CHAPTER 16

16.1 The area and volume can be computed as

$$A = \pi r^2 + 2\pi r h \tag{1}$$
$$V = \pi r^2 h \tag{2}$$

An Excel spreadsheet can be set up to solve the problem as

	A	В
1	radius	1
2	height	1
3		
4	volume	3.141593
5	desired volume	0.5
6		
7	side area	6.283185
8	bottom area	3.141593
9	total area	9.424778

The formulas are

	A	В
1	radius	1
2	height	1
3		
4	volume	=PI()*B1^2*B2
5	desired volume	0.5
6		
7	side area	=2*PI()*B1*B2
8	bottom area	=PI()*B1^2
9	total area	=SUM(B7:B8)

The Solver can be called and set up as

Solver Parameters	
Set Target Cell: \$5\$9	Solve
Equal To: O Max ③ Min O Value of: 0 By Changing Cells:	Close
\$B\$1:\$B\$2	
Subject to the Constraints:	Options
\$8\$4 >= \$8\$5	
	Reset All
Delete	

The resulting solution is

	A	В
1	radius	0.542187
2	height	0.541403
3		
4	volume	0.499999
5	desired volume	0.5
6		
7	side area	1.844378
8	bottom area	0.923525
9	total area	2.767902

Thus, the Solver says that the optimal cylindrical container is one where the radius equals the height. For the case of the desired $V = 0.5 \text{ m}^3$, the dimensions are r = h = 0.542 m.

The general result of r = h can be verified using calculus as follows. First, we can solve the volume equation for h as

$$h = \frac{V}{\pi r^2} \tag{3}$$

This can be substituted into the area equation to give

$$A = \pi r^{2} + 2\pi r \frac{V}{\pi r^{2}} = \pi r^{2} + \frac{2V}{r}$$

We can differentiate this equation with respect to r to yield

$$\frac{dA}{dr} = 2\pi r - \frac{2V}{r^2}$$

which can be set equal to zero and solved for

$$r = \sqrt[3]{\frac{V}{\pi}}$$

This result can then be substituted into Eq. 3 which can be solved for

$$h = \sqrt[3]{\frac{V}{\pi}}$$

Thus, we prove that the optimal container has $r = h = (V/\pi)^{1/3}$. For our desired volume of 0.5 m³, this means that $r = h = (0.5/\pi)^{1/3} = 0.541926$ m, which confirms the result obtained numerically with the Excel Solver.

16.2 (a) The area and volume can be computed as

$$A = \pi r^2 + \pi r \sqrt{r^2 + h^2} \tag{1}$$

$$V = \frac{\pi r^2 h}{3}$$

An Excel spreadsheet can be set up to solve the problem as

	A	В
1	radius	1
2	height	1
3		
4	volume	1.047198
5	desired volume	0.5
6		
7	top area	3.141593
8	side area	4.442883
9	total area	7.584476

The formulas are

	A	В
1	radius	1
2	height	1
3	1.000	
4	volume	=PI()*B1/2*B2/3
5	desired volume	0.5
6		
7	top area	=PI()*B1/2
8	side area	=PI()*B1*SQRT(B1^2+B2^2)
9	total area	=87+88

The Solver can be called and set up as

Solver Parameters	
Set Target Cell: Equal To: <u>Max</u> Min <u>V</u> alue of: By Changing Cells:	Solve Close
\$B\$1:\$B\$2 Guess Subject to the Constraints:	Options
\$B\$4 >= \$B\$5 <u>A</u> dd Change	
Delete	Reset All

The resulting solution is

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. <u>No part of this Manual</u> may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

(2)

1	A	В
1	radius	0.552892
2	height	1.561923
3		
4	volume	0.5
5	desired volume	0.5
6		
7	top area	0.960353
8	side area	2.877962
9	total area	3.838315

(b) For this case, the area and volume can be computed as

$$A = \pi r \sqrt{r^2 + h^2}$$
$$V = \frac{\pi r^2 h}{3}$$

An Excel spreadsheet can be set up to solve the problem in a similar fashion to part (a) with the result: r = 0.6964 m and h = 0.9844 m.

16.3 This problem can be solved in a number of different ways. For example, using the golden section search, the result is

i	Cı	g(c _i)	C 2	g(c ₂)	C 1	g(c 1)	Cu	$g(c_u)$	d	Copt	Ea
1	0.0000	0.0000	3.8197	0.2330	6.1803	0.1310	10.0000	0.0641	6.1803	3.8197	100.00%
2	0.0000	0.0000	2.3607	0.3350	3.8197	0.2330	6.1803	0.1310	3.8197	2.3607	100.00%
3	0.0000	0.0000	1.4590	0.3686	2.3607	0.3350	3.8197	0.2330	2.3607	1.4590	100.00%
4	0.0000	0.0000	0.9017	0.3174	1.4590	0.3686	2.3607	0.3350	1.4590	1.4590	61.80%
5	0.9017	0.3174	1.4590	0.3686	1.8034	0.3655	2.3607	0.3350	0.9017	1.4590	38.20%
6	0.9017	0.3174	1.2461	0.3593	1.4590	0.3686	1.8034	0.3655	0.5573	1.4590	23.61%
7	1.2461	0.3593	1.4590	0.3686	1.5905	0.3696	1.8034	0.3655	0.3444	1.5905	13.38%
8	1.4590	0.3686	1.5905	0.3696	1.6718	0.3688	1.8034	0.3655	0.2129	1.5905	8.27%
9	1.4590	0.3686	1.5403	0.3696	1.5905	0.3696	1.6718	0.3688	0.1316	1.5905	5.11%
10	1.5403	0.3696	1.5905	0.3696	1.6216	0.3694	1.6718	0.3688	0.0813	1.5905	3.16%
11	1.5403	0.3696	1.5713	0.3696	1.5905	0.3696	1.6216	0.3694	0.0502	1.5713	1.98%
12	1.5403	0.3696	1.5595	0.3696	1.5713	0.3696	1.5905	0.3696	0.0311	1.5713	1.22%
13	1.5595	0.3696	1.5713	0.3696	1.5787	0.3696	1.5905	0.3696	0.0192	1.5713	0.75%

Thus, after 13 iterations, the method is converging on the true value of c = 1.5679 which corresponds to a maximum specific growth rate of g = 0.36963.

16.4 (a) The LP formulation is given by

 $Maximize C = 30X + 30Y + 35Z \qquad {Maximize profit}$

subject to

 $6X + 4Y + 12Z \le 2500$ {Raw chemical constraint} $0.05X + 0.1Y + 0.2Z \le 55$ {Time constraint}

$X + Y + Z \le 450$	{Storage constraint}
$X, Y, Z \ge 0$	{Positivity constraints}

Basis	С	Х	Y	Ζ	S1	S2	S3	Solution	Intercept
Р	1	-30	-30	-35	0	0	0	0	
S1	0	6	4	12	1	0	0	2500	208.3333
S2	0	0.05	0.1	0.2	0	1	0	55	275
S3	0	1	1	1	0	0	1	450	450
Basis	С	Х	Y	Ζ	S1	S2	S3	Solution	Intercept
Р	1	-12.5	-18.3333	0	2.91667	0	0	7291.667	
Z	0	0.5	0.33333	1	0.08333	0	0	208.3333	625
S2	0	-0.05	0.03333	0	-0.0167	1	0	13.33333	400
S3	0	0.5	0.66667	0	-0.0833	0	1	241.6667	362.5
Basis	С	Х	Y	Ζ	S1	S2	S3	Solution	Intercept
Р	1	1.25	0	0	0.625	0	27.5	13937.5	
Z	0	0.25	0	1	0.125	0	-0.5	87.5	700
S2	0	-0.075	0	0	-0.0125	1	-0.05	1.25	-100
Y	0	0.75	1	0	-0.125	0	1.5	362.5	-2900

