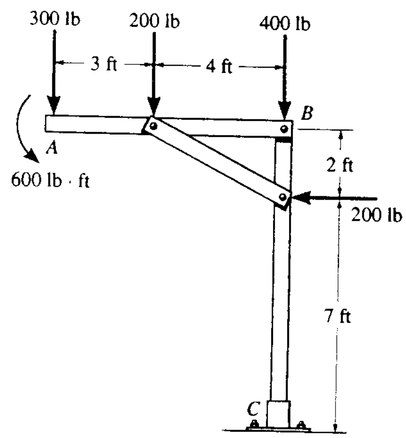
4-114. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member AB, measured from A.



$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;$ $F_{Rx} = -200 \text{ lb} = 200 \text{ lb} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y;$ $F_{Ry} = -300 - 200 - 400 = -900 \text{ lb} = 900 \text{ lb} \downarrow$

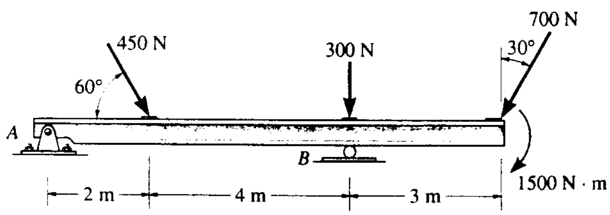
$F = \sqrt{(-200)^2 + (-900)^2} = 922 \text{ lb}$ **Ans**

$\theta = \tan^{-1}(\frac{900}{200}) = 77.5°$ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A;$ $900(x) = 200(3) + 400(7) + 200(2) - 600$

$x = \frac{3200}{900} = 3.56 \text{ ft}$ **Ans**

4-115. Replace the three forces acting on the beam by a single resultant force. Specify where the force acts, measured from end A.



$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;$ $F_{Rx} = 450 \cos 60° - 700 \sin 30° = -125 \text{ N} = 125 \text{ N} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y;$ $F_{Ry} = -450 \sin 60° - 700 \cos 30° - 300 = -1296 \text{ N} = 1296 \text{ N} \downarrow$

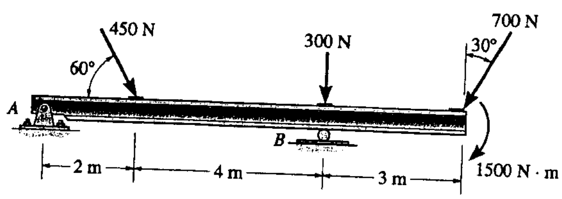
$F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N}$ **Ans**

$\theta = \tan^{-1}(\frac{1296}{125}) = 84.5°$ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A;$ $1296(x) = 450 \sin 60°(2) + 300(6) + 700 \cos 30°(9) + 1500$

$x = 7.36 \text{ m}$ **Ans**

***4-116.** Replace the three forces acting on the beam by a single resultant force. Specify where the force acts, measured from B.



$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;$ $F_{Rx} = 450 \cos 60° - 700 \sin 30° = -125 \text{ N} = 125 \text{ N} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y;$ $F_{Ry} = -450 \sin 60° - 700 \cos 30° - 300 = -1296 \text{ N} = 1296 \text{ N} \downarrow$

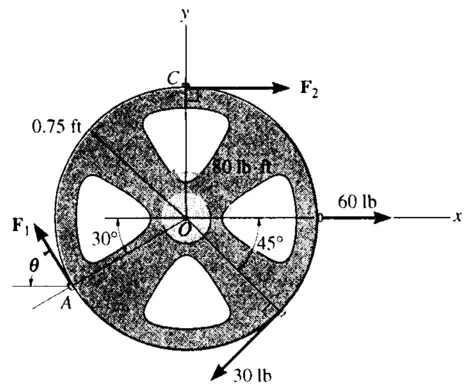
$F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N}$ **Ans**

$\theta = \tan^{-1}(\frac{1296}{125}) = 84.5°$ **Ans**

$\circlearrowleft + M_{RB} = \Sigma M_B;$ $1296(x) = -450 \sin 60°(4) + 700 \cos 30°(3) + 1500$

$x = 1.36 \text{ m (to the right)}$ **Ans**

4-117. Determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ and the direction of $\mathbf{F}_1$ so that the loading creates a zero resultant force and couple moment on the wheel.



Force Summation :

$$\xrightarrow{+} 0 = \Sigma F_x; \quad 0 = F_2 + 60 - F_1 \cos\theta - 30\cos 45°$$
$$F_2 - F_1 \cos\theta = -38.79 \qquad [1]$$

$$+\uparrow 0 = \Sigma F_y; \quad 0 = F_1 \sin\theta - 30\sin 45°$$
$$F_1 \sin\theta = 21.21 \qquad [2]$$

Moment Summation :

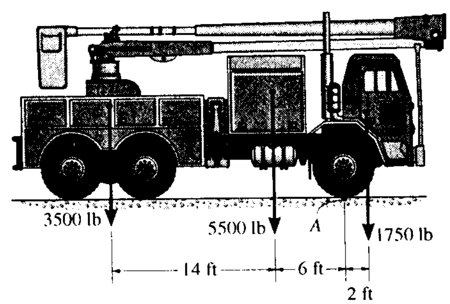
$$\circlearrowleft + \; 0 = \Sigma M_O; \quad 0 = 80 - F_2(0.75) - 30(0.75) - F_1 \sin\theta(0.75\cos 30°) - F_1\cos\theta(0.75\sin 30°)$$

$$0.6495F_1 \sin\theta + 0.375F_1 \cos\theta + 0.75F_2 = 57.5 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields

$F_2 = 25.9$ lb $\quad \theta = 18.1°\quad F_1 = 68.1$ lb **Ans**

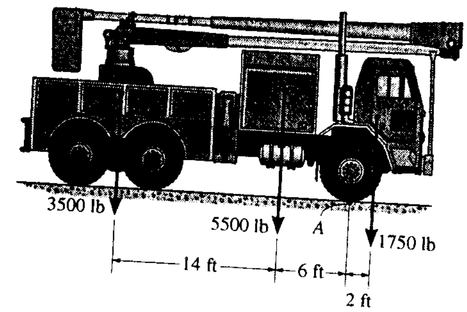
4-118. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and couple moment acting at point A.



$$+\uparrow F_R = \Sigma F_y; \quad F_R = -1750 - 5500 - 3500$$
$$= -10750 \text{ lb} = 10.75 \text{ kip} \downarrow \qquad \textbf{Ans}$$

$$\circlearrowleft + \; M_{R_A} = \Sigma M_A; \quad M_{R_A} = 3500(20) + 5500(6) - 1750(2)$$
$$= 99500 \text{ lb}\cdot\text{ft}$$
$$= 99.5 \text{ kip}\cdot\text{ft} \;\; (\textit{Counterclockwise}) \qquad \textbf{Ans}$$

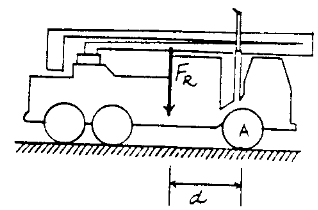
4-119. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from point A.



Equivalent Force :

$$+\uparrow F_R = \Sigma F_y; \quad F_R = -1750 - 5500 - 3500 = -10750 \text{ lb} = 10.75 \text{ kip} \downarrow \qquad \textbf{Ans}$$

Location of Resultant Force From Point A :



$$\curvearrowleft + M_{R_A} = \Sigma M_A; \quad 10750(d) = 3500(20) + 5500(6) - 1750(2)$$

$$d = 9.26 \text{ ft} \qquad \textbf{Ans}$$

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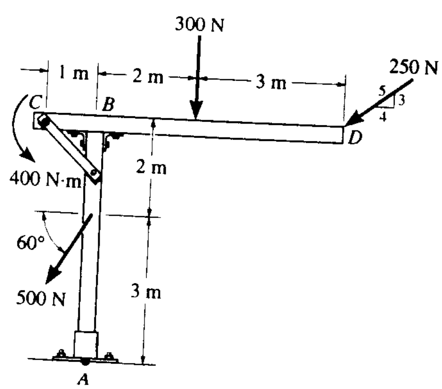
*4-120. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member AB, measured from A.



$$\xrightarrow{+} \Sigma F_x = F_{Rx}; \quad F_{Rx} = -250\left(\frac{4}{5}\right) - 500(\cos 60°) = -450 \text{ N} = 450 \text{ N} \leftarrow$$

$$+\uparrow \Sigma F_y = \Sigma F_y; \quad F_{Ry} = -300 - 250\left(\frac{3}{5}\right) - 500\sin 60° = -883.0127 \text{ N} = 883.0127 \text{ N} \downarrow$$

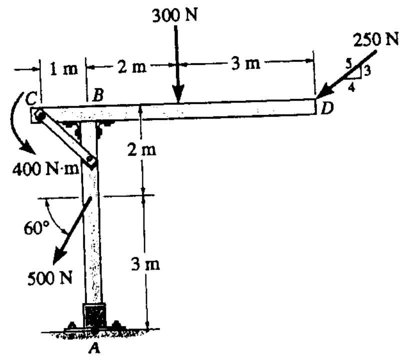
$$F_R = \sqrt{(-450)^2 + (-883.0127)^2} = 991 \text{ N} \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{883.0127}{450}\right) = 63.0°$$

$$\curvearrowleft + M_{RA} = \Sigma M_A; \quad 450y = 400 + (500\cos 60°)(3) + 250\left(\frac{4}{5}\right)(5) - 300(2) - 250\left(\frac{3}{5}\right)(5)$$

$$y = \frac{800}{450} = 1.78 \text{ m} \qquad \textbf{Ans}$$

4-121. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member CD, measured from end C.



$\xrightarrow{+} \Sigma F_x = F_{Rx};\quad F_{Rx} = -250(\frac{4}{5}) - 500(\cos 60°) = -450\text{ N} = 450\text{ N} \leftarrow$

$+\uparrow \Sigma F_y = \Sigma F_y;\quad F_{Ry} = -300 - 250(\frac{3}{5}) - 500\sin 60° = -883.0127\text{ N} = 883.0127\text{ N}\downarrow$

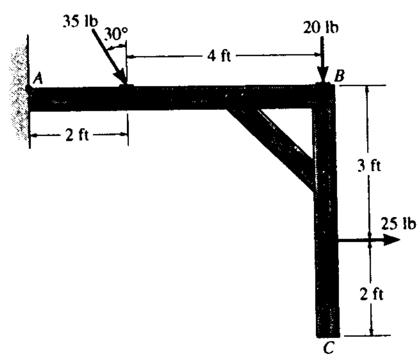
$F_R = \sqrt{(-450)^2 + (-883.0127)^2} = 991\text{ N}$ **Ans**

$\theta = \tan^{-1}(\frac{883.0127}{450}) = 63.0°$

$\circlearrowleft +M_{RA} = \Sigma M_C;\quad 883.0127x = -400 + 300(3) + 250(\frac{3}{5})(6) + 500\cos 60°(2) + (500\sin 60°)(1)$

$x = \frac{2333}{883.0127} = 2.64\text{ m}$ **Ans**

4-122. Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member AB, measured from point A.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 35\sin 30° + 25 = 42.5\text{ lb}$

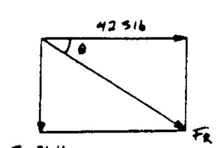
$+\downarrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 35\cos 30° + 20 = 50.31\text{ lb}$

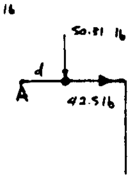
$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9\text{ lb}$ **Ans**

$\theta = \tan^{-1}\left(\frac{50.31}{42.5}\right) = 49.8°$ **Ans**

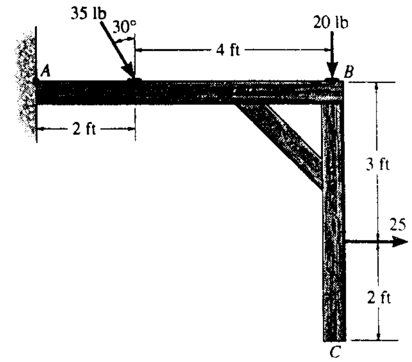
$\circlearrowleft +M_{RA} = \Sigma M_A;\quad 50.31(d) = 35\cos 30°(2) + 20(6) - 25(3)$

$d = 2.10\text{ ft}$ **Ans**





4-123. Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member BC, measured from point B.

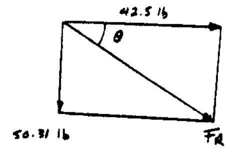


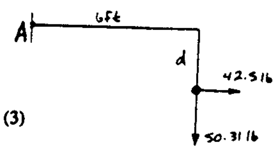
$\xrightarrow{+} F_{Rx} = \Sigma F_x$; $\quad F_{Rx} = 35 \sin 30° + 25 = 42.5$ lb

$+\downarrow F_{Ry} = \Sigma F_y$; $\quad F_{Ry} = 35 \cos 30° + 20 = 50.31$ lb

$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9$ lb **Ans**

$\theta = \tan^{-1}\left(\dfrac{50.31}{42.5}\right) = 49.8°$ **Ans**

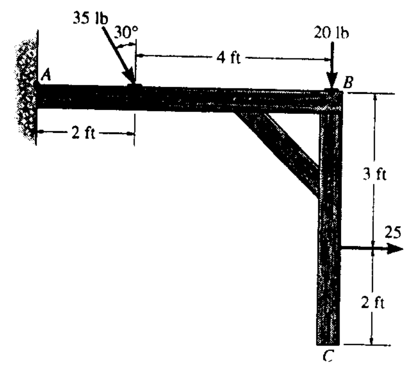




$\curvearrowleft + M_{RA} = \Sigma M_A$; $\quad 50.31(6) - 42.5(d) = 35 \cos 30° (2) + 20(6) - 25(3)$

$d = 4.62$ ft **Ans**

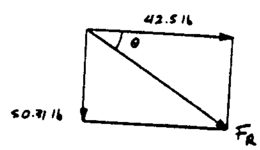
***4-124.** Replace the force system acting on the frame by an equivalent resultant force and couple moment acting at point A.



$\xrightarrow{+} F_{Rx} = \Sigma F_x$; $\quad F_{Rx} = 35 \sin 30° + 25 = 42.5$ lb

$+\downarrow F_{Ry} = \Sigma F_y$; $\quad F_{Ry} = 35 \cos 30° + 20 = 50.31$ lb

$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9$ lb **Ans**

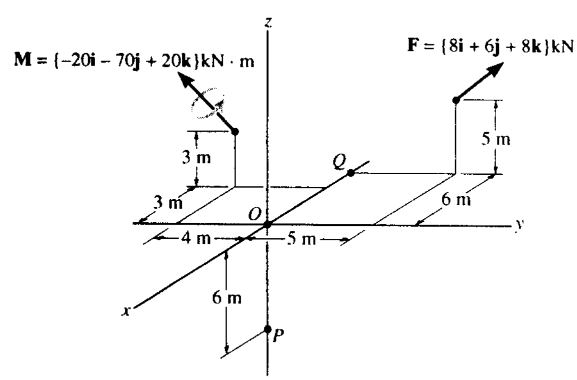


$\theta = \tan^{-1}\left(\dfrac{50.31}{42.5}\right) = 49.8°$ **Ans**

$\curvearrowleft + M_{RA} = \Sigma M_A$; $\quad M_{RA} = 35 \cos 30° (2) + 20(6) - 25(3)$

$M_{RA} = 106$ lb · ft $\circlearrowright$ **Ans**

4-125. Replace the force and couple-moment system by an equivalent resultant force and couple moment at point O. Express the results in Cartesian vector form.

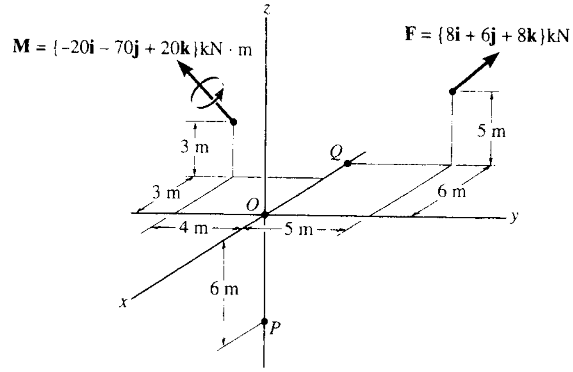


$\mathbf{F}_R = \Sigma \mathbf{F}$; $\quad \mathbf{F}_R = \{8\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}\}$ kN **Ans**

$$\mathbf{M}_{RO} = \Sigma \mathbf{M}_O; \quad \mathbf{M}_{RO} = -20\mathbf{i} - 70\mathbf{j} + 20\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 5 & 5 \\ 8 & 6 & 8 \end{vmatrix}$$

$$= \{-10\mathbf{i} + 18\mathbf{j} - 56\mathbf{k}\} \text{ kN} \cdot \text{m} \quad \textbf{Ans}$$

4-126. Replace the force and couple-moment system by an equivalent resultant force and couple moment at point P. Express the results in Cartesian vector form.

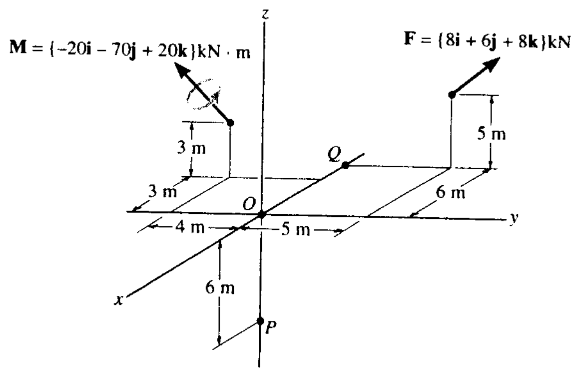


$\mathbf{F}_R = \{8\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}\}$ kN **Ans**

$$\mathbf{M}_{RP} = \Sigma\mathbf{M}_P = -20\mathbf{i} - 70\mathbf{j} + 20\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 5 & 11 \\ 8 & 6 & 8 \end{vmatrix}$$

$= \{-46\mathbf{i} + 66\mathbf{j} - 56\mathbf{k}\}$ kN·m **Ans**

4-127. Replace the force and couple-moment system by an equivalent resultant force and couple moment at point Q. Express the results in Cartesian vector form.

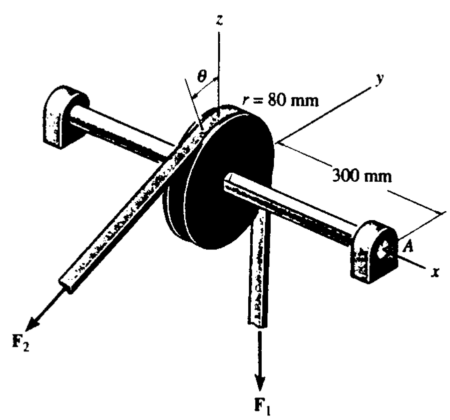


$\mathbf{F}_R = \{8\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}\}$ kN **Ans**

$$\mathbf{M}_{RQ} = -20\mathbf{i} - 70\mathbf{j} + 20\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 5 \\ 8 & 6 & 8 \end{vmatrix}$$

$= \{-10\mathbf{i} - 30\mathbf{j} - 20\mathbf{k}\}$ kN·m **Ans**

***4-128.** The belt passing over the pulley is subjected to forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Set $\theta = 0°$ so that $\mathbf{F}_2$ acts in the $-\mathbf{j}$ direction.



$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

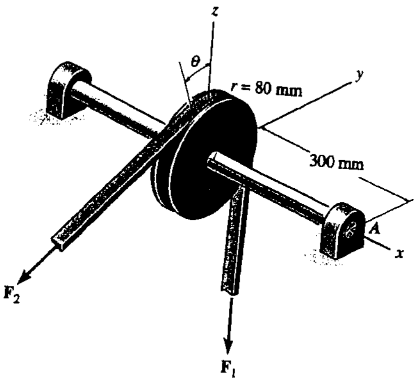
$\mathbf{F}_R = \{-40\mathbf{j} - 40\mathbf{k}\}$N **Ans**

$\mathbf{M}_{RA} = \Sigma(\mathbf{r} \times \mathbf{F})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0 & 0.08 \\ 0 & -40 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \end{vmatrix}$$

$\mathbf{M}_{RA} = \{-12\mathbf{j} + 12\mathbf{k}\}$ N·m **Ans**

4-129. The belt passing over the pulley is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Take $\theta = 45°$.



$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$= -40\cos45°\mathbf{j} + (-40 - 40\sin45°)\mathbf{k}$

$\mathbf{F}_R = \{-28.3\mathbf{j} - 68.3\mathbf{k}\}$ N **Ans**

$\mathbf{r}_{AF1} = \{-0.3\mathbf{i} + 0.08\mathbf{j}\}$ m

$\mathbf{r}_{AF2} = -0.3\mathbf{i} - 0.08\sin45°\mathbf{j} + 0.08\cos45°\mathbf{k}$

$= \{-0.3\mathbf{i} - 0.0566\mathbf{j} + 0.0566\mathbf{k}\}$ m

$\mathbf{M}_{RA} = (\mathbf{r}_{AF1} \times \mathbf{F}_1) + (\mathbf{r}_{AF2} \times \mathbf{F}_2)$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & -0.0566 & 0.0566 \\ 0 & -40\cos45° & -40\sin45° \end{vmatrix}$$

$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\}$ N·m **Ans**

Also,

$M_{RA_x} = \Sigma M_{A_x}$

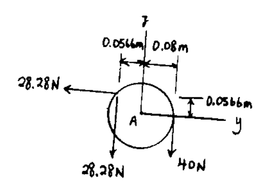
$M_{RA_x} = 28.28(0.0566) + 28.28(0.0566) - 40(0.08)$

$M_{RA_x} = 0$

$M_{RA_y} = \Sigma M_{A_y}$

$M_{RA_y} = -28.28(0.3) - 40(0.3)$

$M_{RA_y} = -20.5$ N·m



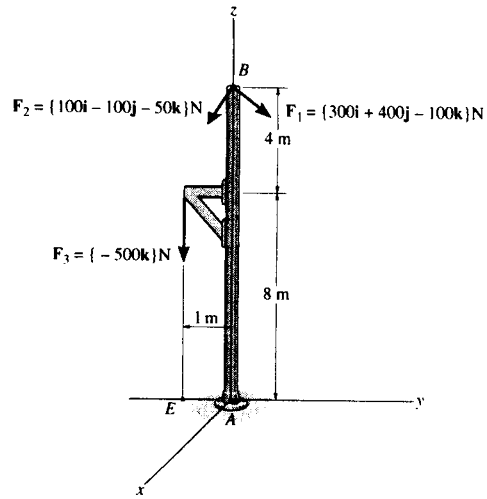
$M_{RA_z} = \Sigma M_{A_z}$

$M_{RA_z} = 28.28(0.3)$

$M_{RA_z} = 8.49$ N·m

$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\}$ N·m **Ans**

4-130. Replace the force system by an equivalent force and couple moment at point A.



$\mathbf{F}_R = \Sigma \mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$= (300 + 100)\mathbf{i} + (400 - 100)\mathbf{j} + (-100 - 50 - 500)\mathbf{k}$

$= \{400\mathbf{i} + 300\mathbf{j} - 650\mathbf{k}\}$ N **Ans**

The position vectors are $\mathbf{r}_{AB} = \{12\mathbf{k}\}$ m and $\mathbf{r}_{AE} = \{-1\mathbf{j}\}$ m.

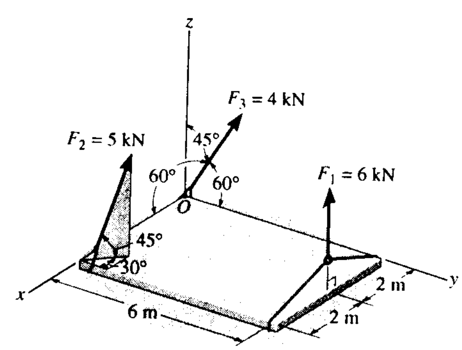
$$\mathbf{M}_{R_A} = \Sigma \mathbf{M}_A;\quad \mathbf{M}_{R_A} = \mathbf{r}_{AB} \times \mathbf{F}_1 + \mathbf{r}_{AB} \times \mathbf{F}_2 + \mathbf{r}_{AE} \times \mathbf{F}_3$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 300 & 400 & -100 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 100 & -100 & -50 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 0 \\ 0 & 0 & -500 \end{vmatrix}$$

$= \{-3100\mathbf{i} + 4800\mathbf{j}\}$ N·m **Ans**

4-131. The slab is to be hoisted using the three slings shown. Replace the system of forces acting on slings by an equivalent force and couple moment at point O. The force $\mathbf{F}_1$, is vertical.



Force Vectors :

$\mathbf{F}_1 = \{6.00\mathbf{k}\}$ kN

$\mathbf{F}_2 = 5(-\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k})$

$= \{-1.768\mathbf{i} + 3.062\mathbf{j} + 3.536\mathbf{k}\}$ kN

$\mathbf{F}_3 = 4(\cos 60°\mathbf{i} + \cos 60°\mathbf{j} + \cos 45°\mathbf{k})$

$= \{2.00\mathbf{i} + 2.00\mathbf{j} + 2.828\mathbf{k}\}$ kN

Equivalent Force and Couple Moment At Point O :

$\mathbf{F}_R = \Sigma \mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$= (-1.768 + 2.00)\mathbf{i} + (3.062 + 2.00)\mathbf{j}$

$+ (6.00 + 3.536 + 2.828)\mathbf{k}$

$= \{0.232\mathbf{i} + 5.06\mathbf{j} + 12.4\mathbf{k}\}$ kN **Ans**

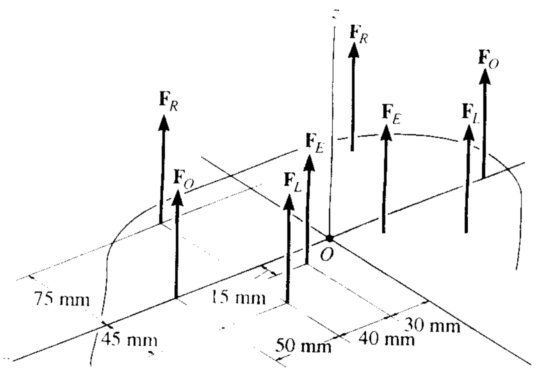
The position vectors are $\mathbf{r}_1 = \{2\mathbf{i} + 6\mathbf{j}\}$ m and $\mathbf{r}_2 = \{4\mathbf{i}\}$ m.

$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_O;\quad \mathbf{M}_{R_O} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 6 & 0 \\ 0 & 0 & 6.00 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 0 \\ -1.768 & 3.062 & 3.536 \end{vmatrix}$$

$= \{36.0\mathbf{i} - 26.1\mathbf{j} + 12.2\mathbf{k}\}$ kN·m **Ans**

***4-132.** A biomechanical model of the lumber region of the human trunk is shown. The forces acting in the four muscle groups consist of $F_R = 35$ N for the rectus, $F_O = 45$ N for the oblique, $F_L = 23$ N for the lumbar latissimus dorsi, and $F_E = 32$ N for the erector spinae. These loadings are symmetric with respect to the y-z plane. Replace this system of parallel forces by an equivalent force and couple moment acting at the spine, point O. Express the results in Cartesian vector form.

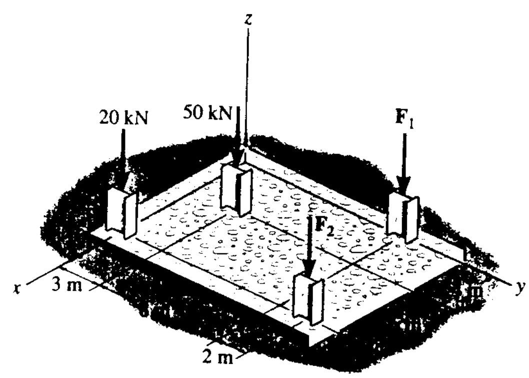


$F_R = \Sigma F_z$; $\mathbf{F}_R = \{2(35 + 45 + 23 + 32)\mathbf{k}\} = \{270\mathbf{k}\}$ N   **Ans**

$\mathbf{M}_{RO_x} = \Sigma \mathbf{M}_{O_x}$; $\mathbf{M}_{RO} = [-2(35)(0.075) + 2(32)(0.015) + 2(23)(0.045)]\mathbf{i}$

$\mathbf{M}_{RO} = \{-2.22\mathbf{i}\}$ N·m   **Ans**

4-133. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 30$ kN, $F_2 = 40$ kN.



