CHAPTER 4

4.1 (a) For this case $x_i = 0$ and $h = x$. Thus,

$$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2} + \cdots$$

$$f(0) = f'(0) = f''(0) = e^0 = 1$$

$$f(x) = 1 + x + \frac{x^2}{2} + \cdots$$

(b)

$$f(x_{i+1}) = e^{-x_i} - e^{-x_i}h + e^{-x_i}\frac{h^2}{2} - e^{-x_i}\frac{h^3}{6} + \cdots$$

for $x_i = 0.2$, $x_{i+1} = 1$ and $h = 0.8$. True value = $e^{-1} = 0.367879$.

<u>zero order:</u>

$$f(1) = e^{-0.2} = 0.818731$$
$$\varepsilon_t = \left|\frac{0.367879 - 0.818731}{0.367879}\right| \times 100\% = 122.55\%$$

<u>first order:</u>

$$f(1) = 0.818731 - 0.818731(0.8) = 0.163746$$
$$\varepsilon_t = \left|\frac{0.367879 - 0.163746}{0.367879}\right| \times 100\% = 55.49\%$$

<u>second order:</u>

$$f(1) = 0.818731 - 0.818731(0.8) + 0.818731\frac{0.8^2}{2} = 0.42574$$
$$\varepsilon_t = \left|\frac{0.367879 - 0.42574}{0.367879}\right| \times 100\% = 15.73\%$$

<u>third order:</u>

$$f(1) = 0.818731 - 0.818731(0.8) + 0.818731\frac{0.8^2}{2} - 0.818731\frac{0.8^3}{6} = 0.355875$$
$$\varepsilon_t = \left|\frac{0.367879 - 0.355875}{0.367879}\right| \times 100\% = 3.26\%$$

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4.2 Use the stopping criterion

$$\varepsilon_s = 0.5 \times 10^{2-2}\% = 0.5\%$$

True value: $\cos(\pi/3) = 0.5$

<u>zero order:</u>

$$\cos\left(\frac{\pi}{3}\right) = 1$$

$$\varepsilon_t = \left|\frac{0.5 - 1}{0.5}\right| \times 100\% = 100\%$$

<u>first order:</u>

$$\cos\left(\frac{\pi}{3}\right) = 1 - \frac{(\pi/3)^2}{2} = 0.451689$$

$$\varepsilon_t = 9.66\% \qquad \varepsilon_a = \left|\frac{0.451689 - 1}{0.451689}\right| \times 100\% = 121.4\%$$

<u>second order:</u>

$$\cos\left(\frac{\pi}{3}\right) = 0.451689 + \frac{(\pi/3)^4}{24} = 0.501796$$

$$\varepsilon_t = 0.359\% \qquad \varepsilon_a = \left|\frac{0.501796 - 0.451689}{0.501796}\right| \times 100\% = 9.986\%$$

<u>third order:</u>

$$\cos\left(\frac{\pi}{3}\right) = 0.501796 - \frac{(\pi/3)^6}{720} = 0.499965$$

$$\varepsilon_t = 0.00709\% \qquad \varepsilon_a = \left|\frac{0.499965 - 0.501796}{0.499965}\right| \times 100\% = 0.366\%$$

Since the approximate error is below 0.5%, the computation can be terminated.

4.3 Use the stopping criterion: $\varepsilon_s = 0.5 \times 10^{2-2}\% = 0.5\%$

True value: $\sin(\pi/3) = 0.866025\ldots$

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<u>zero order:</u>

$$\sin\left(\frac{\pi}{3}\right)=\frac{\pi}{3}=1.047198$$

$$\varepsilon_t=\left|\frac{0.866025-1.047198}{0.866025}\right|\times 100\%=20.92\%$$

<u>first order:</u>

$$\sin\left(\frac{\pi}{3}\right)=1.047198-\frac{(\pi/3)^3}{6}=0.855801$$

$$\varepsilon_t=1.18\% \qquad \varepsilon_a=\left|\frac{0.855801-1.047198}{0.855801}\right|\times 100\%=22.36\%$$

<u>second order:</u>

$$\sin\left(\frac{\pi}{3}\right)=0.855801+\frac{(\pi/3)^5}{120}=0.866295$$

$$\varepsilon_t=0.031\% \qquad \varepsilon_a=\left|\frac{0.866295-0.855801}{0.866295}\right|\times 100\%=1.211\%$$

<u>third order:</u>

$$\sin\left(\frac{\pi}{3}\right)=0.866295-\frac{(\pi/3)^7}{5040}=0.866021$$

$$\varepsilon_t=0.000477\% \qquad \varepsilon_a=\left|\frac{0.866021-0.866295}{0.866021}\right|\times 100\%=0.0316\%$$

Since the approximate error is below 0.5%, the computation can be terminated.

