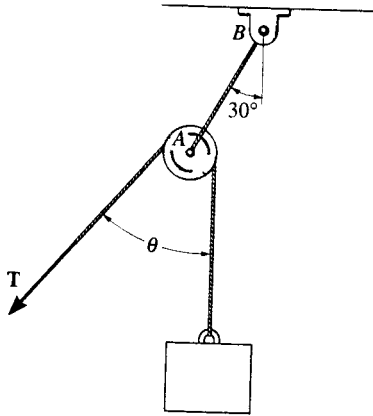


3-22. The block has a weight of 20 lb and is being hoisted at uniform velocity. Determine the angle  $\theta$  for equilibrium and the required force in each cord.



Point A :

$$\rightarrow \Sigma F_x = 0; \quad T_{AB} \cos 60^\circ - 20 \sin \theta = 0$$

$$T_{AB} \cos 60^\circ = 20 \sin \theta$$

$$+\uparrow \Sigma F_y = 0; \quad T_{AB} \sin 60^\circ - 20 - 20 \cos \theta = 0$$

$$T_{AB} \sin 60^\circ = 20(1 + \cos \theta)$$

$$\tan 60^\circ = \frac{1 + \cos \theta}{\sin \theta}$$

$$\tan 60^\circ \sin \theta = 1 + \cos \theta$$

$$\theta = 60^\circ$$

Ans

$$T_{AB} = \frac{20 \sin 60^\circ}{\cos 60^\circ} = 34.6 \text{ lb}$$

Ans

Also :

$$\Sigma F_y = 0; \quad \text{Requires } \frac{\theta}{2} = 30^\circ$$

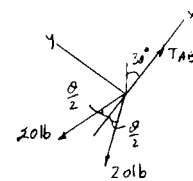
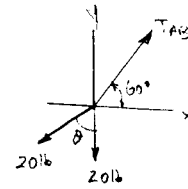
$$\theta = 60^\circ$$

Ans

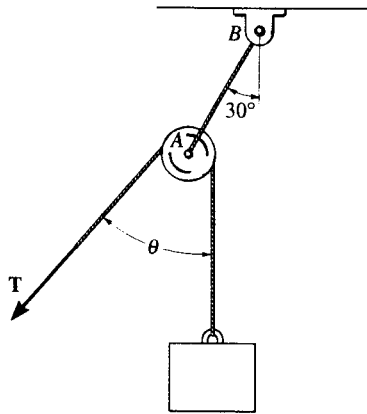
$$\swarrow \Sigma F_x = 0; \quad T_{AB} - 2[20 \cos 30^\circ] = 0$$

$$T_{AB} = 34.6 \text{ lb}$$

Ans



3-23. Determine the maximum weight  $W$  of the block that can be suspended in the position shown if each cord can support a maximum tension of 80 lb. Also, what is the angle  $\theta$  for equilibrium?



1) Assume  $T_{AB} = 80$  lb

$$+\uparrow \Sigma F_y = 0; \quad 80 \sin 60^\circ - W - W \cos \theta = 0$$

$$80 \sin 60^\circ = W(1 + \cos \theta) \quad (1)$$

$$\rightarrow \Sigma F_x = 0; \quad 80 \cos 60^\circ - W \sin \theta = 0$$

$$80 \cos 60^\circ = W \sin \theta \quad (2)$$

$$\tan 60^\circ = \frac{1 + \cos \theta}{\sin \theta}$$

$$\tan 60^\circ \sin \theta = 1 + \cos \theta$$

$$\theta = 60^\circ \quad \text{Ans}$$

$$W = \frac{80 \cos 60^\circ}{\sin 60^\circ} = 46.188 \text{ lb} < 80 \text{ lb} \quad (\text{O.K.})$$

2) Assume  $W = 80$  lb

$$+\uparrow \Sigma F_y = 0; \quad T \sin 60^\circ - 80 - 80 \cos \theta = 0$$

$$T \sin 60^\circ = 80(1 + \cos \theta) \quad (3)$$

$$\rightarrow \Sigma F_x = 0; \quad T \cos 60^\circ - 80 \sin \theta = 0$$

$$T \cos 60^\circ = 80 \sin \theta \quad (4)$$

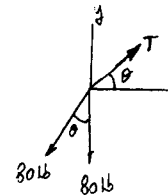
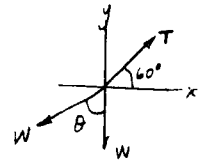
$$\tan 60^\circ = \frac{1 + \cos \theta}{\sin \theta}$$

$$\tan 60^\circ \sin \theta = 1 + \cos \theta$$

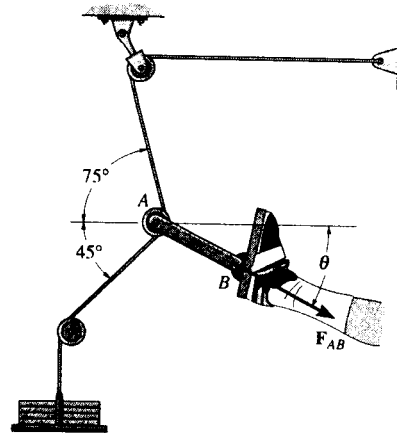
$$\theta = 60^\circ$$

$$T = \frac{80 \sin 60^\circ}{\cos 60^\circ} = 138.6 \text{ lb} > 80 \text{ lb} \quad (\text{N.G.})$$

Thus,  $W = 46.2$  lb      **Ans**



\*3-24. Determine the magnitude and direction  $\theta$  of the equilibrium force  $F_{AB}$  exerted along link  $AB$  by the tractive apparatus shown. The suspended mass is 10 kg. Neglect the size of the pulley at  $A$ .



**Free Body Diagram :** The tension in the cord is the same throughout the cord, that is  $10(9.81) = 98.1$  N.

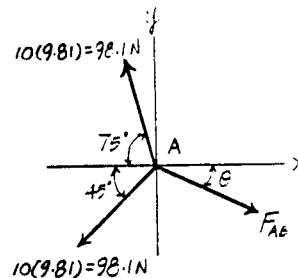
**Equations of Equilibrium :**

$$\begin{aligned} \rightarrow \Sigma F_x = 0; \quad F_{AB} \cos \theta - 98.1 \cos 75^\circ - 98.1 \cos 45^\circ &= 0 \\ F_{AB} \cos \theta &= 94.757 \end{aligned} \quad [1]$$

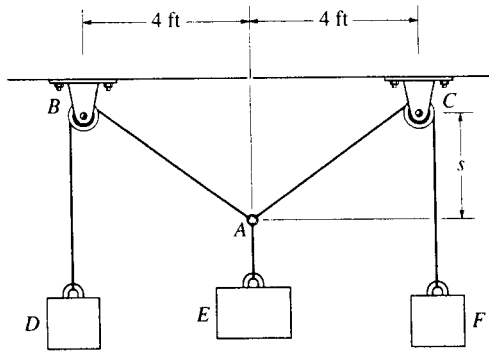
$$\begin{aligned} + \uparrow \Sigma F_y = 0; \quad 98.1 \sin 75^\circ - 98.1 \sin 45^\circ - F_{AB} \sin \theta &= 0 \\ F_{AB} \sin \theta &= 25.390 \end{aligned} \quad [2]$$

Solving Eqs. [1] and [2] yields

$$\theta = 15.0^\circ \quad F_{AB} = 98.1 \text{ N} \quad \text{Ans}$$



3-25. Blocks  $D$  and  $F$  weigh 5 lb each and block  $E$  weighs 8 lb. Determine the sag  $s$  for equilibrium. Neglect the size of the pulleys.



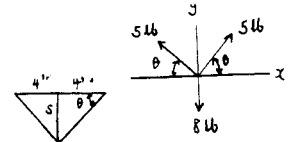
$$+\uparrow \Sigma F_y = 0;$$

$$2(5) \sin \theta - 8 = 0$$

$$\theta = \sin^{-1}(0.8) = 53.13^\circ$$

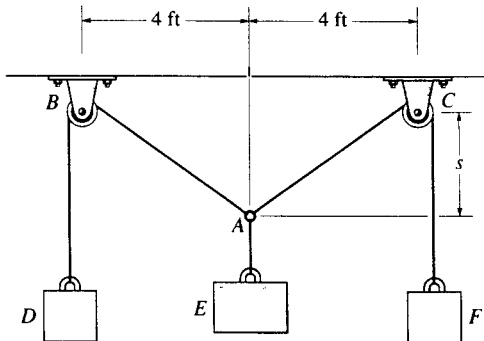
$$\tan \theta = \frac{s}{4}$$

$$s = 4 \tan 53.13^\circ = 5.33 \text{ ft}$$



**Ans**

3-26. If blocks  $D$  and  $F$  weigh 5 lb each, determine the weight of block  $E$  if the sag  $s = 3$  ft. Neglect the size of the pulleys.

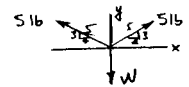


$$+\uparrow \Sigma F_y = 0;$$

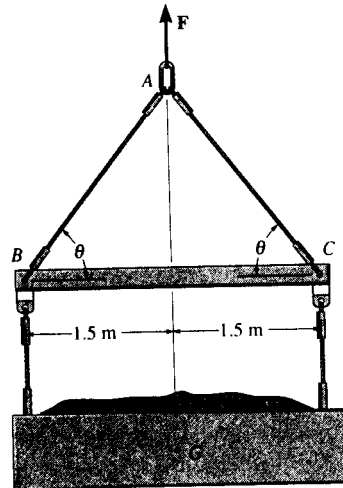
$$2(5)\left(\frac{3}{5}\right) - W = 0$$

$$W = 6 \text{ lb}$$

**Ans**



3-27. The lift sling is used to hoist a container having a mass of 500 kg. Determine the force in each of the cables  $AB$  and  $AC$  as a function of  $\theta$ . If the maximum tension allowed in each cable is 5 kN, determine the shortest lengths of cables  $AB$  and  $AC$  that can be used for the lift. The center of gravity of the container is located at  $G$ .



**Free Body Diagram :** By observation, the force  $F_1$  has to support the entire weight of the container. Thus,  $F_1 = 500(9.81) = 4905 \text{ N}$ .

**Equations of Equilibrium :**

$$\rightarrow \Sigma F_x = 0; \quad F_{AC} \cos \theta - F_{AB} \cos \theta = 0 \quad F_{AC} = F_{AB} = F$$

$$+ \uparrow \Sigma F_y = 0; \quad 4905 - 2F \sin \theta = 0 \quad F = \{2452.5 \csc \theta\} \text{ N}$$

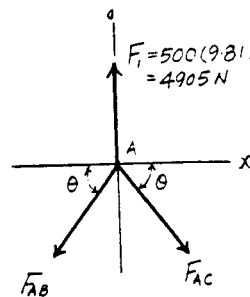
Thus,  $F_{AC} = F_{AB} = F = \{2.45 \csc \theta\} \text{ kN}$       **Ans**

If the maximum allowable tension in the cable is 5 kN, then

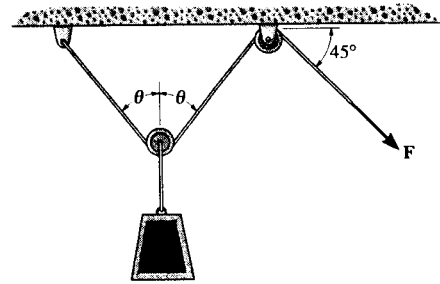
$$2452.5 \csc \theta = 5000 \\ \theta = 29.37^\circ$$

From the geometry,  $l = \frac{1.5}{\cos \theta}$  and  $\theta = 29.37^\circ$ . Therefore

$$l = \frac{1.5}{\cos 29.37^\circ} = 1.72 \text{ m} \quad \text{Ans}$$



\*3-28. The load has a mass of 15 kg and is lifted by the pulley system shown. Determine the force  $F$  in the cord as a function of the angle  $\theta$ . Plot the function of force  $F$  versus the angle  $\theta$  for  $0 \leq \theta \leq 90^\circ$ .



**Free Body Diagram :** The tension force is the same throughout the cord.

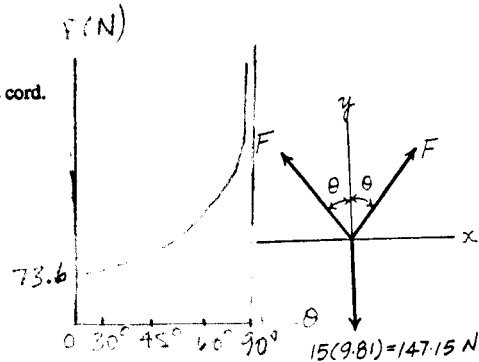
**Equations of Equilibrium :**

$$\rightarrow \Sigma F_x = 0; \quad F \sin \theta - F \sin \theta = 0 \quad (\text{Satisfied!})$$

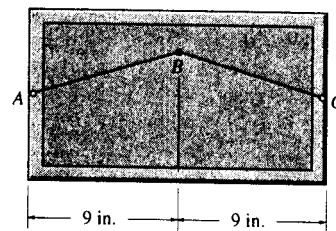
$$+ \uparrow \Sigma F_y = 0; \quad 2F \cos \theta - 147.15 = 0$$

$$F = \{73.6 \sec \theta\} \text{ N}$$

Ans



3-29. The picture has a weight of 10 lb and is to be hung over the smooth pin  $B$ . If a string is attached to the frame at points  $A$  and  $C$ , and the maximum force the string can support is 15 lb, determine the shortest string that can be safely used.



**Free Body Diagram :** Since the pin is smooth, the tension force in the cord is the same throughout the cord.

**Equations of Equilibrium :**

$$\rightarrow \Sigma F_x = 0; \quad T \cos \theta - T \cos \theta = 0 \quad (\text{Satisfied!})$$

$$+ \uparrow \Sigma F_y = 0; \quad 10 - 2T \sin \theta = 0 \quad T = \frac{5}{\sin \theta}$$

If tension in the cord cannot exceed 15 lb, then

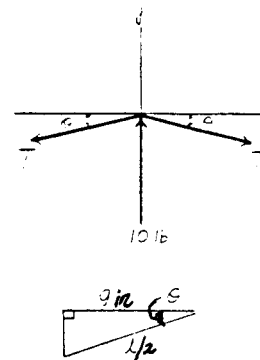
$$\frac{5}{\sin \theta} = 15$$

$$\theta = 19.47^\circ$$

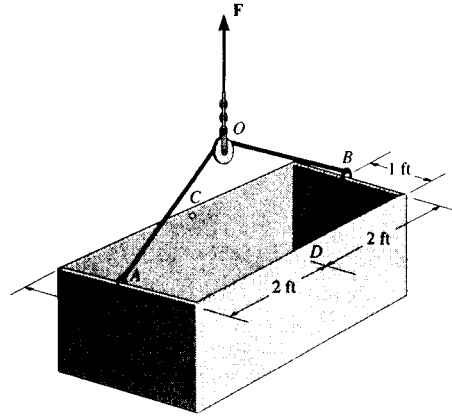
From the geometry,  $\frac{l}{2} = \frac{9}{\cos \theta}$  and  $\theta = 19.47^\circ$ . Therefore

$$l = \frac{18}{\cos 19.47^\circ} = 19.1 \text{ in.}$$

Ans



3-30. The 200-lb uniform tank is suspended by means of a 6-ft-long cable, which is attached to the sides of the tank and passes over the small pulley located at  $O$ . If the cable can be attached at either points  $A$  and  $B$ , or  $C$  and  $D$ , determine which attachment produces the least amount of tension in the cable. What is this tension?



**Free Body Diagram :** By observation, the force  $F$  has to support the entire weight of the tank. Thus,  $F = 200$  lb. The tension in cable is the same throughout the cable.

**Equations of Equilibrium :**

$$\rightarrow \Sigma F_x = 0; \quad T \cos \theta - T \cos \theta = 0 \quad (\text{Satisfied!})$$

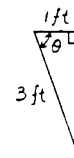
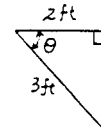
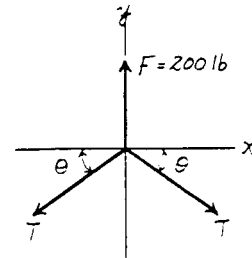
$$+ \uparrow \Sigma F_y = 0; \quad 200 - 2T \sin \theta = 0 \quad T = \frac{100}{\sin \theta} \quad [1]$$

From the function obtained above, one realizes that in order to produce the least amount of tension in the cable,  $\sin \theta$  hence  $\theta$  must be as great as possible. Since the attachment of the cable to point  $C$  and  $D$  produces a greater  $\theta$  ( $\theta = \cos^{-1} \frac{1}{3} = 70.53^\circ$ ) as compared to the attachment of the cable to points  $A$  and  $B$  ( $\theta = \cos^{-1} \frac{2}{3} = 48.19^\circ$ ),

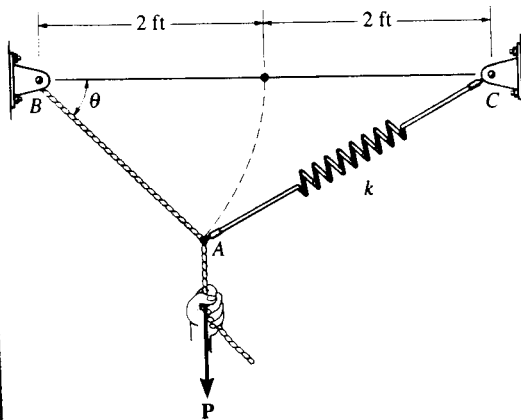
The attachment of the cable to point  $C$  and  $D$  will produce the least amount of tension in the cable. Ans

Thus,

$$T = \frac{100}{\sin 70.53^\circ} = 106 \text{ lb} \quad \text{Ans}$$



■3-31. A vertical force  $P = 10$  lb is applied to the ends of the 2-ft cord  $AB$  and spring  $AC$ . If the spring has an unstretched length of 2 ft, determine the angle  $\theta$  for equilibrium. Take  $k = 15$  lb/ft.



$$\rightarrow \Sigma F_x = 0; \quad F_s \cos \phi - T \cos \theta = 0 \quad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad T \sin \theta + F_s \sin \phi - 10 = 0 \quad (2)$$

$$x = \sqrt{(4)^2 + (2)^2 - 2(4)(2) \cos \theta} = 2\sqrt{5-4 \cos \theta}$$

$$F_s = kx = 2k(\sqrt{5-4 \cos \theta} - 1)$$

$$\text{From Eq. (1): } T = F_s \left( \frac{\cos \phi}{\cos \theta} \right)$$

$$\text{From Eq. (2): } T = 2k(\sqrt{5-4 \cos \theta} - 1) \left( \frac{2 - \cos \theta}{\sqrt{5-4 \cos \theta}} \right) \left( \frac{1}{\cos \theta} \right)$$

$$\frac{2k(\sqrt{5-4 \cos \theta} - 1)(2 - \cos \theta)}{\sqrt{5-4 \cos \theta}} \tan \theta + \frac{2k(\sqrt{5-4 \cos \theta} - 1)2 \sin \theta}{2\sqrt{5-4 \cos \theta}} = 10$$

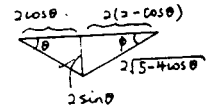
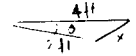
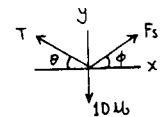
$$\frac{(\sqrt{5-4 \cos \theta} - 1)(2 \tan \theta - \sin \theta + \sin \theta)}{\sqrt{5-4 \cos \theta}} = \frac{10}{2k}$$

$$\frac{\tan \theta (\sqrt{5-4 \cos \theta} - 1)}{\sqrt{5-4 \cos \theta}} = \frac{10}{4k}$$

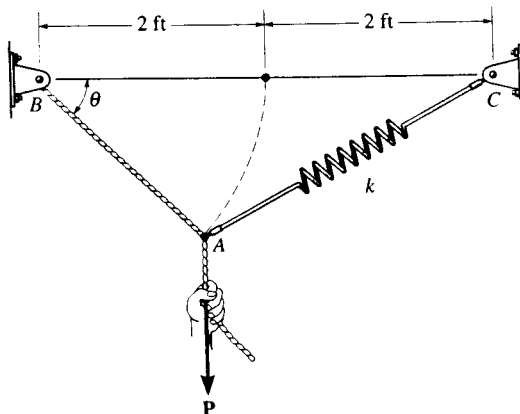
$$\text{Set } k = 15 \text{ lb/ft}$$

Solving for  $\theta$ ,

$$\theta = 35.0^\circ \quad \text{Ans}$$



\*3-32. Determine the unstretched length of spring  $AC$  if a force  $P = 80$  lb causes the angle  $\theta = 60^\circ$  for equilibrium. Cord  $AB$  is 2 ft long. Take  $k = 50$  lb/ft.



$$l = \sqrt{4^2 + 2^2 - 2(2)(4) \cos 60^\circ}$$

$$l = \sqrt{12}$$

$$\frac{\sqrt{12}}{\sin 60^\circ} = \frac{2}{\sin \phi}$$

$$\phi = \sin^{-1} \left( \frac{2 \sin 60^\circ}{\sqrt{12}} \right) = 30^\circ$$

$$+\uparrow \Sigma F_y = 0; \quad T \sin 60^\circ + F_s \sin 30^\circ - 80 = 0$$

$$\rightarrow \Sigma F_x = 0; \quad -T \cos 60^\circ + F_s \cos 30^\circ = 0$$

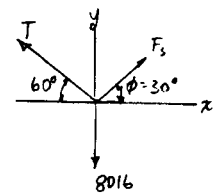
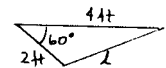
Solving for  $F_s$ ,

$$F_s = 40 \text{ lb}$$

$$F_s = kx$$

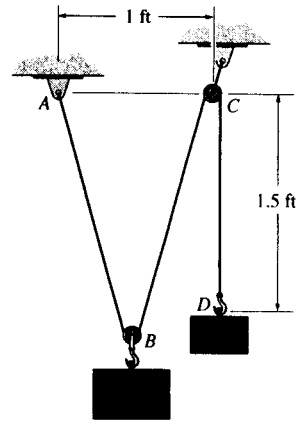
$$40 = 50(\sqrt{12} - l')$$

$$l' = 2.66 \text{ ft} \quad \text{Ans}$$





■3-33. A "scale" is constructed with a 4-ft-long cord and the 10-lb block *D*. The cord is fixed to a pin at *A* and passes over two *small* pulleys at *B* and *C*. Determine the weight of the suspended block *E* if the system is in equilibrium when  $s = 1.5$  ft.



**Free Body Diagram :** The tension force in the cord is the same throughout the cord, that is 10 lb. From the geometry,

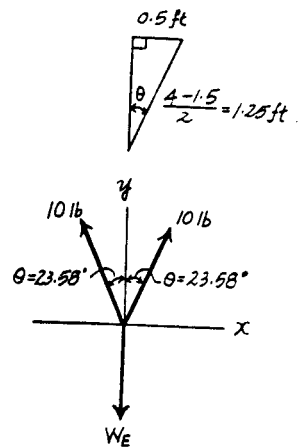
$$\theta = \sin^{-1}\left(\frac{0.5}{1.25}\right) = 23.58^\circ$$

**Equations of Equilibrium :**

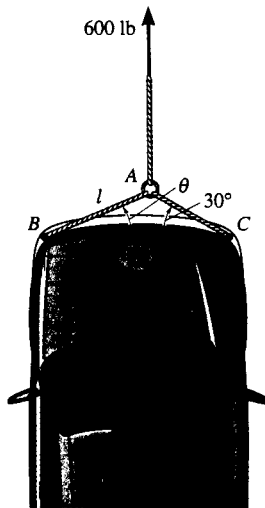
$$\rightarrow \Sigma F_x = 0; \quad 10 \sin 23.58^\circ - 10 \sin 23.58^\circ = 0 \quad (\text{Satisfied!})$$

$$+ \uparrow \Sigma F_y = 0; \quad 2(10) \cos 23.58^\circ - W_E = 0$$

$$W_E = 18.3 \text{ lb} \quad \text{Ans}$$



- 3-34. A car is to be towed using the rope arrangement shown. The towing force required is 600 lb. Determine the minimum length  $l$  of rope  $AB$  so that the tension in either rope  $AB$  or  $AC$  does not exceed 750 lb. *Hint:* Use the equilibrium condition at point  $A$  to determine the required angle  $\theta$  for attachment, then determine  $l$  using trigonometry applied to triangle  $ABC$ .



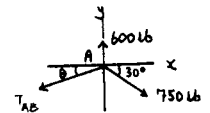
Case 1 : Assume  $T_{AC} = 750$  lb

$$\begin{aligned} \rightarrow \Sigma F_x = 0; & \quad 750 \cos 30^\circ - T_{AB} \cos \theta = 0 \\ + \uparrow \Sigma F_y = 0; & \quad 600 - 750 \sin 30^\circ - T_{AB} \sin \theta = 0 \\ & \quad \theta = 19.107^\circ \end{aligned}$$

$$T_{AB} = 687.39 \text{ lb} < 750 \text{ lb} \quad (\text{O.K!})$$

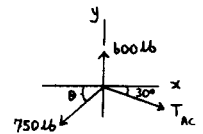
$$\frac{4}{\sin(180^\circ - 30^\circ - 19.107^\circ)} = \frac{l}{\sin 30^\circ}$$

$$l = 2.65 \text{ ft}$$



Case 2 : Assume  $T_{AB} = 750$  lb

$$\begin{aligned} \rightarrow \Sigma F_x = 0; & \quad T_{AC} \cos 30^\circ - 750 \cos \theta = 0 \\ + \uparrow \Sigma F_y = 0; & \quad 600 - T_{AC} \sin 30^\circ - 750 \sin \theta = 0 \\ & \quad 600 - \frac{750 \cos \theta}{\cos 30^\circ} \sin 30^\circ - 750 \sin \theta = 0 \\ & \quad 433.01 \cos \theta + 750 \sin \theta = 600 \end{aligned}$$



An analytic approach to the solution is as follows :

$$(433.01 \sqrt{1 - \sin^2 \theta})^2 = (600 - 750 \sin \theta)^2$$

$$172\,500 - 900\,000 \sin \theta + 750\,000 \sin^2 \theta = 0$$

Solving this quadratic equation for the root of  $\theta$  that gives a positive value for  $T_{AC}$  we get

$$\theta = 13.854^\circ$$

$$T_{AC} = \frac{750 \cos 13.854^\circ}{\cos 30^\circ}$$

$$T_{AC} = 840.83 \text{ lb} > 750 \text{ lb} \quad (\text{N.G!})$$

Thus,  $l = 2.65 \text{ ft}$  Ans

\*3-35. The spring has a stiffness of  $k = 800 \text{ N/m}$  and an unstretched length of  $200 \text{ mm}$ . Determine the force in cables  $BC$  and  $BD$  when the spring is held in the position shown.

**The Force in The Spring :** The spring stretches  $s = l - l_0 = 0.5 - 0.2 = 0.3 \text{ m}$ . Applying Eq. 3-2, we have

$$F_p = ks = 800(0.3) = 240 \text{ N}$$

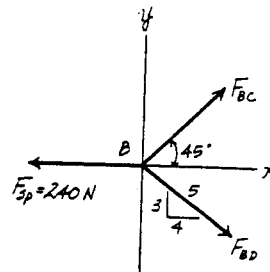
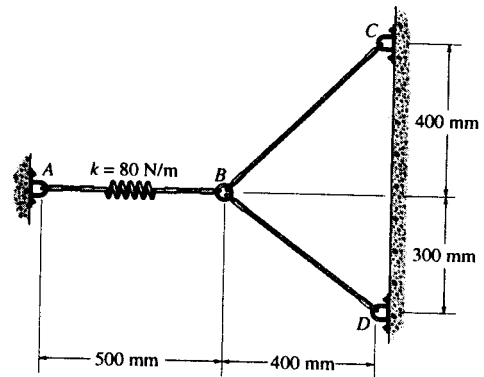
**Equations of Equilibrium :**

$$\begin{aligned} \rightarrow \Sigma F_x = 0; \quad F_{BC} \cos 45^\circ + F_{BD} \left(\frac{4}{5}\right) - 240 = 0 \\ 0.7071 F_{BC} + 0.8 F_{BD} = 240 \end{aligned} \quad [1]$$

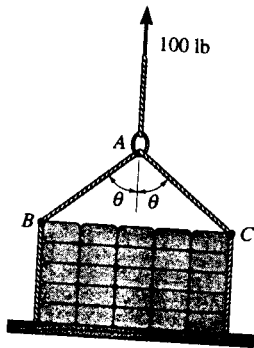
$$\begin{aligned} + \uparrow \Sigma F_y = 0; \quad F_{BC} \sin 45^\circ - F_{BD} \left(\frac{3}{5}\right) = 0 \\ F_{BC} = 0.8485 F_{BD} \end{aligned} \quad [2]$$

Solving Eqs. [1] and [2] yields,

$$F_{BD} = 171 \text{ N} \quad F_{BC} = 145 \text{ N} \quad \text{Ans}$$

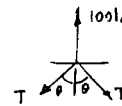


\*3-36. The sling  $BAC$  is used to lift the  $100\text{-lb}$  load with constant velocity. Determine the force in the sling and plot its value  $T$  (ordinate) as a function of its orientation  $\theta$ ,  $+ \uparrow \Sigma F_y = 0$ ; where  $0 \leq \theta \leq 90^\circ$ .

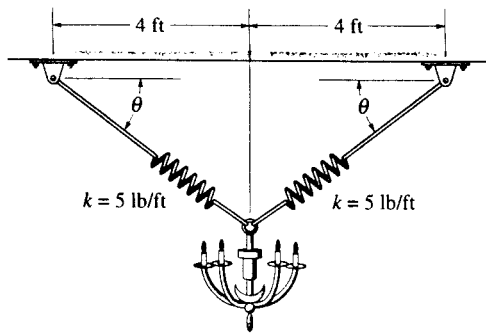


$$100 - 2T \cos \theta = 0$$

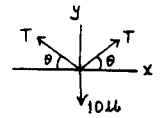
$$T = \frac{50}{\cos \theta} \quad \text{Ans}$$



■3-37. The 10-lb lamp fixture is suspended from two springs, each having an unstretched length of 4 ft and stiffness of  $k = 5 \text{ lb/ft}$ . Determine the angle  $\theta$  for equilibrium.



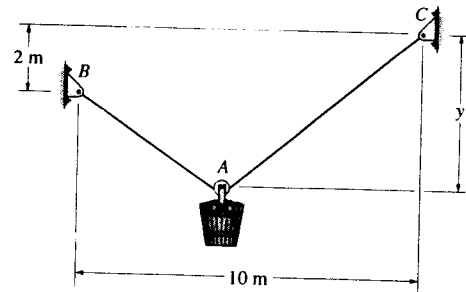
$$\begin{aligned} \rightarrow \Sigma F_x &= 0; & T \cos \theta - T \cos \theta &= 0 \\ + \uparrow \Sigma F_y &= 0; & 2T \sin \theta - 10 &= 0 \\ & & T \sin \theta &= 5 \text{ lb} \\ F &= ks; & T &= 5 \left( \frac{4}{\cos \theta} - 4 \right) \\ & & T &= 20 \left( \frac{1}{\cos \theta} - 1 \right) \\ 20 \left( \frac{\sin \theta}{\cos \theta} - \sin \theta \right) &= 5 \\ \tan \theta - \sin \theta &= 0.25 \end{aligned}$$



Solving by trial and error,

$$\theta = 43.0^\circ \quad \text{Ans}$$

■3-38. The pail and its contents have a mass of 60 kg. If the cable is 15 m long, determine the distance  $y$  of the pulley for equilibrium. Neglect the size of the pulley at  $A$ .



**Free Body Diagram :** Since the pulley is smooth, the tension in the cable is the same throughout the cable.

**Equations of Equilibrium :**

$$\rightarrow \Sigma F_x = 0; \quad T \sin \theta - T \sin \phi = 0 \quad \theta = \phi$$

**Geometry :**

$$l_1 = \sqrt{(10-x)^2 + (y-2)^2} \quad l_2 = \sqrt{x^2 + y^2}$$

Since  $\theta = \phi$ , two triangles are similar.

$$\frac{10-x}{x} = \frac{y-2}{y} = \frac{\sqrt{(10-x)^2 + (y-2)^2}}{\sqrt{x^2 + y^2}}$$

Also,

$$\begin{aligned} l_1 + l_2 &= 15 \\ \sqrt{(10-x)^2 + (y-2)^2} + \sqrt{x^2 + y^2} &= 15 \\ \left( \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2}} \right) \sqrt{(10-x)^2 + (y-2)^2} + \sqrt{x^2 + y^2} &= 15 \end{aligned}$$

However, from Eq. [1]  $\frac{\sqrt{(10-x)^2 + (y-2)^2}}{\sqrt{x^2 + y^2}} = \frac{10-x}{x}$ , Eq. [2] becomes

$$\sqrt{x^2 + y^2} \left( \frac{10-x}{x} \right) + \sqrt{x^2 + y^2} = 15$$

Dividing both sides of Eq. [3] by  $\sqrt{x^2 + y^2}$  yields

$$\frac{10}{x} = \frac{15}{\sqrt{x^2 + y^2}} \quad x = \sqrt{0.8y} \quad [4]$$

From Eq. [1]

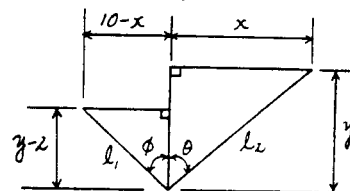
$$\frac{10-x}{x} = \frac{y-2}{y} \quad x = \frac{5y}{y-1} \quad [5]$$

Equating Eq. [1] and [5] yields

$$\begin{aligned} \sqrt{0.8y} &= \frac{5y}{y-1} \\ y &= 6.59 \text{ m} \end{aligned}$$

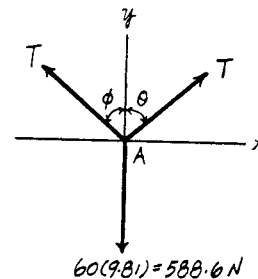
Ans

[1]

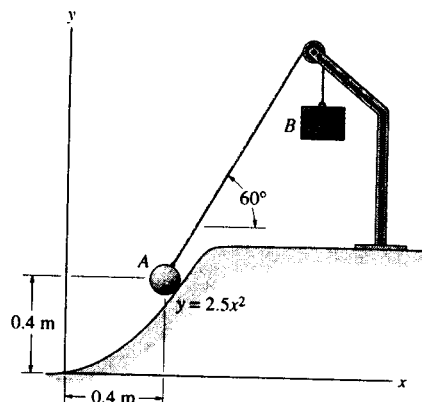


[2]

[3]



3-39. A 4-kg sphere rests on the smooth parabolic surface. Determine the normal force it exerts on the surface and the mass  $m_B$  of block  $B$  needed to hold it in the equilibrium position shown.



**Geometry :** The angle  $\theta$  which the surface make with the horizontal is to be determined first.

$$\tan \theta \Big|_{x=0.4\text{m}} = \frac{dy}{dx} \Big|_{x=0.4\text{m}} = 5.0x \Big|_{x=0.4\text{m}} = 2.00$$

$$\theta = 63.43^\circ$$

**Free Body Diagram :** The tension in the cord is the same throughout the cord and is equal to the weight of block  $B$ ,  $W_B = m_B (9.81)$ .

**Equations of Equilibrium :**

$$\rightarrow \Sigma F_x = 0; \quad m_B (9.81) \cos 60^\circ - N \sin 63.43^\circ = 0$$

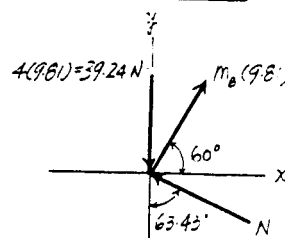
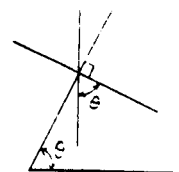
$$N = 5.4840m_B \quad [1]$$

$$+ \uparrow \Sigma F_y = 0; \quad m_B (9.81) \sin 60^\circ + N \cos 63.43^\circ - 39.24 = 0$$

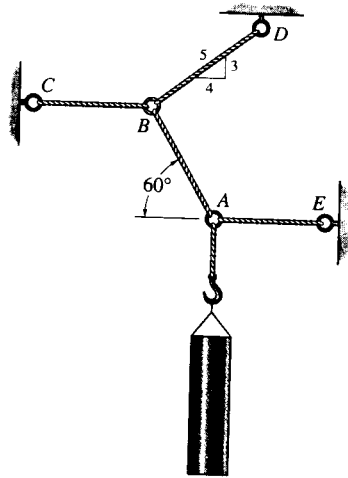
$$8.4957m_B + 0.4472N = 39.24 \quad [2]$$

Solving Eqs. [1] and [2] yields

$$m_B = 3.58 \text{ kg} \quad N = 19.7 \text{ N} \quad \text{Ans}$$

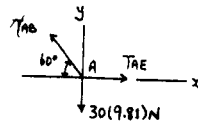


\*3-40. The 30-kg pipe is supported at  $A$  by a system of five cords. Determine the force in each cord for equilibrium.



$$+\uparrow \Sigma F_y = 0; \quad T_{AB} \sin 60^\circ - 30(9.81) = 0$$

$$T_{AB} = 339.83 = 340 \text{ N} \quad \text{Ans}$$

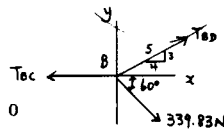


$$\rightarrow \Sigma F_x = 0; \quad T_{AE} - 339.83 \cos 60^\circ = 0$$

$$T_{AE} = 170 \text{ N} \quad \text{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad T_{BD} \left(\frac{3}{5}\right) - 339.83 \sin 60^\circ = 0$$

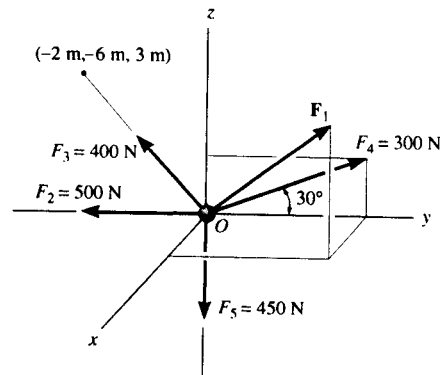
$$T_{BD} = 490.5 = 490 \text{ N} \quad \text{Ans}$$



$$\rightarrow \Sigma F_x = 0; \quad 490.5 \left(\frac{4}{5}\right) + 339.83 \cos 60^\circ - T_{BC} = 0$$

$$T_{BC} = 562 \text{ N} \quad \text{Ans}$$

3-41. Determine the magnitude and direction of  $F_1$  required to keep the concurrent force system in equilibrium.