$+\uparrow F_R = \Sigma F_z$; $F_R = -30 - 50 - 40 - 20 = -140$ kN $= 140$ kN $\downarrow$ **Ans**

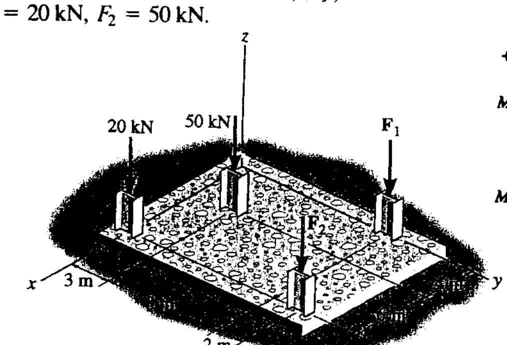
$(M_R)_x = \Sigma M_x$; $-140y = -50(3) - 30(11) - 40(13)$

$y = 7.14$ m   **Ans**

$(M_R)_y = \Sigma M_y$; $140x = 50(4) + 20(10) + 40(10)$

$x = 5.71$ m   **Ans**

4-134. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 20$ kN, $F_2 = 50$ kN.



$+\downarrow F_R = \Sigma F_z$; $F_R = 20 + 50 + 20 + 50 = 140$ kN   **Ans**

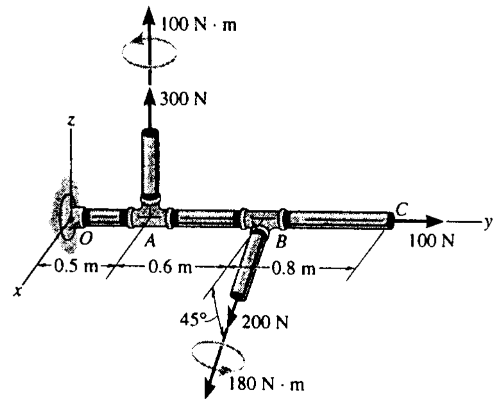
$M_{RO_y} = \Sigma M_y$; $140(x) = (50)(4) + 20(10) + 50(10)$

$x = 6.43$ m   **Ans**

$M_{RO_x} = \Sigma M_x$; $-140(y) = -(50)(3) - 20(11) - 50(13)$

$y = 7.29$ m   **Ans**

***4-135.** Replace the two wrenches and the force, acting on the pipe assembly, by an equivalent resultant force and couple moment at point O.



Force And Moment Vectors :

$\mathbf{F}_1 = \{300\mathbf{k}\}$ N $\qquad \mathbf{F}_3 = \{100\mathbf{j}\}$ N

$\mathbf{F}_2 = 200\{\cos 45°\mathbf{i} - \sin 45°\mathbf{k}\}$ N
$= \{141.42\mathbf{i} - 141.42\mathbf{k}\}$ N

$\mathbf{M}_1 = \{100\mathbf{k}\}$ N · m

$\mathbf{M}_2 = 180\{\cos 45°\mathbf{i} - \sin 45°\mathbf{k}\}$ N · m
$= \{127.28\mathbf{i} - 127.28\mathbf{k}\}$ N · m

Equivalent Force and Couple Moment At Point O :

$\mathbf{F}_R = \Sigma\mathbf{F}; \quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$
$= 141.42\mathbf{i} + 100.0\mathbf{j} + (300 - 141.42)\mathbf{k}$

$= \{141\mathbf{i} + 100\mathbf{j} + 159\mathbf{k}\}$ N **Ans**

The position vectors are $\mathbf{r}_1 = \{0.5\mathbf{j}\}$ m and $\mathbf{r}_2 = \{1.1\mathbf{j}\}$ m.

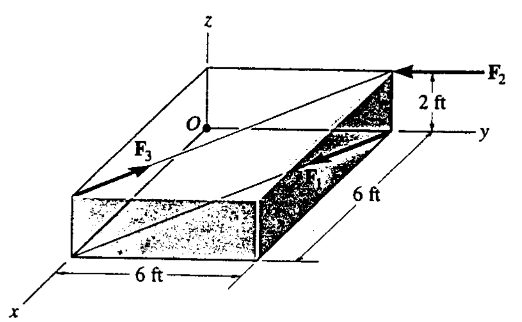
$$\mathbf{M}_{R_O} = \Sigma\mathbf{M}_O; \quad \mathbf{M}_{R_O} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{M}_1 + \mathbf{M}_2$$

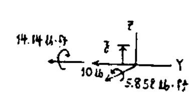
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.5 & 0 \\ 0 & 0 & 300 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.1 & 0 \\ 141.42 & 0 & -141.42 \end{vmatrix}$$

$$+ 100\mathbf{k} + 127.28\mathbf{i} - 127.28\mathbf{k}$$

$= \{122\mathbf{i} - 183\mathbf{k}\}$ N · m **Ans**

***4-136.** The three forces acting on the block each have a magnitude of 10 lb. Replace this system by a wrench and specify the point where the wrench intersects the z axis, measured from point O.





$\mathbf{F}_R = \{-10\mathbf{j}\}$ lb

$\mathbf{M}_O = (6\mathbf{j} + 2\mathbf{k}) \times (-10\mathbf{j}) + 2(10)(-0.707\mathbf{i} - 0.707\mathbf{j})$

$= \{5.858\mathbf{i} - 14.14\mathbf{j}\}$ lb·ft

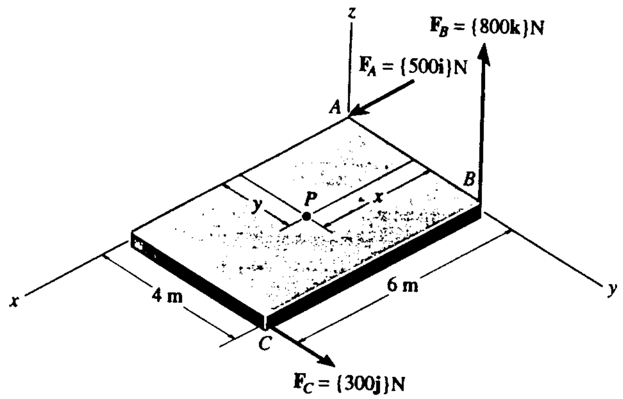
Require

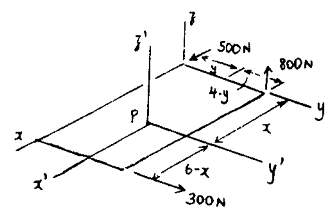
$z = \dfrac{5.858}{10} = 0.586$ ft **Ans**

$\mathbf{F}_w = \{-10\mathbf{j}\}$ lb **Ans**

$\mathbf{M}_w = \{-14.1\mathbf{j}\}$ lb·ft **Ans**

4-137. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.





$\mathbf{F}_R = \{500\mathbf{i} + 300\mathbf{j} + 800\mathbf{k}\}$ N

$F_R = \sqrt{(500)^2 + (300)^2 + (800)^2} = 990$ N **Ans**

$\mathbf{u}_{FR} = \{0.5051\mathbf{i} + 0.3030\mathbf{j} + 0.8081\mathbf{k}\}$

$M_{R_{x'}} = \Sigma M_{x'}$; $M_{R_{x'}} = 800(4-y)$

$M_{R_{y'}} = \Sigma M_{y'}$; $M_{R_{y'}} = 800x$

$M_{R_{z'}} = \Sigma M_{z'}$; $M_{R_{z'}} = 500y + 300(6-x)$

Since $\mathbf{M}_R$ also acts in the direction of $\mathbf{u}_{FR}$,

$M_R(0.5051) = 800(4-y)$

$M_R(0.3030) = 800x$

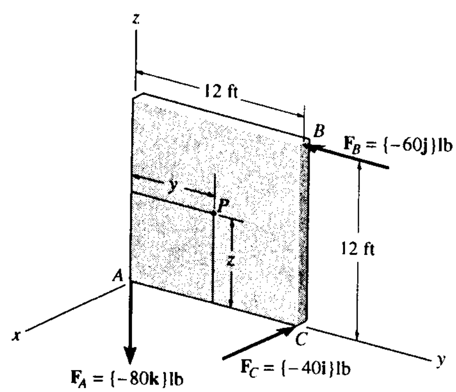
$M_R(0.8081) = 500y + 300(6-x)$

$M_R = 3.07$ kN·m **Ans**

$x = 1.16$ m **Ans**

$y = 2.06$ m **Ans**

4-138. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(y, z)$ where its line of action intersects the plate.



Resultant Force Vector :

$\mathbf{F}_R = \{-40\mathbf{i} - 60\mathbf{j} - 80\mathbf{k}\}$ lb

$F_R = \sqrt{(-40)^2 + (-60)^2 + (-80)^2} = 107.70 \text{ lb} = 108 \text{ lb}$ **Ans**

$$\mathbf{u}_{F_R} = \frac{-40\mathbf{i} - 60\mathbf{j} - 80\mathbf{k}}{107.70} = -0.3714\mathbf{i} - 0.5571\mathbf{j} - 0.7428\mathbf{k}$$

Resultant Moment: The line of action of $\mathbf{M}_R$ of the wrench is parallel to the line of action of $\mathbf{F}_R$. Assume that both $\mathbf{M}_R$ and $\mathbf{F}_R$ have the same sense. Therefore, $\mathbf{u}_{M_R} = -0.3714\mathbf{i} - 0.5571\mathbf{j} - 0.7428\mathbf{k}$.

$(M_R)_{x'} = \Sigma M_{x'}$; $-0.3714M_R = 60(12-z) + 80y$ [1]

$(M_R)_{y'} = \Sigma M_{y'}$; $-0.5571M_R = 40z$ [2]

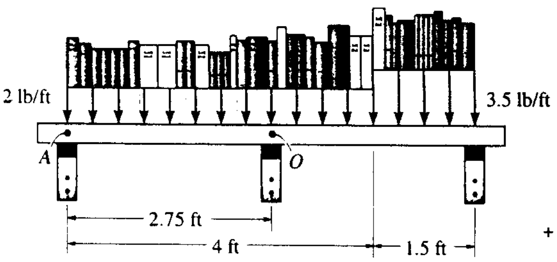
$(M_R)_{z'} = \Sigma M_{z'}$; $-0.7428M_R = 40(12-y)$ [3]

Solving Eqs.[1], [2], and [3] yields :

$M_R = -624$ lb·ft $z = 8.69$ ft $y = 0.414$ ft **Ans**

The negative sign indicates that the line of action for $\mathbf{M}_R$ is directed in the opposite sense to that of $\mathbf{F}_R$.

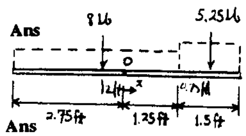
4-139. The loading on the bookshelf is distribut Determine the magnitude of the equivalent resultant location, measured from point O.



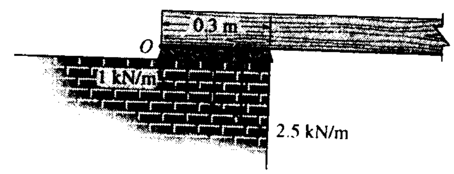
$+\downarrow F_{RO} = \Sigma F;$ $\quad F_{RO} = 8 + 5.25 = 13.25 = 13.2 \text{ lb} \downarrow$ **Ans**

$\curvearrowleft + M_{RO} = \Sigma M_O;$ $\quad 13.25x = 5.25(0.75 + 1.25) - 8(2 - 1.25)$

$x = 0.340 \text{ ft}$ **Ans**



***4-140.** The masonry support creates the loading distribution acting on the end of the beam. Simplify this load to a single resultant force and specify its location measured from point O.



Equivalent Resultant Force :

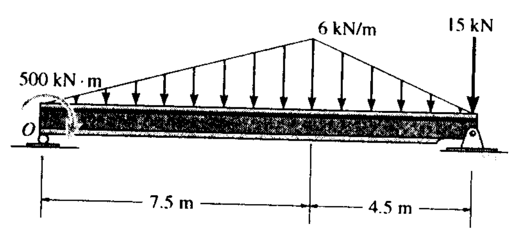
$+\uparrow F_R = \Sigma F_y;$ $\quad F_R = 0.300 + 0.225 = 0.525 \text{ kN} \uparrow$ **Ans**

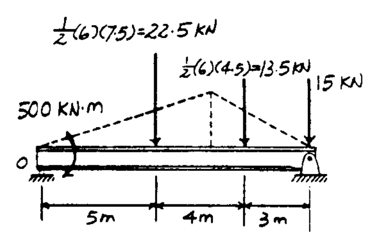
Location of Equivalent Resultant Force :

$\curvearrowleft + (M_R)_O = \Sigma M_O;$ $\quad 0.525(d) = 0.300(0.15) + 0.225(0.2)$

$d = 0.171 \text{ m}$ **Ans**

4-141. Replace the loading by an equivalent force and couple moment acting at point O.





$+\uparrow F_R = \Sigma F_y;$ $\quad F_R = -22.5 - 13.5 - 15.0$

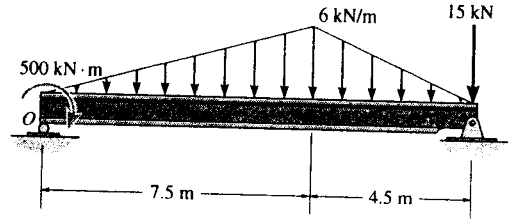
$= -51.0 \text{ kN} = 51.0 \text{ kN} \downarrow$ **Ans**

$\curvearrowleft + M_{R_O} = \Sigma M_O;$ $\quad M_{R_O} = -500 - 22.5(5) - 13.5(9) - 15(12)$

$= -914 \text{ kN} \cdot \text{m}$

$= 914 \text{ kN} \cdot \text{m}$ ***(Clockwise)*** **Ans**

4-142. Replace the loading by a single resultant force, and specify the location of the force on the beam measured from point O.



Equivalent Resultant Force :

$+\uparrow F_R = \Sigma F_y;\quad -F_R = -22.5 - 13.5 - 15$

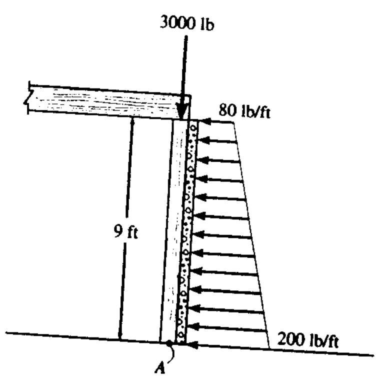
$F_R = 51.0\text{ kN}\downarrow$ **Ans**

Location of Equivalent Resultant Force :

$\circlearrowleft + (M_R)_O = \Sigma M_O;\quad -51.0(d) = -500 - 22.5(5) - 13.5(9) - 15(12)$

$d = 17.9\text{ m}$ **Ans**

4-143. The column is used to support the floor which exerts a force of 3000 lb on the top of the column. The effect of soil pressure along its side is distributed as shown. Replace this loading by an equivalent resultant force and specify where it acts along the column, measured from its base A.



$\overset{+}{\leftarrow}\Sigma F_{Rx} = \Sigma F_x;\quad F_{Rx} = 720 + 540 = 1260\text{ lb}$

$+\downarrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 3000\text{ lb}$

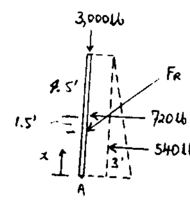
$F_R = \sqrt{(1260)^2 + (3000)^2} = 3254\text{ lb}$

$F_R = 3.25\text{ kip}$ **Ans**

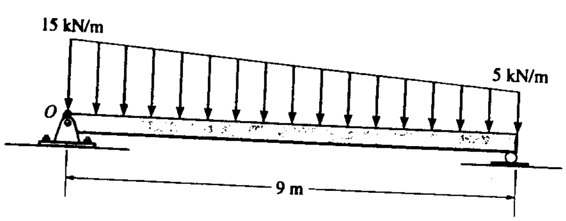
$\theta = \tan^{-1}\left[\frac{3000}{1260}\right] = 67.2°$ **Ans**

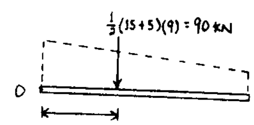
$\circlearrowleft + M_{RA} = \Sigma M_A;\quad 1260x = 540(3) + 720(4.5)$

$x = 3.86\text{ ft}$ **Ans**



***4-144.** Replace the loading by an equivalent force and couple moment acting at point O.

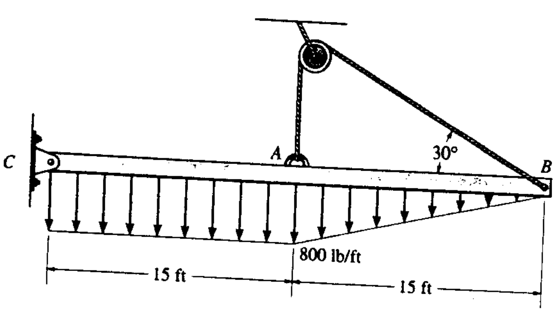




$+\downarrow F_R = \Sigma F;\quad F_R = 90\text{ kN}\downarrow$ **Ans**

$\circlearrowleft + M_{RO} = \Sigma M_O;\quad M_{RO} = 90(3.75) = 338\text{ kN}\cdot\text{m}$ **Ans**

4-145. Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at C.

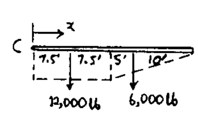


$+\downarrow F_R = \Sigma F;\quad F_R = 12\,000 + 6000 = 18\,000\text{ lb}$

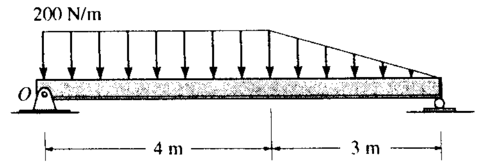
$F_R = 18.0\text{ kip}\downarrow$ **Ans**

$\circlearrowleft + M_{RC} = \Sigma M_C;\quad 18\,000x = 12\,000(7.5) + 6000(20)$

$x = 11.7\text{ ft}$ **Ans**



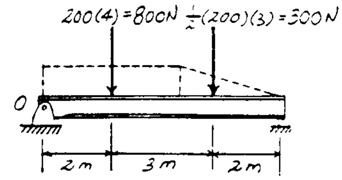
4-146. Replace the loading by an equivalent force and couple moment acting at point O.



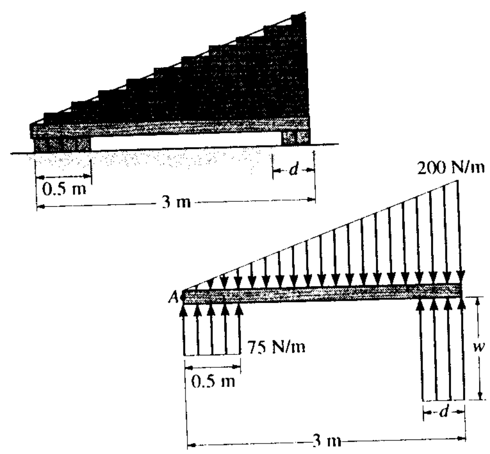
Equivalent Force and Couple Moment At Point O :

$+\uparrow F_R = \Sigma F_y;\quad F_R = -800-300$
$= -1100\text{ N} = 1.10\text{ kN}\downarrow$ **Ans**

$+ M_{R_O} = \Sigma M_O;\quad M_{R_O} = -800(2)-300(5)$
$= -3100\text{ N}\cdot\text{m}$
$= 3.10\text{ kN}\cdot\text{m}$ ***(Clockwise)*** **Ans**



***4-147.** The bricks on top of the beam and the supports at the bottom create the distributed loading shown in the second figure. Determine the required intensity w and dimension d of the right support so that the resultant force and couple moment about point A of the system are both zero.



Require $F_R = 0$.

$+\uparrow F_R = \Sigma F_y;\quad 0 = wd + 37.5 - 300$
$wd = 262.5$ [1]

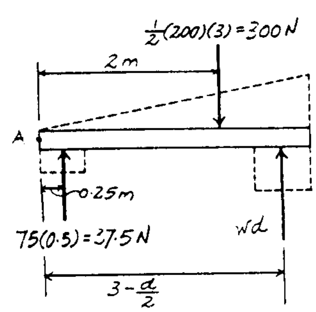
Require $M_{R_A} = 0$.

$+ M_{R_A} = \Sigma M_A;\quad 0 = 37.5(0.25) + wd\left(3-\frac{d}{2}\right) - 300(2)$

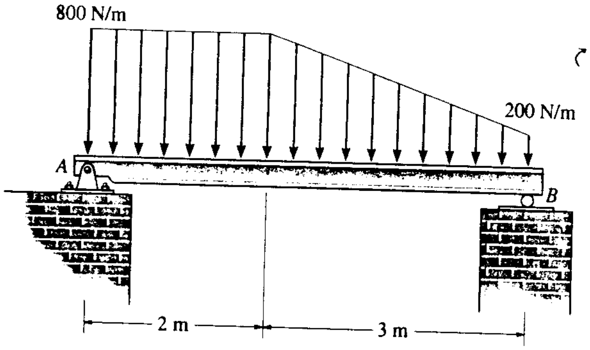
$$3wd - \frac{wd^2}{2} = 590.625 \quad [2]$$

Solving Eqs. [1] and [2] yields

$d = 1.50\text{ m}\qquad w = 175\text{ N/m}$ **Ans**



***4-148.** Replace the distributed loading by an equivalent resultant force and specify its location, measured from point A.

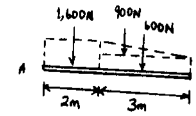


$+\downarrow F_R = \Sigma F;\quad F_R = 1600 + 900 + 600 = 3100\text{ N}$

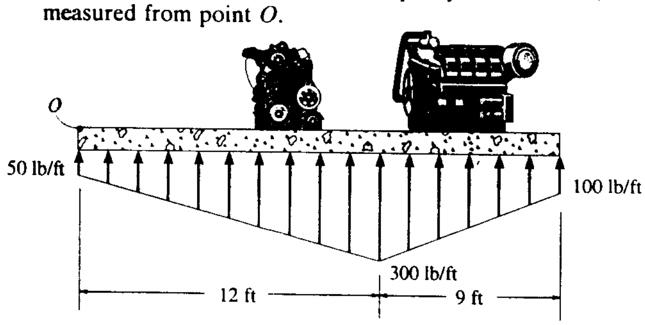
$F_R = 3.10\text{ kN}\downarrow$ **Ans**

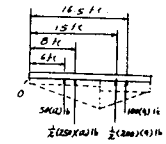
$\curvearrowleft + M_{RA} = \Sigma M_A;\quad x(3100) = 1600(1) + 900(3) + 600(3.5)$

$x = 2.06\text{ m}$ **Ans**



4-149. The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point O.





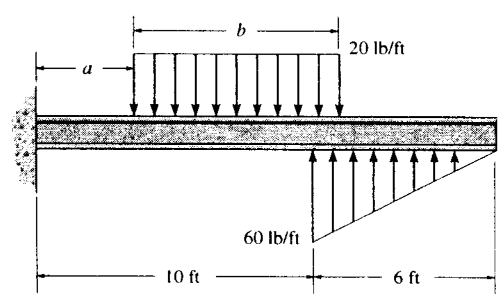
$+\uparrow F_R = \Sigma F_y;\quad F_R = 50(12) + \frac{1}{2}(250)(12) + \frac{1}{2}(200)(9) + 100(9)$

$= 3900\text{ lb} = 3.90\text{ kip}\uparrow$ **Ans**

$\curvearrowright + M_{R_O} = \Sigma M_O;\quad 3900(d) = 50(12)(6) + \frac{1}{2}(250)(12)(8) + \frac{1}{2}(200)(9)(15) + 100(9)(16.5)$

$d = 11.3\text{ ft}$ **Ans**

4-150. The beam is subjected to the distributed loading. Determine the length b of the uniform load and its position a on the beam such that the resultant force and couple moment acting on the beam are zero.



Require $F_R = 0$.

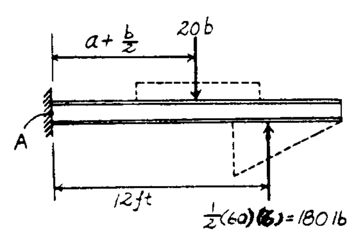
$+\uparrow F_R = \Sigma F_y;\quad 0 = 180 - 20b$

$b = 9.00\text{ ft}$ **Ans**

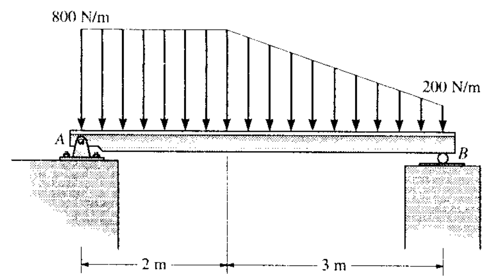
Require $M_{R_A} = 0$. Using the result $b = 9.00$ ft, we have

$\curvearrowleft + M_{R_A} = \Sigma M_A;\quad 0 = 180(12) - 20(9.00)\left(a + \frac{9.00}{2}\right)$

$a = 7.50\text{ ft}$ **Ans**



***4-148.** Replace the distributed loading by an equivalent resultant force and specify its location, measured from point A.

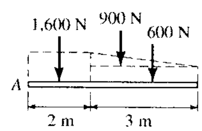


$+\downarrow F_R = \Sigma F;\quad F_R = 1600 + 900 + 600 = 3100 \text{ N}$

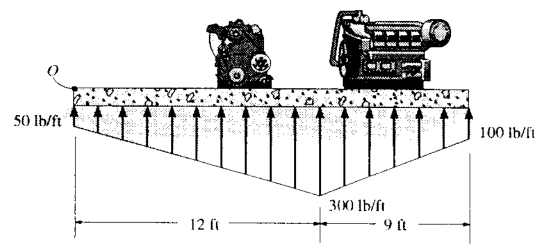
$F_R = 3.10 \text{ kN} \downarrow$ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A;\quad x(3100) = 1600(1) + 900(3) + 600(3.5)$

$x = 2.06 \text{ m}$ **Ans**



4-149. The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point O.



$+\uparrow F_R = \Sigma F_y;\quad F_R = 50(12) + \frac{1}{2}(250)(12)$

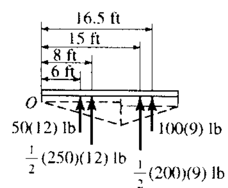
$+ \frac{1}{2}(200)(9) + 100(9)$

$= 3900 \text{ lb} = 3.90 \text{ kip} \uparrow$ **Ans**

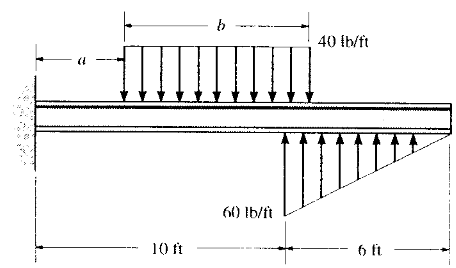
$\circlearrowleft + M_{R_O} = \Sigma M_O;\quad 3900(d) = 50(12)(6) + \frac{1}{2}(250)(12)(8)$

$+ \frac{1}{2}(200)(9)(15) + 100(9)(16.5)$

$d = 11.3 \text{ ft}$ **Ans**



4-150. The beam is subjected to the distributed loading. Determine the length b of the uniform load and its position a on the beam such that the resultant force and couple moment acting on the beam are zero.



Require $F_R = 0$.

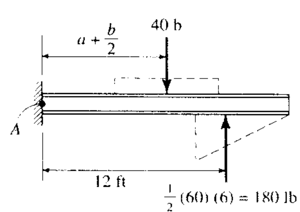
$+\uparrow F_R = \Sigma F_y;\quad 0 = 180 - 40b$

$b = 4.50 \text{ ft}$ **Ans**

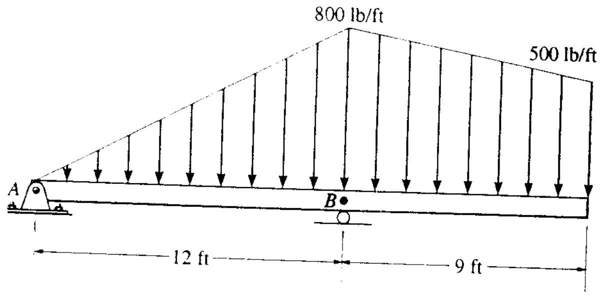
Require $M_{R_A} = 0$. Using the result $b = 4.50$ ft, we have

$\circlearrowleft + M_{R_A} = \Sigma M_A;\quad 0 = 180(12) - 40(4.50)\left(a + \frac{4.50}{2}\right)$

$a = 9.75 \text{ ft}$ **Ans**



4-151. Replace the loading by an equivalent resultant force and specify its location on the beam, measured from point B.