(b) The simplex tableau for the problem can be set up and solved as

(c) An Excel spreadsheet can be set up to solve the problem as

í.	A	В	C	D	E	F
1		Product 1	Product 2	Product 3	Total	Constraint
2	amount	0	0	0		
3	material	6	4	12	0	2500
4	time	0.05	0.1	0.2	0	55
5	storage	1	1	1	0	450
6	profit	30	30	35	0	

The formulas are

1	A	B	C	D	E	F
1	-	Product 1	Product 2	Product 3	Total	Constraint
2	amount	0	0	0		1
3	material	6	4	12	=B3*B\$2+C3*C\$2+D3*D\$2	2500
4	time	0.05	0.1	0.2	=B4*B\$2+C4*C\$2+D4*D\$2	55
5	storage	1	1	1	=B5*B\$2+C5*C\$2+D5*D\$2	450
6	profit	30	30	35	=B6*B\$2+C6*C\$2+D6*D\$2	

The Solver can be called and set up as

5et Target Cell: \$E\$6 💽	Solve
iqual To: <u>Max</u> Min <u>y</u> alue of: By Changing Cells:	: 0 Close
4842/4D42	Guess
4045.4045	
Subject to the Constraints:	Options
subject to the Constraints:	
Subject to the Constraints: $\$B\$2 \ge 0$ $\$C\$2 \ge 0$ $\$C\$2 \ge 0$ $\$D\$2 \ge 0$	<u>A</u> dd
\$ubject to the Constraints: \$B\$2>=0 \$C\$2>=0 \$D\$2>=0 \$D\$2>=0 \$E\$3<=	<u>Add</u> <u>Change</u> Peret 0
$$y_{1},y_{2} \in C$ Subject to the Constraints: $$B$_{2} >= 0$ $$C$_{2} >= 0$ $$b$_{2} >= 0$ $$t$_{3} <= F_{3}$ $$t$_{4} <= t_{4}$	<u>A</u> dd <u>Change</u> <u>Reset /</u>

The resulting solution is

	A	B	C	D	E	F
1	5	Product 1	Product 2	Product 3	Total	Constraint
2	amount	0	362.5	87.5		al and a second second
3	material	6	4	12	2500	2500
4	time	0.05	0.1	0.2	53.75	55
5	storage	1	1	1	450	450
6	profit	30	30	35	13937.5	

In addition, a sensitivity report can be generated as

	A B	C	D	E	F	G	Н
1	Microso	ft Excel 11.0 Sens	sitivity R	eport			
2	Worksh	eet: [Prob1604.xls]Sheet1				
3	Report	Created: 6/29/2005	8:38:14	AM			
4							
5							
6	Adjustal	ole Cells					
7			Final	Reduced	Objective	Allowable	Allowable
8	Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9	\$B\$2	amount Product 1	0	-1.25	30	1.25	1E+30
10	\$C\$2	amount Product 2	362.5	0	30	5	1.666666667
11	\$D\$2	amount Product 3	87.5	0	35	55	5
12							
13	Constra	ints					
14	10 M		Final	Shadow	Constraint	Allowable	Allowable
15	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
16	\$E\$3	material Total	2500	0.625	2500	100	700
17	\$E\$4	time Total	53.75	0	55	1E+30	1.25
18	\$E\$5	storage Total	450	27.5	450	25	241.6666667

(d) The high shadow price for storage from the sensitivity analysis from (c) suggests that increasing storage will result in the best increase in profit.

16.5 An LP formulation for this problem can be set up as

Maximize $P = 2000Z_1 - 75Z_2 + 250Z_3 - 300W$ {Maximize profit}

subject to

$Z_1 + Z_2 \le 7500$	$\{X \text{ material constraint}\}$
$2.5Z_1 + Z_3 \le 12,500$	{ <i>Y</i> material constraint }
$Z_1 - Z_2 - Z_3 - W = 0$	{Waste constraint}

An Excel spreadsheet can be set up to solve the problem as

	A	В	С	D	E	F	G
1		Z1	Z2	Z3	W	total	constraint
2	amount	0	0	0	0		
3	amount X	1	1	0	0	0	7500
4	amount Y	2.5	0	1	0	0	12500
5	amount W	1	-1	-1	-1	0	
6	profit	2000	-75	250	-300	0	

The formulas are

	A	B	C	D	E	F	G
1	35	Z1	Z2	Z3	W	total	constraint
2	amount	0	0	0	0		
3	amount X	1	1	0	0	=B3*B\$2+C3*C\$2+D3*D\$2+E3*E\$2	7500
4	amount Y	2.5	0	1	0	=B4*B\$2+C4*C\$2+D4*D\$2+E4*E\$2	12500
5	amount W	1	-1	-1	-1	=B5*B\$2+C5*C\$2+D5*D\$2+E5*E\$2	
6	profit	2000	-75	250	-300	=B6*B\$2+C6*C\$2+D6*D\$2+E6*E\$2	

The Solver can be called and set up as

Solver Parameters	
Set Target Cell: Equal To: <u>Max</u> <u>Min</u> <u>V</u> alue of: 0	<u>S</u> olve Close
\$B\$2:\$E\$2 Guess Subject to the Constraints: \$C\$2 >= 0	Options
\$D\$2 >= 0	

The resulting solution is

2	A	B	C	D	E	F	G
1	1	Z1	Z2	Z3	W	total	constraint
2	amount	5000	2500	0	2500		-
3	amount X	1	1	0	0	7500	7500
4	amount Y	2.5	0	1	0	12500	12500
5	amount W	1	-1	-1	-1	0	
6	profit	2000	-75	250	-300	9062500	

This is an interesting result which might seem counterintuitive at first. Notice that we create some of the unprofitable Z_2 while producing none of the profitable Z_3 . This occurred because

we used up all of Y in producing the highly profitable Z_1 . Thus, there was none left to produce Z_3 .

16.6 Substitute $x_B = 1 - x_T$ into the pressure equation,

$$(1 - x_T)P_{sat_B} + x_T P_{sat_T} = P$$

and solve for x_T ,

$$x_T = \frac{P - P_{sat_B}}{P_{sat_T} - P_{sat_B}} \tag{1}$$

where the partial pressures are computed as

$$P_{sat_B} = 10^{\left(6.905 - \frac{1211}{T + 221}\right)}$$
$$P_{sat_T} = 10^{\left(6.953 - \frac{1344}{T + 219}\right)}$$

The solution then consists of maximizing Eq. 1 by varying *T* subject to the constraint that $0 \le x_T \le 1$. The Excel Solver can be used to obtain the solution. Here is how the worksheet can be set up:

2	A	В	С	D	E	F	G	Н
1	Prob16.6							
2								
3	T	0						
4								
5	Ρ	800						
6								
7	PsatB	26.62944	<	=10^(6.905	5-1211/(T+)	221))		
8	PsatT	6.546568	<	=10^(6.953	3-1344/(T+)	219))		
9				7 50		1		
10	хT	-38.509	<	=(P-PsatE)/(PsatT-P	'satB)		
11				19.520	5.751	1. 100		
12		C						
13		Solver Par	ameters					
14		Set Target (Iell:	B\$10 🔜	1			Solve
15		Equal To:	0	0.00	2 	0		
16		Equal ro.	• Colley		V value or:			Close
17		by changin	y cells;					
18		\$B\$3				🔚 🛛 🔂 🛄	ss	
19		Subject to I	be Constrain	ter		10 - 2 0		
20		<u>Da</u> bject to t	and constrain	(62)				Options
21		\$B\$10 <=	1			A Ad	±	
22		\$6\$10 >=	U			Cohan		
23							ige	Reset All
24						Dele	te	- Topor Hill
25					1			Help
26		-						

The result is T = 112.8592 as shown below:

	A	В	C	D	E	F
1	Prob16.6					
2	20 10 10					
3	Т	112.8592				
4		Contraction (Contraction of				
5	P	800				
6		in the second se				
7	PsatB	1895.496	<	=10^(6.90	5-1211/(T+2	21))
8	PsatT	800.0004	<	=10^(6.95)	3-1344/(T+2	19))
9				26. 	1. X.	
10	хT	1	<	=(P-PsatE	3)/(PsatT-Ps	atB)

16.7 This is a straightforward problem of varying x_A in order to minimize



First, the function can be plotted versus x_A



The result indicates a minimum between 0.5 and 0.6. Using golden section search or a package like Excel or MATLAB yields a minimum of 0.587683.

