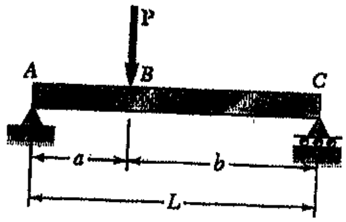
CHAPTER 5

PROBLEM 5.1

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



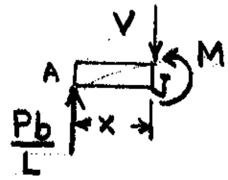
SOLUTION

Reactions

$\Sigma M_C = 0 \quad LA - bP = 0 \quad A = \frac{Pb}{L}$

$\Sigma M_A = 0 \quad LC - aP = 0 \quad C = \frac{Pa}{L}$

From A to B $\quad 0 < x < a$



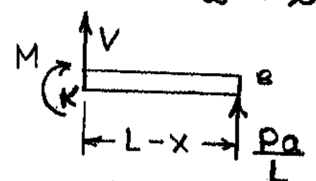
$+\uparrow \Sigma F_y = 0 \quad \frac{Pb}{L} - V = 0$

$V = \frac{Pb}{L}$ ◄

$\circlearrowleft \Sigma M_J = 0 \quad M - \frac{Pb}{L}x = 0$

$M = \frac{Pbx}{L}$ ◄

From B to C $\quad a < x < L$

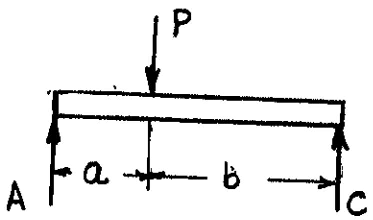


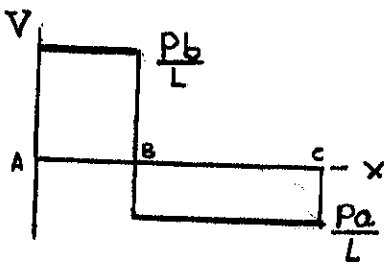
$+\uparrow \Sigma F_y = 0 \quad V + \frac{Pa}{L} = 0$

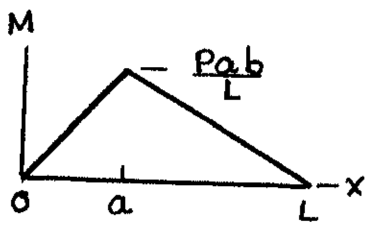
$V = -\frac{Pa}{L}$ ◄

$\circlearrowleft \Sigma M_K = 0 \quad -M + \frac{Pa}{L}(L - x) = 0$

$M = \frac{Pa(L-x)}{L}$ ◄



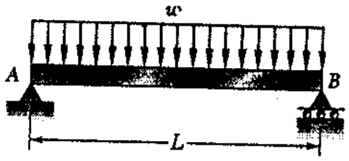




At section B $\quad M = \frac{Pab}{L}$ ◄

PROBLEM 5.2

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

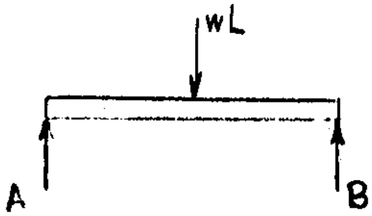


SOLUTION

Reactions

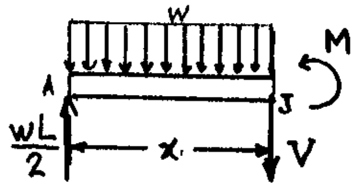
$\circlearrowleft \Sigma M_B = 0 \qquad -AL + wL \cdot \frac{L}{2} = 0 \qquad A = \frac{wL}{2}$

$\circlearrowleft \Sigma M_A = 0 \qquad BL - wL \cdot \frac{L}{2} = 0 \qquad B = \frac{wL}{2}$

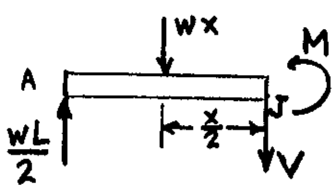


Free body diagram for determining reactions

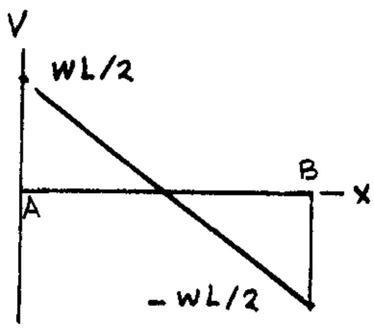
Over whole beam $\quad 0 < x < L$



Place section at x.



Replace distributed load by equivalent concentrated load.



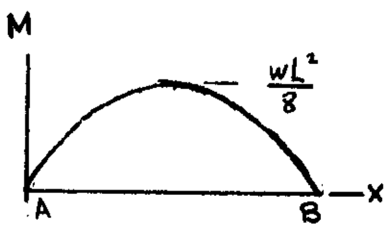
$+\uparrow \Sigma F_y = 0 \qquad \frac{wL}{2} - wx - V = 0$

$$V = w\left(\frac{L}{2} - x\right) \blacktriangleleft$$

$\circlearrowleft \Sigma M_J = 0 \qquad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x) \blacktriangleleft$$

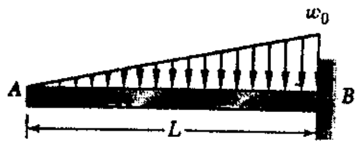


Maximum bending moment occurs at $x = \frac{L}{2}$.

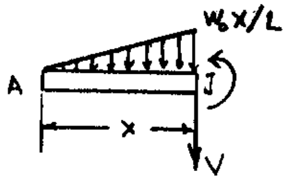
$$M_{max} = \frac{wL^2}{8} \blacktriangleleft$$

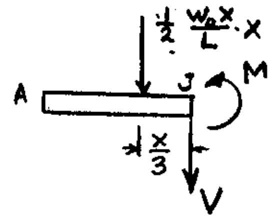
PROBLEM 5.3

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION





$+\uparrow \Sigma F_y = 0 \qquad -\frac{1}{2}\frac{w_0 x}{L}\cdot x - V = 0$

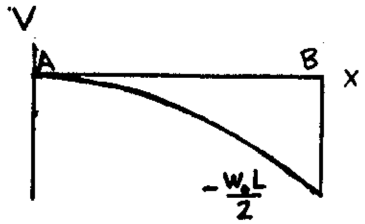
$$V = -\frac{w_0 x^2}{2L}$$

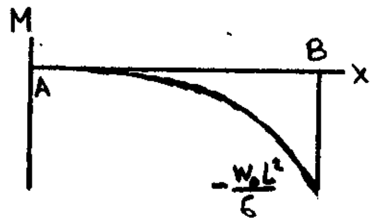
$\circlearrowleft \Sigma M_J = 0 \qquad \frac{1}{2}\frac{w_0 x}{L}\cdot x \cdot \frac{x}{3} + M = 0$

$$M = -\frac{w_0 x^3}{6L}$$

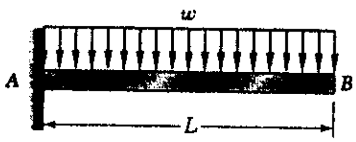
At $x = L \qquad V = -\frac{w_0 L}{2} \qquad |V|_{max} = \frac{w_0 L}{2}$