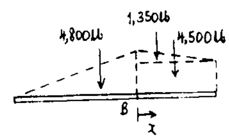


$+\downarrow F_R = \Sigma F;\quad F_R = 4800 + 1350 + 4500 = 10\,650\text{ lb}$

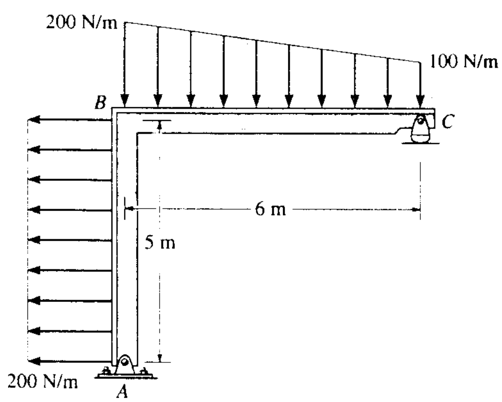
$F_R = 10.6\text{ kip}\downarrow$ **Ans**

$\circlearrowleft + M_{RB} = \Sigma M_B;\quad 10\,650x = -4800(4) + 1350(3) + 4500(4.5)$

$x = 0.479\text{ ft}$ **Ans**



***4-152.** Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects member AB, measured from A.



$\overset{+}{\leftarrow}\Sigma F_{Rx} = \Sigma F_x;\quad F_{Rx} = 1000\text{ N}$

$+\downarrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 900\text{ N}$

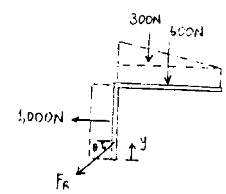
$F_R = \sqrt{(1000)^2 + (900)^2} = 1345\text{ N}$

$F_R = 1.35\text{ kN}$ **Ans**

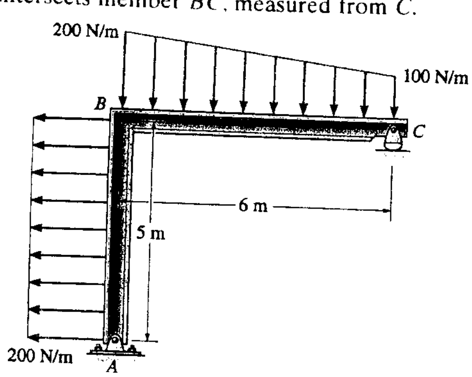
$\theta = \tan^{-1}\left[\frac{900}{1000}\right] = 42.0°$ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A;\quad 1000y = 1000(2.5) - 300(2) - 600(3)$

$y = 0.1\text{ m}$ **Ans**



4-153. Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects member BC, measured from C.



$\overset{+}{\leftarrow}\Sigma F_{Rx} = \Sigma F_x;\quad F_{Rx} = 1000\text{ N}$

$+\downarrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 900\text{ N}$

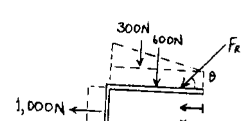
$F_R = \sqrt{(1000)^2 + (900)^2} = 1345\text{ N}$

$F_R = 1.35\text{ kN}$ **Ans**

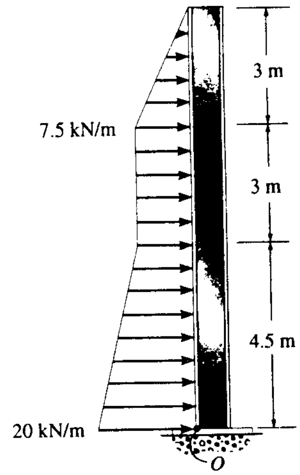
$\theta = \tan^{-1}\left[\frac{900}{1000}\right] = 42.0°$ **Ans**

$\circlearrowleft + M_{RC} = \Sigma M_C;\quad 900x = 600(3) + 300(4) - 1000(2.5)$

$x = 0.556\text{ m}$ **Ans**

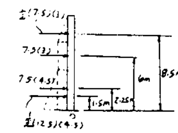


4-154. Replace the loading by an equivalent resultant force and couple moment acting at point O.



$\xrightarrow{+} F_R = \Sigma F_x;\quad F_R = \frac{1}{2}(12.5)(4.5) + 7.5(4.5) + 7.5(3) + \frac{1}{2}(7.5)(3)$

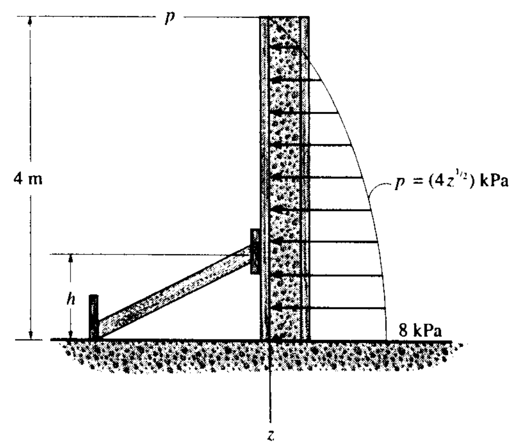
$= 95.6\ \text{kN} \rightarrow$ **Ans**



$\circlearrowleft +M_{R_O} = \Sigma M_O;\quad M_{R_O} = -\frac{1}{2}(12.5)(4.5)(1.5) - 7.5(4.5)(2.25) - 7.5(3)(6) - \frac{1}{2}(7.5)(3)(8.5)$

$= -349\ \text{kN}\cdot\text{m} = 349\ \text{kN}\cdot\text{m}$ ↻ **Ans**

4-155. Wet concrete exerts a pressure distribution along the wall of the form. Determine the resultant force of this distribution and specify the height h where the bracing strut should be placed so that it lies through the line of action of the resultant force. The wall has a width of 5 m.

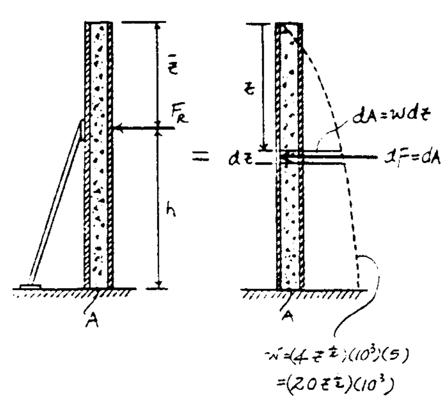


Equivalent Resultant Force :

$$\xrightarrow{+} F_R = \Sigma F_x;\quad -F_R = -\int_A dA = -\int_0^z w\,dz$$

$$F_R = \int_0^{4\text{m}} \left(20z^{\frac{1}{2}}\right)\left(10^3\right) dz$$

$$= 106.67\left(10^3\right)\ \text{N} = 107\ \text{kN} \leftarrow \qquad \textbf{Ans}$$



Location of Equivalent Resultant Force :

$$\bar{z} = \frac{\int_A z\,dA}{\int_A dA} = \frac{\int_0^z zw\,dz}{\int_0^z w\,dz}$$

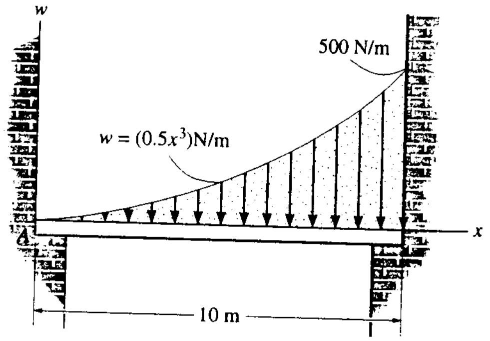
$$= \frac{\int_0^{4\text{m}} z\left[\left(20z^{\frac{1}{2}}\right)(10^3)\right]dz}{\int_0^{4\text{m}}\left(20z^{\frac{1}{2}}\right)(10^3)\,dz}$$

$$= \frac{\int_0^{4\text{m}}\left[\left(20z^{\frac{3}{2}}\right)(10^3)\right]dz}{\int_0^{4\text{m}}\left(20z^{\frac{1}{2}}\right)(10^3)\,dz}$$

$$= 2.40\ \text{m}$$

Thus, $h = 4 - \bar{z} = 4 - 2.40 = 1.60\ \text{m}$ **Ans**

***4-156.** Wind has blown sand over a platform such that the intensity of the load can be approximated by the function $w = (0.5x^3)$ N/m. Simplify this distributed loading to an equivalent resultant force and specify the magnitude and location of the force, measured from A.



$dA = w\,dx$

$F_R = \int dA = \int_0^{10} \frac{1}{2}x^3 dx$

$= \left[\frac{1}{8}x^4\right]_0^{10}$

$= 1250$ N

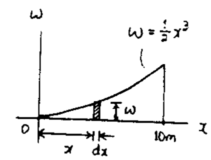
$F_R = 1.25$ kN **Ans**

$\int \tilde{x} dA = \int_0^{10} \frac{1}{2}x^4 dx$

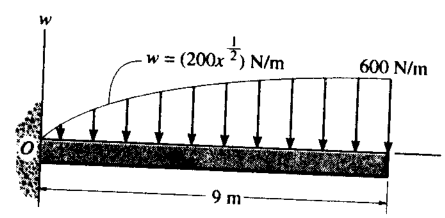
$= \left[\frac{1}{10}x^5\right]_0^{10}$

$= 10\,000$ N·m

$\bar{x} = \frac{10\,000}{1250} = 8.00$ m **Ans**



4-157. Replace the loading by an equivalent force and couple moment acting at point O.

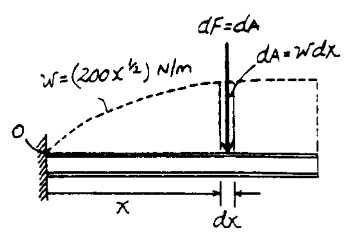


Equivalent Resultant Force And Moment At Point O :

$+\uparrow F_R = \Sigma F_y; \quad F_R = -\int_A dA = -\int_0^x w\,dx$

$F_R = -\int_0^{9\,m}\left(200x^{\frac{1}{2}}\right)dx$

$= -3600$ N $= 3.60$ kN $\downarrow$ **Ans**



$\curvearrowleft + M_{R_O} = \Sigma M_O; \quad M_{R_O} = -\int_0^x xw\,dx$

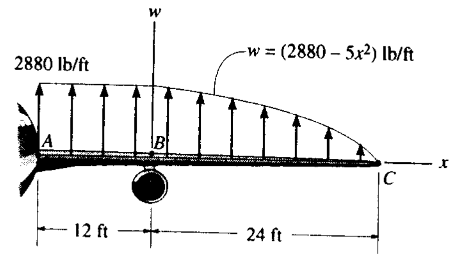
$= -\int_0^{9\,m} x\left(200x^{\frac{1}{2}}\right)dx$

$= -\int_0^{9\,m}\left(200x^{\frac{3}{2}}\right)dx$

$= -19\,440$ N·m

$= 19.4$ kN·m ***(Clockwise)*** **Ans**

***4-158.** The lifting force along the wing of a jet aircraft consists of a uniform distribution along AB, and a semiparabolic distribution along BC with origin at B. Replace this loading by a single resultant force and specify its location measured from point A.



Equivalent Resultant Force :

$$+\uparrow F_R = \Sigma F_y; \quad F_R = 34560 + \int_0^x w\,dx$$

$$F_R = 34560 + \int_0^{24\text{ft}} \left(2880 - 5x^2\right) dx$$

$$= 80640 \text{ lb} = 80.6 \text{ kip} \uparrow \qquad \textbf{Ans}$$

Location of Equivalent Resultant Force :

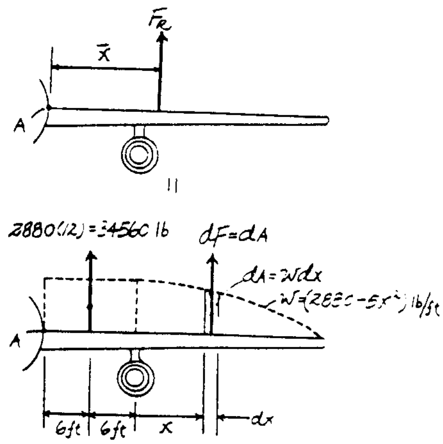
$$\circlearrowleft + M_{R_A} = \Sigma M_A;$$

$$80640\bar{x} = 34560(6) + \int_0^x (x+12)\,w\,dx$$

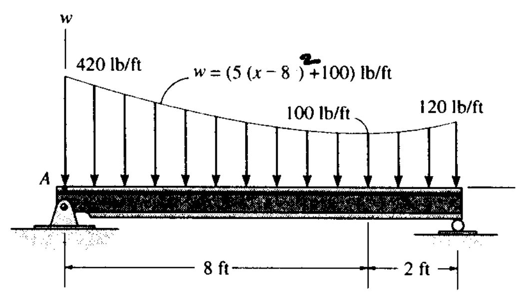
$$80640\bar{x} = 207360 + \int_0^{24\text{ft}} (x+12)\left(2880 - 5x^2\right) dx$$

$$80640\bar{x} = 207360 + \int_0^{24\text{ft}} \left(-5x^3 - 60x^2 + 2880x + 34560\right) dx$$

$$\bar{x} = 14.6 \text{ ft} \qquad \textbf{Ans}$$



4-159. Determine the magnitude of the equivalent resultant force of the distributed load and specify its location on the beam measured from point A.



Equivalent Resultant Force :

$$+\uparrow F_R = \Sigma F_y; \quad -F_R = -\int_A dA = -\int_0^x w\,dx$$

$$F_R = \int_0^{10\text{ft}} \left[5(x-8)^2 + 100\right] dx$$

$$= 1866.67 \text{ lb} = 1.87 \text{ kip} \downarrow \qquad \textbf{Ans}$$

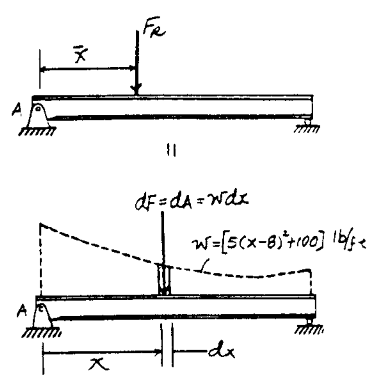
Location of Equivalent Resultant Force :

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_0^x xw\,dx}{\int_0^x w\,dx}$$

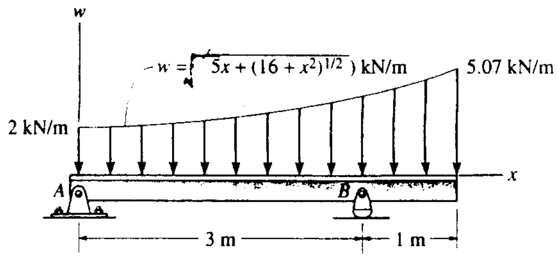
$$= \frac{\int_0^{10\text{ft}} x\left[5(x-8)^2 + 100\right] dx}{\int_0^{10\text{ft}} \left[5(x-8)^2 + 100\right] dx}$$

$$= \frac{\int_0^{10\text{ft}} \left(5x^3 - 80x^2 + 420x\right) dx}{\int_0^{10\text{ft}} \left[5(x-8)^2 + 100\right] dx}$$

$$= 3.66 \text{ ft} \qquad \textbf{Ans}$$



***4-160.** Determine the equivalent resultant force of the distributed loading and its location, measured from point A. Evaluate the integrals using Simpson's rule.



$$F_R = \int w\,dx = \int_0^4 \sqrt{5x + (16 + x^2)^{\frac{1}{2}}}\,dx$$

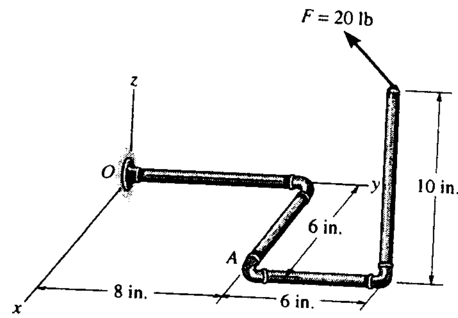
$F_R = 14.9$ kN **Ans**

$$\int_0^4 \tilde{x}\,dF = \int_0^4 (x)\sqrt{5x + (16 + x^2)^{\frac{1}{2}}}\,dx$$

$$= 33.74 \text{ kN}\cdot\text{m}$$

$$\bar{x} = \frac{33.74}{14.9} = 2.27 \text{ m}$$ **Ans**

4-161. Determine the coordinate direction angles α, β, γ of **F**, which is applied to the end A of the pipe assembly, so that the moment of **F** about O is zero.



Require $\mathbf{M}_O = \mathbf{0}$. This happens when force **F** is directed along line OA either from point O to A or from point A to O. The unit vectors $\mathbf{u}_{OA}$ and $\mathbf{u}_{AO}$ are

$$\mathbf{u}_{OA} = \frac{(6-0)\mathbf{i} + (14-0)\mathbf{j} + (10-0)\mathbf{k}}{\sqrt{(6-0)^2 + (14-0)^2 + (10-0)^2}}$$
$$= 0.3293\mathbf{i} + 0.7683\mathbf{j} + 0.5488\mathbf{k}$$

Thus,

$\alpha = \cos^{-1} 0.3293 = 70.8°$ **Ans**

$\beta = \cos^{-1} 0.7683 = 39.8°$ **Ans**

$\gamma = \cos^{-1} 0.5488 = 56.7°$ **Ans**

$$\mathbf{u}_{AO} = \frac{(0-6)\mathbf{i} + (0-14)\mathbf{j} + (0-10)\mathbf{k}}{\sqrt{(0-6)^2 + (0-14)^2 + (0-10)^2}}$$
$$= -0.3293\mathbf{i} - 0.7683\mathbf{j} - 0.5488\mathbf{k}$$

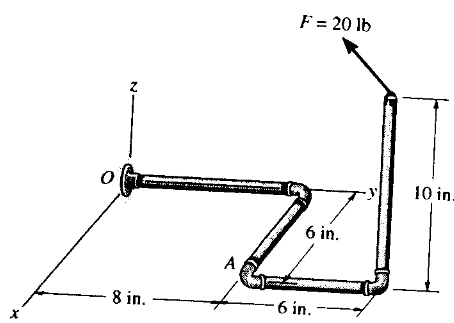
Thus,

$\alpha = \cos^{-1}(-0.3293) = 109°$ **Ans**

$\beta = \cos^{-1}(-0.7683) = 140°$ **Ans**

$\gamma = \cos^{-1}(-0.5488) = 123°$ **Ans**

4-162. Determine the moment of the force **F** about point O. The force has coordinate direction angles of $\alpha = 60°$, $\beta = 120°$, $\gamma = 45°$. Express the result as a Cartesian vector.



Position Vector And Force Vectors :

$$\mathbf{r}_{OA} = \{(6-0)\mathbf{i} + (14-0)\mathbf{j} + (10-0)\mathbf{k}\} \text{ in.}$$
$$= \{6\mathbf{i} + 14\mathbf{j} + 10\mathbf{k}\} \text{ in.}$$

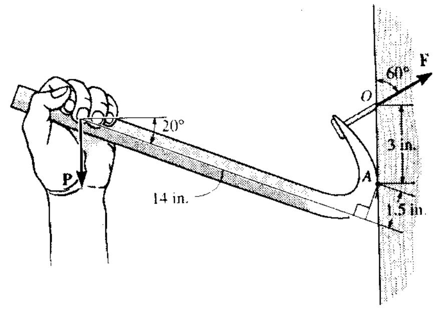
$$\mathbf{F} = 20(\cos 60°\mathbf{i} + \cos 120°\mathbf{j} + \cos 45°\mathbf{k}) \text{ lb}$$
$$= \{10.0\mathbf{i} - 10.0\mathbf{j} + 14.142\mathbf{k}\} \text{ lb}$$

Moment of Force* F *About Point O : Applying Eq. 4 – 7, we have

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 14 & 10 \\ 10.0 & -10.0 & 14.142 \end{vmatrix}$$

$$= \{298\mathbf{i} + 15.1\mathbf{j} - 200\mathbf{k}\} \text{ lb}\cdot\text{in}$$ **Ans**

4-163. If it takes a force of $F = 125$ lb to pull the nail out, determine the smallest vertical force P that must be applied to the handle of the crowbar. *Hint*: This requires the moment of F about point A to be equal to the moment of P about A. Why?

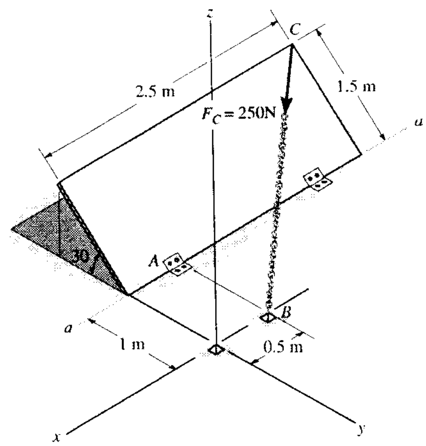


$$\curvearrowleft +M_F = 125(\sin 60^\circ)(3) = 324.7595 \text{ lb} \cdot \text{in.}$$

$$\curvearrowleft +M_P = P(14\cos 20^\circ + 1.5\sin 20^\circ) = M_F = 324.7595 \text{ lb} \cdot \text{in.}$$

$P = 23.8$ lb **Ans**

***4-164.** Determine the moment of the force $\mathbf{F}_c$ about the door hinge at A. Express the result as a Cartesian vector.



Position Vector And Force Vector:

$$\mathbf{r}_{AB} = \{[-0.5-(-0.5)]\mathbf{i} + [0-(-1)]\mathbf{j} + (0-0)\mathbf{k}\} \text{ m} = \{1\mathbf{j}\} \text{ m}$$

$$\mathbf{F}_C = 250\left(\frac{[-0.5-(-2.5)]\mathbf{i} + \{0-[-(1+1.5\cos 30^\circ)]\}\mathbf{j} + (0-1.5\sin 30^\circ)\mathbf{k}}{\sqrt{[-0.5-(-2.5)]^2 + \{0-[-(1+1.5\cos 30^\circ)]\}^2 + (0-1.5\sin 30^\circ)^2}}\right) \text{N}$$

$$= \{159.33\mathbf{i} + 183.15\mathbf{j} - 59.75\mathbf{k}\}\text{N}$$

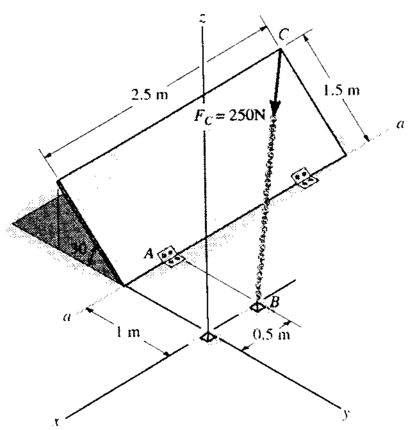
Moment of Force $\mathbf{F}_C$ About Point A: Applying Eq. 4–7, we have

$$\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 159.33 & 183.15 & -59.75 \end{vmatrix}$$

$$= \{-59.7\mathbf{i} - 159\mathbf{k}\}\text{N} \cdot \text{m}$$ **Ans**

4-165. Determine the magnitude of the moment of the force $\mathbf{F}_c$ about the hinged axis aa of the door.



Position Vector And Force Vectors:

$$\mathbf{r}_{AB} = \{[-0.5-(-0.5)]\mathbf{i} + [0-(-1)]\mathbf{j} + (0-0)\mathbf{k}\} \text{ m} = \{1\mathbf{j}\} \text{ m}$$

$$\mathbf{F}_C = 250\left(\frac{[-0.5-(-2.5)]\mathbf{i} + \{0-[-(1+1.5\cos 30^\circ)]\}\mathbf{j} + (0-1.5\sin 30^\circ)\mathbf{k}}{\sqrt{[-0.5-(-2.5)]^2 + \{0-[-(1+1.5\cos 30^\circ)]\}^2 + (0-1.5\sin 30^\circ)^2}}\right) \text{N}$$

$$= \{159.33\mathbf{i} + 183.15\mathbf{j} - 59.75\mathbf{k}\}\text{N}$$

Moment of Force $\mathbf{F}_C$ About a - aAxis: The unit vector along the $a-a$ axis is $\mathbf{i}$. Applying Eq. 4–11, we have

$$M_{a-a} = \mathbf{i} \cdot (\mathbf{r}_{AB} \times \mathbf{F}_C)$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 159.33 & 183.15 & -59.75 \end{vmatrix}$$

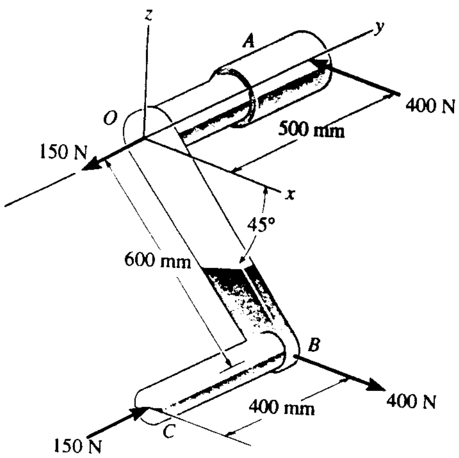
$$= 1[1(-59.75) - (183.15)(0)] - 0 + 0$$

$$= -59.7 \text{ N} \cdot \text{m}$$

The negative sign indicates that $\mathbf{M}_{a-a}$ is directed toward negative x axis.

$M_{a-a} = 59.7$ N · m **Ans**

4-166. Determine the resultant couple moment of the two couples that act on the assembly. Member OB lies in the x-z plane.



For the 400 - N forces :

$$\mathbf{M}_{C1} = \mathbf{r}_{AB} \times (400\mathbf{i})$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6\cos 45° & -0.5 & -0.6\sin 45° \\ 400 & 0 & 0 \end{vmatrix}$$

$$= -169.7\mathbf{j} + 200\mathbf{k}$$

For the 150 - N forces :

$$\mathbf{M}_{C2} = \mathbf{r}_{OB} \times (150\mathbf{j})$$

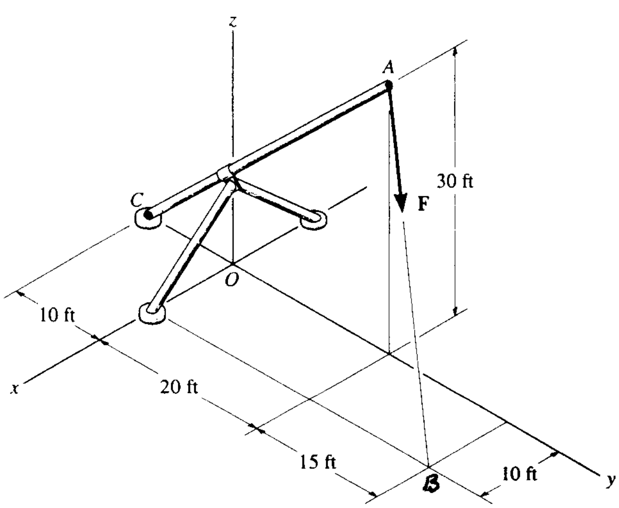
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6\cos 45° & 0 & -0.6\sin 45° \\ 0 & 150 & 0 \end{vmatrix}$$

$$= 63.6\mathbf{i} + 63.6\mathbf{k}$$

$$\mathbf{M}_{CR} = \mathbf{M}_{C1} + \mathbf{M}_{C2}$$

$$\mathbf{M}_{CR} = \{63.6\mathbf{i} - 170\mathbf{j} + 264\mathbf{k}\}\ \text{N}\cdot\text{m}$$ **Ans**

4-167. Replace the force **F** having a magnitude of $F = 50$ lb and acting at point A by an equivalent force and couple moment at point C.



$$\mathbf{F}_R = 50\left[\frac{(10\mathbf{i} + 15\mathbf{j} - 30\mathbf{k})}{\sqrt{(10)^2 + (15)^2 + (-30)^2}}\right]$$

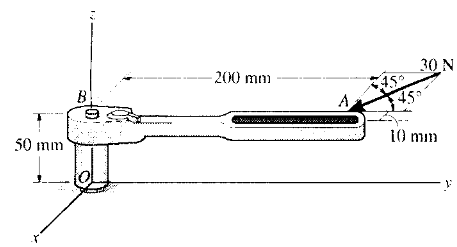
$$\mathbf{F}_R = \{14.3\mathbf{i} + 21.4\mathbf{j} - 42.9\mathbf{k}\}\ \text{lb}$$ **Ans**

$$\mathbf{M}_{RC} = \mathbf{r}_{CB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & 45 & 0 \\ 14.29 & 21.43 & -42.86 \end{vmatrix}$$

$$= \{-1929\mathbf{i} + 428.6\mathbf{j} - 428.6\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$

$$\mathbf{M}_A = \{-1.93\mathbf{i} + 0.429\mathbf{j} - 0.429\mathbf{k}\}\ \text{kip}\cdot\text{ft}$$ **Ans**

***4-168.** The horizontal 30-N force acts on the handle of the wrench. What is the magnitude of the moment of this force about the z axis?