**Cartesian Vector Notation :**

$$F_1 = F_{1x}i + F_{1y}j + F_{1z}k$$

$$F_2 = \{-500j\} \text{ N}$$

$$F_3 = 400 \left( \frac{-2i - 6j + 3k}{\sqrt{(-2)^2 + (-6)^2 + 3^2}} \right) = \{-114.29i - 342.86j + 171.43k\} \text{ N}$$

$$F_4 = 300\{\cos 30^\circ j + \sin 30^\circ k\} \text{ N} = \{259.81j + 150.0k\} \text{ N}$$

$$F_5 = \{-450k\} \text{ N}$$

**Equations of Equilibrium :**

$$\Sigma F = 0; \quad F_1 + F_2 + F_3 + F_4 + F_5 = 0$$

$$\begin{aligned} (F_{1x} - 114.29)i + (F_{1y} - 500 - 342.86 + 259.81)j \\ + (F_{1z} + 171.43 + 150.0 - 450)k = 0 \end{aligned}$$

Equating i, j and k components, we have

$$\begin{aligned} F_{1x} - 114.29 &= 0 & F_{1x} &= 114.29 \text{ N} \\ F_{1y} - 500 - 342.86 + 259.81 &= 0 & F_{1y} &= 583.05 \text{ N} \\ F_{1z} + 171.43 + 150.0 - 450 &= 0 & F_{1z} &= 128.57 \text{ N} \end{aligned}$$

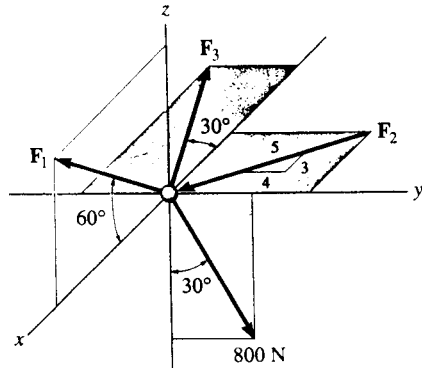
The magnitude of  $F_1$  is

$$\begin{aligned} F_1 &= \sqrt{F_{1x}^2 + F_{1y}^2 + F_{1z}^2} \\ &= \sqrt{114.29^2 + 583.05^2 + 128.57^2} \\ &= 607.89 \text{ N} = 608 \text{ N} \end{aligned} \quad \text{Ans}$$

The coordinate direction angles are

$$\begin{aligned} \alpha &= \cos^{-1} \left( \frac{F_{1x}}{F_1} \right) = \cos^{-1} \left( \frac{114.29}{607.89} \right) = 79.2^\circ & \text{Ans} \\ \beta &= \cos^{-1} \left( \frac{F_{1y}}{F_1} \right) = \cos^{-1} \left( \frac{583.05}{607.89} \right) = 16.4^\circ & \text{Ans} \\ \gamma &= \cos^{-1} \left( \frac{F_{1z}}{F_1} \right) = \cos^{-1} \left( \frac{128.57}{607.89} \right) = 77.8^\circ & \text{Ans} \end{aligned}$$

3-42. Determine the magnitudes of  $F_1$ ,  $F_2$ , and  $F_3$  for equilibrium of the particle.



$$F_1 = F_1 (\cos 60^\circ \mathbf{i} + \sin 60^\circ \mathbf{k})$$

$$= \{0.5F_1 \mathbf{i} + 0.8660F_1 \mathbf{k}\} \text{ N}$$

$$F_2 = F_2 \left(\frac{3}{5} \mathbf{i} - \frac{4}{5} \mathbf{j}\right)$$

$$= \{0.6F_2 \mathbf{i} - 0.8F_2 \mathbf{j}\} \text{ N}$$

$$F_3 = F_3 \{-\cos 30^\circ \mathbf{i} - \sin 30^\circ \mathbf{j}\}$$

$$\{-0.8660F_3 \mathbf{i} - 0.5F_3 \mathbf{j}\} \text{ N}$$

$$\Sigma F_x = 0; \quad 0.5F_1 + 0.6F_2 - 0.8660F_3 = 0$$

$$\Sigma F_y = 0; \quad -0.8F_2 - 0.5F_3 + 800 \sin 30^\circ = 0$$

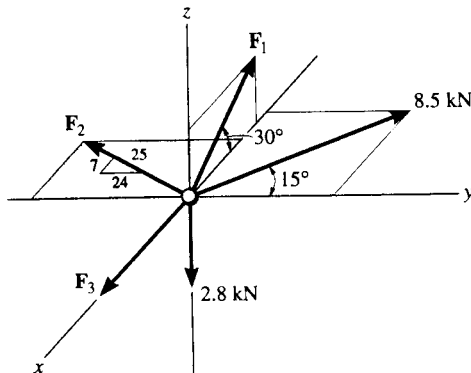
$$\Sigma F_z = 0; \quad 0.8660F_1 - 800 \cos 30^\circ = 0$$

$$F_1 = 800 \text{ N} \quad \text{Ans}$$

$$F_2 = 147 \text{ N} \quad \text{Ans}$$

$$F_3 = 564 \text{ N} \quad \text{Ans}$$

3-43. Determine the magnitudes of  $F_1$ ,  $F_2$ , and  $F_3$  for equilibrium of the particle.



$$\Sigma F_z = 0; \quad F_1 \sin 30^\circ - 2.8 = 0$$

$$F_1 = 5.60 \text{ kN} \quad \text{Ans}$$

$$\Sigma F_y = 0; \quad 8.5 \cos 15^\circ - \left(\frac{24}{25}\right)F_2 = 0$$

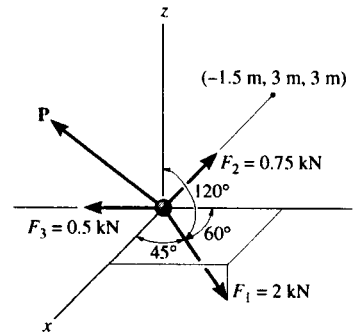
$$F_2 = 8.55 \text{ kN} \quad \text{Ans}$$

$$\Sigma F_x = 0; \quad F_3 - 5.6 \cos 30^\circ - 8.55 \left(\frac{7}{25}\right) - 8.5 \sin 15^\circ = 0$$

$$F_3 = 9.44 \text{ kN} \quad \text{Ans}$$



•\*3-44. Determine the magnitude and direction of the force  $\mathbf{P}$  required to keep the concurrent force system in equilibrium.



**Cartesian Vector Notation :**

$$F_1 = 2\{\cos 45^\circ\mathbf{i} + \cos 60^\circ\mathbf{j} + \cos 120^\circ\mathbf{k}\} \text{ kN} = \{1.414\mathbf{i} + 1.00\mathbf{j} - 1.00\mathbf{k}\} \text{ kN}$$

$$F_2 = 0.75 \left( \frac{-1.5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}}{\sqrt{(-1.5)^2 + 3^2 + 3^2}} \right) = \{-0.250\mathbf{i} + 0.50\mathbf{j} + 0.50\mathbf{k}\} \text{ kN}$$

$$F_3 = \{-0.50\mathbf{j}\} \text{ kN}$$

$$\mathbf{P} = P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k}$$

**Equations of Equilibrium :**

$$\Sigma \mathbf{F} = 0; \quad F_1 + F_2 + F_3 + \mathbf{P} = 0$$

$$(P_x + 1.414 - 0.250)\mathbf{i} + (P_y + 1.00 + 0.50 - 0.50)\mathbf{j} + (P_z - 1.00 + 0.50)\mathbf{k} = 0$$

Equating i, j and k components, we have

$$\begin{aligned} P_x + 1.414 - 0.250 &= 0 & P_x &= -1.164 \text{ kN} \\ P_y + 1.00 + 0.50 - 0.50 &= 0 & P_y &= -1.00 \text{ kN} \\ P_z - 1.00 + 0.50 &= 0 & P_z &= 0.500 \text{ kN} \end{aligned}$$

The magnitude of  $F_1$  is

$$\begin{aligned} P &= \sqrt{P_x^2 + P_y^2 + P_z^2} \\ &= \sqrt{(-1.164)^2 + (-1.00)^2 + (0.500)^2} \\ &= 1.614 \text{ kN} = 1.61 \text{ kN} \end{aligned} \quad \text{Ans}$$

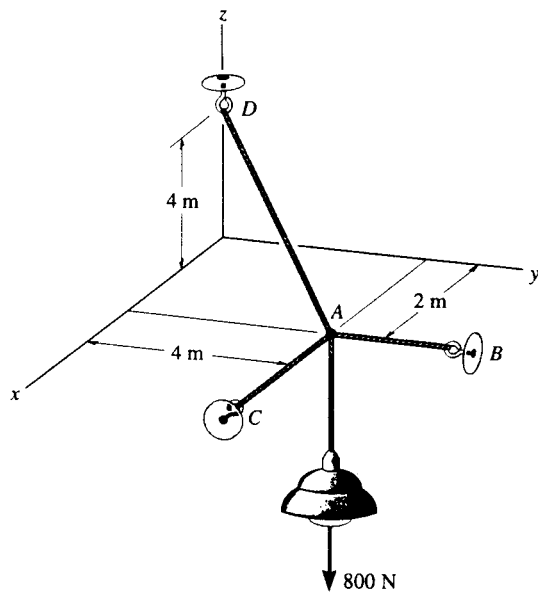
The coordinate direction angles are

$$\alpha = \cos^{-1} \left( \frac{P_x}{P} \right) = \cos^{-1} \left( \frac{-1.164}{1.614} \right) = 136^\circ \quad \text{Ans}$$

$$\beta = \cos^{-1} \left( \frac{P_y}{P} \right) = \cos^{-1} \left( \frac{-1.00}{1.614} \right) = 128^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1} \left( \frac{P_z}{P} \right) = \cos^{-1} \left( \frac{0.500}{1.614} \right) = 72.0^\circ \quad \text{Ans}$$

3-45. The three cables are used to support the 800-N lamp. Determine the force developed in each cable for equilibrium.



$$\Sigma F_z = 0; \quad \frac{4}{6} F_{AD} - 800 = 0$$

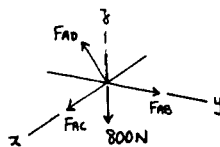
$$F_{AD} = 1.20 \text{ kN} \quad \text{Ans}$$

$$\Sigma F_x = 0; \quad -\frac{2}{6}(1200) + F_{AC} = 0$$

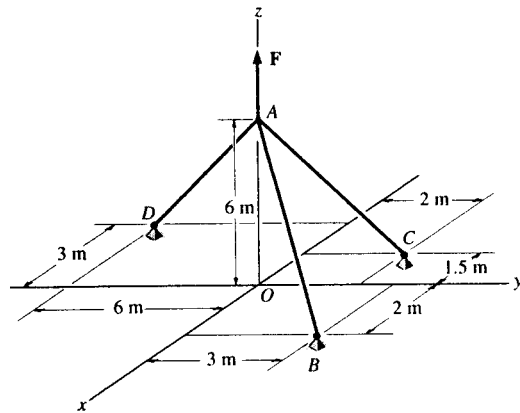
$$F_{AC} = 0.40 \text{ kN} \quad \text{Ans}$$

$$\Sigma F_y = 0; \quad -\left(\frac{4}{6}\right)(1200) + F_{AB} = 0$$

$$F_{AB} = 0.80 \text{ kN} \quad \text{Ans}$$



3-46. If cable  $AB$  is subjected to a tension of 700 N, determine the tension in cables  $AC$  and  $AD$  and the magnitude of the vertical force  $F$ .



**Cartesian Vector Notation :**

$$F_{AB} = 700 \left( \frac{2i + 3j - 6k}{\sqrt{2^2 + 3^2 + (-6)^2}} \right) = \{200i + 300j - 600k\} \text{ N}$$

$$F_{AC} = F_{AC} \left( \frac{-1.5i + 2j - 6k}{\sqrt{(-1.5)^2 + 2^2 + (-6)^2}} \right) = -0.2308F_{AC}i + 0.3077F_{AC}j - 0.9231F_{AC}k$$

$$F_{AD} = F_{AD} \left( \frac{-3i - 6j - 6k}{\sqrt{(-3)^2 + (-6)^2 + (-6)^2}} \right) = -0.3333F_{AD}i - 0.6667F_{AD}j - 0.6667F_{AD}k$$

$$F = Fk$$

**Equations of Equilibrium :**

$$\Sigma F = 0; \quad F_{AB} + F_{AC} + F_{AD} + F = 0$$

$$(200 - 0.2308F_{AC} - 0.3333F_{AD})i + (300 + 0.3077F_{AC} - 0.6667F_{AD})j + (-600 - 0.9231F_{AC} - 0.6667F_{AD} + F)k = 0$$

Equating i, j and k components, we have

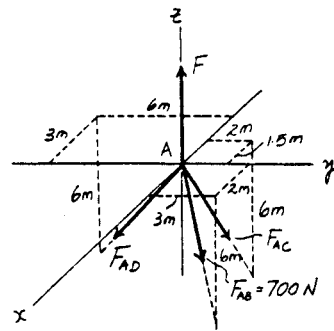
$$200 - 0.2308F_{AC} - 0.3333F_{AD} = 0 \quad [1]$$

$$300 + 0.3077F_{AC} - 0.6667F_{AD} = 0 \quad [2]$$

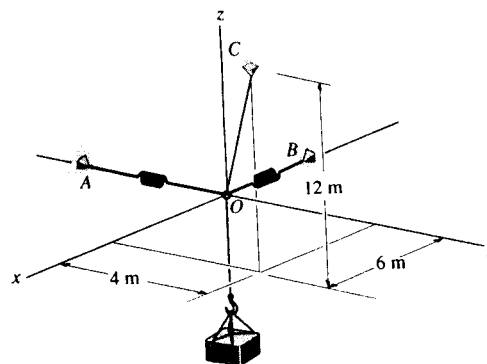
$$-600 - 0.9231F_{AC} - 0.6667F_{AD} + F = 0 \quad [3]$$

Solving Eqs. [1], [2] and [3] yields

$$F_{AC} = 130 \text{ N} \quad F_{AD} = 510 \text{ N} \quad F = 1060 \text{ N} = 1.06 \text{ kN} \quad \text{Ans}$$



3-47. Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of  $k = 300 \text{ N/m}$ .



**Cartesian Vector Notation :**

$$F_{OC} = F_{OC} \left( \frac{6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}}{\sqrt{6^2 + 4^2 + 12^2}} \right) = \frac{3}{7}F_{OC}\mathbf{i} + \frac{2}{7}F_{OC}\mathbf{j} + \frac{6}{7}F_{OC}\mathbf{k}$$

$$F_{OA} = -F_{OA}\mathbf{j} \quad F_{OB} = -F_{OB}\mathbf{i}$$

$$\mathbf{F} = (-196.2\mathbf{k}) \text{ N}$$

**Equations of Equilibrium :**

$$\Sigma \mathbf{F} = 0; \quad F_{OC} + F_{OA} + F_{OB} + \mathbf{F} = 0$$

$$\left( \frac{3}{7}F_{OC} - F_{OB} \right)\mathbf{i} + \left( \frac{2}{7}F_{OC} - F_{OA} \right)\mathbf{j} + \left( \frac{6}{7}F_{OC} - 196.2 \right)\mathbf{k} = 0$$

Equating i, j and k components, we have

$$\frac{3}{7}F_{OC} - F_{OB} = 0 \quad [1]$$

$$\frac{2}{7}F_{OC} - F_{OA} = 0 \quad [2]$$

$$\frac{6}{7}F_{OC} - 196.2 = 0 \quad [3]$$

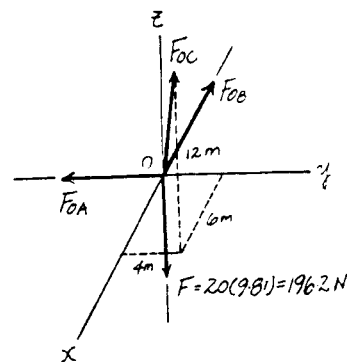
Solving Eqs. [1], [2] and [3] yields

$$F_{OC} = 228.9 \text{ N} \quad F_{OB} = 98.1 \text{ N} \quad F_{OA} = 65.4 \text{ N}$$

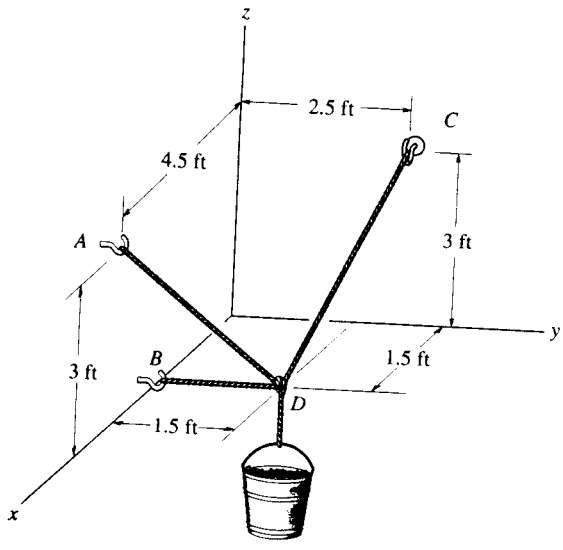
**Spring Elongation :** Using spring formula, Eq. 3-2, the spring elongation is  $s = \frac{F}{k}$ .

$$s_{OB} = \frac{98.1}{300} = 0.327 \text{ m} = 327 \text{ mm} \quad \text{Ans}$$

$$s_{OA} = \frac{65.4}{300} = 0.218 \text{ m} = 218 \text{ mm} \quad \text{Ans}$$



**\*3-48.** If the bucket and its contents have a total weight of 20 lb, determine the force in the supporting cables DA, DB, and DC.



$$u_{DA} = \left\{ \frac{3}{4.5}i - \frac{1.5}{4.5}j + \frac{3}{4.5}k \right\}$$

$$u_{DC} = \left\{ -\frac{1.5}{3.5}i + \frac{1}{3.5}j + \frac{3}{3.5}k \right\}$$

$$\Sigma F_x = 0; \quad \frac{3}{4.5}F_{DA} - \frac{1.5}{3.5}F_{DC} = 0$$

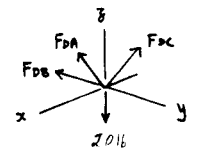
$$\Sigma F_y = 0; \quad -\frac{1.5}{4.5}F_{DA} - F_{DB} + \frac{1}{3.5}F_{DC} = 0$$

$$\Sigma F_z = 0; \quad \frac{3}{4.5}F_{DA} + \frac{3}{3.5}F_{DC} - 20 = 0$$

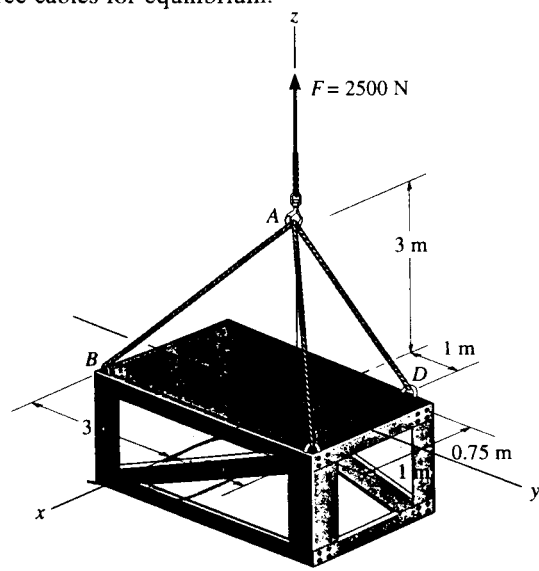
$$F_{DA} = 10.0 \text{ lb} \quad \text{Ans}$$

$$F_{DB} = 1.11 \text{ lb} \quad \text{Ans}$$

$$F_{DC} = 15.6 \text{ lb} \quad \text{Ans}$$



**3-49.** The 2500-N crate is to be hoisted with constant velocity from the hold of a ship using the cable arrangement shown. Determine the tension in each of the three cables for equilibrium.



$$F_{AD} = F_{AD} \left( \frac{-0.75i + j - 3k}{\sqrt{(-0.75)^2 + 1^2 + (-3)^2}} \right) = -0.2308F_{AD}i + 0.3077F_{AD}j - 0.9231F_{AD}k$$

$$F_{AC} = F_{AC} \left( \frac{1i + 1.5j - 3k}{\sqrt{1^2 + 1.5^2 + (-3)^2}} \right) = 0.2857F_{AC}i + 0.4286F_{AC}j - 0.8571F_{AC}k$$

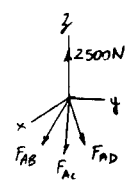
$$F_{AB} = F_{AB} \left( \frac{1i - 3j - 3k}{\sqrt{1^2 + (-3)^2 + (-3)^2}} \right) = 0.2294F_{AB}i - 0.6882F_{AB}j - 0.6882F_{AB}k$$

$$F = \{2.5k\} \text{ kN}$$

$$\Sigma F = 0; \quad F_{AD} + F_{AB} + F_{AC} + F = 0$$

$$(-0.2308F_{AD}i + 0.3077F_{AD}j - 0.9231F_{AD}k) + (0.2294F_{AB}i - 0.6882F_{AB}j - 0.6882F_{AB}k) + (0.2857F_{AC}i + 0.4286F_{AC}j - 0.8571F_{AC}k) + (2.5k) = 0$$

$$(-0.2308F_{AD} + 0.2294F_{AB} + 0.2857F_{AC})i + (0.3077F_{AD} - 0.6882F_{AB} + 0.4286F_{AC})j + (-0.9231F_{AD} - 0.6882F_{AB} - 0.8571F_{AC} + 2.5)k = 0$$



$$\Sigma F_x = 0; \quad -0.2308F_{AD} + 0.2294F_{AB} + 0.2857F_{AC} = 0 \quad [1]$$

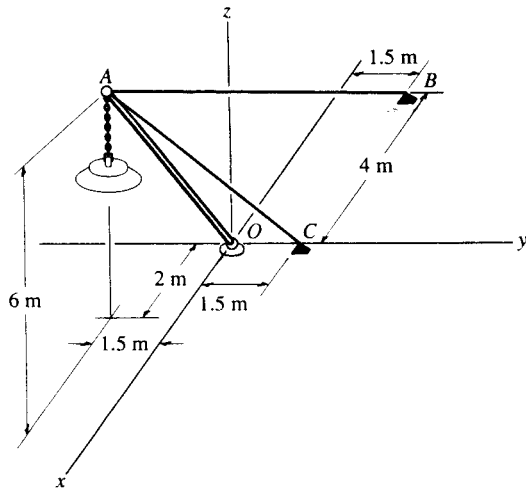
$$\Sigma F_y = 0; \quad 0.3077F_{AD} - 0.6882F_{AB} + 0.4286F_{AC} = 0 \quad [2]$$

$$\Sigma F_z = 0; \quad -0.9231F_{AD} - 0.6882F_{AB} - 0.8571F_{AC} + 2.5 = 0 \quad [3]$$

Solving Eqs. [1], [2] and [3] yields :

$$F_{AB} = 0.980 \text{ kN} \quad F_{AC} = 0.463 \text{ kN} \quad F_{AD} = 1.55 \text{ kN} \quad \text{Ans}$$

■3-50. The lamp has a mass of 15 kg and is supported by a pole  $AO$  and cables  $AB$  and  $AC$ . If the force in the pole acts along its axis, determine the forces in  $AO$ ,  $AB$ , and  $AC$  for equilibrium.



$$\mathbf{F}_{AO} = F_{AO} \left\{ \frac{2}{6.5}\mathbf{i} - \frac{1.5}{6.5}\mathbf{j} + \frac{6}{6.5}\mathbf{k} \right\} \text{ N}$$

$$\mathbf{F}_{AB} = F_{AB} \left\{ -\frac{6}{9}\mathbf{i} + \frac{3}{9}\mathbf{j} - \frac{6}{9}\mathbf{k} \right\} \text{ N}$$

$$\mathbf{F}_{AC} = F_{AC} \left\{ -\frac{2}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \right\} \text{ N}$$

$$\mathbf{W} = 15(9.81)\mathbf{k} = \{-147.15\mathbf{k}\} \text{ N}$$

$$\Sigma F_x = 0; \quad 0.3077F_{AO} - 0.6667F_{AB} - 0.2857F_{AC} = 0$$

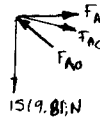
$$\Sigma F_y = 0; \quad -0.2308F_{AO} + 0.3333F_{AB} + 0.4286F_{AC} = 0$$

$$\Sigma F_z = 0; \quad 0.9231F_{AO} - 0.667F_{AB} - 0.8571F_{AC} - 147.15 = 0$$

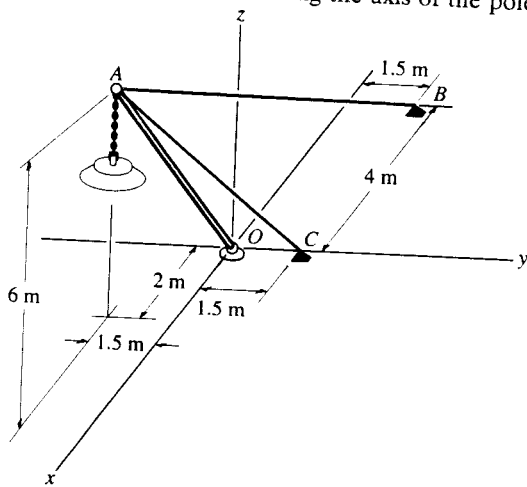
$$F_{AO} = 319 \text{ N} \quad \text{Ans}$$

$$F_{AB} = 110 \text{ N} \quad \text{Ans}$$

$$F_{AC} = 85.8 \text{ N} \quad \text{Ans}$$



3-51. Cables  $AB$  and  $AC$  can sustain a maximum tension of 500 N, and the pole can support a maximum compression of 300 N. Determine the maximum weight of the lamp that can be supported in the position shown. The force in the pole acts along the axis of the pole.



$$F_{Ao} = F_{Ao} \left( \frac{2}{6.5}i - \frac{1.5}{6.5}j + \frac{6}{6.5}k \right) \text{ N}$$

$$F_{Ab} = F_{Ab} \left( -\frac{6}{9}i + \frac{3}{9}j - \frac{6}{9}k \right) \text{ N}$$

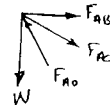
$$F_{Ac} = F_{Ac} \left( -\frac{2}{7}i + \frac{3}{7}j - \frac{6}{7}k \right) \text{ N}$$

$$W = \{-Wk\} \text{ N}$$

$$\Sigma F_x = 0; \quad \frac{2}{6.5}F_{Ao} - \frac{6}{9}F_{Ab} - \frac{2}{7}F_{Ac} = 0$$

$$\Sigma F_y = 0; \quad -\frac{1.5}{6.5}F_{Ao} + \frac{3}{9}F_{Ab} + \frac{3}{7}F_{Ac} = 0$$

$$\Sigma F_z = 0; \quad \frac{6}{6.5}F_{Ao} - \frac{6}{9}F_{Ab} - \frac{6}{7}F_{Ac} - W = 0$$



1) Assume  $F_{Ab} = 500 \text{ N}$

$$\frac{2}{6.5}F_{Ao} - \frac{6}{9}(500) - \frac{2}{7}F_{Ac} = 0$$

$$-\frac{1.5}{6.5}F_{Ao} + \frac{3}{9}(500) + \frac{3}{7}F_{Ac} = 0$$

$$\frac{6}{6.5}F_{Ao} - \frac{6}{9}(500) - \frac{6}{7}F_{Ac} - W = 0$$

Solving,

$$F_{Ao} = 1444.462 \text{ N} > 300 \text{ N (N.G!)}$$

$$F_{Ac} = 388.902 \text{ N}$$

$$W = 666.677 \text{ N}$$

2) Assume  $F_{Ac} = 500 \text{ N}$

$$\frac{2}{6.5}F_{Ao} - \frac{6}{9}F_{Ab} - \frac{2}{7}(500) = 0$$

$$-\frac{1.5}{6.5}F_{Ao} + \frac{3}{9}F_{Ab} + \frac{3}{7}(500) = 0$$

$$\frac{6}{6.5}F_{Ao} - \frac{6}{9}F_{Ab} - \frac{6}{7}(500) - W = 0$$

Solving,

$$F_{Ao} = 1857.143 \text{ N} > 300 \text{ N (N.G!)}$$

$$F_{Ab} = 642.857 \text{ N} > 500 \text{ N (N.G!)}$$

3) Assume  $F_{Ao} = 300 \text{ N}$

$$\frac{2}{6.5}(300) - \frac{6}{9}F_{Ab} - \frac{2}{7}F_{Ac} = 0$$

$$-\frac{1.5}{6.5}(300) + \frac{3}{9}F_{Ab} + \frac{3}{7}F_{Ac} = 0$$

$$\frac{6}{6.5}(300) - \frac{6}{9}F_{Ab} - \frac{6}{7}F_{Ac} - W = 0$$

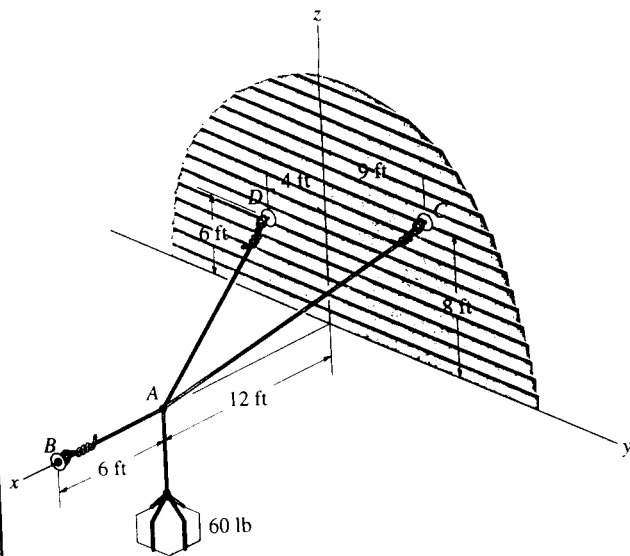
Solving,

$$F_{Ac} = 80.8 \text{ N}$$

$$F_{Ab} = 104 \text{ N}$$

$$W = 138 \text{ N} \quad \text{Ans}$$

\*3-52. Determine the tension in cables  $AB$ ,  $AC$ , and  $AD$ , required to hold the 60-lb crate in equilibrium.

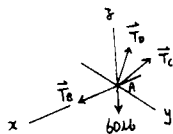


$$\mathbf{W} = -60\mathbf{k}$$

$$\mathbf{T}_B = T_B\mathbf{i}$$

$$\begin{aligned} \mathbf{T}_C &= T_C\left(-\frac{12}{17}\mathbf{i} + \frac{9}{17}\mathbf{j} + \frac{8}{17}\mathbf{k}\right) \\ &= -0.706T_C\mathbf{i} + 0.529T_C\mathbf{j} + 0.471T_C\mathbf{k} \end{aligned}$$

$$\begin{aligned} \mathbf{T}_D &= T_D\left(-\frac{12}{14}\mathbf{i} - \frac{4}{14}\mathbf{j} + \frac{6}{14}\mathbf{k}\right) \\ &= -0.857T_D\mathbf{i} - 0.286T_D\mathbf{j} + 0.429T_D\mathbf{k} \end{aligned}$$



$$\Sigma F_x = 0; \quad T_B - 0.706T_C - 0.857T_D = 0$$

$$\Sigma F_y = 0; \quad 0.529T_C - 0.286T_D = 0$$

$$\Sigma F_z = 0; \quad -60 + 0.471T_C + 0.429T_D = 0$$

Solving,

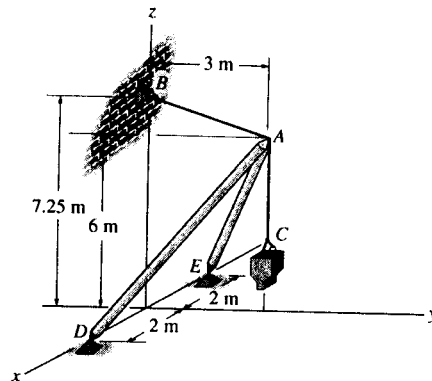
$$T_B = 109 \text{ lb} \quad \text{Ans}$$

$$T_C = 47.4 \text{ lb} \quad \text{Ans}$$

$$T_D = 87.9 \text{ lb} \quad \text{Ans}$$



3-53. The boom supports a bucket and contents, which have a total mass of 300 kg. Determine the forces developed in struts  $AD$  and  $AE$  and the tension in cable  $AB$  for equilibrium. The force in each strut acts along its axis.



Cartesian Vector Notation :

$$F_{AB} = F_{AB} \left( \frac{-3\mathbf{j} + 1.25\mathbf{k}}{\sqrt{(-3)^2 + 1.25^2}} \right) = -\frac{12}{13}F_{AB}\mathbf{j} + \frac{5}{13}F_{AB}\mathbf{k}$$

$$F_{AD} = F_{AD} \left( \frac{-2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}}{\sqrt{(-2)^2 + 3^2 + 6^2}} \right) = -\frac{2}{7}F_{AD}\mathbf{i} + \frac{3}{7}F_{AD}\mathbf{j} + \frac{6}{7}F_{AD}\mathbf{k}$$

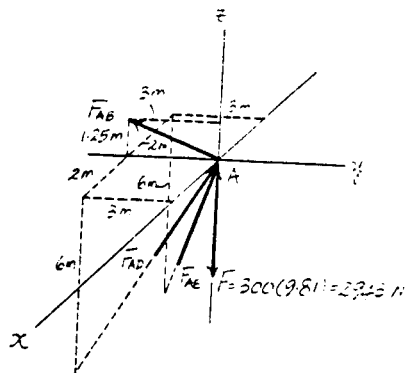
$$F_{AE} = F_{AE} \left( \frac{2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}}{\sqrt{2^2 + 3^2 + 6^2}} \right) = \frac{2}{7}F_{AE}\mathbf{i} + \frac{3}{7}F_{AE}\mathbf{j} + \frac{6}{7}F_{AE}\mathbf{k}$$

$$\mathbf{F} = (-2943\mathbf{k}) \text{ N}$$

Equations of Equilibrium :

$$\Sigma \mathbf{F} = 0; \quad F_{AB} + F_{AD} + F_{AE} + \mathbf{F} = 0$$

$$\left( -\frac{2}{7}F_{AD} + \frac{2}{7}F_{AE} \right) \mathbf{i} + \left( -\frac{12}{13}F_{AB} + \frac{3}{7}F_{AD} + \frac{3}{7}F_{AE} \right) \mathbf{j} + \left( \frac{5}{13}F_{AB} + \frac{6}{7}F_{AD} + \frac{6}{7}F_{AE} - 2943 \right) \mathbf{k} = 0$$



Equating  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  components, we have

$$-\frac{2}{7}F_{AD} + \frac{2}{7}F_{AE} = 0 \quad [1]$$

$$-\frac{12}{13}F_{AB} + \frac{3}{7}F_{AD} + \frac{3}{7}F_{AE} = 0 \quad [2]$$

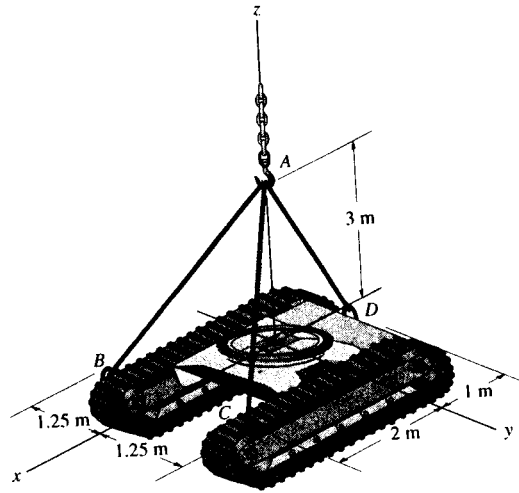
$$\frac{5}{13}F_{AB} + \frac{6}{7}F_{AD} + \frac{6}{7}F_{AE} - 2943 = 0 \quad [3]$$

Solving Eqs. [1], [2] and [3] yields

$$F_{AE} = F_{AD} = 1420.76 \text{ N} = 1.42 \text{ kN} \quad \text{Ans}$$

$$F_{AB} = 1319.28 \text{ N} = 1.32 \text{ kN} \quad \text{Ans}$$

3-54. Determine the force in each of the three cables needed to lift the tractor which has a mass of 8 Mg.



**Cartesian Vector Notation :**

$$F_{AB} = F_{AB} \left( \frac{2i - 1.25j - 3k}{\sqrt{2^2 + (-1.25)^2 + (-3)^2}} \right) = 0.5241F_{AB}i - 0.3276F_{AB}j - 0.7861F_{AB}k$$

$$F_{AC} = F_{AC} \left( \frac{2i + 1.25j - 3k}{\sqrt{2^2 + 1.25^2 + (-3)^2}} \right) = 0.5241F_{AC}i + 0.3276F_{AC}j - 0.7861F_{AC}k$$

$$F_{AD} = F_{AD} \left( \frac{-1i - 3k}{\sqrt{(-1)^2 + (-3)^2}} \right) = -0.3162F_{AD}i - 0.9487F_{AD}k$$

$$F = (78.48k) \text{ kN}$$

**Equations of Equilibrium**

$$\Sigma F = 0; \quad F_{AB} + F_{AC} + F_{AD} + F = 0$$

$$(0.5241F_{AB} + 0.5241F_{AC} - 0.3162F_{AD})i + (-0.3276F_{AB} + 0.3276F_{AC})j + (-0.7861F_{AB} - 0.7861F_{AC} - 0.9487F_{AD} + 78.48)k = 0$$

Equating i, j and k components, we have

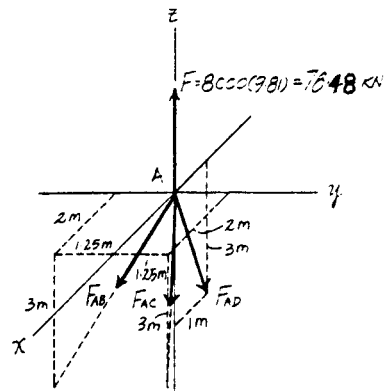
$$0.5241F_{AB} + 0.5241F_{AC} - 0.3162F_{AD} = 0 \quad [1]$$

$$-0.3276F_{AB} + 0.3276F_{AC} = 0 \quad [2]$$

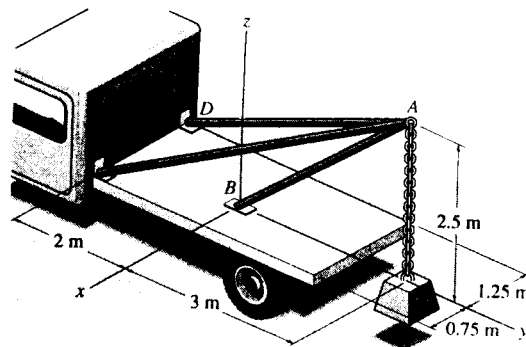
$$-0.7861F_{AB} - 0.7861F_{AC} - 0.9487F_{AD} + 78.48 = 0 \quad [3]$$

Solving Eqs. [1], [2] and [3] yields

$$F_{AB} = F_{AC} = 16.6 \text{ kN} \quad F_{AD} = 55.2 \text{ kN} \quad \text{Ans}$$



3-55. Determine the force acting along the axis of each of the three struts needed to support the 500-kg block.



$$\mathbf{F}_B = F_B \left( \frac{3 \mathbf{j} + 2.5 \mathbf{k}}{3.905} \right)$$

$$= 0.7682 F_B \mathbf{j} + 0.6402 F_B \mathbf{k}$$

$$\mathbf{F}_C = F_C \left( \frac{0.75 \mathbf{i} - 5 \mathbf{j} - 2.5 \mathbf{k}}{5.640} \right)$$

$$= 0.1330 F_C \mathbf{i} - 0.8865 F_C \mathbf{j} - 0.4432 F_C \mathbf{k}$$

$$\mathbf{F}_D = F_D \left( \frac{-1.25 \mathbf{i} - 5 \mathbf{j} - 2.5 \mathbf{k}}{5.728} \right)$$

$$= -0.2182 F_D \mathbf{i} - 0.8729 F_D \mathbf{j} - 0.4364 F_D \mathbf{k}$$

$$\mathbf{W} = -500(9.81) \mathbf{k} = -4905 \mathbf{k}$$

$$\Sigma \mathbf{F} = \mathbf{0}; \quad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$

$$\Sigma F_x = 0; \quad 0.1330 F_C - 0.2182 F_D = 0$$

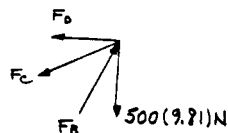
$$\Sigma F_y = 0; \quad 0.7682 F_B - 0.8865 F_C - 0.8729 F_D = 0$$

$$\Sigma F_z = 0; \quad 0.6402 F_B - 0.4432 F_C - 0.4364 F_D - 4905 = 0$$

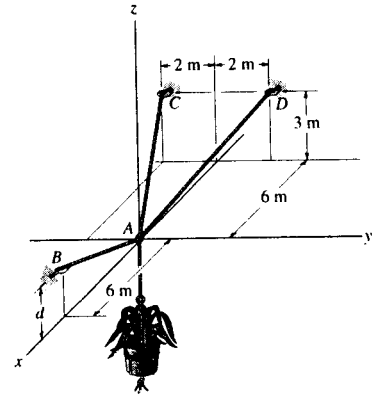
$$F_B = 19.2 \text{ kN} \quad \text{Ans}$$

$$F_C = 10.4 \text{ kN} \quad \text{Ans}$$

$$F_D = 6.32 \text{ kN} \quad \text{Ans}$$



\*3-56. The 50-kg pot is supported from A by the three cables. Determine the force acting in each cable for equilibrium. Take  $d = 2.5$  m.