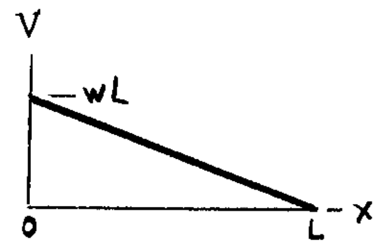
$M = -\frac{w_0 L^2}{6} \qquad |M|_{max} = \frac{w_0 L^2}{6}$

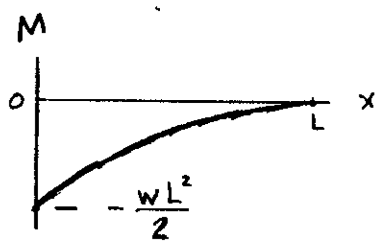




PROBLEM 5.4



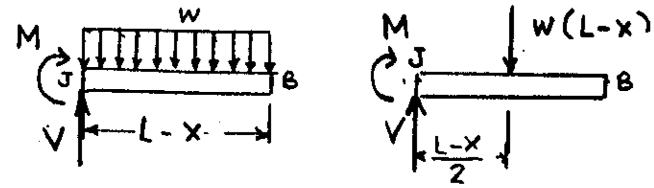




5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION

Use portion to the right of the section as the free body.



Replace distributed load by equivalent concentred load.

$+\uparrow \Sigma F_y = 0 \qquad V - w(L-x) = 0$

$$V = w(L-x)$$

$\circlearrowleft \Sigma M_J = 0 \qquad -M - w(L-x)\left(\frac{L-x}{2}\right) = 0$

$$M = -\frac{w}{2}(L-x)^2$$

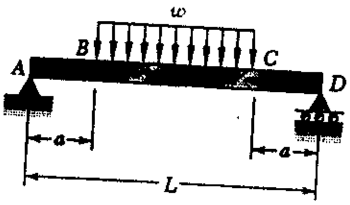
Largest negative bending moment occurs at $x = 0$.

$$M_{min} = -\frac{wL^2}{2}$$

Thus, $\qquad |M|_{max} = \frac{wL^2}{2}$

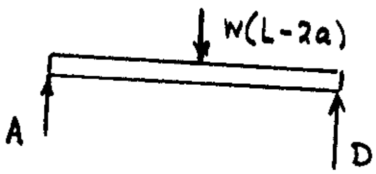
PROBLEM 5.5

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



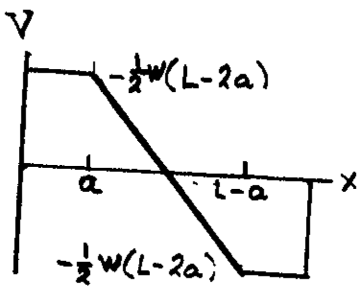
SOLUTION

Calculate reactions after replacing distributed load by an equivalent concentrated load.

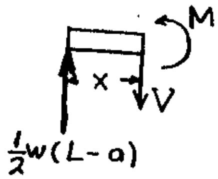


Reactions are

$$A = D = \tfrac{1}{2}w(L-2a)$$



From A to B $\qquad 0 < x < a$

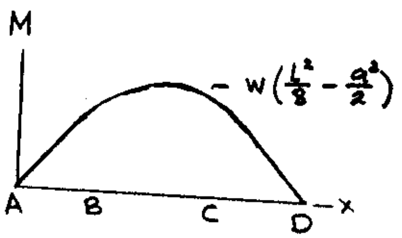


$+\uparrow \Sigma F_y = 0 \qquad \frac{1}{2}w(L-2a) - V = 0$

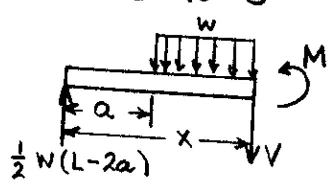
$$V = \tfrac{1}{2}w(L-2a)$$

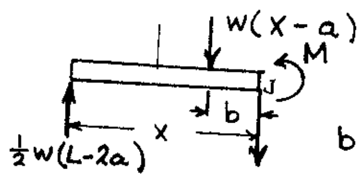
$\circlearrowleft \Sigma M = 0 \qquad -\frac{1}{2}w(L-2a) + M = 0$

$$M = \tfrac{1}{2}w(L-2a)x$$



From B to C $\qquad a < x < L-a$





$b = \dfrac{x-a}{2}$

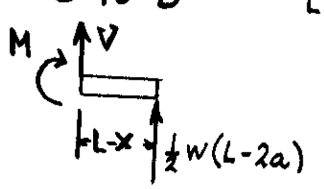
Place section cut at x. Replace distributed load by equiv. conc. load.

$+\uparrow \Sigma F_y = 0 \qquad \frac{1}{2}w(L-2a) - w(x-a) - V = 0 \qquad V = w\left(\frac{L}{2} - x\right)$

$\circlearrowleft M_J = 0 \qquad -\frac{1}{2}w(L-2a)x + w(x-a)\left(\frac{x-a}{2}\right) + M = 0$

$$M = \tfrac{1}{2}w\left[(L-2a)x - (x-a)^2\right]$$

From C to D $\qquad L-a < x < L$



$+\uparrow \Sigma F_y = 0 \qquad V + \frac{1}{2}w(L-2a) = 0$

$$V = -\frac{w}{2}(L-2a)$$

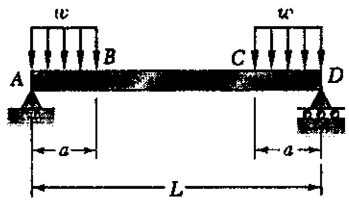
$\circlearrowleft \Sigma M_J = 0 \qquad -M + \frac{1}{2}w(L-2a)(L-x) = 0$

$$M = \tfrac{1}{2}w(L-2a)(L-x)$$

At $x = \frac{L}{2} \qquad M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$

PROBLEM 5.6

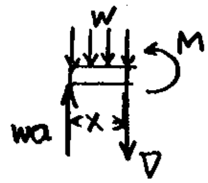
5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