Position Vector And Force Vectors:

$\mathbf{r}_{BA} = \{-0.01\mathbf{i} + 0.2\mathbf{j}\}$ m

$\mathbf{r}_{OA} = \{(-0.01 - 0)\mathbf{i} + (0.2 - 0)\mathbf{j} + (0.05 - 0)\mathbf{k}\}$ m

$= \{-0.01\mathbf{i} + 0.2\mathbf{j} + 0.05\mathbf{k}\}$ m

$\mathbf{F} = 30(\sin 45°\mathbf{i} - \cos 45°\mathbf{j})$ N

$= \{21.213\mathbf{i} - 21.213\mathbf{j}\}$ N

Moment of Force* F *About z Axis: The unit vector along the z axis is k. Applying Eq. 4–11, we have

$M_z = \mathbf{k} \cdot (\mathbf{r}_{BA} \times \mathbf{F})$

$$= \begin{vmatrix} 0 & 0 & 1 \\ -0.01 & 0.2 & 0 \\ 21.213 & -21.213 & 0 \end{vmatrix}$$

$= 0 - 0 + 1[(-0.01)(-21.213) - 21.213(0.2)]$

$= -4.03$ N · m **Ans**

Or

$M_z = \mathbf{k} \cdot (\mathbf{r}_{OA} \times \mathbf{F})$

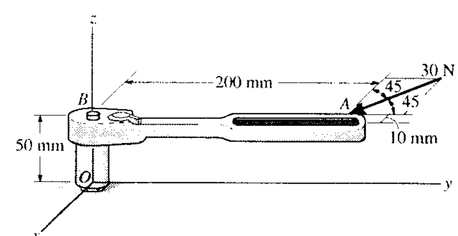
$$= \begin{vmatrix} 0 & 0 & 1 \\ -0.01 & 0.2 & 0.05 \\ 21.213 & -21.213 & 0 \end{vmatrix}$$

$= 0 - 0 + 1[(-0.01)(-21.213) - 21.213(0.2)]$

$= -4.03$ N · m **Ans**

The negative sign indicates that $\mathbf{M}_z$ is directed along the negative z axis.

4-169. The horizontal 30-N force acts on the handle of the wrench. Determine the moment of this force about point O. Specify the coordinate direction angles α, β, γ of the moment axis.



Position Vector And Force Vectors:

$\mathbf{r}_{OA} = \{(-0.01 - 0)\mathbf{i} + (0.2 - 0)\mathbf{j} + (0.05 - 0)\mathbf{k}\}$ m

$= \{-0.01\mathbf{i} + 0.2\mathbf{j} + 0.05\mathbf{k}\}$ m

$\mathbf{F} = 30(\sin 45°\mathbf{i} - \cos 45°\mathbf{j})$ N

$= \{21.213\mathbf{i} - 21.213\mathbf{j}\}$ N

Moment of Force* F *About Point O: Applying Eq. 4–7, we have

$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.01 & 0.2 & 0.05 \\ 21.213 & -21.213 & 0 \end{vmatrix}$$

$= \{1.061\mathbf{i} + 1.061\mathbf{j} - 4.031\mathbf{k}\}$ N · m

$= \{1.06\mathbf{i} + 1.06\mathbf{j} - 4.03\mathbf{k}\}$ N · m **Ans**

The magnitude of $\mathbf{M}_O$ is

$$M_O = \sqrt{1.061^2 + 1.061^2 + (-4.031)^2} = 4.301 \text{ N} \cdot \text{m}$$

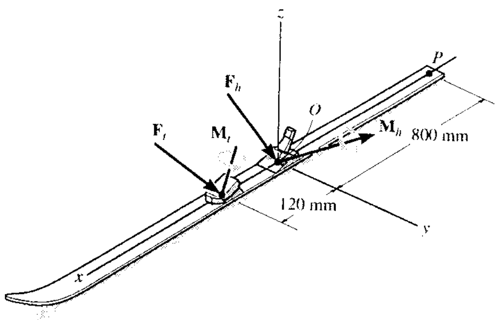
The coordinate direction angles for $\mathbf{M}_O$ are

$\alpha = \cos^{-1}\left(\frac{1.061}{4.301}\right) = 75.7°$ **Ans**

$\beta = \cos^{-1}\left(\frac{1.061}{4.301}\right) = 75.7°$ **Ans**

$\gamma = \cos^{-1}\left(\frac{-4.031}{4.301}\right) = 160°$ **Ans**

4-170. The forces and couple moments that are exerted on the toe and heel plates of a snow ski are $\mathbf{F}_t = \{-50\mathbf{i}+80\mathbf{j}-158\mathbf{k}\}$N, $\mathbf{M}_t = \{-6\mathbf{i}+4\mathbf{j}+2\mathbf{k}\}$N · m, and $\mathbf{F}_h = \{-20\mathbf{i}+60\mathbf{j}-250\mathbf{k}\}$N, $\mathbf{M}_h = \{-20\mathbf{i}+8\mathbf{j}+3\mathbf{k}\}$ N · m, respectively. Replace this system by an equivalent force and couple moment acting at point P. Express the results in Cartesian vector form.



$$\mathbf{F}_R = \mathbf{F}_t + \mathbf{F}_h = \{-70\mathbf{i} + 140\mathbf{j} - 408\mathbf{k}\}\text{ N}$$ **Ans**

$$\mathbf{M}_{RP} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.8 & 0 & 0 \\ -20 & 60 & -250 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.92 & 0 & 0 \\ -50 & 80 & -158 \end{vmatrix} + (-6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) + (-20\mathbf{i} + 8\mathbf{j} + 3\mathbf{k})$$

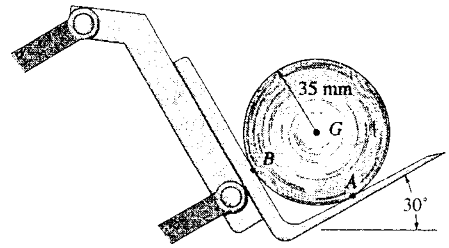
$$\mathbf{M}_{RP} = (200\mathbf{j} + 48\mathbf{k}) + (145.36\mathbf{j} + 73.6\mathbf{k})$$

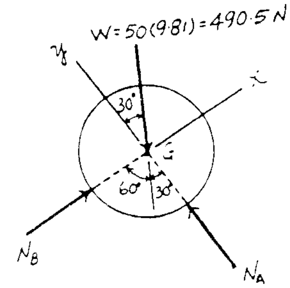
$$+ (-6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) + (-20\mathbf{i} + 8\mathbf{j} + 3\mathbf{k})$$

$$\mathbf{M}_{RP} = \{-26\mathbf{i} + 357.36\mathbf{j} + 126.6\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$\mathbf{M}_{RP} = \{-26\mathbf{i} + 357\mathbf{j} + 127\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans**

5-1. Draw the free-body diagram of the 50-kg paper roll which has a center of mass at G and rests on the smooth blade of the paper hauler. Explain the significance of each force acting on the diagram. (See Fig. 5–7b.)



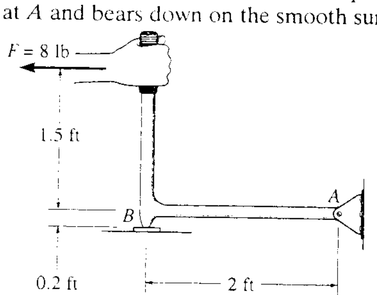


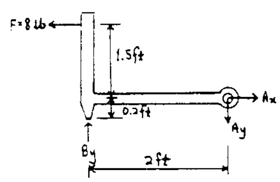
The Significance of Each Force :

W is the effect of gravity (weight) on the paper roll.

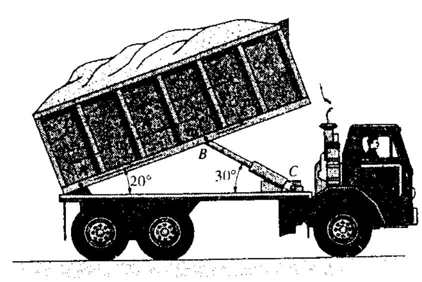
N_A and N_B are the smooth blade reactions on the paper roll.

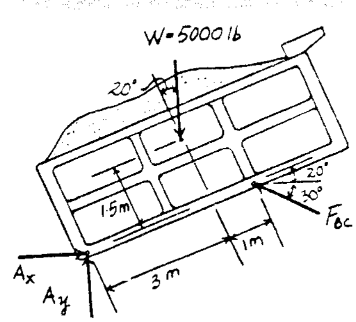
5-2. Draw the free-body diagram of the hand punch, which is pinned at A and bears down on the smooth surface at B.





5-3. Draw the free-body diagram of the dumpster D of the truck, which has a weight of 5000 lb and a center of gravity at G. It is supported by a pin at A and a pin-connected hydraulic cylinder BC (short link) Explain the significance of each force on the diagram. (See Fig. 5–7b.)





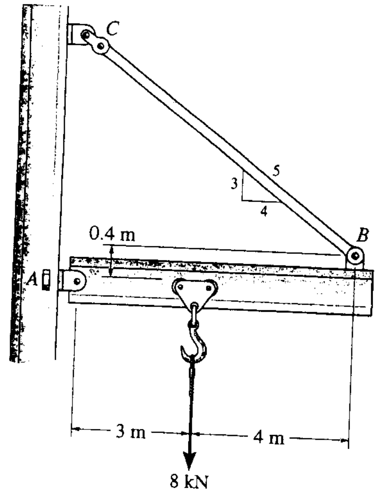
The Significance of Each Force :

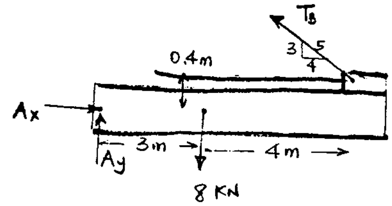
W is the effect of gravity (weight) on the dumpster.

A_y and A_x are the pin A reactions on the dumpster.

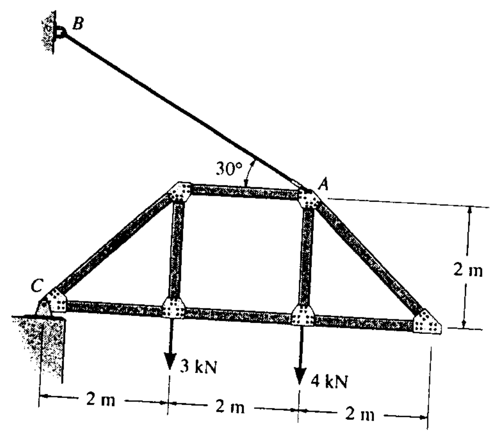
F_{BC} is the hydraulic cylinder BC reaction on the dumpster.

*5-4. Draw the free-body diagram of the jib crane AB, which is pin-connected at A and supported by member (link) BC.





5-5. Draw the free-body diagram of the truss that is supported by the cable AB and pin C. Explain the significance of each force acting on the diagram. (See Fig. 5-7b.)

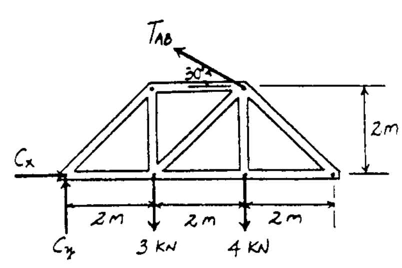


The Significance of Each Force :

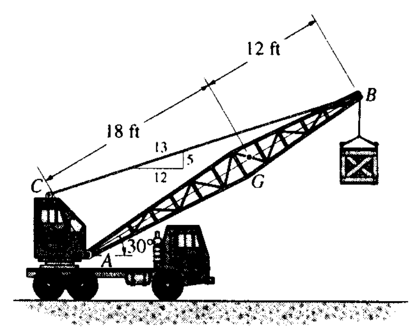
C_y and C_x are the pin C reactions on the truss.

T_{AB} is the cable AB tension on the truss.

3 kN and 4 kN force are the effect of external applied forces on the truss.



5-6. Draw the free-body diagram of the crane boom AB which has a weight of 650 lb and center of gravity at G. The boom is supported by a pin at A and cable BC. The load of 1250 lb is suspended from a cable attached at B. Explain the significance of each force acting on the diagram. (See Fig. 5–7b.)



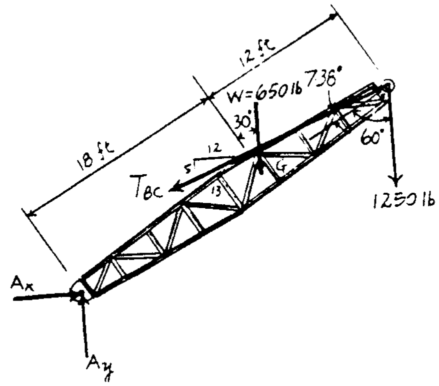
The Significance of Each Force :

W is the effect of gravity (weight) on the boom.

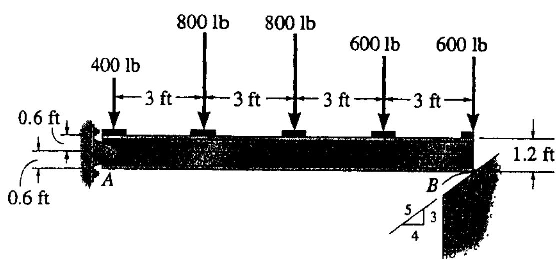
A_y and A_x are the pin A reactions on the boom.

T_{BC} is the cable BC force reactions on the boom.

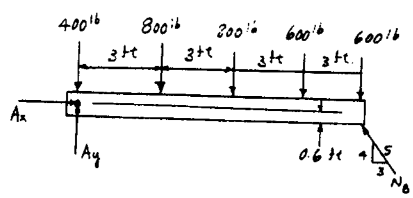
1250 lb force is the suspended load reaction on the boom.



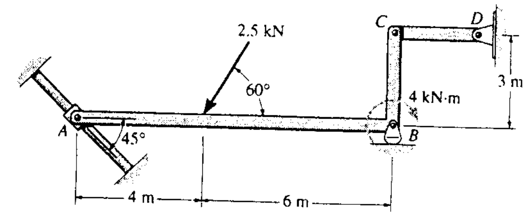
5-7. Draw the free-body diagram of the beam, which is pin-supported at A and rests on the smooth incline at B.

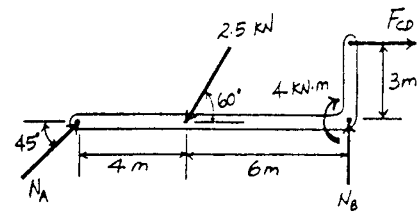


Prob. 5–7



*5-8. Draw the free-body diagram of member ABC which is supported by a smooth collar at A, roller at B, and short link CD. Explain the significance of each force acting on the diagram. (See Fig. 5–7b.)





The Significance of Each Force :

N_A is the smooth collar reaction on member ABC.

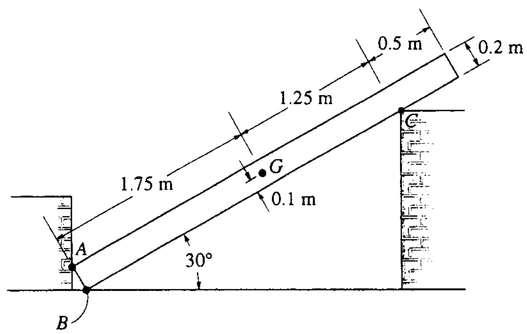
N_B is the roller support B reaction on member ABC.

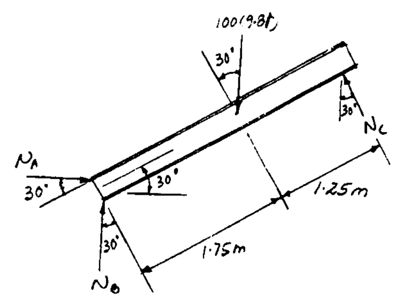
F_{CD} is the short link reaction on member ABC.

2.5 kN is the effect of external applied force on member ABC.

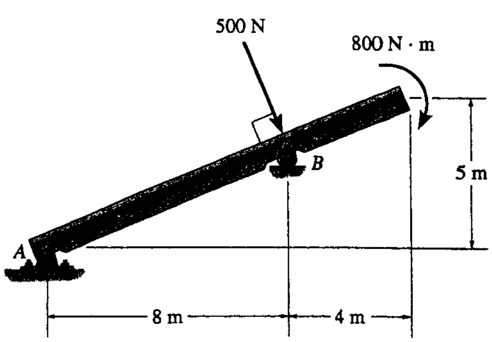
4 kN · m is the effect of external applied couple moment on member ABC.

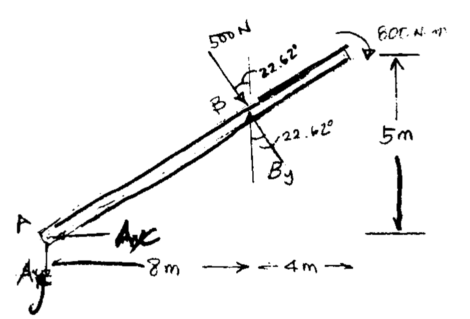
5-9. Draw the free-body diagram of the uniform bar, which has a mass of 100 kg and a center of mass at G. The supports A, B, and C are smooth.



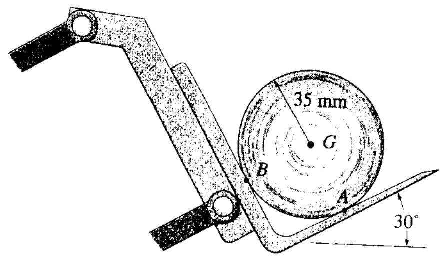


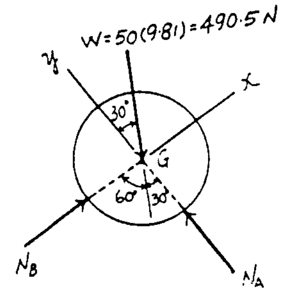
5-10. Draw the free-body diagram of the beam, which is pin-connected at A and rocker-supported at B.





5-11. Determine the reactions at the supports in Prob. 5–1.



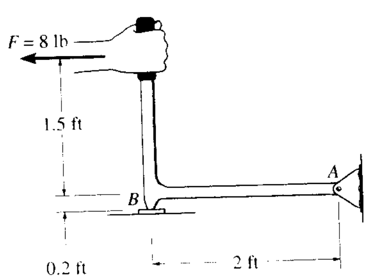


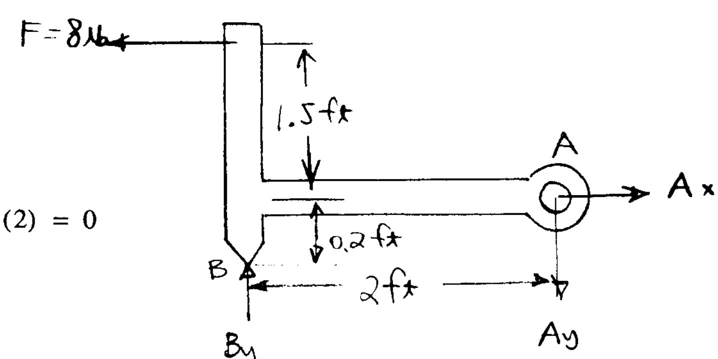
Equations of Equilibrium : By setting up the x and y axes in the manner shown, one can obtain the direct solution for N_A and N_B.

$\nearrow^{+} \Sigma F_x = 0;\quad N_B - 490.5\sin 30° = 0 \quad N_B = 245\text{ N}$ **Ans**

$\nwarrow^{+} \Sigma F_y = 0;\quad N_A - 490.5\cos 30° = 0 \quad N_A = 425\text{ N}$ **Ans**

***5-12.** Determine the magnitude of the resultant force acting at A of the handpunch in Prob. 5–2.





$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - 8 = 0$

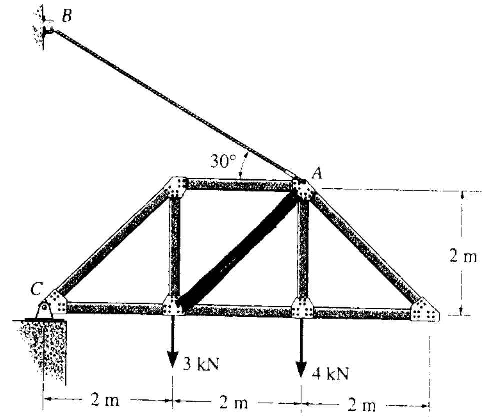
$A_x = 8\text{ lb}$

$\circlearrowleft + \Sigma M_B = 0;\quad 8(1.5) - A_y(2) = 0$

$A_y = 6\text{ lb}$

$F_A = \sqrt{(8)^2 + (6)^2} = 10\text{ lb}$ **Ans**

5-13. Determine the reactions at the supports for the truss in Prob. 5–5.



Equations of Equilibrium : The tension in the cable can be obtained directly by summing moments about point C.

$$\circlearrowleft + \Sigma M_C = 0; \quad T_{AB}\cos 30°(2) + T_{AB}\sin 30°(4) - 3(2) - 4(4) = 0$$

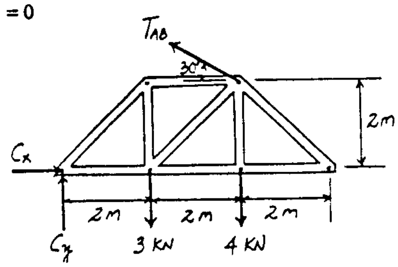
$$T_{AB} = 5.89 \text{ kN} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad C_x - 5.89\cos 30° = 0$$

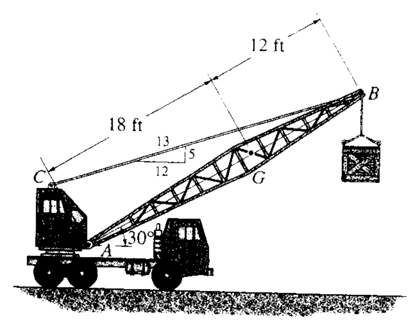
$$C_x = 5.11 \text{ kN} \quad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad C_y + 5.89\sin 30° - 3 - 4 = 0$$

$$C_y = 4.05 \text{ kN} \quad \textbf{Ans}$$



5-14. Determine the reactions on the boom in Prob. 5–6.



Equations of Equilibrium : The force in cable BC can be obtained directly by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \quad T_{BC}\sin 7.380°(30) - 650\cos 30°(18) - 1250\sin 60°(30) = 0$$

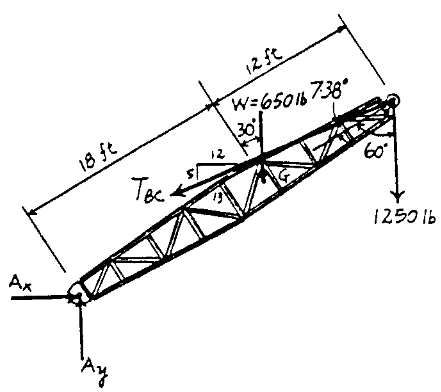
$$T_{BC} = 11056.9 \text{ lb} = 11.1 \text{ kip} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 11056.9\left(\frac{12}{13}\right) = 0$$

$$A_x = 10206.4 \text{ lb} = 10.2 \text{ kip} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 650 - 1250 - 11056.9\left(\frac{5}{13}\right) = 0$$

$$A_y = 6152.7 \text{ lb} = 6.15 \text{ kip} \qquad \textbf{Ans}$$



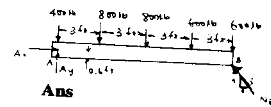
5-15. Determine the support reactions on the beam in Prob. 5–7.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad \frac{4}{5}N_B(12) - \frac{3}{5}N_B(0.6) - 800(3) - 800(6) - 600(9) - 600(12) = 0$

$N_B = 2142.86 \text{ lb} = 2.14 \text{ kip}$ **Ans**

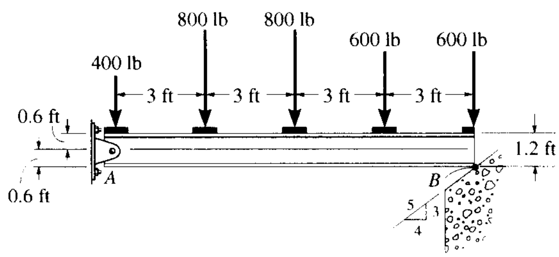
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - \frac{3}{5}(2142.86) = 0$

$A_x = 1286 \text{ lb} = 1.29 \text{ kip}$ **Ans**

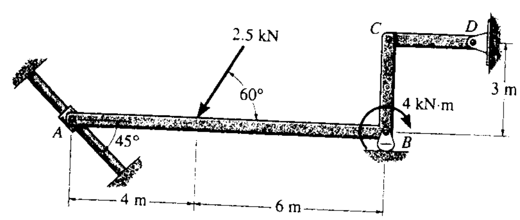


$+\uparrow \Sigma F_y = 0;$ $\quad A_y + \frac{4}{5}(2142.86) - 400 - 800 - 800 - 600 - 600 = 0$

$A_y = 1486 \text{ lb} = 1.49 \text{ kip}$ **Ans**



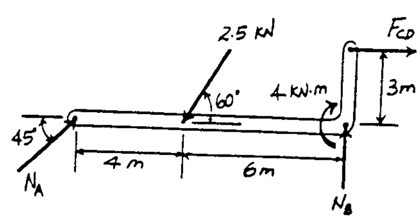
***5-16.** Determine the reactions on the member *A*, *B*, *C* in Prob. 5–8.



Equations of Equilibrium : The normal reaction N_A can be obtained directly by summing moments about point *C*.

$\circlearrowleft + \Sigma M_C = 0;$ $\quad 2.5\sin 60°(6) - 2.5\cos 60°(3) - 4 + N_A\cos 45°(3) - N_A \sin 45°(10) = 0$

$N_A = 1.059 \text{ kN} = 1.06 \text{ kN}$ **Ans**



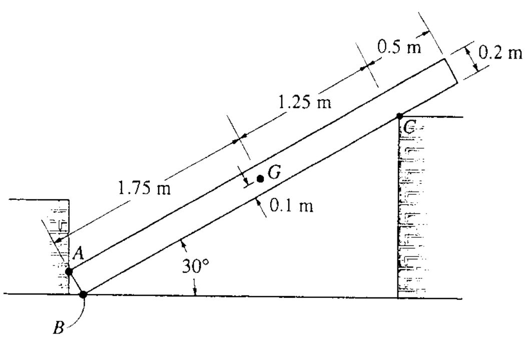
$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 1.059\cos 45° - 2.5\cos 60° + F_{CD} = 0$

$F_{CD} = 0.501 \text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $\quad N_B + 1.059\sin 45° - 2.5\sin 60° = 0$

$N_B = 1.42 \text{ kN}$ **Ans**

5-17. Determine the reactions at the points of contact at A, B, and C of the bar in Prob. 5–9.



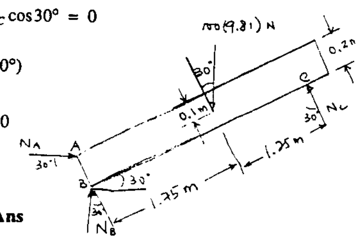
$\circlearrowleft +\Sigma M_A = 0;\quad -100(9.81)(\cos 30°)(1.75) - 100(9.81)(\sin 30°)(0.1) + N_B(\sin 30°)(0.2) + N_C(3) = 0$

$-1535.7991 + 0.1N_B + 3N_C = 0$

$+\uparrow \Sigma F_y = 0;\quad N_B - 100(9.81) + N_C\cos 30° = 0$

$N_B = 981 - N_C(\cos 30°)$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_A - N_C(\sin 30°) = 0$



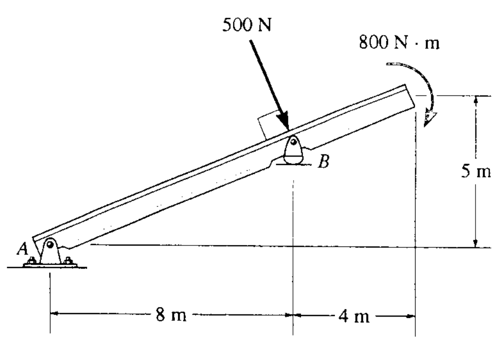
Solving;

$N_C = 493$ N **Ans**

$N_B = 554$ N **Ans**

$N_A = 247$ N **Ans**

5-18. Determine the reactions at the pin A and at the roller at B of the beam in Prob. 5–10.