4.4 True value: $f(3) = 554$.

<u>zero order:</u>

$$f(3)=f(1)=-62$$

$$\varepsilon_t=\left|\frac{554-(-62)}{554}\right|\times 100\%=111.191\%$$

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first order:

$$f(3) = -62 + f'(1)(3-1) = -62 + 70(2) = 78 \qquad \varepsilon_t = 85.921\%$$

second order:

$$f(3) = 78 + \frac{f''(1)}{2}(3-1)^2 = 78 + \frac{138}{2}4 = 354 \qquad \varepsilon_t = 36.101\%$$

third order:

$$f(3) = 354 + \frac{f^{(3)}(1)}{6}(3-1)^3 = 354 + \frac{150}{6}8 = 554 \qquad \varepsilon_t = 0\%$$

Thus, the third-order result is perfect because the original function is a third-order polynomial.

4.5 True value: $f(2.5) = \ln(2.5) = 0.916291...$

zero order:

$$f(2.5) = f(1) = 0$$

$$\varepsilon_t = \left|\frac{0.916291 - 0}{0.916291}\right| \times 100\% = 100\%$$

first order:

$$f(2.5) = f(1) + f'(1)(2.5-1) = 0 + 1(1.5) = 1.5$$

$$\varepsilon_t = \left|\frac{0.916291 - 1.5}{0.916291}\right| \times 100\% = 63.704\%$$

second order:

$$f(2.5) = 1.5 + \frac{f''(1)}{2}(2.5-1)^2 = 1.5 + \frac{-1}{2}1.5^2 = 0.375$$

$$\varepsilon_t = \left|\frac{0.916291 - 0.375}{0.916291}\right| \times 100\% = 59.074\%$$

third order:

$$f(2.5) = 0.375 + \frac{f^{(3)}(1)}{6}(2.5-1)^3 = 0.375 + \frac{2}{6}1.5^3 = 1.5$$

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$$\varepsilon_t = \left|\frac{0.916291 - 1.5}{0.916291}\right| \times 100\% = 63.704\%$$

<u>fourth order:</u>

$$f(2.5) = 1.5 + \frac{f^{(4)}(1)}{24}(2.5-1)^4 = 1.5 + \frac{-6}{24}1.5^4 = 0.234375$$

$$\varepsilon_t = \left|\frac{0.916291 - 0.234375}{0.916291}\right| \times 100\% = 74.421\%$$

Thus, the process seems to be diverging suggesting that a smaller step would be required for convergence.

4.6 True value:

$$f'(x) = 75x^2 - 12x + 7$$

$$f'(2) = 75(2)^2 - 12(2) + 7 = 283$$

function values:

$x_{i-1} = 1.8$	$f(x_{i-1}) = 50.96$
$x_i = 2$	$f(x_i) = 102$
$x_{i+1} = 2.2$	$f(x_{i+1}) = 164.56$

<u>forward:</u>

$$f'(2) = \frac{164.56 - 102}{0.2} = 312.8$$

$$\varepsilon_t = \left|\frac{283 - 312.8}{283}\right| \times 100\% = 10.53\%$$

<u>backward:</u>

$$f'(2) = \frac{102 - 50.96}{0.2} = 255.2$$

$$\varepsilon_t = \left|\frac{283 - 255.2}{283}\right| \times 100\% = 9.823\%$$

<u>centered:</u>

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$$f'(2)=\frac{164.56-50.96}{2(0.2)}=284$$

$$\varepsilon_t=\left|\frac{283-284}{283}\right|\times 100\%=0.353\%$$

Both the forward and backward have errors that can be approximated by (recall Eq. 4.15),