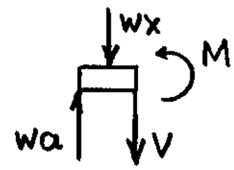


SOLUTION

Reactions: $A = D = wa$

From A to B $\quad 0 < x < a$



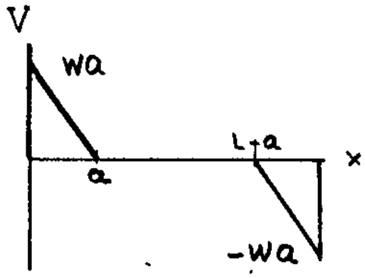


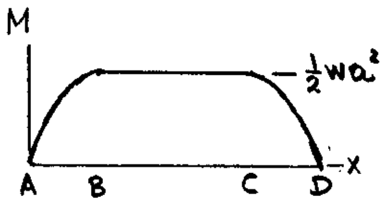
$+\uparrow \Sigma F_y = 0 \qquad wa - wx - V = 0$

$$V = w(a - x)$$

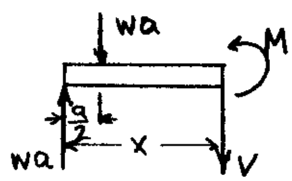
$\circlearrowleft \Sigma M_J = 0 \qquad -wax + (wx)\frac{x}{2} + M = 0$

$$M = w\left(ax - \frac{x^2}{2}\right)$$





From B to C $\quad a < x < L - a$



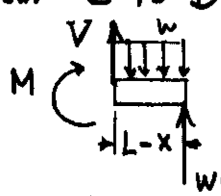
$\Sigma F_y = 0$

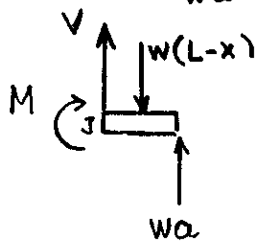
$wa - wa - V = 0$

$$V = 0$$

$\circlearrowleft \Sigma M_J = 0 \qquad -wax + wa\left(x - \frac{a}{2}\right) + M = 0 \qquad M = \frac{1}{2}wa^2$

From C to D $\quad L - a < x < L$





$+\uparrow \Sigma F_y = 0 \qquad V - w(L - x) + wa = 0$

$$V = w(L - x - a)$$

$\circlearrowleft \Sigma M_J = 0 \qquad -M - w(L-x)\left(\frac{L-x}{2}\right) + wa(L-x) = 0$

$$M = w\left[a(L-x) - \frac{1}{2}(L-x)^2\right]$$

PROBLEM 5.7

5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point A.)

SOLUTION

See PROBLEM 5.1

PROBLEM 5.8

5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point A.)

SOLUTION

See PROBLEM 5.2

PROBLEM 5.9

5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point A.)

SOLUTION

See PROBLEM 5.3

PROBLEM 5.10

5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point A.)

SOLUTION

See PROBLEM 5.4

PROBLEM 5.11

5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point A.)

SOLUTION

See PROBLEM 5.5

PROBLEM 5.12

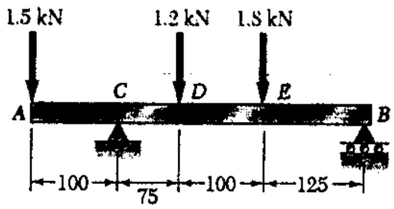
5.7 through 5.12 Determine the equations of the shear and bending- moment curves for the beam and loading shown. (Place the origin at point A.)

SOLUTION

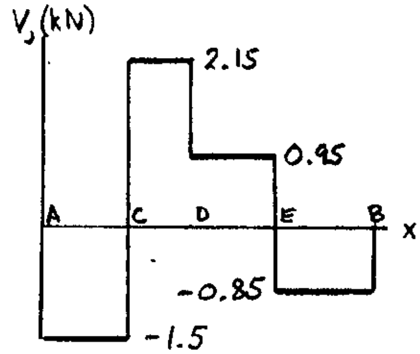
See PROBLEM 5.6

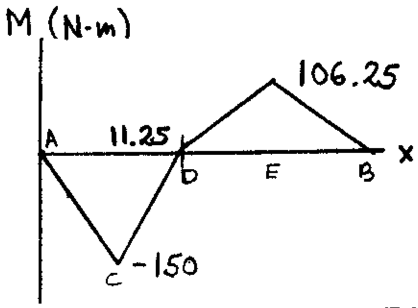
PROBLEM 5.13

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the



Dimensions in mm





SOLUTION

Calculate reactions

$\circlearrowleft \Sigma M_B = 0$

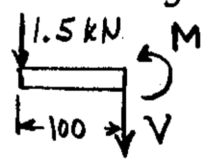
$(400)(1.5) - 300\,C + (225)(1.2) + (125)(1.8) = 0$

$C = 3.65 \text{ kN}$

$\circlearrowleft \Sigma M_C = 0 \qquad B = 0.85 \text{ kN}$

At A $\qquad V = -1.5 \text{ kN}, \quad M = 0$

At C^-

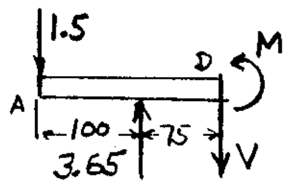


$\Sigma F_y = 0 \qquad -1.5 - V = 0 \qquad V = -1.5 \text{ kN}$

$\Sigma M_C = 0 \qquad (100)(1.5) + M = 0$

$M = -150 \text{ N·m}$ (b)

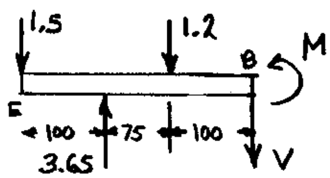
At D^-



$\Sigma F_y = 0 \qquad -1.5 + 3.65 - V = 0, \quad V = 2.15 \text{ kN}$ (a)

$\circlearrowleft \Sigma M_D = 0, \quad (175)(1.5) - (75)(3.65) + M = 0 \qquad M = 11.25 \text{ N·m}$

At E^-



$\Sigma F_y = 0$

$-1.5 + 3.65 - 1.2 - V = 0$

$V = 0.95 \text{ kN}$

$\circlearrowleft \Sigma M_E = 0 \qquad (275)(1.5) - (175)(3.65) + (100)(1.2) + M = 0$