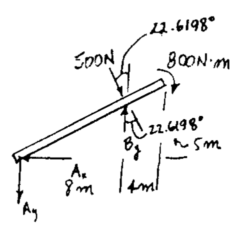


$\circlearrowleft +\Sigma M_A = 0;\quad -500\left(\frac{8}{\cos 22.6198°}\right) - 800 + B_y(8) = 0$

$B_y = 641.6667 = 642$ N **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -A_x + 500(\sin 22.6198°) = 0$

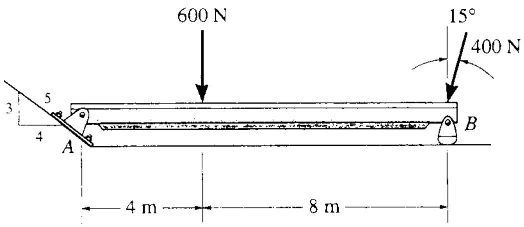
$A_x = 192$ N **Ans**



$+\uparrow \Sigma F_y = 0;\quad -A_y - 500(\cos 22.6198°) + 641.6667 = 0$

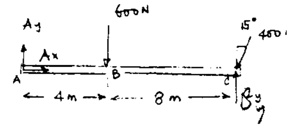
$A_y = 180$ N **Ans**

5-19. Determine the magnitude of the reactions on the beam at A and B. Neglect the thickness of the beam.



$\circlearrowleft + \Sigma M_A = 0;\quad B_y(12) - (400\cos 15°)(12) - 600(4) = 0$

$B_y = 586.37 = 586\text{ N}$ **Ans**



$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - 400\sin 15° = 0$

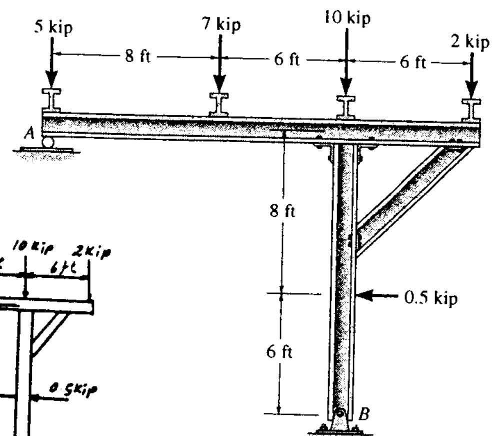
$A_x = 103.528\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad A_y - 600 - 400\cos 15° + 586.37 = 0$

$A_y = 400\text{ N}$

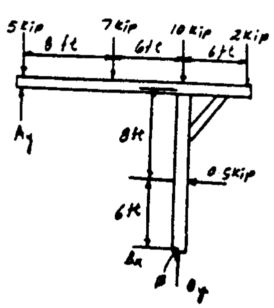
$F_A = \sqrt{(103.528)^2 + (400)^2} = 413\text{ N}$ **Ans**

***5-20.** Determine the reactions at the supports A and B of the frame.



$\circlearrowleft + \Sigma M_B = 0;\quad 5(14) + 7(6) + 0.5(6) - 2(6) - A_y(14) = 0$

$A_y = 7.357\text{ kip} = 7.36\text{ kip}$ **Ans**

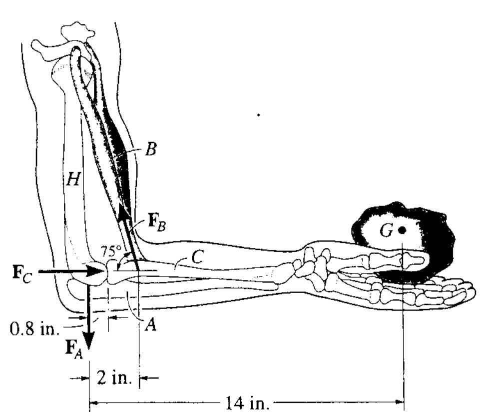


$\xrightarrow{+} \Sigma F_x = 0;\quad B_x - 0.5 = 0 \qquad B_x = 0.5\text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad B_y + 7.357 - 5 - 7 - 10 - 2 = 0$

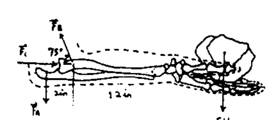
$B_y = 16.6\text{ kip}$ **Ans**

5-21. When holding the 5-lb stone in equilibrium, the humerus H, assumed to be smooth, exerts normal forces $\mathbf{F}_C$ and $\mathbf{F}_A$ on the radius C and ulna A as shown. Determine these forces and the force $\mathbf{F}_B$ that the biceps B exerts on the radius for equilibrium. The stone has a center of mass at G. Neglect the weight of the arm.



$\circlearrowleft + \Sigma M_B = 0;\quad -5(12) + F_A(2) = 0$

$F_A = 30\text{ lb}$ **Ans**



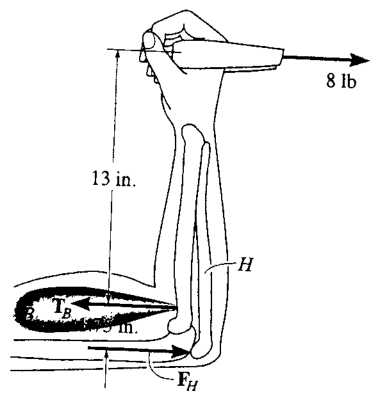
$+\uparrow \Sigma F_y = 0;\quad F_B \sin 75° - 5 - 30 = 0$

$F_B = 36.2\text{ lb}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad F_C - 36.2\cos 75° = 0$

$F_C = 9.38\text{ lb}$ **Ans**

5-22. The man is pulling a load of 8 lb with one arm held as shown. Determine the force $\mathbf{F}_H$ this exerts on the humerus bone H, and the tension developed in the biceps muscle B. Neglect the weight of the man's arm.

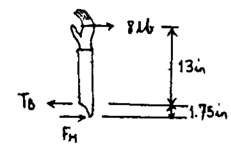


$\curvearrowleft +\Sigma M_B = 0;\quad -8(13) + F_H(1.75) = 0$

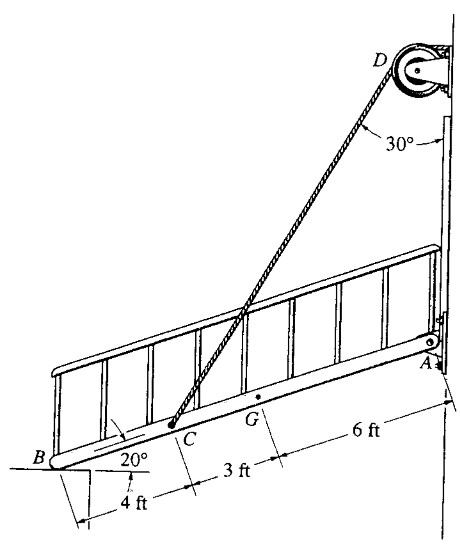
$F_H = 59.43 = 59.4 \text{ lb}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad 8 - T_B + 59.43 = 0$

$T_B = 67.4 \text{ lb}$ **Ans**



5-23. The ramp of a ship has a weight of 200 lb and a center of gravity at G. Determine the cable force in CD needed to just start lifting the ramp, (i.e., so the reaction at B becomes zero). Also, determine the horizontal and vertical components of force at the hinge (pin) at A.

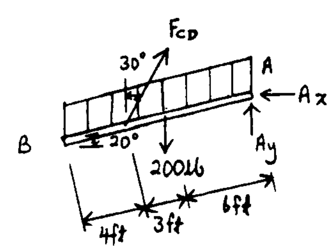


$\curvearrowleft +\Sigma M_A = 0;\quad -F_{CD}\cos 30°(9\cos 20°) + F_{CD}\sin 30°(9\sin 20°) + 200(6\cos 20°) = 0$

$F_{CD} = 194.9 = 195 \text{ lb}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;\quad 194.9\sin 30° - A_x = 0$

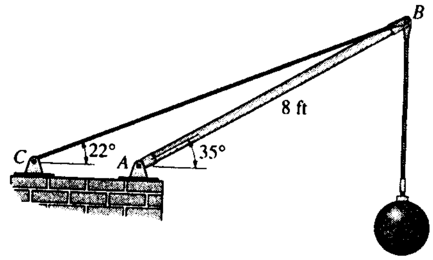
$A_x = 97.4 \text{ lb}$ **Ans**

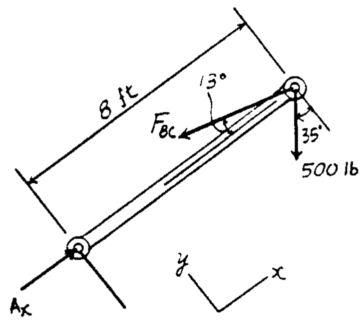


$+\uparrow \Sigma F_y = 0;\quad A_y - 200 + 194.9\cos 30° = 0$

$A_y = 31.2 \text{ lb}$ **Ans**

***5-24.** Determine the magnitude of force at the pin A and in the cable BC needed to support the 500-lb load. Neglect the weight of the boom AB.





Equations of Equilibrium : The force in cable BC can be obtained directly by summing moments about point A.

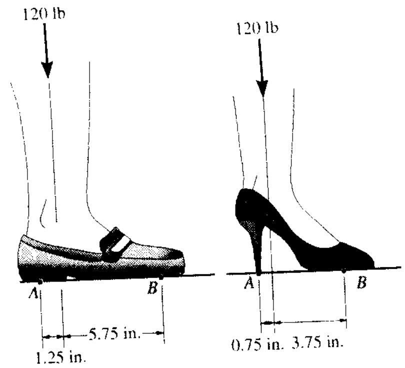
$$\circlearrowleft + \Sigma M_A = 0; \quad F_{BC}\sin 13°(8) - 500\cos 35°(8) = 0$$
$$F_{BC} = 1820.7 \text{ lb} = 1.82 \text{ kip} \qquad \textbf{Ans}$$

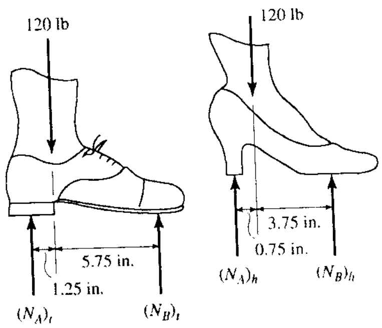
$$+\Sigma F_x = 0; \quad A_x - 1820.7\cos 13° - 500\sin 35° = 0$$
$$A_x = 2060.9 \text{ lb}$$

$$+\Sigma F_y = 0; \quad A_y + 1820.7\sin 13° - 500\cos 35° = 0$$
$$A_y = 0$$

Thus, $\quad F_A = A_x = 2060.9 \text{ lb} = 2.06 \text{ kip}$ **Ans**

5-25. Compare the force exerted on the toe and heel of a 120-lb woman when she is wearing regular shoes and stiletto heels. Assume all her weight is placed on one foot and the reactions occur at points A and B as shown.





Equations of Equilibrium: Regular shoe, we have

$\curvearrowleft + \Sigma M_B = 0;\quad 120(5.75) - (N_A)_r(7) = 0$

$(N_A)_r = 98.6 \text{ lb}$ **Ans**

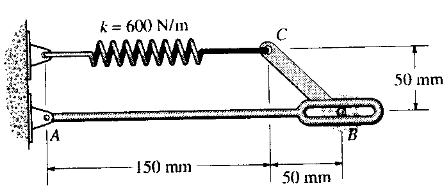
Stiletto heel shoe,

$\curvearrowleft + \Sigma M_B = 0;\quad 120(3.75) - (N_A)_h(4.5) = 0$

$(N_A)_h = 100 \text{ lb}$ **Ans**

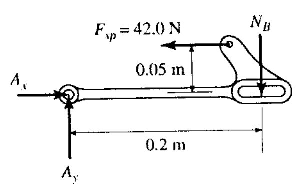
The heal of the stiletto shoe is subjected to a greater force than that of the heel of the regular shoe. Actually the force per area (stress) under the stiletto heel will be much greater than that of the regular shoe. It is this stress that can cause damage to soft flooring.

5-26. Determine the reactions at the pins A and B. The spring has an unstretched length of 80 mm.



Spring Force: The spring stretches $x = 0.15 - 0.08 = 0.07$ m. Applying the spring formula, we have

$F_{sp} = kx = 600(0.07) = 42.0 \text{ N}$



Equations of Equilibrium: The normal reaction N_B can be obtained directly by summing moments about point A.

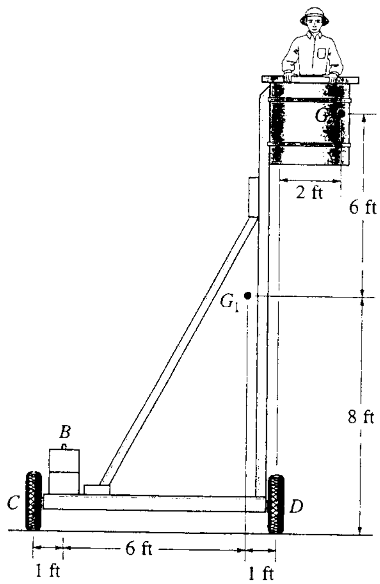
$\curvearrowleft + \Sigma M_A = 0;\quad 42.0(0.05) - N_B(0.2) = 0$

$N_B = 10.5 \text{ N}$ **Ans**

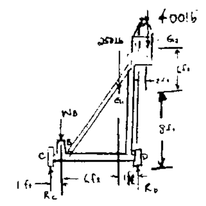
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - 42.0 = 0 \quad A_x = 42.0 \text{ N}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y - 10.5 = 0 \quad A_y = 10.5 \text{ N}$ **Ans**

5-27. The platform assembly has a weight of 250 lb and center of gravity at G_1. If it is intended to support a maximum load of 400 lb placed at point G_2, determine the smallest counterweight W that should be placed at B in order to prevent the platform from tipping over.



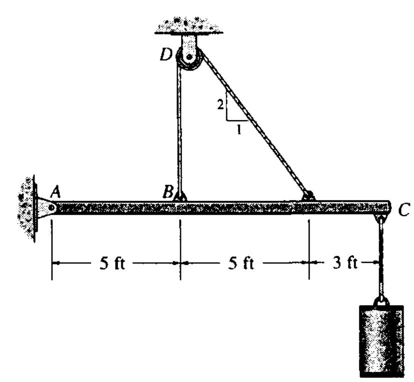
When tipping occurs, $R_C = 0$



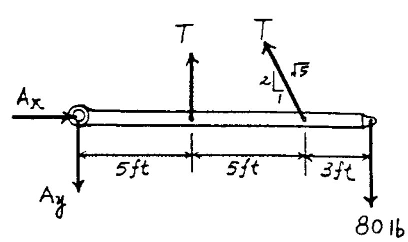
$\circlearrowleft + \Sigma M_D = 0; \quad -400(2) + 250(1) + W_B(7) = 0$

$W_B = 78.6 \text{ lb}$ **Ans**

***5-28.** Determine the tension in the cable and the horizontal and vertical components of reaction of the pin A. The pulley at D is frictionless and the cylinder weighs 80 lb.



Equations of Equilibrium : The tension force developed in the cable is the same throughout the whole cable. The force in the cable can be obtained directly by summing moments about point A.



$\circlearrowleft + \Sigma M_A = 0; \quad T(5) + T\left(\frac{2}{\sqrt{5}}\right)(10) - 80(13) = 0$

$T = 74.583 \text{ lb} = 74.6 \text{ lb}$ **Ans**

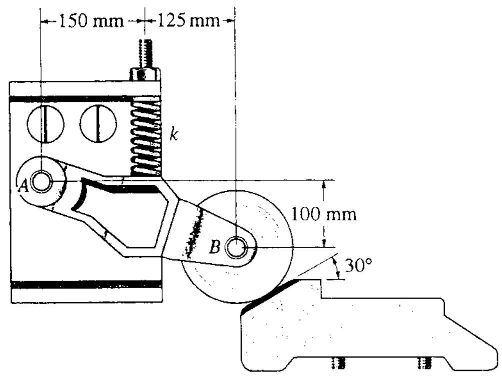
$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 74.583\left(\frac{1}{\sqrt{5}}\right) = 0$

$A_x = 33.4 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad 74.583 + 74.583\left(\frac{2}{\sqrt{5}}\right) - 80 - B_y = 0$

$A_y = 61.3 \text{ lb}$ **Ans**

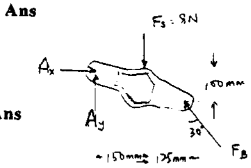
5-29. The device is used to hold an elevator door open. If the spring has a stiffness of $k = 40$ N/m and it is compressed 0.2 m, determine the horizontal and vertical components of reaction at the pin A and the resultant force at the wheel bearing B.



$F_s = ks = (40)(0.2) = 8$ N

$\circlearrowleft + \Sigma M_A = 0;\quad -(8)(150) + F_B(\cos 30°)(275) - F_B(\sin 30°)(100) = 0$

$F_B = 6.37765 \text{ N} = 6.38 \text{ N}$ **Ans**



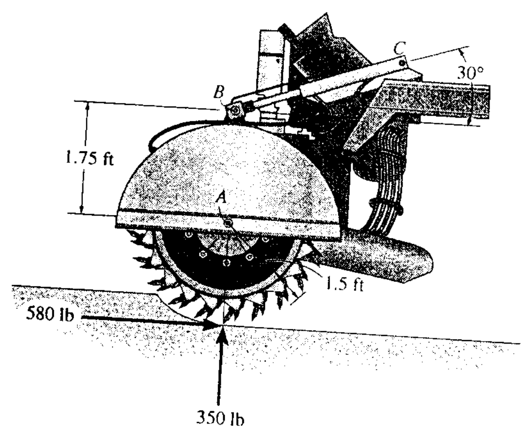
$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - 6.37765 \sin 30° = 0$

$A_x = 3.19$ N **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y - 8 + 6.37765 \cos 30° = 0$

$A_y = 2.48$ N **Ans**

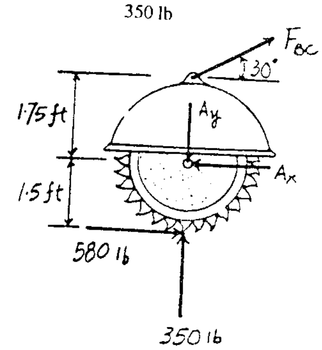
5-30. The cutter is subjected to a horizontal force of 580 lb and a normal force of 350 lb. Determine the horizontal and vertical components of force acting on the pin A and the force along the hydraulic cylinder BC (a two-force member).



Equations of Equilibrium : The force in hydraulic cylinder BC can be obtained directly by summing moments about point A.

$\circlearrowleft + \Sigma M_A = 0;\quad 580(1.5) - F_{BC} \cos 30°(1.75) = 0$

$F_{BC} = 574.05 \text{ lb} = 574 \text{ lb}$ **Ans**



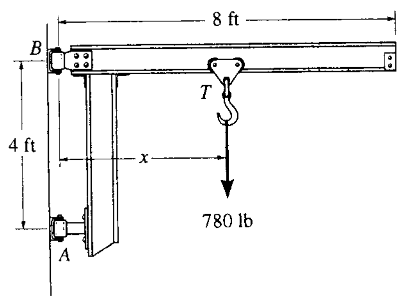
$\xrightarrow{+} \Sigma F_x = 0;\quad 574.05 \cos 30° + 580 - A_x = 0$

$A_x = 1077 \text{ lb} = 1.08 \text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad 574.05 \sin 30° + 350 - A_y = 0$

$A_y = 637$ lb **Ans**

5-31. The cantilevered jib crane is used to support the load of 780 lb. If the trolley T can be placed anywhere between $1.5 \text{ ft} \le x \le 7.5 \text{ ft}$, determine the maximum magnitude of reaction at the supports A and B. Note that the supports are collars that allow the crane to rotate freely about the vertical axis. The collar at B supports a force in the vertical direction, whereas the one at A does not.



Require $x = 7.5$ ft

$\circlearrowleft + \Sigma M_A = 0; \quad -780(7.5) + B_x(4) = 0$

$B_x = 1462.5$ lb

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 1462.5 = 0$

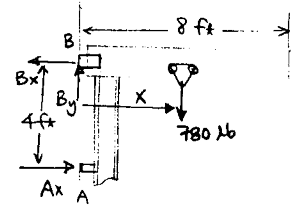
$A_x = 1462.5 = 1462$ lb **Ans**

$+\uparrow \Sigma F_y = 0; \quad B_y - 780 = 0$

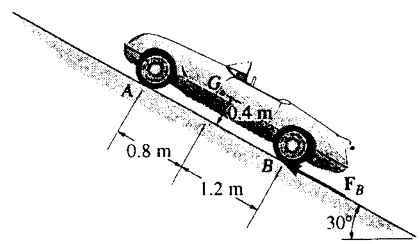
$B_y = 780$ lb

$F_B = \sqrt{(1462.5)^2 + (780)^2}$

$= 1657.5 \text{ lb} = 1.66 \text{ kip}$ **Ans**



***5-32.** The sports car has a mass of 1.5 Mg and mass center at G. If the front two springs each have a stiffness of $k_A = 58$ kN/m and the rear two springs each have a stiffness of $k_B = 65$ kN/m, determine their compression when the car is parked on the 30° incline. Also, what friction force $\mathbf{F}_B$ must be applied to each of the rear wheels to hold the car in equilibrium? *Hint:* First determine the normal force at A and B, then determine the compression in the springs.



Equations of Equilibrium : The normal reaction N_A can be obtained directly by summing moments about point B.

$\circlearrowleft + \Sigma M_B = 0; \quad 14\,715\cos 30°(1.2) - 14\,715\sin 30°(0.4) - 2N_A(2) = 0$

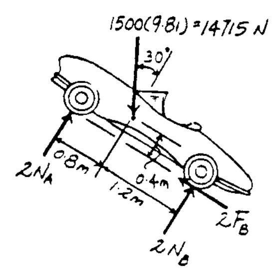
$N_A = 3087.32$ N

$\searrow + \Sigma F_{x'} = 0; \quad 2F_B - 14\,715\sin 30° = 0$

$F_B = 3678.75 \text{ N} = 3.68 \text{ kN}$ **Ans**

$\nearrow + \Sigma F_{y'} = 0; \quad 2N_B + 2(3087.32) - 14\,715\cos 30° = 0$

$N_B = 3284.46$ N

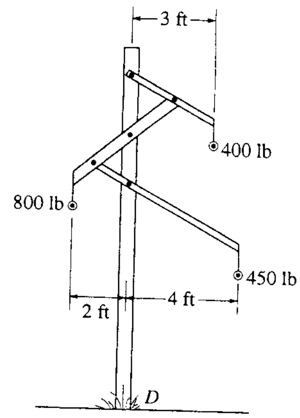


Spring Force Formula : The compression of the sping can be determined using the spring formula $x = \frac{F_{sp}}{k}$.

$x_A = \frac{3087.32}{58(10^3)} = 0.05323 \text{ m} = 53.2 \text{ mm}$ **Ans**

$x_B = \frac{3284.46}{65(10^3)} = 0.05053 \text{ m} = 50.5 \text{ mm}$ **Ans**

5-33. The power pole supports the three lines, each line exerting a vertical force on the pole due to its weight as shown. Determine the reactions at the fixed support D. If it is possible for wind or ice to snap the lines, determine which line(s) when removed create(s) a condition for the greatest moment reaction at D.



$\xrightarrow{+} \Sigma F_x = 0;\quad D_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad D_y - 1650 = 0$

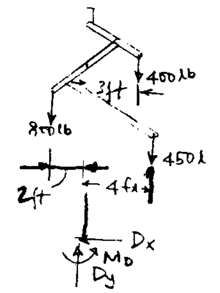
$D_y = 1.65 \text{ kip}$ **Ans**

$\circlearrowleft + \Sigma M_D = 0;\quad -450(4) - 400(3) + 800(2) + M_D = 0$

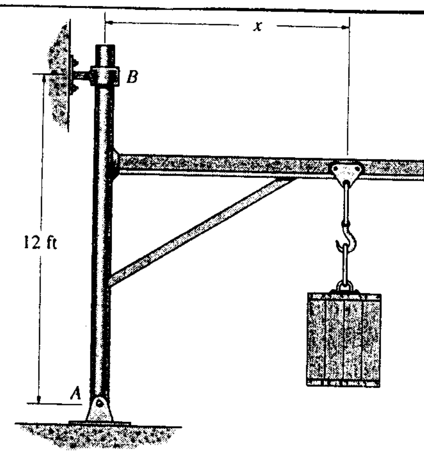
$M_D = 1.40 \text{ kip}\cdot\text{ft}$ **Ans**

Require 800 lb line to snap.

$(M_D)_{max} = 3.00 \text{ kip}\cdot\text{ft}$ **Ans**



5-34. The jib crane is pin-connected at A and supported by a smooth collar at B. Determine the roller placement x of the 5000-lb load so that it gives the maximum and minimum reactions at the supports. Calculate these reactions in each case. Neglect the weight of the crane. Require 4 ft $\le x \le$ 10 ft.



Equations of Equilibrium :

$\circlearrowleft + \Sigma M_A = 0;\quad N_B(12) - 5x = 0 \quad N_B = 0.4167x$ [1]

$+\uparrow \Sigma F_y = 0;\quad A_y - 5 = 0 \quad A_y = 5.00 \text{ kip}$ [2]

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - 0.4167x = 0 \quad A_x = 0.4167x$ [3]

By observation, the **maximum support reactions** occur when

$x = 10$ ft **Ans**

With $x = 10$ ft, from Eqs.[1], [2] and [3], the **maximum support reactions** are

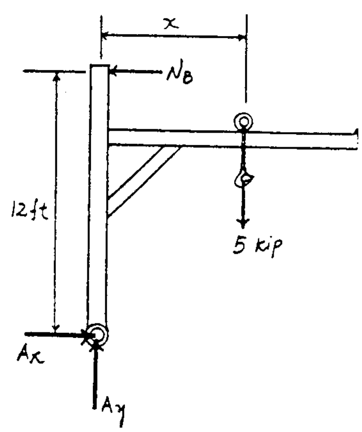
$A_x = N_B = 4.17 \text{ kip} \quad A_y = 5.00 \text{ kip}$ **Ans**

By observation, the **minimum support reactions** occur when

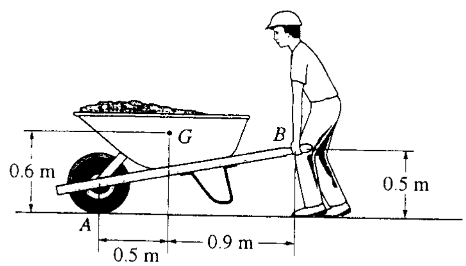
$x = 4$ ft **Ans**

With $x = 4$ ft, from Eqs.[1], [2] and [3], the **minimum support reactions** are

$A_x = N_B = 1.67 \text{ kip} \quad A_y = 5.00 \text{ kip}$ **Ans**

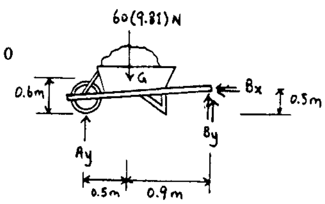


5-35. If the wheelbarrow and its contents have a mass of 60 kg and center of mass at G, determine the magnitude of the resultant force which the man must exert on *each* of the two handles in order to hold the wheelbarrow in equilibrium.



$\curvearrowleft +\Sigma M_B = 0;$ $\quad -A_y(1.4) + 60(9.81)(0.9) = 0$

$A_y = 378.39$ N



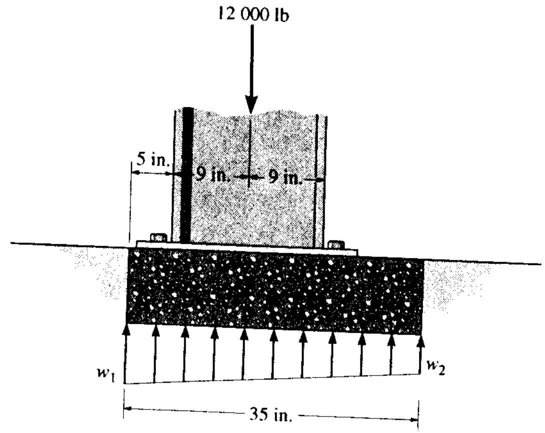
$+\uparrow \Sigma F_y = 0;$ $\quad 378.39 - 60(9.81) + 2B_y = 0$

$B_y = 105.11$ N

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad B_x = 0$

$F_B = 105$ N **Ans**

*5-36. The pad footing is used to support the load of 12 000 lb. Determine the intensities w_1 and w_2 of the distributed loading acting on the base of the footing for the equilibrium.