16.8 This is a case of constrained nonlinear optimization. The conversion factors range between 0 and 1. In addition, the cost function can not be evaluated for certain combinations of X_{A1} and X_{A2} . The problem is the second term,

$$\left(\frac{1-\frac{x_{A1}}{x_{A2}}}{(1-x_{A2})^2}\right)^{0.6}$$

If $x_{A1} > x_{A2}$, the numerator will be negative and the term cannot be evaluated.

Excel Solver can be used to solve the problem:

	B4	-	∱ =(XA1/	XA2/(1-XA1))^2)^0.6+((1-	-(XA1/XA2)))/(1-XA2)^2)'	0.6+6*(1/	XA2)^0.6
1	A	В	С	D	E	F	G	Н	L I
1	XA1	0.5							
2	XA2	0.6							
3									
4	Cost	11.23606			_				
5		14						·	1 10
6			Solver Pa	rameters					X
7									
8			Set Target	Cell:	;B\$4 📃 💽				Solve
9			Equal To:	O Max	Min	O Value of:	0		Class)
10		-	By Changi	ng Cells:					Close
11			det 1. det	+2		6			
12		_	\$0\$1:\$03	p2				5	
13		_	Subject to	the Constrain	nts:			1	Options
14		_	XA1 <= 1	1			Add		
15		_	XA1 <= 3	KA2		9			
16			XA1 >= 0				Chan	ge _	
17			$XA2 \leq = 1$ XA2 > = 1)				_ L	Reset All
18				-		13	Delet		Help
19	1	-							
20		- 1 C - 1 C - 1	199						

The result is

	A	В
1	XA1	0.368949
2	XA2	0.627265
3	10	
4	Cost	11.12039

16.9 This problem can be set up on Excel and the answer generated with Solver. Note that we have named the cells with the labels in the adjacent left columns.

	A	В	С	D	E	F	G	Н	1	J	K	L
1	Prob. 16.9											
2	0) 19672					Solver Dar	amotore					
3	K	2				Solver Fai	ameters					
4	BO	100				Set Target	Cell:	rofit 🛛 📑	1		ſ	Solve
5	CA	-1				Equal Top	A	0		0		
6	CC	10				Du Chapele			Value or:	L.		Close
7						by changin	y cells;			14		
8	AO	200				\$B\$8:\$B\$9	9			Gue:	55	
9	С	90				Subject to I	the Constrain	here.				
10						<u>Su</u> bject to		90				Options
11	A	20	<	=A0-2*C		A >= 0			2	<u>A</u> do	1	
12	В	10	<	=B0-C		$A0 \ge 0$ $B \ge 0$				Cohan		
13						C_>=0				Linan		Reset All
14	Kcalc	0.0225	<	=C /(A^2*B)		K = Kcalc				Dele	te	
15						L						Help
16	Profit	700	<	=CA*AD+CC*C_		0 k						

The solution is

	A	В	C	D
1	Prob. 16.9			
2	200 1. an 170			
3	K	2		
4	B0	100		
5	CA	-1		
6	CC	10		
7				
8	AO	208.085		
9	С	99.41872		
10	10 10 mm			
11	A	9.247574	<	=A0-2*C_
12	В	0.581276	<	=B0-C_
13	1			
14	Kcalc	2	<	=C_/(A^2*B)
15				
16	Profit	786.1022	<	=CA*AD+CC*C

16.10 The problem can be set up in Excel Solver. Note that we have named the cells with the labels in the adjacent left columns.

	А	В	C	D	E	F	G	Н		J
1	Prob. 16.10		1	Solver Param	eters					
2										
3	Unitcost1	\$ 0.50) \$/L	Set Target Cell:	To	tal_cosi 📑				Solve
4	Unitcost2	\$ 1.00) \$/L	Equal To: (Max	Min	O Value of:	0		
5	Unitcost3	\$ 1.20) \$/L	By Changing C	ells:		<u></u>	1000		Close
6	1			gy changing c			-	-		
7	conc1	13	5 mg/L	\$B\$15:\$B\$17				Gu	BSS	
8	conc2	10	0 mg/L	Subject to the	Constraint	s)				Options
9	conc3	7	5 mg/L	Comp II. A				-		
10			2.8		Concr poly1		~	<u>A</u>	bb	
11	Supply1	200000	D L/d	Flow2 <= Sup	ply2			Cha	DOR	
12	Supply2	100000	D L/d	Flow3 <= Sup	ply3				ingo	Reset All
13	Supply3	50000	D L/d	Flowt >= Flow	vr		~	Del	ete	
14										Help
15	Flow1	30000	0 L/d							
16	Flow2	30000	D L/d				· · · · · · · · · · · · · · · · · · ·			
17	Flow3	30000	D L/d							
18										
19	Flowt	90000	D L/d	=Flow1+Flow2	2+Flow3					
20										
21	Flowr	100000	0 L/d							
22	1									
23	ConcBulk	103.333333	3 mg/L	=(Flow1*conc	1+Flow2*	*conc2+F	low3*conc3)/F	lowt		
24			-				1			-
25	Concr	10	0 mg/L							
26	N		2.8		-					
27	Total cost	\$ 810,000.00		=Unitcost1*FI	ow1+Unit	tcost2*Fl	ow2+Unitcost	3*Flow3		

The solution is

15	Flow1	357143	L/d
16	Flow2	142857	L/d
17	Flow3	500000	L/d
18			
19	Flowt	100000	L/d
20			
21	Flowr	1000000	L/d
22			
23	ConcBulk	100.0000001	mg/L
24			ri - 36
25	Concr	100	mg/L
26			C - 385
27	Total cost	\$ 921,428.57	

16.11 Here is a diagram for this problem:



The following formulas can be developed:

$$\theta = \tan^{-1} \frac{1}{s} \tag{1}$$

$$P = 2d\sqrt{1+s^2} \tag{2}$$

$$A = sd^2 \tag{3}$$

Then the following Excel worksheet and Solver application can be set up:

	A	В	С	D	E	F	G	Н		J	K
1	S	0.5			Solver Day	amotore					
2	d	5			Joiver Par	ameters					
3					Set Target	Cell:	3B\$7				Solve
4	A	12.5	<	=B1*B2^2	Equal To:		0		0	10.0	
5	Agoal	50			Equal TO.			Value or:	0		Close
6		1			by changin	iy cells:					
7	P	11.18034	<	=2*SQRT(1+B1^2)*B2	\$B\$1:\$B\$	2			💁 🛛 📴 🖬	55	
8	÷.		-	2 2	Subject to	the Constrain	ster		-		
9	angle (radians)	1.107149	<	=ATAN(1/B1)	<u>Dabject to</u>	che constrait	1051				Options
10	angle (degrees)	63.43495	<	=B9*180/PI()	\$B\$4 = \$E	3\$5		2	<u>A</u> d	d	
11											
12										ige	Reset All
13								12	Dele	te	
14								12			Help
15	1	1		1	L.						

Our goal is to minimize the wetted perimeter by varying the side slope and the depth. We apply the constraint that the computed area must equal the desired area. The result is

	A	В	С	D
1	S	1.000088		
2	d	7.070756		
3				
4	A	50	<	=B1*B2^2
5	Agoal	50		
6	1			
7	P	20	<	=2*SQRT(1+B1^2)*B2
8				
9	angle (radians)	0.785354	<	=ATAN(1/B1)
10	angle (degrees)	44.99747	<	=B9*180/PI()

Thus, this specific application indicates that a 45° angle yields the minimum wetted perimeter.

The verification that this result is universal can be attained inductively or deductively. The inductive approach involves trying several different desired areas in conjunction with our solver solution. As long as the desired area is greater than 0, the result for the optimal design will be 45° .