**Cartesian Vector Notation :**

$$\mathbf{F}_{AB} = F_{AB} \left( \frac{6\mathbf{i} + 2.5\mathbf{k}}{\sqrt{6^2 + 2.5^2}} \right) = \frac{12}{13}F_{AB}\mathbf{i} + \frac{5}{13}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC} \left( \frac{-6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{(-6)^2 + (-2)^2 + 3^2}} \right) = -\frac{6}{7}F_{AC}\mathbf{i} - \frac{2}{7}F_{AC}\mathbf{j} + \frac{3}{7}F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD} \left( \frac{-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{\sqrt{(-6)^2 + 2^2 + 3^2}} \right) = -\frac{6}{7}F_{AD}\mathbf{i} + \frac{2}{7}F_{AD}\mathbf{j} + \frac{3}{7}F_{AD}\mathbf{k}$$

$$\mathbf{F} = \{-490.5\mathbf{k}\} \text{ N}$$

**Equations of Equilibrium :**

$$\Sigma \mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = \mathbf{0}$$

$$\left( \frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} - \frac{6}{7}F_{AD} \right)\mathbf{i} + \left( -\frac{2}{7}F_{AC} + \frac{2}{7}F_{AD} \right)\mathbf{j} + \left( \frac{5}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{3}{7}F_{AD} - 490.5 \right)\mathbf{k} = \mathbf{0}$$

Equating i, j and k components, we have

$$\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} - \frac{6}{7}F_{AD} = 0 \quad [1]$$

$$-\frac{2}{7}F_{AC} + \frac{2}{7}F_{AD} = 0 \quad [2]$$

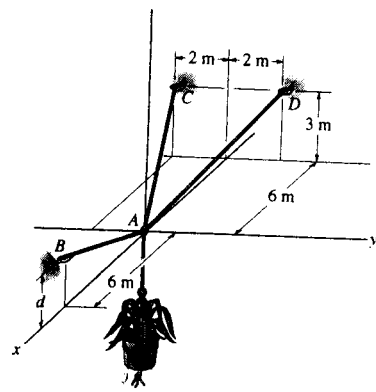
$$\frac{5}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{3}{7}F_{AD} - 490.5 = 0 \quad [3]$$

Solving Eqs. [1], [2] and [3] yields

$$F_{AC} = F_{AD} = 312 \text{ N}$$

$$F_{AB} = 580 \text{ N} \quad \text{Ans}$$

3-57. Determine the height  $d$  of cable  $AB$  so that the force in cables  $AD$  and  $AC$  is one-half as great as the force in cable  $AB$ . What is the force in each cable for this case? The flower pot has a mass of 50 kg.



**Cartesian Vector Notation :**

$$\mathbf{F}_{AB} = (F_{AB})_x \mathbf{i} + (F_{AB})_z \mathbf{k}$$

$$\mathbf{F}_{AC} = \frac{F_{AB}}{2} \left( \frac{-6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{(-6)^2 + (-2)^2 + 3^2}} \right) = -\frac{3}{7}F_{AB}\mathbf{i} - \frac{1}{7}F_{AB}\mathbf{j} + \frac{3}{14}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AD} = \frac{F_{AB}}{2} \left( \frac{-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{\sqrt{(-6)^2 + 2^2 + 3^2}} \right) = -\frac{3}{7}F_{AB}\mathbf{i} + \frac{1}{7}F_{AB}\mathbf{j} + \frac{3}{14}F_{AB}\mathbf{k}$$

$$\mathbf{F} = \{-490.5\mathbf{k}\} \text{ N}$$

**Equations of Equilibrium :**

$$\Sigma \mathbf{F} = 0; \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F} = 0$$

$$\begin{aligned} & \left( (F_{AB})_x - \frac{3}{7}F_{AB} - \frac{3}{7}F_{AB} \right) \mathbf{i} + \left( -\frac{1}{7}F_{AB} + \frac{1}{7}F_{AB} \right) \mathbf{j} \\ & + \left( (F_{AB})_z + \frac{3}{14}F_{AB} + \frac{3}{14}F_{AB} - 490.5 \right) \mathbf{k} = 0 \end{aligned}$$

Equating i, j and k components, we have

$$(F_{AB})_x - \frac{3}{7}F_{AB} - \frac{3}{7}F_{AB} = 0 \quad (F_{AB})_x = \frac{6}{7}F_{AB} \quad [1]$$

$$-\frac{1}{7}F_{AB} + \frac{1}{7}F_{AB} = 0 \quad (\text{Satisfied!})$$

$$(F_{AB})_z + \frac{3}{14}F_{AB} + \frac{3}{14}F_{AB} - 490.5 = 0 \quad (F_{AB})_z = 490.5 - \frac{3}{7}F_{AB} \quad [2]$$

However,  $F_{AB}^2 = (F_{AB})_x^2 + (F_{AB})_z^2$ , then substitute Eqs. [1] and [2] into this expression yields

$$F_{AB}^2 = \left( \frac{6}{7}F_{AB} \right)^2 + \left( 490.5 - \frac{3}{7}F_{AB} \right)^2$$

Solving for positive root, we have

$$F_{AB} = 519.79 \text{ N} = 520 \text{ N} \quad \text{Ans}$$

$$\text{Thus, } F_{AC} = F_{AD} = \frac{1}{2}(519.79) = 260 \text{ N} \quad \text{Ans}$$

Also,

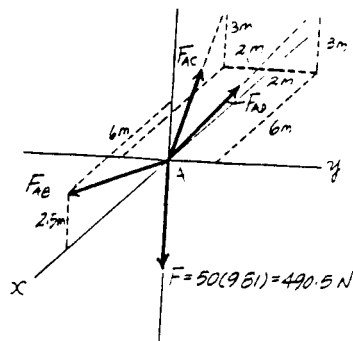
$$(F_{AB})_x = \frac{6}{7}(519.79) = 445.53 \text{ N}$$

$$(F_{AB})_z = 490.5 - \frac{3}{7}(519.79) = 267.73 \text{ N}$$

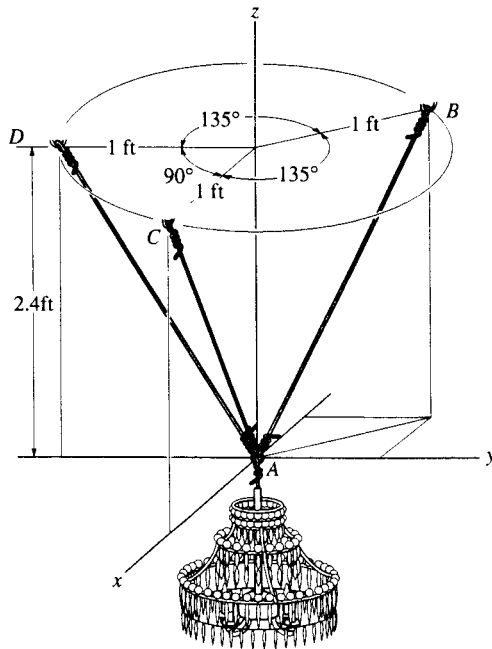
then,

$$\theta = \tan^{-1} \left[ \frac{(F_{AB})_z}{(F_{AB})_x} \right] = \tan^{-1} \left( \frac{267.73}{445.53} \right) = 31.00^\circ$$

$$d = 6 \tan \theta = 6 \tan 31.00^\circ = 3.61 \text{ m} \quad \text{Ans}$$



**3-58.** The 80-lb chandelier is supported by three wires as shown. Determine the force in each wire for equilibrium.



$$\Sigma F_x = 0; \quad \frac{1}{2.6}F_{AC} - \frac{1}{2.6}F_{AB} \cos 45^\circ = 0$$

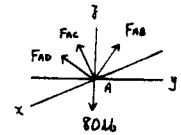
$$\Sigma F_y = 0; \quad -\frac{1}{2.6}F_{AD} + \frac{1}{2.6}F_{AB} \sin 45^\circ = 0$$

$$\Sigma F_z = 0; \quad \frac{2.4}{2.6}F_{AC} + \frac{2.4}{2.6}F_{AD} + \frac{2.4}{2.6}F_{AB} - 80 = 0$$

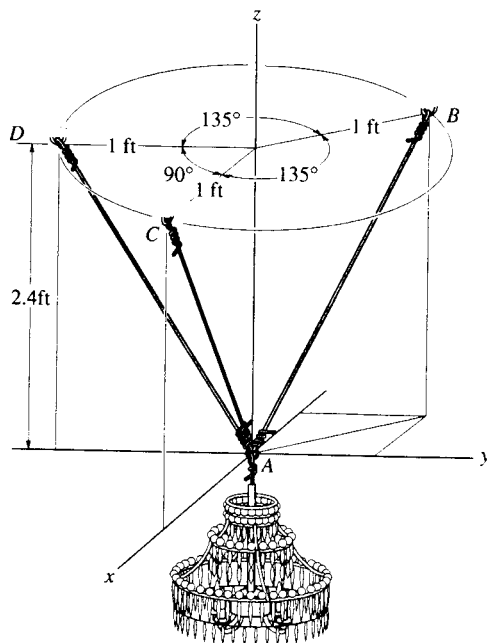
Solving,

$$F_{AB} = 35.9 \text{ lb} \quad \text{Ans}$$

$$F_{AC} = F_{AD} = 25.4 \text{ lb} \quad \text{Ans}$$



**3-59.** If each wire can sustain a maximum tension of 120 lb before it fails, determine the greatest weight of the chandelier the wires will support in the position shown.



$$\Sigma F_x = 0; \quad \frac{1}{2.6}F_{AC} - \frac{1}{2.6}F_{AB} \cos 45^\circ = 0 \quad (1)$$

$$\Sigma F_y = 0; \quad -\frac{1}{2.6}F_{AD} + \frac{1}{2.6}F_{AB} \sin 45^\circ = 0 \quad (2)$$

$$\Sigma F_z = 0; \quad \frac{2.4}{2.6}F_{AC} + \frac{2.4}{2.6}F_{AD} + \frac{2.4}{2.6}F_{AB} - W = 0 \quad (3)$$

Assume  $F_{AC} = 120$  lb. From Eq. (1)

$$\frac{1}{2.6}(120) - \frac{1}{2.6}F_{AB} \cos 45^\circ = 0$$

$$F_{AB} = 169.71 > 120 \text{ lb (N.G.)}$$

Assume  $F_{AB} = 120$  lb. From Eqs. (1) and (2)

$$\frac{1}{2.6}F_{AC} - \frac{1}{2.6}(120)(\cos 45^\circ) = 0$$

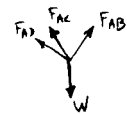
$$F_{AC} = 84.853 \text{ lb} < 120 \text{ lb (O.K.)}$$

$$-\frac{1}{2.6}F_{AD} + \frac{1}{2.6}(120) \sin 45^\circ = 0$$

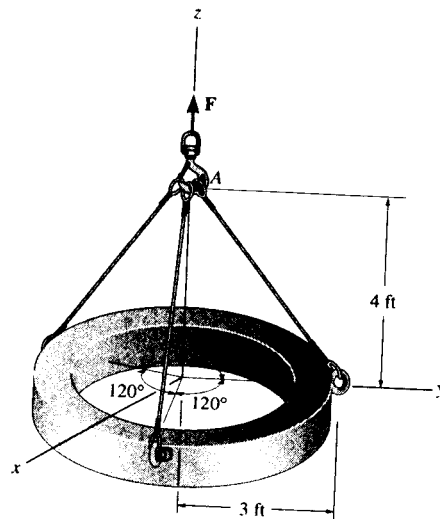
$$F_{AD} = 84.853 \text{ lb} < 120 \text{ lb (O.K.)}$$

Thus,

$$W = \frac{2.4}{2.6}(F_{AC} + F_{AD} + F_{AB}) = 267.42 = 267 \text{ lb} \quad \text{Ans}$$



\*3-60. Three cables are used to support a 900-lb ring. Determine the tension in each cable for equilibrium.



**Cartesian Vector Notation :**

$$F_{AB} = F_{AB} \left( \frac{3j - 4k}{\sqrt{3^2 + (-4)^2}} \right) = 0.6F_{AB}j - 0.8F_{AB}k$$

$$F_{AC} = F_{AC} \left( \frac{3\cos 30^\circ i - 3\sin 30^\circ j - 4k}{\sqrt{(3\cos 30^\circ)^2 + (-3\sin 30^\circ)^2 + (-4)^2}} \right)$$

$$= 0.5196F_{AC}i - 0.3F_{AC}j - 0.8F_{AC}k$$

$$F_{AD} = F_{AD} \left( \frac{-3\cos 30^\circ i - 3\sin 30^\circ j - 4k}{\sqrt{(-3\cos 30^\circ)^2 + (-3\sin 30^\circ)^2 + (-4)^2}} \right)$$

$$= -0.5196F_{AD}i - 0.3F_{AD}j - 0.8F_{AD}k$$

$$F = (900k) \text{ lb}$$

**Equations of Equilibrium :**

$$\Sigma F = 0; \quad F_{AB} + F_{AC} + F_{AD} + F = 0$$

$$(0.5196F_{AC} - 0.5196F_{AD})i + (0.6F_{AB} - 0.3F_{AC} - 0.3F_{AD})j$$

$$+ (-0.8F_{AB} - 0.8F_{AC} - 0.8F_{AD} + 900)k = 0$$

Equating i, j and k components, we have

$$0.5196F_{AC} - 0.5196F_{AD} = 0 \quad [1]$$

$$0.6F_{AB} - 0.3F_{AC} - 0.3F_{AD} = 0 \quad [2]$$

$$-0.8F_{AB} - 0.8F_{AC} - 0.8F_{AD} + 900 = 0 \quad [3]$$

Solving Eqs. [1], [2] and [3] yields

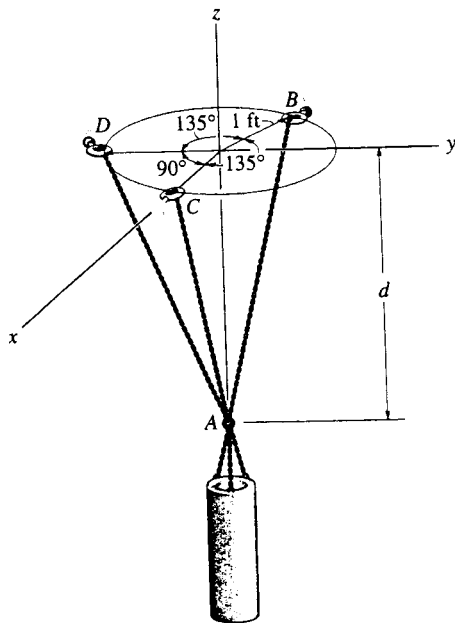
$$F_{AB} = F_{AC} = F_{AD} = 375 \text{ lb}$$

**Ans**

This problem also can be easily solved if one realizes that due to symmetry all cables are subjected to a same tensile force, that is  $F_{AB} = F_{AC} = F_{AD} = F$ . Summing forces along z axis yields

$$\Sigma F_z = 0; \quad 900 - 3F\left(\frac{4}{5}\right) = 0 \quad F = 375 \text{ lb}$$

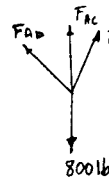
3-61. The 800-lb cylinder is supported by three chains as shown. Determine the force in each chain for equilibrium. Take  $d = 1$  ft.



$$\mathbf{F}_{AD} = F_{AD} \left( \frac{-1\mathbf{j} + 1\mathbf{k}}{\sqrt{(-1)^2 + 1^2}} \right) = -0.7071F_{AD}\mathbf{j} + 0.7071F_{AD}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC} \left( \frac{1\mathbf{i} + 1\mathbf{k}}{\sqrt{1^2 + 1^2}} \right) = 0.7071F_{AC}\mathbf{i} + 0.7071F_{AC}\mathbf{k}$$

$$\begin{aligned} \mathbf{F}_{AB} &= F_{AB} \left( \frac{-0.7071\mathbf{i} + 0.7071\mathbf{j} + 1\mathbf{k}}{\sqrt{(-0.7071)^2 + 0.7071^2 + 1^2}} \right) \\ &= -0.5F_{AB}\mathbf{i} + 0.5F_{AB}\mathbf{j} + 0.7071F_{AB}\mathbf{k} \\ \mathbf{F} &= (-800\mathbf{k}) \text{ lb} \end{aligned}$$



$$\begin{aligned} \Sigma \mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{AD} + \mathbf{F}_{AC} + \mathbf{F}_{AB} + \mathbf{F} = \mathbf{0} \\ (-0.7071F_{AD}\mathbf{j} + 0.7071F_{AD}\mathbf{k}) + (0.7071F_{AC}\mathbf{i} + 0.7071F_{AC}\mathbf{k}) \\ + (-0.5F_{AB}\mathbf{i} + 0.5F_{AB}\mathbf{j} + 0.7071F_{AB}\mathbf{k}) + (-800\mathbf{k}) = \mathbf{0} \\ (0.7071F_{AC} - 0.5F_{AB})\mathbf{i} + (-0.7071F_{AD} + 0.5F_{AB})\mathbf{j} \\ + (0.7071F_{AD} + 0.7071F_{AC} + 0.7071F_{AB} - 800)\mathbf{k} = \mathbf{0} \end{aligned}$$

$$\Sigma F_x = 0; \quad 0.7071F_{AC} - 0.5F_{AB} = 0$$

$$\Sigma F_y = 0; \quad -0.7071F_{AD} + 0.5F_{AB} = 0$$

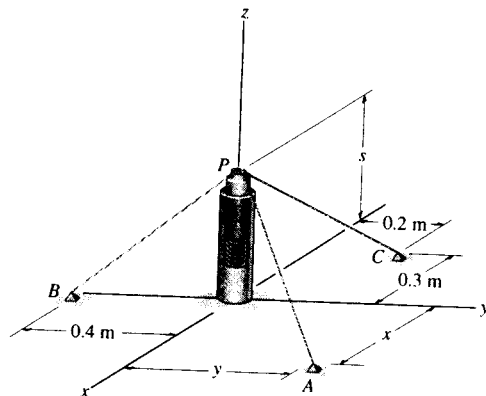
$$\Sigma F_z = 0; \quad 0.7071F_{AD} + 0.7071F_{AC} + 0.7071F_{AB} - 800 = 0$$

Solving Eqs. [1], [2] and [3] yields :

$$F_{AB} = 469 \text{ lb} \quad F_{AC} = F_{AD} = 331 \text{ lb} \quad \text{Ans}$$



3-62. A small peg  $P$  rests on a spring that is contained inside the smooth pipe. When the spring is compressed so that  $s = 0.15$  m, the spring exerts an upward force of 60 N on the peg. Determine the point of attachment  $A(x, y, 0)$  of cord  $PA$  so that the tension in cords  $PB$  and  $PC$  equals 30 N and 50 N, respectively.



**Cartesian Vector Notation :**

$$\mathbf{F}_{PA} = (F_{PA})_x \mathbf{i} + (F_{PA})_y \mathbf{j} + (F_{PA})_z \mathbf{k}$$

$$\mathbf{F}_{PB} = 30 \left( \frac{-0.4\mathbf{j} - 0.15\mathbf{k}}{\sqrt{(-0.4)^2 + (-0.15)^2}} \right) = \{-28.09\mathbf{j} - 10.53\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_{PC} = 50 \left( \frac{-0.3\mathbf{i} + 0.2\mathbf{j} - 0.15\mathbf{k}}{\sqrt{(-0.3)^2 + 0.2^2 + (-0.15)^2}} \right) = \{-38.4\mathbf{i} + 25.6\mathbf{j} - 19.2\mathbf{k}\} \text{ N}$$

$$\mathbf{F} = \{60\mathbf{k}\} \text{ N}$$

**Equations of Equilibrium :**

$$\Sigma \mathbf{F} = \mathbf{0}; \quad \mathbf{F}_{PA} + \mathbf{F}_{PB} + \mathbf{F}_{PC} + \mathbf{F} = \mathbf{0}$$

$$\begin{aligned} [(F_{PA})_x - 38.4]\mathbf{i} + [(F_{PA})_y - 28.09 + 25.6]\mathbf{j} \\ + [(F_{PA})_z - 10.53 - 19.2 + 60]\mathbf{k} = \mathbf{0} \end{aligned}$$

Equating  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  components, we have

$$\begin{aligned} (F_{PA})_x - 38.4 &= 0 & (F_{PA})_x &= 38.4 \text{ N} \\ (F_{PA})_y - 28.09 + 25.6 &= 0 & (F_{PA})_y &= 2.48 \text{ N} \\ (F_{PA})_z - 10.53 - 19.2 + 60 &= 0 & (F_{PA})_z &= -30.26 \text{ N} \end{aligned}$$

The magnitude of  $\mathbf{F}_{PA}$  is

$$\begin{aligned} F_{PA} &= \sqrt{(F_{PA})_x^2 + (F_{PA})_y^2 + (F_{PA})_z^2} \\ &= \sqrt{38.41^2 + 2.48^2 + (-30.26)^2} = 48.96 \text{ N} \end{aligned}$$

The coordinate direction angles are

$$\begin{aligned} \alpha &= \cos^{-1} \left[ \frac{(F_{PA})_x}{F_{PA}} \right] = \cos^{-1} \left( \frac{38.41}{48.96} \right) = 38.32^\circ \\ \beta &= \cos^{-1} \left[ \frac{(F_{PA})_y}{F_{PA}} \right] = \cos^{-1} \left( \frac{2.48}{48.96} \right) = 87.09^\circ \\ \gamma &= \cos^{-1} \left[ \frac{(F_{PA})_z}{F_{PA}} \right] = \cos^{-1} \left( \frac{-30.26}{48.96} \right) = 128.17^\circ \end{aligned}$$

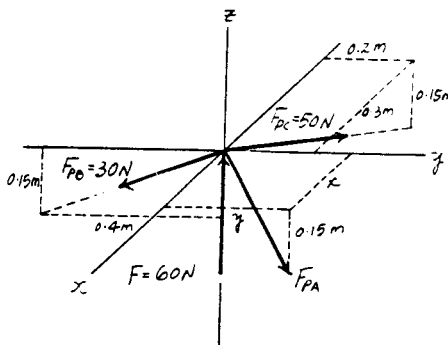
The wire  $PA$  has a length of

$$PA = \frac{(PA)_z}{\cos \gamma} = \frac{-0.15}{\cos 128.17^\circ} = 0.2427 \text{ m}$$

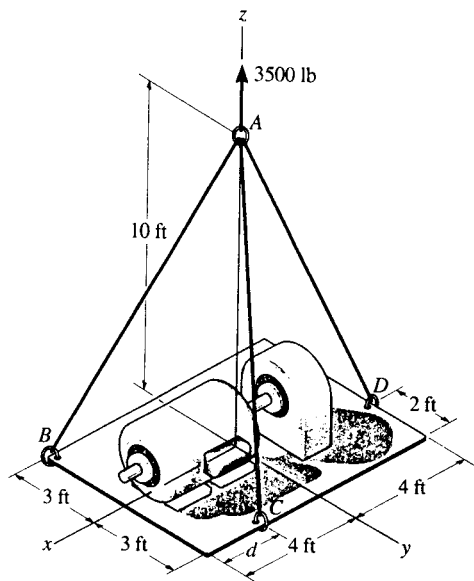
Thus,

$$\begin{aligned} x &= PA \cos \alpha = 0.2427 \cos 38.32^\circ = 0.190 \text{ m} \\ y &= PA \cos \beta = 0.2427 \cos 87.09^\circ = 0.0123 \text{ m} \end{aligned}$$

Ans  
Ans

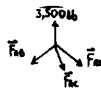


3-63. Determine the force in each cable support the 3500-lb platform. Set  $d = 4$  ft.



$$\mathbf{F}_{AD} = F_{AD} \left\{ \frac{-4}{\sqrt{117}} \mathbf{i} + \frac{1}{\sqrt{117}} \mathbf{j} - \frac{10}{\sqrt{117}} \mathbf{k} \right\}$$

$$= \{-0.3698F_{AD} \mathbf{i} + 0.09245F_{AD} \mathbf{j} - 0.9245F_{AD} \mathbf{k}\} \text{ lb}$$



$$\mathbf{F}_{AC} = F_{AC} \left\{ \frac{3}{\sqrt{109}} \mathbf{j} - \frac{10}{\sqrt{109}} \mathbf{k} \right\}$$

$$= \{0.2873F_{AC} \mathbf{j} - 0.9578F_{AC} \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_{AB} = F_{AB} \left\{ \frac{4}{\sqrt{125}} \mathbf{i} - \frac{3}{\sqrt{125}} \mathbf{j} - \frac{10}{\sqrt{125}} \mathbf{k} \right\}$$

$$= \{0.3578F_{AB} \mathbf{i} - 0.2683F_{AB} \mathbf{j} - 0.8944F_{AB} \mathbf{k}\} \text{ lb}$$

$$\Sigma F_x = 0; \quad -0.3698F_{AD} + 0.3578F_{AB} = 0$$

$$\Sigma F_y = 0; \quad 0.09245F_{AD} + 0.2873F_{AC} - 0.2683F_{AB} = 0$$

$$\Sigma F_z = 0; \quad -0.9245F_{AD} - 0.9578F_{AC} - 0.8944F_{AB} + 3500 = 0$$

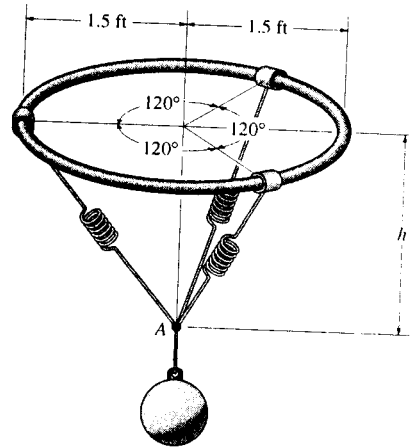
Solving,

$$F_{AD} = 1.42 \text{ kip} \quad \text{Ans}$$

$$F_{AC} = 0.914 \text{ kip} \quad \text{Ans}$$

$$F_{AB} = 1.47 \text{ kip} \quad \text{Ans}$$

3-64. The 80-lb ball is suspended from the horizontal ring using three springs each having an unstretched length of 1.5 ft and stiffness of 50 lb/ft. Determine the vertical distance  $h$  from the ring to point A for equilibrium.



**Equation of Equilibrium:** This problem also can be easily solved if one realizes that due to symmetry all springs are subjected to a same tensile force of  $F_{sp}$ . Summing forces along z axis yields

$$\Sigma F_z = 0; \quad 3F_{sp} \cos \gamma - 80 = 0 \quad [1]$$

**Spring Force:** Applying Eq. 3-2, we have

$$F_{sp} = ks = k(l - l_0) = 50 \left( \frac{1.5}{\sin \gamma} - 1.5 \right) = \frac{75}{\sin \gamma} - 75 \quad [2]$$

Substituting Eq. [2] into [1] yields

$$3 \left( \frac{75}{\sin \gamma} - 75 \right) \cos \gamma - 80 = 0$$

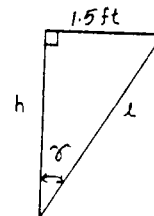
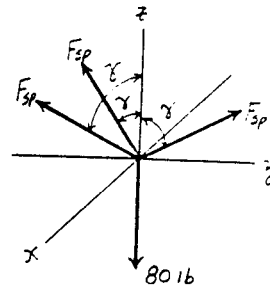
$$\tan \gamma = \frac{45}{16} (1 - \sin \gamma)$$

Solving by trial and error, we have

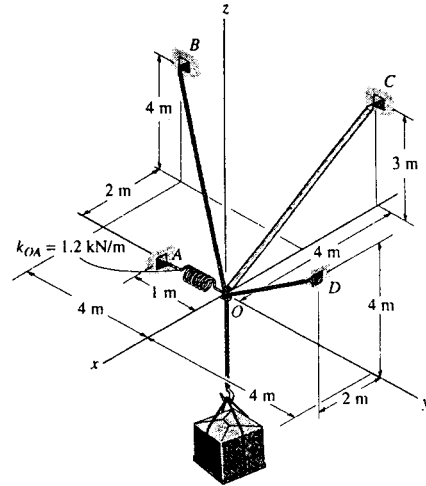
$$\gamma = 42.4425^\circ$$

**Geometry:**

$$h = \frac{1.5}{\tan \gamma} = \frac{1.5}{\tan 42.4425^\circ} = 1.64 \text{ ft} \quad \text{Ans}$$



3-65. Determine the tension developed in cables  $OD$  and  $OB$  and the strut  $OC$ , required to support the 50-kg crate. The spring  $OA$  has an unstretched length of 0.8 m and a stiffness  $k_{OA} = 1.2 \text{ kN/m}$ . The force in the strut acts along the axis of the strut.



**Free Body Diagram :** The spring stretches  $s = l - l_0 = 1 - 0.8 = 0.2 \text{ m}$ . Hence, the spring force is  $F_{sp} = ks = 1.2(0.2) = 0.24 \text{ kN} = 240 \text{ N}$ .

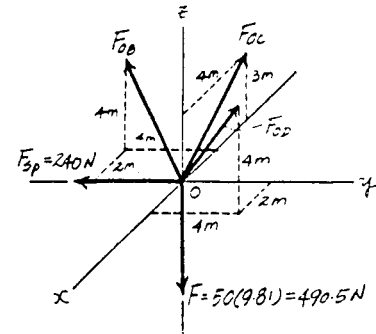
**Cartesian Vector Notation :**

$$\mathbf{F}_{OB} = F_{OB} \left( \frac{-2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}}{\sqrt{(-2)^2 + (-4)^2 + 4^2}} \right) = -\frac{1}{3}F_{OB}\mathbf{i} - \frac{2}{3}F_{OB}\mathbf{j} + \frac{2}{3}F_{OB}\mathbf{k}$$

$$\mathbf{F}_{OC} = F_{OC} \left( \frac{-4\mathbf{i} + 3\mathbf{k}}{\sqrt{(-4)^2 + 3^2}} \right) = -\frac{4}{5}F_{OC}\mathbf{i} + \frac{3}{5}F_{OC}\mathbf{k}$$

$$\mathbf{F}_{OD} = F_{OD} \left( \frac{2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}}{\sqrt{2^2 + 4^2 + 4^2}} \right) = \frac{1}{3}F_{OD}\mathbf{i} + \frac{2}{3}F_{OD}\mathbf{j} + \frac{2}{3}F_{OD}\mathbf{k}$$

$$\mathbf{F}_{sp} = \{-240\mathbf{j}\} \text{ N} \quad \mathbf{F} = \{-490.5\mathbf{k}\} \text{ N}$$



**Equations of Equilibrium :**

$$\Sigma \mathbf{F} = 0; \quad \mathbf{F}_{OB} + \mathbf{F}_{OC} + \mathbf{F}_{OD} + \mathbf{F}_{sp} + \mathbf{F} = 0$$

$$\left( -\frac{1}{3}F_{OB} - \frac{4}{5}F_{OC} + \frac{1}{3}F_{OD} \right) \mathbf{i} + \left( -\frac{2}{3}F_{OB} + \frac{2}{3}F_{OD} - 240 \right) \mathbf{j} + \left( \frac{2}{3}F_{OB} + \frac{3}{5}F_{OC} + \frac{2}{3}F_{OD} - 490.5 \right) \mathbf{k} = 0$$

Equating i, j and k components, we have

$$-\frac{1}{3}F_{OB} - \frac{4}{5}F_{OC} + \frac{1}{3}F_{OD} = 0 \quad [1]$$

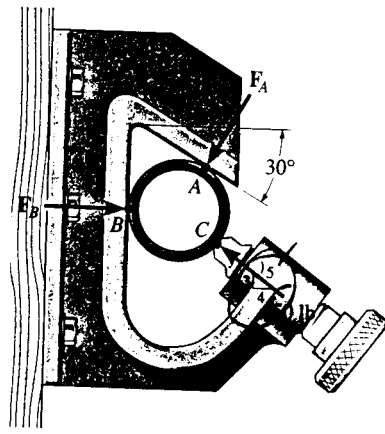
$$-\frac{2}{3}F_{OB} + \frac{2}{3}F_{OD} - 240 = 0 \quad [2]$$

$$\frac{2}{3}F_{OB} + \frac{3}{5}F_{OC} + \frac{2}{3}F_{OD} - 490.5 = 0 \quad [3]$$

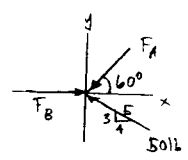
Solving Eqs. [1], [2] and [3] yields

$$F_{OB} = 120 \text{ N} \quad F_{OC} = 150 \text{ N} \quad F_{OD} = 480 \text{ N} \quad \text{Ans}$$

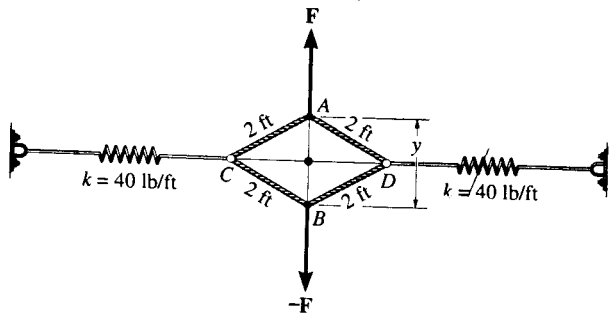
3-66. The pipe is held in place by the vice. If the bolt exerts a force of 50 lb on the pipe in the direction shown, determine the forces  $F_A$  and  $F_B$  that the smooth contacts at A and B exert on the pipe.



$$\begin{aligned} \rightarrow \Sigma F_x = 0; & \quad F_B - F_A \cos 60^\circ - 50\left(\frac{4}{5}\right) = 0 \\ + \uparrow \Sigma F_y = 0; & \quad -F_A \sin 60^\circ + 50\left(\frac{3}{5}\right) = 0 \\ & \quad F_A = 34.6 \text{ lb} \quad \text{Ans} \\ & \quad F_B = 57.3 \text{ lb} \quad \text{Ans} \end{aligned}$$



3-67. When  $y$  is zero, the springs sustain a force of 60 lb. Determine the magnitude of the applied vertical forces  $F$  and  $-F$  required to pull point A away from point B a distance of  $y = 2$  ft. The ends of cords CAD and CBD are attached to rings at C and D.



Initial spring stretch:

$$s_1 = \frac{60}{40} = 1.5 \text{ ft}$$

$$+ \uparrow \Sigma F_y = 0; \quad F - 2\left(\frac{1}{2}T\right) = 0; \quad F = T$$

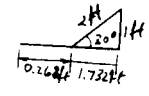
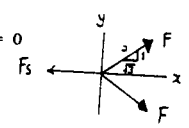
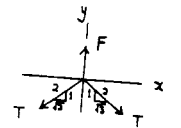
$$\rightarrow \Sigma F_x = 0; \quad -F + 2\left(\frac{\sqrt{3}}{2}\right)F = 0$$

$$F_x = 1.732F$$

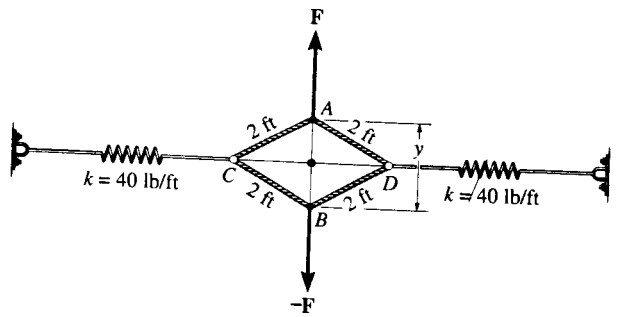
Final stretch is  $1.5 + 0.268 = 1.768 \text{ ft}$

$$40(1.768) = 1.732F$$

$$F = 40.8 \text{ lb} \quad \text{Ans}$$



\*3-68. When  $y$  is zero, the springs are each stretched 1.5 ft. Determine the distance  $y$  if a force of  $F = 60$  lb is applied to points A and B as shown. The ends of cords CAD and CBD are attached to rings at C and D.



$$+ \uparrow \Sigma F_y = 0; \quad 2T \sin \theta = 60$$

$$T \sin \theta = 30$$

$$\rightarrow \Sigma F_x = 0; \quad 2T \cos \theta = F$$

$$F \tan \theta = 60 \quad (1)$$

$$F = kx$$

$$F = 40(1.5 + 2 - 2 \cos \theta)$$

Substitute  $F$  in Eq. 1

$$40(1.5 + 2 - 2 \cos \theta) \tan \theta = 60$$

$$(3.5 - 2 \cos \theta) \tan \theta = 1.5$$

$$3.5 \tan \theta - 2 \sin \theta = 1.5$$

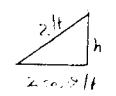
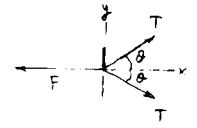
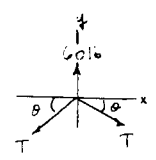
$$1.75 \tan \theta - \sin \theta = 0.75$$

By trial and error:

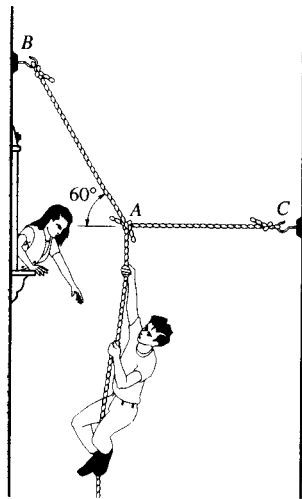
$$\theta = 37.9^\circ$$

$$\frac{y}{2} = 2 \sin 37.9^\circ$$

$$y = 2.46 \text{ ft} \quad \text{Ans}$$



**3-69.** Romeo tries to reach Juliet by climbing with constant velocity up a rope which is knotted at point A. Any of the three segments of the rope can sustain a maximum force of 2 kN before it breaks. Determine if Romeo, who has a mass of 65 kg, can climb the rope, and if so, can he along with his Juliet, who has a mass of 60 kg, climb down with constant velocity?



$$+\uparrow \Sigma F_y = 0; \quad T_{AB} \sin 60^\circ - 65(9.81) = 0$$

$$T_{AB} = 736.29 \text{ N} < 2000 \text{ N}$$

$$\rightarrow \Sigma F_x = 0; \quad T_{AC} - 736.29 \cos 60^\circ = 0$$

$$T_{AC} = 368.15 \text{ N} < 2000 \text{ N}$$

Yes, Romeo can climb up the rope. **Ans**

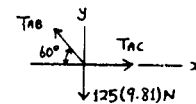
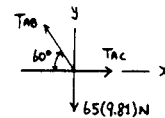
$$+\uparrow \Sigma F_y = 0; \quad T_{AB} \sin 60^\circ - 125(9.81) = 0$$

$$T_{AB} = 1415.95 \text{ N} < 2000 \text{ N}$$

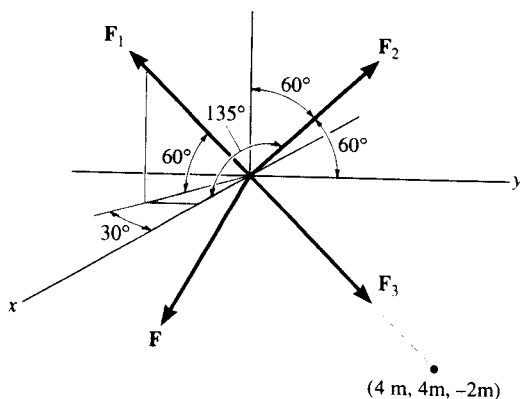
$$\rightarrow \Sigma F_x = 0; \quad T_{AC} - 1415.95 \cos 60^\circ = 0$$

$$T_{AC} = 708 \text{ N} < 2000 \text{ N}$$

Yes, Romeo and Juliet can climb down. **Ans**



**3-70.** Determine the magnitudes of forces  $\mathbf{F}_1$ ,  $\mathbf{F}_2$ , and  $\mathbf{F}_3$  necessary to hold the force  $\mathbf{F} = \{-9\mathbf{i} - 8\mathbf{j} - 5\mathbf{k}\}$  kN in equilibrium.



$$\Sigma F_x = 0; \quad F_1 \cos 60^\circ \cos 30^\circ + F_2 \cos 135^\circ + \frac{4}{6}F_3 - 9 = 0$$

$$\Sigma F_y = 0; \quad -F_1 \cos 60^\circ \sin 30^\circ + F_2 \cos 60^\circ + \frac{4}{6}F_3 - 8 = 0$$

$$\Sigma F_z = 0; \quad F_1 \sin 60^\circ + F_2 \cos 60^\circ - \frac{2}{6}F_3 - 5 = 0$$

$$0.433F_1 - 0.707F_2 + 0.667F_3 = 9$$

$$-0.250F_1 + 0.500F_2 + 0.667F_3 = 8$$

$$0.866F_1 + 0.500F_2 - 0.333F_3 = 5$$

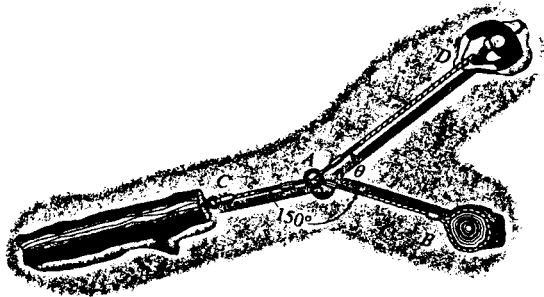
Solving,

$$F_1 = 8.26 \text{ kN} \quad \mathbf{Ans}$$

$$F_2 = 3.84 \text{ kN} \quad \mathbf{Ans}$$

$$F_3 = 12.2 \text{ kN} \quad \mathbf{Ans}$$

3-71. The man attempts to pull the log at  $C$  by using the three ropes. Determine the direction  $\theta$  in which he should pull on his rope with a force of 80 lb, so that he exerts a maximum force on the log. What is the force on the log for this case? Also, determine the direction in which he should pull in order to maximize the force in the rope attached to  $B$ . What is this maximum force?



$$\rightarrow \Sigma F_x = 0; \quad F_{AB} + 80 \cos \theta - F_{AC} \sin 60^\circ = 0$$

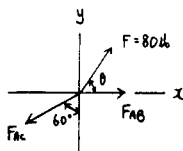
$$+ \uparrow \Sigma F_y = 0; \quad 80 \sin \theta - F_{AC} \cos 60^\circ = 0$$

$$F_{AC} = 160 \sin \theta$$

$$\frac{dF_{AC}}{d\theta} = 160 \cos \theta = 0$$

$$\theta = 90^\circ \quad \text{Ans}$$

$$F_{AC} = 160 \text{ lb} \quad \text{Ans}$$



$$F_{AC} \sin 60^\circ = F_{AB} + 80 \cos \theta$$

$$80 \sin \theta \sin 60^\circ = (F_{AB} + 80 \cos \theta) \cos 60^\circ$$

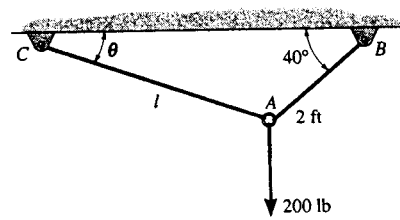
$$F_{AB} = 138.6 \sin \theta - 80 \cos \theta$$

$$\frac{dF_{AB}}{d\theta} = 138.6 \cos \theta + 80 \sin \theta = 0$$

$$\theta = \tan^{-1} \left[ \frac{138.6}{-80} \right] = 120^\circ \quad \text{Ans}$$

$$F_{AB} = 138.6 \sin 120^\circ - 80 \cos 120^\circ = 160 \text{ lb} \quad \text{Ans}$$

■3-72. The ring of negligible size is subjected to a vertical force of 200 lb. Determine the required length  $l$  of cord  $AC$  such that the tension acting in  $AC$  is 160 lb. Also, what is the force acting in cord  $AB$ ? *Hint:* Use the equilibrium condition to determine the required angle  $\theta$  for attachment, then determine  $l$  using trigonometry applied to  $\triangle ABC$ .