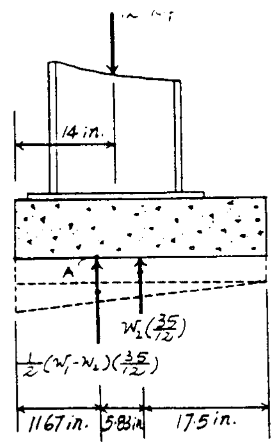
Equations of Equilibrium : The load intensity w_2 can be determined directly by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \quad w_2\left(\frac{35}{12}\right)(17.5 - 11.67) - 12(14 - 11.67) = 0$$

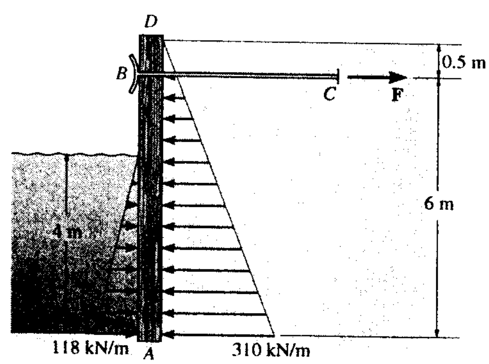
$$w_2 = 1.646 \text{ kip/ft} = 1.65 \text{ kip/ft} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{1}{2}(w_1 - 1.646)\left(\frac{35}{12}\right) + 2.743\left(\frac{35}{12}\right) - 12 = 0$$

$$w_1 = 6.58 \text{ kip/ft} \qquad \textbf{Ans}$$



5-37. The bulk head AD is subjected to both water and soil-backfill pressures. Assuming AD is "pinned" to the ground at A, determine the horizontal and vertical reactions there and also the required tension in the ground anchor BC necessary for equilibrium. The bulk head has a mass of 800 kg.



Equations of Equilibrium : The force in ground anchor BC can be obtained directly by summing moments about point A.

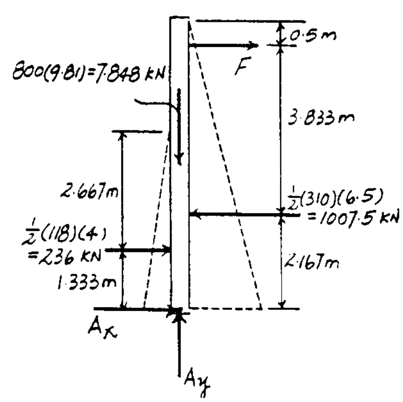
$$\circlearrowleft + \Sigma M_A = 0; \quad 1007.5(2.167) - 236(1.333) - F(6) = 0$$

$$F = 311.375 \text{ kN} = 311 \text{ kN} \qquad \textbf{Ans}$$

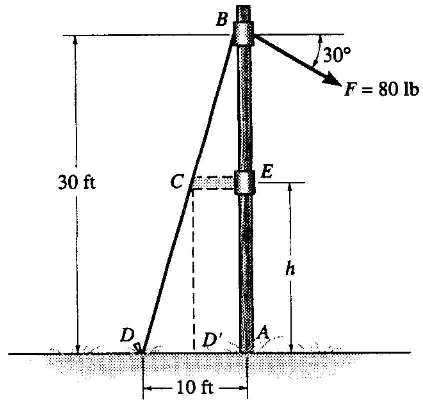
$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x + 311.375 + 236 - 1007.5 = 0$$

$$A_x = 460 \text{ kN} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 7.848 = 0 \quad A_y = 7.85 \text{ kN} \qquad \textbf{Ans}$$



5-38. The telephone pole of negligible thickness is subjected to the force of 80 lb directed as shown. It is supported by the cable BCD and can be assumed pinned at its base A. In order to provide clearance for a sidewalk right of way, where D is located, a strut CE is attached at C, as shown by the dashed lines (cable segment CD is removed). If the tension in CD' is to be twice the tension in BCD, determine the height h for placement of the strut CE.



$\circlearrowleft + \Sigma M_A = 0; \quad -80(30)\cos 30° + \frac{1}{\sqrt{10}} T_{BCD}(30) = 0$

$T_{BCD} = 219.089$ lb

Require $T_{CD'} = 2(219.089) = 438.178$ lb

$+\Sigma M_A = 0; \quad 438.178(d) - 80\cos 30°(30) = 0$

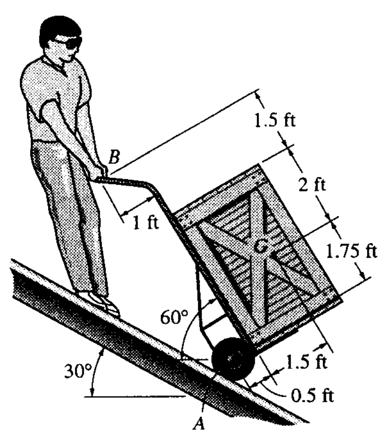
$d = 4.7434$ ft

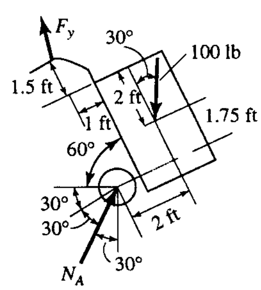
$\frac{30 - h}{4.7434} = \frac{30}{10}$

$300 - 10h = 142.3025$

$h = 15.8$ ft **Ans**

5-39. The worker uses the hand truck to move material down the ramp. If the truck and its contents are held in the position shown and have a weight of 100 lb with center of gravity at G, determine the resultant normal force of both wheels on the ground A and the magnitude of the force required at the grip B.





$\circlearrowleft + \Sigma M_B = 0; \quad (N_A \cos 30°)(5.25) + N_A \sin 30°(0.5)$

$- 100\sin 30°(3.5) - 100\cos 30°(2.5) = \bullet$

$N_A = 81.621 \text{ lb} = 81.6 \text{ lb}$ **Ans**

$+\searrow \Sigma F_x = 0; \quad -B_x + 100\cos 30° - 81.621\sin 30° = 0$

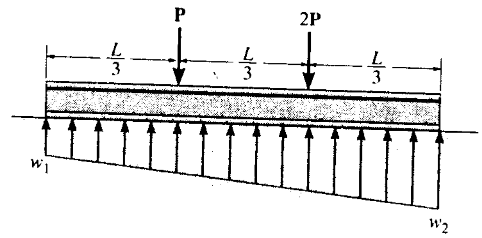
$B_x = 45.792$ lb

$\nearrow + \Sigma F_y = 0; \quad B_y - 100\sin 30° + 81.621\cos 30° = 0$

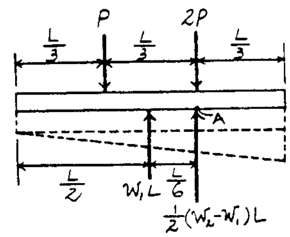
$B_y = -20.686$ lb

$F_B = \sqrt{(45.792)^2 + (-20.686)^2} = 50.2$ lb **Ans**

*5-40. The beam is subjected to the two concentrated loads as shown. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities w_1 and w_2 for equilibrium (a) in terms of the parameters shown; (b) set $P = 500$ lb, $L = 12$ ft.



Equations of Equilibrium : The load intensity w_1 can be determined directly by summing moments about point A.



$$\circlearrowleft + \Sigma M_A = 0; \quad P\left(\frac{L}{3}\right) - w_1 L\left(\frac{L}{6}\right) = 0$$

$$w_1 = \frac{2P}{L} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{1}{2}\left(w_2 - \frac{2P}{L}\right)L + \frac{2P}{L}(L) - 3P = 0$$

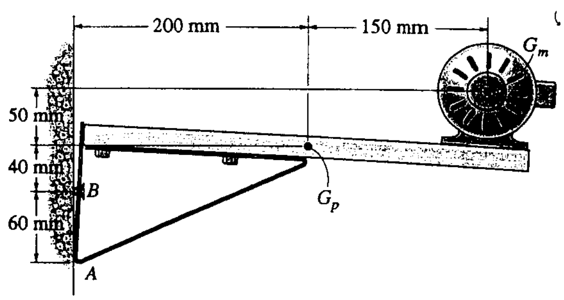
$$w_2 = \frac{4P}{L} \qquad \textbf{Ans}$$

If $P = 500$ lb and $L = 12$ft,

$$w_1 = \frac{2(500)}{12} = 83.3 \text{ lb/ft} \qquad \textbf{Ans}$$

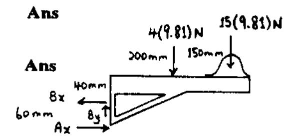
$$w_2 = \frac{4(500)}{12} = 167 \text{ lb/ft} \qquad \textbf{Ans}$$

5-41. The shelf supports the electric motor which has a mass of 15 kg and mass center at G_m. The platform upon which it rests has a mass of 4 kg and mass center at G_p. Assuming that a single bolt B holds the shelf up and the bracket bears against the smooth wall at A, determine this normal force at A and the horizontal and vertical components of reaction of the bolt on the bracket.



$$\circlearrowleft + \Sigma M_A = 0; \quad B_x(60) - 4(9.81)(200) - 15(9.81)(350) = 0$$

$$B_x = 989.18 = 989 \text{ N} \qquad \textbf{Ans}$$

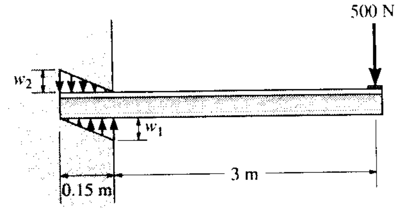


$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 989.18 = 989 \text{ N} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad B_y = 4(9.81) + 15(9.81)$$

$$B_y = 186.39 = 186 \text{ N} \qquad \textbf{Ans}$$

5-42. A cantilever beam, having an extended length of 3 m, is subjected to a vertical force of 500 N. Assuming that the wall resists this load with linearly varying distributed loads over the 0.15-m length of the beam portion inside the wall, determine the intensities w_1 and w_2 for equilibrium.



$$+\uparrow \Sigma F_y = 0; \quad \frac{1}{2}(w_1)(0.15) - \frac{1}{2}(w_2)(0.15) - 500 = 0$$

$$+\Sigma M_A = 0; \quad -(500)3 - \frac{1}{2}(w_1)(0.15)(0.05) + \frac{1}{2}(w_2)(0.15)(0.1) = 0$$

These equations become

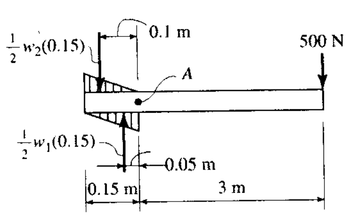
$w_1 - w_2 = 6666.7$

$2w_2 - w_1 = 400\,000$

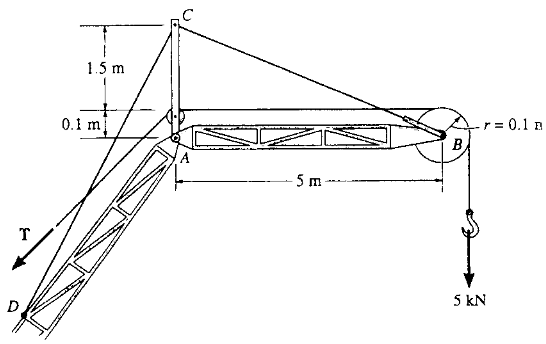
Solving,

$w_1 = 413$ kN/m **Ans**

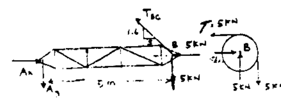
$w_2 = 407$ kN/m **Ans**



5-43. The upper portion of the crane boom consists of the jib AB, which is supported by the pin at A, the guy line BC, and the backstay CD, each cable being separately attached to the mast at C. If the 5-kN load is supported by the hoist line, which passes over the pulley at B, determine the magnitude of the resultant force the pin exerts on the jib at A for equilibrium, the tension in the guy line BC, and the tension T in the hoist line. Neglect the weight of the jib. The pulley at B has a radius of 0.1 m.



From pulley, tension in the hoist line is



$\circlearrowleft +\Sigma M_B = 0;\quad T(0.1) - 5(0.1) = 0;$

$T = 5 \text{ kN}$ **Ans**

From the jib,

$\circlearrowleft +\Sigma M_A = 0;\quad -5(5) + T_{BC}\left(\frac{1.6}{\sqrt{27.56}}\right)(5) = 0$

$T_{BC} = 16.4055 = 16.4 \text{ kN}$ **Ans**

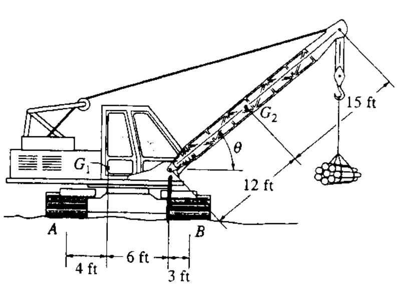
$+\uparrow \Sigma F_y = 0;\quad -A_y + (16.4055)\left(\frac{1.6}{\sqrt{27.56}}\right) - 5 = 0$

$A_y = 0$

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - 16.4055\left(\frac{5}{\sqrt{27.56}}\right) - 5 = 0$

$F_A = A_x = 20.6 \text{ kN}$ **Ans**

***5-44.** The mobile crane has a weight of 120,000 lb and center of gravity at G_1; the boom has a weight of 30,000 lb and center of gravity at G_2. Determine the smallest angle of tilt θ of the boom, without causing the crane to overturn if the suspended load is W = 40,000 lb. Neglect the thickness of the tracks at A and B.

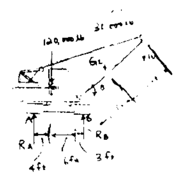


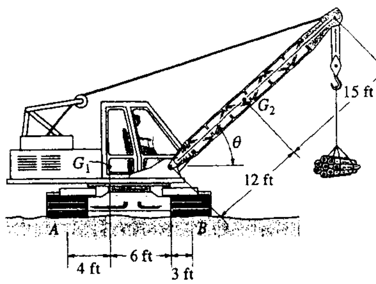
When tipping occurs, $R_A = 0$

$\circlearrowleft +\Sigma M_B = 0;\quad -(30\,000)(12\cos\theta - 3) - (40\,000)(27\cos\theta - 3) + (120\,000)(9) = 0$

$\theta = \cos^{-1}(0.896) = 26.4°$ **Ans**

5-45. The mobile crane has a weight of 120,000 lb and center of gravity at G_1; the boom has a weight of 30,000 lb and center of gravity at G_2. If the suspended load has a weight of W = 16,000 lb, determine the normal reactions at the tracks A and B. For the calculation, neglect the thickness of the tracks and take $\theta = 30°$.





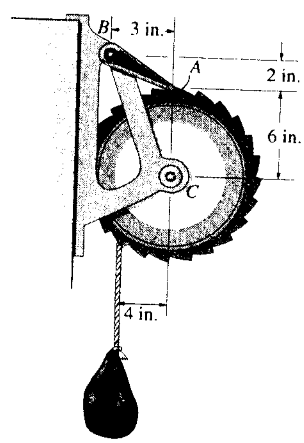
$\curvearrowleft + \Sigma M_B = 0;$ $\quad -(30\,000)(12\cos 30° - 3) - (16\,000)(27\cos 30° - 3) - R_A(13) + (120\,000)(9) = 0$

$R_A = 40\,931 \text{ lb} = 40.9 \text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $\quad 40\,931 + R_B - 120\,000 - 30\,000 - 16\,000 = 0$

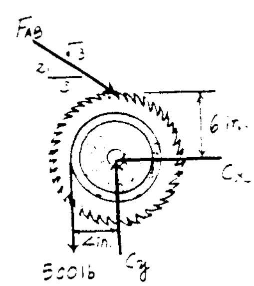
$R_B = 125 \text{ kip}$ **Ans**

5-46. The winch consists of a drum radius 4 in., which is pin-connected at its center C. At its outer rim is a ratchet gear having a mean radius of 6 in. The pawl AB serves as a two-force member (short link) and holds the drum from rotating. If the suspended load is 500 lb, determine the horizontal and vertical components of reaction at the pin C.



Equations of Equilibrium : The force in short link AB can be obtained directly by summing moments about point C.

$\curvearrowleft + \Sigma M_C = 0;$ $\quad 500(4) - F_{AB}\left(\frac{3}{\sqrt{13}}\right)(6) = 0 \qquad F_{AB} = 400.62 \text{ lb}$



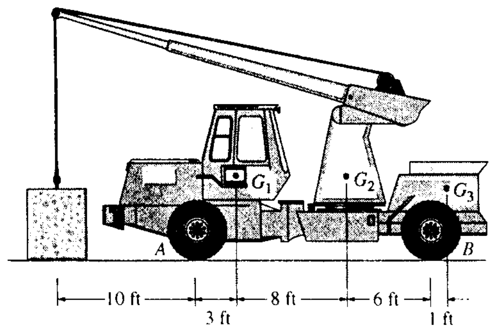
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 400.62\left(\frac{3}{\sqrt{13}}\right) - C_x = 0$

$C_x = 333 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $\quad C_y - 500 - 400.62\left(\frac{2}{\sqrt{13}}\right) = 0$

$C_y = 722 \text{ lb}$ **Ans**

5-47. The crane consists of three parts, which have weights of $W_1 = 3500$ lb, $W_2 = 900$ lb, $W_3 = 1500$ lb and centers of gravity at G_1, G_2, and G_3, respectively. Neglecting the weight of the boom, determine (a) the reactions on each of the four tires if the load is hoisted at constant velocity and has a weight of 800 lb, and (b), with the boom held in the position shown, the maximum load the crane can lift without tipping over.



Equations of Equilibrium : The normal reaction N_B can be obtained directly by summing moments about point A.

$$\curvearrowleft + \Sigma M_A = 0; \quad 2N_B(17) + W(10) - 3500(3) - 900(11) - 1500(18) = 0$$

$$N_B = 1394.12 - 0.2941W \qquad [1]$$

Using the result $N_B = 2788.24 - 0.5882W$,

$$+\uparrow \Sigma F_y = 0; \quad 2N_A + (2788.24 - 0.5882W) - W - 3500 - 900 - 1500 = 0$$

$$N_A = 0.7941W + 1555.88 \qquad [2]$$

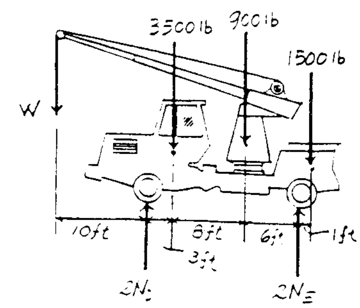
a) Set $W = 800$ lb and substitute into Eqs. [1] and [2] yields

$N_A = 0.7941(800) + 1555.88 = 2191.18 \text{ lb} = 2.19 \text{ kip}$ **Ans**

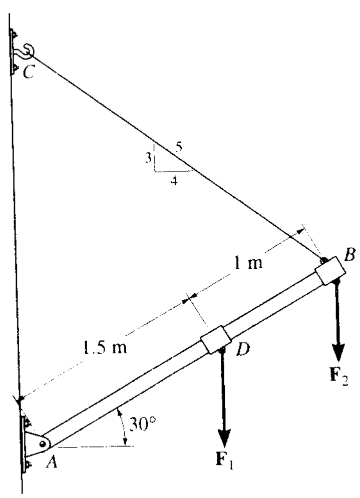
$N_B = 1394.12 - 0.2941(800) = 1158.82 \text{ lb} = 1.16 \text{ kip}$ **Ans**

b) When the crane is about to tip over, the normal reaction on $N_E = 0$. From Eq. [1],

$$N_B = 0 = 1394.12 - 0.2941W$$
$$W = 4740 \text{ lb} = 4.74 \text{ kip}$$ **Ans**



***5-48.** The boom supports the two vertical loads. Neglect the size of the collars at D and B and the thickness of the boom, and compute the horizontal and vertical components of force at the pin A and the force in cable CB. Set $F_1 = 800$ N and $F_2 = 350$ N.



$$\curvearrowleft + \Sigma M_A = 0; \quad -800(1.5\cos 30°) - 350(2.5\cos 30°) + \frac{4}{5}F_{CB}(2.5\sin 30°) + \frac{3}{5}F_{CB}(2.5\cos 30°) = 0$$

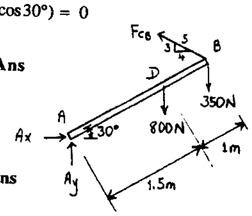
$F_{CB} = 781.6 = 782$ N **Ans**

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - \frac{4}{5}(781.6) = 0$$

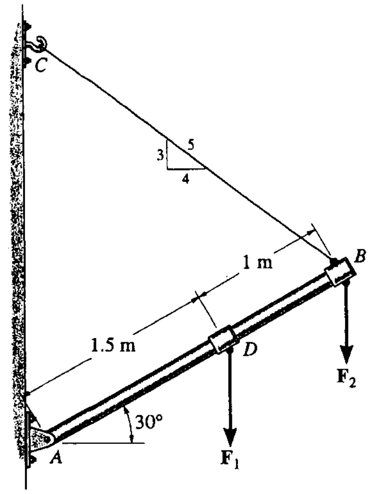
$A_x = 625$ N **Ans**

$$+\uparrow \Sigma F_y = 0; \quad A_y - 800 - 350 + \frac{3}{5}(781.6) = 0$$

$A_y = 681$ N **Ans**



5-49. The boom is intended to support two vertical loads, $\mathbf{F}_1$ and $\mathbf{F}_2$. If the cable CB can sustain a maximum load of 1500 lb before it fails, determine the critical loads if $F_1 = 2F_2$. Also, what is the magnitude of the maximum reaction at pin A?

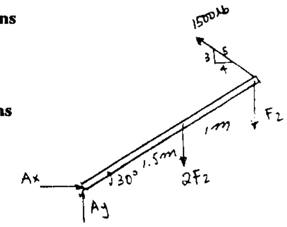


$\circlearrowleft + \Sigma M_A = 0;\quad -2F_2(1.5\cos 30°) - F_2(2.5\cos 30°) + \frac{4}{5}(1500)(2.5\sin 30°) + \frac{3}{5}(1500)(2.5\cos 30°) = 0$

$F_2 = 724$ lb **Ans**

$F_1 = 2F_2 = 1448$ lb

$F_1 = 1.45$ kip **Ans**



$\xrightarrow{+}\Sigma F_x = 0;\quad A_x - \frac{4}{5}(1500) = 0$

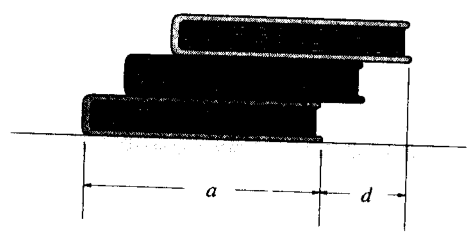
$A_x = 1200$ lb

$+\uparrow \Sigma F_y = 0;\quad A_y - 724 - 1448 + \frac{3}{5}(1500) = 0$

$A_y = 1272$ lb

$F_A = \sqrt{(1200)^2 + (1272)^2} = 1749 \text{ lb} = 1.75 \text{ kip}$ **Ans**

5-50. Three uniform books, each having a weight W and length a, are stacked as shown. Determine the maximum distance d that the top book can extend out from the bottom one so the stack does not topple over.

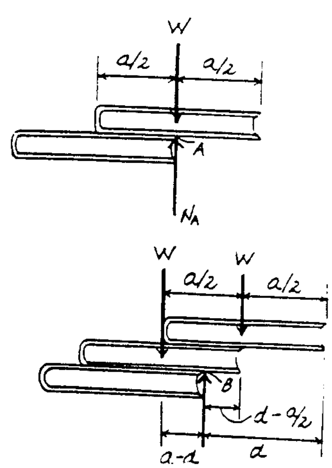


Equilibrium : For top two books, the upper book will topple when the center of gravity of this book is to the right of point A. Therefore, the maximum distance from the right edge of this book to point A is $a/2$.

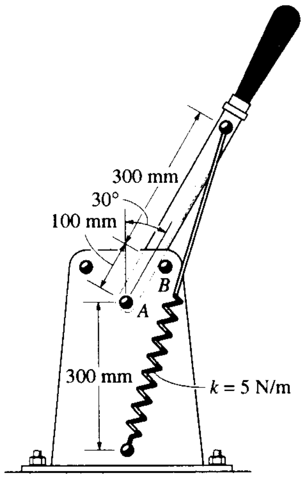
Equation of Equilibrium : For the entire three books, the top two books will topple about point B.

$$+\Sigma M_B = 0;\quad W(a-d) - W\left(d - \frac{a}{2}\right) = 0$$

$$d = \frac{3a}{4}$$ **Ans**



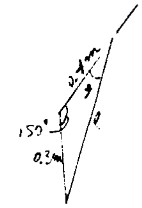
5-51. The toggle switch consists of a cocking lever that is pinned to a fixed frame at A and held in place by the spring which has an unstretched length of 200 mm. Determine the magnitude of the resultant force at A and the normal force on the peg at B when the lever is in the position shown.



$l = \sqrt{(0.3)^2 + (0.4)^2 - 2(0.3)(0.4)\cos 150^\circ} = 0.67664 \text{ m}$

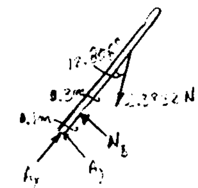
$\frac{\sin\theta}{0.3} = \frac{\sin 150^\circ}{0.67664}; \quad \theta = 12.808^\circ$

$F_s = ks = 5(0.67664 - 0.2) = 2.3832 \text{ N}$



$\circlearrowleft + \Sigma M_A = 0; \quad -(2.3832\sin 12.808^\circ)(0.4) + N_B(0.1) = 0$

$N_B = 2.11327 \text{ N} = 2.11 \text{ N}$ **Ans**



$\nearrow + \Sigma F_x = 0; \quad A_x - 2.3832\cos 12.808^\circ = 0$

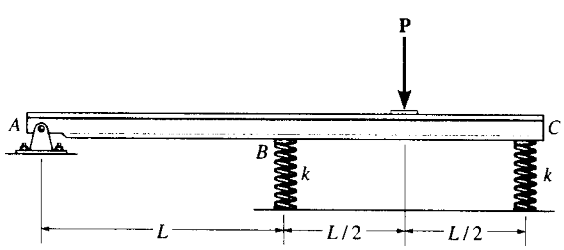
$A_x = 2.3239 \text{ N}$

$+\nwarrow \Sigma F_y = 0; \quad A_y + 2.11327 - 2.3832\sin 12.808^\circ = 0$

$A_y = -1.5850 \text{ N}$

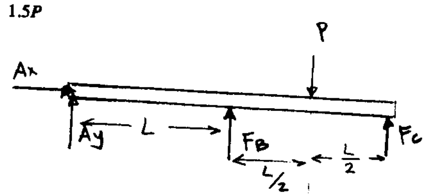
$F_A = \sqrt{(2.3239)^2 + (-1.5850)^2} = 2.81 \text{ N}$ **Ans**

***5-52.** The rigid beam of negligible weight is supported horizontally by two springs and a pin. If the springs are uncompressed when the load is removed, determine the force in each spring when the load **P** is applied. Also, compute the vertical deflection of end C. Assume the spring stiffness k is large enough so that only small deflections occur. *Hint:* The beam rotates about A so the deflections in the springs can be related.



$+\Sigma M_A = 0; \quad F_B(L) + F_C(2L) - P(\frac{3}{2}L) = 0$

$F_B + 2F_C = 1.5P$

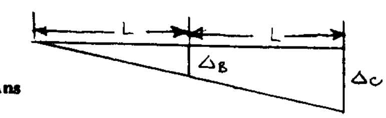


$\frac{L}{\Delta_B} = \frac{2L}{\Delta_C}$

$\Delta_C = 2\Delta_B$

$\frac{F_C}{k} = \frac{2F_B}{k}$

$F_C = 2F_B$



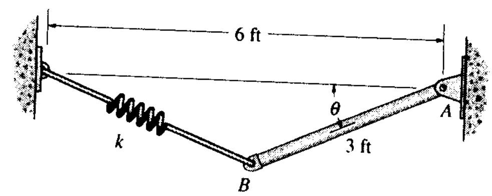
$5F_B = 1.5P$

$F_B = 0.3P$ **Ans**

$F_C = 0.6P$ **Ans**

Deflection $\quad x_C = \frac{0.6P}{k}$ **Ans**

5-53. The uniform rod AB has a weight of 15 lb and the spring is unstretched when $\theta = 0°$. If $\theta = 30°$, determine the stiffness k of the spring.