The deductive verification involves calculus. First, Eq. 3 can be solved for d and the result substituted into Eq. 2 to give

$$P = 2\sqrt{A\left(s + \frac{1}{s}\right)} \tag{4}$$

The minimum wetted perimeter should occur when the derivative of the perimeter with respect to *s* flattens out. That is, the slope is zero. Setting the derivative of Eq. 4 to zero yields,

$$\frac{dP}{ds} = \frac{1 - \frac{1}{s^2}}{\sqrt{s + \frac{1}{s}}} = 0$$
(5)

We can see that the derivative is zero if s = 1. According to Eq. 1, this corresponds to $\theta = 45^{\circ}$. Thus, the result obtained numerically is shown to be universal.

16.12 Here is a diagram for this problem:



The following formulas can be developed:

$$\theta = \tan^{-1} \frac{1}{s} \tag{1}$$

$$P = b + 2d\sqrt{1 + s^2} \tag{2}$$

$$A = (b + sd)d\tag{3}$$

Then the following Excel worksheet and Solver application can be set up:

	A	В	С	D	E	F	G	Н	1	J	K
1	b	1			Solver Par	ameters					X
2	S	1	-					-			
3	d	1			Set Target (Cell:	B\$8 🛛 💽				Solve
4		-			Equal To:	O Max	Min	O Value of:	0		_
5	A	2	<	=(B1+B2*B3)*B3	By Changin	n Cells:	✓ 1 = 12	C 7000 000			Close
6	Agoal	100			Ey analight	9		-			
7					\$B\$1:\$B\$	3				is l	
8	Р	3.828427	<	=B1+2*SQRT(1+B2^2)*B3	Subject to	the Constrain	its:			- r	Options
9						ult a		100		L	Options
10	angle (radians)	0.785398	<	=ATAN(1/B2)	\$6\$5 = \$6	іфБ		3	Add		
11	angle (degrees)	45	<	=B10*180/PI()					Chan	ne)	
12									Gridi	ge j	Reset All
13									Dele	ie j	
14								12		_ (Help
15					10						

Our goal is to minimize the wetted perimeter by varying the depth, side slope and bottom width. We apply the constraint that the computed area must equal the desired area. The result is

	A	В	С	D
1	b	8.773829		
2	S	0.577343		
3	d	7.598379		
4				
5	A	100	<	=(B1+B2*B3)*B3
6	Agoal	100		
7				
8	P	26.32148	<	=B1+2*SQRT(1+B2*2)*B3
9				2 (2) (2)
10	angle (radians)	1.047203	<	=ATAN(1/B2)
11	angle (degrees)	60.0003	<	=B10*180/PI()

Thus, this specific application indicates that a 60° angle yields the minimum wetted perimeter.

The verification of whether this result is universal can be attained inductively or deductively. The inductive approach involves trying several different desired areas in conjunction with our solver solution. As long as the desired area is greater than 0, the result for the optimal design will be 60° .

The deductive verification involves calculus. First, we can solve Eq. 3 for b and substitute the result into Eq. 2 to give,

$$P = \frac{A}{d} + d\left(2\sqrt{1+s^2} - s\right) \tag{4}$$

If both A and d are constants and s is a variable, the condition for the minimum perimeter is dP/ds = 0. Differentiating Eq. 4 with respect to s and setting the resulting equation to zero,

$$\frac{dP}{ds} = d\left(\frac{2s}{\sqrt{1+s^2}} - 1\right) = 0\tag{4}$$

Therefore, we obtain $s = 1/\sqrt{3}$. Using Eq. 1, this corresponds to $\theta = 60^{\circ}$.

16.13

$$A_{ends} = 2\pi r^{2}$$

$$A_{side} = 2\pi rh$$

$$A_{total} = A_{ends} + A_{side}$$

$$V_{computed} = \pi r^{2}h$$

 $Cost = F_{ends}A_{ends} + F_{side}A_{side} + F_{coating}A_{coating}$

Then the following Excel worksheet and Solver application can be set up:

	CostTotal	▼ f3	=SUM(CostEnd:Cost(Dp)								
	A	В	C	D	E	F	G	Н		J	K	L	M
1	Prob. 16.13					1	Salura Da	matara					
2							Solver Pa	rameters		40			
3	hside	1	m	Vdesired	10	m3	Set Target	Cell:	ostTotal 張	1			Solve
4	dend	1	m	Vcomputed	0.785398163	m3	Equal To:	0		0 U-1 C	lo.		
5							Du Chapair			O value or:	0		Close
6	dend:hside	1	1	Aend	1.570796327	m2	by Changi	ig cells:					
7	1			Aside	3.141592654	m2	\$B\$3:\$B\$;4			. <u>G</u> ue	855	
8	rend	0.5	m	Atotal	4.71238898		Subject to	the Constrain	sheri		12		
9						1	D <u>u</u> bject to	une constrail	1051		- T		Options
10	FEnd	\$ 200.00	\$/m2	CostEnd	\$ 314.16		Vdesired	= Vcomputed		1	Ad	ld	
11	FSide	\$ 100.00	\$/m2	CostSide	\$ 314.16						Char		
12	FCoat	\$ 50.00	\$/m2	CostCoat	\$ 235.62						<u>C</u> <u>u</u> a	iye	Reset All
13						12				10	Dele	ete	
14				CostTotal	\$ 863.94		-						Help
15	3					- C	A						

which results in the following solution:

	A	В	C	D	E	F
1	Prob. 16.13					1
2						
3	hside	3.282282	m	Vdesired	10	m3
4	dend	1.96955	m	Vcomputed	10	m3
5	1					
6	dend:hside	0.600055		Aend	6.093321721	m2
7				Aside	20.30920276	m2
8	rend	0.984775	m	Atotal	26.40252448	
9	1					
10	FEnd	\$ 200.00	\$/m2	CostEnd	\$ 1,218.66	
11	FSide	\$ 100.00	\$/m2	CostSide	\$ 2,030.92	
12	FCoat	\$ 50.00	\$/m2	CostCoat	\$ 1,320.13	
13						
14				CostTotal	\$ 4,569.71	2

16.14 As shown below, Excel Solver gives: x = 0.5, y = 0.8 and $f_{\min} = -0.85$.

	A	В	С	D	E	F	G	Н	1	J
1	X	1	1	Solver Par	ameters					
2 3 4 5	y f(x,y)			Set Target Equal To: By Changir	Cell:	B\$4.] ○⊻alue of:	0		<u>S</u> olve Close
6 7 8 9				\$B\$1:\$B\$ Subject to	2 the Constrair	its:			ss (Options
11 12 13 14									nge (Reset All



16.15 An Excel spreadsheet can be set up to solve the problem as

	A	В	С	D	E
1	Parameters	-		Sec. S.	
2	c1	4		d1	1
3	c2	2		d2	10
4	Н	275		t1	0.1
5	Р	2000		t2	1
6	E	900000			
7	rho	0.0025			
8	sigmamax	550			
9	1 36				
10	Decision var	iables			
11	t	0.5			
12	d	10			
13	1				
14	Computed q	uantities		goals:	
15	W	10.79922			
16	1	196.8404			
17	sigma	127.324	<	550	
18	sigmab	1471.876			
19	1.045-0				
20	Objective fur	nction			
21	C	63.1969			

The formulas are

	A	В	С	D	E
1	Parameters				
2	c1	4		d1	1
3	c2	2		d2	10
4	H	275		t1	0.1
5	P	2000		t2	1
6	E	900000			
7	rho	0.0025			
8	sigmamax	550			
9					
10	Decision va				
11	t	0.5			
12	d	10			
13		-			
14	Computed of			goals:	
15	W	=PI()*B12*B11*B4*B7			
16	1	=PI()/8*B12*B11*(B12*2+B11*2)			
17	sigma	=B5/PI()/B12/B11	<	550	
18	sigmab	=PI()*B6*B16/B4^2/B12/B11		1	
19					
20	Objective fu				
21	C	=B2*B15+B3*B12			

