CHAPTER 12

12.1 Flow balances can be used to determine

$Q_{01} = 6$	$Q_{15} = 3$	$Q_{12} = 4$	$Q_{31} = 1$	$Q_{03} = 8$
$Q_{25} = 1$	$Q_{23} = 1$	$Q_{54} = 2$	$Q_{55} = 2$	$Q_{24} = 2$
$Q_{34} = 8$	$Q_{44} = 12$			

Mass balances can be used to determine the following simultaneous equations,

$$\begin{bmatrix} 7 & 0 & -1 & 0 & 0 \\ -4 & 4 & 0 & 0 & 0 \\ 0 & -1 & 9 & 0 & 0 \\ 0 & -2 & -8 & 12 & -2 \\ -3 & -1 & 0 & 0 & 4 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{Bmatrix} = \begin{Bmatrix} 240 \\ 0 \\ 80 \\ 0 \\ 0 \end{Bmatrix}$$

The solution and the matrix inverse can then be developed. For example, using MATLAB,

```
>> A=[7 0 -1 0 0;
-4 4 0 0 0;
0 -1 9 0 0;
0 -2 -8 12 -2;
-3 -1 0 0 4];
>> B=[240;0;80;0;0];
>> C=A\B

C =
   36.1290
   36.1290
   12.9032
   20.6452
   36.1290

>> inv(A)

ans =

    0.1452    0.0040    0.0161         0         0
    0.1452    0.2540    0.0161         0         0
    0.0161    0.0282    0.1129         0         0
    0.0591    0.0722    0.0806    0.0833    0.0417
    0.1452    0.0665    0.0161         0    0.2500
```

12.2 The relevant coefficients of the matrix inverse are $a_{13}^{-1} = 0.018868$ and $a_{43}^{-1} = 0.087479$. Therefore a 25% change in the input to reactor 3 will lead to the following concentration changes to reactors 1 and 4:

$$\Delta c_1 = 0.018868(0.25 \times 160) = 0.754717$$

$$\Delta c_4 = 0.087479(0.25 \times 160) = 3.499142$$

These can be expressed as percent changes,

$$\frac{\Delta c_1}{c_1} \times 100\% = \frac{0.754717}{11.50943} \times 100\% = 6.56\%$$

$$\frac{\Delta c_4}{c_4} \times 100\% = \frac{3.499142}{16.99828} \times 100\% = 20.59\%$$

12.3 Because of conservation of flow:

$$Q_{01} + Q_{03} = Q_{44} + Q_{55}$$

12.4 Mass balances can be used to determine the following simultaneous equations,

$$\begin{bmatrix} 8 & 0 & -3 & 0 & 0 \\ -4 & 4 & 0 & 0 & 0 \\ 0 & -2 & 10 & 0 & 0 \\ 0 & 0 & -7 & 10 & -3 \\ -4 & -2 & 0 & 0 & 6 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{Bmatrix} = \begin{Bmatrix} 50 \\ 0 \\ 160 \\ 0 \\ 0 \end{Bmatrix}$$

The solution can then be developed. For example, using MATLAB,

```
>> A=[8 0 -3 0 0;
-4 4 0 0 0;
0 -2 10 0 0;
0 0 -7 10 -3;
-4 -2 0 0 6];
>> B=[50;0;160;0;0];
>> C=A\B

C =
   13.2432
   13.2432
   18.6486
   17.0270
   13.2432
```

12.5 Flow balances can be used to determine

$Q_{01} = 5$	$Q_{15} = 3$	$Q_{12} = 0$	$Q_{31} = -2$	$Q_{03} = 8$
$Q_{25} = 0$	$Q_{23} = -7$	$Q_{54} = 0$	$Q_{55} = 3$	$Q_{24} = 7$
$Q_{34} = 3$	$Q_{44} = 10$			

Mass balances can be used to determine the following simultaneous equations,

$$\begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 7 & -7 & 0 & 0 \\ -2 & 0 & 10 & 0 & 0 \\ 0 & -7 & -3 & 10 & 0 \\ -3 & 0 & 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{Bmatrix} = \begin{Bmatrix} 50 \\ 0 \\ 160 \\ 0 \\ 0 \end{Bmatrix}$$

The solution can then be developed. For example, using MATLAB,

```
>> A=[5 0 0 0 0;
0 7 -7 0 -1;
-2 0 10 0 0;
0 -7 -3 10 0;
-3 0 0 0 3];
>> B=[50;0;160;0;0];
>> C=A\B

C =
   10.0000
   18.0000
   18.0000
   18.0000
   10.0000
```

12.6 Mass balances can be written for each of the reactors as

$$500 - Q_{13}c_1 - Q_{12}c_1 + Q_{21}c_2 = 0$$
$$Q_{12}c_1 - Q_{21}c_2 - Q_{23}c_2 = 0$$
$$200 + Q_{13}c_1 + Q_{23}c_2 - Q_{33}c_3 = 0$$

Values for the flows can be substituted and the system of equations can be written in matrix form as

$$\begin{bmatrix} 130 & -30 & 0 \\ -90 & 90 & 0 \\ -40 & -60 & 120 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} = \begin{Bmatrix} 500 \\ 0 \\ 200 \end{Bmatrix}$$

The solution can then be developed. For example, using MATLAB,

```
>> A=[130 -30 0;-90 90 0;-40 -60 120];
>> B=[500;0;200];
>> C=A\B

C =

    5.0000
    5.0000
    5.8333
```

12.7 Mass balances can be written for each of the lakes as

Superior, c_1:

$$180 = 67c_1$$

Michigan, c_2:

$$710 = 36c_2$$

Huron, c_3:

$$740 + 67c_1 + 36c_2 = 161c_3$$

Erie, c_4:

$$3850 + 161c_3 = 182c_4$$

Ontario, c_5:

$$4720 + 182c_4 = 212c_5$$

The system of equations can be written in matrix form as

$$\begin{bmatrix} 67 & 0 & 0 & 0 & 0 \\ 0 & 36 & 0 & 0 & 0 \\ -67 & -36 & 161 & 0 & 0 \\ 0 & 0 & -161 & 182 & 0 \\ 0 & 0 & 0 & -182 & 212 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{Bmatrix} = \begin{Bmatrix} 180 \\ 710 \\ 740 \\ 3850 \\ 4720 \end{Bmatrix}$$

The solution can then be developed. For example, using MATLAB,

```
>> A=[67 0 0 0 0;
0 36 0 0 0;
-67 -36 161 0 0;
0 0 -161 182 0;
0 0 0 -182 212];
>> B=[180 710 740 3850 4720]';
>> C=A\B

C =
    2.6866
   19.7222
   10.1242
   30.1099
   48.1132
```

12.8 (a) The solution can be developed using your own software or a package. For example, using MATLAB,

```
>> A=[13.422 0 0 0;
-13.422 12.252 0 0;
0 -12.252 12.377 0;
0 0 -12.377 11.797];
>> W=[750.5 300 102 30]';
>> AI=inv(A)

AI =
    0.0745         0         0         0
    0.0816    0.0816         0         0
    0.0808    0.0808    0.0808         0
```

```
    0.0848    0.0848    0.0848    0.0848

>> C=AI*W

C =
   55.9157
   85.7411
   93.1163
  100.2373
```

(b) The element of the matrix that relates the concentration of Havasu (lake 4) to the loading of Powell (lake 1) is $a_{41}^{-1} = 0.084767$. This value can be used to compute how much the loading to Lake Powell must be reduced in order for the chloride concentration of Lake Havasu to be 75 as

$$\Delta W_1 = \frac{\Delta c_4}{a_{41}^{-1}} = \frac{100.2373 - 75}{0.084767} = 297.725$$

(c) First, normalize the matrix to give

$$[A] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -0.91283 & 0 & 0 \\ 0 & -0.9899 & 1 & 0 \\ 0 & 0 & 1 & -0.95314 \end{bmatrix}$$

The column-sum norm for this matrix is 2. The inverse of the matrix can be computed as

$$[A]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1.095495 & -1.09549 & 0 & 0 \\ 1.084431 & -1.08443 & 1 & 0 \\ 1.137747 & -1.13775 & 1.049165 & -1.04917 \end{bmatrix}$$

The column-sum norm for the inverse can be computed as 4.317672. The condition number is, therefore, 2(4.317672) = 8.635345. This means that less than 1 digit is suspect [$\log_{10}(8.635345) = 0.93628$]. Interestingly, if the original matrix is unscaled, the same condition number results.