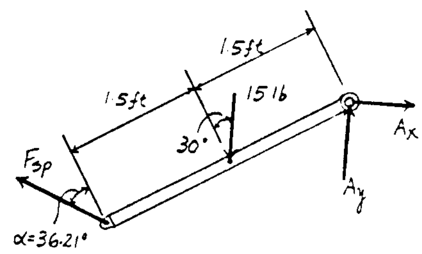


Geometry : From triangle CDE, the cosine law gives

$$l = \sqrt{2.536^2 + 1.732^2 - 2(2.536)(1.732)\cos 120°} = 3.718 \text{ ft}$$

Using the sine law,

$$\frac{\sin\alpha}{2.536} = \frac{\sin 120°}{3.718} \qquad \alpha = 36.21°$$



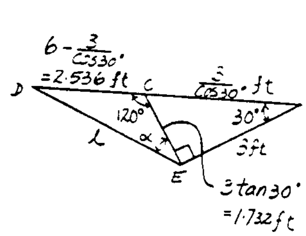
Equations of Equilibrium : The force in the spring can be obtained directly by summing moments about point A.

$$\curvearrowleft + \Sigma M_A = 0; \qquad 15\cos 30°(1.5) - F_{sp}\cos 36.21°(3) = 0$$

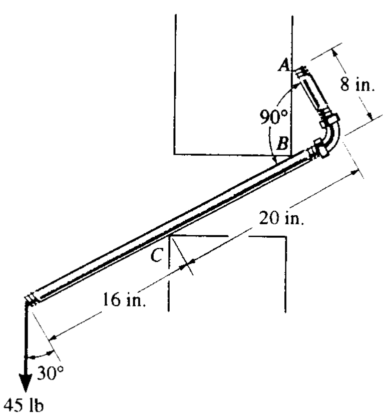
$$F_{sp} = 8.050 \text{ lb}$$

Spring Force Formula : The spring stretches $x = 3.718 - 3 = 0.718$ ft.

$$k = \frac{F_{sp}}{x} = \frac{8.050}{0.718} = 11.2 \text{ lb/ft} \qquad \textbf{Ans}$$



5-54. The smooth pipe rests against the wall at the points of contact A, B, and C. Determine the reactions at these points needed to support the vertical force of 45 lb. Neglect the pipe's thickness in the calculation.



$\curvearrowleft + \Sigma M_A = 0; \quad 45\cos 30°(36) - 45\sin 30°(8) - R_C(20) + R_B(8\tan 30°) = 0$

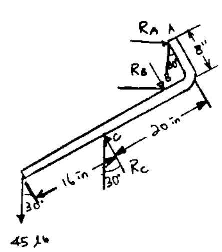
$+\uparrow \Sigma F_y = 0; \quad R_C\cos 30° - R_B\cos 30° - 45 = 0$

$R_C = 63.91 = 63.9$ lb **Ans**

$R_B = 11.95 = 11.9$ lb **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad R_A + 11.95\sin 30° - 63.91\sin 30° = 0$

$R_A = 26.0$ lb **Ans**



Also;

$+\Sigma F_{x'} = 0; \quad 45\sin 30° - R_A\cos 30° = 0$

$R_A = 26.0$ lb **Ans**

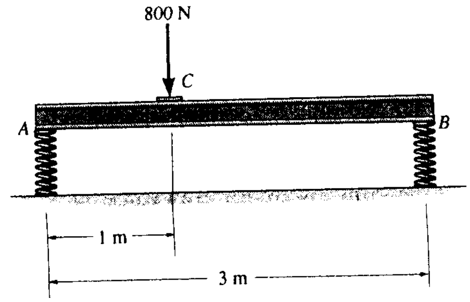
$+\Sigma F_{y'} = 0; \quad -45\cos 30° + R_C - R_B - 25.98\sin 30° = 0$

$\curvearrowleft + \Sigma M_C = 0; \quad 45\cos 30°(16) - R_B(20 - 8\tan 30°) - 25.98(8\cos 30° + 20\sin 30°) = 0$

$R_B = 11.9$ lb **Ans**

$R_C = 63.9$ lb **Ans**

***5-55.** The horizontal beam is supported by springs at its ends. Each spring has a stiffness of $k = 5$ kN/m and is originally unstretched so that the beam is in the horizontal position. Determine the angle of tilt of the beam if a load of 800 N is applied at point C as shown.



Equations of Equilibrium : The spring force at A and B can be obtained directly by summing moments about points B and A, respectively.

$\curvearrowleft + \Sigma M_B = 0;\quad 800(2) - F_A(3) = 0 \quad F_A = 533.33\text{ N}$

$\curvearrowleft + \Sigma M_A = 0;\quad F_B(3) - 800(1) = 0 \quad F_B = 266.67\text{ N}$

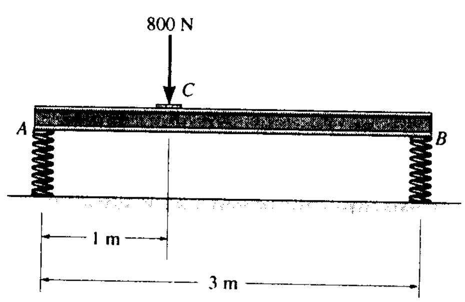
Spring Formula : Applying $\Delta = \frac{F}{k}$, we have

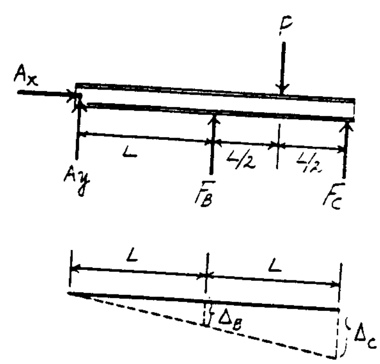
$$\Delta_A = \frac{533.33}{5(10^3)} = 0.1067\text{ m}$$
$$\Delta_B = \frac{266.67}{5(10^3)} = 0.05333\text{ m}$$

Geometry : The angle of tilt α is

$$\alpha = \tan^{-1}\left(\frac{0.05333}{3}\right) = 1.02^\circ \qquad \textbf{Ans}$$

***5-56.** The horizontal beam is supported by springs at its ends. If the stiffness of the spring at A is $k_A = 5$kN/m, determine the required stiffness of the spring at B so that if the beam is loaded with the 800 N it remains in the horizontal position. The springs are originally constructed so that the beam is in the horizontal position when it is unloaded.





Equations of Equilibrium : The spring forces at A and B can be obtained directly by summing moments about points B and A respectively.

$\curvearrowleft + \Sigma M_B = 0;\quad 800(2) - F_A(3) = 0 \quad F_A = 533.33\text{ N}$

$\curvearrowleft + \Sigma M_A = 0;\quad F_B(3) - 800(1) = 0 \quad F_B = 266.67\text{ N}$

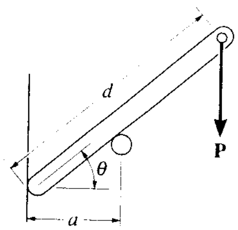
Spring Formula : Applying $\Delta = \frac{F}{k}$, we have

$$\Delta_A = \frac{533.33}{5(10^3)} = 0.1067\text{ m} \qquad \Delta_B = \frac{266.67}{k_B}$$

Geometry : Requires, $\Delta_B = \Delta_A$. Then

$$\frac{266.67}{k_B} = 0.1067$$
$$k_B = 2500\text{ N/m} = 2.50\text{ kN/m} \qquad \textbf{Ans}$$

5-57. Determine the distance d for placement of the load **P** for equilibrium of the smooth bar in the position θ as shown. Neglect the weight of the bar.

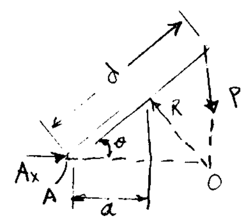


$+\uparrow \Sigma F_y = 0;$ $\quad R\cos\theta - P = 0$

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -P(d\cos\theta) + R(\frac{a}{\cos\theta}) = 0$

$$Rd\cos^2\theta = R(\frac{a}{\cos\theta})$$

$$d = \frac{a}{\cos^3\theta} \quad \textbf{Ans}$$



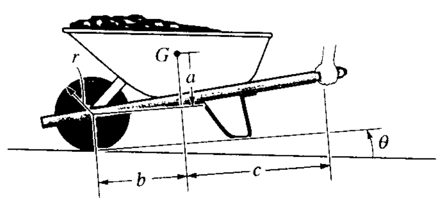
Also;

Require forces to be concurrent at point O.

$$AO = d\cos\theta = \frac{a/\cos\theta}{\cos\theta}$$

thus $\quad d = \frac{a}{\cos^3\theta} \quad$ **Ans**

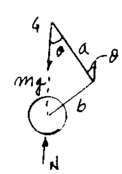
5-58. The wheelbarrow and its contents have a mass m and center of mass at G. Determine the greatest angle of tilt θ without causing the wheelbarrow to tip over.



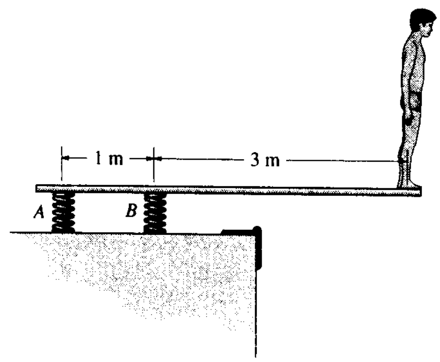
Require point G to be over the wheel axle for tipping. Thus

$$b\cos\theta = a\sin\theta$$

$$\theta = \tan^{-1}\frac{b}{a} \quad \textbf{Ans}$$



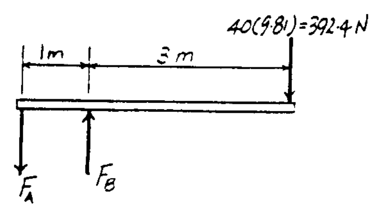
5-59. A man stands out at the end of the diving board, which is supported by two springs A and B, each having a stiffness of $k = 15$ kN/m. In the position shown the board is horizontal. If the man has a mass of 40 kg, determine the angle of tilt which the board makes with the horizontal after he jumps off. Neglect the weight of the board and assume it is rigid.



Equations of Equilibrium : The spring force at A and B can be obtained directly by summing moments about points B and A, respectively.

$$\curvearrowleft + \Sigma M_B = 0; \quad F_A(1) - 392.4(3) = 0 \quad F_A = 1177.2 \text{ N}$$

$$\curvearrowleft + \Sigma M_A = 0; \quad F_B(1) - 392.4(4) = 0 \quad F_B = 1569.6 \text{ N}$$



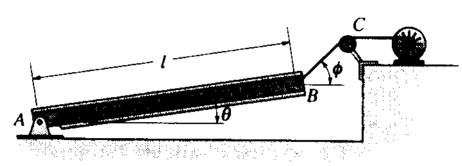
Spring Formula : Applying $\Delta = \frac{F}{k}$, we have

$$\Delta_A = \frac{1177.2}{15(10^3)} = 0.07848 \text{ m} \qquad \Delta_B = \frac{1569.6}{15(10^3)} = 0.10464 \text{ m}$$

Geometry : The angle of tilt α is

$$\alpha = \tan^{-1}\left(\frac{0.10464 + 0.07848}{1}\right) = 10.4° \qquad \textbf{Ans}$$

***5-60.** The uniform beam has a weight W and length l and is supported by a pin at A and a cable BC. Determine the horizontal and vertical components of reaction at A and the tension in the cable necessary to hold the beam in the position shown.

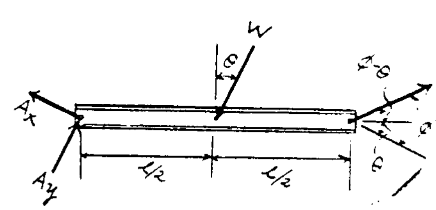


Equations of Equilibrium : The tension the cable can be obtained directly by summing moments about point A.

$$\curvearrowleft + \Sigma M_A = 0; \quad T\sin(\phi - \theta)l - W\cos\theta\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W\cos\theta}{2\sin(\phi - \theta)} \qquad \textbf{Ans}$$

Using the result $T = \dfrac{W\cos\theta}{2\sin(\phi-\theta)}$



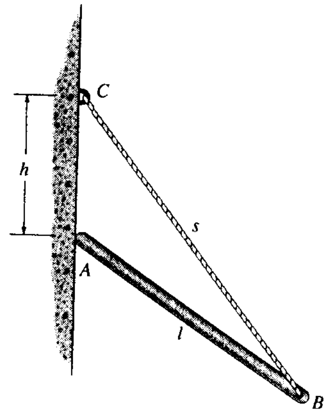
$$\xrightarrow{+} \Sigma F_x = 0; \quad \left(\frac{W\cos\theta}{2\sin(\phi - \theta)}\right)\cos\phi - A_x = 0$$

$$A_x = \frac{W\cos\phi\cos\theta}{2\sin(\phi - \theta)} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + \left(\frac{W\cos\theta}{2\sin(\phi - \theta)}\right)\sin\phi - W = 0$$

$$A_y = \frac{W(\sin\phi\cos\theta - 2\cos\phi\sin\theta)}{2\sin(\phi - \theta)} \qquad \textbf{Ans}$$

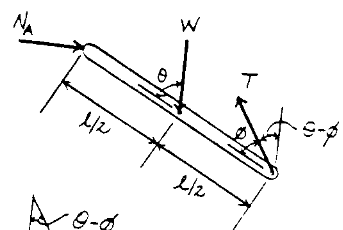
5-61. The uniform rod has a length l and weight W. It is supported at one end A by a smooth wall and the other end by a cord of length s which is attached to the wall as shown. Show that for equilibrium it is required that $h = [(s^2 - l^2)/3]^{1/2}$.



Equations of Equilibrium : The tension in the cable can be obtained directly by summing moments about point A.

$$\curvearrowleft + \Sigma M_A = 0; \quad T\sin\phi\,(l) - W\sin\theta\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W\sin\theta}{2\sin\phi}$$

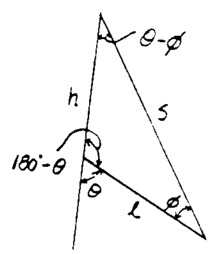


Using the result $T = \dfrac{W\sin\theta}{2\sin\phi}$,

$$+\uparrow \Sigma F_y = 0; \quad \frac{W\sin\theta}{2\sin\phi}\cos(\theta - \phi) - W = 0$$

$$\sin\theta\cos(\theta - \phi) - 2\sin\phi = 0 \qquad [1]$$

Geometry : Applying the sine law with $\sin(180° - \theta) = \sin\theta$, we have



$$\frac{\sin\phi}{h} = \frac{\sin\theta}{s} \qquad \sin\phi = \frac{h}{s}\sin\theta \qquad [2]$$

Substituting Eq.[2] into [1] yields

$$\cos(\theta - \phi) = \frac{2h}{s} \qquad [3]$$

Using the cosine law,

$$l^2 = h^2 + s^2 - 2hs\cos(\theta - \phi)$$

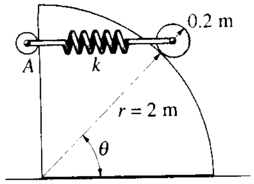
$$\cos(\theta - \phi) = \frac{h^2 + s^2 - l^2}{2hs} \qquad [4]$$

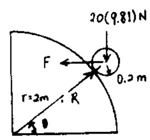
Equating Eqs.[3] and [4] yields

$$\frac{2h}{s} = \frac{h^2 + s^2 - l^2}{2hs}$$

$$h = \sqrt{\frac{s^2 - l^2}{3}} \qquad (\textit{Q.E.D})$$

5-62. The disk has a mass of 20 kg and is supported on the smooth cylindrical surface by a spring having a stiffness of $k = 400$ N/m and unstretched length of $l_0 = 1$ m. The spring remains in the horizontal position since its end A is attached to the small roller guide which has negligible weight. Determine the angle θ to the nearest degree for equilibrium of the roller.





$+\uparrow \Sigma F_y = 0;\quad R\sin\theta - 20(9.81) = 0$

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad R\cos\theta - F = 0$

$$\tan\theta = \frac{20(9.81)}{F}$$

Since $\quad \cos\theta = \dfrac{1.0 + \frac{F}{400}}{2.2}$

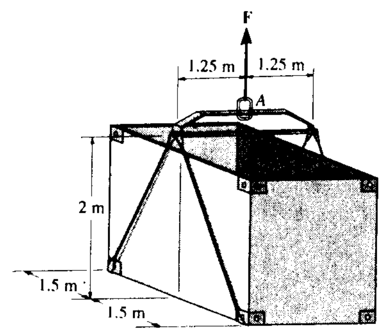
$$2.2\cos\theta = 1.0 + \frac{20(9.81)}{400\tan\theta}$$

$$880\sin\theta = 400\tan\theta + 20(9.81)$$

Solving,

$\theta = 27.1°$ and $\theta = 50.2°$ **Ans**

5-63. The uniform load has a mass of 600 kg and is lifted using a uniform 30-kg strongback beam and four wire ropes as shown. Determine the tension in each segment of rope and the force that must be applied to the sling at A.



Prob. 5-64

Equations of Equilibrium : Due to symmetry, all wires are subjected to the same tension. This condition statisfies moment equilibrium about the x and y axes and force equilibrium along y axis.

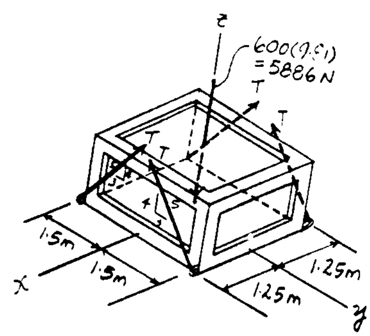
$\Sigma F_z = 0;\quad 4T\left(\frac{4}{5}\right) - 5886 = 0$

$T = 1839.375$ N $= 1.84$ kN **Ans**

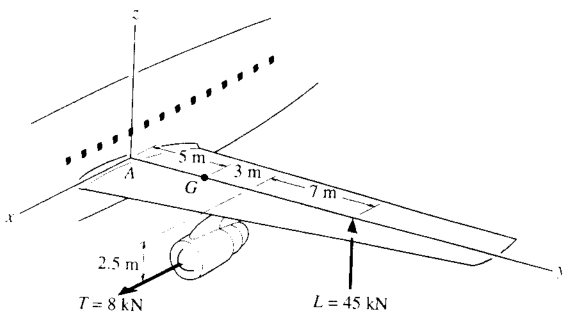
The force **F** applied to the sling A must support the weight of the load and strongback beam. Hence

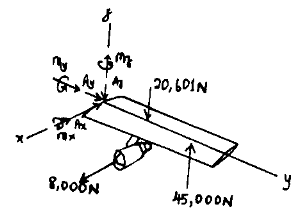
$\Sigma F_z = 0;\quad F - 600(9.81) - 30(9.81) = 0$

$F = 6180.3$ N $= 6.18$ kN **Ans**



*5-64. The wing of the jet aircraft is subjected to a thrust of $T = 8$ kN from its engine and the resultant lift force $L = 45$ kN. If the mass of the wing is 2.1 Mg and the mass center is at G, determine the x, y, z components of reaction where the wing is fixed to the fuselage at A.





$\Sigma F_x = 0;\quad -A_x + 8000 = 0$

$A_x = 8.00$ kN **Ans**

$\Sigma F_y = 0;\quad A_y = 0$ **Ans**

$\Sigma F_z = 0;\quad -A_z - 20\,601 + 45\,000 = 0$

$A_z = 24.4$ kN **Ans**

$\Sigma M_y = 0;\quad M_y - 2.5(8000) = 0$

$M_y = 20.0$ kN·m **Ans**

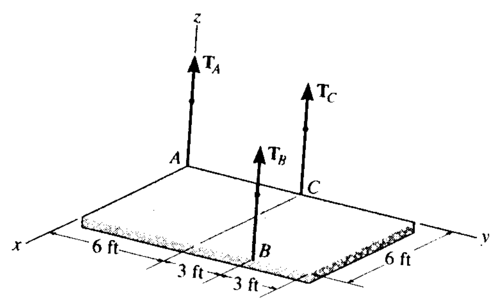
$\Sigma M_x = 0;\quad 45\,000(15) - 20\,601(5) - M_x = 0$

$M_x = 572$ kN·m **Ans**

$\Sigma M_z = 0;\quad M_z - 8000(8) = 0$

$M_z = 64.0$ kN·m **Ans**

5-65. The uniform concrete slab has a weight of 5500 lb. Determine the tension in each of the three parallel supporting cables when the slab is held in the horizontal plane as shown.



Equations of Equilibrium : The cable tension T_B can be obtained directly by summing moments about the y axis.

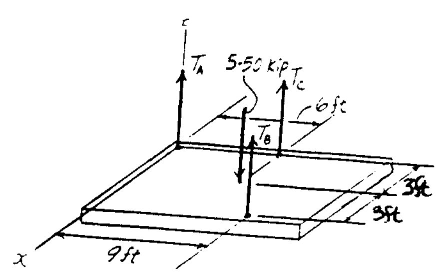
$\Sigma M_y = 0;\quad 5.50(3) - T_B(6) = 0 \quad T_B = 2.75$ kip **Ans**

$\Sigma M_x = 0;\quad T_C(6) + 2.75(9) - 5.50(6) = 0$

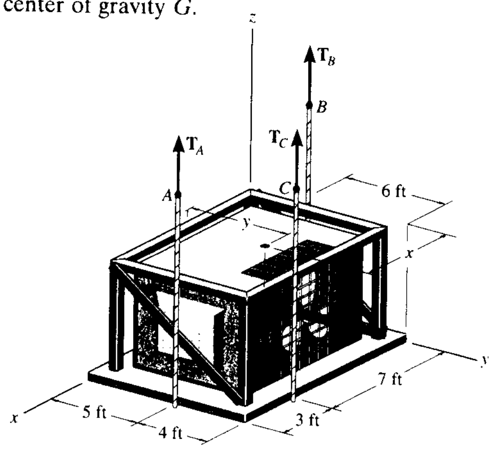
$T_C = 1.375$ kip **Ans**

$\Sigma F_z = 0;\quad T_A + 2.75 + 1.375 - 5.50 = 0$

$T_A = 1.375$ kip **Ans**



5-66. The air-conditioning unit is hoisted to the roof of a building using the three cables. If the tensions in the cables are $T_A = 250$ lb, $T_B = 300$ lb, and $T_C = 200$ lb, determine the weight of the unit and the location (x, y) of its center of gravity G.



$\Sigma F_z = 0;$ $250 + 300 + 200 - W = 0$

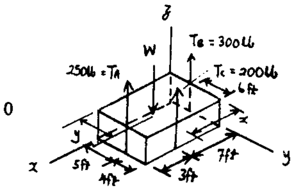
$W = 750$ lb **Ans**

$\Sigma M_y = 0;$ $750(x) - 250(10) - 200(7) = 0$

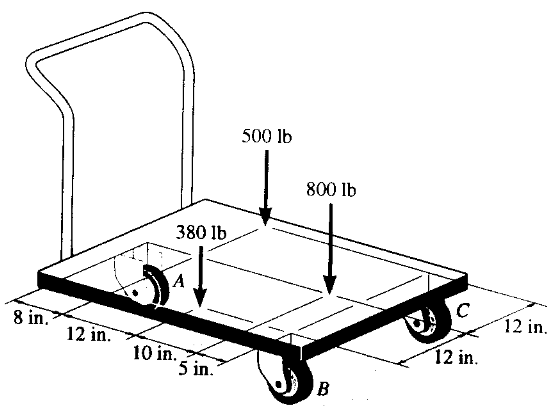
$x = 5.20$ ft **Ans**

$\Sigma M_x = 0;$ $250(5) + 300(3) + 200(9) - 750(y) = 0$

$y = 5.27$ ft **Ans**



5-67. The platform truck supports the three loadings shown. Determine the normal reactions on each of its three wheels.



$\Sigma M_x = 0;$ $380(15) + 500(27) + 800(5) - F_A(35) = 0$

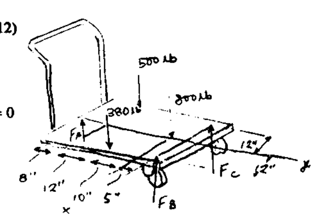
$F_A = 662.8571 = 663$ lb **Ans**

$\Sigma M_y = 0;$ $380(12) - F_B(12) - 500(12) + F_C(12)$

$F_C - F_B = 120$

$\Sigma F_z = 0;$ $F_B + F_C - 500 + 663 - 380 - 800 = 0$

$F_B + F_C = 1017.1429$

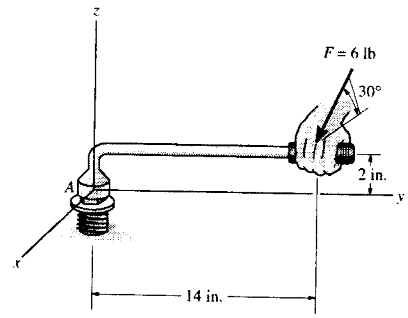


Solving,

$F_C = 569$ lb **Ans**

$F_B = 449$ lb **Ans**

*5-68. The wrench is used to tighten the bolt at A. If the force $F = 6$ lb is applied to the handle as shown, determine the magnitudes of the resultant force and moment that the bolt head exerts on the wrench. The force **F** is in a plane parallel to the $x-z$ plane.



Equations of Equilibrium:

$\Sigma F_x = 0;\quad 6\cos 30° - A_x = 0 \quad A_x = 5.196 \text{ lb}$

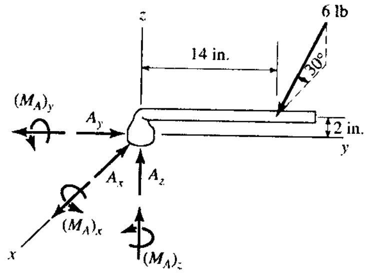
$\Sigma F_y = 0;\quad A_y = 0$

$\Sigma F_z = 0;\quad A_z - 6\sin 30° = 0 \quad A_z = 3.00 \text{ lb}$

$\Sigma M_x = 0;\quad (M_A)_x - 6\sin 30°(14) = 0 \quad (M_A)_x = 42.0 \text{ lb}\cdot\text{in}$

$\Sigma M_y = 0;\quad 6\cos 30°(2) - (M_A)_y = 0 \quad (M_A)_y = 10.39 \text{ lb}\cdot\text{in}$

$\Sigma M_z = 0;\quad (M_A)_z - 6\cos 30°(14) = 0 \quad (M_A)_z = 72.75 \text{ lb}\cdot\text{in}$



The magnitude of force and moment reactions are

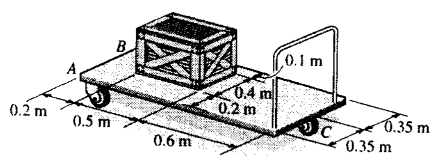
$F_A = \sqrt{A_x^2 + A_z^2} = \sqrt{5.196^2 + 3.00^2} = 6.00 \text{ lb}$ **Ans**

$M_A = \sqrt{(M_A)_x^2 + (M_A)_y^2 + (M_A)_z^2}$

$= \sqrt{42.0^2 + 10.39^2 + 72.75^2}$

$= 84.64 \text{ lb}\cdot\text{in} = 7.05 \text{ lb}\cdot\text{ft}$ **Ans**

5-69. The cart supports the uniform crate having a mass of 85 kg. Determine the vertical reactions on the three casters at A, B, and C. The caster at B is not shown. Neglect the mass of the cart.



Equations of Equilibrium: The normal reaction N_C can be obtained directly by summing moments about x axis.

$\Sigma M_x = 0;\quad N_C(1.3) - 833.85(0.45) = 0$

$N_C = 288.64 \text{ N} = 289 \text{ N}$ **Ans**

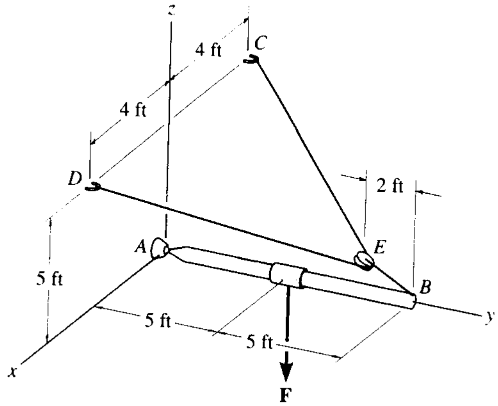
$\Sigma M_y = 0;\quad 833.85(0.3) - 288.64(0.35) - N_A(0.7) = 0$

$N_A = 213.04 \text{ N} = 213 \text{ N}$ **Ans**

$\Sigma F_z = 0;\quad N_B + 288.64 + 213.04 - 833.85 = 0$

$N_B = 332 \text{ N}$ **Ans**

5-70. The boom AB is held in equilibrium by a ball-and-socket joint A and a pulley and cord system as shown. Determine the x, y, z components of reaction at A and the tension in cable DEC if $\mathbf{F} = \{-1500\mathbf{k}\}$ lb.