The Solver can be called and set up as

S <u>e</u> t Target Cell: \$B\$21 	Solve
Equal To: <u>Max</u> O Min <u>V</u> alue of: By Changing Cells:	Close
\$B\$11:\$B\$12	Guess
Subject to the Constraints:	 Options
Subject to the Constraints: \$B\$11 <= \$E\$5	Add
Subject to the Constraints: \$B\$11 <= \$E\$5	Add
Subject to the Constraints: \$B\$11 <= \$E\$5	Add Change
Subject to the Constraints: \$B\$11 <= \$E\$5	Add Change Reset All

The resulting solution is

	A	В	С	D	E
1	Parameters			Sec. Sec.	
2	c1	4		d1	1
3	c2	2		d2	10
4	Н	275		t1	0.1
5	Р	2000		t2	1
6	E	900000			
7	rho	0.0025			
8	sigmamax	550			
9	1 36				
10	Decision var	iables			
11	t	0.189207			
12	d	6.117589			
13					
14	Computed q	uantities		goals:	
15	W	2.5			
16	1	17.02759			
17	sigma	550	<	550	
18	sigmab	550			
19	1.0224				
20	Objective fur	nction			
21	C	22.23518			
22					

16.16 A plot of the function indicates a minimum at about t = 2.2.



The Excel Solver can be used to determine that a minimum of o = 1.699 occurs at a value of t = 2.2023.



PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. <u>No part of this Manual</u> may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

16.17 This problem can be solved graphically by using a software package to generate a contour plot of the function. For example, the following plot can be developed with Excel. As can be seen, a minimum occurs at approximately x = 1 and y = 7.



We can use a software package like Excel to determine the maximum precisely as x = 1.034593 and y = 6.64868.

	B3	-	f ≈ =7.7+0	l.15*B1+0.22	2*B2-0.05*B	31^2-0.016*	B2^2-0.007	*B1*B2		
	A	В	С	D	Е	F	G	Н	I.	J
1	X	1.034593		Solver Par	ameters					
2 3 4 5 6	у с(х,у)	8.50895		S <u>e</u> t Target (Equal To: <u>By</u> Changin	Cell: \$	B\$3 💽] O <u>V</u> alue of:	0		<u>S</u> olve Close
7 8 9 10				\$B\$1:\$B\$2 Subject to t	2 the Constrain	ts:			55	Options
11 12 13 14 15									ge te	Reset All

16.18 (a) The problem consists of

$$\min P = B + 2 * H$$

Subject to

$$\frac{1}{n}BH\left(\frac{BH}{B+2H}\right)^{2/3}S^{1/2} = Q$$

The problem can be set up and solved with the Excel Solver as in

	A	В	C	D	Е	F	G	Н	1	J	K	L
1	Problem 18	6.18				Solver Par	ameters					
3	n S	0.03				S <u>e</u> t Target Equal To:	Cell:	Min) O Value of:	0		
5	Q	1				By Changin	ng Cells:					Close
7	В	2.13514				\$B\$7:\$B\$	8				ss	
8	Н	1.067614				Subject to	the Constrain	its:				Ontines
9	B/H	1.999917						696. 	100	1		Options
10						Qmanning	1 = Q		1	<u>A</u> de		
11	Ac	2.279507	=B*H							Chan	ine]	
12	P	4.270369	=B+2*H							<u>C</u> ridin	ι <mark>ge</mark> Γ	Reset All
13	R	0.533796	=Ac/P						~	Dele	te	
14									100		_ (Help
15	Qmanning	1	=1/n*Ac*R	^(2/3)*S^0.5		19						

As can be seen, the result shows that the dimensions for the minimum wetted perimeter correspond to having the bottom width that is twice the length of each vertical side.

(b) Now we can redo the problem as a cost minimization:

 $\min C = 100A_{c} + 50P$

Subject to

$$\frac{1}{n}BH\left(\frac{BH}{B+2H}\right)^{2/3}S^{1/2} = Q$$

The problem can be set up and solved with the Excel Solver as in

Ĩ.	Cost	-	£ =100*A	:+50*P								
	A	В	С	D	E	F	G	Н	Î	J	K	Ĺ
1	Problem 18	5.18				Solver Pa	rameters					
2												
3	n	0.03				S <u>e</u> t Target	Cell:	ost 💦 💽)			Solve
4	S	0.0004				Equal To:	O Max	Min	O Value of:	0		
5	Q	1				By Changi	na Cells:	0.00	O Targo ou	1	_	Close
6							18 A. F. 1990		-	-		
7	В	2.135512	2.000614			\$B\$7:\$B\$	8				ss	
8	Н	1.067429				Subject to	the Constrain	lts:				Options
9	B/H	2.000614				Omanning	0		102		-	
10						Qinanining	1 = Q			Add		
11	Ac	2.279507	=B*H							Chan	ine	
12	Р	4.27037	=B+2*H								<u></u>	Reset All
13	R	0.533796	=Ac/P						V	<u>D</u> ele	te	
14												Help
15	Cost	441.4692				1						
16		-				645 I						
17	Qmanning	1	=1/n*Ac*R	^(2/3)*S^0.5								

Very interestingly, the result is identical to that obtained when cost was not an issue!!!

(c) The constraint can be rewritten as

$$\frac{(BH)^{5/2}}{B+2H} = \left(\frac{nQ}{S^{1/2}}\right)^{3/2} = \text{constant}$$

 $BH = \text{constant} \times (B + 2H)^{2/5}$

Therefore, both A_c and P are minimized simultaneously. This is great, because the excavation costs will be proportional to the cross-sectional area. Hence, by having the bottom width twice the length of each vertical side, we will minimize both excavation and lining costs simultaneously!!!

16.19 Using Excel Solver,

	A	В	С	D	E	F	G	Н		J	К	L	M
1	P	3.00E+06	Ν				Solver	Parameters					
2	E	2.00E+11	N/m2				Sources	ranameters				212	
3	Vgoal	0.075	m3				S <u>e</u> t Tar	rget Cell: P	Ye [<u>.</u>		ſ	Solve
4	1			-			Equal T	0: O Max	O Min	Value of:	3000000		
5	L	4.14267	m				By Cha	anaina Cells:	<u> </u>	O.Tage 611	11		Close
6	radius	0.075913	m							(=	1		
7							\$B\$5	:\$B\$6				55	
8	1	2.61E-05	m4	<	=PI()*radiu	s^4/4	Subjec	t to the Constrain	nts:			6	Options
9							Volum	e – Vacel	esetter.	102			options
10	Volume	0.075	m3	<	=PI()*radiu	s^2*L	Voiun	ie – vydai			Had		
11	-										Chan	ae	
12	Pc	3000000	Ν	<	=PI()*2*E*I	/L^2							Reset All
13	,		-							V	Dele	te	
14											-		Help
15				1									

An alternative solution can be developed by maximizing L subject to Volume $\leq 0.075 \text{ m}^3$ and $P_c \geq 3,000,000 \text{ N},$

	A	В	С	D	E	F	G	Н		J	K	L	M
1	P	3.00E+06	N				Solver	Darameters					
2	E	2.00E+11	N/m2				3010-51	Fuldimeters	20	4.55.51			
3	Vgoal	0.075	m3				Set Targ	get Cell:	. 6				Solve
4	10						Equal To	n: May	Min	Value of:	3000000		
5	L	4.14267	m				By Cha	naina Cells:	0.00				Close
6	radius	0.075913	m				by chia	riging color		-	-		
7							\$B\$5::	\$B\$6				55	
8	1	2.61E-05	m4	<	=PI()*radiu:	s^4/4	Subject	to the Constrain	nts:				Ontions
9							- /	0000000	2004 - C	100			Options
10	Volume	0.075	m3	<	=PI()*radiu:	s^2*L	PC >=	: 3000000 e <= Vooal		-	Add		
11							, c.d.iii				Chan	ne	
12	Pc	3000000	N	<	=PI()^2*E*I.	/L^2						90	Reset All
13											Dele	te	
14													Help
15													

