\cos\theta = \frac{1}{4}(h - 3d)\cos\theta.$$

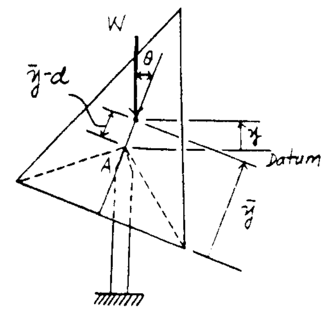
$$V = W\left[\frac{1}{4}(h-3d)\cos\theta\right]\cos\theta = \frac{W(h-3d)}{4}\cos\theta$$



Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$

$$\frac{dV}{d\theta} = -\frac{W(h-3d)}{4}\sin\theta = 0$$

$$\theta = 0 \qquad \theta = 0^\circ$$



Stability : To have neutral equilibrium at $\theta = 0^\circ$, $\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = 0.$

$$\frac{d^2V}{d\theta^2} = -\frac{W(h-3d)}{4}\cos\theta$$

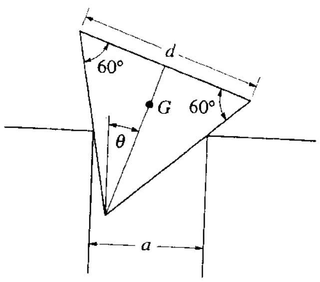
$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = -\frac{W(h-3d)}{4}\cos 0^\circ = 0$$

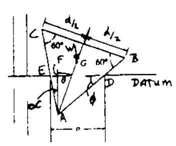
$$-\frac{W(h-3d)}{4} = 0$$

$$d = \frac{h}{3} \qquad \textbf{Ans}$$

Note : By substituting $d = \frac{h}{3}$ into Eq.[1], one realizes that the fulcrum must be at the center of gravity for neutral equilibrium.

*11-44. The triangular block of weight W rests on the smooth corners which are a distance a apart. If the block has three equal sides of length d, determine the angle θ for equilibrium.





$$AF = AD\sin\phi = AD\sin(60^\circ - \theta)$$

$$\frac{AD}{\sin\alpha} = \frac{a}{\sin 60^\circ}$$

$$AD = \frac{a}{\sin 60^\circ}(\sin(60^\circ + \theta))$$

$$AF = \frac{a}{\sin 60^\circ}(\sin(60^\circ + \theta))\sin(60^\circ - \theta)$$

$$= \frac{a}{\sin 60^\circ}(0.75\cos^2\theta - 0.25\sin^2\theta)$$

$$V = Wy$$

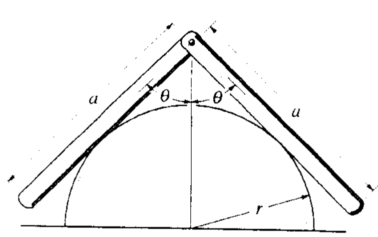
$$\frac{dV}{d\theta} = W(-0.5774d)\sin\theta - \frac{a}{\sin 60^\circ}(-1.5\sin\theta\cos\theta - 0.5\sin\theta\cos\theta) = 0$$

Require, $\sin\theta = 0$ $\qquad \theta = 0^\circ$ **Ans**

and $-0.5774\,d - \frac{a}{\sin 60^\circ}(-2\cos\theta) = 0$

$$\theta = \cos^{-1}\left(\frac{d}{4a}\right)$$ **Ans**

11-45. Two uniform bars, each having a weight W, are pin-connected at their ends. If they are placed over a smooth cylindrical surface, show that the angle θ for equilibrium must satisfy the equation $\cos\theta/\sin^3\theta = a/2r$.

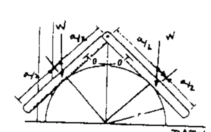


$$V = 2W\left(r\csc\theta - \frac{a}{2}\cos\theta\right)$$

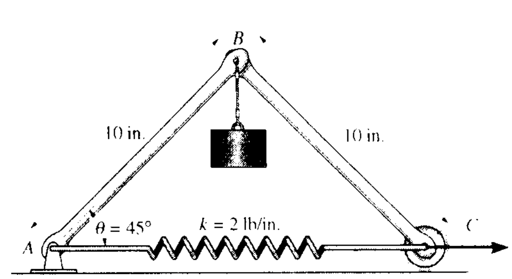
$$\frac{dV}{d\theta} = 2W\left(-r\csc\theta\cot\theta + \frac{a}{2}\sin\theta\right) = 0$$

$$r\left(\frac{\cos\theta}{\sin^2\theta}\right) = \frac{a}{2}\sin\theta$$

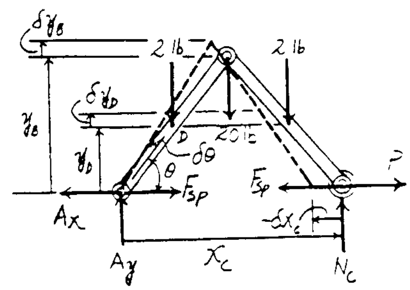
$$\frac{\cos\theta}{\sin^3\theta} = \frac{a}{2r}$$ **Ans**



11-46. The uniform links AB and BC each weigh 2 lb and the cylinder weighs 20 lb. Determine the horizontal force **P** required to hold the mechanism in the position when $\theta = 45°$. The spring has an unstretched length of 6 in.



Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only the spring force F_{sp}, the weight of links (2 lb), 20 lb force and force **P** do work.



Virtual Displacements : The positions of points B, D and C are measured from the fixed point A using position coordinates y_B, y_D and x_C respectively.

$$y_B = 10\sin\theta \qquad \delta y_B = 10\cos\theta\,\delta\theta \qquad [1]$$
$$y_D = 5\sin\theta \qquad \delta y_D = 5\cos\theta\,\delta\theta \qquad [2]$$
$$x_C = 2(10\cos\theta) \qquad \delta x_C = -20\sin\theta\,\delta\theta \qquad [3]$$

Virtual - Work Equation : When points B, D and C undergo positive virtual displacements δy_B, δy_D and δx_C, spring force F_{sp} that acts at point C, the weight of links (2 lb) and 20 lb force do negative work while force **P** does positive work.

$$\delta U = 0; \qquad -F_{sp}\,\delta x_C - 2(2\delta y_D) - 20\delta y_B + P\delta x_C = 0 \qquad [4]$$

Substituting Eqs.[1], [2] and [3] into [4] yields

$$(20F_{sp}\sin\theta - 20P\sin\theta - 220\cos\theta)\,\delta\theta = 0 \qquad [5]$$

However, from the spring formula, $F_{sp} = kx = 2[2(10\cos\theta) - 6] = 40\cos\theta - 12$. Substituting this value into Eq.[5] yields

$$(800\sin\theta\cos\theta - 240\sin\theta - 220\cos\theta - 20P\sin\theta)\,\delta\theta = 0$$

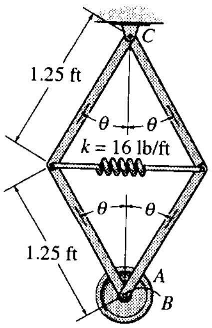
Since $\delta\theta \neq 0$, then

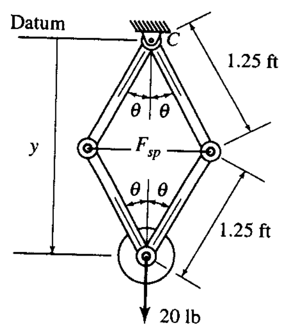
$$800\sin\theta\cos\theta - 240\sin\theta - 220\cos\theta - 20P\sin\theta = 0$$
$$P = 40\cos\theta - 11\cot\theta - 12$$

At the equilibrium position, $\theta = 45°$. Then

$$P = 40\cos 45° - 11\cot 45° - 12 = 5.28 \text{ lb} \qquad \textbf{Ans}$$

11-47. The spring attached to the mechanism has an unstretched length when $\theta = 90°$. Determine the position θ for equilibrium and investigate the stability of the mechanism at this position. Disk A is pin-connected to the frame at B and has a weight of 20 lb. Neglect the weight of the bars.





Potential Function: The datum is established at point C. Since the center of gravity of the disk is below the datum, its potential energy is negative. Here, $y = 2(1.25\cos\theta) = 2.5\cos\theta$ ft and the spring compresses $x = (2.5 - 2.5\sin\theta)$ ft.

$$V = V_e + V_g$$

$$= \frac{1}{2}kx^2 - Wy$$

$$= \frac{1}{2}(16)(2.5 - 2.5\sin\theta)^2 - 20(2.5\cos\theta)$$

$$= 50\sin^2\theta - 100\sin\theta - 50\cos\theta + 50$$

Equilibrium Position: The system is in equilibrium if $\frac{dV}{d\theta} = 0$.

$$\frac{dV}{d\theta} = 100\sin\theta\cos\theta - 100\cos\theta + 50\sin\theta = 0$$

$$\frac{dV}{d\theta} = 50\sin 2\theta - 100\cos\theta + 50\sin\theta = 0$$

Solving by trial and error,

$\theta = 37.77° = 37.8°$ **Ans**

Stability:

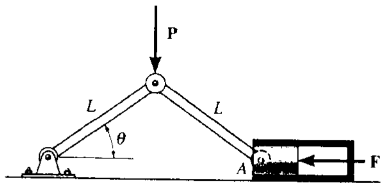
$$\frac{d^2V}{d\theta^2} = 100\cos 2\theta + 100\sin\theta + 50\cos\theta$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=37.77°} = 100\cos 75.54° + 100\sin 37.77° + 50\cos 37.77°$$

$$= 125.7 > 0$$

Thus, the system is in **stable equilibrium** at $\theta = 37.8°$ **Ans**

***11-48.** The toggle joint is subjected to the load **P**. Determine the compressive force F it creates on the cylinder at A as a function of θ.