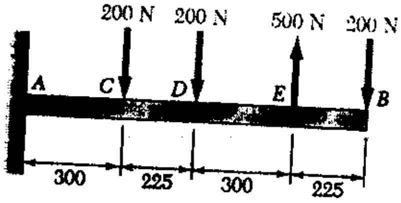
$M = 106.25 \text{ Nm}$

At B $\qquad V = -B = -0.85 \text{ kN}$

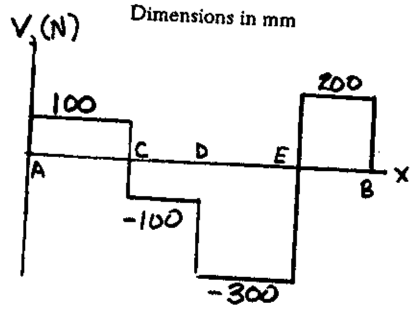
$M = 0$

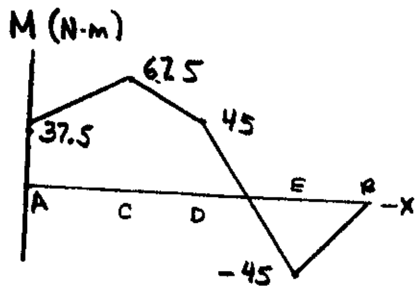
PROBLEM 5.14

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Dimensions in mm

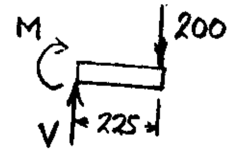




SOLUTION

At B $\qquad V = 200\ \text{N},\quad M = 0$

At E^+

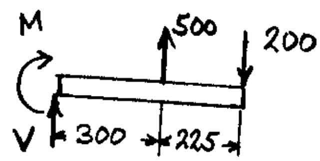


$+\uparrow \Sigma F_y = 0 \qquad V - 200 = 0 \qquad V = 200\ \text{N}$

$+\circlearrowleft \Sigma M_E = 0 \qquad -M - (0.225)(200) = 0$

$M = -45\ \text{N}\cdot\text{m}$

At D^+



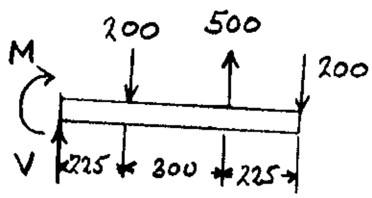
$+\uparrow \Sigma F_y = 0, \qquad V + 500 - 200 = 0$

$V = -300\ \text{N}$ (a)

$+\circlearrowleft \Sigma M_D = 0 \qquad -M + (0.3)(500) - (0.525)(200) = 0$

$M = 45\ \text{N}\cdot\text{m}$

At C^+

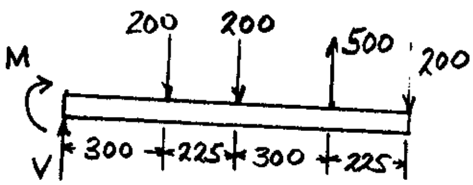


$+\uparrow \Sigma F_y = 0 \qquad V - 200 + 500 - 200 = 0 \qquad V = -100\ \text{N}$

$+\circlearrowleft \Sigma M_C = 0 \qquad -M - (0.225)(200) + (0.525)(500) - (0.75)(200) = 0$

$M = 67.5\ \text{N}\cdot\text{m}$ (b)

At A



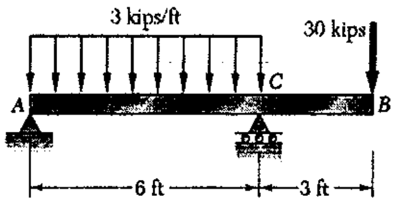
$+\uparrow \Sigma F_y = 0 \qquad V - 200 - 200 + 500 - 200 = 0 \qquad V = 100\ \text{N}$

$+\circlearrowleft \Sigma M_A = 0 \qquad -M - (0.3)(200) - (0.525)(200) + (0.825)(500) - (1.05)(200) = 0$

$M = 37.5\ \text{N}\cdot\text{m}$

PROBLEM 5.15

5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

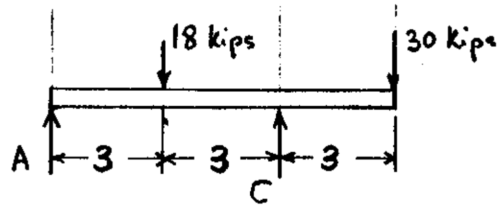


SOLUTION

Reactions

$\circlearrowleft \Sigma M_C = 0 \qquad -6A + (3)(18) - (3)(30) = 0$

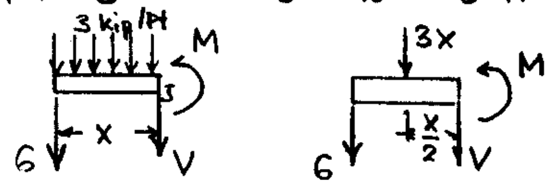
$A = -6 \text{ kips} \quad \text{i.e. } 6 \text{ kips} \downarrow$

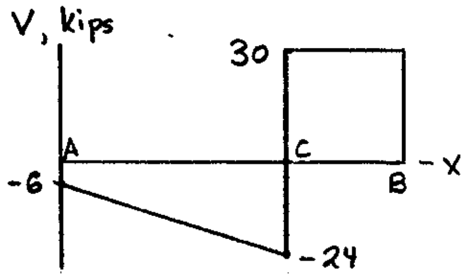


$\circlearrowleft \Sigma M_A = 0 \qquad 6C - (3)(18) - (9)(30) = 0$

$C = 54 \text{ kips} \uparrow$

A to C $\qquad 0 < x < 6$ ft.



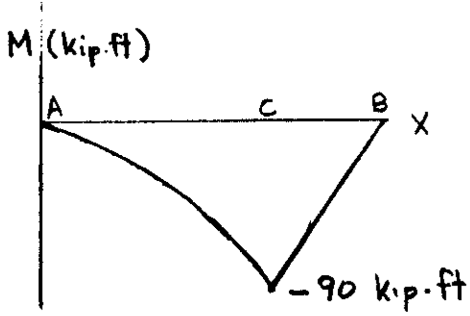


$+\uparrow \Sigma F_y = 0 \qquad -6 - 3x - V = 0$

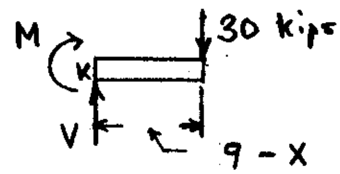
$V = -6 - 3x \text{ kips.}$

$\circlearrowleft \Sigma M_J = 0 \qquad -6x - (3x)(\frac{x}{2}) - M = 0$

$M = -6x - 1.5x^2 \text{ kip·ft}$



C to B $\qquad 6 \text{ ft} < x < 9 \text{ ft}$



$+\uparrow \Sigma F_y = 0 \qquad V - 30 = 0$

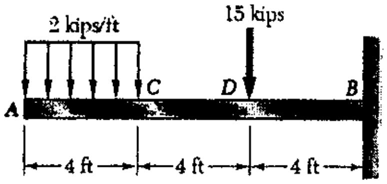
$V = 30 \text{ kips}$ ◄ (a)

$\circlearrowleft \Sigma M_K = 0 \qquad -M - (9 - x)(30) = 0$

$M = 30x - 270 \text{ kip·ft}$

(b) $\qquad |M|_{max} = 90 \text{ kip·ft}$ ◄

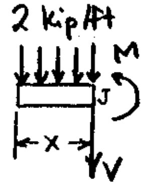
PROBLEM 5.16

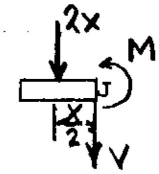


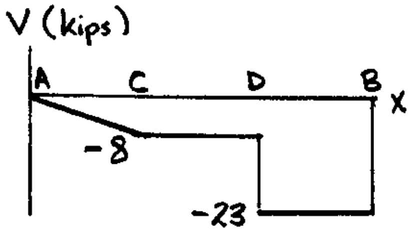
5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