12.9 For the first stage, the mass balance can be written as

$$F_1 y_{\text{in}} + F_2 x_2 = F_2 x_1 + F_1 x_1$$

Substituting $x = Ky$ and rearranging gives

$$-\left(1 + \frac{F_2}{F_1} K\right) y_1 + \frac{F_2}{F_1} K y_2 = -y_{\text{in}}$$

Using a similar approach, the equation for the last stage is

$$y_4 - \left(1 + \frac{F_2}{F_1} K\right) y_5 = -\frac{F_2}{F_1} x_{\text{in}}$$

For interior stages,

$$y_{\text{i-1}} - \left(1 + \frac{F_2}{F_1} K\right) y_{\text{i}} + \frac{F_2}{F_1} K y_{i+1} = 0$$

These equations can be used to develop the following system,

$$\begin{bmatrix} 9 & -8 & 0 & 0 & 0 \\ -1 & 9 & -8 & 0 & 0 \\ 0 & -1 & 9 & -8 & 0 \\ 0 & 0 & -1 & 9 & -8 \\ 0 & 0 & 0 & -1 & 9 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{Bmatrix} = \begin{Bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

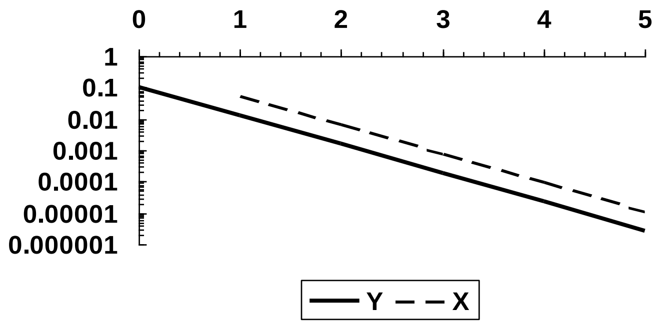
The solution can be developed in a number of ways. For example, using MATLAB,

```
>> format long
>> A=[9 -8 0 0 0;
-1 9 -8 0 0;
0 -1 9 -8 0;
0 0 -1 9 -8;
0 0 0 -1 9];
>> B=[0.1;0;0;0;0];
>> Y=A\B

Y =
   0.01249966621272
   0.00156212448931
   0.00019493177388
   0.00002403268445
   0.00000267029827
```

Note that the corresponding values of X can be computed as

```
>> X=4*Y

X =
   0.04999866485086
   0.00624849795722
   0.00077972709552
   0.00009613073780
   0.00001068119309
```

Therefore, y_{out} = 0.0000026703 and x_{out} = 0.05. In addition, here is a logarithmic plot of the simulation results versus stage,



12.10 Steady-state mass balances for *A* in each reactor can be written as

$$
\begin{aligned}
&Q_{\text{in}} c_{A,\text{in}} - Q_{\text{in}} c_{A,1} - k_1 V_1 c_{A,1} = 0 \\
&Q_{\text{in}} c_{A,1} + Q_{32} c_{A,3} - (Q_{\text{in}} + Q_{32}) c_{A,2} - k_2 V_2 c_{A,2} = 0 \\
&(Q_{\text{in}} + Q_{32}) c_{A,2} + Q_{43} c_{A,4} - (Q_{\text{in}} + Q_{43}) c_{A,3} - k_3 V_3 c_{A,3} = 0 \\
&(Q_{\text{in}} + Q_{43}) c_{A,3} - (Q_{\text{in}} + Q_{43}) c_{A,4} - k_4 V_4 c_{A,4} = 0
\end{aligned}
$$

Steady-state mass balances for *B* in each reactor can be written as

$$
\begin{aligned}
&-Q_{\text{in}} c_{B,1} + k_1 V_1 c_{A,1} = 0 \\
&Q_{\text{in}} c_{B,1} + Q_{32} c_{B,3} - (Q_{\text{in}} + Q_{32}) c_{B,2} + k_2 V_2 c_{A,2} = 0 \\
&(Q_{\text{in}} + Q_{32}) c_{B,2} + Q_{43} c_{B,4} - (Q_{\text{in}} + Q_{43}) c_{B,3} + k_3 V_3 c_{A,3} = 0 \\
&(Q_{\text{in}} + Q_{43}) c_{B,3} - (Q_{\text{in}} + Q_{43}) c_{B,4} + k_4 V_4 c_{A,4} = 0
\end{aligned}
$$

Values for the parameters can be substituted and the system of equations can be written in matrix form as

$$
\begin{bmatrix}
11.875 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1.875 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\
-10 & 0 & 26.25 & 0 & -5 & 0 & 0 & 0 \\
0 & -10 & -11.25 & 15 & 0 & -5 & 0 & 0 \\
0 & 0 & -15 & 0 & 53 & 0 & -3 & 0 \\
0 & 0 & 0 & -15 & -40 & 13 & 0 & -3 \\
0 & 0 & 0 & 0 & -13 & 0 & 15.5 & 0 \\
0 & 0 & 0 & 0 & 0 & -13 & -2.5 & 13
\end{bmatrix}
\begin{Bmatrix}
c_{A,1} \\ c_{B,1} \\ c_{A,2} \\ c_{B,2} \\ c_{A,3} \\ c_{B,3} \\ c_{A,4} \\ c_{B,4}
\end{Bmatrix}
=
\begin{Bmatrix}
10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{Bmatrix}
$$

The solution can then be developed. For example, using MATLAB,

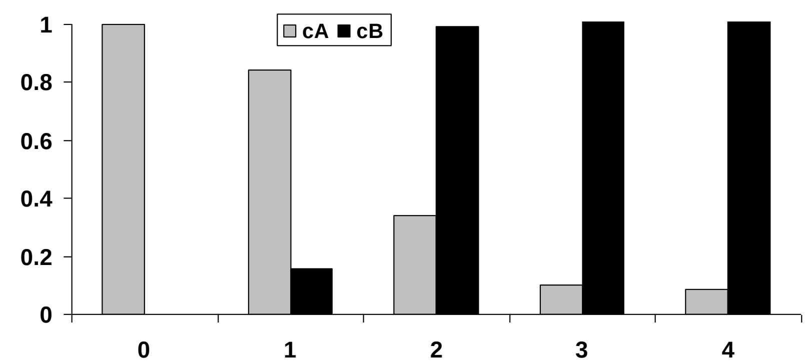
```
>> A=[11.875 0 0 0 0 0 0 0;
-1.875 10 0 0 0 0 0 0;
-10 0 26.25 0 -5 0 0 0;
0 -10 -11.25 15 0 -5 0 0;
0 0 -15 0 53 0 -3 0;
0 0 0 -15 40 13 0 -3;
0 0 0 0 -13 0 15.5 0;
0 0 0 0 0 -13 -2.5 13];
```

```
>> B=[10 0 0 0 0 0 0 0]';
>> C=A\B

C =
    0.8421
    0.1579
    0.3400
    0.9933
    0.1010
    1.8990
    0.0847
    1.9153
```

Therefore, to summarize the results

reactor	*A*	*B*
inflow	1	0
1	0.842105	0.157895
2	0.340047	0.993286
3	0.101036	1.898964
4	0.084740	1.915260

Here is a plot of the results:



12.11 Assuming a unit flow for Q_1, the simultaneous equations can be written in matrix form as

$$\begin{bmatrix} -2 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & -2 & 3 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix} \begin{Bmatrix} Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$$

These equations can then be solved. For example, using MATLAB,

```
>> A=[-2 1 2 0 0 0;
0 0 -2 1 2 0;
0 0 0 0 -2 3;
```