$$|E_t|\approx\frac{f''(x_i)}{2}h$$

$$f''(2)=150x-12=150(2)-12=288$$

$$|E_t|\approx\frac{288}{2}0.2=28.8$$

This is very close to the actual error that occurred in the approximations

forward: $|E_t|\approx|283-312.8|=29.8$
backward: $|E_t|\approx|283-255.2|=27.8$

The centered approximation has an error that can be approximated by,

$$E_t\approx-\frac{f^{(3)}(x_i)}{6}h^2=-\frac{150}{6}0.2^2=-1$$

which is exact: E_t = 283 – 284 = –1. This result occurs because the original function is a cubic equation which has zero fourth and higher derivatives.

4.7 True value:

$$f''(x)=150x-12$$

$$f''(2)=150(2)-12=288$$

<u>h = 0.25:</u>

$$f''(2)=\frac{f(2.25)-2f(2)+f(1.75)}{0.25^2}=\frac{182.1406-2(102)+39.85938}{0.25^2}=288$$

<u>h = 0.125:</u>

$$f''(2)=\frac{f(2.125)-2f(2)+f(1.875)}{0.125^2}=\frac{139.6738-2(102)+68.82617}{0.125^2}=288$$

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Both results are exact because the errors are a function of 4th and higher derivatives which are zero for a 3rd-order polynomial.

4.8

$$\frac{\partial v}{\partial c} = \frac{cgte^{-(c/m)t} - gm\left(1 - e^{-(c/m)t}\right)}{c^2} = -1.38666$$

$$\Delta v(\tilde{c}) = \left|\frac{\partial v}{\partial c}\right| \Delta\tilde{c} = 1.38666(1.5) = 2.079989$$

$$v(12.5) = \frac{9.8(50)}{12.5}\left(1 - e^{-12.5(6)/50}\right) = 30.4533$$

$$v = 30.4533 \pm 2.079989$$

Thus, the bounds computed with the first-order analysis range from 28.3733 to 32.5333. This result can be verified by computing the exact values as

$$v(c - \Delta c) = \frac{9.8(50)}{11}\left(1 - e^{-(11/50)6}\right) = 32.6458$$

$$v(c + \Delta c) = \frac{9.8(50)}{14}\left(1 - e^{-(14/50)6}\right) = 28.4769$$

Thus, the range of ±2.0844 is close to the first-order estimate.

4.9

$$v(12.5) = \frac{9.8(50)}{12.5}\left(1 - e^{-12.5(6)/50}\right) = 30.4533$$

$$\Delta v(\tilde{c}, \tilde{m}) = \left|\frac{\partial v}{\partial c}\right| \Delta\tilde{c} + \left|\frac{\partial v}{\partial m}\right| \Delta\tilde{m}$$

$$\frac{\partial v}{\partial c} = \frac{cgte^{-(c/m)t} - gm\left(1 - e^{-(c/m)t}\right)}{c^2} = -1.38666$$

$$\frac{\partial v}{\partial m} = \frac{gt}{m} e^{-(c/m)t} + \frac{g}{c}\left(1 - e^{-(c/m)t}\right) = 0.871467$$

$$\Delta v(\tilde{c}, \tilde{m}) = |-1.38666|(1.5) + |0.871467|(2) = 2.079989 + 1.742934 = 3.822923$$

$$v = 30.4533 \pm 3.822923$$

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4.10 For $\Delta\tilde{T}$ =20,

$$\Delta H(\tilde{T}) = \left|\frac{\partial H}{\partial T}\right| \Delta\tilde{T}$$

$$\frac{\partial H}{\partial T} = 4Ae\sigma T^3 = 4(0.15)0.9(5.67\times 10^{-8})650^3 = 8.408$$

$$\Delta H(\tilde{T}) = 8.408(20) = 168.169$$

Exact error:

$$\Delta H_{\text{true}} = \frac{H(670) - H(630)}{2} = \frac{1542.468 - 1205.81}{2} = 168.3286$$

Thus, the first-order approximation is close to the exact result.