**Equations of Equilibrium :**

$$\rightarrow \Sigma F_x = 0; \quad F_{AB} \cos 40^\circ - 160 \cos \theta = 0 \quad [1]$$

$$+ \uparrow \Sigma F_y = 0; \quad F_{AB} \sin 40^\circ + 160 \sin \theta - 200 = 0 \quad [2]$$

Solving Eqs. [1] and [2] yields

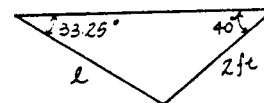
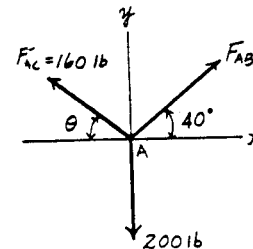
$$\theta = 33.25^\circ \quad \text{Ans}$$

$$F_{AB} = 175 \text{ lb}$$

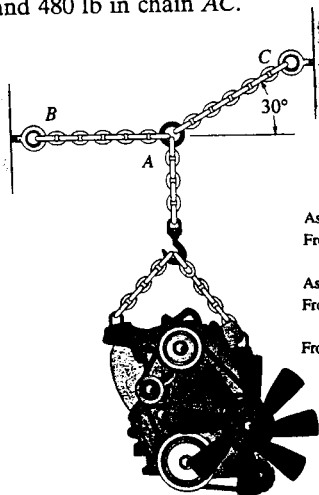
**Geometry :** Applying law of sines, we have

$$\frac{l}{\sin 40^\circ} = \frac{2}{\sin 33.25^\circ}$$

$$l = 2.34 \text{ ft} \quad \text{Ans}$$



3-73. Determine the maximum weight of the engine that can be supported without exceeding a tension of 450 lb in chain  $AB$  and 480 lb in chain  $AC$ .



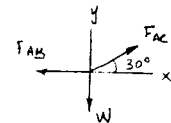
$$\rightarrow \Sigma F_x = 0; \quad F_{AC} \cos 30^\circ - F_{AB} = 0 \quad [1]$$

$$+ \uparrow \Sigma F_y = 0; \quad F_{AC} \sin 30^\circ - W = 0 \quad [2]$$

Assuming cable  $AB$  reaches the maximum tension  $F_{AB} = 450 \text{ lb}$ .  
From Eq. [1]  $F_{AC} \cos 30^\circ - 450 = 0$   $F_{AC} = 519.6 \text{ lb} > 480 \text{ lb}$  (N.G!)

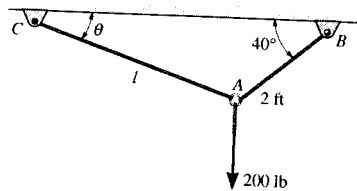
Assuming cable  $AC$  reaches the maximum tension  $F_{AC} = 480 \text{ lb}$ .  
From Eq. [1]  $480 \cos 30^\circ - F_{AB} = 0$   $F_{AB} = 415.7 \text{ lb} < 450 \text{ lb}$  (O.K!)

$$\text{From Eq. [2]} \quad 480 \sin 30^\circ - W = 0 \quad W = 240 \text{ lb} \quad \text{Ans}$$





■3-72. The ring of negligible size is subjected to a vertical force of 200 lb. Determine the required length  $l$  of cord  $AC$  such that the tension acting in  $AC$  is 160 lb. Also, what is the force acting in cord  $AB$ ? *Hint:* Use the equilibrium condition to determine the required angle  $\theta$  for attachment, then determine  $l$  using trigonometry applied to  $\triangle ABC$ .



**Equations of Equilibrium:**

$$\rightarrow \Sigma F_x = 0; \quad F_{AB} \cos 40^\circ - 160 \cos \theta = 0 \quad [1]$$

$$+ \uparrow \Sigma F_y = 0; \quad F_{AB} \sin 40^\circ + 160 \sin \theta - 200 = 0 \quad [2]$$

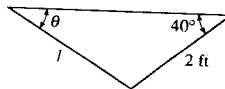
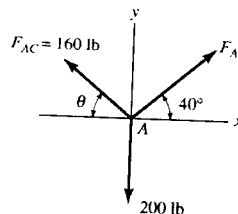
Solving Eqs. [1] and [2] yields

$$\begin{aligned} \theta = 33.25^\circ & \quad \text{or} \quad \theta = 66.75^\circ \\ F_{AB} = 175 \text{ lb} & \quad \text{or} \quad F_{AB} = 82.4 \text{ lb} \end{aligned} \quad \text{Ans}$$

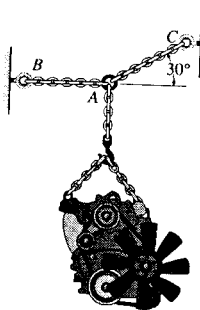
**Geometry:** Applying law of sines, we have

$$\frac{l}{\sin 40^\circ} = \frac{2}{\sin 33.25^\circ} \quad \text{or} \quad \frac{l}{\sin 40^\circ} = \frac{2}{\sin 66.75^\circ}$$

$$l = 2.34 \text{ ft} \quad \text{or} \quad l = 1.40 \text{ ft} \quad \text{Ans}$$



3-73. Determine the maximum weight of the engine that can be supported without exceeding a tension of 450 lb in chain  $AB$  and 480 lb in chain  $AC$ .



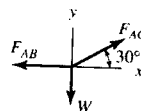
$$\rightarrow \Sigma F_x = 0; \quad F_{AC} \cos 30^\circ - F_{AB} = 0 \quad [1]$$

$$+ \uparrow \Sigma F_y = 0; \quad F_{AC} \sin 30^\circ - W = 0 \quad [2]$$

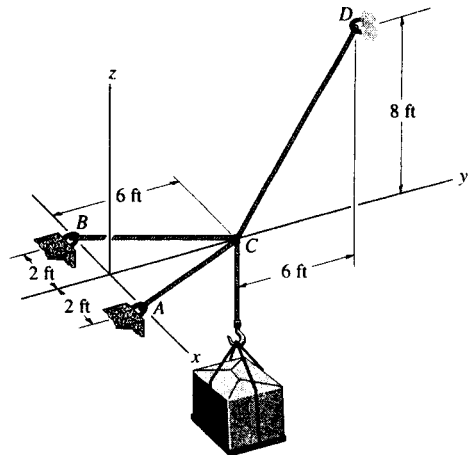
Assuming cable  $AB$  reaches the maximum tension  $F_{AB} = 450$  lb.  
From Eq. [1]  $F_{AC} \cos 30^\circ - 450 = 0$   $F_{AC} = 519.6 \text{ lb} > 480 \text{ lb}$  (N.G!)

Assuming cable  $AC$  reaches the maximum tension  $F_{AC} = 480$  lb.  
From Eq. [1]  $480 \cos 30^\circ - F_{AB} = 0$   $F_{AB} = 415.7 \text{ lb} < 450 \text{ lb}$  (O.K!)

$$\text{From Eq. [2]} \quad 480 \sin 30^\circ - W = 0 \quad W = 240 \text{ lb} \quad \text{Ans}$$



\*3-74. Determine the force in each cable needed to support the 500-lb load.



**Equation of Equilibrium :**

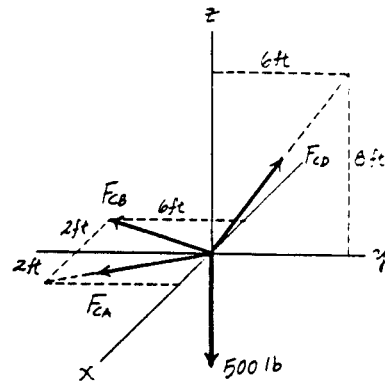
$$\Sigma F_z = 0; \quad F_{CD} \left( \frac{4}{5} \right) - 500 = 0 \quad F_{CD} = 625 \text{ lb} \quad \text{Ans}$$

Using the results  $F_{CD} = 625$  lb and then summing forces along  $x$  and  $y$  axes we have

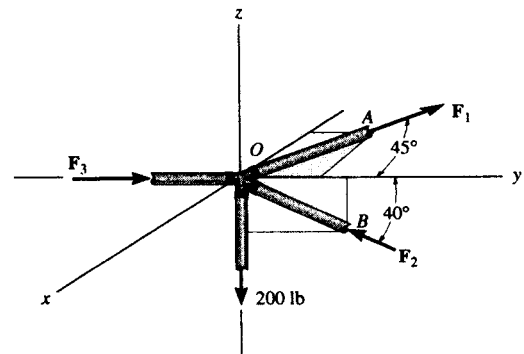
$$\Sigma F_x = 0; \quad F_{CA} \left( \frac{2}{\sqrt{40}} \right) - F_{CB} \left( \frac{2}{\sqrt{40}} \right) = 0 \quad F_{CA} = F_{CB} = F$$

$$\Sigma F_y = 0; \quad 2F \left( \frac{6}{\sqrt{40}} \right) - 625 \left( \frac{3}{5} \right) = 0$$

$$F_{CA} = F_{CB} = F = 198 \text{ lb} \quad \text{Ans}$$



3-75. The joint of a space frame is subjected to four member forces. Member  $OA$  lies in the  $x - y$  plane and member  $OB$  lies in the  $y - z$  plane. Determine the forces acting in each of the members required for equilibrium of the joint.



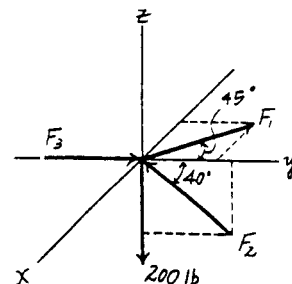
**Equation of Equilibrium :**

$$\Sigma F_x = 0; \quad F_1 \sin 45^\circ = 0 \quad F_1 = 0 \quad \text{Ans}$$

$$\Sigma F_z = 0; \quad F_2 \sin 40^\circ - 200 = 0 \quad F_2 = 311.14 \text{ lb} = 311 \text{ lb} \quad \text{Ans}$$

Using the results  $F_1 = 0$  and  $F_2 = 311.14$  lb and then summing forces along the  $y$  axis, we have

$$\Sigma F_y = 0; \quad F_3 - 311.14 \cos 40^\circ = 0 \quad F_3 = 238 \text{ lb} \quad \text{Ans}$$



4-1. If  $A$ ,  $B$ , and  $D$  are given vectors, prove the distributive law for the vector cross product, i.e.,  $A \times (B + D) = (A \times B) + (A \times D)$ .

Consider the three vectors: with  $A$  vertical.

Note  $abd$  is perpendicular to  $A$ .

$$od = |A \times (B + D)| = |A||B + D| \sin \theta_3$$

$$ob = |A \times B| = |A||B| \sin \theta_1$$

$$bd = |A \times D| = |A||D| \sin \theta_2$$



Also, these three cross products all lie in the plane  $abd$  since they are all perpendicular to  $A$ . As noted the magnitude of each cross product is proportional to the length of each side of the triangle.

The three vector cross-products also form a closed triangle  $o'b'd'$  which is similar to triangle  $abd$ . Thus from the figure,

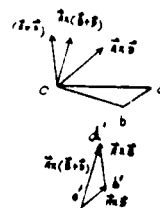
$$A \times (B + D) = A \times B + A \times D \quad (\text{QED})$$

Note also,

$$A = A_x i + A_y j + A_z k$$

$$B = B_x i + B_y j + B_z k$$

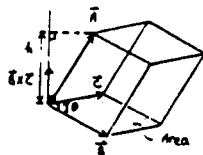
$$D = D_x i + D_y j + D_z k$$



$$\begin{aligned} A \times (B + D) &= \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x + D_x & B_y + D_y & B_z + D_z \end{vmatrix} \\ &= [A_y(B_z + D_z) - A_z(B_y + D_y)]i \\ &\quad - [A_x(B_z + D_z) - A_z(B_x + D_x)]j \\ &\quad + [A_x(B_y + D_y) - A_y(B_x + D_x)]k \\ &= [(A_y B_z - A_z B_y) + (A_y D_z - A_z D_y)]i \\ &\quad + [(A_x B_z - A_z B_x) + (A_x D_z - A_z D_x)]j \\ &\quad + [(A_x B_y - A_y B_x) + (A_x D_y - A_y D_x)]k \end{aligned}$$

$$\begin{aligned} &= \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{vmatrix} \\ &= (A \times B) + (A \times D) \quad (\text{QED}) \end{aligned}$$

4-2. Prove the triple scalar product identity  $A \cdot B \times C = A \times B \cdot C$ .



As shown in the figure

$$\text{Area} = B(C \sin \theta) = |B \times C|$$

Thus,

$$\text{Volume of parallelepiped is } |B \times C| |A|$$

But,

$$|A| = |A \cdot \frac{B \times C}{|B \times C|}| = \left| A \cdot \left( \frac{B \times C}{|B \times C|} \right) \right|$$

Thus,

$$\text{Volume} = |A \cdot B \times C|$$

Since  $|A \times B \cdot C|$  represents this same volume then

$$A \cdot B \times C = A \times B \cdot C \quad (\text{QED})$$

Also,

$$LHS = A \cdot B \times C$$

$$= (A_x i + A_y j + A_z k) \cdot \begin{vmatrix} i & j & k \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$\begin{aligned} &= A_x(B_y C_z - B_z C_y) - A_y(B_x C_z - B_z C_x) + A_z(B_x C_y - B_y C_x) \\ &= A_x B_y C_z - A_x B_z C_y - A_y B_x C_z + A_y B_z C_x + A_z B_x C_y - A_z B_y C_x \end{aligned}$$

$$RHS = A \times B \cdot C$$

$$= \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \cdot (C_x i + C_y j + C_z k)$$

$$\begin{aligned} &= C_x(A_y B_z - A_z B_y) - C_y(A_x B_z - A_z B_x) + C_z(A_x B_y - A_y B_x) \\ &= A_x B_y C_z - A_x B_z C_y - A_y B_x C_z + A_y B_z C_x + A_z B_x C_y - A_z B_y C_x \end{aligned}$$

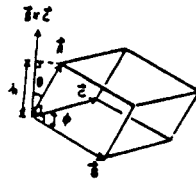
Thus,  $LHS = RHS$

$$A \cdot B \times C = A \times B \cdot C \quad (\text{QED})$$

4-3. Given the three nonzero vectors **A**, **B**, and **C**, show that if  $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$ , the three vectors *must* lie in the same plane.

Consider,

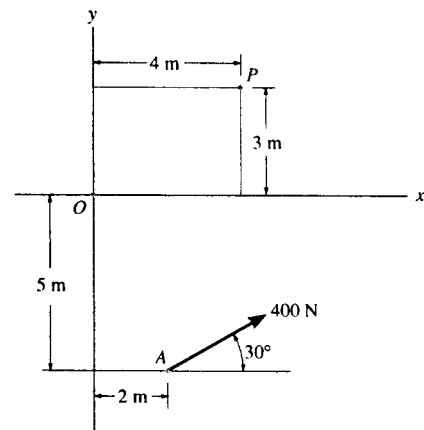
$$\begin{aligned} |\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})| &= |\mathbf{A}| |\mathbf{B} \times \mathbf{C}| \cos \theta \\ &= (|\mathbf{A}| \cos \theta) |\mathbf{B} \times \mathbf{C}| \\ &= |h| |\mathbf{B} \times \mathbf{C}| \\ &= BC |h| \sin \phi \\ &= \text{volume of parallelepiped.} \end{aligned}$$



If  $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$ , then the volume equals zero, so that **A**, **B**, and **C** are coplanar.

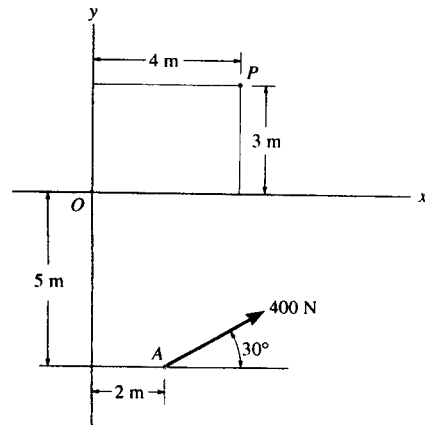
\*4-4. Determine the magnitude and directional sense of the moment of the force at **A** about point **O**.

$$\begin{aligned} \curvearrowright M_O &= 400 \cos 30^\circ (5) + 400 \sin 30^\circ (2) \\ &= 2132 \text{ N} \cdot \text{m} \\ &= 2.13 \text{ kN} \cdot \text{m} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$



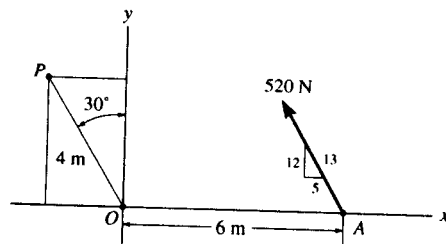
4-5. Determine the magnitude and directional sense of the moment of the force at **A** about point **P**.

$$\begin{aligned} \curvearrowright M_P &= 400 \cos 30^\circ (8) - 400 \sin 30^\circ (2) \\ &= 2371 \text{ N} \cdot \text{m} \\ &= 2.37 \text{ kN} \cdot \text{m} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$



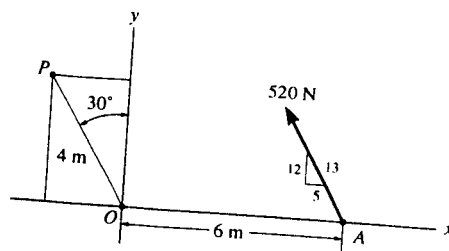
4-6. Determine the magnitude and directional sense of the moment of the force at  $A$  about point  $O$ .

$$\begin{aligned} \zeta + M_O &= 520 \left( \frac{12}{13} \right) (6) \\ &= 2880 \text{ N}\cdot\text{m} = 2.88 \text{ kN}\cdot\text{m} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$

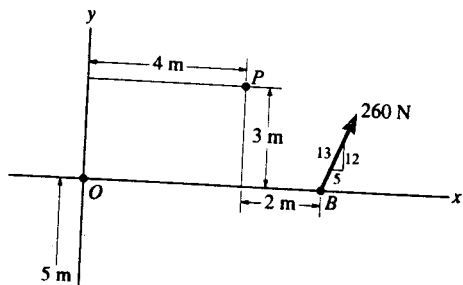


4-7. Determine the magnitude and directional sense of the moment of the force at  $A$  about point  $P$ .

$$\begin{aligned} \zeta + M_P &= 520 \left( \frac{12}{13} \right) (6 + 4 \sin 30^\circ) - 520 \left( \frac{5}{13} \right) (4 \cos 30^\circ) \\ &= 3147 \text{ N}\cdot\text{m} \\ &= 3.15 \text{ kN}\cdot\text{m} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$



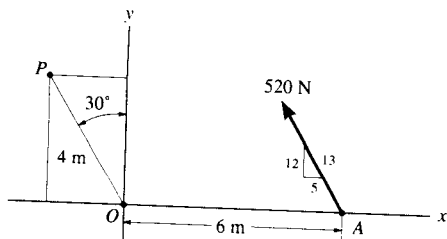
\*4-8. Determine the magnitude and directional sense of the resultant moment of the forces about point  $O$ .



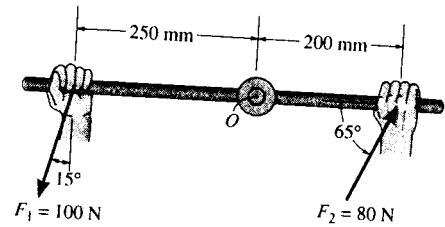
$$\begin{aligned} \zeta + M_O &= 400 \sin 30^\circ (2) + 400 \cos 30^\circ (5) + 260 \left( \frac{12}{13} \right) (6) \\ &= 3572.1 \text{ N}\cdot\text{m} = 3.57 \text{ kN}\cdot\text{m} \quad \text{Ans} \end{aligned}$$

4-9. Determine the magnitude and directional sense of the resultant moment of the forces about point  $P$ .

$$\begin{aligned} \zeta + M_P &= 260 \left( \frac{5}{13} \right) (3) + 260 \left( \frac{12}{13} \right) (2) - 400 \sin 30^\circ (2) + 400 \cos 30^\circ (8) \\ &= 3151 \text{ N}\cdot\text{m} = 3.15 \text{ kN}\cdot\text{m} \quad \text{Ans} \end{aligned}$$



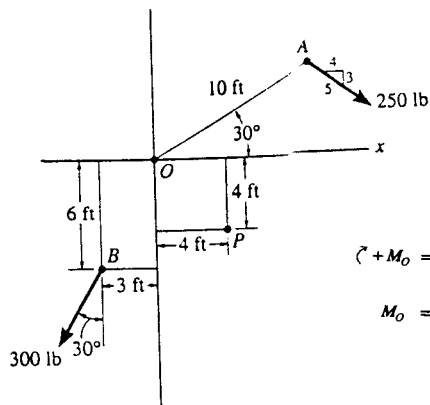
4-10. The wrench is used to loosen the bolt. Determine the moment of each force about the bolt's axis passing through point  $O$ .



$$\begin{aligned} \zeta^+ (M_{F_1})_O &= 100 \cos 15^\circ (0.25) \\ &= 24.1 \text{ N}\cdot\text{m} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$

$$\begin{aligned} \zeta^+ (M_{F_2})_O &= 80 \sin 65^\circ (0.2) \\ &= 14.5 \text{ N}\cdot\text{m} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$

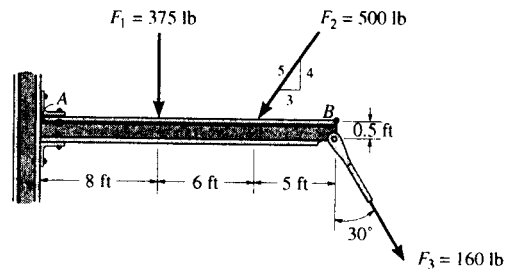
4-11. Determine the magnitude and directional sense of the resultant moment of the forces about point  $O$ .



$$\zeta^+ M_O = 250\left(\frac{4}{5}\right)(10 \sin 30^\circ) + 250\left(\frac{3}{5}\right)(10 \cos 30^\circ) + 300(\sin 30^\circ)(6) - 300(\cos 30^\circ)(3)$$

$$M_O = 2419.62 \text{ lb}\cdot\text{ft} = 2.42 \text{ kip}\cdot\text{ft} \quad \text{Ans}$$

\*4-12. Determine the moment about point  $A$  of each of the three forces acting on the beam.



$$\begin{aligned} \zeta^+ (M_{F_1})_A &= -375(8) \\ &= -3000 \text{ lb}\cdot\text{ft} = 3.00 \text{ kip}\cdot\text{ft} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$

$$\begin{aligned} \zeta^+ (M_{F_2})_A &= -500\left(\frac{4}{5}\right)(14) \\ &= -5600 \text{ lb}\cdot\text{ft} = 5.60 \text{ kip}\cdot\text{ft} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$

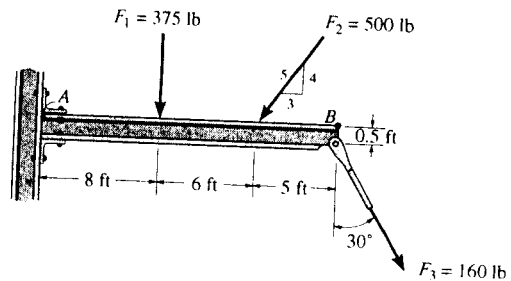
$$\begin{aligned} \zeta^+ (M_{F_3})_A &= -160(\cos 30^\circ)(19) + 160 \sin 30^\circ (0.5) \\ &= -2593 \text{ lb}\cdot\text{ft} = 2.59 \text{ kip}\cdot\text{ft} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$

4-13. Determine the moment about point  $B$  of each of the three forces acting on the beam.

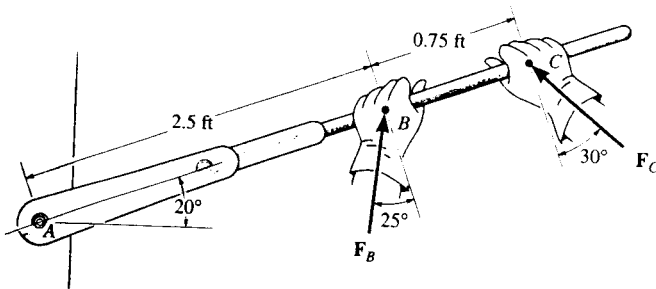
$$\begin{aligned} \curvearrowleft (M_{F_1})_B &= 375(11) \\ &= 4125 \text{ lb}\cdot\text{ft} = 4.125 \text{ kip}\cdot\text{ft} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$

$$\begin{aligned} \curvearrowleft (M_{F_2})_B &= 500\left(\frac{4}{5}\right)(5) \\ &= 2000 \text{ lb}\cdot\text{ft} = 2.00 \text{ kip}\cdot\text{ft} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$

$$\begin{aligned} \curvearrowleft (M_{F_3})_B &= 160\sin 30^\circ(0.5) - 160\cos 30^\circ(0) \\ &= 40.0 \text{ lb}\cdot\text{ft} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$



4-14. Determine the moment of each force about the bolt located at  $A$ . Take  $F_B = 40 \text{ lb}$ ,  $F_C = 50 \text{ lb}$ .



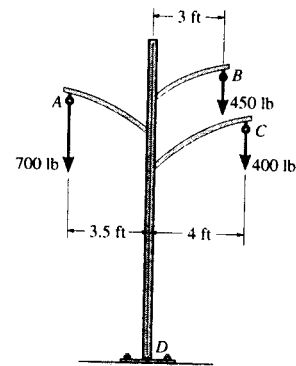
$$\curvearrowleft +M_B = 40 \cos 25^\circ(2.5) = 90.6 \text{ lb}\cdot\text{ft} \quad \text{Ans}$$

$$\curvearrowleft +M_C = 50 \cos 30^\circ(3.25) = 141 \text{ lb}\cdot\text{ft} \quad \text{Ans}$$

4-15. If  $F_B = 30 \text{ lb}$  and  $F_C = 45 \text{ lb}$ , determine the resultant moment about the bolt located at  $A$ .

$$\begin{aligned} \curvearrowleft +M_A &= 30 \cos 25^\circ(2.5) + 45 \cos 30^\circ(3.25) \\ &= 195 \text{ lb}\cdot\text{ft} \quad \text{Ans} \end{aligned}$$

\*4-16. The power pole supports the three lines, each line exerting a vertical force on the pole due to its weight as shown. Determine the resultant moment at the base  $D$  due to all of these forces. If it is possible for wind or ice to snap the lines, determine which line(s) when removed create(s) a condition for the greatest moment about the base. What is this resultant moment?

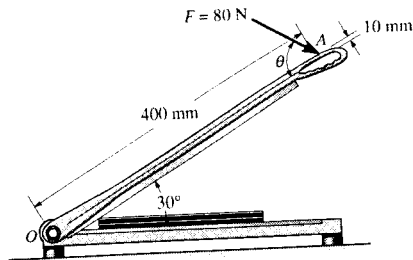


$$\begin{aligned} \curvearrowleft +M_{R_D} = \Sigma Fd; \quad M_{R_D} &= 700(3.5) - 450(3) - 400(4) \\ &= -500 \text{ lb}\cdot\text{ft} = 500 \text{ lb}\cdot\text{ft} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$

When the cable at  $A$  is removed it will create the greatest moment at point  $D$ . Ans

$$\begin{aligned} \curvearrowleft + (M_{R_D})_{\max} &= \Sigma Fd; \\ (M_{R_D})_{\max} &= -450(3) - 400(4) \\ &= -2950 \text{ lb}\cdot\text{ft} = 2.95 \text{ kip}\cdot\text{ft} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$

4-17. A force of 80 N acts on the handle of the paper cutter at A. Determine the moment created by this force about the hinge at O, if  $\theta = 60^\circ$ . At what angle  $\theta$  should the force be applied so that the moment it creates about point O is a maximum (clockwise)? What is this maximum moment?



$$\begin{aligned} \curvearrowright +M_o &= \Sigma Fd; \quad M_o = -80 \cos \theta (0.01) - 80 \sin \theta (0.4) \\ &= -(0.800 \cos \theta + 32.0 \sin \theta) \text{ N} \cdot \text{m} \\ &= (0.800 \cos \theta + 32.0 \sin \theta) \text{ N} \cdot \text{m} \text{ (Clockwise)} \end{aligned}$$

$$\begin{aligned} \text{At } \theta = 60^\circ, \quad M_o &= 0.800 \cos 60^\circ + 32.0 \sin 60^\circ \\ &= 28.1 \text{ N} \cdot \text{m} \text{ (Clockwise)} \quad \text{Ans} \end{aligned}$$

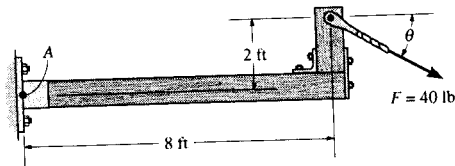
In order to produce the maximum and minimum moment about point A,  $\frac{dM_o}{d\theta} = 0$

$$\begin{aligned} \frac{dM_o}{d\theta} &= 0 = -0.800 \sin \theta + 32.0 \cos \theta \\ \theta &= 88.568^\circ = 88.6^\circ \quad \text{Ans} \end{aligned}$$

$$\begin{aligned} \frac{d^2 M_o}{d\theta^2} &= -0.800 \cos \theta - 32.0 \sin \theta \\ \text{Since } \left. \frac{d^2 M_o}{d\theta^2} \right|_{\theta=88.568^\circ} &= -0.800 \cos 88.568^\circ - 32.0 \sin 88.568^\circ = \\ &= -32.00 \text{ is a negative value, indeed at } \theta = 88.568^\circ, \text{ the } 80 \text{ N produces a maximum clockwise moment at O. This maximum clockwise moment is} \end{aligned}$$

$$\begin{aligned} (M_o)_{\max} &= 0.800 \cos 88.568^\circ + 32.0 \sin 88.568^\circ \\ &= 32.0 \text{ N} \cdot \text{m} \text{ (Clockwise)} \quad \text{Ans} \end{aligned}$$

4-18. Determine the direction  $\theta$  ( $0^\circ \leq \theta \leq 180^\circ$ ) of the force  $F = 40 \text{ lb}$  so that it produces (a) the maximum moment about point A and (b) the minimum moment about point A. Compute the moment in each case.



$$(a) \quad \curvearrowright + (M_A)_{\max} = 40(\sqrt{8^2 + 2^2}) = 330 \text{ lb} \cdot \text{ft} \quad \text{Ans}$$

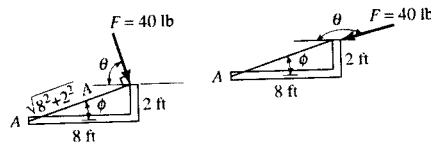
$$\phi = \tan^{-1} \left( \frac{2}{8} \right) = 14.04^\circ$$

$$\theta = 90^\circ - 14.04^\circ = 76.0^\circ \quad \text{Ans}$$

$$(b) \quad \curvearrowright + (M_A)_{\min} = 0 \quad \text{Ans}$$

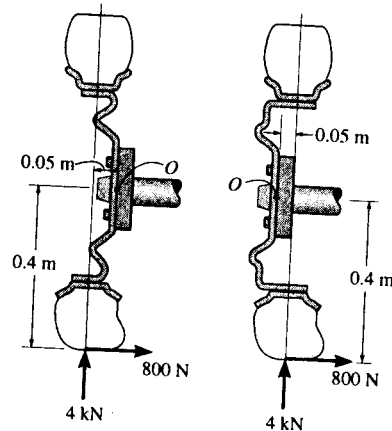
$$\phi = \tan^{-1} \left( \frac{2}{8} \right) = 14.04^\circ$$

$$\theta = 180^\circ - 14.04^\circ = 166^\circ \quad \text{Ans}$$





\*4-19. The hub of the wheel can be attached to the axle either with negative offset (left) or with positive offset (right). If the tire is subjected to both a normal and radial load as shown, determine the resultant moment of these loads about the axle, point  $O$  for both cases.



For case 1 with negative offset, we have

$$\begin{aligned} \zeta + M_O &= 800(0.4) - 4000(0.05) \\ &= 120 \text{ N}\cdot\text{m} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$

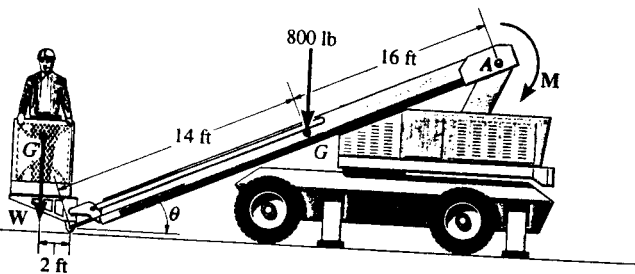
For case 2 with positive offset, we have

$$\begin{aligned} \zeta + M_O &= 800(0.4) + 4000(0.05) \\ &= 520 \text{ N}\cdot\text{m} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$

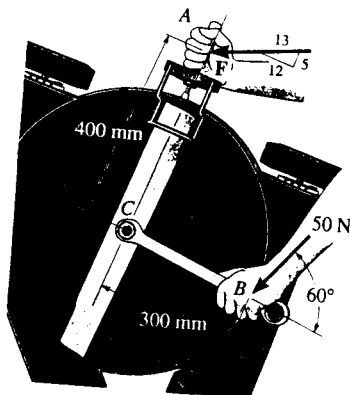
\*4-20. The boom has a length of 30 ft, a weight of 800 lb, and mass center at  $G$ . If the maximum moment that can be developed by the motor at  $A$  is  $M = 20(10^3)$  lb·ft, determine the maximum load  $W$ , having a mass center at  $G'$ , that can be lifted. Take  $\theta = 30^\circ$ .

$$20(10^3) = 800(16 \cos 30^\circ) + W(30 \cos 30^\circ + 2)$$

$$W = 319 \text{ lb} \quad \text{Ans}$$



4-21. The tool at  $A$  is used to hold a power lawnmower blade stationary while the nut is being loosened with the wrench. If a force of 50 N is applied to the wrench at  $B$  in the direction shown, determine the moment it creates about the nut at  $C$ . What is the magnitude of force  $F$  at  $A$  so that it creates the opposite moment about  $C$ ?



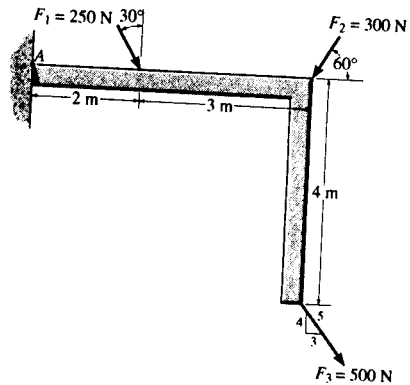
$$(a) \zeta + M_A = 50 \sin 60^\circ (0.3)$$

$$M_A = 12.99 = 13.0 \text{ N}\cdot\text{m} \quad \text{Ans}$$

$$(b) \zeta + M_A = 0; \quad -12.99 + F\left(\frac{12}{13}\right)(0.4) = 0$$

$$F = 35.2 \text{ N} \quad \text{Ans}$$

4-22. Determine the moment of each of the three forces about point A. Solve the problem first by using each force as a whole, and then by using the principle of moments.