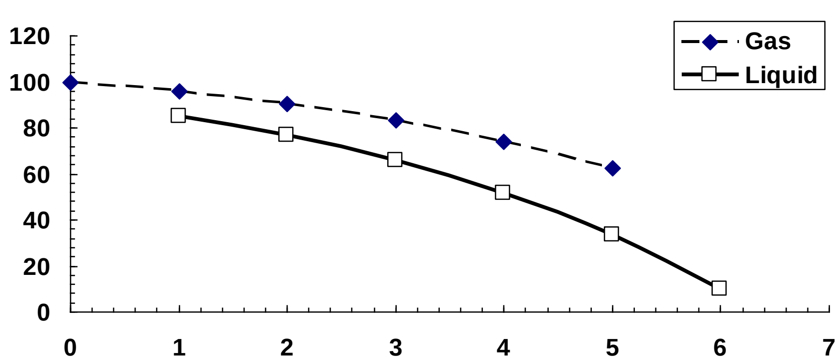
```
1 1 0 0 0 0;
0 1 -1 -1 0 0;
0 0 0 1 -1 -1];
>> B=[0 0 0 1 0 0 ]';
>> Q=A\B

Q =
    0.5059
    0.4941
    0.2588
    0.2353
    0.1412
    0.0941
```

12.12 The mass balances can be expressed in matrix form as

$$\begin{bmatrix} 2.8 & 0 & 0 & 0 & 0 & -0.8 & 0 & 0 & 0 & 0 \\ -2 & 2.8 & 0 & 0 & 0 & 0 & -0.8 & 0 & 0 & 0 \\ 0 & -2 & 2.8 & 0 & 0 & 0 & 0 & -0.8 & 0 & 0 \\ 0 & 0 & -2 & 2.8 & 0 & 0 & 0 & 0 & -0.8 & 0 \\ 0 & 0 & 0 & -2 & 2.8 & 0 & 0 & 0 & 0 & -0.8 \\ -0.8 & 0 & 0 & 0 & 0 & 1.8 & -1 & 0 & 0 & 0 \\ 0 & -0.8 & 0 & 0 & 0 & 0 & 1.8 & -1 & 0 & 0 \\ 0 & 0 & -0.8 & 0 & 0 & 0 & 0 & 1.8 & -1 & 0 \\ 0 & 0 & 0 & -0.8 & 0 & 0 & 0 & 0 & 1.8 & -1 \\ 0 & 0 & 0 & 0 & -0.8 & 0 & 0 & 0 & 0 & 1.8 \end{bmatrix} \begin{Bmatrix} c_{G1} \\ c_{G2} \\ c_{G3} \\ c_{G4} \\ c_{G5} \\ c_{L1} \\ c_{L2} \\ c_{L3} \\ c_{L4} \\ c_{L5} \end{Bmatrix} = \begin{Bmatrix} 200 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 10 \end{Bmatrix}$$

These equations can then be solved. The results are tabulated and plotted below:

Reactor	Gas	Liquid
0	100	
1	95.73328	85.06649
2	90.2475	76.53306
3	83.19436	65.5615
4	74.12603	51.45521
5	62.46675	33.31856
6		10

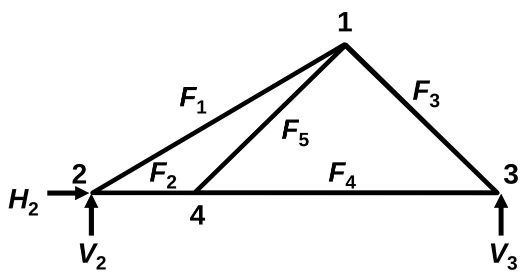


12.13 Let x_i = the volume taken from pit i. Therefore, the following system of equations must hold

$$
\begin{aligned}
0.55x_1 + 0.25x_2 + 0.25x_3 &= 4800 \\
0.30x_1 + 0.45x_2 + 0.20x_3 &= 5800 \\
0.15x_1 + 0.30x_2 + 0.55x_3 &= 5700
\end{aligned}
$$

These can then be solved for x_1 = 2416.667, x_2 = 9193.333, and x_3 = 4690.

12.14 We can number the nodes as



Node 1:

$$\Sigma F_H = 0 = -F_1 \cos 30° - F_5 \cos 45° + F_3 \cos 45° + 1200$$

$$\Sigma F_V = 0 = -F_1 \sin 30° - F_5 \sin 45° - F_3 \sin 45° - 600$$

Node 2:

$$\Sigma F_H = 0 = H_2 + F_2 + F_1 \cos 30°$$

$$\Sigma F_V = 0 = F_1 \sin 30° + V_2$$

Node 3:

$$\Sigma F_H = 0 = -F_4 - F_3 \cos 45°$$

$$\Sigma F_V = 0 = V_3 + F_3 \sin 45°$$

Node 4:

$$\Sigma F_H = 0 = -F_2 + F_4 + F_5 \cos 45°$$

$$\Sigma F_V = 0 = F_5 \sin 45° - 500$$

These balances can then be expressed in matrix form as

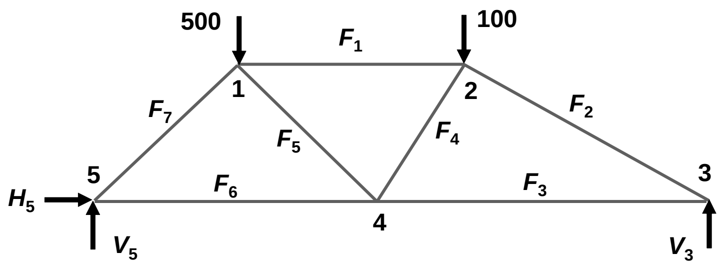
$$
\begin{bmatrix}
0.866 & 0 & -0.707 & 0 & 0.707 & 0 & 0 & 0 \\
0.5 & 0 & -0.707 & 0 & 0.707 & 0 & 0 & 0 \\
-0.866 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\
-0.5 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0.707 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.707 & 0 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & -1 & -0.707 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.707 & 0 & 0 & 0
\end{bmatrix}
\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ H_2 \\ V_2 \\ V_3 \end{Bmatrix}
=
\begin{Bmatrix} 1200 \\ -600 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -500 \end{Bmatrix}
$$

This system can be solved for

$F_1 = -292.82$	$F_2 = 1453.59$	$F_3 = -1348.58$	$F_4 = 953.5898$
$F_5 = 707.1068$	$H_2 = -1200$	$V_2 = 146.4102$	$V_3 = 953.5898$

Note that the horizontal reactions ($H_2 = -1200$) and the vertical reactions ($V_2 + V_3 = 146.4102 + 953.5898 = 1100$) are equal to the negative of the imposed loads. This is a good check that the computation is correct.

12.15 We can number the nodes as



Node 1:

$$\Sigma F_H = 0 = F_1 + F_5 \cos 45° - F_7 \cos 45°$$

$$\Sigma F_V = 0 = -F_5 \sin 45° - F_7 \sin 45° - 500$$

Node 2:

$$\Sigma F_H = 0 = -F_1 + F_2 \cos 30° - F_4 \cos 60°$$

$$\Sigma F_V = 0 = -F_2 \sin 30° - F_4 \sin 60° - 100$$

Node 3:

$$\Sigma F_H = 0 = -F_2 \cos 30° - F_3$$

$$\Sigma F_V = 0 = V_3 + F_2 \sin 30°$$

Node 4:

$$\Sigma F_H = 0 = F_3 + F_4 \cos 60° - F_5 \cos 45° - F_6$$

$$\Sigma F_V = 0 = F_4 \sin 60° + F_5 \sin 45°$$

Node 5:

$$\Sigma F_H = 0 = F_6 + F_7 \cos 45° + H_5$$

$$\Sigma F_V = 0 = F_7 \sin 45° + V_5$$

These balances can then be expressed in matrix form as

$$\begin{bmatrix} -1 & 0 & 0 & 0 & -0.707 & 0 & 0.707 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.707 & 0 & 0.707 & 0 & 0 & 0 \\ 1 & -0.866 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0.866 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.866 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.5 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -0.5 & 0.707 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.866 & -0.707 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -0.707 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.707 & 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ V_3 \\ H_5 \\ V_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -500 \\ 0 \\ -100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