$x = 2L\cos\theta$

$\delta x = -2L\sin\theta\ \delta\theta$

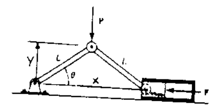
$y = L\sin\theta$

$\delta y = L\cos\theta\ \delta\theta$

$\delta U = 0; \quad -P\delta y - F\delta x = 0$

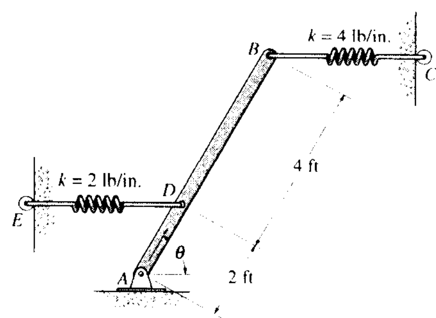
$-PL\cos\theta\ \delta\theta - F(-2L\sin\theta)\delta\theta = 0$

$-P\cos\theta + 2F\sin\theta = 0$



$$F = \frac{P}{2\tan\theta} \qquad \textbf{Ans}$$

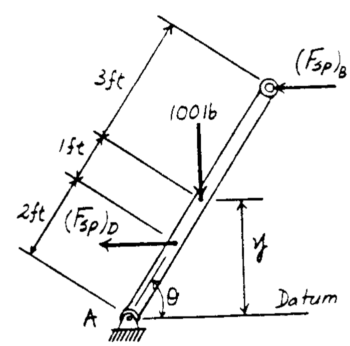
11-49. The uniform beam AB weighs 100 lb. If both springs DE and BC are unstretched when $\theta = 90°$, determine the angle θ for equilibrium using the principle of potential energy. Investigate the stability at the equilibrium position. Both springs always act in the horizontal position because of the roller guides at C and E.



Potential Function : The datum is established at point A. Since the center of gravity of the beam is above the datum, its potential energy is positive. Here, $y = (3\sin\theta)$ ft, the spring at D stretches $x_D = (2\cos\theta)$ ft and the spring at B compresses $x = (6\cos\theta)$ ft.

$$\begin{aligned} V &= V_e + V_g \\ &= \Sigma\frac{1}{2}kx^2 + Wy \\ &= \frac{1}{2}(24)(2\cos\theta)^2 + \frac{1}{2}(48)(6\cos\theta)^2 + 100(3\sin\theta) \\ &= 912\cos^2\theta + 300\sin\theta \end{aligned}$$

Equilibrium Position : The system is in equilibrium if $\frac{dV}{d\theta} = 0$.



$$\frac{dV}{d\theta} = -1824\sin\theta\cos\theta + 300\cos\theta = 0$$

$$\frac{dV}{d\theta} = -912\sin 2\theta + 300\cos\theta = 0$$

Solving,

$$\theta = 90° \quad \text{or} \quad \theta = 9.467° = 9.47° \qquad \textbf{Ans}$$

Stability :

$$\frac{d^2V}{d\theta^2} = -1824\cos 2\theta - 300\sin\theta$$

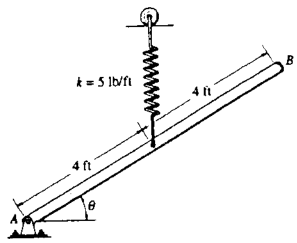
$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=90°} = -1824\cos 180° - 300\sin 90° = 1524 > 0$$

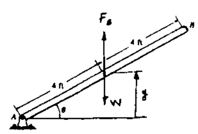
Thus, the system is in **stable equilibrium** at $\theta = 90°$ **Ans**

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=9.467°} = -1824\cos 18.933° - 300\sin 9.467° = -1774.7 < 0$$

Thus, the system is in **unstable equilibrium** at $\theta = 9.47°$ **Ans**

11-50. The uniform bar AB weighs 10 lb. If the attached spring is unstretched when $\theta = 90°$, use the method of virtual work and determine the angle θ for equilibrium. Note that the spring always remains in the vertical position due to the roller guide.