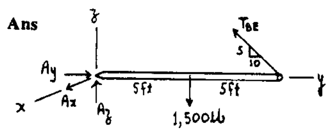


From FBD of boom,

$\Sigma M_x = 0;\quad \frac{5}{\sqrt{125}}T_{BE}(10) - 1500(5) = 0$

$T_{BE} = 1677.05$ lb

$\Sigma F_x = 0;\quad A_x = 0$ **Ans**



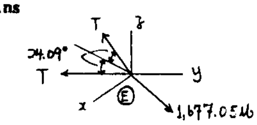
$\Sigma F_y = 0;\quad A_y - \frac{10}{\sqrt{125}}(1677.05) = 0$

$A_y = 1500$ lb $= 1.50$ kip **Ans**

$\Sigma F_z = 0;\quad A_z - 1500 + \frac{5}{\sqrt{125}}(1677.05) = 0$

$A_z = 750$ lb **Ans**

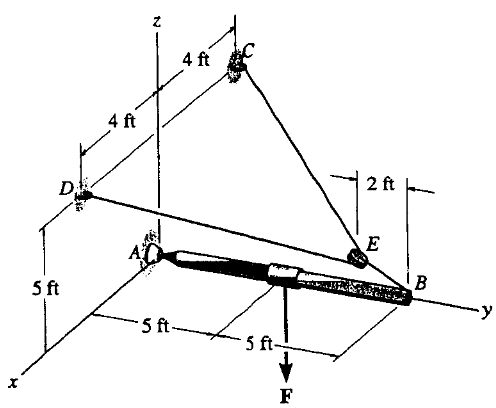
From FBD of pulley,



$\Sigma F_z = 0;\quad 2(\frac{4}{\sqrt{96}})T - \frac{1}{\sqrt{5}}(1677.05) = 0$

$T = 918.56 = 919$ lb **Ans**

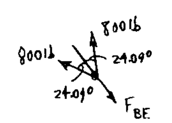
5-71. The cable CED can sustain a maximum tension of 800 lb before it fails. Determine the greatest vertical force F that can be applied to the boom. Also, what are the x, y, z components of reaction at the ball-and-socket joint A?



From FBD of pulley;

$\Sigma F_{x'} = 0;\quad 2(800)\cos 24.09° - F_{BE} = 0$

$F_{BE} = 1460.59$ lb

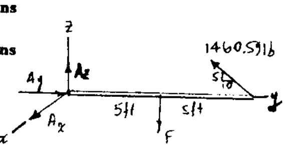


From FBD of boom;

$\Sigma M_x = 0;\quad \frac{5}{\sqrt{125}}(1460.59)(10) - F(5) = 0$

$F = 1306.39$ lb $= 1.31$ kip **Ans**

$\Sigma F_x = 0;\quad A_x = 0$ **Ans**



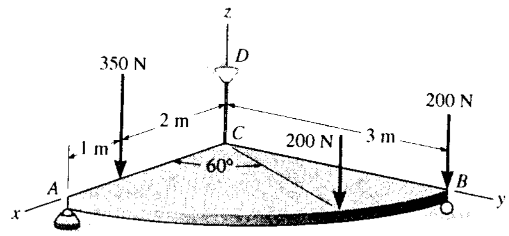
$\Sigma F_y = 0;\quad A_y - \frac{10}{\sqrt{125}}(1460.59) = 0$

$A_y = 1306.39$ lb $= 1.31$ kip **Ans**

$\Sigma F_z = 0;\quad A_z - 1306.39 + \frac{5}{\sqrt{125}}(1460.59) = 0$

$A_z = 653$ lb **Ans**

*5-72. Determine the force components acting on the ball-and-socket at A, the reaction at the roller B and the tension on the cord CD needed for equilibrium of the quarter circular plate.



Equations of Equilibrium : The normal reaction N_B and A_z can be obtained directly by summing moments about the x and y axes respectively.

$\Sigma M_x = 0;$ $N_B(3) - 200(3) - 200(3\sin 60°) = 0$

$N_B = 373.21 \text{ N} = 373 \text{ N}$ **Ans**

$\Sigma M_y = 0;$ $350(2) + 200(3\cos 60°) - A_z(3) = 0$

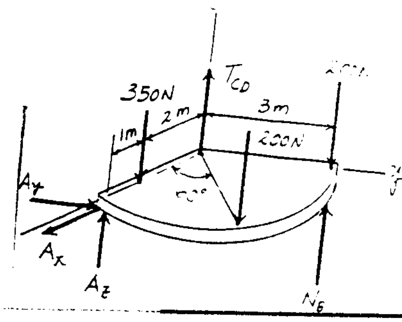
$A_z = 333.33 \text{ N} = 333 \text{ N}$ **Ans**

$\Sigma F_z = 0;$ $T_{CD} + 373.21 + 333.33 - 350 - 200 - 200 = 0$

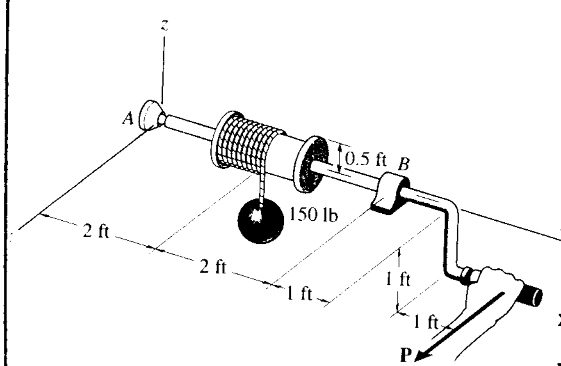
$T_{CD} = 43.5 \text{ N}$ **Ans**

$\Sigma F_x = 0;$ $A_x = 0$ **Ans**

$\Sigma F_y = 0;$ $A_y = 0$ **Ans**



5-73. The windlass is subjected to a load of 150 lb. Determine the horizontal force **P** needed to hold the handle in the position shown, and the components of reaction at the ball-and-socket joint A and the smooth journal bearing B. The bearing at B is in proper alignment and exerts only force reactions on the windlass.



$\Sigma M_y = 0;$ $(150)(0.5) - P(1) = 0$

$P = 75 \text{ lb}$ **Ans**

$\Sigma F_y = 0;$ $A_y = 0$ **Ans**

$\Sigma M_x = 0;$ $-(150)(2) + B_z(4) = 0$

$B_z = 75 \text{ lb}$ **Ans**

$\Sigma F_z = 0;$ $A_z + 75 - 150 = 0$

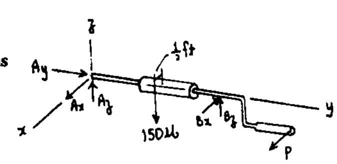
$A_z = 75 \text{ lb}$ **Ans**

$\Sigma M_z = 0;$ $B_x(4) - 75(6) = 0$

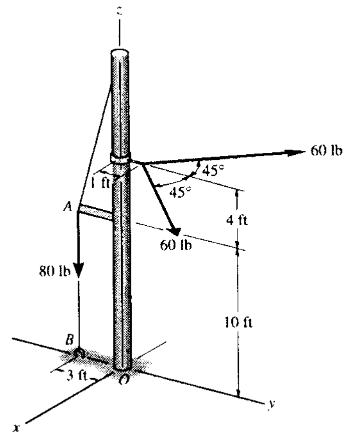
$B_x = 112.5 = 112 \text{ lb}$ **Ans**

$\Sigma F_x = 0;$ $A_x - 112.5 + 75 = 0$

$A_x = 37.5 \text{ lb}$ **Ans**



5-74. The pole for a power line is subjected to the two cable forces of 60 lb, each force lying in a plane parallel to the $x-y$ plane. If the tension in the guy wire AB is 80 lb, determine the x, y, z components of reaction at the fixed base of the pole, O.



Equations of Equilibrium:

$\Sigma F_x = 0;\quad O_x + 60\sin 45° - 60\sin 45° = 0$

$O_x = 0$ **Ans**

$\Sigma F_y = 0;\quad O_y + 60\cos 45° + 60\cos 45° = 0$

$O_y = -84.9\ \text{lb}$ **Ans**

$\Sigma F_z = 0;\quad O_z - 80 = 0 \quad O_z = 80.0\ \text{lb}$ **Ans**

$\Sigma M_x = 0;\quad (M_O)_x + 80(3) - 2[60\cos 45°(14)] = 0$

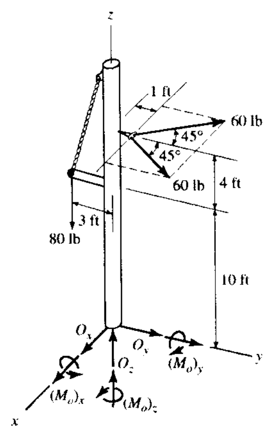
$(M_O)_x = 948\ \text{lb}\cdot\text{ft}$ **Ans**

$\Sigma M_y = 0;\quad (M_O)_y + 60\sin 45°(14) - 60\sin 45°(14) = 0$

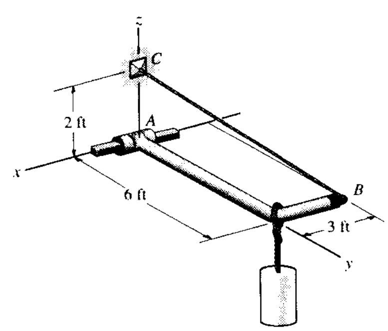
$(M_O)_y = 0$ **Ans**

$\Sigma M_z = 0;\quad (M_O)_y + 60\sin 45°(1) - 60\sin 45°(1) = 0$

$(M_O)_z = 0$ **Ans**



5-75. Member AB is supported by a cable BC and at A by a *square* rod which fits loosely through the square hole at the end joint of the member as shown. Determine the components of reaction at A and the tension in the cable needed to hold the 800-lb cylinder in equilibrium.



$\mathbf{F}_{BC} = F_{BC}\left(\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$

$\Sigma F_x = 0;\quad F_{BC}\left(\frac{3}{7}\right) = 0$

$F_{BC} = 0$ **Ans**

$\Sigma F_y = 0;\quad A_y = 0$ **Ans**

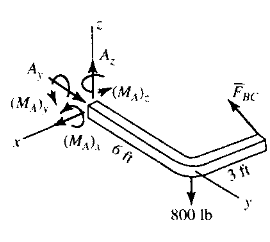
$\Sigma F_z = 0;\quad A_z = 800\ \text{lb}$ **Ans**

$\Sigma M_x = 0;\quad (M_A)_x - 800(6) = 0$

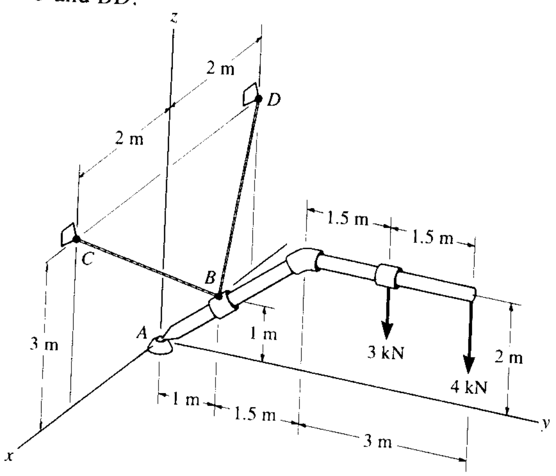
$(M_A)_x = 4.80\ \text{kip}\cdot\text{ft}$ **Ans**

$\Sigma M_y = 0;\quad (M_A)_y = 0$ **Ans**

$\Sigma M_z = 0;\quad (M_A)_z = 0$ **Ans**



*5-76. The pipe assembly supports the vertical loads shown. Determine the components of reaction at the ball-and-socket joint A and the tension in the supporting cables BC and BD.



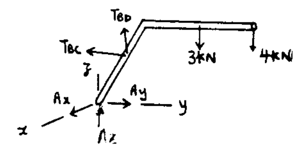
$$\mathbf{T}_{BD} = T_{BD}\left(\frac{-2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$\mathbf{T}_{BC} = T_{BC}\left(\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$\Sigma M_x = 0;\quad -3(4) - 4(5.5) + \frac{2}{3}T_{BD}(1) + \frac{2}{3}T_{BC}(1) + \frac{1}{3}T_{BD}(1) + \frac{1}{3}T_{BC}(1) = 0$

$T_{BD} + T_{BC} = 34$

$\Sigma M_y = 0;\quad \frac{2}{3}T_{BC}(1) - \frac{2}{3}T_{BD} = 0$



$T_{BC} = T_{BD}$

$T_{BC} = T_{BD} = 17\text{ kN}$ **Ans**

$\Sigma F_y = 0;\quad A_y - 17(\frac{1}{3}) - 17(\frac{1}{3}) = 0$

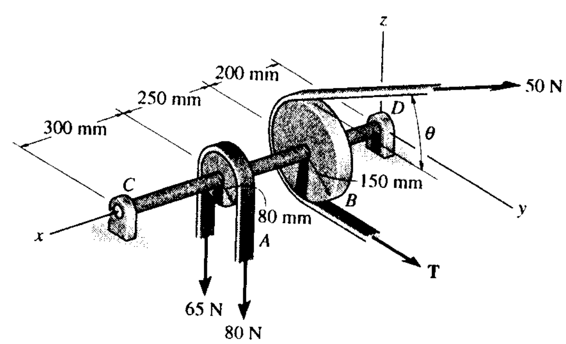
$A_y = 11.3\text{ kN}$ **Ans**

$\Sigma F_x = 0;\quad A_x = 0$ **Ans**

$\Sigma F_z = 0;\quad A_z + 17(\frac{2}{3}) + 17(\frac{2}{3}) - 3 - 4 = 0$

$A_z = -15.7\text{ kN}$ **Ans**

5-77. Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley A is transmitted to pulley B. Determine the horizontal tension $\mathbf{T}$ in the belt on pulley B and the x, y, z components of reaction at the journal bearing C and thrust bearing D if $\theta = 0°$. The bearings are in proper alignment and exert only force reactions on the shaft.



Equations of Equilibrium :

$\Sigma M_x = 0;\quad 65(0.08) - 80(0.08) + T(0.15) - 50(0.15) = 0$

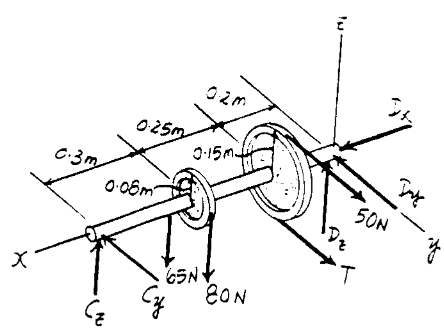
$T = 58.0\text{ N}$ **Ans**

$\Sigma M_y = 0;\quad (65 + 80)(0.45) - C_z(0.75) = 0$

$C_z = 87.0\text{ N}$ **Ans**

$\Sigma M_z = 0;\quad (50 + 58.0)(0.2) - C_y(0.75) = 0$

$C_y = 28.8\text{ N}$ **Ans**



$\Sigma F_x = 0;\quad D_x = 0$ **Ans**

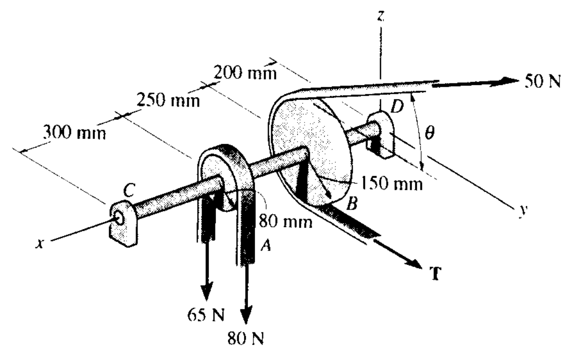
$\Sigma F_y = 0;\quad D_y + 28.8 - 50 - 58.0 = 0$

$D_y = 79.2\text{ N}$ **Ans**

$\Sigma F_z = 0;\quad D_z + 87.0 - 80 - 65 = 0$

$D_z = 58.0\text{ N}$ **Ans**

5-78. Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley A is transmitted to pulley B. Determine the horizontal tension **T** in the belt on pulley B and the x, y, z components of reaction at the journal bearing C and thrust bearing D if $\theta = 45°$. The bearings are in proper alignment and exert only force reactions on the shaft.



Equations of Equilibrium :

$\Sigma M_x = 0;\quad 65(0.08) - 80(0.08) + T(0.15) - 50(0.15) = 0$

$T = 58.0\text{ N}$ **Ans**

$\Sigma M_y = 0;\quad (65 + 80)(0.45) - 50\sin 45°(0.2) - C_z(0.75) = 0$

$C_z = 77.57\text{ N} = 77.6\text{ N}$ **Ans**

$\Sigma M_z = 0;\quad 58.0(0.2) + 50\cos 45°(0.2) - C_y(0.75) = 0$

$C_y = 24.89\text{ N} = 24.9\text{ N}$ **Ans**

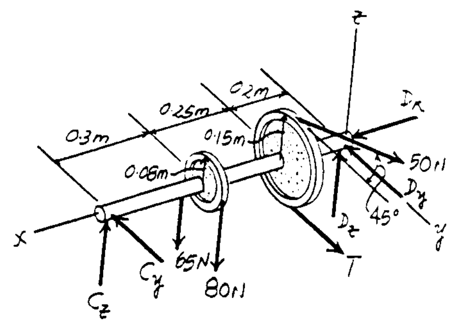
$\Sigma F_x = 0;\quad D_x = 0$ **Ans**

$\Sigma F_y = 0;\quad D_y + 24.89 - 50\cos 45° - 58.0 = 0$

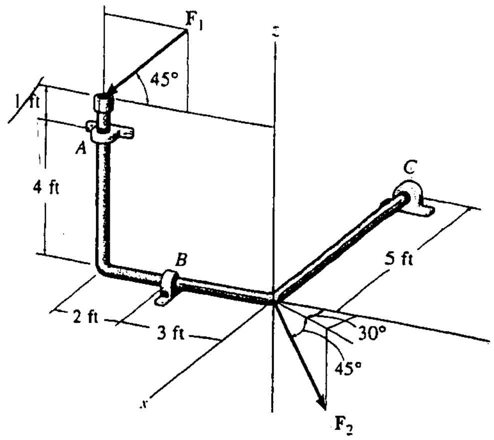
$D_y = 68.5\text{ N}$ **Ans**

$\Sigma F_z = 0;\quad D_z + 77.57 + 50\sin 45° - 80 - 65 = 0$

$D_z = 32.1\text{ N}$ **Ans**



5-79. The bent rod is supported at A, B, and C by smooth journal bearings. Compute the x, y, z components of reaction at the bearings if the rod is subjected to forces $F_1 = 300$ lb and $F_2 = 250$ lb. F_1 lies in the y-z plane. The bearings are in proper alignment and exert only force reactions on the rod.

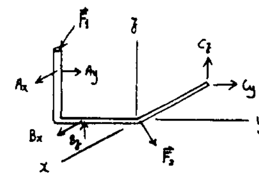


$\mathbf{F}_1 = (-300\cos 45°\mathbf{j} - 300\sin 45°\mathbf{k})$

$= \{-212.1\mathbf{j} - 212.1\mathbf{k}\}$ lb

$\mathbf{F}_2 = (250\cos 45°\sin 30°\mathbf{i} + 250\cos 45°\cos 30°\mathbf{j} - 250\sin 45°\mathbf{k})$

$= \{88.39\mathbf{i} + 153.1\mathbf{j} - 176.8\mathbf{k}\}$ lb



$\Sigma F_x = 0;\quad A_x + B_x + 88.39 = 0$

$\Sigma F_y = 0;\quad A_y + C_y - 212.1 + 153.1 = 0$

$\Sigma F_z = 0;\quad B_z + C_z - 212.1 - 176.8 = 0$

$\Sigma M_x = 0;\quad -B_z(3) - A_y(4) + 212.1(5) + 212.1(5) = 0$

$\Sigma M_y = 0;\quad C_z(5) + A_x(4) = 0$

$\Sigma M_z = 0;\quad A_x(5) + B_x(3) - C_y(5) = 0$

$A_x = 633$ lb **Ans**

$A_y = -141$ lb **Ans**

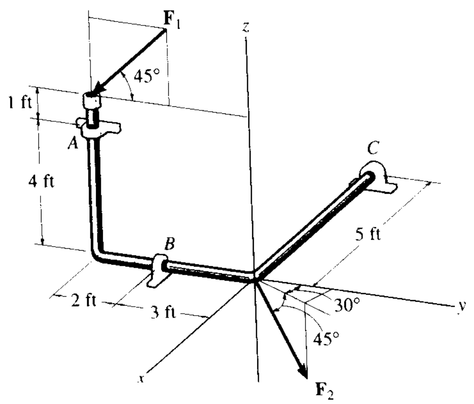
$B_x = -721$ lb **Ans**

$B_z = 895$ lb **Ans**

$C_y = 200$ lb **Ans**

$C_z = -506$ lb **Ans**

***5-80.** The bent rod is supported at A, B, and C by smooth journal bearings. Determine the magnitude of $\mathbf{F}_2$ which will cause the reaction $\mathbf{C}_y$ at the bearing C to be equal to zero. The bearings are in proper alignment and exert only force reactions on the rod. Set $F_1 = 300$ lb.

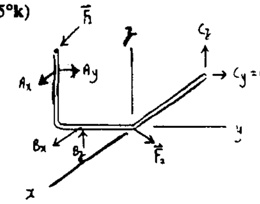


$$\mathbf{F}_1 = (-300\cos 45°\mathbf{j} - 300\sin 45°\mathbf{k})$$

$$= \{-212.1\mathbf{j} - 212.1\mathbf{k}\}\text{lb}$$

$$\mathbf{F}_2 = (F_2\cos 45°\sin 30°\mathbf{i} + F_2\cos 45°\cos 30°\mathbf{j} - F_2\sin 45°\mathbf{k})$$

$$= \{0.3536F_2\mathbf{i} + 0.6124F_2\mathbf{j} - 0.7071F_2\mathbf{k}\}\text{lb}$$



$\Sigma F_x = 0;\quad A_x + B_x + 0.3536F_2 = 0$

$\Sigma F_y = 0;\quad A_y + 0.6124F_2 - 212.1 = 0$

$\Sigma F_z = 0;\quad B_z + C_z - 0.7071F_2 - 212.1 = 0$

$\Sigma M_x = 0;\quad -B_z(3) - A_y(4) + 212.1(5) + 212.1(5) = 0$

$\Sigma M_y = 0;\quad C_z(5) + A_x(4) = 0$

$\Sigma M_z = 0;\quad A_x(5) + B_x(3) = 0$

$A_x = 357$ lb

$A_y = -200$ lb

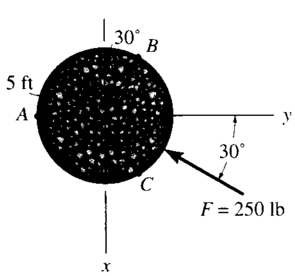
$B_x = -596$ lb

$B_z = 974$ lb

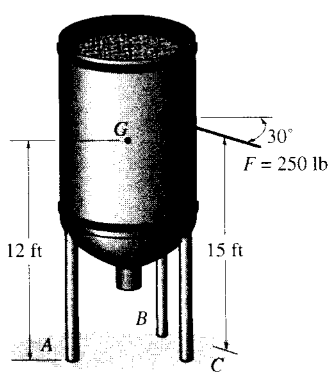
$C_z = -286$ lb

$F_2 = 674$ lb **Ans**

5-81. The silo has a weight of 3500 lb and a center of gravity at G. Determine the vertical component of force that each of the three struts at A, B, and C exerts on the silo if it is subjected to a resultant wind loading of 250 lb which acts in the direction shown.



Top view



Set the coordinate- axes system at the base of the silo with the origin at point O.

$\Sigma M_y = 0;\quad B_z(5\sin 60°) - C_z(5\sin 60°) - 250\sin 30°(15) = 0$

$4.330B_z - 4.330C_z - 1875 = 0 \qquad [1]$

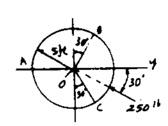
$\Sigma M_x = 0;\quad B_z(5\cos 60°) + C_z(5\cos 60°) - A_z(5) + 250\cos 30°(15) = 0$

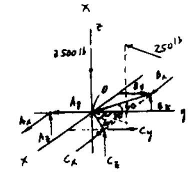
$2.5B_z + 2.5C_z - 5A_z + 3247.6 = 0 \qquad [2]$

$\Sigma F_z = 0;\quad A_z + B_z + C_z - 3500 = 0 \qquad [3]$

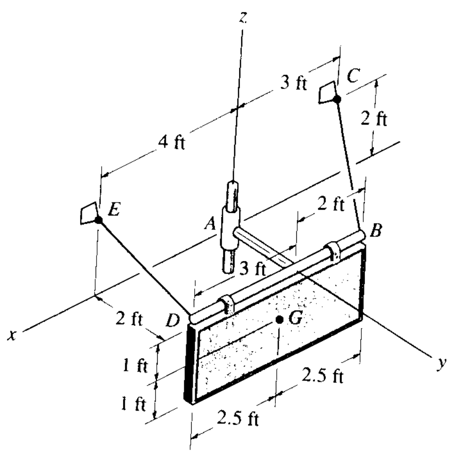
Solving Eqs.[1], [2] and [3] yields :

$B_z = 1167$ lb $\quad C_z = 734$ lb $\quad A_z = 1600$ lb **Ans**



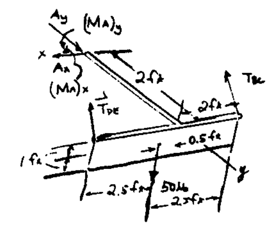


5-82. Determine the tensions in the cables and the components of reaction acting on the smooth collar at A necessary to hold the 50-lb sign in equilibrium. The center of gravity for the sign is at G.



$$\mathbf{T}_{DE} = T_{DE}\left(\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$\mathbf{T}_{BC} = T_{BC}\left(\frac{-1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$



$\Sigma F_x = 0;\quad \frac{1}{3}T_{DE} - \frac{1}{3}T_{BC} + A_x = 0$

$\Sigma F_z = 0;\quad \frac{2}{3}T_{DE} + \frac{2}{3}T_{BC} - 50 = 0$

$\Sigma F_y = 0;\quad -\frac{2}{3}T_{DE} - \frac{2}{3}T_{BC} + A_y = 0$

$\Sigma M_x = 0;\quad (M_A)_x + \frac{2}{3}T_{DE}(2) + \frac{2}{3}T_{BC}(2) - 50(2) = 0$

$\Sigma M_y = 0;\quad (M_A)_y - \frac{2}{3}T_{DE}(3) + \frac{2}{3}T_{BC}(2) + 50(0.5) = 0$

$\Sigma M_z = 0;\quad -\frac{1}{3}T_{DE}(2) - \frac{2}{3}T_{DE}(3) + \frac{1}{3}T_{BC}(2) + \frac{2}{3}T_{BC}(2) = 0$

Solving;

$T_{DE} = 32.1429 = 32.1\ \text{lb}$ **Ans**

$T_{BC} = 42.8571 = 42.9\ \text{lb}$ **Ans**

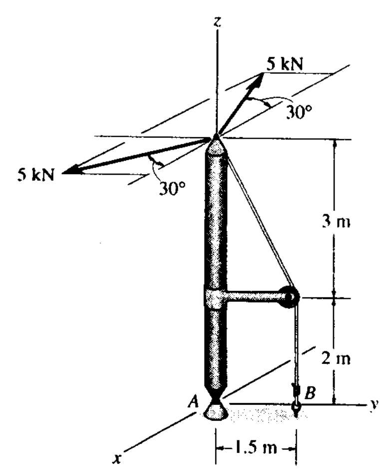
$A_x = 3.5714 = 3.57\ \text{lb}$ **Ans**

$A_y = 50\ \text{lb}$ **Ans**

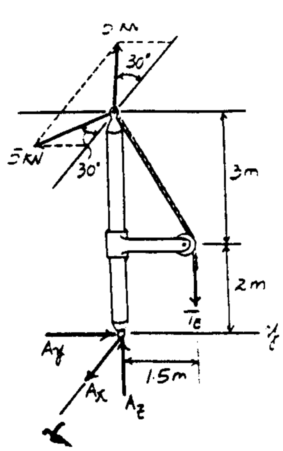
$(M_A)_x = 0$ **Ans**

$(M_A)_y = -17.8571 = -17.9\ \text{lb·ft}$ **Ans**

5-83. The boom is supported by a ball-and-socket joint at A and a guy wire at B. If the 5-kN loads lie in a plane which is parallel to the x–y plane, determine the x, y, z components of reaction at A and the tension in the cable at B.



Equations of Equilibrium :



$\Sigma M_x = 0;\quad 2[5\sin 30°(5)] - T_B(1.5) = 0$
$T_B = 16.67\ \text{kN} = 16.7\ \text{kN}$ **Ans**

$\Sigma M_y = 0;\quad 5\cos 30°(5) - 5\cos 30°(5) = 0$ ***(Statisfied!)***

$\Sigma F_x = 0;\quad A_x + 5\cos 30° - 5\cos 30° = 0$
$A_x = 0$ **Ans**

$\Sigma F_y = 0;\quad A_y - 2(5\sin 30°) = 0$
$A_y = 5.00\ \text{kN}$ **Ans**

$\Sigma F_z = 0;\quad A_z - 16.67 = 0 \quad A_z = 16.7\ \text{kN}$ **Ans**