The moment arm measured perpendicular to each force from point A is

$$d_1 = 2 \sin 60^\circ = 1.732 \text{ m}$$

$$d_2 = 5 \sin 60^\circ = 4.330 \text{ m}$$

$$d_3 = 2 \sin 53.13^\circ = 1.60 \text{ m}$$

Using each force where  $M_A = Fd$ , we have

$$\begin{aligned} \zeta + (M_{F_1})_A &= -250(1.732) \\ &= -433 \text{ N} \cdot \text{m} = 433 \text{ N} \cdot \text{m} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$

$$\begin{aligned} \zeta + (M_{F_2})_A &= -300(4.330) \\ &= -1299 \text{ N} \cdot \text{m} = 1.30 \text{ kN} \cdot \text{m} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$

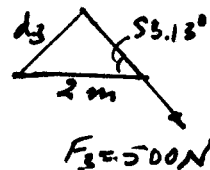
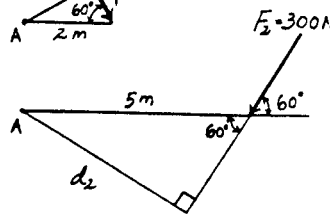
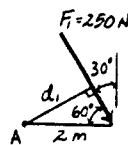
$$\begin{aligned} \zeta + (M_{F_3})_A &= -500(1.60) \\ &= -800 \text{ N} \cdot \text{m} = 800 \text{ N} \cdot \text{m} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$

Using principle of moments, we have

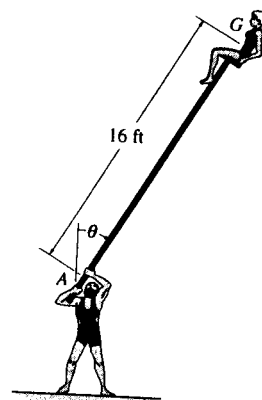
$$\begin{aligned} \zeta + (M_{F_1})_A &= -250 \cos 30^\circ (2) \\ &= -433 \text{ N} \cdot \text{m} = 433 \text{ N} \cdot \text{m} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$

$$\begin{aligned} \zeta + (M_{F_2})_A &= -300 \sin 60^\circ (5) \\ &= -1299 \text{ N} \cdot \text{m} = 1.30 \text{ kN} \cdot \text{m} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$

$$\begin{aligned} \zeta + (M_{F_3})_A &= 500 \left(\frac{3}{5}\right)(4) - 500 \left(\frac{4}{5}\right)(5) \\ &= -800 \text{ N} \cdot \text{m} = 800 \text{ N} \cdot \text{m} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$



4-23. As part of an acrobatic stunt, a man supports a girl who has a weight of 120 lb and is seated on a chair on top of the pole. If her center of gravity is at G, and if the maximum counterclockwise moment the man can exert on the pole at A is 250 lb·ft, determine the maximum angle of tilt,  $\theta$ , which will not allow the girl to fall, i.e., so her clockwise moment about A does not exceed 250 lb·ft.



In order to prevent the girl from falling down, the clockwise moment produced by the girl's weight must not exceed 250 lb·ft.

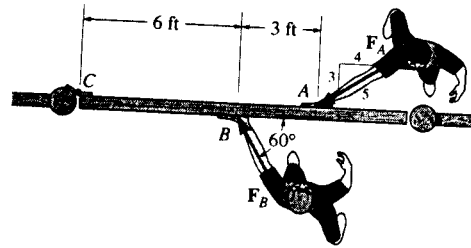
$$M_A = 120(16 \sin \theta) \leq 250$$

$$\sin \theta \leq 0.1302$$

$$\theta = 7.48^\circ$$

Ans

4-24. The two boys push on the gate with forces of  $F_A = 30$  lb and  $F_B = 50$  lb as shown. Determine the moment of each force about  $C$ . Which way will the gate rotate, clockwise or counterclockwise? Neglect the thickness of the gate.

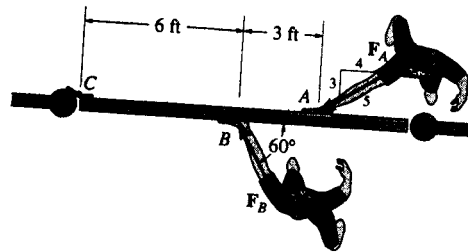


$$\begin{aligned} (+) (M_{F_A})_C &= -30\left(\frac{3}{5}\right)(9) \\ &= -162 \text{ lb} \cdot \text{ft} = 162 \text{ lb} \cdot \text{ft} \quad (\text{Clockwise}) \quad \text{Ans} \end{aligned}$$

$$\begin{aligned} (+) (M_{F_B})_C &= 50(\sin 60^\circ)(6) \\ &= 260 \text{ lb} \cdot \text{ft} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$

Since  $(M_{F_B})_C > (M_{F_A})_C$ , the gate will rotate *Counterclockwise*. Ans

4-25. Two boys push on the gate as shown. If the boy at  $B$  exerts a force of  $F_B = 30$  lb, determine the magnitude of the force  $F_A$  the boy at  $A$  must exert in order to prevent the gate from turning. Neglect the thickness of the gate.

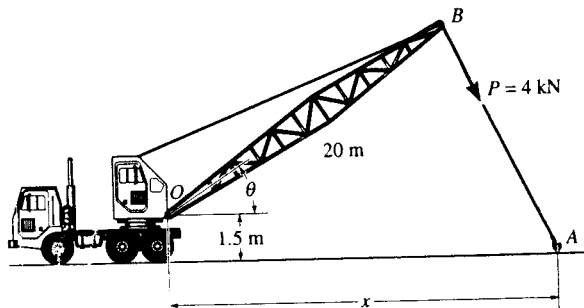


In order to prevent the gate from turning, the resultant moment about point  $C$  must be equal to zero.

$$+ M_{R_C} = \Sigma Fd; \quad M_{R_C} = 0 = 30 \sin 60^\circ (6) - F_A \left(\frac{3}{5}\right)(9)$$

$$F_A = 28.9 \text{ lb} \quad \text{Ans}$$

4-26. The towline exerts a force of  $P = 4 \text{ kN}$  at the end of the 20-m-long crane boom. If  $\theta = 30^\circ$ , determine the placement  $x$  of the hook at  $A$  so that this force creates a maximum moment about point  $O$ . What is this moment?



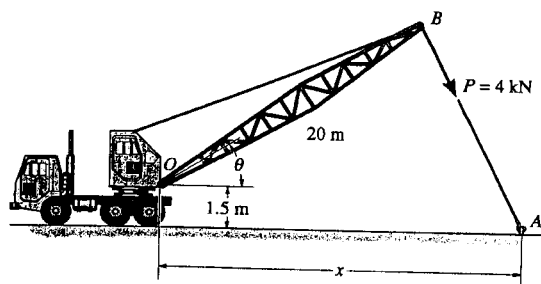
Maximum moment,  $OB \perp BA$

$$\zeta^+ (M_O)_{max} = 4000(20) = 80 \text{ kN}\cdot\text{m} \quad \text{Ans}$$

$$4 \text{ kN} \sin 60^\circ(x) - 4 \text{ kN} \cos 60^\circ(1.5) = 80 \text{ kN}\cdot\text{m}$$

$$x = 24.0 \text{ m} \quad \text{Ans}$$

4-27. The towline exerts a force of  $P = 4 \text{ kN}$  at the end of the 20-m-long crane boom. If  $x = 25 \text{ m}$ , determine the position  $\theta$  of the boom so that this force creates a maximum moment about point  $O$ . What is this moment?



Maximum moment,  $OB \perp BA$

$$\zeta^+ (M_O)_{max} = 4000(20) = 80\,000 \text{ N}\cdot\text{m} = 80.0 \text{ kN}\cdot\text{m} \quad \text{Ans}$$

$$4000 \sin \phi(25) - 4000 \cos \phi(1.5) = 80\,000$$

$$25 \sin \phi - 1.5 \cos \phi = 20$$

$$\phi = 56.43^\circ$$

$$\theta = 90^\circ - 56.43^\circ = 33.6^\circ \quad \text{Ans}$$

Also,

$$(1.5)^2 + z^2 = y^2$$

$$2.25 + z^2 = y^2$$

Similar triangles

$$\frac{20+y}{z} = \frac{25+z}{y}$$

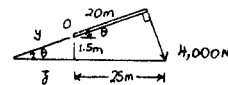
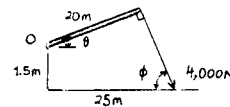
$$20y + y^2 = 25z + z^2$$

$$20(\sqrt{2.25 + z^2}) + 2.25 + z^2 = 25z + z^2$$

$$z = 2.259 \text{ m}$$

$$y = 2.712 \text{ m}$$

$$\theta = \cos^{-1}\left(\frac{2.259}{2.712}\right) = 33.6^\circ \quad \text{Ans}$$



\*4-28. Determine the direction  $\theta$  for  $0^\circ \leq \theta \leq 180^\circ$  of the force  $F$  so that  $F$  produces (a) the maximum moment about point  $A$  and (b) the minimum moment about point  $A$ . Calculate the moment in each case.

a)

$$\zeta +M_A = 400 \sqrt{(3)^2 + (2)^2} = 1442 \text{ N} \cdot \text{m}$$

$$M_A = 1.44 \text{ kN} \cdot \text{m} \quad \text{Ans}$$

$$\phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69^\circ$$

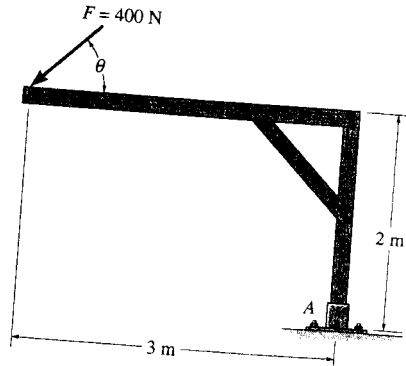
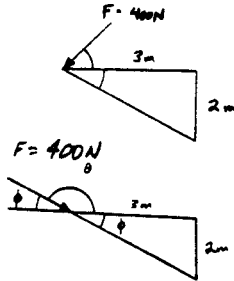
$$\theta = 90^\circ - 33.69^\circ = 56.3^\circ \quad \text{Ans}$$

b)

$$\zeta +M_A = 0 \quad \text{Ans}$$

$$\phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69^\circ$$

$$\theta = 180^\circ - 33.69^\circ = 146^\circ \quad \text{Ans}$$



4-29. Determine the moment of the force  $F$  about point  $A$  as a function of  $\theta$ . Plot the results of  $M$  (ordinate) versus  $\theta$  (abscissa) for  $0^\circ \leq \theta \leq 180^\circ$ .

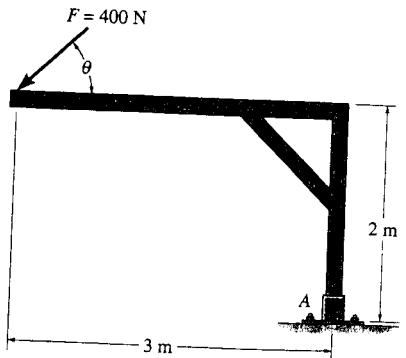
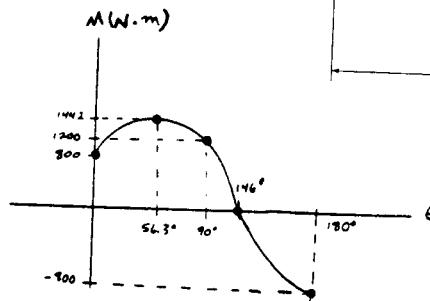
$$\zeta +M_A = 400 \sin\theta(3) + 400 \cos\theta(2)$$

$$= 1200 \sin\theta + 800 \cos\theta \quad \text{Ans}$$

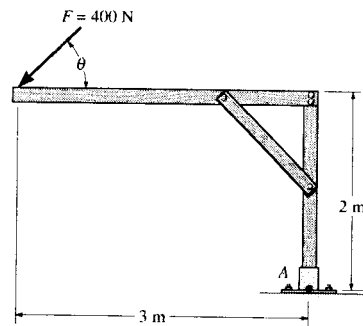
$$\frac{dM_A}{d\theta} = 1200 \cos\theta - 800 \sin\theta = 0$$

$$\theta = \tan^{-1}\left(\frac{1200}{800}\right) = 56.3^\circ$$

$$(M_A)_{\max} = 1200 \sin 56.3^\circ + 800 \cos 56.3^\circ = 1442 \text{ N} \cdot \text{m}$$



**\*4-28.** Determine the direction  $\theta$  for  $0^\circ \leq \theta \leq 180^\circ$  of the force  $F$  so that  $F$  produces (a) the maximum moment about point  $A$  and (b) the minimum moment about point  $A$ . Calculate the moment in each case.



(a)  $\curvearrowleft +M_A = 400\sqrt{(3)^2 + (2)^2} = 1442 \text{ N} \cdot \text{m}$

$M_A = 1.44 \text{ kN} \cdot \text{m}$  Ans

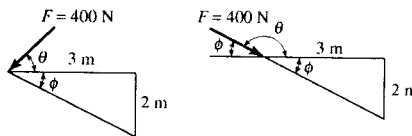
$\phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69^\circ$

$\theta = 90^\circ - 33.69^\circ = 56.3^\circ$  Ans

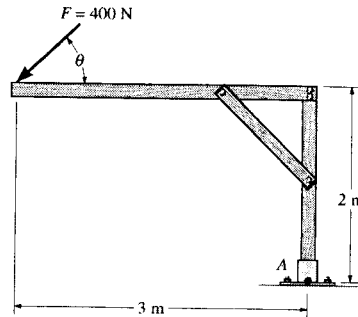
(b)  $\curvearrowleft +M_A = 0$  Ans

$\phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69^\circ$

$\theta = 180^\circ - 33.69^\circ = 146^\circ$  Ans



**4-29.** Determine the moment of the force  $F$  about point  $A$  as a function of  $\theta$ . Plot the results of  $M$  (ordinate) versus  $\theta$  (abscissa) for  $0^\circ \leq \theta \leq 180^\circ$ .



$\curvearrowleft +M_A = 400 \sin \theta(3) + 400 \cos \theta(2)$

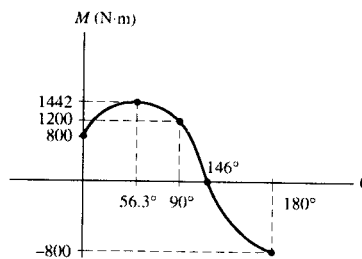
$= 1200 \sin \theta + 800 \cos \theta$

$\frac{dM_A}{d\theta} = 1200 \cos \theta - 800 \sin \theta = 0$

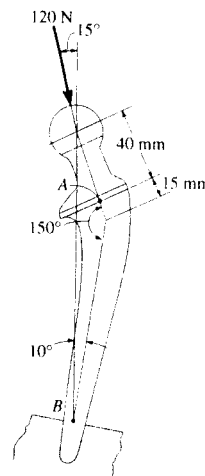
$\theta = \tan^{-1}\left(\frac{1200}{800}\right) = 56.3^\circ$

$(M_A)_{\max} = 1200 \sin 56.3^\circ + 800 \cos 56.3^\circ = 1442 \text{ N} \cdot \text{m}$

Ans



4-30. The total hip replacement is subjected to a force of  $F = 120 \text{ N}$ . Determine the moment of this force about the neck at  $A$  and at the stem  $B$ .



**Moment About Point A :** The angle between the line of action of the load and the neck axis is  $20^\circ - 15^\circ = 5^\circ$ .

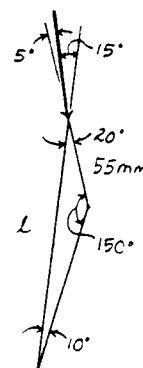
$$\begin{aligned} (+ M_A &= 120 \sin 5^\circ (0.04) \\ &= 0.418 \text{ N} \cdot \text{m} \quad (\text{Counterclockwise}) \end{aligned} \quad \text{Ans}$$

**Moment About Point B :** The dimension  $l$  can be determined using the law of sines.

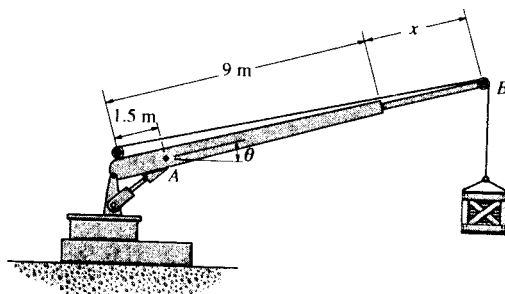
$$\frac{l}{\sin 150^\circ} = \frac{55}{\sin 10^\circ} \quad l = 158.4 \text{ mm} = 0.1584 \text{ m}$$

Then,

$$\begin{aligned} + M_B &= -120 \sin 15^\circ (0.1584) \\ &= -4.92 \text{ N} \cdot \text{m} = 4.92 \text{ N} \cdot \text{m} \quad (\text{Clockwise}) \end{aligned} \quad \text{Ans}$$



\*4-31. The crane can be adjusted for any angle  $0^\circ \leq \theta \leq 90^\circ$  and any extension  $0 \leq x \leq 5 \text{ m}$ . For a suspended mass of  $120 \text{ kg}$ , determine the moment developed at  $A$  as a function of  $x$  and  $\theta$ . What values of both  $x$  and  $\theta$  develop the maximum possible moment at  $A$ ? Compute this moment. Neglect the size of the pulley at  $B$ .

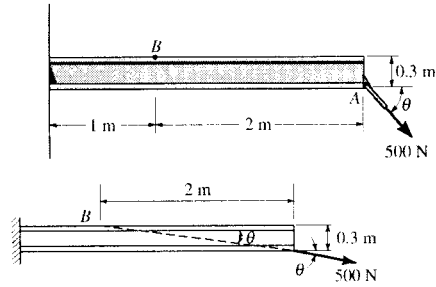


$$\begin{aligned} (+ M_A &= -120(9.81)(7.5+x) \cos \theta \\ &= \{-1177.2 \cos \theta (7.5+x)\} \text{ N} \cdot \text{m} \\ &= \{1.18 \cos \theta (7.5+x)\} \text{ kN} \cdot \text{m} \quad (\text{clockwise}) \end{aligned} \quad \text{Ans}$$

The maximum moment at  $A$  occurs when  $\theta = 0^\circ$  and  $x = 5 \text{ m}$ . Ans

$$\begin{aligned} (+ (M_A)_{\max} &= \{-1177.2 \cos 0^\circ (7.5+5)\} \text{ N} \cdot \text{m} \\ &= -14715 \text{ N} \cdot \text{m} \\ &= 14.7 \text{ kN} \cdot \text{m} \quad (\text{clockwise}) \end{aligned} \quad \text{Ans}$$

**\*4-32.** Determine the angle  $\theta$  at which the 500-N force must act at  $A$  so that the moment of this force about point  $B$  is equal to zero.



This problem requires that the resultant moment about point  $B$  be equal to zero.

$$\curvearrowleft +M_R = \Sigma Fd; \quad M_R = 0 = 500 \cos \theta(0.3) - 500 \sin \theta(2)$$

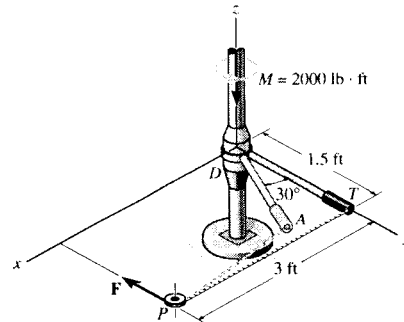
$$\theta = 8.53^\circ$$

**Ans**

Also note that if the line of action of the 500 N force passes through point  $B$ , it produces zero moment about  $B$ . Hence, from the geometry

$$\theta = \tan^{-1} \left( \frac{0.3}{2} \right) = 8.53^\circ$$

**4-33.** Segments of drill pipe  $D$  for an oil well are tightened a prescribed amount by using a set of tongs  $T$ , which grip the pipe, and a hydraulic cylinder (not shown) to regulate the force  $F$  applied to the tongs. This force acts along the cable which passes around the small pulley  $P$ . If the cable is originally perpendicular to the tongs as shown, determine the magnitude of force  $F$  which must be applied so that the moment about the pipe is  $M = 2000 \text{ lb} \cdot \text{ft}$ . In order to maintain this same moment what magnitude of  $F$  is required when the tongs rotate  $30^\circ$  to the dashed position? *Note:* The angle  $DAP$  is not  $90^\circ$  in this position.



This problem requires that the moment produced by  $F$  and  $F'$  about the  $z$  axis is  $2000 \text{ lb} \cdot \text{ft}$ .

$$M_z = 2000 = F(1.5)$$

$$F = 1333.3 \text{ lb} = 1.33 \text{ kip}$$

**Ans**

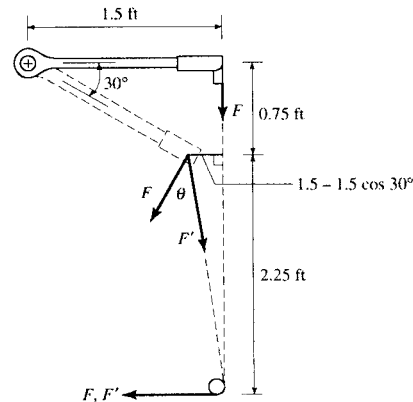
$$F = F' \cos \theta, \text{ where}$$

$$\theta = 30^\circ + \tan^{-1} \left( \frac{1.5 - 1.5 \cos 30^\circ}{2.25} \right)$$

$$= 35.104^\circ$$

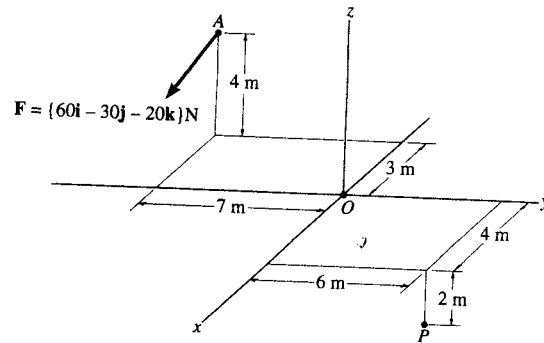
$$F' = \frac{1333.33}{\cos 35.104^\circ} = 1.63 \text{ kip}$$

**Ans**





- 4-34. Determine the moment of the force at  $A$  about point  $O$ . Express the result as a Cartesian vector.



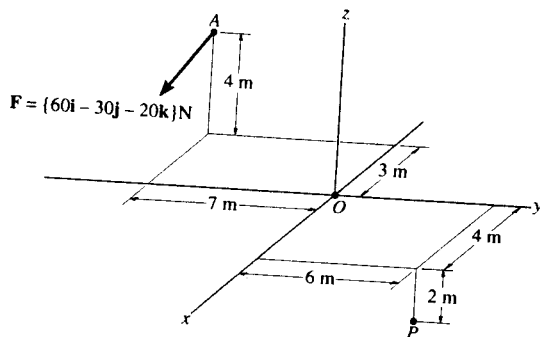
**Position Vector :**

$$\begin{aligned} r_{OA} &= \{(-3-0)\mathbf{i} + (-7-0)\mathbf{j} + (4-0)\mathbf{k}\} \text{ m} \\ &= \{-3\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}\} \text{ m} \end{aligned}$$

**Moment of Force  $F$  About Point  $O$  :** Applying Eq. 4-7, we have

$$\begin{aligned} M_O &= r_{OA} \times F \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -7 & 4 \\ 60 & -30 & -20 \end{vmatrix} \\ &= \{260\mathbf{i} + 180\mathbf{j} + 510\mathbf{k}\} \text{ N}\cdot\text{m} \end{aligned} \quad \text{Ans}$$

- 4-35. Determine the moment of the force at  $A$  about point  $P$ . Express the result as a Cartesian vector.



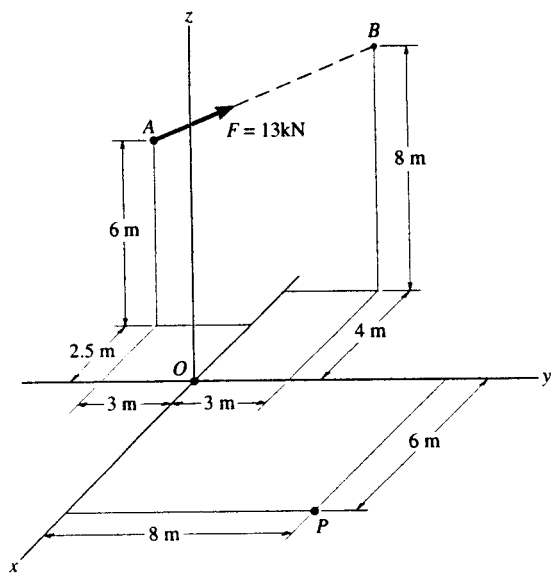
**Position Vector :**

$$\begin{aligned} r_{PA} &= \{(-3-4)\mathbf{i} + (-7-6)\mathbf{j} + [4-(-2)]\mathbf{k}\} \text{ m} \\ &= \{-7\mathbf{i} - 13\mathbf{j} + 6\mathbf{k}\} \text{ m} \end{aligned}$$

**Moment of Force  $F$  About Point  $O$  :** Applying Eq. 4-7, we have

$$\begin{aligned} M_O &= r_{PA} \times F \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7 & -13 & 6 \\ 60 & -30 & -20 \end{vmatrix} \\ &= \{440\mathbf{i} + 220\mathbf{j} + 990\mathbf{k}\} \text{ N}\cdot\text{m} \end{aligned} \quad \text{Ans}$$

\*4-36. Determine the moment of the force  $F$  at  $A$  about point  $O$ . Express the result as a Cartesian vector.



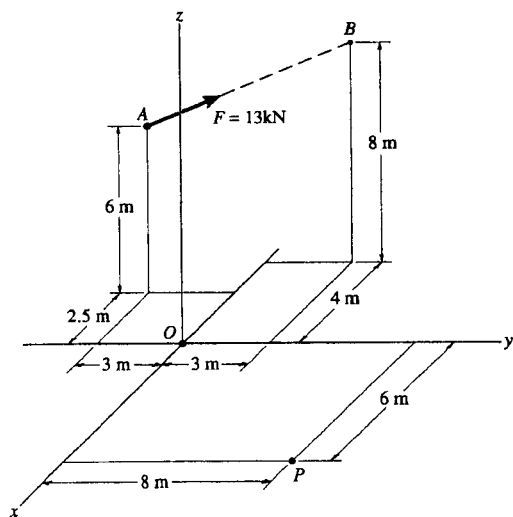
$$\mathbf{r}_{AB} = \{-1.5\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}\} \text{ m}$$

$$r_{AB} = \sqrt{(-1.5)^2 + 6^2 + 2^2} = 6.5 \text{ m}$$

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2.5 & -3 & 6 \\ -\frac{1.5}{6.5}(13) & \frac{6}{6.5}(13) & \frac{2}{6.5}(13) \end{vmatrix}$$

$$\mathbf{M}_O = \{-84\mathbf{i} - 8\mathbf{j} - 39\mathbf{k}\} \text{ kN}\cdot\text{m} \quad \text{Ans}$$

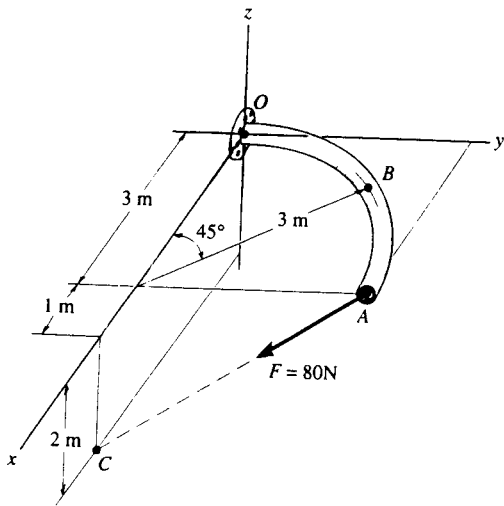
4-37. Determine the moment of the force  $F$  at  $A$  about point  $P$ . Express the result as a Cartesian vector.



$$\mathbf{M}_P = \mathbf{r}_{PA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8.5 & -11 & 6 \\ -\frac{1.5}{6.5}(13) & \frac{6}{6.5}(13) & \frac{2}{6.5}(13) \end{vmatrix}$$

$$\mathbf{M}_P = \{-116\mathbf{i} + 16\mathbf{j} - 135\mathbf{k}\} \text{ kN}\cdot\text{m} \quad \text{Ans}$$

4-38. The curved rod lies in the  $x$ - $y$  plane and has a radius of 3 m. If a force of  $F = 80$  N acts at its end as shown, determine the moment of this force about point  $O$ .



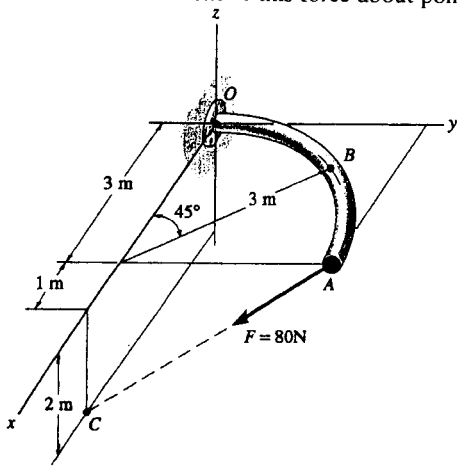
$$r_{AC} = \{1i - 3j - 2k\} \text{ m}$$

$$r_{AC} = \sqrt{(1)^2 + (-3)^2 + (-2)^2} = 3.742 \text{ m}$$

$$M_O = r_{OC} \times F = \begin{vmatrix} i & j & k \\ 4 & 0 & -2 \\ \frac{1}{3.742}(80) & -\frac{3}{3.742}(80) & -\frac{2}{3.742}(80) \end{vmatrix}$$

$$M_O = \{-128i + 128j - 257k\} \text{ N}\cdot\text{m} \quad \text{Ans}$$

4-39. The curved rod lies in the  $x$ - $y$  plane and has a radius of 3 m. If a force of  $F = 80$  N acts at its end as shown, determine the moment of this force about point  $B$ .



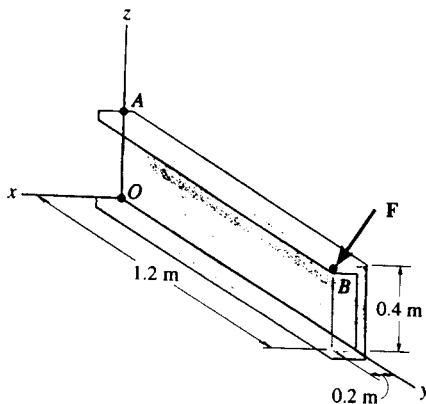
$$r_{AC} = \{1i - 3j - 2k\} \text{ m}$$

$$r_{AC} = \sqrt{(1)^2 + (-3)^2 + (-2)^2} = 3.742 \text{ m}$$

$$M_B = r_{BA} \times F = \begin{vmatrix} i & j & k \\ 3\cos 45^\circ & (3 - 3\sin 45^\circ) & 0 \\ \frac{1}{3.742}(80) & -\frac{3}{3.742}(80) & -\frac{2}{3.742}(80) \end{vmatrix}$$

$$M_B = \{-37.6i + 90.7j - 155k\} \text{ N}\cdot\text{m} \quad \text{Ans}$$

\*4-40. The force  $F = \{600i + 300j - 600k\}$  N acts at the end of the beam. Determine the moment of the force about point  $A$ .

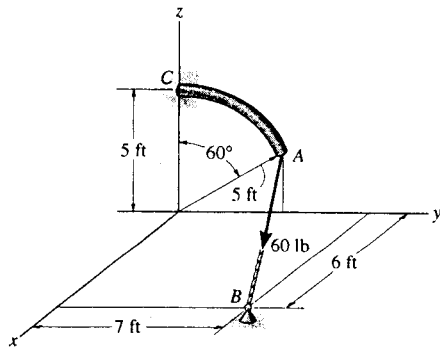


$$r = \{0.2i + 1.2j\} \text{ m}$$

$$M_A = r_{AB} \times F = \begin{vmatrix} i & j & k \\ 0.2 & 1.2 & 0 \\ 600 & 300 & -600 \end{vmatrix}$$

$$M_A = \{-720i + 120j - 660k\} \text{ N}\cdot\text{m} \quad \text{Ans}$$

4-41. The curved rod has a radius of 5 ft. If a force of 60 lb acts at its end as shown, determine the moment of this force about point C.



Position Vector and Force Vector :

$$\mathbf{r}_{CA} = \{(5 \sin 60^\circ - 0)\mathbf{j} + (5 \cos 60^\circ - 5)\mathbf{k}\} \text{ m}$$

$$= \{4.330\mathbf{j} - 2.50\mathbf{k}\} \text{ m}$$

$$\mathbf{F}_{AB} = 60 \left\{ \frac{(6-0)\mathbf{i} + (7-5 \sin 60^\circ)\mathbf{j} + (0-5 \cos 60^\circ)\mathbf{k}}{\sqrt{(6-0)^2 + (7-5 \sin 60^\circ)^2 + (0-5 \cos 60^\circ)^2}} \right\} \text{ lb}$$

$$= \{51.231\mathbf{i} + 22.797\mathbf{j} - 21.346\mathbf{k}\} \text{ lb}$$

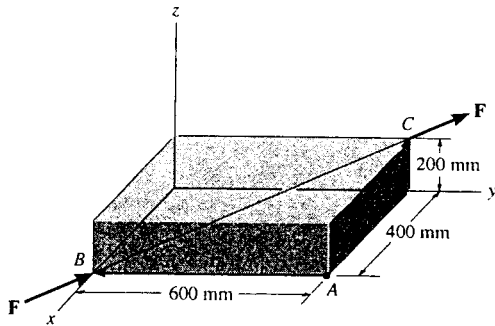
Moment of Force  $\mathbf{F}_{AB}$  About Point C : Applying Eq. 4-7, we have

$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}_{AB}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.330 & -2.50 \\ 51.231 & 22.797 & -21.346 \end{vmatrix}$$

$$= \{-35.4\mathbf{i} - 128\mathbf{j} - 222\mathbf{k}\} \text{ lb} \cdot \text{ft} \quad \text{Ans}$$

4-42. A force  $\mathbf{F}$  having a magnitude of  $F = 100 \text{ N}$  along the diagonal of the parallelepiped. Determine the moment of  $\mathbf{F}$  about point A, using  $\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F}$  and  $\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F}$ .



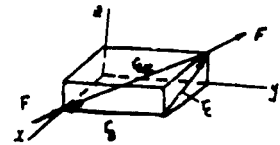
$$\mathbf{F} = 100 \left( \frac{-0.4\mathbf{i} + 0.6\mathbf{j} + 0.2\mathbf{k}}{0.7483} \right)$$

$$\mathbf{F} = \{-53.5\mathbf{i} + 80.2\mathbf{j} + 26.7\mathbf{k}\} \text{ N}$$

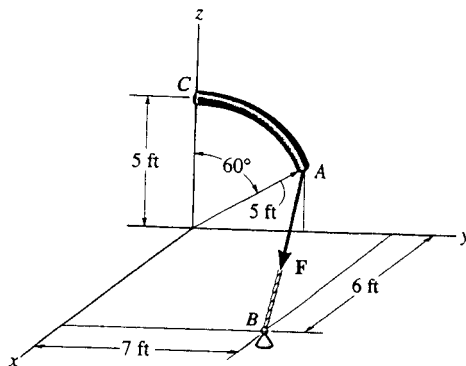
$$\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.6 & 0 \\ -53.5 & 80.2 & 26.7 \end{vmatrix} = \{-16.0\mathbf{i} - 32.1\mathbf{k}\} \text{ N} \cdot \text{m} \quad \text{Ans}$$

Also,

$$\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.4 & 0 & 0.2 \\ -53.5 & 80.2 & 26.7 \end{vmatrix} = \{-16.0\mathbf{i} - 32.1\mathbf{k}\} \text{ N} \cdot \text{m} \quad \text{Ans}$$



4-43. Determine the smallest force  $F$  that must be applied along the rope in order to cause the curved rod, which has a radius of 5 ft, to fail at the support C. This requires a moment of  $M = 80 \text{ lb} \cdot \text{ft}$  to be developed at C.



$$\mathbf{r}_{CA} = \{4.330\mathbf{j} - 2.5\mathbf{k}\} \text{ ft}$$

$$\mathbf{F}_{AB} = F_{AB} \left\{ \frac{6\mathbf{i} + (7-5 \sin 60^\circ)\mathbf{j} - 5 \cos 60^\circ \mathbf{k}}{\sqrt{(6)^2 + (7-5 \sin 60^\circ)^2 + (-5 \cos 60^\circ)^2}} \right\}$$

$$\mathbf{F}_{AB} = F_{AB} (0.8538\mathbf{i} + 0.3799\mathbf{j} - 0.3558\mathbf{k})$$

$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}_{AB}$$

$$\mathbf{M}_C = F_{AB} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.330 & -2.5 \\ 0.8538 & 0.3799 & -0.3558 \end{vmatrix}$$

$$\mathbf{M}_C = F_{AB} (-0.5909\mathbf{i} + 2.135\mathbf{j} - 3.697\mathbf{k})$$

$$M_C = F_{AB} \sqrt{(-0.5909)^2 + (2.135)^2 + (-3.697)^2}$$

$$80 = F_{AB} (4.310)$$

$$F_{AB} = \frac{80}{4.310} = 18.5618 \text{ lb}$$

$$F_{AB} = 18.6 \text{ lb} \quad \text{Ans}$$

\*4-44. The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point A.

**Position Vector And Force Vector :**

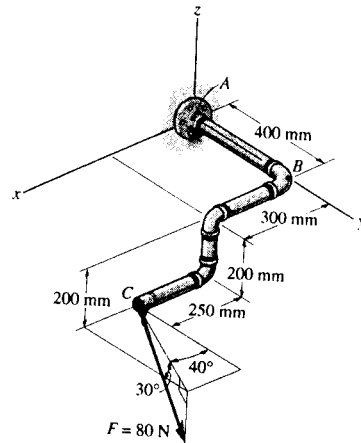
$$\begin{aligned} r_{AC} &= \{(0.55-0)\mathbf{i} + (0.4-0)\mathbf{j} + (-0.2-0)\mathbf{k}\} \text{ m} \\ &= \{0.55\mathbf{i} + 0.4\mathbf{j} - 0.2\mathbf{k}\} \text{ m} \end{aligned}$$

$$\begin{aligned} \mathbf{F} &= 80(\cos 30^\circ \sin 40^\circ \mathbf{i} + \cos 30^\circ \cos 40^\circ \mathbf{j} - \sin 30^\circ \mathbf{k}) \text{ N} \\ &= \{44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}\} \text{ N} \end{aligned}$$

**Moment of Force F About Point A :** Applying Eq. 4-7, we have

$$\begin{aligned} \mathbf{M}_A &= r_{AC} \times \mathbf{F} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix} \\ &= \{-5.39\mathbf{i} + 13.1\mathbf{j} + 11.4\mathbf{k}\} \text{ N} \cdot \text{m} \end{aligned}$$

**Ans**



4-45. The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point B.