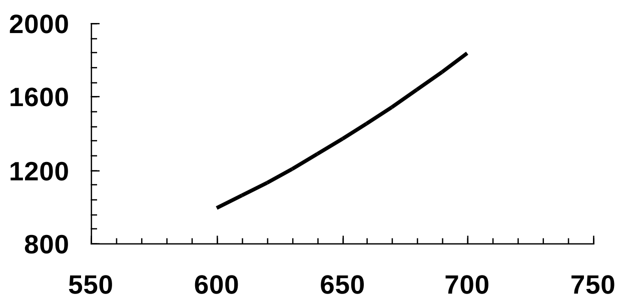
For $\Delta\tilde{T}$ =40,

$$\Delta H(\tilde{T}) = 8.408(40) = 336.3387$$

Exact error:

$$\Delta H_{\text{true}} = \frac{H(690) - H(610)}{2} = \frac{1735.055 - 1059.83}{2} = 337.6124$$

Again, the first-order approximation is close to the exact result. The results are good because $H(T)$ is nearly linear over the ranges we are examining. This is illustrated by the following plot.



4.11 For a sphere, $A = 4\pi r^2$. Therefore,

$$H = 4\pi r^2 e\sigma T^4$$

At the mean values of the parameters,

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$$H(0.15, 0.9, 550) = 4\pi(0.15)^2\, 0.90(5.67\times 10^{-8})(550)^4 = 1320.288$$

$$\Delta H = \left|\frac{\partial H}{\partial r}\right|\Delta\tilde{r} + \left|\frac{\partial H}{\partial e}\right|\Delta\tilde{e} + \left|\frac{\partial H}{\partial T}\right|\Delta\tilde{T}$$

$$\frac{\partial H}{\partial r} = 8\pi r e\sigma T^4 = 17{,}603.84$$

$$\frac{\partial H}{\partial e} = 4\pi r^2\sigma T^4 = 1466.987$$

$$\frac{\partial H}{\partial T} = 16\pi r^2 e\sigma T^3 = 9.6021$$

$$\Delta H = 17603.84(0.01) + 1466.987(0.05) + 9.6021(20) = 441.4297$$

To check this result, we can compute

$$H(0.14, 0.85, 530) = 4\pi(0.14)^2\, 0.85(5.67\times 10^{-8})(530)^4 = 936.6372$$

$$H(0.16, 0.95, 570) = 4\pi(0.16)^2\, 0.95(5.67\times 10^{-8})(570)^4 = 1829.178$$

$$\Delta H_{\text{true}} = \frac{1829.178 - 936.6372}{2} = 446.2703$$

4.12 The condition number is computed as

$$CN = \frac{\tilde{x} f'(\tilde{x})}{f(\tilde{x})}$$

(a) $$CN = \frac{1.00001\left[\dfrac{1}{2\sqrt{1.00001-1}}\right]}{\sqrt{1.00001-1}+1} = \frac{1.00001(158.1139)}{1.003162} = 157.617$$

The result is ill-conditioned because the derivative is large near $x = 1$.

(b) $$CN = \frac{10(-e^{-10})}{e^{-10}} = \frac{10(-4.54\times 10^{-5})}{4.54\times 10^{-5}} = -10$$

The result is ill-conditioned because x is large.

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$$\textbf{(c)}\ CN = \frac{300\left[\dfrac{300}{\sqrt{300^2+1}} - 1\right]}{\sqrt{300^2+1} - 300} = \frac{300(-5.555556 \times 10^{-6})}{0.0016667} = -0.99999444$$

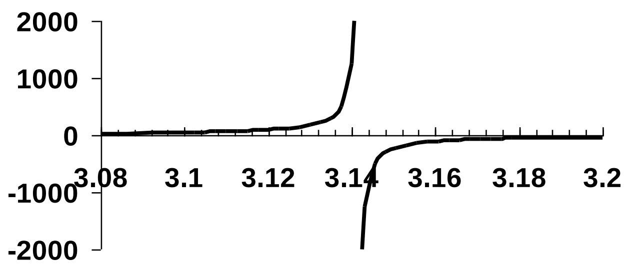
The result is well-conditioned.

$$\textbf{(d)}\ CN = \frac{x\dfrac{-xe^{-x} - e^{-x} + 1}{x^2}}{\left(\dfrac{e^{-x}-1}{x}\right)} = \frac{0.001(0.499667)}{-0.9995} = -0.0005$$

The result is well-conditioned.

$$\textbf{(e)}\ CN = \frac{x\dfrac{(1+\cos x)\cos x + \sin x(\sin x)}{(1+\cos x)^2}}{\dfrac{\sin x}{1+\cos x}} = \frac{3.141907(20{,}264{,}237)}{-6366.2} = -10{,}001$$

The result is ill-conditioned because, as in the following plot, the function has a singularity at $x = \pi$.