This system can be solved for

$F_1 = -348.334$ $F_2 = -351.666$ $F_3 = 304.5517$ $F_4 = 87.56443$ $F_5 = -107.244$
$F_6 = 424.167$ $F_7 = -599.863$ $V_3 = 175.833$ $H_5 = 0$ $V_5 = 424.167$

12.16 The first two columns of the inverse provide the information to solve this problem

	F_{1H}	F_{1V}
F_1	0.866025	0.500000
F_2	0.250000	–0.433013
F_3	–0.500000	0.866025
H_2	–1.000000	0.000000
V_2	–0.433013	–0.250000
V_3	0.433013	–0.750000

$F_1 = 2000(0.866025) – 2500(0.5) = 482.0508$
$F_2 = 2000(0.25) – 2500(–0.433013) = 1582.532$
$F_3 = 2000(–0.5) – 2500(0.866025) = –3165.06$
$H_2 = 2000(–1)– 2500(0) = –2000$
$V_2 = 2000(–0.433013) – 2500(–0.25) = –241.025$
$V_3 = 2000(0.433013) – 2500(–0.75) = 2741.025$

12.17

$\Sigma F_y = 0$ $V_2 + V_3 = 1000$

$\Sigma M = 0$ $1000(\cos 30°)L_1 - V_3 L_2$

Geometry $\cos 30° L_1 + \cos 60° L_3 = L_2$

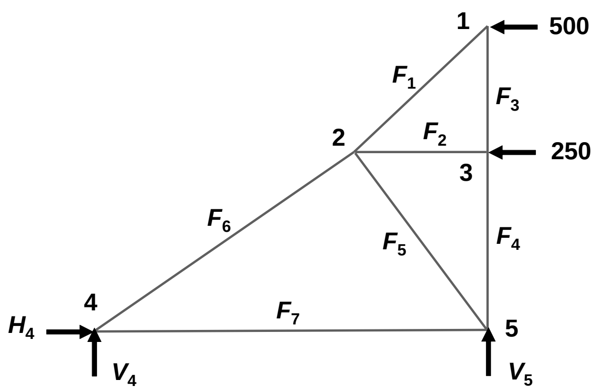
Since $V_2 = 250$ and $V_3 = 750$,

$$866L_1 - 750L_2 = 0$$
$$0.866L_1 + 0.5L_3 = L_2$$

Therefore,

$$L_3 = \frac{L_2 - 0.866L_1}{0.5}$$

12.18 We can number the nodes as



Node 1:

$\Sigma F_H = 0 = -F_1 \cos 45° - 500$

$\Sigma F_V = 0 = -F_1 \sin 45° - F_3$

Node 2:

$\Sigma F_H = 0 = F_1 \cos 45° + F_2 + F_5 \cos 60° - F_6 \cos 30°$

$\Sigma F_V = 0 = F_1 \sin 45° - F_5 \sin 60° - F_6 \sin 30°$

Node 3:

$\Sigma F_H = 0 = -F_2 - 250$

$\Sigma F_V = 0 = F_3 - F_4$

Node 4:

$\Sigma F_H = 0 = F_6 \cos 30° + F_7 + H_4$

$\Sigma F_V = 0 = F_6 \sin 30° + V_4$

Node 5:

$\Sigma F_H = 0 = -F_7 - F_5 \cos 60°$

$\Sigma F_V = 0 = F_4 + F_5 \sin 60° + V_5$

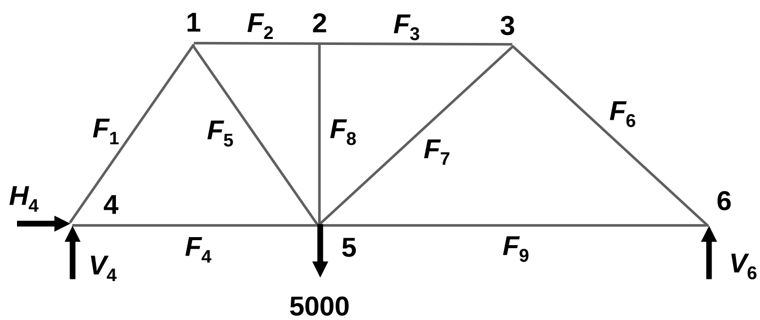
These balances can then be expressed in matrix form as

$$\begin{bmatrix} 0.707 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.707 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.707 & -1 & 0 & 0 & -0.5 & 0.866 & 0 & 0 & 0 & 0 \\ -0.707 & 0 & 0 & 0 & 0.866 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.866 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -0.866 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ H_4 \\ V_4 \\ V_5 \end{Bmatrix} = \begin{Bmatrix} -500 \\ 0 \\ 0 \\ 0 \\ -250 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

This system can be solved for

$F_1 = -707.107$ $F_2 = -250$ $F_3 = 500$ $F_4 = 500$ $F_5 = -58.0127$
$F_6 = -899.519$ $F_7 = 29.00635$ $H_4 = 750$ $V_4 = 449.7595$ $V_5 = -449.76$

12.19 We can number the nodes as



Node 1:
$\Sigma F_H = 0 = -F_1 \cos 60° + F_2 + F_5 \cos 60°$
$\Sigma F_V = 0 = -F_1 \sin 60° - F_5 \sin 60°$
Node 2:
$\Sigma F_H = 0 = -F_2 + F_3$
$\Sigma F_V = 0 = -F_8$
Node 3:
$\Sigma F_H = 0 = -F_3 + F_6 \cos 45° - F_7 \cos 45°$
$\Sigma F_V = 0 = -F_6 \sin 45° - F_7 \sin 45°$
Node 4:
$\Sigma F_H = 0 = F_1 \cos 30° + F_4 + H_4$
$\Sigma F_V = 0 = F_1 \sin 60° + V_4$
Node 5:
$\Sigma F_H = 0 = -F_4 - F_5 \cos 60° + F_7 \cos 45° + F_9$
$\Sigma F_V = 0 = F_5 \sin 60° + F_8 + F_7 \sin 45° - 5000$

Node 6:

$$\Sigma F_H = 0 = -F_6 \cos 45° - F_9$$

$$\Sigma F_V = 0 = F_6 \sin 45° + V_6$$

Note that $F_8 = 0$. Thus, the middle member is unnecessary unless there is a load with a nonzero vertical component at node 2. These balances can then be expressed in matrix form as

$$\begin{bmatrix} 0.5 & -1 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.866 & 0 & 0 & 0 & 0.866 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -0.707 & 0.707 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.707 & 0.707 & 0 & 0 & 0 & 0 \\ -0.5 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ -0.866 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0.5 & 0 & -0.707 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.866 & 0 & -0.707 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.707 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.707 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_9 \\ H_4 \\ V_4 \\ V_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -5000 \\ 0 \\ 0 \end{Bmatrix}$$

This system can be solved for

$F_1 = -3660.25$	$F_2 = -3660.25$	$F_3 = -3660.25$	$F_4 = 1830.127$	$F_5 = -3660.25$
$F_6 = -2588.19$	$F_7 = 2588.19$	$F_9 = 1830.13$	$H_4 = 0$	$V_4 = 3169.87$
$V_6 = 1830.13$				

12.20 (a)

<u>Room 1:</u>

$$0 = W_{\text{smoker}} + Q_a c_a - Q_a c_1 + E_{13}(c_3 - c_1)$$

<u>Room 2:</u>

$$0 = Q_b c_b + (Q_a - Q_d)c_4 - Q_c c_2 + E_{24}(c_4 - c_2)$$

<u>Room 3:</u>

$$0 = W_{\text{grill}} + Q_a c_1 + E_{13}(c_1 - c_3) + E_{34}(c_4 - c_3) - Q_a c_3$$