**Position Vector And Force Vector :**

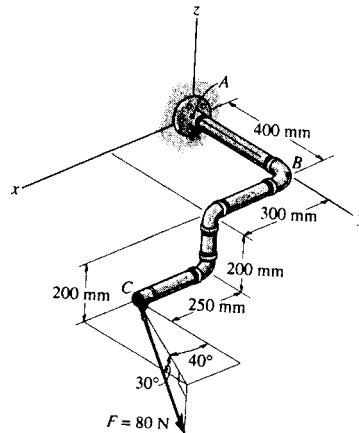
$$\begin{aligned} r_{BC} &= \{(0.55-0)\mathbf{i} + (0.4-0.4)\mathbf{j} + (-0.2-0)\mathbf{k}\} \text{ m} \\ &= \{0.55\mathbf{i} - 0.2\mathbf{k}\} \text{ m} \end{aligned}$$

$$\begin{aligned} \mathbf{F} &= 80(\cos 30^\circ \sin 40^\circ \mathbf{i} + \cos 30^\circ \cos 40^\circ \mathbf{j} - \sin 30^\circ \mathbf{k}) \text{ N} \\ &= \{44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}\} \text{ N} \end{aligned}$$

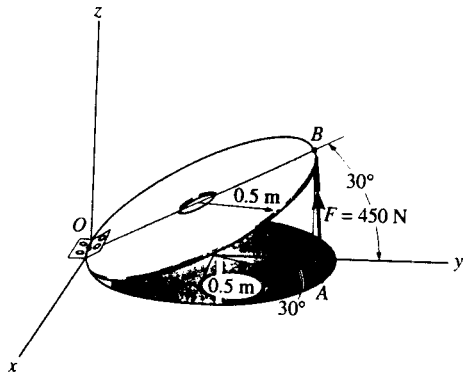
**Moment of Force F About Point B :** Applying Eq. 4-7, we have

$$\begin{aligned} \mathbf{M}_B &= r_{BC} \times \mathbf{F} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix} \\ &= \{10.6\mathbf{i} + 13.1\mathbf{j} + 29.2\mathbf{k}\} \text{ N} \cdot \text{m} \end{aligned}$$

**Ans**



4-46. Strut  $AB$  of the 1-m-diameter hatch door exerts a force of 450 N on point  $B$ . Determine the moment of this force about point  $O$ .



**Position Vector And Force Vector :**

$$\begin{aligned} \mathbf{r}_{OB} &= \{(0-0)\mathbf{i} + (1\cos 30^\circ - 0)\mathbf{j} + (1\sin 30^\circ - 0)\mathbf{k}\} \text{ m} \\ &= \{0.8660\mathbf{j} + 0.5\mathbf{k}\} \text{ m} \end{aligned}$$

$$\begin{aligned} \mathbf{r}_{OA} &= \{(0.5\sin 30^\circ - 0)\mathbf{i} + (0.5 + 0.5\cos 30^\circ - 0)\mathbf{j} + (0-0)\mathbf{k}\} \text{ m} \\ &= \{0.250\mathbf{i} + 0.9330\mathbf{j}\} \text{ m} \end{aligned}$$

$$\begin{aligned} \mathbf{F} &= 450 \left\{ \frac{(0 - 0.5\sin 30^\circ)\mathbf{i} + [1\cos 30^\circ - (0.5 + 0.5\cos 30^\circ)]\mathbf{j} + (1\sin 30^\circ - 0)\mathbf{k}}{\sqrt{(0 - 0.5\sin 30^\circ)^2 + [1\cos 30^\circ - (0.5 + 0.5\cos 30^\circ)]^2 + (1\sin 30^\circ - 0)^2}} \right\} \text{ N} \\ &= \{-199.82\mathbf{i} - 53.54\mathbf{j} + 399.63\mathbf{k}\} \text{ N} \end{aligned}$$

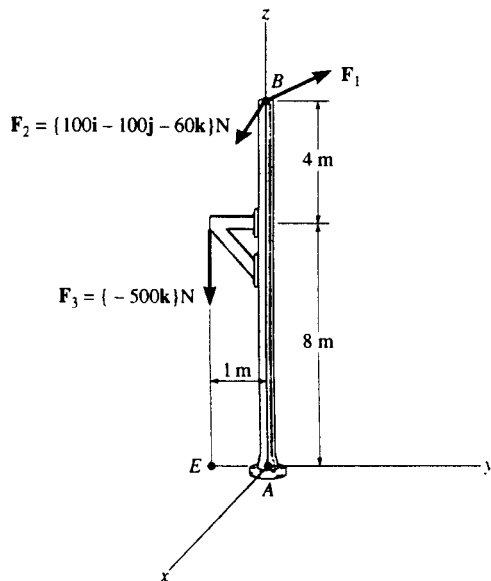
**Moment of Force  $\mathbf{F}$  About Point  $O$  :** Applying Eq. 4-7, we have

$$\begin{aligned} \mathbf{M}_O &= \mathbf{r}_{OB} \times \mathbf{F} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.8660 & 0.5 \\ -199.82 & -53.54 & 399.63 \end{vmatrix} \\ &= \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\} \text{ N}\cdot\text{m} \quad \text{Ans} \end{aligned}$$

Or

$$\begin{aligned} \mathbf{M}_O &= \mathbf{r}_{OA} \times \mathbf{F} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.250 & 0.9330 & 0 \\ -199.82 & -53.54 & 399.63 \end{vmatrix} \\ &= \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\} \text{ N}\cdot\text{m} \end{aligned}$$

4-47. Using Cartesian vector analysis, determine the resultant moment of the three forces about the base of the column at  $A$ . Take  $\mathbf{F}_1 = \{400\mathbf{i} + 300\mathbf{j} + 120\mathbf{k}\} \text{ N}$ .



$$(\mathbf{M}_A)_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 400 & 300 & 120 \end{vmatrix} = \{-3.6\mathbf{i} + 4.8\mathbf{j}\} \text{ kN}\cdot\text{m}$$

$$(\mathbf{M}_A)_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 100 & -100 & -60 \end{vmatrix} = \{1.2\mathbf{i} + 1.2\mathbf{j}\} \text{ kN}\cdot\text{m}$$

$$(\mathbf{M}_A)_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 0 \\ 0 & 0 & -500 \end{vmatrix} = \{0.5\mathbf{i}\} \text{ kN}\cdot\text{m}$$

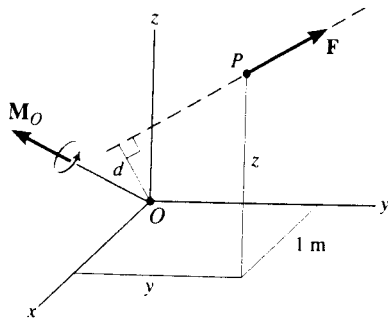
$$M_{Ax} = -3.6 + 1.2 + 0.5 = -1.90 \text{ kN}\cdot\text{m}$$

$$M_{Ay} = 4.8 + 1.2 = 6.00 \text{ kN}\cdot\text{m}$$

$$M_{Az} = 0$$

$$\mathbf{M}_R = \{-1.90\mathbf{i} + 6.00\mathbf{j}\} \text{ kN}\cdot\text{m} \quad \text{Ans}$$

\*4-48. A force of  $\mathbf{F} = \{6\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\}$  kN produces a moment of  $\mathbf{M}_O = \{4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}\}$  kN·m about the origin of coordinates, point  $O$ . If the force acts at a point having an  $x$  coordinate of  $x = 1$  m, determine the  $y$  and  $z$  coordinates.



$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & -2 & 1 \end{vmatrix}$$

$$4 = y + 2z$$

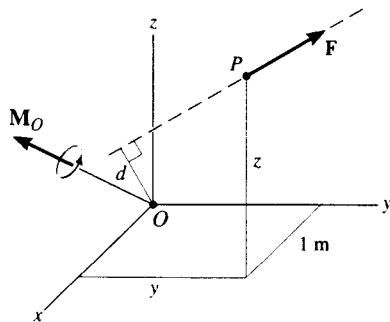
$$5 = -1 + 6z$$

$$-14 = -2 - 6y$$

$$y = 2 \text{ m} \quad \text{Ans}$$

$$z = 1 \text{ m} \quad \text{Ans}$$

4-49. The force  $\mathbf{F} = \{6\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}\}$  N creates a moment about point  $O$  of  $\mathbf{M}_O = \{-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}\}$  N·m. If the force passes through a point having an  $x$  coordinate of 1 m, determine the  $y$  and  $z$  coordinates of the point. Also, realizing that  $M_O = Fd$ , determine the perpendicular distance  $d$  from point  $O$  to the line of action of  $\mathbf{F}$ .



$$-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & 8 & 10 \end{vmatrix}$$

$$-14 = 10y - 8z$$

$$8 = -10 + 6z$$

$$2 = 8 - 6y$$

$$y = 1 \text{ m} \quad \text{Ans}$$

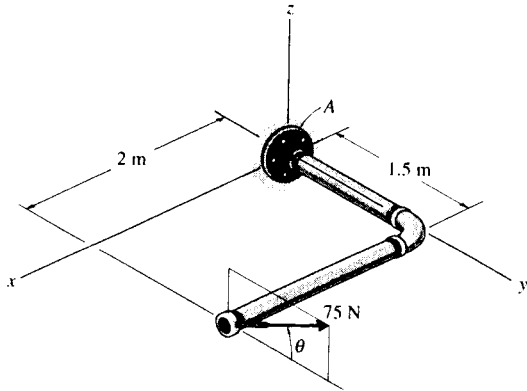
$$z = 3 \text{ m} \quad \text{Ans}$$

$$M_O = \sqrt{(-14)^2 + (8)^2 + (2)^2} = 16.25 \text{ N}\cdot\text{m}$$

$$F = \sqrt{(6)^2 + (8)^2 + (10)^2} = 14.14 \text{ N}$$

$$d = \frac{16.25}{14.14} = 1.15 \text{ m} \quad \text{Ans}$$

4-50. Using a ring collar the 75-N force can act in the vertical plane at various angles  $\theta$ . Determine the magnitude of the moment it produces about point A, plot the result of  $M$  (ordinate) versus  $\theta$  (abscissa) for  $0^\circ \leq \theta \leq 180^\circ$ , and specify the angles that give the maximum and minimum moment.



$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1.5 & 0 \\ 0 & 75 \cos \theta & 75 \sin \theta \end{vmatrix}$$

$$= 112.5 \sin \theta \mathbf{i} - 150 \sin \theta \mathbf{j} + 150 \cos \theta \mathbf{k}$$

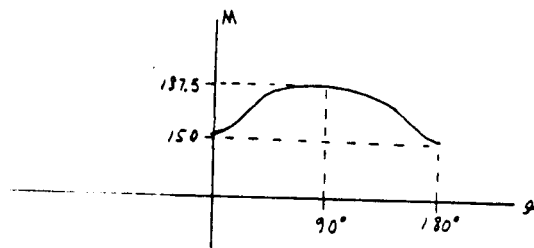
$$M_A = \sqrt{(112.5 \sin \theta)^2 + (-150 \sin \theta)^2 + (150 \cos \theta)^2} = \sqrt{12\,656.25 \sin^2 \theta + 22\,500}$$

$$\frac{dM_A}{d\theta} = \frac{1}{2} (12\,656.25 \sin^2 \theta + 22\,500)^{-1/2} (12\,656.25)(2 \sin \theta \cos \theta) = 0$$

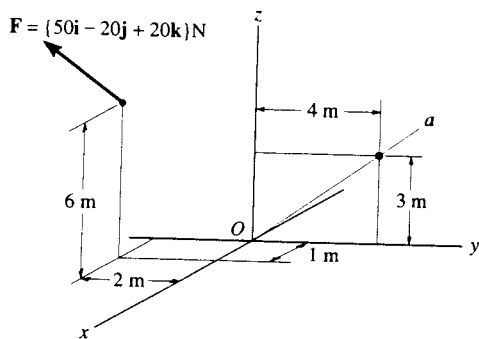
$$\sin \theta \cos \theta = 0; \quad \theta = 0^\circ, 90^\circ, 180^\circ \quad \text{Ans}$$

$$M_{\max} = 187.5 \text{ N} \cdot \text{m} \text{ at } \theta = 90^\circ$$

$$M_{\min} = 150 \text{ N} \cdot \text{m} \text{ at } \theta = 0^\circ, 180^\circ$$



4-51. Determine the moment of the force  $\mathbf{F}$  about the  $Oa$  axis. Express the result as a Cartesian vector.



$$\mathbf{u}_{Oa} = \frac{4}{5} \mathbf{j} + \frac{3}{5} \mathbf{k}$$

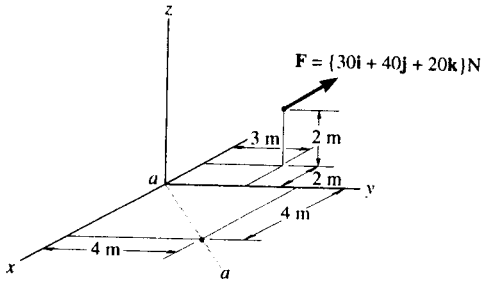
$$(\mathbf{M}_{Oa})_P = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 1 & -2 & 6 \\ 50 & -20 & 20 \end{vmatrix} = 272 \text{ N} \cdot \text{m}$$

$$\begin{aligned} (\mathbf{M}_{Oa})_P &= (\mathbf{M}_{Oa})_P \mathbf{u}_{Oa} \\ &= 272 \left( \frac{4}{5} \mathbf{j} + \frac{3}{5} \mathbf{k} \right) \end{aligned}$$

$$(\mathbf{M}_{Oa})_P = \{ 218\mathbf{j} + 163\mathbf{k} \} \text{ N} \cdot \text{m} \quad \text{Ans}$$



\*4-52. Determine the moment of the force  $F$  about the  $aa$  axis. Express the result as a Cartesian vector.



**Position Vector :**

$$r = \{(-2-0)\mathbf{i} + (3-0)\mathbf{j} + (2-0)\mathbf{k}\} \text{ m} = \{-2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}\} \text{ m}$$

**Unit Vector Along a-a Axis :**

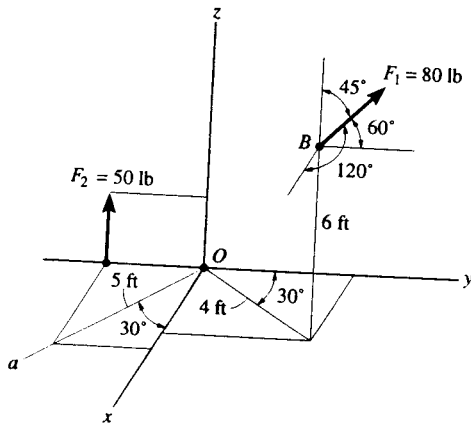
$$u_{aa} = \frac{(4-0)\mathbf{i} + (4-0)\mathbf{j}}{\sqrt{(4-0)^2 + (4-0)^2}} = 0.7071\mathbf{i} + 0.7071\mathbf{j}$$

**Moment of Force  $F$  About a-a Axis :** With  $F = \{30\mathbf{i} + 40\mathbf{j} + 20\mathbf{k}\}$  N, applying Eq. 4-11, we have

$$\begin{aligned} M_{aa} &= u_{aa} \cdot (r \times F) \\ &= \begin{vmatrix} 0.7071 & 0.7071 & 0 \\ -2 & 3 & 2 \\ 30 & 40 & 20 \end{vmatrix} \\ &= 0.7071[3(20) - 40(2)] - 0.7071[(-2)(20) - 30(2)] + 0 \\ &= 56.6 \text{ N} \cdot \text{m} \end{aligned}$$

**Ans**

4-53. Determine the resultant moment of the two forces about the  $Oa$  axis. Express the result as a Cartesian vector.



$$F_1 = 80(\cos 120^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 45^\circ \mathbf{k}) = \{-40\mathbf{i} + 40\mathbf{j} + 56.569\mathbf{k}\} \text{ lb}$$

$$F_2 = \{50\mathbf{k}\} \text{ lb}$$

$$\begin{aligned} r_1 &= (4 \sin 30^\circ - 0)\mathbf{i} + (4 \cos 30^\circ - 0)\mathbf{j} + (6 - 0)\mathbf{k} \\ &= \{2\mathbf{i} + 3.464\mathbf{j} + 6\mathbf{k}\} \text{ ft} \end{aligned}$$

$$r_2 = (-5 \sin 30^\circ)\mathbf{j} = \{-2.5\mathbf{j}\} \text{ ft}$$

$$M_R = r_1 \times F_1 + r_2 \times F_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3.464 & 6 \\ -40 & 40 & 56.569 \end{vmatrix} + (-2.5\mathbf{j}) \times (50\mathbf{k})$$

$$\begin{aligned} &= [3.464(56.569) - 40(6)]\mathbf{i} - [2(56.569) - (-40)(6)]\mathbf{j} + [2(40) - (-40)(3.464)]\mathbf{k} - 125\mathbf{i} \\ &= \{-169.044\mathbf{i} - 353.138\mathbf{j} + 218.560\mathbf{k}\} \text{ lb} \cdot \text{ft} \end{aligned}$$

$$u_{Oa} = \cos 30^\circ \mathbf{i} - \sin 30^\circ \mathbf{j} = 0.8660\mathbf{i} - 0.5\mathbf{j}$$

$$(M_R)_{Oa} = u_{Oa} \cdot M_R = (0.8660\mathbf{i} - 0.5\mathbf{j}) \cdot (-169.044\mathbf{i} - 353.138\mathbf{j} + 218.560\mathbf{k})$$

$$\begin{aligned} &= (0.8660)(-169.044) + (-0.5)(-353.138) + 0(218.560) \\ &= 30.173 \text{ lb} \cdot \text{ft} \end{aligned}$$

$$(M_R)_{Oa} = (M_R)_{Oa} u_{Oa} = 30.173(0.8660\mathbf{i} - 0.5\mathbf{j})$$

$$= \{26.1\mathbf{i} - 15.1\mathbf{j}\} \text{ lb} \cdot \text{ft}$$

**Ans**

**\*4-52.** Determine the moment of the force  $F$  about the  $aa$  axis. Express the result as a Cartesian vector.

**Position Vector:**

$$\mathbf{r} = [(-2 - 0)\mathbf{i} + (3 - 0)\mathbf{j} + (2 - 0)\mathbf{k}] \text{ m} = \{-2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}\} \text{ m}$$

**Unit Vector Along  $a - a$  Axis:**

$$\mathbf{u}_{aa} = \frac{(4 - 0)\mathbf{i} + (4 - 0)\mathbf{j}}{\sqrt{(4 - 0)^2 + (4 - 0)^2}} = 0.7071\mathbf{i} + 0.7071\mathbf{j}$$

**Moment of Force  $F$  About  $a - a$  Axis:** With  $\mathbf{F} = \{30\mathbf{i} + 40\mathbf{j} + 20\mathbf{k}\}$  N, applying Eq. 4-11, we have

$$M_{aa} = \mathbf{u}_{aa} \cdot (\mathbf{r} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.7071 & 0.7071 & 0 \\ -2 & 3 & 2 \\ 30 & 40 & 20 \end{vmatrix}$$

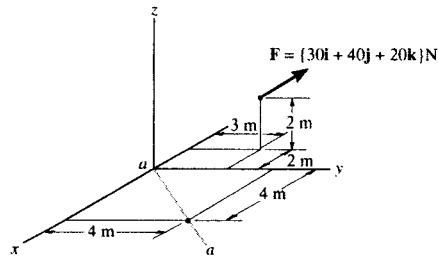
$$= 0.7071[3(20) - 40(2)] - 0.7071[(-2)(20) - 30(2)] + 0$$

$$= 56.6 \text{ N} \cdot \text{m} \quad \text{Ans}$$

$$\mathbf{M}_{aa} = M_{aa}\mathbf{u}_{aa}$$

$$= 56.57(0.7071\mathbf{i} + 0.7071\mathbf{j})$$

$$= \{40\mathbf{i} + 40\mathbf{j}\} \text{ N} \cdot \text{m} \quad \text{Ans}$$



**4-53.** Determine the resultant moment of the two forces about the  $Oa$  axis. Express the result as a Cartesian vector.

$$\mathbf{F}_1 = 80(\cos 120^\circ\mathbf{i} + \cos 60^\circ\mathbf{j} + \cos 45^\circ\mathbf{k}) = \{-40\mathbf{i} + 40\mathbf{j} + 56.569\mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_2 = \{50\mathbf{k}\} \text{ lb}$$

$$\mathbf{r}_1 = (4 \sin 30^\circ - 0)\mathbf{i} + (4 \cos 30^\circ - 0)\mathbf{j} + (6 - 0)\mathbf{k}$$

$$= \{2\mathbf{i} + 3.464\mathbf{j} + 6\mathbf{k}\} \text{ ft}$$

$$\mathbf{r}_2 = (-5 \sin 30^\circ)\mathbf{j} = \{-2.5\mathbf{j}\} \text{ ft}$$

$$\mathbf{M}_R = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3.464 & 6 \\ -40 & 40 & 56.569 \end{vmatrix} + (-2.5\mathbf{j}) \times (50\mathbf{k})$$

$$= [3.464(56.569) - 40(6)]\mathbf{i} - [2(56.569) - (-40)(6)]\mathbf{j} + [2(40) - (-40)(3.464)]\mathbf{k} - 125\mathbf{i}$$

$$= \{-169.044\mathbf{i} - 353.138\mathbf{j} + 218.560\mathbf{k}\} \text{ lb} \cdot \text{ft}$$

$$\mathbf{u}_{Oa} = \cos 30^\circ\mathbf{i} - \sin 30^\circ\mathbf{j} = 0.8660\mathbf{i} - 0.5\mathbf{j}$$

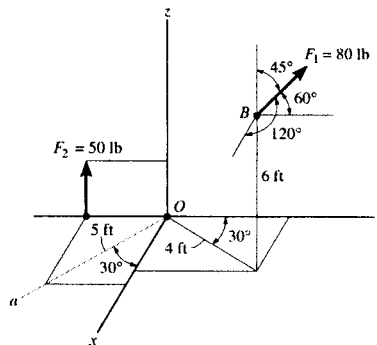
$$(\mathbf{M}_R)_{Oa} = \mathbf{u}_{Oa} \cdot \mathbf{M}_R = (0.8660\mathbf{i} - 0.5\mathbf{j}) \cdot (-169.044\mathbf{i} - 353.138\mathbf{j} + 218.560\mathbf{k})$$

$$= (0.8660)(-169.044) + (-0.5)(-353.138) + 0(218.560)$$

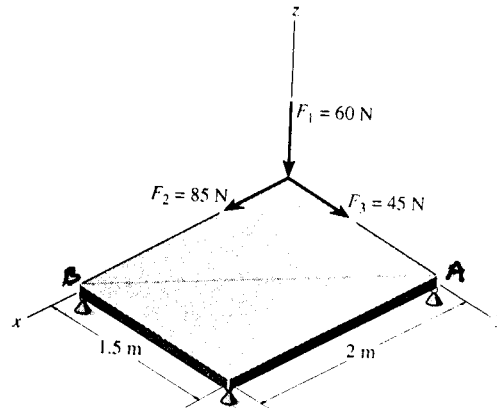
$$= 30.173 \text{ lb} \cdot \text{ft}$$

$$(\mathbf{M}_R)_{Oa} = (\mathbf{M}_R)_{Oa}\mathbf{u}_{Oa} = 30.173(0.8660\mathbf{i} - 0.5\mathbf{j})$$

$$= \{26.1\mathbf{i} - 15.1\mathbf{j}\} \text{ lb} \cdot \text{ft} \quad \text{Ans}$$



4-54. Determine the magnitude of the moment of each of the three forces about the axis  $AB$ . Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.



**a) Vector Analysis**

**Position Vector and Force Vector :**

$$\begin{aligned} \mathbf{r}_1 &= \{-1.5\mathbf{j}\} \text{ m} & \mathbf{r}_2 &= \mathbf{r}_3 = \mathbf{0} \\ \mathbf{F}_1 &= \{-60\mathbf{k}\} \text{ N} & \mathbf{F}_2 &= \{85\mathbf{i}\} \text{ N} & \mathbf{F}_3 &= \{45\mathbf{j}\} \text{ N} \end{aligned}$$

**Unit Vector Along  $AB$  Axis :**

$$\mathbf{u}_{AB} = \frac{(2-0)\mathbf{i} + (0-1.5)\mathbf{j}}{\sqrt{(2-0)^2 + (0-1.5)^2}} = 0.8\mathbf{i} - 0.6\mathbf{j}$$

**Moment of Each Force About  $AB$  Axis :** Applying Eq. 4-11, we have

$$(M_{AB})_1 = \mathbf{u}_{AB} \cdot (\mathbf{r}_1 \times \mathbf{F}_1)$$

$$= \begin{vmatrix} 0.8 & -0.6 & 0 \\ 0 & -1.5 & 0 \\ 0 & 0 & -60 \end{vmatrix}$$

$$= 0.8[(-1.5)(-60) - 0] - 0 + 0 = 72.0 \text{ N} \cdot \text{m} \quad \text{Ans}$$

$$(M_{AB})_2 = \mathbf{u}_{AB} \cdot (\mathbf{r}_2 \times \mathbf{F}_2)$$

$$= \begin{vmatrix} 0.8 & -0.6 & 0 \\ 0 & 0 & 0 \\ 85 & 0 & 0 \end{vmatrix} = 0$$

**Ans**

$$(M_{AB})_3 = \mathbf{u}_{AB} \cdot (\mathbf{r}_3 \times \mathbf{F}_3)$$

$$= \begin{vmatrix} 0.8 & -0.6 & 0 \\ 0 & 0 & 0 \\ 0 & 45 & 0 \end{vmatrix} = 0$$

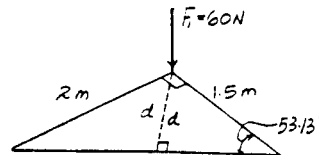
**Ans**

**b) Scalar Analysis :** Since moment arm from force  $F_2$  and  $F_3$  is equal to zero, Hence

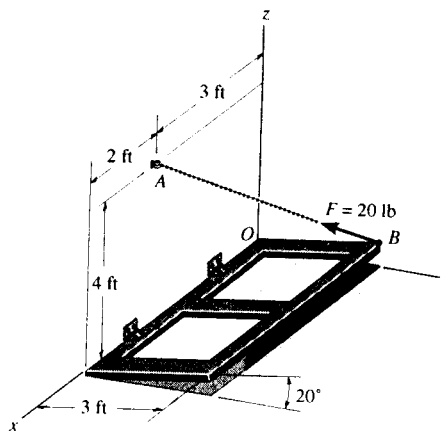
$$(M_{AB})_2 = (M_{AB})_3 = 0 \quad \text{Ans}$$

Moment arm  $d$  from force  $F_1$  to axis  $AB$  is  $d = 1.5 \sin 53.13^\circ = 1.20 \text{ m}$ , Hence

$$(M_{AB})_1 = F_1 d = 60(1.20) = 72.0 \text{ N} \cdot \text{m} \quad \text{Ans}$$



4-55. The chain  $AB$  exerts a force of 20 lb on the door at  $B$ . Determine the magnitude of the moment of this force along the hinged axis  $x$  of the door.



**Position Vector and Force Vector :**

$$\begin{aligned} \mathbf{r}_{OA} &= \{(3-0)\mathbf{i} + (4-0)\mathbf{k}\} \text{ ft} = \{3\mathbf{i} + 4\mathbf{k}\} \text{ ft} \\ \mathbf{r}_{OB} &= \{(0-0)\mathbf{i} + (3\cos 20^\circ - 0)\mathbf{j} + (3\sin 20^\circ - 0)\mathbf{k}\} \text{ ft} \\ &= \{2.8191\mathbf{j} + 1.0261\mathbf{k}\} \text{ ft} \end{aligned}$$

$$\begin{aligned} \mathbf{F} &= 20 \left( \frac{(3-0)\mathbf{i} + (0-3\cos 20^\circ)\mathbf{j} + (4-3\sin 20^\circ)\mathbf{k}}{\sqrt{(3-0)^2 + (0-3\cos 20^\circ)^2 + (4-3\sin 20^\circ)^2}} \right) \text{ lb} \\ &= \{11.814\mathbf{i} - 11.102\mathbf{j} + 11.712\mathbf{k}\} \text{ lb} \end{aligned}$$

**Moment of Force  $\mathbf{F}$  About  $x$  Axis :** The unit vector along the  $x$  axis is  $\mathbf{i}$ . Applying Eq. 4-11, we have

$$\begin{aligned} M_x &= \mathbf{i} \cdot (\mathbf{r}_{OA} \times \mathbf{F}) \\ &= \begin{vmatrix} 1 & 0 & 0 \\ 3 & 0 & 4 \\ 11.814 & -11.102 & 11.712 \end{vmatrix} \end{aligned}$$

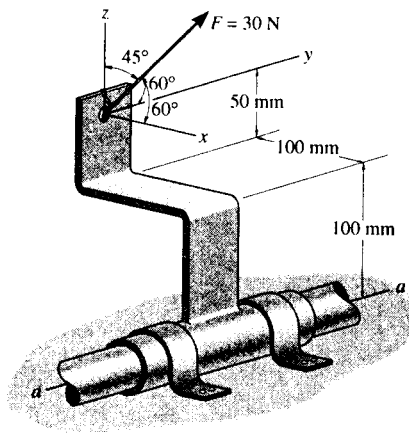
$$\begin{aligned} &= 1[0(11.712) - (-11.102)(4)] - 0 + 0 \\ &= 44.4 \text{ lb} \cdot \text{ft} \end{aligned}$$

**Ans**

Or

$$\begin{aligned} M_x &= \mathbf{i} \cdot (\mathbf{r}_{OB} \times \mathbf{F}) \\ &= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2.8191 & 1.0261 \\ 11.814 & -11.102 & 11.712 \end{vmatrix} \\ &= 1[2.8191(11.712) - (-11.102)(1.0261)] - 0 + 0 \\ &= 44.4 \text{ lb} \cdot \text{ft} \end{aligned}$$

**\*4-56.** The force of  $F = 30 \text{ N}$  acts on the bracket as shown. Determine the moment of the force about the  $a$ - $a$  axis of the pipe. Also, determine the coordinate direction angles of  $F$  in order to produce the maximum moment about the  $a$ - $a$  axis. What is this moment?



$$\mathbf{F} = 30 (\cos 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 45^\circ \mathbf{k})$$

$$= (15 \mathbf{i} + 15 \mathbf{j} + 21.21 \mathbf{k}) \text{ N}$$

$$\mathbf{r} = (-0.1 \mathbf{i} + 0.15 \mathbf{k}) \text{ m}$$

$$\mathbf{u} = \mathbf{j}$$

$$M_a = \begin{vmatrix} 0 & 1 & 0 \\ -0.1 & 0 & 0.15 \\ 15 & 15 & 21.21 \end{vmatrix} = 4.37 \text{ N} \cdot \text{m} \quad \text{Ans}$$

$\mathbf{F}$  must be perpendicular to  $\mathbf{u}$  and  $\mathbf{r}$ .

$$\mathbf{u}_F = \frac{0.15}{0.1803} \mathbf{i} + \frac{0.1}{0.1803} \mathbf{k}$$

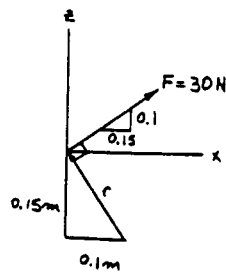
$$= 0.8321 \mathbf{i} + 0.5547 \mathbf{k}$$

$$\alpha = \cos^{-1} 0.8321 = 33.7^\circ \quad \text{Ans}$$

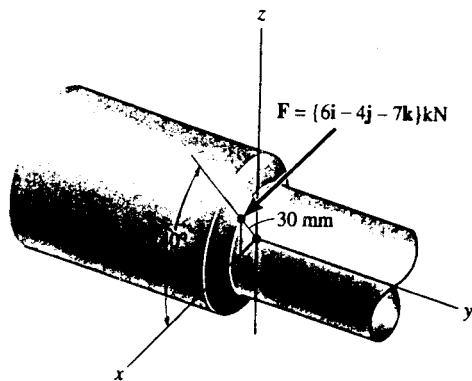
$$\beta = \cos^{-1} 0 = 90^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1} 0.5547 = 56.3^\circ \quad \text{Ans}$$

$$M = 30 (0.1803) = 5.41 \text{ N} \cdot \text{m} \quad \text{Ans}$$



4-57. The cutting tool on the lathe exerts a force  $F$  on the shaft in the direction shown. Determine the moment of this force about the  $y$  axis of the shaft.



$$M_y = \mathbf{u}_y \cdot (\mathbf{r} \times \mathbf{F})$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ 0.03 \cos 40^\circ & 0 & 0.03 \sin 40^\circ \\ 6 & -4 & -7 \end{vmatrix}$$

$$M_y = 276.57 \text{ N}\cdot\text{mm} = 0.277 \text{ N}\cdot\text{m} \quad \text{Ans}$$

4-58. The hood of the automobile is supported by the strut  $AB$ , which exerts a force of  $F = 24 \text{ lb}$  on the hood. Determine the moment of this force about the hinged axis  $y$ .

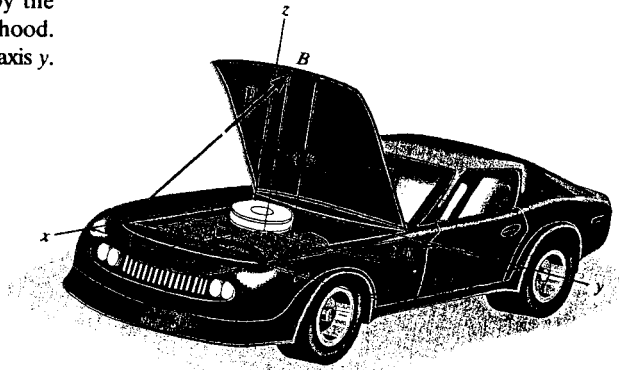
$$\mathbf{r} = \{4\mathbf{i}\} \text{ m}$$

$$\mathbf{F} = 24 \left( \frac{-2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}}{\sqrt{(-2)^2 + (2)^2 + (4)^2}} \right)$$

$$= \{-9.80\mathbf{i} + 9.80\mathbf{j} + 19.60\mathbf{k}\} \text{ lb}$$

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ 4 & 0 & 0 \\ -9.80 & 9.80 & 19.60 \end{vmatrix} = -78.4 \text{ lb}\cdot\text{ft}$$

$$M_y = \{-78.4\mathbf{j}\} \text{ lb}\cdot\text{ft} \quad \text{Ans}$$



4-59. Determine the magnitude of the moments of the force  $F$  about the  $x$ ,  $y$ , and  $z$  axes. Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.

a) **Vector Analysis**

Position Vector :

$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\} \text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\} \text{ ft}$$

Moment of Force  $F$  About  $x$ ,  $y$  and  $z$  Axes : The unit vectors along  $x$ ,  $y$  and  $z$  axes are  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  respectively. Applying Eq. 4-11, we have

$$M_x = \mathbf{i} \cdot (\mathbf{r}_{AB} \times \mathbf{F})$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 1[3(-3) - (12)(-2)] - 0 + 0 = 15.0 \text{ lb}\cdot\text{ft} \quad \text{Ans}$$

$$M_y = \mathbf{j} \cdot (\mathbf{r}_{AB} \times \mathbf{F})$$

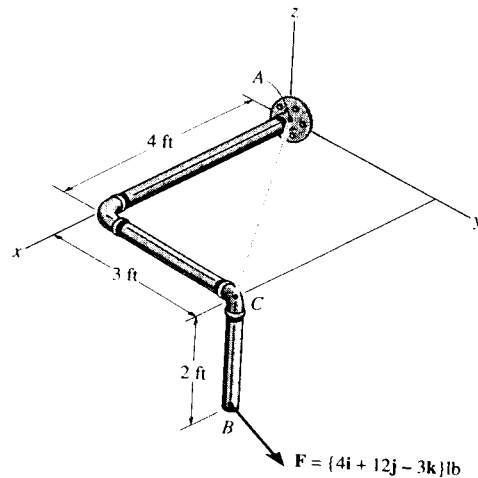
$$= \begin{vmatrix} 0 & 1 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 1[4(-3) - (4)(-2)] + 0 = 4.00 \text{ lb}\cdot\text{ft} \quad \text{Ans}$$

$$M_z = \mathbf{k} \cdot (\mathbf{r}_{AB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 0 + 1[4(12) - 4(3)] = 36.0 \text{ lb}\cdot\text{ft} \quad \text{Ans}$$



b) **Scalar Analysis**

$$M_x = \Sigma M_x; \quad M_x = 12(2) - 3(3) = 15.0 \text{ lb}\cdot\text{ft} \quad \text{Ans}$$

$$M_y = \Sigma M_y; \quad M_y = -4(2) + 3(4) = 4.00 \text{ lb}\cdot\text{ft} \quad \text{Ans}$$

$$M_z = \Sigma M_z; \quad M_z = -4(3) + 12(4) = 36.0 \text{ lb}\cdot\text{ft} \quad \text{Ans}$$

**\*4-60.** Determine the moment of the force  $\mathbf{F}$  about an axis extending between  $A$  and  $C$ . Express the result as a Cartesian vector.

**Position Vector :**

$$\mathbf{r}_{CB} = \{-2\mathbf{k}\} \text{ ft}$$

$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\} \text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\} \text{ ft}$$

**Unit Vector Along AC Axis :**

$$\mathbf{u}_{AC} = \frac{(4-0)\mathbf{i} + (3-0)\mathbf{j}}{\sqrt{(4-0)^2 + (3-0)^2}} = 0.8\mathbf{i} + 0.6\mathbf{j}$$

**Moment of Force  $\mathbf{F}$  About AC Axis :** With  $\mathbf{F} = \{4\mathbf{i} + 12\mathbf{j} - 3\mathbf{k}\}$  lb, applying Eq. 4-11, we have

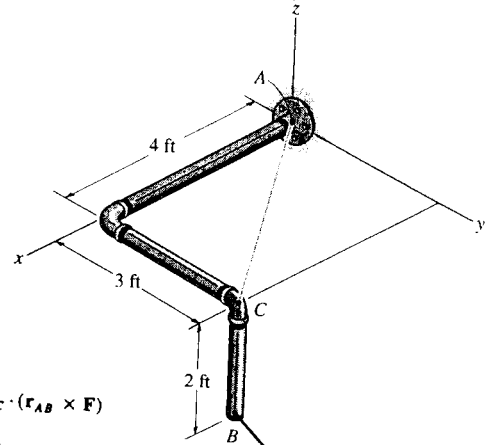
$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{CB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 0 & 0 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(0)(-3) - 12(-2)] - 0.6[0(-3) - 4(-2)] + 0$$

$$= 14.4 \text{ lb} \cdot \text{ft}$$

Or



$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{AB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(3)(-3) - 12(-2)] - 0.6[4(-3) - 4(-2)] + 0$$

$$= 14.4 \text{ lb} \cdot \text{ft}$$

Expressing  $M_{AC}$  as a Cartesian vector yields

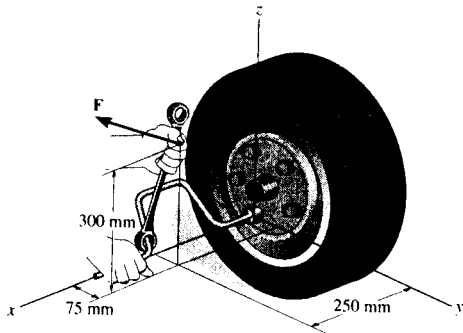
$$M_{AC} = M_{AC} \mathbf{u}_{AC}$$

$$= 14.4(0.8\mathbf{i} + 0.6\mathbf{j})$$

$$= \{11.5\mathbf{i} + 8.64\mathbf{j}\} \text{ lb} \cdot \text{ft}$$

Ans

**4-61.** The lug and box wrenches are used in combination to remove the lug nut from the wheel hub. If the applied force on the end of the box wrench is  $\mathbf{F} = \{4\mathbf{i} - 12\mathbf{j} + 2\mathbf{k}\}$  N, determine the magnitude of the moment of this force about the  $x$  axis which is effective in unscrewing the lug nut.