16.20 The total flow in the river: $F = 2 \times 10^6 \text{ m}^3/\text{d}.$

The flow into the channels:

 $f_1 + f_2 \le 0.7F = 1.4 \times 10^6 \text{ m}^3/\text{d}$

Minimum channel flows for navigation:

 $f_1 \ge 0.3 \times 10^6 \text{ m}^3/\text{d}$

or

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. <u>No part of this Manual</u> may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

$$f_2 \ge 0.2 \times 10^6 \text{ m}^3/\text{d}$$

Political constraints:

$$\frac{\left|f_1 - f_2\right|}{f_1 + f_2} \le 0.4$$

leads to

$$f_2 \ge \frac{3}{7} f_1$$
$$f_2 \le \frac{7}{3} f_1$$

Maintenance cost per year, $C \le \$1.8 \times 10^6$

Channel 1: $C_1 = 1.1f_1$ Channel 2: $C_2 = 1.4f_2$

leads to

 $1.1f_1 + 1.4f_2 \le 1.8 \times 10^6$

Power revenue (revenue per year):

Channel 1: $r_{p1} = 4f_1$ Channel 2: $r_{p2} = 3f_2$

Irrigation revenue (revenue per year):

Channel 1: loss, $\alpha_1 = 0.3$ value/yr: $i_1 = 3.2(1 - \alpha) f_1 = 2.24 f_1$

Channel 2: loss, $\alpha_2 = 0.2$ value/yr: $i_2 = 3.2(1 - \alpha) f_2 = 2.56 f_2$

Net revenue = Revenue - losses

 $P = 4f_1 + 3f_2 + 2.24f_1 + 2.56f_2 - 1.1f_1 - 1.4f_2$

 $P = 5.14f_1 + 4.16f_2$

Therefore, the problem is formulated as

Decision variables: f_1 : flow in channel 1 f_2 : flow in channel 2

Maximize: $P = 5.14f_1 + 4.16f_2$

Subject to

$f_1 + f_2 \le 1.4 \times 10^6$	channel flow
$1.1f_1 + 1.4f_2 \le 1.8 \times 10^6$	maintenance
$0.43f_1 - f_2 \le 0$	political constraint 1
$-2.33f_1 + f_2 \le 0$	political constraint 2
$f_1 \ge 0.3 \times 10^6$	minimum channel flow 1
$f_2 \ge 0.2 \times 10^6$	minimum channel flow 2

The problem can then be set up and solved with a tool such as Excel:

	A	В	C	D	E
1	8	Channel 1	Channel 2	total	constraint
2	Flow	0	0		
3		1	1	0	1.40E+06
4		1.1	1.4	0	1.80E+06
5		0.43	-1	0	0
6		-2.33	1	0	0
7		1		0	3.00E+05
8	1		1	0	2.00E+05
9	1				
10	Profit	5.14	4.16	0	

The cell formulas are

1	A	В	C	D	E
1	-	Channel 1	Channel 2	total	constraint
2	Flow	0	0		
3		1	1	=B3*B\$2+C3*C\$2	1400000
4		1.1	1.4	=B4*B\$2+C4*C\$2	1800000
5		0.43	-1	=B5*B\$2+C5*C\$2	0
6		-2.33	1	=B6*B\$2+C6*C\$2	0
7		1		=B7*B\$2+C7*C\$2	300000
8			1	=B8*B\$2+C8*C\$2	200000
9					
10	Profit	5.14	4.16	=B10*B\$2+C10*C\$2	

The Excel Solver can be invoked as

Set Target Cell: \$D\$10	<u>S</u> olve
Equal To: <u>Max</u> Min <u>Value of:</u> By Changing Cells:	Close
\$B\$2:\$C\$2	Guess
Anderska (TE	Guess
Subject to the Constraints:	Quess Options
Subject to the Constraints: \$D\$3 <= \$E\$3	Add
\$Ubject to the Constraints: \$D\$3 <= \$E\$3	<u>A</u> dd
\$ubject to the Constraints: \$ubj4 <= \$E\$3	<u>Add</u>
\$U\$piect to the Constraints: \$D\$3 <= \$E\$3	Add Options Change Reset All

The resulting solution is

A	В	C	D	Е
8 	Channel 1	Channel 2	total	constraint
Flow	979021	420979		
	1	1	1400000	1.40E+06
	1.1	1.4	1666294	1.80E+06
	0.43	-1	0	0
	-2.33	1	-1860140	0
	1		979021	3.00E+05
10 A		1	420979	2.00E+05
1				
Profit	5.14	4.16	6783441	
	A Flow Profit	A B Channel 1 Flow 979021 1 1.1 0.43 -2.33 1 Profit 5.14	A B C Channel 1 Channel 2 Channel 2 Flow 979021 420979 1 1 1 1 1.1 1.4 0.43 -1 -1 -2.33 1 1 1 1 1 Profit 5.14 4.16	A B C D Channel 1 Channel 2 total Flow 979021 420979 1 1 1400000 1.1 1.4 1666294 0.43 -1 0 -2.33 1 -1860140 1 1 979021 1 1 1400000 1.1 1.4 1666294 0.43 -1 0 -2.33 1 -1860140 1 1 979021 979021 1 1 1 420979 Profit 5.14 4.16 6783441

16.21 The weight of the truss is equal to

$$W = \rho(L_1A_c + L_2A_t + L_3A_c)$$

where $\rho = \text{density}$, $L_i = \text{length of member } i$, $A_c = \text{cross-sectional area of compression member,}$ and $A_t = \text{cross-sectional area of tension member}$. The lengths of the 3 members can be determined as $L_1 = 43.3013$, $L_2 = 50$, and $L_3 = 25$. Therefore, the solution can be formulated as a linear programming problem as

Minimize: $W = 3.5(43.3013A_c + 50A_t + 25A_c)$

subject to

$$A_c \ge 50$$
$$A_c \ge 43.3$$

The solution can be developed in Excel using the Solver tool,

	B4	* 3	f ∡ =3.5*i	(43.3013*Ac+	-50*At+25*/	Ac)				
	A	В	С	D	E	F	G	Н	1	J
1	Ac	50		Solver Par	ameters					
2	At	43.3		Source i van	ametera				1.2	
3				S <u>e</u> t Target (Cell: \$8	3\$4 🛛 🔣			1	Solve
4	weight	19530.23		Equal To:	O May	Min (Walue of:	0		
5		2	6	By Changin	a Cells:					Close
6					P 70001		-	-	_	
7				\$B\$1:\$B\$2	2				s	
8				Subject to t	he Constrain	ts:			1	Options
9				$a_c > -50$			1.22	1		
10				At >= 30 At >= 43.	3		-	AOO		
11								Chane	je l	
12	0									Reset All
13							~	Delet	e	
14	-			L						Help
15										0

16.22 The solution can be developed in Excel using the Solver tool,

	B7	-	£ <mark>√ =1/(4*</mark> F	'PI()*e0)*q*QQ*B6/(B6^2+rad^2)^1.5									
	A	В	С	D	E	F	G	Н	1	J			
1	eO	8.85E-12		Solver Dar	amotore								
2	QQ	2.00E-05		Solver Par	unieters								
3	q	2.00E-05		Set Target (Cell: \$	B\$7 💽	1			Solve			
4	rad	0.90		Equal To:	(Marcil	O Min	C University	0					
5				Pu Chapain		O MI⊡		18		Close			
6	x	0.636396		by changin	y cells:			_					
7	F	1.70911		\$B\$6				Gue:	ss				
8			(Subject to I	the Constrain	ts:							
9													
10							2	<u>A</u> de	±				
11								Chan					
12								<u>C</u> nan	ige	Reset All			
13							12	Dele	te				
14							12		_	<u>H</u> elp			
15													

16.23 The problem can be formulated as

Minimize

 $C = 2p_1 + 10p_2 + 2$

subject to

$$0.6p_1 + 0.4p_2 \ge 30$$

$$p_1 \le 42$$

Using the Excel Solver:

2	6	

	A	В	С	D	E	F	G	Н	1	J	K	L	M
1	Individual power				Constraint		Solver Par	ameters					
3	p1	42			<	42	Set Target	Cell:	B\$13	1			Solve
4	p2	12					Equal To:	O M	() Min	Value of	0		Fourte
5	Losses						Ry Chapgin		€ MI <u>I</u>			-	Close
6	L1	9.6	<	=0.2*B3+0.1*B4			by changin	ig cells,					
7	L2	14.4	<	=0.2*B3+0.5*B4			\$B\$3:\$B\$	4				ss	
8	Total power	30	<		=	30	Subject to	the Constrain	its:				Ontions
9								Le Lo				_	Options
10	Costs						\$B\$3 <= :	\$F\$3 #F#8		1	Ad	d	
11	F1	86	<	=2*B3+2	1		\$D\$0 >= .	рі ро			Chan		
12	F2	120	<	=10*B4								ige	Reset All
13	Total cost	206	<	=B11+B12						K	Dele	te	
14			8									_	Help
15							<u></u>						

16.24 This is a trick question. Because of the presence of (1 - s) in the denominator, the function will experience a division by zero at the maximum. This can be rectified by merely canceling the (1 - s) terms in the numerator and denominator to give

$$T = \frac{15s}{4s^2 - 3s + 4}$$

Any of the optimizers described in this section can then be used to determine that the maximum of T = 3 occurs at s = 1.

16.25 (a) An LP formulation for this problem can be set up as

Maximize $P = 500x_1 + 400x_2$

subject to

$$300x_1 + 400x_2 \le 127,000$$

$$20x_1 + 10x_2 \le 4,270$$

$$x_1, x_2 \ge 0$$

An Excel spreadsheet can be set up to solve the problem as

	A	В	С	D	E	F	G	Н	1	J	K	L	M
1	Device	Capital (\$/unit)	Labor (hrs/unit)	Profit			Solver Da	amotore					
2	Scanner	300	20	500		1	JUIVET Fa	ameters					
3	Printer	400	10	400			Set Target	Cell:	:B\$11 📃 💽]			Solve
4							Equal To:	() May	Min	Value of	0		
5	Devise1	88				1	By Changi	na Celler				_	Close
6	Devise2	251					By chongi	ig color		_	-		
7						Constraint	\$B\$5:\$B\$	6				ss	
8	Capital	126800	<	=B2*B5+B3*B6	<	127000	Subject to	the Constrain	hts:				Continue
9	Labor	4270	<	=C2*B5+C3*B6	<	4270	Links				-		Options
10						1	\$B\$5 = IF \$B\$5 \-	teger 0		3	Ad	d	
11	Profit	144400	<	=D2*B5+D3*B6			\$B\$6 = in	teger			Char		
12							\$B\$6 >=	0			Ciridi	ige j	Reset All
13							\$B\$8 <=	\$F\$8 #E#0		-	Dele	te	
14							\$U\$9 <-	p1 p2					Help
15						1	36					9	

(b) This problem can be formulated as

Maximize $P = 500x_1 + (400 - x_2)x_2$

PROPRIETARY MATERIAL. © The McGraw-Hill Companies, Inc. All rights reserved. <u>No part of this Manual</u> may be displayed, reproduced or distributed in any form or by any means, without the prior written permission of the publisher, or used beyond the limited distribution to teachers and educators permitted by McGraw-Hill for their individual course preparation. If you are a student using this Manual, you are using it without permission.

subject to

$$300x_1 + 400x_2 \le 127,000$$
$$20x_1 + 10x_2 \le 4,270$$
$$x_1, x_2 \ge 0$$

An Excel spreadsheet can be set up to solve the problem as

	A	В	С	D	E	F	G	Н	1	J	K	L	M
1	Device	Capital (\$/unit)	Labor (hrs/unit)	Profit			Solver Da	ramotore					
2	Scanner	300	20	500			Joiver Pa	Tumeters					
З	Printer	400	10	325	<	=400-B6	S <u>e</u> t Target	Cell:	3\$11 📑				Solve
4							Equal To:	May	O Min	Value of	0		
5	Devise1	176					By Changin		O MIL			-	Close
6	Devise2	75					By cridingin	ig color					
7						Constraint	\$B\$5:\$B\$	6			Gue	55	
8	Capital	82800	<	=B2*B5+B3*B6	<	127000	Subject to	the Constraint	s:				Continue
9	Labor	4270	<	=C2*B5+C3*B6	<	4270					-		Options
10							\$B\$5 = in #P#5 >	iteger 0			<u>A</u> d		
11	Profit	112375	<	=D2*B5+D3*B6			\$B\$6 = in	teger			Char		
12							\$B\$6 >=	0			L Gridi	ige	Reset All
13							\$B\$8 <=	\$F\$8			Dele	te	
14							\$0\$9 <=	\$F\$3					[<u>H</u> elp]
15							15						

16.26 An LP formulation for this problem can be set up as

Decision variables: x_{ri} = chips produced in regular time for month i x_{oi} = chips produced in overtime for month i x_{si} = chips stored for month i

Minimize $C = 100x_{r1} + 100x_{r2} + 120x_{r3} + 110x_{o1} + 120x_{o2} + 130x_{o3} + 5x_{s1} + 5x_{s2}$

subject to

$$\begin{aligned} x_{r1} + x_{o1} - x_{s1} \ge 1,000 \\ x_{s1} + x_{r2} + x_{o2} - x_{s2} \ge 2,500 \\ x_{s2} + x_{r3} + x_{o3} \ge 2,200 \\ 1.5x_{r1} \le 2,400 \\ 1.5x_{r2} \le 2,400 \\ 1.5x_{r3} \le 2,400 \\ 1.5x_{o1} \le 720 \\ 1.5x_{o2} \le 720 \\ 1.5x_{o3} \le 720 \\ x_{r1}, x_{r2}, x_{r3}, x_{o1}, x_{o2}, x_{o3}, x_{s1}, x_{s2} \ge 0 \end{aligned}$$

An Excel spreadsheet can be set up to solve the problem as

	B19 ▼ <i>f</i> x =B3*B10	I+C3*B11	+D3*B12+	-B4*B13+C	4*B14+D4	*B15+B7*	B16+B7*B1	7						
	A	В	С	D	E	F	G	Н	I.I.		J		<	L
1		Month 1	Month 2	Month 3	-	Salvar Da	ramotore							
2	Chips required	1000	2500	2200	_	SUIVEI Pa	in dimeters							
З	Cost regular time (\$/chip)	100	100	120		Set Target	: Cell:	B\$19	6.]				ſ	Solve
4	Cost overtime (\$/chip)	110	120	130		Equal To:	O Mary	(Marine)		6 .	0		5	
5	Regular operation time (hrs)	2400	2400	2400		-By Chang				3 01 1				Close
6	Overtime (hrs)	720	720	720		by criary	ing cells,			_				
7	Storage cost	5				\$B\$10:\$I	B\$17				G	uess		
8	Production time	1.5		1		Subject to	the Constrai	ate:			2001 AC		- 6	
9							o cho conseren	icon.		-			L	Options
10	Regular time production month 1	1600				\$C\$22 >	= \$E\$22			^		Add		
11	Regular time production month 2	1600				\$C\$24 >	= \$E\$24				CC			
12	Regular time production month 3	1600				\$C\$25 <	= \$E\$25					lange	ſ	Reset All
13	Overtime production month 1	480		1		\$C\$26 <	= \$E\$26			-	D	elete	L	
14	Overtime production month 2	420				\$C\$27 <	.= \$E\$27			((20)				Help
15	Overtime production month 3	0				·							-110	
16	Chips in storage month 1	1080		1										
17	Chips in storage month 2	600												
18	5													
19	Cost	623600												
20		(a												
21	Constraints													
22	=B10+B13-B16	>	1000	Ŧ	1000									
23	=B16+B11+B14-B17	>	2500	=	2500									
24	=B17+B12+B15	>	2200	1 = 1	2200		1	1	1	1		1		1
25	=B8*B10	>	2400	<=	2400									
26	=B8*B11	>	2400	<=	2400									
27	=B8*B12	>	2400	<=	2400									
28	=B8*B13	>	720	<=	720									
29	=B8*B14	>	630	<=	720									
30	=B8*B15	>	0	<=	720									
31	=B16	>	1080	<=	2080	<	=B10+B1	3						
32	=B17	>	600	<=	3100	<	=B11+B1	4+B16	1	1		1		1