A to C
$0 < x < 4$ ft





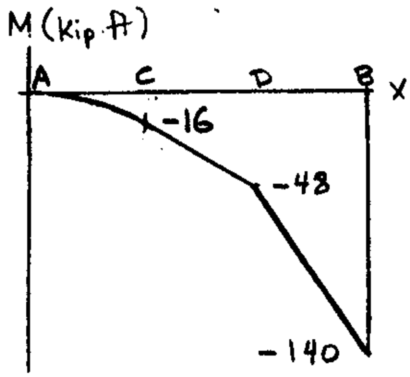


$+\uparrow \Sigma F_y = 0 \qquad -V - 2x = 0 \qquad V = -2x$ kips

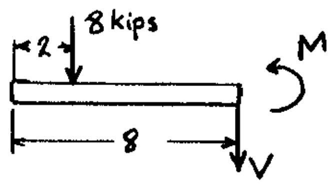
$\circlearrowleft \Sigma M_J = 0 \qquad M + (2x)(\frac{x}{2}) = 0$

$M = -x$ kip·ft

At C $\quad V = -8$ kips $\qquad M = -16$ kip·ft



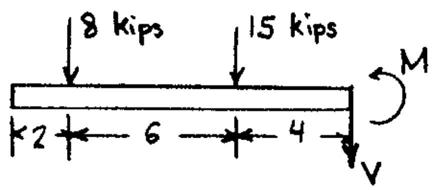
At D^-



$+\uparrow \Sigma F_y = 0 \qquad -8 - V = 0 \qquad V = -8$ kips

$\circlearrowleft \Sigma M_o = 0 \qquad (6)(8) - M = 0 \qquad M = -48$ kip ft

At B^-



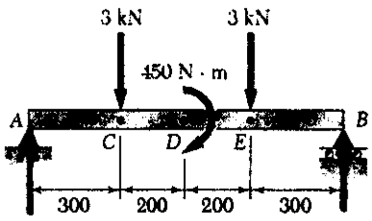
$+\uparrow \Sigma F_y = 0 \qquad -8 - 15 - V = 0$

$V = -23$ kips ◄ (a)

$\circlearrowleft \Sigma M_B = 0 \qquad -(10)(8) - (4)(15) - M = 0$

$M = -140$ kip ft ◄ (b)

PROBLEM 5.17



Dimensions in mm

5.17 and *5.18* Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$\circlearrowleft \Sigma M_B = 0 \qquad (700)(3) - 450 + (300)(3) - 1000A = 0$

$A = 2.55 \text{ kN} \uparrow$

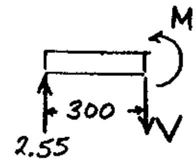
$\circlearrowleft \Sigma M_A = 0 \qquad -(300)(3) - 450 - (700)(3) + 1000B = 0$

$B = 3.45 \text{ kN} \uparrow$

At A $\quad V = 2.55 \text{ kN} \qquad M = 0$

A to C $\quad V = 2.55 \text{ kN}$

At C



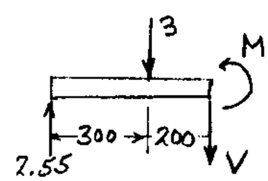
$\circlearrowleft \Sigma M_c = 0$

$-(300)(2.55) + M = 0$

$M = 765 \text{ N}\cdot\text{m}$

C to E $\quad V = -0.45 \text{ N}\cdot\text{m}$

At D^-

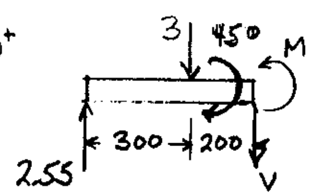


$\circlearrowleft \Sigma M_D = 0$

$-(500)(2.55) + (200)(3) + M = 0$

$M = 675 \text{ N}\cdot\text{m}$

At D^+



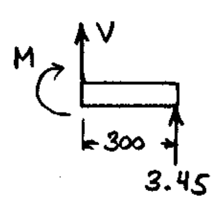
$\circlearrowleft \Sigma M_D = 0$

$-(500)(2.55) + (200)(3) - 450 + M = 0$

$M = 1125 \text{ N}\cdot\text{m}$

E to B $\quad V = -3.45 \text{ kN}$

At E



$\circlearrowleft \Sigma M_E = 0$

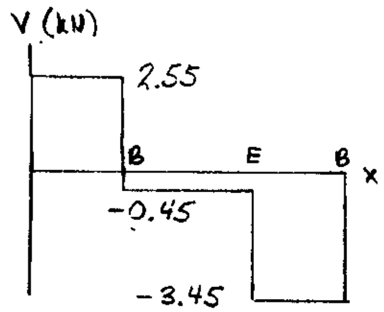
$-M + (300)(3.45) = 0$

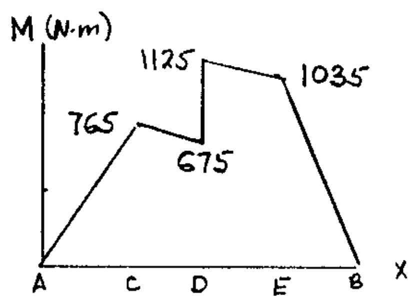
$M = 1035 \text{ N}\cdot\text{m}.$

At B $\quad V = 3.45 \text{ N}\cdot\text{m}, \quad M = 0$

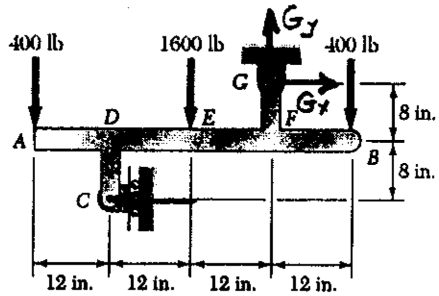
Maximum $|V| = 3.45 \text{ kN}$ ◄

Maximum $|M| = 1125 \text{ N}\cdot\text{m}$ ◄





PROBLEM 5.18



5.17 and *5.18* Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$\circlearrowleft \Sigma M_G = 0$

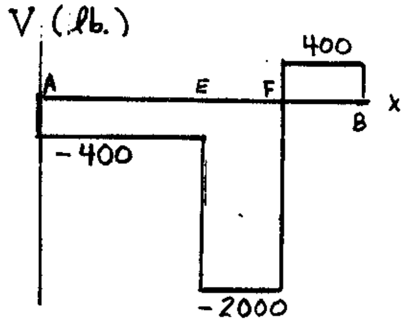
$-16C + (36)(400) + (12)(1600) - (12)(400) = 0$

$C = 1800$ lb.

$\xrightarrow{+} \Sigma F_x = 0 \qquad -C + G_x = 0 \qquad G_x = 1800$ lb.

$+\uparrow \Sigma F_y = 0 \qquad -400 - 1600 + G_y - 400 = 0$

$G_y = 2400$ lb.

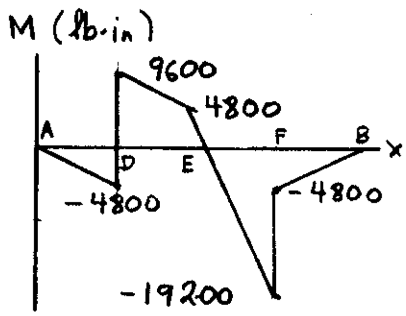


A to E $\qquad V = -400$ lb.