4.13 Addition and subtraction:

$$f(u,v) = u + v$$

$$\Delta f = \left|\frac{\partial f}{\partial u}\right|\Delta\tilde{u} + \left|\frac{\partial f}{\partial v}\right|\Delta\tilde{v}$$

$$\left|\frac{\partial f}{\partial u}\right| = 1 \qquad \left|\frac{\partial f}{\partial v}\right| = 1$$

$$f(\tilde{u},\tilde{v}) = \Delta\tilde{u} + \Delta\tilde{v}$$

Multiplication:

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$$f(u,v) = u \cdot v$$

$$\left|\frac{\partial f}{\partial u}\right| = v \qquad \left|\frac{\partial f}{\partial v}\right| = u$$

$$f(\tilde{u},\tilde{v}) = |\tilde{v}|\Delta\tilde{u} + |\tilde{u}|\Delta\tilde{v}$$

Division:

$$f(u,v) = u / v$$

$$\left|\frac{\partial f}{\partial u}\right| = \frac{1}{v} \qquad \left|\frac{\partial f}{\partial v}\right| = \frac{u}{v^2}$$

$$f(\tilde{u},\tilde{v}) = \left|\frac{1}{v}\right|\Delta\tilde{u} + \left|\frac{u}{v^2}\right|\Delta\tilde{v}$$

$$f(\tilde{u},\tilde{v}) = \frac{|v|\Delta\tilde{u} + |u|\Delta\tilde{v}}{|v^2|}$$

4.14

$$f(x) = ax^2 + bx + c$$

$$f'(x) = 2ax + b$$

$$f''(x) = 2a$$

Substitute these relationships into Eq. (4.4),

$$ax_{i+1}^2 + bx_{i+1} + c = ax_i^2 + bx_i + c + (2ax_i + b)(x_{i+1} - x_i) + \frac{2a}{2!}(x_{i+1}^2 - 2x_{i+1}x_i + x_i^2)$$

Collect terms

$$ax_{i+1}^2 + bx_{i+1} + c = ax_i^2 + 2ax_i(x_{i+1} - x_i) + a(x_{i+1}^2 - 2x_{i+1}x_i + x_i^2) + bx_i + b(x_{i+1} - x_i) + c$$

$$ax_{i+1}^2 + bx_{i+1} + c = ax_i^2 + 2ax_i x_{i+1} - 2ax_i^2 + ax_{i+1}^2 - 2ax_{i+1}x_i + ax_i^2 + bx_i + bx_{i+1} - bx_i + c$$

$$ax_{i+1}^2 + bx_{i+1} + c = (ax_i^2 - 2ax_i^2 + ax_i^2) + ax_{i+1}^2 + (2ax_i x_{i+1} - 2ax_{i+1}x_i) + (bx_i - bx_i) + bx_{i+1} + c$$

$$ax_{i+1}^2 + bx_{i+1} + c = ax_{i+1}^2 + bx_{i+1} + c$$

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4.15 The first-order error analysis can be written as

$$\Delta Q = \left|\frac{\partial Q}{\partial n}\right|\Delta n + \left|\frac{\partial Q}{\partial S}\right|\Delta S$$

$$\frac{\partial Q}{\partial n} = -\frac{1}{n^2}\frac{(BH)^{5/3}}{(B+2H)^{2/3}}S^{0.5} = -50.74 \qquad \frac{\partial Q}{\partial S} = \frac{1}{n}\frac{(BH)^{5/3}}{(B+2H)^{2/3}}\frac{1}{2S^{0.5}} = 2536.9$$

$$\Delta Q = |-50.74|0.003 + |2536.9|0.00003 = 0.152 + 0.076 = 0.228$$

The error from the roughness is about 2 times the error caused by the uncertainty in the slope. Thus, improving the precision of the roughness measurement would be the best strategy.