<u>Room 4:</u>

$$0 = Q_a c_3 + E_{34}(c_3 - c_4) + E_{24}(c_2 - c_4) - Q_a c_4$$

Substituting the parameters yields

$$\begin{bmatrix} 225 & 0 & -25 & 0 \\ 0 & 175 & 0 & -125 \\ -225 & 0 & 275 & -50 \\ 0 & -25 & -250 & 275 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} 1400 \\ 100 \\ 2000 \\ 0 \end{Bmatrix}$$

These can be solved for

$$\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} 8.0996 \\ 12.3448 \\ 16.8966 \\ 16.4828 \end{Bmatrix}$$

(b) The matrix inverse can be determined as

$$[A]^{-1} = \begin{bmatrix} 0.004996 & 0.0000153 & 0.000552 & 0.000107 \\ 0.003448 & 0.006207 & 0.003448 & 0.003448 \\ 0.004966 & 0.000138 & 0.004966 & 0.000966 \\ 0.004828 & 0.00069 & 0.004828 & 0.004828 \end{bmatrix}$$

The percent of the carbon monoxide in the kids' section due to each source can be computed as

(*i*) the smokers

$$c_{2,\text{smokers}} = a_{21}^{-1} W_{\text{smokers}} = 0.003448(1000) = 3.448$$

$$\%_{\text{smokers}} = \frac{3.448}{12.3448} \times 100\% = 27.93\%$$

(*ii*) the grill

$$c_{2,\text{grill}} = a_{31}^{-1} W_{\text{grill}} = 0.003448(2000) = 6.897$$

$$\%_{\text{grill}} = \frac{6.897}{12.3448} \times 100\% = 55.87\%$$

(*iii*) the intakes

$$c_{2,\text{intakes}} = a_{21}^{-1} Q_a c_a + a_{22}^{-1} Q_b c_b = 0.003448(200)2 + 0.006207(50)2 = 1.37931 + 0.62069 = 2$$

$$\%_{\text{grill}} = \frac{2}{12.3448} \times 100\% = 16.20\%$$

(c) If the smoker and grill loads are increased by 1000 and 3000 mg/hr, respectively, the concentration in the kids' section will be increased by

$$\Delta c_2 = a_{21}^{-1} \Delta W_{\text{smoker}} + a_{23}^{-1} \Delta W_{\text{grill}} = 0.003448(2000-1000) + 0.003448(5000-2000)$$
$$= 3.448 + 10.3448 = 13.7931$$

(d) If the mixing between the kids' area and zone 4 is decreased to 5, the system of equations is changed to

$$\begin{bmatrix} 225 & 0 & -25 & 0 \\ 0 & 155 & 0 & -105 \\ -225 & 0 & 275 & -50 \\ 0 & -5 & -250 & 255 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} 1400 \\ 100 \\ 2000 \\ 0 \end{Bmatrix}$$

which can be solved for

$$\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix} = \begin{Bmatrix} 8.1084 \\ 12.0800 \\ 16.9760 \\ 16.8800 \end{Bmatrix}$$

Therefore, the concentration in the kids' area would be decreased 0.26483 mg/m^3 or 2.145%.

12.21 The coordinates of the connection points are

D: (0, 0, 2.4)
A: (0.8, –0.6, 0)
B: (–0.8, –0.6, 0)
C: (0, 1, 0)

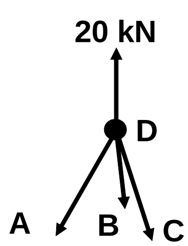
The lengths of the legs can be computed as

$$DA = \sqrt{0.8^2 + 0.6^2 + 2.4^2} = 2.6$$

$$DB = \sqrt{0.8^2 + 0.6^2 + 2.4^2} = 2.6$$

$$DC = \sqrt{0^2 + 1^2 + 2.4^2} = 2.6$$

Assume that each leg is in tension, which mean that each pulls on point *D*.



The direction cosines of the vectors A, B and C can be determined as

A: $\left(\frac{4}{13}, -\frac{3}{13}, -\frac{12}{13}\right)$

B: $\left(-\frac{4}{13}, -\frac{3}{13}, -\frac{12}{13}\right)$

C: $\left(0, \frac{5}{13}, -\frac{12}{13}\right)$

Force balances for point *D* can then be written as

$$\sum F_x = \frac{4}{13}A - \frac{4}{13}B = 0$$

$$\sum F_y = -\frac{3}{13}A - \frac{3}{13}B + \frac{5}{13}C = 0$$

$$\sum F_z = -\frac{12}{13}A - \frac{12}{13}B - \frac{12}{13}C + 20 = 0$$

Thus, the solution amounts to solving the following system of linear algebraic equations

$$\begin{aligned} 0.30769A - 0.30769B \qquad\qquad &= 0 \\ -0.23077A - 0.23077B + 0.38462C &= 0 \\ -0.92308A - 0.92308B - 0.92308C &= -20 \end{aligned}$$

These equations can be solved with Gauss elimination for A = 6.7708, B = 6.7708, and C = 8.125.

12.22 The solution can be generated in a number of ways. For example, using MATLAB,

```
>> A=[1 0 0 0 0 0 0 0 1 0;
      0 0 1 0 0 0 0 1 0 0;
      0 1 0 3/5 0 0 0 0 0 0;
      -1 0 0 -4/5 0 0 0 0 0 0;
      0 -1 0 0 0 0 3/5 0 0 0;
      0 0 0 0 -1 0 -4/5 0 0 0;
      0 0 -1 -3/5 0 1 0 0 0 0;
      0 0 0 4/5 1 0 0 0 0 0;
      0 0 0 0 0 -1 -3/5 0 0 0;
      0 0 0 0 0 0 4/5 0 0 1];
>> B=[0 0 -74 0 0 24 0 0 0 0]';
>> x=A\B

x =
   37.3333
  -46.0000
   74.0000
  -46.6667
   37.3333
   46.0000
  -76.6667
  -74.0000
  -37.3333
```

```
   61.3333
```

Therefore, in kN

AB = 37.3333	BC = –46	AD = 74	BD = –46.6667	CD = 37.3333
DE = 46	CE = –76.6667	A_x = –74	A_y = –37.33333	E_y = 61.3333

12.23 The simultaneous equations are

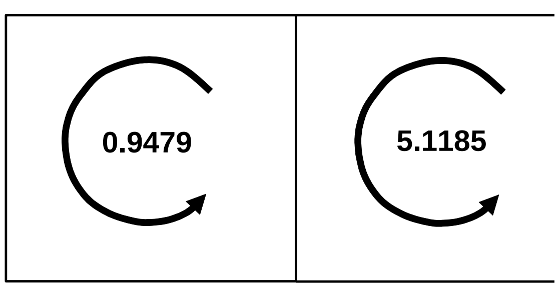
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 5 & -15 & 0 & -5 & -2 \\ 10 & -5 & 0 & -25 & 0 & 0 \end{bmatrix} \begin{Bmatrix} i_{12} \\ i_{52} \\ i_{32} \\ i_{65} \\ i_{54} \\ i_{43} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 200 \end{Bmatrix}$$

This system can be solved in a number of ways. For example, using MATLAB,

```
>> A=[1 1 1 0 0 0;
0 -1 0 1 -1 0;
0 0 -1 0 0 1;
0 0 0 0 1 -1;
0 5 -15 0 -5 -2;
10 -5 0 -25 0 0];
>> B=[0 0 0 0 0 200]';
>> I=A\B

I =
    5.1185
   -4.1706
   -0.9479
   -5.1185
   -0.9479
   -0.9479
```

i_{21} = 5.1185 i_{52} = –4.1706 i_{32} = –0.9479 i_{65} = –5.1185 i_{54} = –0.9479 i_{43} = –0.9479

Here are the resulting currents superimposed on the circuit:



12.24 The current equations can be written as

$$-i_{21} - i_{23} + i_{52} = 0$$
$$i_{23} - i_{35} + i_{43} = 0$$
$$-i_{43} + i_{54} = 0$$
$$i_{35} - i_{52} + i_{65} - i_{54} = 0$$