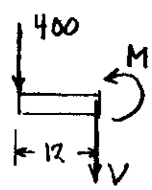
E to F $\qquad V = -2000$ lb.

F to B $\qquad V = 400$ lb.

At A and B $\qquad M = 0$



At D^-

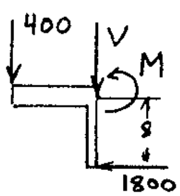


$\circlearrowleft \Sigma M_D = 0$

$(12)(400) + M = 0$

$M = -4800$ lb·in.

At D^+

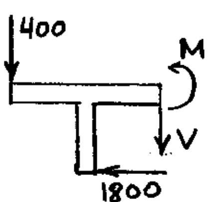


$\circlearrowleft \Sigma M_D = 0$

$(12)(400) - (8)(1800) + M = 0$

$M = 9600$ lb·in.

At E

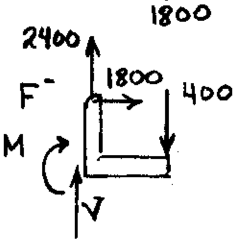


$\circlearrowleft \Sigma M_E = 0$

$(24)(400) - (8)(1800) + M = 0$

$M = 4800$ lb·in

At F^-

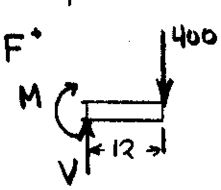


$\circlearrowleft \Sigma M_F = 0$

$-M - (8)(1800) - (12)(400) = 0$

$M = -19200$ lb·in

At F^+



$\Sigma M_F = 0$

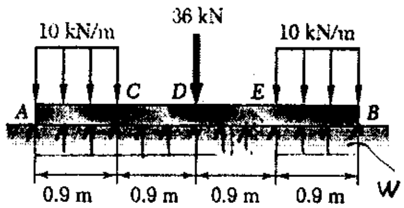
$-M - (12)(400) = 0$

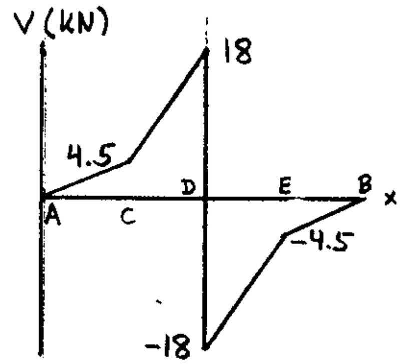
$M = -4800$ lb·in.

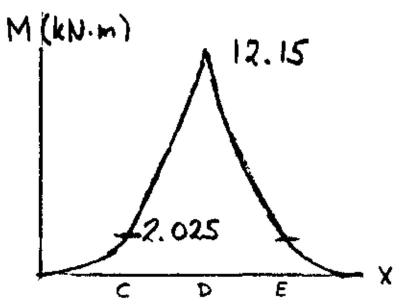
(a) Maximum $|V|$
$= 2000$ lb ◀

(b) Maximum $|M|$
$= 19200$ lb·in. ◀

PROBLEM 5.19







(a) Maximum |V| = 18 kN ◄

(b) Maximum |M| = 12.15 kN·m ◄

5.19 and *5.20* Assuming the upward reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam *AB* and determine the maximum value (*a*) of the shear, (*b*) of the bending moment.

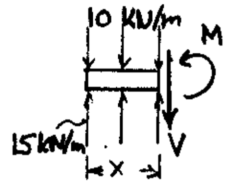
SOLUTION

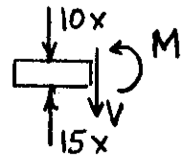
Over whole beam $+\uparrow \Sigma F_y = 0$

$$3.6\,w - (0.9)(10) - 36 - (0.9)(10) = 0$$

$$w = 15 \text{ kN/m}$$

A to C $\quad 0 < x < 0.9$ m





$+\uparrow \Sigma F_y = 0 \qquad 15x - 10x - V = 0$

$$V = 5x$$

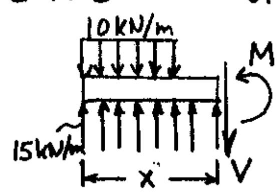
$\circlearrowleft \Sigma M_J = 0 \qquad -(15x)\frac{x}{2} + (10x)\frac{x}{2} + M = 0$

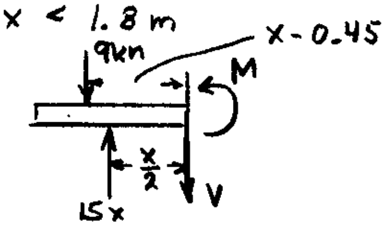
$$M = 2.5x^2$$

At $x = C \qquad V = 4.5$ kN

$M = 2.025$ kN·m

C to D $\quad 0.9 \text{ m} < x < 1.8 \text{ m}$





$+\uparrow \Sigma F_y = 0 \qquad 15x - 9 - V = 0$

$$V = 15x - 9$$

$\circlearrowleft \Sigma M_J = 0 \qquad -(15x)\left(\frac{x}{2}\right) + 9(x - 0.45) + M = 0$

$$M = 7.5x^2 - 9x + 4.05 = 0$$

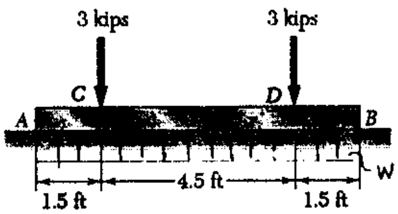
At D^- $\qquad V = 18$ kN

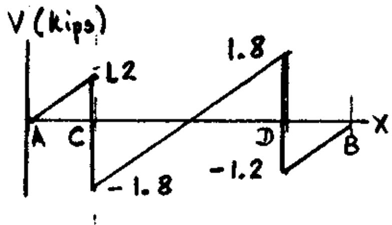
$M = 12.15$ kN·m

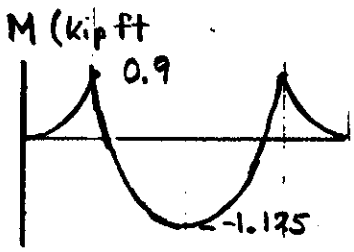
D to B $\qquad$ Use symmetry to calculate the shear and bending moment.

PROBLEM 5.20

5.19 and *5.20* Assuming the upward reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam *AB* and determine the maximum value (*a*) of the shear, (*b*) of the bending moment.