$y = 4\sin\theta$

$\delta y = 4\cos\theta\ \delta\theta$

$F_s = 5(4 - 4\sin\theta)$

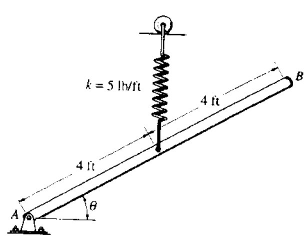
$\delta U = 0;\qquad -10\delta y + F_s\delta y = 0$

$[-10 + 20(1-\sin\theta)](4\cos\theta\ \delta\theta) = 0$

$\cos\theta = 0$ and $10 - 20\sin\theta = 0$

$\theta = 90°$ $\qquad\theta = 30°$ **Ans**

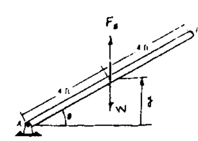
11-51. Solve Prob. 11-50 using the principle of potential energy. Investigate the stability of the bar when it is in the equilibrium position.



$y = 4\sin\theta$

$V = 10(4\sin\theta) + \frac{1}{2}(5)(4 - 4\sin\theta)^2$

$\frac{dV}{d\theta} = 40\cos\theta + 5(4-4\sin\theta)(-4\cos\theta)$



Require, $\frac{dV}{d\theta} = 0$

$40\cos\theta - 20(4 - 4\sin\theta)\cos\theta = 0$

$\cos\theta = 0$ or $40 - 80(1-\sin\theta) = 0$

$\theta = 90°$, or $\theta = 30°$ **Ans**

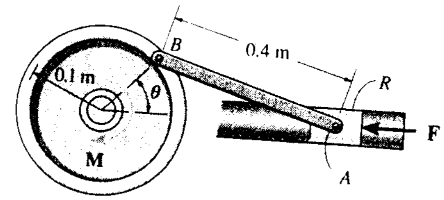
$\frac{d^2V}{d\theta^2} = -40\sin\theta + 5(4-4\sin\theta)(4\sin\theta) + 5(-4\cos\theta)(-4\cos\theta)$

$\frac{d^2V}{d\theta^2} = -40\sin\theta + 80(1-\sin\theta)\sin\theta + 80\cos^2\theta$

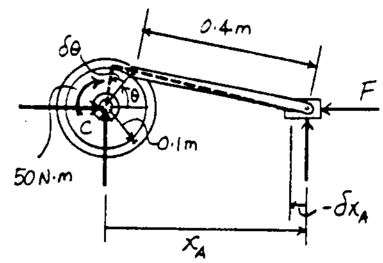
$\theta = 90°$, $\frac{d^2V}{d\theta^2} = -40 < 0$ Unstable **Ans**

$\theta = 30°$, $\frac{d^2V}{d\theta^2} = 60 > 0$ Stable **Ans**

*11-52. The punch press consists of the ram R, connecting rod AB, and a flywheel. If a torque of $M = 50\ \text{N}\cdot\text{m}$ is applied to the flywheel, determine the force **F** applied at the ram to hold the rod in the position $\theta = 60°$.



Free Body Diagram : The system has only one degree of freedom defined by the independent coordinate θ. When θ undergoes a positive displacement $\delta\theta$, only force **F** and 50 N · m couple moment do work.



Virtual Displacements : The force **F** is located from the fixed point A using the position coordinate x_A. Using the law of cosines,

$$0.4^2 = x_A^2 + 0.1^2 - 2(x_A)(0.1)\cos\theta \qquad [1]$$

Differentiating the above expression, we have

$$0 = 2x_A\,\delta x_A - 0.2\delta x_A\cos\theta + 0.2x_A\sin\theta\,\delta\theta$$

$$\delta x_A = \frac{0.2x_A\sin\theta}{0.2\cos\theta - 2x_A}\delta\theta \qquad [2]$$

Virtual - Work Equation : When point A undergoes positive virtual displacement δx_A, force **F** does negative work. The 50 N · m couple moment does negative work when the flywheel undergoes a positive virtual rotation $\delta\theta$.

$$\delta U = 0; \qquad -F\delta x_A - 50\delta\theta = 0 \qquad [3]$$

Substituting Eq.[2] into [3] yields

$$\left(-\frac{0.2x_A\sin\theta}{0.2\cos\theta - 2x_A}F - 50\right)\delta\theta = 0$$

Since $\delta\theta \neq 0$, then

$$-\frac{0.2x_A\sin\theta}{0.2\cos\theta - 2x_A}F - 50 = 0$$

$$F = -\frac{50(0.2\cos\theta - 2x_A)}{0.2x_A\sin\theta} \qquad [4]$$

At the equilibrium position, $\theta = 60°$. Substituting into Eq.[1], we have

$$0.4^2 = x_A^2 + 0.1^2 - 2(x_A)(0.1)\cos 60°$$

$$x_A = 0.4405\ \text{m}$$

Substituting the above results into Eq. [4], we have

$$F = -\frac{50[0.2\cos 60° - 2(0.4405)]}{0.2(0.4405)\sin 60°} = 512\ \text{N} \qquad \textbf{Ans}$$