4.16 Use the stopping criterion

$$\varepsilon_s = 0.5 \times 10^{2-2}\% = 0.5\%$$

True value: 1/(1 – 0.1) = 1.111111….

<u>zero order:</u>

$$\frac{1}{1-x} = 1$$

$$\varepsilon_t = \left|\frac{1.11111 - 1}{1.11111}\right| \times 100\% = 10\%$$

<u>first order:</u>

$$\frac{1}{1-x} = 1 + 0.1 = 1.1$$

$$\varepsilon_t = 1\% \qquad \varepsilon_a = \left|\frac{1.1-1}{1.1}\right| \times 100\% = 9.0909\%$$

<u>second order:</u>

$$\frac{1}{1-x} = 1 + 0.1 + 0.01 = 1.11$$

$$\varepsilon_t = 0.1\% \qquad \varepsilon_a = \left|\frac{1.11-1.1}{1.11}\right| \times 100\% = 0.9009009\%$$

<u>third order:</u>

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$$\frac{1}{1-x}=1+0.1+0.01+0.001=1.111$$

$$\varepsilon_t=0.01\% \qquad \varepsilon_a=\left|\frac{1.111-1.11}{1.111}\right|\times 100\%=0.090009\%$$

Since the approximate error is below 0.5%, the computation can be terminated.

4.17

$$\Delta(\sin\phi_0)=\left|\frac{d\sin\phi_0}{d\alpha}\right|\Delta\alpha$$

$$\frac{d\sin\phi_0}{d\alpha}=\frac{-\beta}{2\sqrt{(1+\alpha)(1+\alpha-\alpha\beta)}}+\sqrt{1-\frac{\alpha\beta}{1+\alpha}}$$

where $\beta=(v_e/v_0)^2=4$ and $\alpha=0.25$ to give,

$$\frac{d\sin\phi_0}{d\alpha}=\frac{-4}{2\sqrt{(1+0.25)(1+0.25-0.25(4))}}+\sqrt{1-\frac{0.25(4)}{1+0.25}}=-3.1305$$

$$\Delta(\sin\phi_0)=3.1305\Delta\alpha$$

For $\Delta\alpha=0.25(0.02)=0.005$,

$$\Delta(\sin\phi_0)=3.1305(0.005)=0.015652$$

$$\sin\phi_0=(1+0.25)\sqrt{1-\frac{0.25}{1+0.25}4}=0.559017$$

Therefore,

$$\max\sin\phi_0=0.559017+0.015652=0.574669$$

$$\min\sin\phi_0=0.559017-0.015652=0.543365$$

$$\max\phi_0=\arcsin(0.574669)\times\frac{180}{\pi}=35.076^\circ$$

$$\min\phi_0=\arcsin(0.543365)\times\frac{180}{\pi}=32.913^\circ$$

4.18 The derivatives can be computed as

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$$f(x) = x - 1 - 0.5\sin x$$

$$f'(x) = 1 - 0.5\cos x$$

$$f''(x) = 0.5\sin x$$

$$f^{(3)}(x) = 0.5\cos x$$

$$f^{(4)}(x) = -0.5\sin x$$

The first through fourth-order Taylor series expansions can be computed based on Eq. 4.5 as

First-order:

$$f_1(x) = f(a) + f'(a)(x-a)$$

$$f_1(x) = \frac{\pi}{2} - 1 - 0.5\sin\frac{\pi}{2} + \left[1 - 0.5\cos\frac{\pi}{2}\right]\left(x - \frac{\pi}{2}\right) = x - 1.5$$

Second-order:

$$f_2(x) = f_1(x) + \frac{f''(a)}{2}(x-a)^2$$

$$f_2(x) = x - 1.5 + 0.25\sin(\pi/2)(x - \pi/2)^2$$

Third-order:

$$f_3(x) = f_2(x) + \frac{f^{(3)}(a)}{6}(x-a)^3$$

$$f_3(x) = x - 1.5 + 0.25\sin(\pi/2)(x - \pi/2)^2 + \frac{0.5\cos(\pi/2)}{6}(x-a)^3$$

$$f_3(x) = x - 1.5 + 0.25\sin(\pi/2)(x - \pi/2)^2$$

Fourth-order:

$$f_4(x) = f_3(x) + \frac{f^{(4)}(a)}{24}(x-a)^4$$

$$f_4(x) = x - 1.5 + 0.25\sin(\pi/2)(x - \pi/2)^2 - \frac{0.5\sin(\pi/2)}{24}(x - \pi/2)^4$$