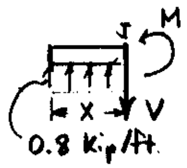
SOLUTION

Over the whole beam

$+\uparrow \Sigma F_y = 0 \qquad 7.5w - 3 - 3 = 0$

$w = 0.8 \text{ kip/ft}$

A to C $\qquad 0 < x < 1.5 \text{ ft}$



$+\uparrow \Sigma F_y = 0 \qquad 0.8x - V = 0$

$V = 0.8x$

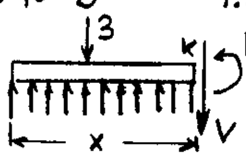
$\circlearrowleft \Sigma M_J = 0$

$-(0.8x)\left(\frac{x}{2}\right) + M = 0$

$M = 0.4x^2$

At C^- $\qquad V = 1.2$ kips, $\quad M = 0.9$ kip·ft

C to D $\qquad 1.5 \text{ ft} < x < 6 \text{ ft}$



$+\uparrow \Sigma F_y = 0$

$0.8x - 3 - V = 0$

$V = 0.8x - 3$

$\circlearrowleft \Sigma M_K = 0 \qquad -(0.8x)\left(\frac{x}{2}\right) + 3(x - 1.5) + M = 0$

$M = 0.4x^2 - 3x + 4.5$

At the center of the beam $\qquad x = 3.75$ ft

$V = 0, \quad M = -1.125$ kip ft

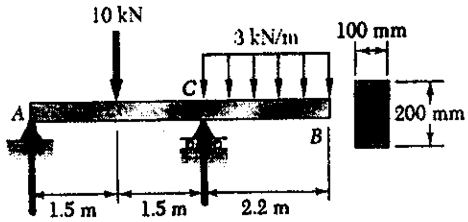
At C^+ $\qquad V = -1.8$ kip $\qquad M = 0.9$ kip ft

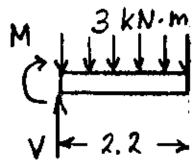
(a) Maximum $|V| = 1.8$ kips ◄

(b) Maximum $|M| = 1.125$ kip·ft ◄

PROBLEM 5.21

5.21 For the beam and loading shown, determine the maximum normal stress on a transverse section at C.





SOLUTION

Using CB as a free body

$$\circlearrowleft \Sigma M_C = 0$$

$$-M + (2.2)(3\times10^3)(1.1) = 0$$

$$M = 7.26\times10^3 \text{ N}\cdot\text{m}$$

Section modulus for rectangle

$$S = \frac{1}{6} b h^2$$

$$= \frac{1}{6}(100)(200)^2 = 666.7\times10^3 \text{ mm}^3$$

$$= 666.7\times10^{-6} \text{ m}^3$$

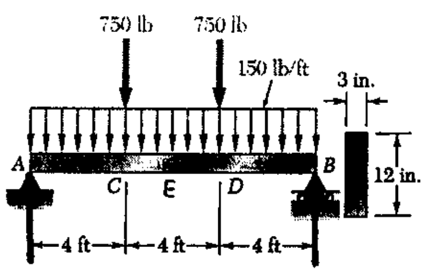
Normal stress

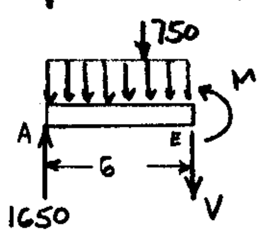
$$\sigma = \frac{M}{S} = \frac{7.26\times10^3}{666.7\times10^{-6}} = 10.89\times10^6 \text{ Pa}$$

$$\sigma = 10.89 \text{ MPa}$$ ◄

PROBLEM 5.22

5.22 For the beam and loading shown, determine the maximum normal stress on a transverse section at the center of the beam





SOLUTION

Reactions: $C = A$ by symmetry

$$+\uparrow \Sigma F_y = 0 \qquad A + C - (2)(750) - (12)(150) = 0$$

$$A = C = 1650 \text{ lb.}$$

Use left half of beam as free body

$$\circlearrowleft \Sigma M_E = 0$$

$$-(1650)(6) + (750)(2) + (150)(6)(3) + M = 0$$

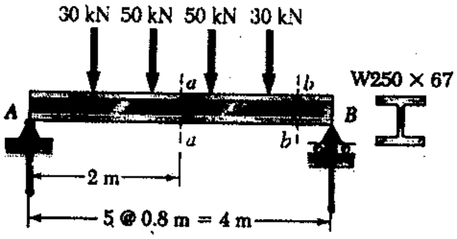
$$M = 5700 \text{ lb}\cdot\text{ft} = 68.4\times10^3 \text{ lb}\cdot\text{in}$$

Section modulus $\quad S = \frac{1}{6} b h^2 = (\frac{1}{6})(3)(12)^2 = 72 \text{ in}^3$

Normal stress $\quad \sigma = \frac{M}{S} = \frac{68.4\times10^3}{72} = 950 \text{ psi}$ ◄

PROBLEM 5.23

5.23 For the beam and loading shown, determine the maximum normal stress on section *a-a*.

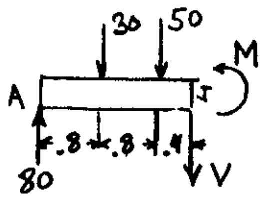


SOLUTION

Reactions: By symmetry $A = B$

$+\uparrow \Sigma F_y = 0 \qquad A = B = 80\ \text{kN}$

Using left half of beam as free body



$\circlearrowleft \Sigma M_J = 0$

$-(80)(2) + (30)(1.2) + (50)(0.4) + M = 0$

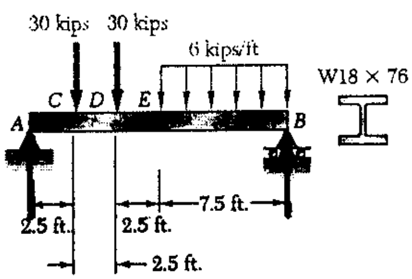
$M = 104\ \text{kN}\cdot\text{m} = 104 \times 10^3\ \text{N}\cdot\text{m}$

For W250 × 67 $\quad S = 809 \times 10^3\ \text{mm}^3 = 809 \times 10^{-6}\ \text{m}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{104 \times 10^3}{809 \times 10^{-6}} = 128.6 \times 10^6\ \text{Pa} = 128.6\ \text{MPa}$ ◄

PROBLEM 5.24

5.24 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.



SOLUTION

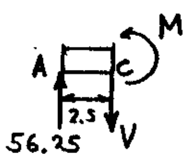
Use entire beam as free body

$\circlearrowleft \Sigma M_B = 0$

$-15A + (12.5)(30) + (10)(30) + (6)(7.5)(3.75) = 0$

$A = 56.25\ \text{kips}$

Use portion AC as free body



$\circlearrowleft \Sigma M_C = 0 \qquad -(56.25)(2.5) + M = 0$

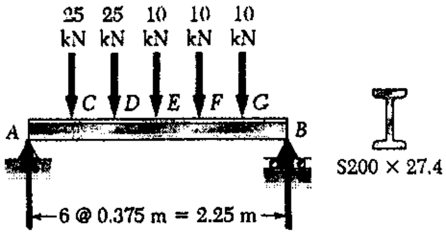
Bending moment at C $\quad M = 140.625\ \text{kip ft} = 1687.5\ \text{kip}\cdot\text{in}$

For W18 × 76 $\quad S = 146\ \text{in}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{1687.5}{146} = 11.56\ \text{ksi}$ ◄

PROBLEM 5.25

5.25 **and** *5.26* For the beam and loading shown, determine the maximum normal stress on a transverse section at C.