Note that before depressing the Solve button, the Options button should be depressed and the following boxes should be selected: "Assume Linear Model" and "Assume Non-Negative."

olver Option	5			
4ax <u>T</u> ime:	100 seconds	ОК		
erations:	100	Cancel		
recision:	0.000001	Load Model		
ol <u>e</u> rance:	5 %	Save Model		
on <u>v</u> ergence:	0.0001	Help		
Assume Line	ar <u>M</u> odel	se Automatic Scaling		
Assume Non	-Negative 📃 S	how Iteration <u>R</u> esults		
stimates	Derivatives	Search		
Tangent	Eorward	Newton		
	O Central	Conjugate		

16.27 A tool such as the Excel Solver can be used to determine the solution as

	B5	-	∱ =0.01*s	=0.01*sigma*B4^2+0.95/sigma*(VV/B4)^2									
	A	В	С	C D E F G H									
1 2	sigma W	0.6	Solver Pa	rameters									
3 4 5 6	V D	509.8181 3118.974	S <u>e</u> t Target Equal To: <u>By</u> Changi	Cell: [O <u>M</u> ax ng Cells:	\$B\$5 🚺 📑		. 0		<u>S</u> olve Close				
7 8 9 10			\$B\$4 Subject to	the Constra	ints:			ess d	Options				
11 12 13 14 15	1 1 1 1 2 1 3 1 4 1 5 1						<u>_</u> ha 	nge ete	Reset All				

The approach can be implemented to evaluate other values of W with a constant σ to yield the following results:

W	V	D
12000	441.5154	2339.231
13000	459.5438	2534.167
14000	476.8912	2729.102
15000	493.6293	2924.038
16000	509.8181	3118.974
17000	525.5085	3313.910
18000	540.7438	3508.846
19000	555.5614	3703.782
20000	569.9940	3898.718

The optimal velocity along with the minimal drag can be plotted versus weight. As shown below, the relationship is fairly linear for the specified range.



16.28 A tool such as the Excel Solver can be used to determine the solution as

	B4	- t	🖌 =cons	st/SQRT(1+x [/]	2)-SQRT(1	+x^2)*(1-co	onst/(1+x^2))+x		
1	A	В	С	D	E	F	G	Н	L L	J
1	const	0.4		Solver Par	ameters					X
- 3 4 5 6 7	x 1.05185	f(x) 3 <mark>. 0.151724)</mark>		Set Target Equal To: By Changin	Cell:	B\$4 O Mi <u>n</u>) O <u>V</u> alue of:		55	<u>S</u> olve Close
8 9 10 11				Subject to	the Constrain	its:	2		<u> </u>	Options
12 13 14							8	<u>C</u> han Dele	ge te	Reset All

16.29 An LP formulation for this problem can be set up as

Minimize	C = 0.05X + 0.025Y + 0.15Z	{Minimize cost}
----------	----------------------------	-----------------

subject to

$X + Y + Z \ge 6$	{Performance constraint}
X + Y < 2.5	{Safety constraint}
$X - Y \ge 0$	{X-Y Relationship constraint}
$Z - 0.5Y \ge 0$	{ <i>Y</i> - <i>Z</i> Relationship constraint}

An Excel spreadsheet can be set up to solve the problem as

	A	B	C	D	E	F	G
1		X	Y	Z	Total		Constraint
2	Amount	0	0	0			
3	Performance	1	1	1	0	>=	6
4	Safety	1	1	0	0	<=	2.5
5	X-Y	1	-1	0	0	>=	0
6	Z-0.5*Y	0	-0.5	1	0	>=	0
7	Cost	0.05	0.025	0.15	0		

The formulas are

	A	B	C	D	E	F	G
1		X	Y	Z	Total		Constraint
2	Amount	0	0	0			
3	Performance	1	1	1	=B3*B\$2+C3*C\$2+D3*D\$2	>=	6
4	Safety	1	1	0	=B4*B\$2+C4*C\$2+D4*D\$2	<=	2.5
5	X-Y	1	-1	0	=B5*B\$2+C5*C\$2+D5*D\$2	>=	0
6	Z-0.5*Y	0	-0.5	1	=B6*B\$2+C6*C\$2+D6*D\$2	>=	0
7	Cost	0.05	0.025	0.15	=B7*B\$2+C7*C\$2+D7*D\$2		

The Solver can be called and set up as

S <u>e</u> t Target Cell:	\$E\$7	<u>k.</u>		Solve
Equal To: O <u>M</u> ax By Changing Cells:	<mark>⊙</mark> Mi <u>n</u>	O ⊻alue of:	0	Close
\$B\$2:\$D\$2			Guess	
S <u>u</u> bject to the Constra	ints:			Options
Subject to the Constra \$B\$2 >= 0	ints:	<u>^</u>		Options
Subject to the Constra \$B\$2 >= 0 \$C\$2 >= 0 \$D\$2 >= 0	ints:	^		
Subject to the Constra \$B\$2 >= 0 \$C\$2 >= 0 \$D\$2 >= 0 \$E\$3 >= \$G\$3	ints:	<u> </u>	<u>A</u> dd	Options Reset All

The resulting solution is

	A	B	C	D	E	F	G
1		X	Y	Z	Total		Constraint
2	Amount	1.25	1.25	3.5			
3	Performance	1	1	1	6	>=	6
4	Safety	1	1	0	2.5	<=	2.5
5	X-Y	1	-1	0	0	>=	0
6	Z-0.5*Y	0	-0.5	1	2.875	>=	0
7	Cost	0.05	0.025	0.15	0.61875		

16.30 An LP formulation for this problem can be set up as

Decision variables: x_i = quantity of part *i*

Minimize $P = 375x_A + 275x_B + 475x_C + 325x_D$ subject to $2.5x_A + 1.5x_B + 2.75x_C + 2x_D \le 640$

 $3.5x_A + 3x_B + 3x_C + 2x_D \le 960$

A tool such as the Excel Solver can be used to determine the solution as

	A	В	C	D	E	F	G	Н	1	J	K	L	M
1		Part					Solver Dar	motore					
2		A	В	С	D		Solver Pare	ameters		- 10			
3	Fabrication time (hr/100 units)	2.5	1.5	2.75	2		Set Target C	ell:	\$C\$9 📑	1			Solve
4	Finishing time (hr/100 units)	3.5	3	3	2		Equal To:	@ May	O Min		0		
5	Profit (\$/100 units)	375	275	475	325		By Chapging		O MIL			_	Close
6	L.						by cridinging	(COID)		-]	
7	Quantity	0	192	128	0		\$B\$7:\$E\$7					55	
8	Profit						Subject to t	he Constrai	nts:				Continue
9	=B5*B7+C5*C7+D5*D7+E5*E7	>	113600)						0.80	2.42 1.50	-		Options
10	Constraints:						\$B\$/ >= 0	45412		1	<u>A</u> d	d	
11	Fabr Time						\$C\$14 <=	\$E\$14			Char		
12	=B3*B\$7+C3*C\$7+D3*D\$7+E3*E\$7	>	640	<=	640		\$C\$7 >= 0					ige	Reset All
13	Finish time						\$D\$7 >= 0			2	Dele	te	
14	=B4*B\$7+C4*C\$7+D4*D\$7+E4*E\$7	>	960	<=	960		[p_p/ >= 0						Help
15							-						

Thus, the results indicate that if we produce none of parts A and D and 192 and 128 of B and C, respectively, we will generate a maximum profit of \$113,600.