Voltage equations:

$$i_{21} = \frac{V_2 - 10}{35} \qquad i_{54} = \frac{V_5 - V_4}{15}$$

$$i_{23} = \frac{V_2 - V_3}{30} \qquad i_{35} = \frac{V_3 - V_5}{7}$$

$$i_{43} = \frac{V_4 - V_3}{8} \qquad i_{52} = \frac{V_5 - V_2}{10}$$

$$i_{65} = \frac{150 - V_5}{5}$$

$$\begin{bmatrix} -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 35 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 10 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} i_{21} \\ i_{23} \\ i_{52} \\ i_{35} \\ i_{43} \\ i_{54} \\ i_{65} \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 150 \end{Bmatrix}$$

This system can be solved for

i_{21} = 2.9291 i_{23} = –0.6457 i_{52} = 2.2835 i_{35} = –0.4950 i_{43} = 0.1507
i_{54} = 0.1507 i_{65} = 2.9291 V_2 = 112.5196 V_3 = 131.8893 V_4 = 133.0945
V_5 = 135.3543

12.25 The current equations can be written as

$$i_{32} - i_{25} + i_{12} = 0$$
$$-i_{32} - i_{34} + i_{63} = 0$$
$$i_{34} - i_{47} = 0$$
$$i_{25} + i_{65} - i_{58} = 0$$

$$i_{76} - i_{63} - i_{65} = 0$$
$$i_{47} - i_{76} + i_{97} = 0$$
$$i_{58} - i_{89} - i_{80} = 0$$
$$i_{89} - i_{97} = 0$$

Voltage equations:

$$-20i_{25} + 10i_{65} - 5i_{63} - 5i_{32} = 0$$
$$5i_{63} + 20i_{76} + 5i_{47} + 20i_{34} = 0$$
$$-50i_{58} - 15i_{89} - 0i_{97} - 20i_{76} - 10i_{65} = 0$$
$$120 - 20i_{25} - 50i_{58} = 40$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 5 & 20 & 0 & 0 & 5 & 0 & -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -20 & -5 & -5 & 0 & 0 & -20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10 & 50 & 20 & 0 & 15 & 0 \\ 0 & 20 & 0 & 0 & 0 & 0 & 0 & 50 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} i_{32} \\ i_{25} \\ i_{12} \\ i_{34} \\ i_{63} \\ i_{47} \\ i_{65} \\ i_{58} \\ i_{76} \\ i_{97} \\ i_{89} \\ i_{80} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 80 \end{Bmatrix}$$

This system can be solved for

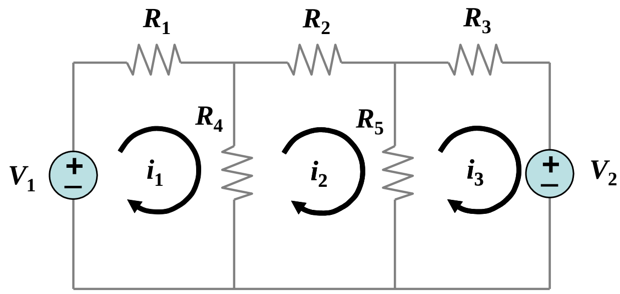
$i_{32} = -2.5670$	$i_{25} = 1.0449$	$i_{12} = 3.6119$	$i_{34} = 1.2287$	$i_{63} = -1.3384$
$i_{47} = 1.2287$	$i_{65} = 0.1371$	$i_{58} = 1.1820$	$i_{76} = -1.2012$	$i_{97} = -2.4299$
$i_{89} = -2.4299$	$i_{80} = 3.6119$			

12.26 Let c_i = component *i*. Therefore, the following system of equations must hold

$$15c_1 + 17c_2 + 19c_3 = 3890$$
$$0.30c_1 + 0.40c_2 + 0.55c_3 = 95$$
$$1.0c_1 + 1.2c_2 + 1.5c_3 = 282$$

These can then be solved for $c_1 = 90$, $c_2 = 60$, and $c_3 = 80$.

12.27 First, we can number the loops and assume that the currents are clockwise.



Kirchhoff's voltage law can be applied to each loop.

$$-V_1 + R_1 i_1 + R_4(i_1 - i_2) = 0$$
$$R_4(i_2 - i_1) + R_2 i_2 + R_5(i_2 - i_3) = 0$$
$$R_5(i_3 - i_2) + R_3 i_3 + V_2 = 0$$

Collecting terms, the system can be written in matrix form as

$$\begin{bmatrix} 20 & -15 & 0 \\ -15 & 50 & -25 \\ 0 & -25 & 45 \end{bmatrix} \begin{Bmatrix} i_1 \\ i_2 \\ i_3 \end{Bmatrix} = \begin{Bmatrix} 80 \\ 0 \\ -50 \end{Bmatrix}$$

This can be solved with a tool like MATLAB,

```
> A=[20 -15 0;-15 50 -25;0 -25 45];
>> B=[80;0;-50];
>> I=A\B

I =
    4.9721
    1.2961
   -0.3911
```

Therefore, $I_1 = 4.9721$, $I_2 = 1.2961$, and $I_3 = -0.3911$.

12.28 This problem can be solved by applying Kirchhoff's voltage law to each loop.

$$-20 + 4(i_1 - i_2) + 2(i_1 - i_3) = 0$$
$$4(i_2 - i_1) + 6i_2 + 8(i_2 - i_3) = 0$$
$$8(i_3 - i_2) + 5i_3 + 2(i_3 - i_1) = 0$$

Collecting terms, the system can be written in matrix form as

$$\begin{bmatrix} 6 & -4 & -2 \\ -4 & 18 & -8 \\ -2 & -8 & 15 \end{bmatrix} \begin{Bmatrix} i_1 \\ i_2 \\ i_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 0 \\ 0 \end{Bmatrix}$$

This can be solved with a tool like MATLAB,

```
> A=[6 -4 -2;-4 18 -8;-2 -8 15];
>> B=[20;0;0];
>> I=A\B

I =
    5.1759
    1.9095
    1.7085
```

Therefore, $I_1 = 5.1759$, $I_2 = 1.9095$, and $I_3 = 1.7085$.

12.29 This problem can be solved directly on a calculator capable of doing matrix operations or on MATLAB.

```
>> b=[-200;-250;100];
>> a=[55 0 -25;0 -37 -4;-25 -4 29];
>> b=[-200;-250;100];
>> x=a\b

x =
   -2.7278
    6.5407
    1.9989
```

Therefore, $I_1 = -2.7278$ A, $I_3 = 6.5407$ A, and $I_4 = 1.9989$ A

12.30 This problem can be solved directly on a calculator capable of doing matrix operations or on MATLAB.

```
>> a=[60 -40 0
   -40 150 -100
   0 -100 130];
>> b=[200
   0
   230];
>> x=a\b

x =
    7.7901
    6.6851
    6.9116
```

Therefore, $I_1 = 7.79$ A, $I_2 = 6.69$ A, and $I_3 = 6.91$ A.

12.31 At steady state, the force balances can be written as

$$\begin{aligned} 4kx_1 - 3kx_2 \qquad\quad &= m_1 g \\ -3kx_1 + 4kx_2 - kx_3 &= m_2 g \\ -kx_2 + kx_3 &= m_3 g \end{aligned}$$

Substituting the parameter values

$$\begin{bmatrix} 120 & -90 & 0 \\ -90 & 120 & -30 \\ 0 & -30 & 30 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 19.6 \\ 29.4 \\ 24.5 \end{Bmatrix}$$

The solution is x_1 = 2.45, x_2 = 3.049, and x_3 = 3.866.

12.32 At steady state, the force balances can be written as

$$\begin{bmatrix} 30 & -20 & 0 \\ -20 & 30 & -10 \\ 0 & -10 & -10 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 98 \\ 34.3 \\ 19.6 \end{Bmatrix}$$

The solution is x_1 = 15.19, x_2 = 17.885, and x_3 = 19.845.