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$$f_4(x) = x - 1.5 + 0.25\sin(\pi/2)(x - \pi/2)^2 - \frac{1}{48}(x - \pi/2)^4$$

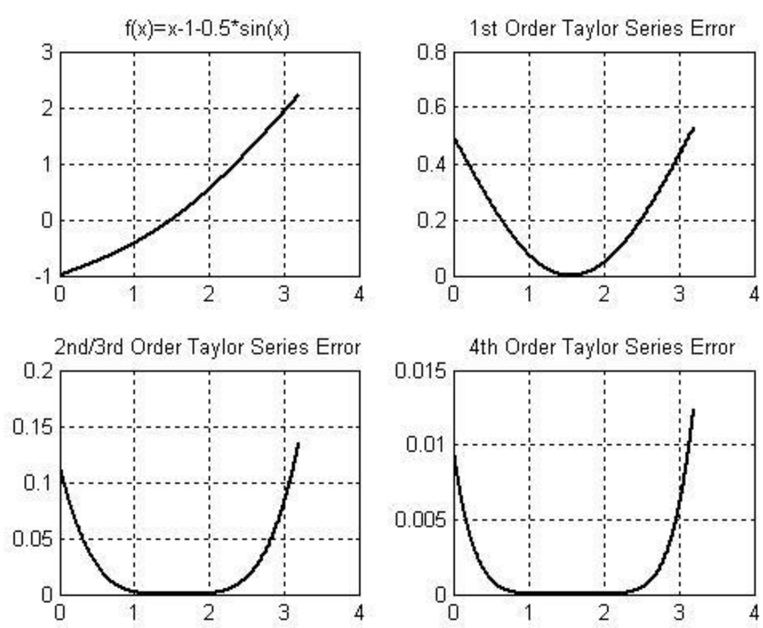
Note the 2nd and 3rd Order Taylor Series functions are the same. The following MATLAB script file which implements and plots each of the functions indicates that the 4th-order expansion satisfies the problem requirements.

```
x=0:0.001:3.2;
f=x-1-0.5*sin(x);
subplot(2,2,1);
plot(x,f);grid;title('f(x)=x-1-0.5*sin(x)');hold on

f1=x-1.5;
e1=abs(f-f1);         %Calculates the absolute value of the
difference/error
subplot(2,2,2);
plot(x,e1);grid;title('1st Order Taylor Series Error');

f2=x-1.5+0.25.*((x-0.5*pi).^2);
e2=abs(f-f2);
subplot(2,2,3);
plot(x,e2);grid;title('2nd/3rd Order Taylor Series Error');

f4=x-1.5+0.25.*((x-0.5*pi).^2)-(1/48)*((x-0.5*pi).^4);
e4=abs(f4-f);
subplot(2,2,4);
plot(x,e4);grid;title('4th Order Taylor Series Error');hold off
```