**Position Vector and Force Vector :**

$$\mathbf{r} = \{(0.075-0)\mathbf{j} + (0.3-0)\mathbf{k}\} \text{ m} = \{0.075\mathbf{j} + 0.3\mathbf{k}\} \text{ m}$$

**Moment of Force  $\mathbf{F}$  About  $x$  Axis :** The unit vector along  $x$  axis is  $\mathbf{i}$ . With  $\mathbf{F} = \{4\mathbf{i} - 12\mathbf{j} + 2\mathbf{k}\}$  N, applying Eq. 4-11, we have

$$M_x = \mathbf{i} \cdot (\mathbf{r} \times \mathbf{F})$$

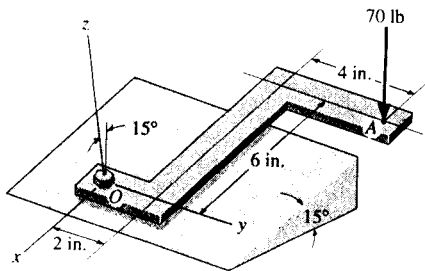
$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.075 & 0.3 \\ 4 & -12 & 2 \end{vmatrix}$$

$$= 1[0.075(2) - (-12)(0.3)] - 0 + 0$$

$$= 3.75 \text{ N} \cdot \text{m}$$

Ans

4-62. A 70-lb force acts vertically on the "Z" bracket. Determine the magnitude of the moment of this force about the bolt axis (z axis).



**Position Vector And Force Vector :**

$$r_{OA} = \{(-6-0)\mathbf{i} + (6-0)\mathbf{j}\} \text{ in.} = \{-6\mathbf{i} + 6\mathbf{j}\} \text{ in.}$$

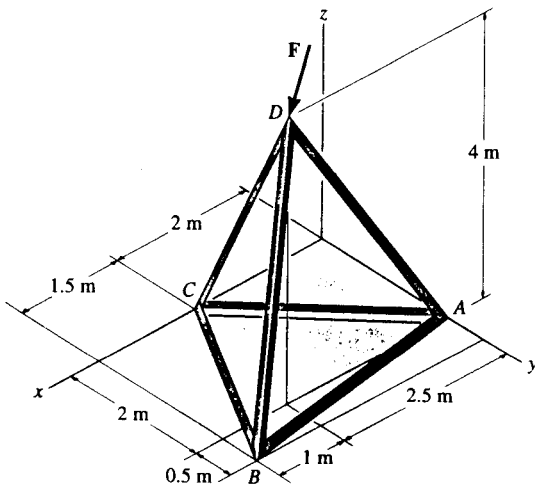
$$\mathbf{F} = 70(\sin 15^\circ\mathbf{i} - \cos 15^\circ\mathbf{k}) \text{ lb} = \{18.117\mathbf{i} - 67.615\mathbf{k}\} \text{ lb}$$

**Moment of Force F About z Axis :** The unit vector along z axis is  $\mathbf{k}$ . Applying Eq. 4-11, we have

$$\begin{aligned} M_z &= \mathbf{k} \cdot (r_{OA} \times \mathbf{F}) \\ &= \begin{vmatrix} 0 & 0 & 1 \\ -6 & 6 & 0 \\ 18.117 & 0 & -67.615 \end{vmatrix} \\ &= 0 - 0 + 1[(-6)(0) - (6)(18.117)] \\ &= -109 \text{ lb} \cdot \text{in} \end{aligned}$$

Negative sign indicates that  $M_z$  is directed toward negative z axis.  
 $M_z = 109 \text{ lb} \cdot \text{in}$

4-63. Determine the magnitude of the moment of the force  $\mathbf{F} = \{50\mathbf{i} - 20\mathbf{j} - 80\mathbf{k}\} \text{ N}$  about the base line CA of the tripod.



$$u_{CA} = \frac{\{-2\mathbf{i} + 2\mathbf{j}\}}{\sqrt{(-2)^2 + (2)^2}}$$

$$u_{CA} = \{-0.707\mathbf{i} + 0.707\mathbf{j}\}$$

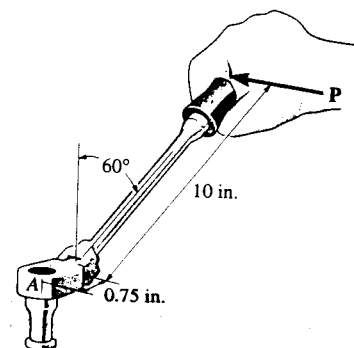
$$M_{CA} = u_{CA} \cdot (r_{AD} \times \mathbf{F}) = \begin{vmatrix} -0.707 & 0.707 & 0 \\ 2.5 & 0 & 4 \\ 50 & -20 & -80 \end{vmatrix}$$

$$|M_{CA}| = 226 \text{ N} \cdot \text{m} \quad \text{Ans}$$

\*4-64. The flex-headed ratchet wrench is subjected to a force of  $P = 16 \text{ lb}$ , applied perpendicular to the handle as shown. Determine the moment or torque this imparts along the vertical axis of the bolt at A.

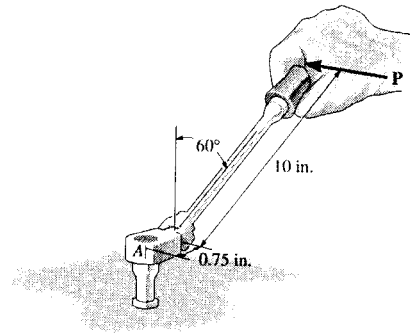
$$M = 16(0.75 + 10\sin 60^\circ)$$

$$M = 151 \text{ lb} \cdot \text{in.} \quad \text{Ans}$$





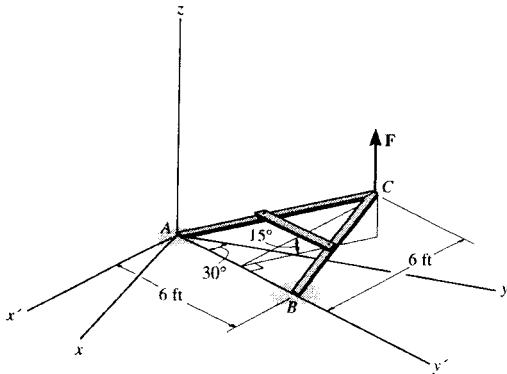
4-65. If a torque or moment of 80 lb · in is required to looser the bolt at A, determine the force P that must be applied perpendicular to the handle of the flex-headed ratchet wrench.



$$80 = P(0.75 + 10 \sin 60^\circ)$$

$$P = \frac{80}{9.41} = 8.50 \text{ lb} \quad \text{Ans}$$

4-66. The A-frame is being hoisted into an upright position by the vertical force of  $F = 80 \text{ lb}$ . Determine the moment of this force about the  $y$  axis when the frame is in the position shown.



Using  $x', y', z$ :

$$u_x = -\sin 30^\circ i' + \cos 30^\circ j'$$

$$r_{AC} = -6 \cos 15^\circ i' + 3j' + 6 \sin 15^\circ k$$

$$F = 80k$$

$$M_y = \begin{vmatrix} -\sin 30^\circ & \cos 30^\circ & 0 \\ -6 \cos 15^\circ & 3 & 6 \sin 15^\circ \\ 0 & 0 & 80 \end{vmatrix} = -120 + 401.52 + 0$$

$$M_y = 282 \text{ lb} \cdot \text{ft} \quad \text{Ans}$$

Also, using  $x, y, z$ :

Coordinates of point C:

$$x = 3 \sin 30^\circ - 6 \cos 15^\circ \cos 30^\circ = -3.52 \text{ ft}$$

$$y = 3 \cos 30^\circ + 6 \cos 15^\circ \sin 30^\circ = 5.50 \text{ ft}$$

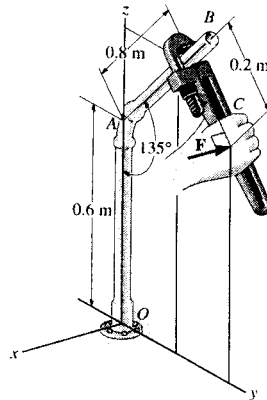
$$z = 6 \sin 15^\circ = 1.55 \text{ ft}$$

$$r_{AC} = -3.52i + 5.50j + 1.55k$$

$$F = 80k$$

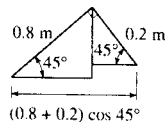
$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix} = 282 \text{ lb} \cdot \text{ft} \quad \text{Ans}$$

4-67. A horizontal force of  $F = \{-50i\} \text{ N}$  is applied perpendicular to the handle of the pipe wrench. Determine the moment that this force exerts along the axis OA ( $z$  axis) of the pipe assembly. Both the wrench and pipe assembly OABC lie in the  $y$ - $z$  plane. *Suggestion:* Use a scalar analysis.

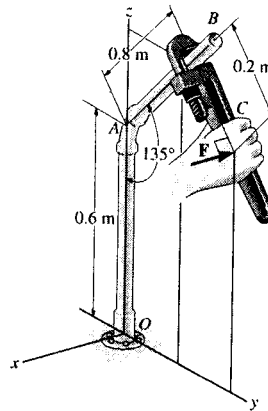


$$M_z = 50(0.8 + 0.2) \cos 45^\circ = 35.36 \text{ N} \cdot \text{m}$$

$$M_z = \{35.4k\} \text{ N} \cdot \text{m} \quad \text{Ans}$$



**\*4-68.** Determine the magnitude of the horizontal force  $\mathbf{F} = -F\mathbf{i}$  acting on the handle of the wrench so that this force produces a component of moment along the  $OA$  axis ( $z$  axis) of the pipe assembly of  $M_z = \{4\mathbf{k}\}\text{N}\cdot\text{m}$ . Both the wrench and the pipe assembly,  $OABC$ , lie in the  $y-z$  plane. *Suggestion:* Use a scalar analysis.



$$M_z = F(0.8 + 0.2) \cos 45^\circ = 4$$

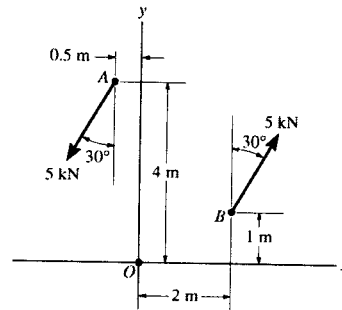
$$F = 5.66 \text{ N} \quad \text{Ans}$$

**4-69.** Determine the magnitude and sense of the couple moment.

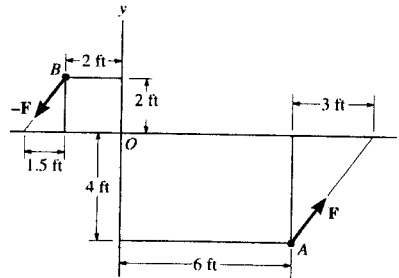
About point  $A$ ,

$$\curvearrowleft +M_C = 5 \cos 30^\circ (2.5) + 5 \sin 30^\circ (3)$$

$$M_C = 18.3 \text{ kN}\cdot\text{m} \quad \curvearrowright \quad \text{Ans}$$



**4-70.** Determine the magnitude and sense of the couple moment. Each force has a magnitude of  $F = 65 \text{ lb}$ .

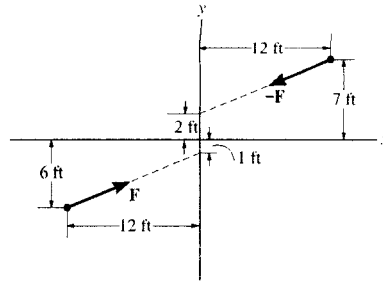


$$\begin{aligned} \curvearrowleft +M_C = \Sigma M_B: \quad M_C &= 65 \left(\frac{4}{5}\right) (6+2) + 65 \left(\frac{3}{5}\right) (4+2) \\ &= 650 \text{ lb}\cdot\text{ft} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$

**4-71.** Determine the magnitude and sense of the couple moment. Each force has a magnitude of  $F = 8 \text{ kN}$ .

$$\begin{aligned} \curvearrowleft +M_C = \Sigma M_B: \quad M_C &= 8 \left(\frac{3}{5}\right) (5+4) - 8 \left(\frac{4}{5}\right) (3+1) \\ &= 17.6 \text{ kN}\cdot\text{m} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$

\*4-72. If the couple moment has a magnitude of  $300 \text{ lb} \cdot \text{ft}$ , determine the magnitude  $F$  of the couple forces.

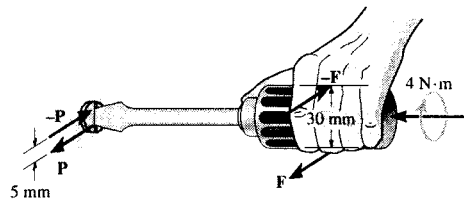


$$300 = F \left( \frac{12}{13} \right) (13) - F \left( \frac{5}{13} \right) (24)$$

$$F = 108 \text{ lb}$$

Ans

4-73. A twist of  $4 \text{ N} \cdot \text{m}$  is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces  $F$  exerted on the handle and  $P$  exerted on the blade.



For the handle

$$M_C = \Sigma M_x; \quad F(0.03) = 4$$

$$F = 133 \text{ N}$$

Ans

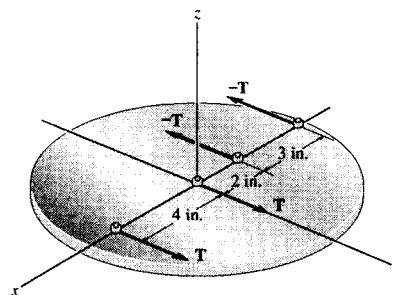
For the blade,

$$M_C = \Sigma M_x; \quad P(0.005) = 4$$

$$P = 800 \text{ N}$$

Ans

4-74. The resultant couple moment created by the two couples acting on the disk is  $\mathbf{M}_R = \{10\mathbf{k}\} \text{ kip} \cdot \text{in}$ . Determine the magnitude of force  $T$ .

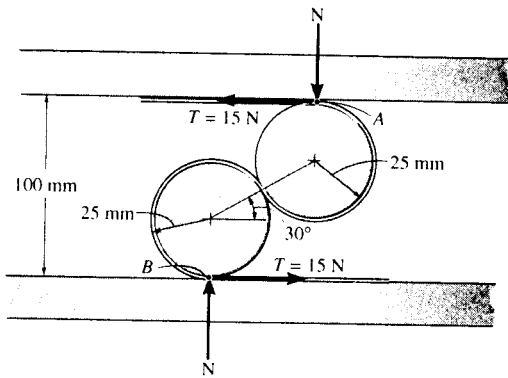


$$M_R = \Sigma M_z; \quad 10 = T(9) + T(2)$$

$$T = 0.909 \text{ kip}$$

Ans

4-75. A device called a rolamite is used in various ways to replace slipping motion with rolling motion. If the belt, which wraps between the rollers, is subjected to a tension of 15 N, determine the reactive forces  $N$  of the top and bottom plates on the rollers so that the resultant couple acting on the rollers is equal to zero.

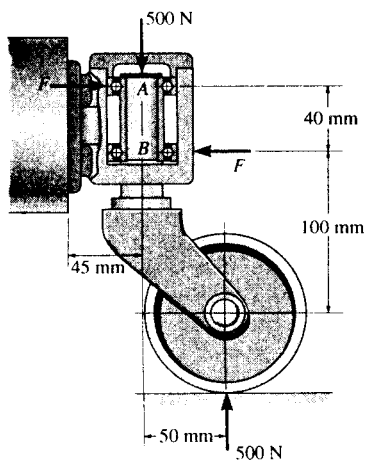


$$\left( +\Sigma M_A = 0; \quad 15(50 + 50\sin 30^\circ) - N(50\cos 30^\circ) = 0 \right.$$

$$N = 26.0 \text{ N}$$

Ans

4-76. The caster wheel is subjected to the two couples. Determine the forces  $F$  that the bearings create on the shaft so that the resultant couple moment on the caster is zero.

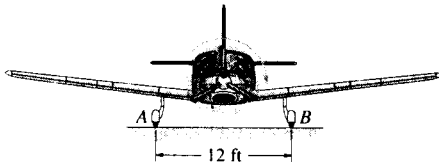


$$\left( +\Sigma M_A = 0; \quad 500(50) - F(40) = 0 \right.$$

$$F = 625 \text{ N}$$

Ans

4-77. When the engine of the plane is running, the vertical reaction that the ground exerts on the wheel at  $A$  is measured as 650 lb. When the engine is turned off, however, the vertical reactions at  $A$  and  $B$  are 575 lb each. The difference in readings at  $A$  is caused by a couple acting on the propeller when the engine is running. This couple tends to overturn the plane counterclockwise, which is opposite to the propeller's clockwise rotation. Determine the magnitude of this couple and the magnitude of the reaction force exerted at  $B$  when the engine is running.



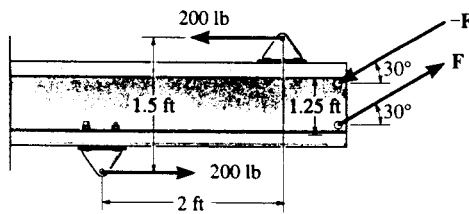
When the engine of the plane is turned on, the resulting couple moment exerts an additional force of  $F = 650 - 575 = 75.0$  lb on wheel  $A$  and a lesser the reactive force on wheel  $B$  of  $F = 75.0$  lb as well. Hence,

$$M = 75.0(12) = 900 \text{ lb} \cdot \text{ft} \quad \text{Ans}$$

The reactive force at wheel  $B$  is

$$R_B = 575 - 75.0 = 500 \text{ lb} \quad \text{Ans}$$

4-78. Two couples act on the beam. Determine the magnitude of  $F$  so that the resultant couple moment is  $450 \text{ lb} \cdot \text{ft}$ , counterclockwise. Where on the beam does the resultant couple moment act?

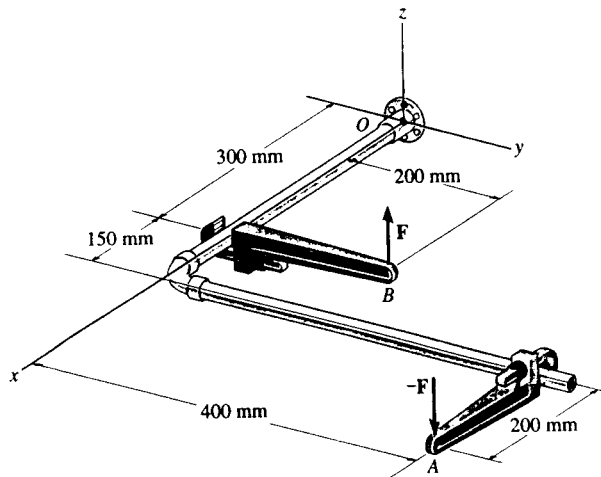


$$(+M_R = \Sigma M; \quad 450 = 200(1.5) + F \cos 30^\circ(1.25)$$

$$F = 139 \text{ lb} \quad \text{Ans}$$

The resultant couple moment is a free vector. It can act at any point on the beam.

4-79. Express the moment of the couple acting on the pipe assembly in Cartesian vector form. Solve the problem (a) using Eq. 4-13, and (b) summing the moment of each force about point  $O$ . Take  $F = \{25\mathbf{k}\}$  N.



$$(a) \quad M_C = r_{AB} \times (25\mathbf{k})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix}$$

$$M_C = \{-5\mathbf{i} + 8.75\mathbf{j}\} \text{ N}\cdot\text{m} \quad \text{Ans}$$

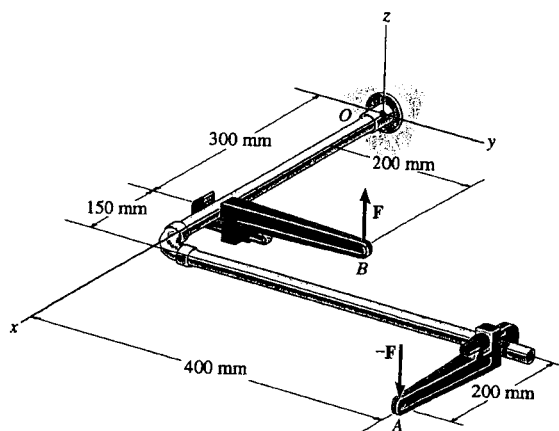
$$(b) \quad M_C = r_{OB} \times (25\mathbf{k}) + r_{OA} \times (-25\mathbf{k})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0.4 & 0 \\ 0 & 0 & -25 \end{vmatrix}$$

$$M_C = (5 - 10)\mathbf{i} + (-7.5 + 16.25)\mathbf{j}$$

$$M_C = \{-5\mathbf{i} + 8.75\mathbf{j}\} \text{ N}\cdot\text{m} \quad \text{Ans}$$

\*4-80. If the couple moment acting on the pipe has a magnitude of  $400 \text{ N}\cdot\text{m}$ , determine the magnitude  $F$  of the vertical force applied to each wrench.



$$(a) \quad M_C = r_{AB} \times (F\mathbf{k})$$

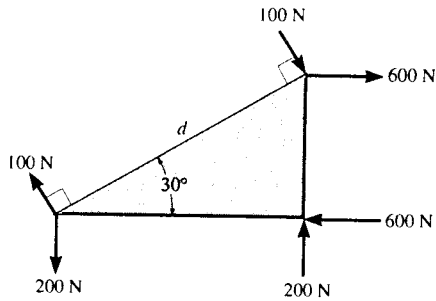
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & F \end{vmatrix}$$

$$M_C = \{-0.2F\mathbf{i} + 0.35F\mathbf{j}\} \text{ N}\cdot\text{m}$$

$$M_C = \sqrt{(-0.2F)^2 + (0.35F)^2} = 400$$

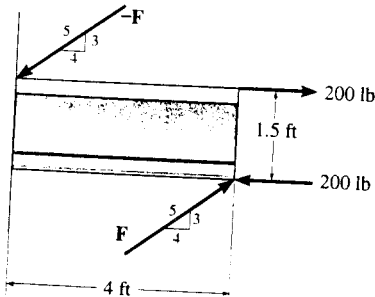
$$F = \frac{400}{\sqrt{(-0.2)^2 + (0.35)^2}} = 992 \text{ N} \quad \text{Ans}$$

4-81. The ends of the triangular plate are subjected to three couples. Determine the plate dimension  $d$  so that the resultant couple is 350 N·m clockwise.



$$\begin{aligned} \curvearrowright + M_R = \Sigma M_A: \quad -350 &= 200(d \cos 30^\circ) - 600(d \sin 30^\circ) - 100d \\ d &= 1.54 \text{ m} \quad \text{Ans} \end{aligned}$$

4-82. Two couples act on the beam as shown. Determine the magnitude of  $F$  so that the resultant couple moment is 300 lb·ft counterclockwise. Where on the beam does the resultant couple act?

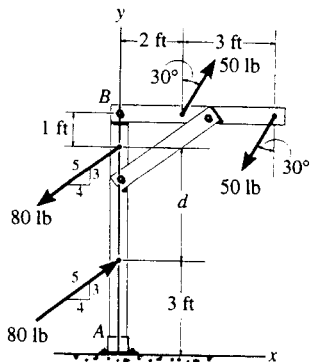


$$\curvearrowright + (M_C)_R = \frac{3}{5}F(4) + \frac{4}{5}F(1.5) - 200(1.5) = 300$$

$$F = 167 \text{ lb} \quad \text{Ans}$$

Resultant couple can act anywhere. Ans

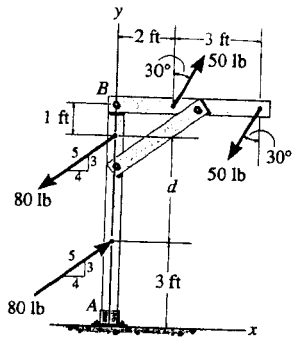
4-83. Two couples act on the frame. If the resultant couple moment is to be zero, determine the distance  $d$  between the 80-lb couple forces.



$$\curvearrowright + M_C = -50 \cos 30^\circ(3) + \frac{4}{5}(80)(d) = 0$$

$$d = 2.03 \text{ ft} \quad \text{Ans}$$

\*4-84. Two couples act on the frame. If  $d = 4$  ft, determine the resultant couple moment. Compute the result by resolving each force into  $x$  and  $y$  components and (a) finding the moment of each couple (Eq. 4-13) and (b) summing the moments of all the force components about point  $A$ .



$$(a) \quad \mathbf{M}_C = \Sigma(\mathbf{r} \times \mathbf{F})$$

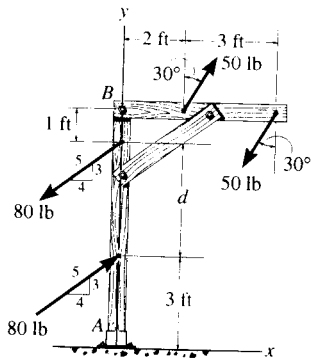
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -50 \sin 30^\circ & -50 \cos 30^\circ & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & 0 \\ -\frac{4}{5}(80) & -\frac{3}{5}(80) & 0 \end{vmatrix}$$

$$\mathbf{M}_C = \{126\mathbf{k}\} \text{ lb}\cdot\text{ft} \quad \text{Ans}$$

$$(b) \quad \Sigma +M_C = -\frac{4}{5}(80)(3) + \frac{4}{5}(80)(7) + 50 \cos 30^\circ(2) - 50 \cos 30^\circ(5)$$

$$M_C = 126 \text{ lb}\cdot\text{ft} \quad \text{Ans}$$

4-85. Two couples act on the frame. If  $d = 4$  ft, determine the resultant couple moment. Compute the result by resolving each force into  $x$  and  $y$  components and (a) finding the moment of each couple (Eq. 4-13) and (b) summing the moments of all the force components about point  $B$ .



$$(a) \quad \mathbf{M}_C = \Sigma(\mathbf{r} \times \mathbf{F})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -50 \sin 30^\circ & -50 \cos 30^\circ & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 0 \\ \frac{4}{5}(80) & \frac{3}{5}(80) & 0 \end{vmatrix}$$

$$\mathbf{M}_C = \{126\mathbf{k}\} \text{ lb}\cdot\text{ft} \quad \text{Ans}$$

$$(b) \quad \Sigma +M_C = 50 \cos 30^\circ(2) - 50 \cos 30^\circ(5) - \frac{4}{5}(80)(1) + \frac{4}{5}(80)(5)$$

$$M_C = 126 \text{ lb}\cdot\text{ft} \quad \text{Ans}$$

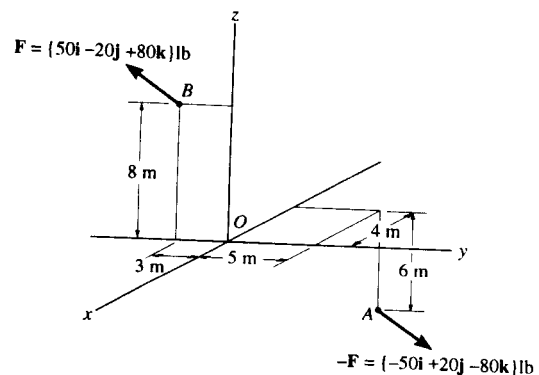
4-86. Determine the couple moment. Express the result as a Cartesian vector.

Position Vector:

$$\begin{aligned} \mathbf{r}_{AB} &= \{[0 - (-4)]\mathbf{i} + [-3 - 5]\mathbf{j} + [8 - (-6)]\mathbf{k}\} \text{ ft} \\ &= \{4\mathbf{i} - 8\mathbf{j} + 14\mathbf{k}\} \text{ ft} \end{aligned}$$

Couple Moment: With  $\mathbf{F} = \{50\mathbf{i} - 20\mathbf{j} + 80\mathbf{k}\}$  lb, applying Eq. 4-15, we have

$$\begin{aligned} \mathbf{M}_C &= \mathbf{r}_{AB} \times \mathbf{F} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -8 & 14 \\ 50 & -20 & 80 \end{vmatrix} \\ &= \{-360\mathbf{i} + 380\mathbf{j} + 320\mathbf{k}\} \text{ lb}\cdot\text{ft} \quad \text{Ans} \end{aligned}$$





4-87. Determine the couple moment. Express the result as a Cartesian vector. Each force has a magnitude of  $F = 120$  lb.

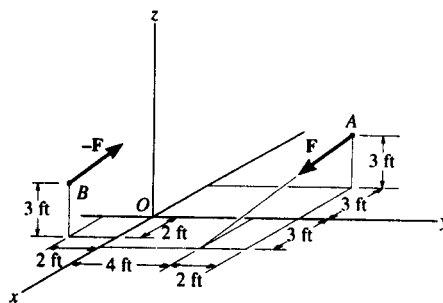
**Position Vector and Force Vector :**

$$\begin{aligned} \mathbf{r}_{BA} &= ((-3-2)\mathbf{i} + [6-(-2)]\mathbf{j} + (3-3)\mathbf{k}) \text{ ft} \\ &= (-5\mathbf{i} + 8\mathbf{j}) \text{ ft} \end{aligned}$$

$$\begin{aligned} \mathbf{F} &= 120 \left( \frac{[3-(-3)]\mathbf{i} + (4-6)\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{[3-(-3)]^2 + (4-6)^2 + (0-3)^2}} \right) \\ &= \{102.86\mathbf{i} - 34.26\mathbf{j} - 51.43\mathbf{k}\} \text{ lb} \end{aligned}$$

**Couple Moment :** Applying Eq. 4-15, we have

$$\begin{aligned} \mathbf{M}_C &= \mathbf{r}_{BA} \times \mathbf{F} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 8 & 0 \\ 102.86 & -34.26 & -51.43 \end{vmatrix} \\ &= \{-411\mathbf{i} - 257\mathbf{j} - 651\mathbf{k}\} \text{ lb}\cdot\text{ft} \end{aligned} \quad \text{Ans}$$



\*4-88. The gear reducer is subjected to the four couple moments. Determine the magnitude of the resultant couple moment and its coordinate direction angles.

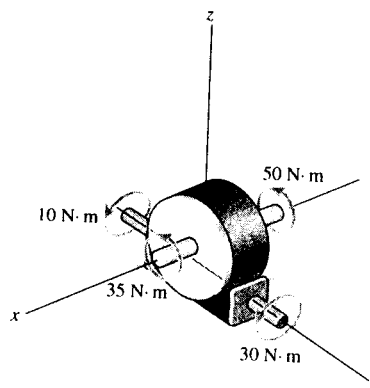
$$\begin{aligned} (M_R)_x &= \Sigma M_x; & (M_R)_x &= 35 + 50 = 85.0 \text{ N}\cdot\text{m} \\ (M_R)_y &= \Sigma M_y; & (M_R)_y &= 30 + 10 = 40.0 \text{ N}\cdot\text{m} \end{aligned}$$

The magnitude of the resultant couple moment is

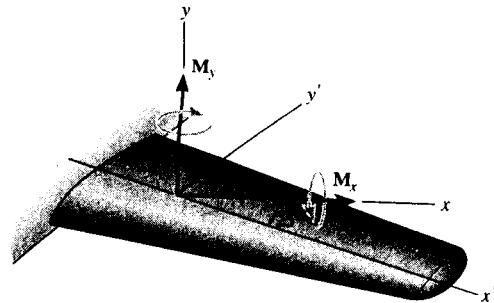
$$\begin{aligned} M_R &= \sqrt{(M_R)_x^2 + (M_R)_y^2} \\ &= \sqrt{85.0^2 + 40.0^2} \\ &= 93.941 \text{ N}\cdot\text{m} = 93.9 \text{ N}\cdot\text{m} \end{aligned} \quad \text{Ans}$$

The coordinate direction angles are

$$\begin{aligned} \alpha &= \cos^{-1} \left[ \frac{(M_R)_x}{M_R} \right] = \cos^{-1} \left( \frac{85.0}{93.941} \right) = 25.2^\circ & \text{Ans} \\ \beta &= \cos^{-1} \left[ \frac{(M_R)_y}{M_R} \right] = \cos^{-1} \left( \frac{40.0}{93.941} \right) = 64.8^\circ & \text{Ans} \\ \gamma &= \cos^{-1} \left[ \frac{(M_R)_z}{M_R} \right] = \cos^{-1} \left( \frac{0}{93.941} \right) = 90.0^\circ & \text{Ans} \end{aligned}$$



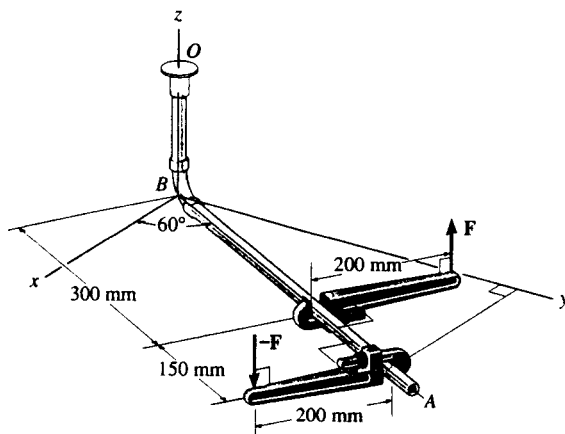
4-89. The main beam along the wing of an airplane is swept back at an angle of  $25^\circ$ . From load calculations it is determined that the beam is subjected to couple moments  $M_x = 17 \text{ kip}\cdot\text{ft}$  and  $M_y = 25 \text{ kip}\cdot\text{ft}$ . Determine the resultant couple moments created about the  $x'$  and  $y'$  axes. The axes all lie in the same horizontal plane.



$$(M_R)_{x'} = \Sigma M_x; \quad (M_R)_{x'} = 17 \cos 25^\circ - 25 \sin 25^\circ = 4.84 \text{ kip}\cdot\text{ft} \quad \text{Ans}$$

$$(M_R)_{y'} = \Sigma M_y; \quad (M_R)_{y'} = 17 \sin 25^\circ + 25 \cos 25^\circ = 29.8 \text{ kip}\cdot\text{ft} \quad \text{Ans}$$

4-90. If  $\mathbf{F} = \{100\mathbf{k}\} \text{ N}$ , determine the couple moment that acts on the assembly. Express the result as a Cartesian vector. Member  $BA$  lies in the  $x$ - $y$  plane.



$$\phi = \tan^{-1}\left(\frac{2}{3}\right) - 30^\circ = 3.69^\circ$$

$$\mathbf{r}_1 = \{-360.6 \sin 3.69^\circ \mathbf{i} + 360.6 \cos 3.69^\circ \mathbf{j}\} = \{-23.21 \mathbf{i} + 359.8 \mathbf{j}\} \text{ mm}$$

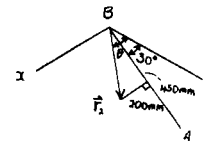
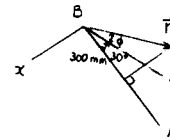
$$\theta = \tan^{-1}\left(\frac{2}{4.5}\right) + 30^\circ = 53.96^\circ$$

$$\mathbf{r}_2 = \{492.4 \sin 53.96^\circ \mathbf{i} + 492.4 \cos 53.96^\circ \mathbf{j}\} = \{398.2 \mathbf{i} + 289.7 \mathbf{j}\} \text{ mm}$$

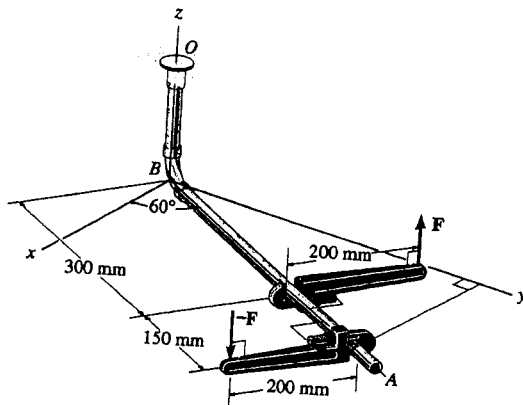
$$\mathbf{M}_C = (\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -421.4 & 70.10 & 0 \\ 0 & 0 & 100 \end{vmatrix}$$

$$\mathbf{M}_C = \{7.01 \mathbf{i} + 42.1 \mathbf{j}\} \text{ N}\cdot\text{m} \quad \text{Ans}$$



4-91. If the magnitude of the resultant couple moment is  $15 \text{ N}\cdot\text{m}$ , determine the magnitude  $F$  of the forces applied to the wrenches.



$$\phi = \tan^{-1}\left(\frac{2}{3}\right) - 30^\circ = 3.69^\circ$$

$$\begin{aligned} \mathbf{r}_1 &= \{-360.6 \sin 3.69^\circ \mathbf{i} + 360.6 \cos 3.69^\circ \mathbf{j}\} \\ &= \{-23.21 \mathbf{i} + 359.8 \mathbf{j}\} \text{ mm} \end{aligned}$$

$$\theta = \tan^{-1}\left(\frac{2}{4.5}\right) + 30^\circ = 53.96^\circ$$

$$\begin{aligned} \mathbf{r}_2 &= \{492.4 \sin 53.96^\circ \mathbf{i} + 492.4 \cos 53.96^\circ \mathbf{j}\} \\ &= \{398.2 \mathbf{i} + 289.7 \mathbf{j}\} \text{ mm} \end{aligned}$$

$$\mathbf{M}_C = (\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -421.4 & 70.10 & 0 \\ 0 & 0 & F \end{vmatrix}$$

$$\mathbf{M}_C = \{0.07F \mathbf{i} + 0.421F \mathbf{j}\} \text{ N}\cdot\text{m}$$

$$M_C = \sqrt{(0.07F)^2 + (0.421F)^2} = 15$$

$$F = \frac{15}{\sqrt{(0.07)^2 + (0.421)^2}} = 35.1 \text{ N} \quad \text{Ans}$$

Also, align  $y'$  axis along  $BA$ .

$$\mathbf{M}_C = -F(0.15) \mathbf{i}' - F(0.4) \mathbf{j}'$$

$$15 = \sqrt{(F(0.15))^2 + (F(0.4))^2}$$

$$F = 35.1 \text{ N} \quad \text{Ans}$$

4-92. The gear reducer is subjected to the couple moments shown. Determine the resultant couple moment and specify its magnitude and coordinate direction angles.