12.33 The force balances can be written as

$$\begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ F \end{Bmatrix}$$

Substituting the parameter values

$$\begin{bmatrix} 150 & -50 & 0 & 0 \\ -50 & 130 & -80 & 0 \\ 0 & -80 & 280 & -200 \\ 0 & 0 & -200 & 200 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1500(9.8) \end{Bmatrix}$$

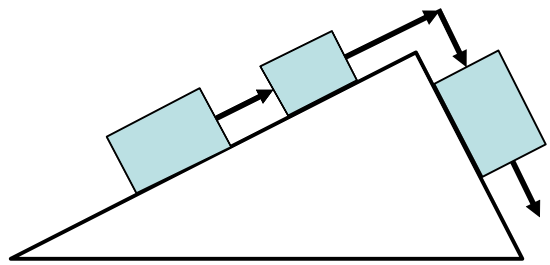
The solution is x_1 = 147, x_2 = 441, x_3 = 624.75, and x_4 = 698.25.

12.34 The equations can be solved in a number of ways. For example, using MATLAB,

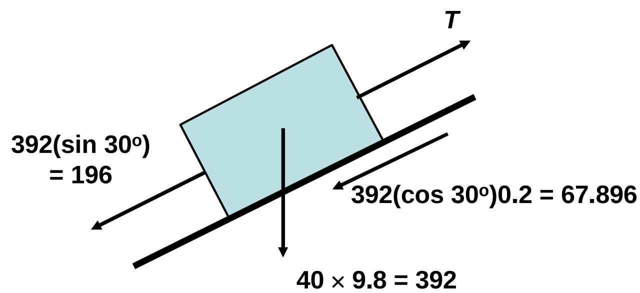
```
>> A=[100 1 0;50 -1 1;25 0 -1];
>> B=[519.72;216.55;108.27];
>> x=A\B

x =
    4.8259
   37.1257
   12.3786
```

Therefore, a = 4.8259, T = 37.1257, and R = 12.3786.

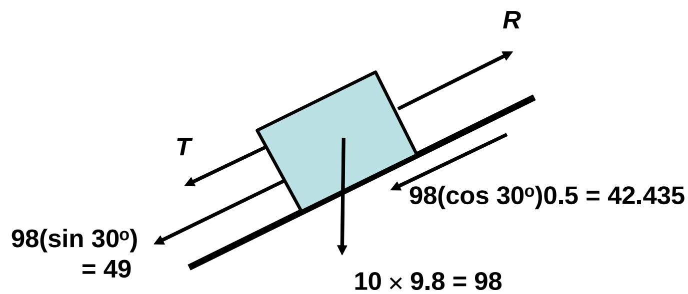
12.35 In order to solve this problem, we must assume the direction that the blocks are moving. For example, we can assume that the blocks are moving from left to right as shown



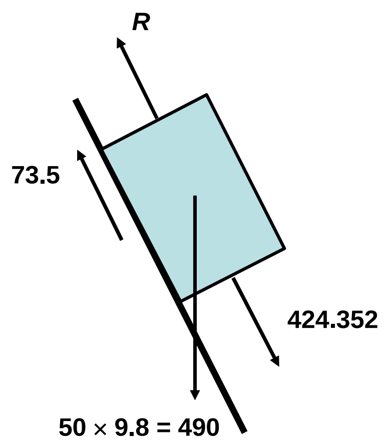
Force balances can be written for each block:



$-196 - 67.896 + T = 40a$



$-49 - T + R - 42.435 = 10a$



$424.352 - 73.5 - R = 50a$

Therefore, the system of equations to be solved can be written in matrix form as

$$\begin{bmatrix} 40 & -1 & 0 \\ 10 & 1 & -1 \\ 50 & 0 & 1 \end{bmatrix} \begin{Bmatrix} a \\ T \\ R \end{Bmatrix} = \begin{Bmatrix} -263.8964 \\ -91.43524 \\ 350.852 \end{Bmatrix}$$

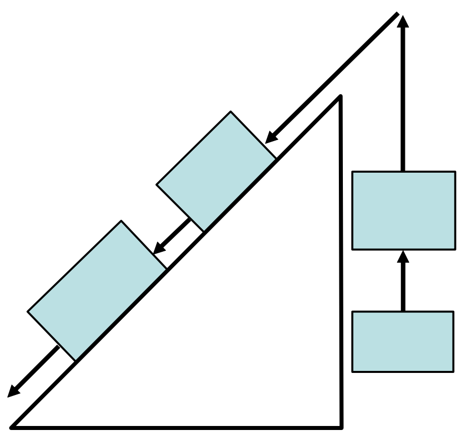
The solution is a = –0.044792, T = 262.1047, and R = 353.092.

Note that if we had assumed that the blocks were moving from right to left, the system of equations would have been

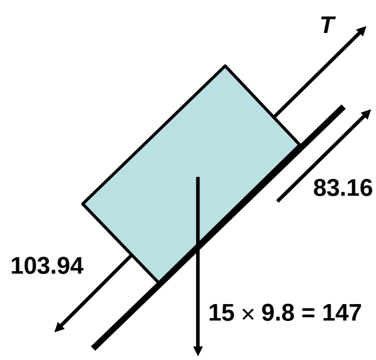
$$\begin{bmatrix} 40 & 1 & 0 \\ 10 & -1 & 1 \\ 50 & 0 & -1 \end{bmatrix} \begin{Bmatrix} a \\ T \\ R \end{Bmatrix} = \begin{Bmatrix} 128.1036 \\ 6.564755 \\ -497.852 \end{Bmatrix}$$

The solution for this case is a = –3.63184, T = 273.3772, and R = 316.2604.

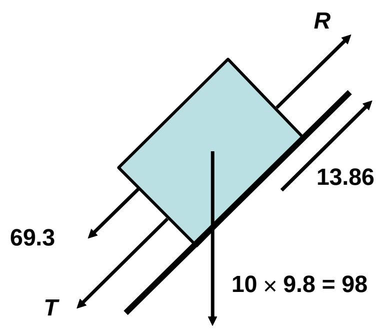
12.36 In order to solve this problem, we must assume the direction that the blocks are moving. For example, we can assume that the blocks are moving from right to left as shown



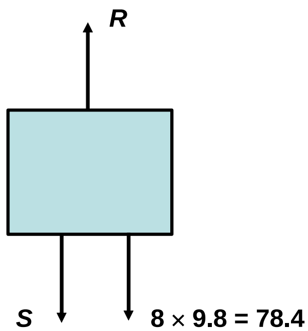
Force balances can be written for each block:



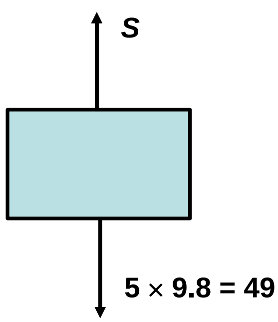
$103.94 - T - 83.16 = 15a$



$T + 69.3 - R - 13.86 = 10a$



$R - 78.4 - S = 8a$



$S - 49 = 5a$

Therefore, the system of equations to be solved can be written in matrix form as

$$\begin{bmatrix} 15 & 1 & 0 & 0 \\ 10 & -1 & 1 & 0 \\ 8 & 0 & -1 & 1 \\ 5 & 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} a \\ T \\ R \\ S \end{Bmatrix} = \begin{Bmatrix} 20.789 \\ 55.437 \\ -78.4 \\ -49 \end{Bmatrix}$$

The solution is $a = -1.347$, $T = 40.989$, $R = 109.893$, and $S = 42.267$.

Note that if we had assumed that the blocks were moving from left to right, the system of equations would have been

$$\begin{bmatrix} 15 & -1 & 0 & 0 \\ 10 & 1 & -1 & 0 \\ 8 & 0 & 1 & -1 \\ 5 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} a \\ T \\ S \\ R \end{Bmatrix} = \begin{Bmatrix} -187.1005 \\ -83.15576 \\ 78.4 \\ 49 \end{Bmatrix}$$

The solution for this case is a = –3.759374, T = 130.7098, R = 176.27186, and S = 67.79687.