4.19 Here are Excel worksheets and charts that have been set up to solve this problem:

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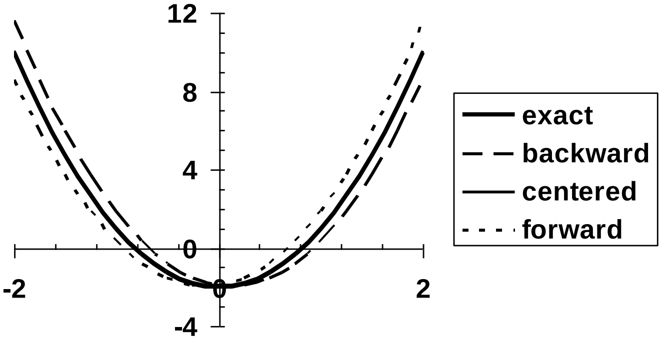
	A	B	C	D	E	F	G	H
1	dx	0.25						
2								
3	First	f(x)	f(x-dx)	f(x+dx)	f'(x)-exact	f'(x)-back	f'(x)-cent	f'(x)-forw
4	-2	0	-2.89063	2.140625	10	11.5625	10.0625	8.5625
5	-1.75	2.140625	0	3.625	7.1875	8.5625	7.25	5.9375
6	-1.5	3.625	2.140625	4.546875	4.75	5.9375	4.8125	3.6875
7	-1.25	4.546875	3.625	5	2.6875	3.6875	2.75	1.8125
8	-1	5	4.546875	5.078125	1	1.8125	1.0625	0.3125
9	-0.75	5.078125	5	4.875	-0.3125	0.3125	-0.25	-0.8125
10	-0.5	4.875	5.078125	4.484375	-1.25	-0.8125	-1.1875	-1.5625
11	-0.25	4.484375	4.875	4	-1.8125	-1.5625	-1.75	-1.9375
12	0	4	4.484375	3.515625	-2	-1.9375	-1.9375	-1.9375
13	0.25	3.515625	4	3.125	-1.8125	-1.9375	-1.75	-1.5625
14	0.5	3.125	3.515625	2.921875	-1.25	-1.5625	-1.1875	-0.8125
15	0.75	2.921875	3.125	3	-0.3125	-0.8125	-0.25	0.3125
16	1	3	2.921875	3.453125	1	0.3125	1.0625	1.8125
17	1.25	3.453125	3	4.375	2.6875	1.8125	2.75	3.6875
18	1.5	4.375	3.453125	5.859375	4.75	3.6875	4.8125	5.9375
19	1.75	5.859375	4.375	8	7.1875	5.9375	7.25	8.5625
20	2	8	5.859375	10.89063	10	8.5625	10.0625	11.5625



	A	B	C	D	E	F	G	H	I	J
1	dx	0.25								
2										
3	x	f(x)	f(x-dx)	f(x+dx)	f(x-2dx)	f(x+2dx)	f''(x)-exact	f''(x)-back	f''(x)-cent	f''(x)-forw
4	-2	0	-2.89063	2.140625	-6.625	3.625	-12	-13.5	-12	-10.5
5	-1.75	2.140625	0	3.625	-2.89063	4.546875	-10.5	-12	-10.5	-9
6	-1.5	3.625	2.140625	4.546875	0	5	-9	-10.5	-9	-7.5
7	-1.25	4.546875	3.625	5	2.140625	5.078125	-7.5	-9	-7.5	-6
8	-1	5	4.546875	5.078125	3.625	4.875	-6	-7.5	-6	-4.5
9	-0.75	5.078125	5	4.875	4.546875	4.484375	-4.5	-6	-4.5	-3
10	-0.5	4.875	5.078125	4.484375	5	4	-3	-4.5	-3	-1.5
11	-0.25	4.484375	4.875	4	5.078125	3.515625	-1.5	-3	-1.5	0
12	0	4	4.484375	3.515625	4.875	3.125	0	-1.5	0	1.5
13	0.25	3.515625	4	3.125	4.484375	2.921875	1.5	0	1.5	3
14	0.5	3.125	3.515625	2.921875	4	3	3	1.5	3	4.5
15	0.75	2.921875	3.125	3	3.515625	3.453125	4.5	3	4.5	6
16	1	3	2.921875	3.453125	3.125	4.375	6	4.5	6	7.5
17	1.25	3.453125	3	4.375	2.921875	5.859375	7.5	6	7.5	9
18	1.5	4.375	3.453125	5.859375	3	8	9	7.5	9	10.5
19	1.75	5.859375	4.375	8	3.453125	10.89063	10.5	9	10.5	12
20	2	8	5.859375	10.89063	4.375	14.625	12	10.5	12	13.5

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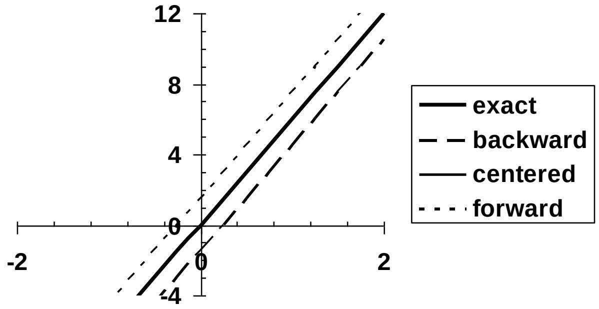

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