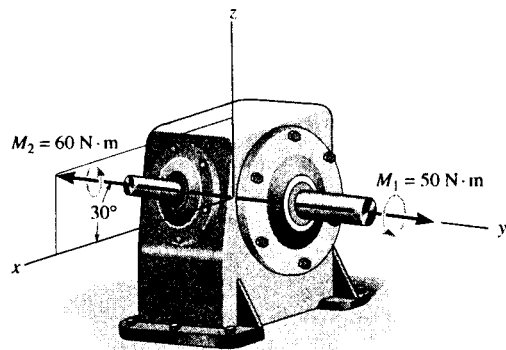
Express Each Couple Moment as a Cartesian Vector :

$$\mathbf{M}_1 = \{50 \mathbf{j}\} \text{ N}\cdot\text{m}$$

$$\mathbf{M}_2 = 60(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{k}) \text{ N}\cdot\text{m} = \{51.96 \mathbf{i} + 30.0 \mathbf{k}\} \text{ N}\cdot\text{m}$$

Resultant Couple Moment :

$$\begin{aligned} \mathbf{M}_R &= \Sigma \mathbf{M}; \quad \mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2 \\ &= \{51.96 \mathbf{i} + 50.0 \mathbf{j} + 30.0 \mathbf{k}\} \text{ N}\cdot\text{m} \end{aligned}$$



The magnitude of the resultant couple moment is

$$\begin{aligned} M_R &= \sqrt{51.96^2 + 50.0^2 + 30.0^2} \\ &= 78.102 \text{ N}\cdot\text{m} = 78.1 \text{ N}\cdot\text{m} \quad \text{Ans} \end{aligned}$$

The coordinate direction angles are

$$\alpha = \cos^{-1}\left(\frac{51.96}{78.102}\right) = 48.3^\circ \quad \text{Ans}$$

$$\beta = \cos^{-1}\left(\frac{50.0}{78.102}\right) = 50.2^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{30.0}{78.102}\right) = 67.4^\circ \quad \text{Ans}$$

**4-91.** If the magnitude of the resultant couple moment is  $15 \text{ N} \cdot \text{m}$  determine the magnitude  $F$  of the forces applied to the wrenches.

$$\phi = \tan^{-1}\left(\frac{2}{3}\right) - 30^\circ = 3.69^\circ$$

$$\begin{aligned} \mathbf{r}_1 &= [-360.6 \sin 3.69^\circ \mathbf{i} + 360.6 \cos 3.69^\circ \mathbf{j}] \\ &= [-23.21 \mathbf{i} + 359.8 \mathbf{j}] \text{ mm} \end{aligned}$$

$$\theta = \tan^{-1}\left(\frac{2}{4.5}\right) + 30^\circ = 53.96^\circ$$

$$\begin{aligned} \mathbf{r}_2 &= [492.4 \sin 53.96^\circ \mathbf{i} + 492.4 \cos 53.96^\circ \mathbf{j}] \\ &= [398.2 \mathbf{i} + 289.7 \mathbf{j}] \text{ mm} \end{aligned}$$

$$\mathbf{M}_C = (\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -421.4 & 70.10 & 0 \\ 0 & 0 & F \end{vmatrix}$$

$$\mathbf{M}_C = \{0.07F \mathbf{i} + 0.421F \mathbf{j}\} \text{ N} \cdot \text{m}$$

$$M_C = \sqrt{(0.07F)^2 + (0.421F)^2} = 15$$

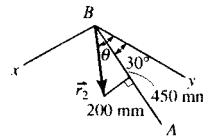
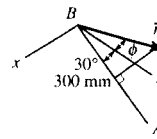
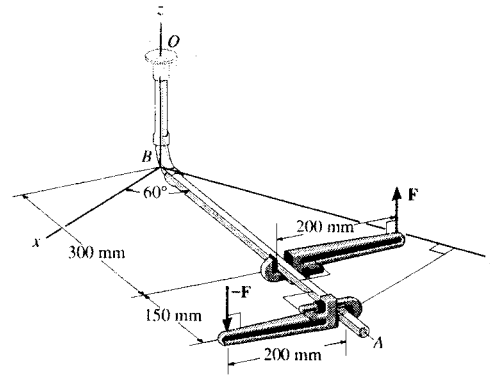
$$F = \frac{15}{\sqrt{(0.07)^2 + (0.421)^2}} = 35.1 \text{ N} \quad \text{Ans}$$

Also, align  $y'$  axis along  $BA$ .

$$\mathbf{M}_C = -F(0.15)\mathbf{i}' + F(0.4)\mathbf{j}'$$

$$15 = \sqrt{(F(0.15))^2 + (F(0.4))^2}$$

$$F = 35.1 \text{ N} \quad \text{Ans}$$



**4-92.** The gear reducer is subjected to the couple moments shown. Determine the resultant couple moment and specify its magnitude and coordinate direction angles.

*Express Each Couple Moment as a Cartesian Vector:*

$$\mathbf{M}_1 = \{50\mathbf{j}\} \text{ N} \cdot \text{m}$$

$$\mathbf{M}_2 = 60(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{k}) \text{ N} \cdot \text{m} = \{51.96\mathbf{i} + 30.0\mathbf{k}\} \text{ N} \cdot \text{m}$$

*Resultant Couple Moment:*

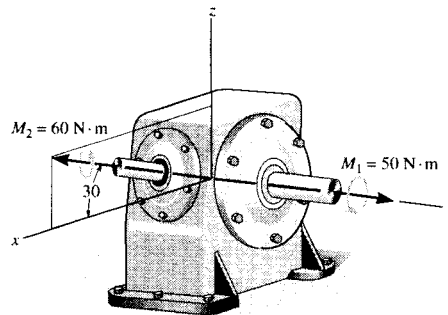
$$\mathbf{M}_R = \Sigma \mathbf{M}; \quad \mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

$$= \{51.96\mathbf{i} + 50.0\mathbf{j} + 30.0\mathbf{k}\} \text{ N} \cdot \text{m}$$

The magnitude of the resultant couple moment is

$$M_R = \sqrt{51.96^2 + 50.0^2 + 30.0^2}$$

$$= 78.102 \text{ N} \cdot \text{m} = 78.1 \text{ N} \cdot \text{m} \quad \text{Ans}$$



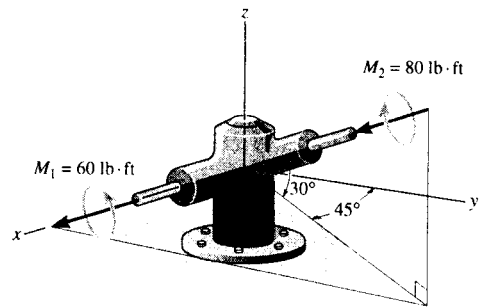
The coordinate direction angles are

$$\alpha = \cos^{-1}\left(\frac{51.96}{78.102}\right) = 48.3^\circ \quad \text{Ans}$$

$$\beta = \cos^{-1}\left(\frac{50.0}{78.102}\right) = 50.2^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{30.0}{78.102}\right) = 67.4^\circ \quad \text{Ans}$$

4-93. The gear reducer is subject to the couple moments shown. Determine the resultant couple moment and specify its magnitude and coordinate direction angles.



**Express Each :**

$$M_1 = \{60\mathbf{i}\} \text{ lb} \cdot \text{ft}$$

$$M_2 = 80(-\cos 30^\circ \sin 45^\circ \mathbf{i} - \cos 30^\circ \cos 45^\circ \mathbf{j} - \sin 30^\circ \mathbf{k}) \text{ lb} \cdot \text{ft}$$

$$= \{-48.99\mathbf{i} - 48.99\mathbf{j} - 40.0\mathbf{k}\} \text{ lb} \cdot \text{ft}$$

**Resultant Couple Moment :**

$$M_R = \Sigma M; \quad M_R = M_1 + M_2$$

$$= \{(60 - 48.99)\mathbf{i} - 48.99\mathbf{j} - 40.0\mathbf{k}\} \text{ lb} \cdot \text{ft}$$

$$= \{11.01\mathbf{i} - 48.99\mathbf{j} - 40.0\mathbf{k}\} \text{ lb} \cdot \text{ft}$$

$$= \{11.0\mathbf{i} - 49.0\mathbf{j} - 40.0\mathbf{k}\} \text{ lb} \cdot \text{ft} \quad \text{Ans}$$

The magnitude of the resultant couple moment is

$$M_R = \sqrt{11.01^2 + (-48.99)^2 + (-40.0)^2}$$

$$= 64.20 \text{ lb} \cdot \text{ft} = 64.2 \text{ lb} \cdot \text{ft} \quad \text{Ans}$$

The coordinate direction angles are

$$\alpha = \cos^{-1}\left(\frac{11.01}{64.20}\right) = 80.1^\circ \quad \text{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-48.99}{64.20}\right) = 140^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{-40.0}{64.20}\right) = 129^\circ \quad \text{Ans}$$

4-94. The meshed gears are subjected to the couple moments shown. Determine the magnitude of the resultant couple moment and specify its coordinate direction angles.

$$M_1 = \{50\mathbf{k}\} \text{ N} \cdot \text{m}$$

$$M_2 = 20(-\cos 20^\circ \sin 30^\circ \mathbf{i} - \cos 20^\circ \cos 30^\circ \mathbf{j} + \sin 20^\circ \mathbf{k}) \text{ N} \cdot \text{m}$$

$$= \{-9.397\mathbf{i} - 16.276\mathbf{j} + 6.840\mathbf{k}\} \text{ N} \cdot \text{m}$$

**Resultant Couple Moment :**

$$M_R = \Sigma M; \quad M_R = M_1 + M_2$$

$$= \{-9.397\mathbf{i} - 16.276\mathbf{j} + (50 + 6.840)\mathbf{k}\} \text{ N} \cdot \text{m}$$

$$= \{-9.397\mathbf{i} - 16.276\mathbf{j} + 56.840\mathbf{k}\} \text{ N} \cdot \text{m}$$

The magnitude of the resultant couple moment is

$$M_R = \sqrt{(-9.397)^2 + (-16.276)^2 + (56.840)^2}$$

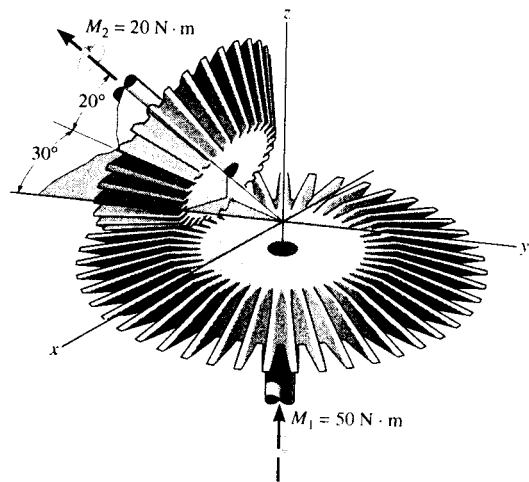
$$= 59.867 \text{ N} \cdot \text{m} = 59.9 \text{ N} \cdot \text{m} \quad \text{Ans}$$

The coordinate direction angles are

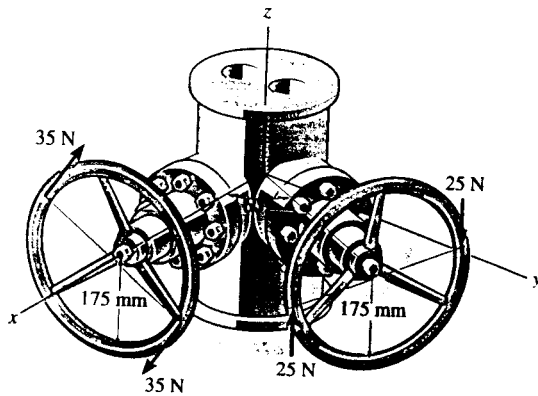
$$\alpha = \cos^{-1}\left(\frac{-9.397}{59.867}\right) = 99.0^\circ \quad \text{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-16.276}{59.867}\right) = 106^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{56.840}{59.867}\right) = 18.3^\circ \quad \text{Ans}$$



4-95. A couple acts on each of the handles of the minidial valve. Determine the magnitude and coordinate direction angles of the resultant couple moment.



$$M_x = -35(0.35) - 25(0.35) \cos 60^\circ = -16.625$$

$$M_y = -25(0.35) \sin 60^\circ = -7.5777 \text{ N}\cdot\text{m}$$

$$|M| = \sqrt{(-16.625)^2 + (-7.5777)^2} = 18.2705 = 18.3 \text{ N}\cdot\text{m}$$

$$\alpha = \cos^{-1}\left(\frac{-16.625}{18.2705}\right) = 155^\circ \quad \text{Ans}$$

$$\beta = \cos^{-1}\left(\frac{-7.5777}{18.2705}\right) = 115^\circ \quad \text{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{0}{18.2705}\right) = 90^\circ \quad \text{Ans}$$

\*4-96. Determine the resultant couple moment of the two couples that act on the pipe assembly. The distance from A to B is  $d = 400$  mm. Express the result as a Cartesian vector.

**Vector Analysis**

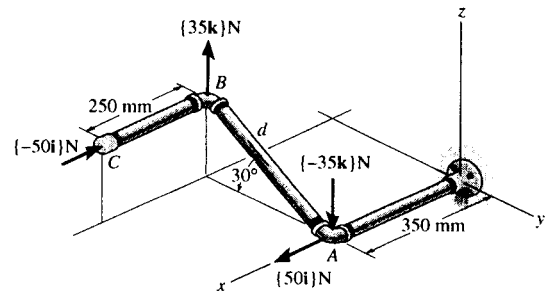
**Position Vector :**

$$\begin{aligned} r_{AB} &= \{(0.35 - 0.35)\mathbf{i} + (-0.4 \cos 30^\circ - 0)\mathbf{j} + (0.4 \sin 30^\circ - 0)\mathbf{k}\} \text{ m} \\ &= \{-0.3464\mathbf{j} + 0.20\mathbf{k}\} \text{ m} \end{aligned}$$

**Couple Moments :** With  $F_1 = \{35\mathbf{k}\}$  N and  $F_2 = \{-50\mathbf{i}\}$  N, applying Eq. 4-15, we have

$$\begin{aligned} (M_C)_1 &= r_{AB} \times F_1 \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.3464 & 0.20 \\ 0 & 0 & 35 \end{vmatrix} = \{-12.12\mathbf{i}\} \text{ N}\cdot\text{m} \end{aligned}$$

$$\begin{aligned} (M_C)_2 &= r_{AB} \times F_2 \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.3464 & 0.20 \\ -50 & 0 & 0 \end{vmatrix} = \{-10.0\mathbf{j} - 17.32\mathbf{k}\} \text{ N}\cdot\text{m} \end{aligned}$$



**Resultant Couple Moment :**

$$\begin{aligned} \mathbf{M}_R &= \Sigma \mathbf{M}; \quad \mathbf{M}_R = (\mathbf{M}_C)_1 + (\mathbf{M}_C)_2 \\ &= \{-12.1\mathbf{i} - 10.0\mathbf{j} - 17.3\mathbf{k}\} \text{ N}\cdot\text{m} \quad \text{Ans} \end{aligned}$$

**Scalar Analysis :** Summing moments about  $x$ ,  $y$  and  $z$  axes, we have

$$\begin{aligned} (M_R)_x &= \Sigma M_x; & (M_R)_x &= -35(0.4 \cos 30^\circ) = -12.12 \text{ N}\cdot\text{m} \\ (M_R)_y &= \Sigma M_y; & (M_R)_y &= -50(0.4 \sin 30^\circ) = -10.0 \text{ N}\cdot\text{m} \\ (M_R)_z &= \Sigma M_z; & (M_R)_z &= -50(0.4 \cos 30^\circ) = -17.32 \text{ N}\cdot\text{m} \end{aligned}$$

Express  $\mathbf{M}_R$  as a Cartesian vector, we have

$$\mathbf{M}_R = \{-12.1\mathbf{i} - 10.0\mathbf{j} - 17.3\mathbf{k}\} \text{ N}\cdot\text{m}$$

4-97. Determine the distance  $d$  between  $A$  and  $B$  so that the resultant couple moment has a magnitude of  $M_R = 20 \text{ N}\cdot\text{m}$ .

**Position Vector :**

$$\mathbf{r}_{AB} = \{(0.35 - 0.35)\mathbf{i} + (-d\cos 30^\circ - 0)\mathbf{j} + (d\sin 30^\circ - 0)\mathbf{k}\} \text{ m}$$

$$= \{-0.8660d\mathbf{j} + 0.50d\mathbf{k}\} \text{ m}$$

**Couple Moments :** With  $\mathbf{F}_1 = \{35\mathbf{k}\} \text{ N}$  and  $\mathbf{F}_2 = \{-50\mathbf{i}\} \text{ N}$ , applying Eq. 4-15, we have

$$(\mathbf{M}_C)_1 = \mathbf{r}_{AB} \times \mathbf{F}_1$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.8660d & 0.50d \\ 0 & 0 & 35 \end{vmatrix} = \{-30.31d\mathbf{i}\} \text{ N}\cdot\text{m}$$

$$(\mathbf{M}_C)_2 = \mathbf{r}_{AB} \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.8660d & 0.50d \\ -50 & 0 & 0 \end{vmatrix} = \{-25.0d\mathbf{j} - 43.30d\mathbf{k}\} \text{ N}\cdot\text{m}$$

**Resultant Couple Moment :**

$$\mathbf{M}_R = \Sigma \mathbf{M}; \quad \mathbf{M}_R = (\mathbf{M}_C)_1 + (\mathbf{M}_C)_2$$

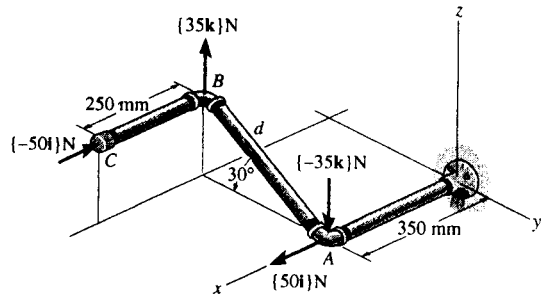
$$= \{-30.31d\mathbf{i} - 25.0d\mathbf{j} - 43.30d\mathbf{k}\} \text{ N}\cdot\text{m}$$

The magnitude of  $\mathbf{M}_R$  is  $20 \text{ N}\cdot\text{m}$  thus

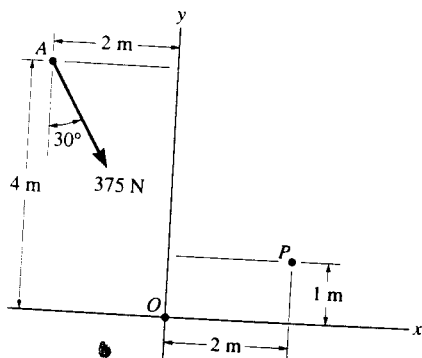
$$20 = \sqrt{(-30.31d)^2 + (-25.0d)^2 + (43.30d)^2}$$

$$d = 0.3421 \text{ m} = 342 \text{ mm}$$

**Ans**



4-98. Replace the force at  $A$  by an equivalent force and couple moment at point  $O$ .



$$F = 375 \text{ N} \quad \text{Ans}$$

$$(+M_O = 375 \cos 30^\circ(2) - 375 \sin 30^\circ(4))$$

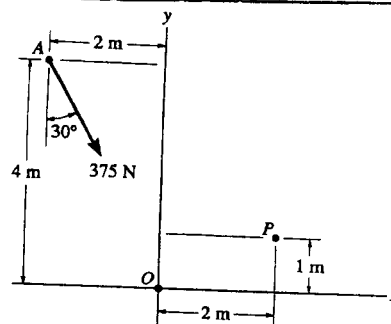
$$M_O = -100.48 = 100 \text{ N}\cdot\text{m} \quad \text{Ans}$$

4-99. Replace the force at  $A$  by an equivalent force and couple moment at point  $P$ .

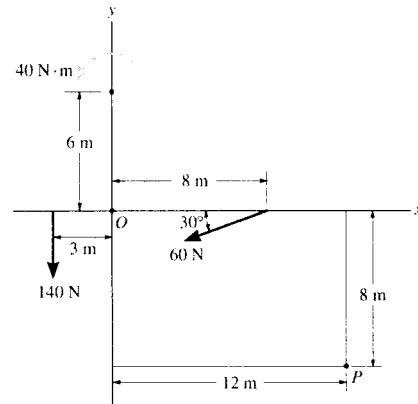
$$F_P = 375 \text{ N} \quad \text{Ans}$$

$$(+M_P = 375 \cos 30^\circ(4) - 375 \sin 30^\circ(3))$$

$$M_P = 737 \text{ N}\cdot\text{m} \quad \text{Ans}$$



**4-100.** Replace the force and couple moment system by an equivalent force and couple moment acting at point  $O$ .



$$\rightarrow F_{R_x} = \Sigma F_x; \quad F_{R_x} = -60 \cos 30^\circ = -51.96 \text{ N} = 51.96 \text{ N} \leftarrow$$

$$+ \uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = -60 \sin 30^\circ - 140 \\ = -170.0 \text{ N} = 170.0 \text{ N} \downarrow$$

Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{51.96^2 + 170.0^2} = 178 \text{ N} \quad \text{Ans}$$

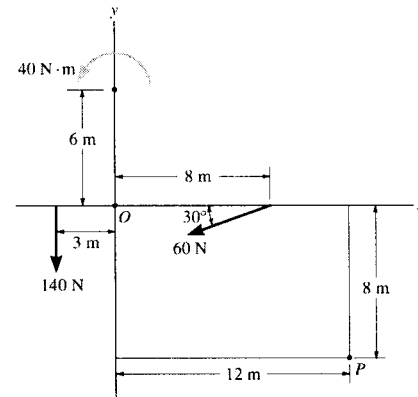
and

$$\theta = \tan^{-1} \left( \frac{F_{R_y}}{F_{R_x}} \right) = \tan^{-1} \left( \frac{170.0}{51.96} \right) = 73.0^\circ \quad \text{Ans}$$

$$\curvearrowright + M_{R_O} = \Sigma M_O; \quad M_{R_O} = -60 \sin 30^\circ (8) + 40 + 140(3)$$

$$= 220 \text{ N} \cdot \text{m} \quad (\text{Counterclockwise}) \quad \text{Ans}$$

**4-101.** Replace the force and couple moment system by an equivalent force and couple moment acting at point  $P$ .



$$\rightarrow F_{R_x} = \Sigma F_x; \quad F_{R_x} = -60 \cos 30^\circ = -51.96 \text{ N} = 51.96 \text{ N} \leftarrow$$

$$+ \uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = -60 \sin 30^\circ - 140 \\ = -170.0 \text{ N} = 170.0 \text{ N} \downarrow$$

Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{51.96^2 + 170.0^2} = 178 \text{ N} \quad \text{Ans}$$

and

$$\theta = \tan^{-1} \left( \frac{F_{R_y}}{F_{R_x}} \right) = \tan^{-1} \left( \frac{170.0}{51.96} \right) = 73.0^\circ \quad \text{Ans}$$

$$\curvearrowright + M_{R_P} = \Sigma M_P; \quad M_{R_P} = 60 \sin 30^\circ (12 - 8) + 60 \cos 30^\circ (8)$$

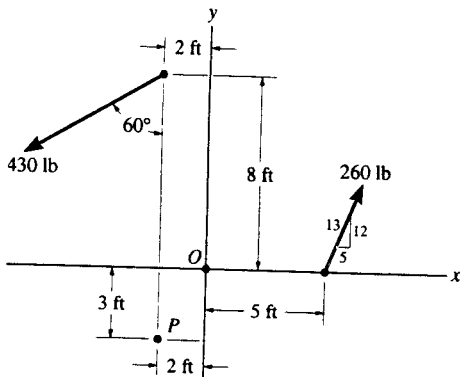
$$+ 40 + 140(3 + 12)$$

$$= 2676 \text{ N} \cdot \text{m}$$

$$= 2.68 \text{ kN} \cdot \text{m} \quad (\text{Counterclockwise}) \quad \text{Ans}$$



4-102. Replace the force system by an equivalent force and couple moment at point  $O$ .



$$\begin{aligned} \rightarrow \Sigma F_{Rx} = \Sigma F_x; \quad F_{Rx} &= 260\left(\frac{5}{13}\right) - 430 \sin 60^\circ \\ &= -272.39 \text{ lb} \end{aligned}$$

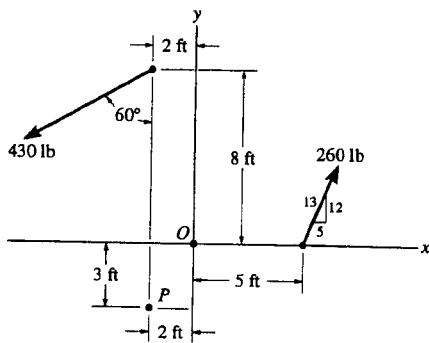
$$\begin{aligned} + \uparrow \Sigma F_{Ry} = \Sigma F_y; \quad F_{Ry} &= 260\left(\frac{12}{13}\right) - 430 \cos 60^\circ \\ &= 25 \text{ lb} \end{aligned}$$

$$F_R = \sqrt{(-272.39)^2 + (25)^2} = 274 \text{ lb} \quad \text{Ans}$$

$$\theta = \tan^{-1}\left[\frac{25}{272.39}\right] = 5.24^\circ \quad \text{e}_\Delta \quad \text{Ans}$$

$$\begin{aligned} (+) M_O = \Sigma M_O; \quad M_O &= 430 \cos 60^\circ(2) + 430 \sin 60^\circ(8) + 260\left(\frac{12}{13}\right)(5) \\ M_O &= 4609 \text{ lb}\cdot\text{ft} = 4.61 \text{ kip}\cdot\text{ft} \quad \text{Ans} \end{aligned}$$

4-103. Replace the force system by an equivalent force and couple moment at point  $P$ .



$$\begin{aligned} \rightarrow \Sigma F_{Rx} = \Sigma F_x; \quad F_{Rx} &= 260\left(\frac{5}{13}\right) - 430 \sin 60^\circ \\ &= -272.39 \text{ lb} \end{aligned}$$

$$\begin{aligned} + \uparrow \Sigma F_{Ry} = \Sigma F_y; \quad F_{Ry} &= 260\left(\frac{12}{13}\right) - 430 \cos 60^\circ \\ &= 25 \text{ lb} \end{aligned}$$

$$F_R = \sqrt{(-272.39)^2 + (25)^2} = 274 \text{ lb} \quad \text{Ans}$$

$$\theta = \tan^{-1}\left[\frac{25}{272.39}\right] = 5.24^\circ \quad \text{e}_\Delta \quad \text{Ans}$$

$$\begin{aligned} (+) M_P = \Sigma M_P; \quad M_P &= 430 \sin 60^\circ(11) + 260\left(\frac{12}{13}\right)(7) - 260\left(\frac{5}{13}\right)(3) \\ M_P &= 5.48 \text{ kip}\cdot\text{ft} \quad \text{Ans} \end{aligned}$$

**\*4-104.** Replace the force and couple system by an equivalent force and couple moment acting at point O.

Note that the 6 kN pair of forces form a couple.

$$\begin{aligned} \rightarrow F_{R_x} &= \Sigma F_x; & F_{R_x} &= 5 \cos 45^\circ = 3.536 \text{ kN} \rightarrow \\ + \uparrow F_{R_y} &= \Sigma F_y; & F_{R_y} &= -5 \sin 45^\circ - 2 \\ & & &= -5.536 \text{ kN} = 5.536 \text{ kN} \downarrow \end{aligned}$$

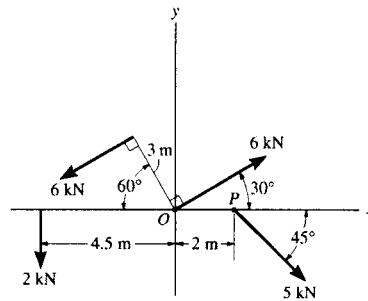
Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{3.536^2 + 5.536^2} = 6.57 \text{ kN} \quad \text{Ans}$$

and

$$\theta = \tan^{-1} \left( \frac{F_{R_y}}{F_{R_x}} \right) = \tan^{-1} \left( \frac{5.536}{3.536} \right) = 57.4^\circ \quad \text{Ans}$$

$$\begin{aligned} \curvearrowright +M_{R_o} &= \Sigma M_o; & M_{R_o} &= 6(3) + 2(4.5) - 5 \sin 45^\circ(2) \\ & & &= 19.9 \text{ kN} \cdot \text{m} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$



**4-105.** Replace the force and couple system by an equivalent force and couple moment acting at point P.

Note that the 6 kN pair of forces form a couple.

$$\begin{aligned} \rightarrow F_{R_x} &= \Sigma F_x; & F_{R_x} &= 5 \cos 45^\circ = 3.536 \text{ kN} \rightarrow \\ + \uparrow F_{R_y} &= \Sigma F_y; & F_{R_y} &= -5 \sin 45^\circ - 2 \\ & & &= -5.536 \text{ kN} = 5.536 \text{ kN} \downarrow \end{aligned}$$

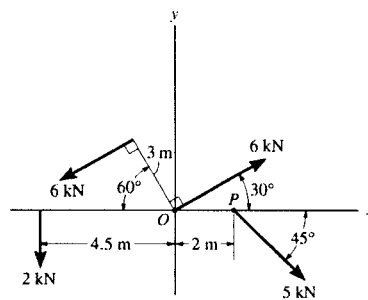
Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{3.536^2 + 5.536^2} = 6.57 \text{ kN} \quad \text{Ans}$$

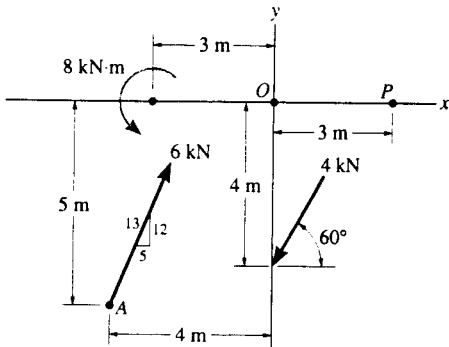
and

$$\theta = \tan^{-1} \left( \frac{F_{R_y}}{F_{R_x}} \right) = \tan^{-1} \left( \frac{5.536}{3.536} \right) = 57.4^\circ \quad \curvearrowright \quad \text{Ans}$$

$$\begin{aligned} \curvearrowright +M_{R_p} &= \Sigma M_p; & M_{R_p} &= 6(3) + 2(4.5 + 2) - 5(0) \\ & & &= 31.0 \text{ kN} \cdot \text{m} \quad (\text{Counterclockwise}) \quad \text{Ans} \end{aligned}$$



4-106. Replace the force and couple system by an equivalent force and couple moment at point  $O$ .



$$\begin{aligned} \rightarrow \Sigma F_{Rx} = \Sigma F_x; \quad F_{Rx} &= 6\left(\frac{5}{13}\right) - 4 \cos 60^\circ \\ &= 0.30769 \text{ kN} \end{aligned}$$

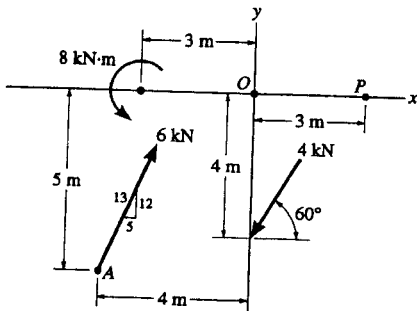
$$\begin{aligned} + \uparrow \Sigma F_{Ry} = \Sigma F_y; \quad F_{Ry} &= 6\left(\frac{12}{13}\right) - 4 \sin 60^\circ \\ &= 2.0744 \text{ kN} \end{aligned}$$

$$F_R = \sqrt{(0.30769)^2 + (2.0744)^2} = 2.10 \text{ kN} \quad \text{Ans}$$

$$\theta = \tan^{-1}\left[\frac{2.0744}{0.30769}\right] = 81.6^\circ \angle \theta \quad \text{Ans}$$

$$\begin{aligned} (+) M_O = \Sigma M_O; \quad M_O &= 8 - 6\left(\frac{12}{13}\right)(4) + 6\left(\frac{5}{13}\right)(5) - 4 \cos 60^\circ(4) \\ M_O &= -10.62 \text{ kN}\cdot\text{m} = 10.6 \text{ kN}\cdot\text{m} \quad \text{Ans} \end{aligned}$$

4-107. Replace the force and couple system by an equivalent force and couple moment at point  $P$ .



$$\begin{aligned} \rightarrow \Sigma F_{Rx} = \Sigma F_x; \quad F_{Rx} &= 6\left(\frac{5}{13}\right) - 4 \cos 60^\circ \\ &= 0.30769 \text{ kN} \end{aligned}$$

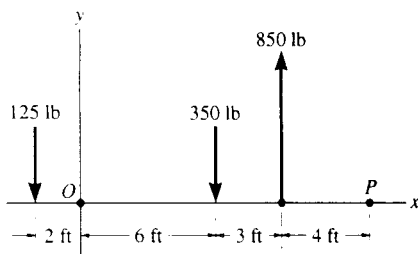
$$\begin{aligned} + \uparrow \Sigma F_{Ry} = \Sigma F_y; \quad F_{Ry} &= 6\left(\frac{12}{13}\right) - 4 \sin 60^\circ \\ &= 2.0744 \text{ kN} \end{aligned}$$

$$F_R = \sqrt{(0.30769)^2 + (2.0744)^2} = 2.10 \text{ kN} \quad \text{Ans}$$

$$\theta = \tan^{-1}\left[\frac{2.0744}{0.30769}\right] = 81.6^\circ \angle \theta \quad \text{Ans}$$

$$\begin{aligned} (+) M_P = \Sigma M_P; \quad M_P &= 8 - 6\left(\frac{12}{13}\right)(7) + 6\left(\frac{5}{13}\right)(5) - 4 \cos 60^\circ(4) + 4 \sin 60^\circ(3) \\ M_P &= -16.8 \text{ kN}\cdot\text{m} = 16.8 \text{ kN}\cdot\text{m} \quad \text{Ans} \end{aligned}$$

\*4-108. Replace the force system by a single force resultant and specify its point of application, measured along the  $x$  axis from point  $O$ .



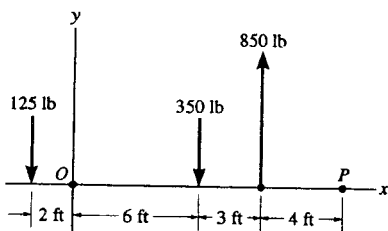
$$+\uparrow \Sigma F_{Ry} = \Sigma F_y; \quad F_{Ry} = 850 - 350 - 125 = 375 \text{ lb}$$

$$F_R = 375 \text{ lb } \uparrow \quad \text{Ans}$$

$$(+M_{Ro} = \Sigma M_o; \quad 375(x) = 850(9) - 350(6) + 125(2)$$

$$x = 15.5 \text{ ft} \quad \text{Ans}$$

4-109. Replace the force system by a single force resultant and specify its point of application, measured along the  $x$  axis from point  $P$ .



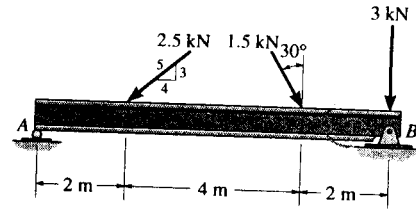
$$+\uparrow \Sigma F_{Ry} = \Sigma F_y; \quad F_{Ry} = 850 - 350 - 125 = 375 \text{ lb}$$

$$F_R = 375 \text{ lb } \uparrow \quad \text{Ans}$$

$$(+M_{Rp} = \Sigma M_p; \quad 375(x) = 350(7) - 850(4) + 125(15)$$

$$x = 2.47 \text{ ft} \quad \text{Ans} \quad (\text{to the left})$$

4-110. Replace the force system acting on the beam by an equivalent force and couple moment at point A.



$$\begin{aligned} \rightarrow F_{R_x} = \Sigma F_x; \quad F_{R_x} &= 1.5 \sin 30^\circ - 2.5 \left(\frac{4}{5}\right) \\ &= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow \end{aligned}$$

$$\begin{aligned} + \uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} &= -1.5 \cos 30^\circ - 2.5 \left(\frac{3}{5}\right) - 3 \\ &= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow \end{aligned}$$

Thus,

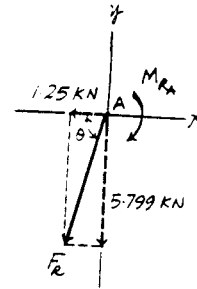
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN}$$

and

$$\theta = \tan^{-1} \left( \frac{F_{R_y}}{F_{R_x}} \right) = \tan^{-1} \left( \frac{5.799}{1.25} \right) = 77.8^\circ$$

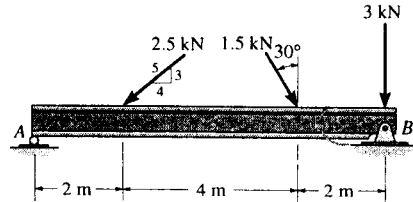
Ans

Ans



$$\begin{aligned} \curvearrowright + M_{R_A} = \Sigma M_A; \quad M_{R_A} &= -2.5 \left(\frac{3}{5}\right)(2) - 1.5 \cos 30^\circ(6) - 3(8) \\ &= -34.8 \text{ kN} \cdot \text{m} = 34.8 \text{ kN} \cdot \text{m} \text{ (Clockwise)} \end{aligned} \quad \text{Ans}$$

4-111. Replace the force system acting on the beam by an equivalent force and couple moment at point B.



$$\begin{aligned} \rightarrow F_{R_x} = \Sigma F_x; \quad F_{R_x} &= 1.5 \sin 30^\circ - 2.5 \left(\frac{4}{5}\right) \\ &= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow \end{aligned}$$

$$\begin{aligned} + \uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} &= -1.5 \cos 30^\circ - 2.5 \left(\frac{3}{5}\right) - 3 \\ &= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow \end{aligned}$$

Thus,

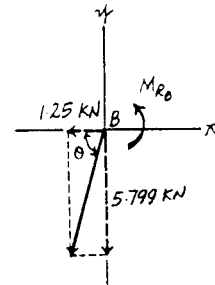
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN}$$

and

$$\theta = \tan^{-1} \left( \frac{F_{R_y}}{F_{R_x}} \right) = \tan^{-1} \left( \frac{5.799}{1.25} \right) = 77.8^\circ$$

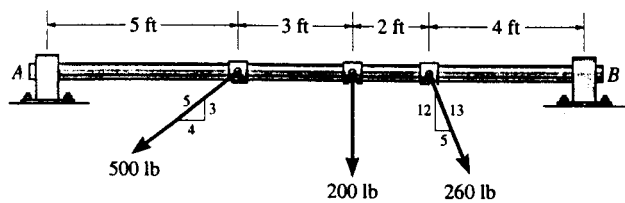
Ans

Ans



$$\begin{aligned} \curvearrowleft + M_{R_B} = \Sigma M_B; \quad M_{R_B} &= 1.5 \cos 30^\circ(2) + 2.5 \left(\frac{3}{5}\right)(6) \\ &= 11.6 \text{ kN} \cdot \text{m} \text{ (Counterclockwise)} \end{aligned} \quad \text{Ans}$$

**\*4-112.** Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end A.



$$\rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = -500\left(\frac{4}{5}\right) + 260\left(\frac{5}{13}\right) = -300 \text{ lb} = 300 \text{ lb} \leftarrow$$

$$+\uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = -500\left(\frac{3}{5}\right) - 200 - 260\left(\frac{12}{13}\right) = -740 \text{ lb} = 740 \text{ lb} \downarrow$$

$$F = \sqrt{(-300)^2 + (-740)^2} = 798 \text{ lb} \quad \text{Ans}$$

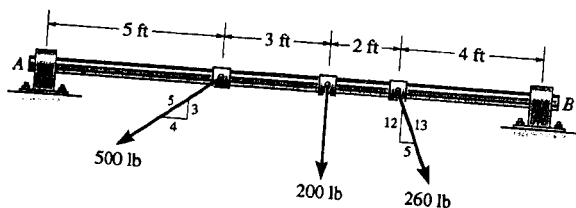
$$\theta = \tan^{-1}\left(\frac{740}{300}\right) = 67.9^\circ \text{ } \overleftrightarrow{xy} \quad \text{Ans}$$

$$+\circlearrowleft M_{RA} = \Sigma M_A; \quad 740(x) = 500\left(\frac{3}{5}\right)(5) + 200(8) + 260\left(\frac{12}{13}\right)(10)$$

$$740(x) = 5500$$

$$x = 7.43 \text{ ft} \quad \text{Ans}$$

**4-113.** Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end B.



$$\rightarrow F_{Rx} = \Sigma F_x; \quad F_{Rx} = -500\left(\frac{4}{5}\right) + 260\left(\frac{5}{13}\right) = -300 \text{ lb} = 300 \text{ lb} \leftarrow$$

$$+\uparrow F_{Ry} = \Sigma F_y; \quad F_{Ry} = -500\left(\frac{3}{5}\right) - 200 - 260\left(\frac{12}{13}\right) = -740 \text{ lb} = 740 \text{ lb} \downarrow$$

$$F = \sqrt{(-300)^2 + (-740)^2} = 798 \text{ lb} \quad \text{Ans}$$

$$\theta = \tan^{-1}\left(\frac{740}{300}\right) = 67.9^\circ \text{ } \overleftrightarrow{xy} \quad \text{Ans}$$

$$(+\circlearrowleft M_{RB} = \Sigma M_B; \quad 740(x) = 500\left(\frac{3}{5}\right)(9) + 200(6) + 260\left(\frac{12}{13}\right)(4)$$

$$x = 6.57 \text{ ft} \quad \text{Ans}$$