12.37 This problem can be solved in a number of ways. For example, using MATLAB,

```
%prob1237.m

k1=10;k2=30;k3=30;k4=10;
m1=2;m2=2;m3=2;
km=[(1/m1)*(k2+k1),-(k2/m1),0;
    -(k2/m2),(1/m2)*(k2+k3),-(k3/m2);
    0,-(k3/m3),(1/m3)*(k3+k4)]
x=[0.05;0.04;0.03]
kmx=km*x

>> prob1237

km =
    20   -15     0
   -15    30   -15
     0   -15    20


x =
    0.0500
    0.0400
    0.0300


kmx =
    0.4000
         0
         0
```

Therefore, $\ddot{x}_1$ = –0.4, $\ddot{x}_2$ = 0, and $\ddot{x}_3$ = 0 m/s^2.

12.38 (a) Substituting the parameters gives

$$\frac{d^2T}{dx^2} + h'(T_a - T) = 0$$

An analytical solution can be derived in a number of ways. One way is to assume a solution of the form

$$T = A + Be^{\lambda x} + Ce^{\lambda x}$$

Differentiating twice gives

$$T'' = \lambda^2 Be^{\lambda x} + \lambda^2 Ce^{\lambda x}$$

Substituting these into the original differential equation yields

$$\lambda^2 Be^{\lambda x} + \lambda^2 Ce^{\lambda x} + h'(T_a - A - Be^{\lambda x} - Ce^{\lambda x}) = 0$$

Equating like terms yields

$$\lambda^2 Be^{\lambda x} = h' Be^{\lambda T}$$
$$\lambda^2 Ce^{\lambda x} = h' Ce^{\lambda T}$$
$$h' T_a = h' A$$

The first two equations give $\lambda = \pm\sqrt{h'}$. The equation third gives $A = T_a$. Therefore, the solution is

$$T = T_a + Be^{\sqrt{h'}x} + Ce^{-\sqrt{h'}x}$$

The unknown constants can be evaluated from the boundary conditions

$$40 = 20 + B + C$$
$$200 = 20 + Be^{\sqrt{0.02}(10)} + Ce^{-\sqrt{0.02}(10)}$$

These simultaneous equations can be solved for B = 45.25365 and C = –25.25365. Therefore, the analytical solution is

$$T = 20 + 45.25365 e^{0.141421x} - 25.25365 e^{-0.141421x}$$

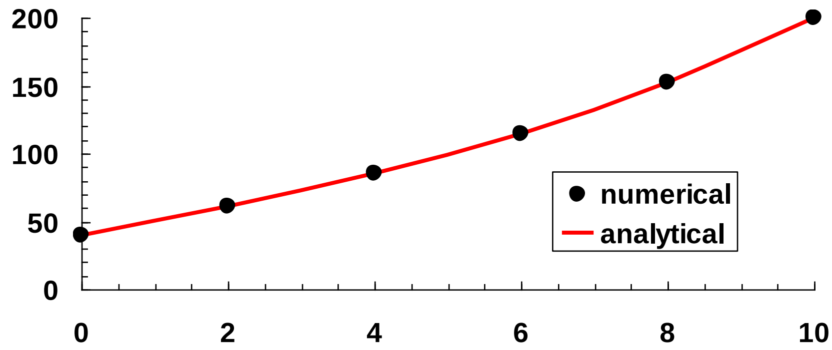
(b) Substituting the parameters into the finite-difference equation gives

$$-T_{i-1} + 2.08T_i - T_{i+1} = 1.6$$

An analytical solution can be derived in a number of ways. A nice approach is to employ L This equation can be written for each node to yield the following system of equations,

$$\begin{bmatrix} 2.08 & -1 & 0 & 0 \\ -1 & 2.08 & -1 & 0 \\ 0 & -1 & 2.08 & -1 \\ 0 & 0 & -1 & 2.08 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 41.6 \\ 1.6 \\ 1.6 \\ 201.6 \end{Bmatrix}$$

These can be solved for T_1 = 61.0739, T_2 = 85.4338, T_3 = 115.0283, and T_4 = 152.2252. A plot of the results is shown below (circles). In addition, the plot also shows the analytical solution (line) that was developed in **(a)**:



12.39 Substituting centered difference finite differences, the Laplace equation can be written for the node (1, 1) as

$$0 = \frac{T_{21} - 2T_{11} + T_{01}}{\Delta x^2} + \frac{T_{12} - 2T_{11} + T_{10}}{\Delta y^2}$$

Because the grid is square ($\Delta x = \Delta y$), this equation can be expressed as

$$0 = T_{21} - 4T_{11} + T_{01} + T_{12} + T_{10}$$

The boundary node values (T_{01} = 100 and T_{10} = 75) can be substituted to give

$$4T_{11} - T_{12} - T_{21} = 175$$

The same approach can be written for the other interior nodes. When this is done, the following system of equations results

$$\begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix} \begin{Bmatrix} T_{11} \\ T_{12} \\ T_{21} \\ T_{22} \end{Bmatrix} = \begin{Bmatrix} 175 \\ 125 \\ 75 \\ 25 \end{Bmatrix}$$

These equations can be solved using the Gauss-Seidel method. For example, the first iteration would be

$$T_{11} = \frac{175 + T_{12} + T_{21}}{4} = \frac{175 + 0 + 0}{4} = 43.75$$

$$T_{12} = \frac{125 + T_{11} + T_{22}}{4} = \frac{125 + 43.75 + 0}{4} = 42.1875$$

$$T_{21} = \frac{75 + T_{11} + T_{22}}{4} = \frac{75 + 43.75 + 0}{4} = 29.6875$$

$$T_{22} = \frac{25 + T_{12} + T_{21}}{4} = \frac{25 + 42.1875 + 29.6875}{4} = 24.21875$$

The computation can be continued as follows:

iteration	unknown	value	ε_a	maximum ε_a
1	x_1	43.75	100.00%	
	x_2	42.1875	100.00%	
	x_3	29.6875	100.00%	
	x_4	24.21875	100.00%	100.00%
2	x_1	61.71875	29.11%	
	x_2	52.73438	20.00%	
	x_3	40.23438	26.21%	
	x_4	29.49219	17.88%	29.11%
3	x_1	66.99219	7.87%	
	x_2	55.37109	4.76%	
	x_3	42.87109	6.15%	
	x_4	30.81055	4.28%	7.87%
4	x_1	68.31055	1.93%	
	x_2	56.03027	1.18%	
	x_3	43.53027	1.51%	
	x_4	31.14014	1.06%	1.93%
5	x_1	68.64014	0.48%	
	x_2	56.19507	0.29%	
	x_3	43.69507	0.38%	
	x_4	31.22253	0.26%	0.48%

Thus, after 5 iterations, the maximum error is 0.48% and we are converging on the final result: T_{11} = 68.64, T_{12} = 56.195, T_{21} = 43.695, and T_{22} = 31.22.

12.40 Find the unit vectors:

$$A\left(\frac{1\hat{i} - 2\hat{j} - 4\hat{k}}{\sqrt{1^2 + 2^2 + 4^2}}\right) = 0.218\hat{i} - 0.436\hat{j} - 0.873\hat{k}$$

$$B\left(\frac{2\hat{i} + 1\hat{j} - 4\hat{k}}{\sqrt{1^2 + 2^2 + 4^2}}\right) = 0.436\hat{i} + 0.218\hat{j} - 0.873\hat{k}$$

Sum moments about the origin:

$$\sum M_{ox} = 50(2) - 0.436B(4) - 0.218A(4) = 0$$

$$\sum M_{oy} = 0.436A(4) - 0.218B(4) = 0$$

Solve for *A* and *B* using equations 9.10 and 9.11:

In the form $\begin{array}{l} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_2 + a_{22}x_2 = b_2 \end{array}$

$$\begin{array}{l} -0.872A - 1.744B = -100 \\ 1.744A - 0.872B = 0 \end{array}$$

Plug into equations 9.10 and 9.11:

$$A = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}} = \frac{87.2}{3.80192} = 22.94 \ \ N$$

$$B = \frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{12}a_{21}} = \frac{174.4}{3.80192} = 45.87 \ \ N$$