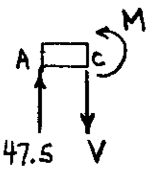
SOLUTION

Use entire beam as free body

$\circlearrowleft + \Sigma M_B = 0$

$$2.25A - (1.875)(25) - (1.5)(25) - (1.125)(10) - (0.75)(10) - (0.375)(10) = 0$$

$$A = 47.5 \text{ kN}$$



Use portion AC as free body

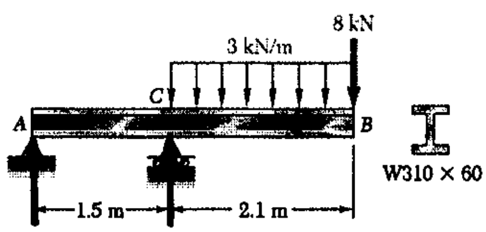
$$-(0.375)(47.5) + M = 0 \qquad M = 17.8125 \text{ kN}\cdot\text{m}$$

For S 200 × 27.4 $\quad S = 235 \times 10^3 \text{ mm}^3 = 235 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{17.8125 \times 10^3}{235 \times 10^{-6}} = 75.8 \times 10^6 \text{ Pa} = 75.8 \text{ MPa}$ ◄

PROBLEM 5.26

5.25 **and** *5.26* For the beam and loading shown, determine the maximum normal stress on a transverse section at C.



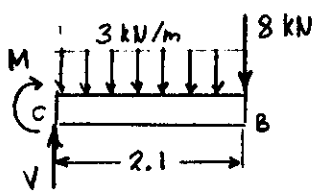
SOLUTION

Use portion CB as free body.

$\circlearrowleft + \Sigma M_C = 0$

$$-M + (3)(2.1)(1.05) + (8)(2.1) = 0$$

$$M = -23.415 \text{ kN}\cdot\text{m}$$

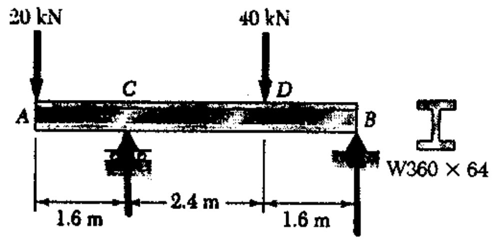


For W 310 × 60 $\quad S = 851 \times 10^3 \text{ mm}^3 = 851 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{23.415 \times 10^3}{851 \times 10^{-6}} = 27.5 \times 10^6 \text{ Pa} = 27.5 \text{ MPa}$ ◄

PROBLEM 5.27

5.27 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$\circlearrowleft \Sigma M_C = 0$

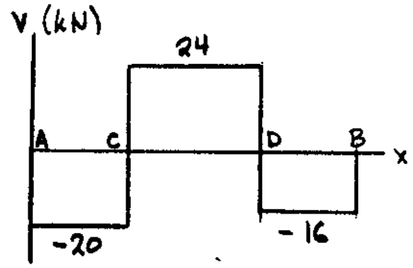
$(1.6)(20) - (2.4)(40) + (4.0)B = 0$

$B = 16 \text{ kN}$

$\circlearrowleft \Sigma M_B = 0$

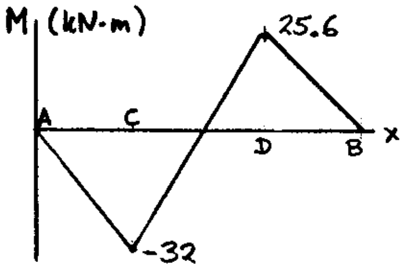
$(5.6)(20) - (4.0)C + (1.6)(40) = 0$

$C = 44 \text{ kN}$



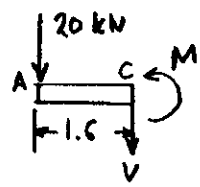
Shear

A to C^-	$V = -20 \text{ kN}$
C^+ to D^-	$V = 24 \text{ kN}$
D^+ to B	$V = -16 \text{ kN}$



Bending moment

At C

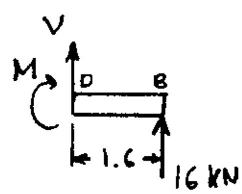


$\circlearrowleft \Sigma M_C = 0$

$(1.6)(20) + M = 0$

$M = -32 \text{ kN}\cdot\text{m}$

At D



$\circlearrowleft \Sigma M_D = 0$

$-M + (1.6)(16) = 0$

$M = 25.6 \text{ kN}\cdot\text{m}$

max $|M| = 32 \text{ kN}\cdot\text{m} = 32 \times 10^3 \text{ N}\cdot\text{m}$ ◄

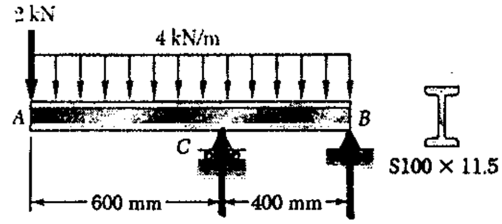
For rolled steel section W 360 × 64 $S = 1030 \times 10^3 \text{ mm}^3 = 1030 \times 10^{-6} \text{ m}^3$

Normal stress $\sigma = \dfrac{|M|}{S} = \dfrac{32 \times 10^3}{1030 \times 10^{-6}} = 31.1 \times 10^6 \text{ Pa}$

$= 31.1 \text{ MPa}$ ◄

PROBLEM 5.28

5.28 and 5.29 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$\circlearrowleft^{+} \Sigma M_C = 0$

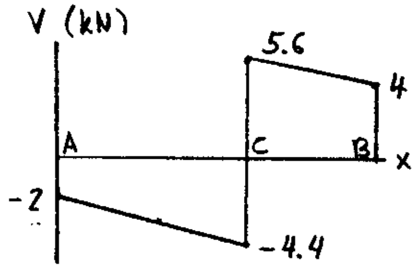
$(0.6)(2) + (0.1)(4) + (0.4)B = 0$

$B = -4 \text{ kN}$

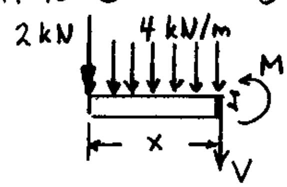
$\circlearrowleft^{+} \Sigma M_B = 0$

$(1.0)(2) + (0.5)(4) - (0.4)C = 0$

$C = 10 \text{ kN}$



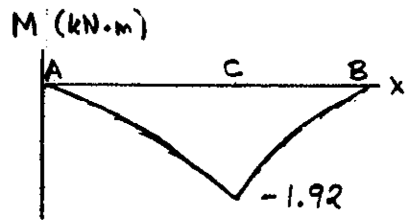
A to C $\quad 0 < x < 0.6 \text{ m}$



$+\uparrow \Sigma F_y = 0$

$-2 - 4x - V = 0$

$V = -2 - 4x$



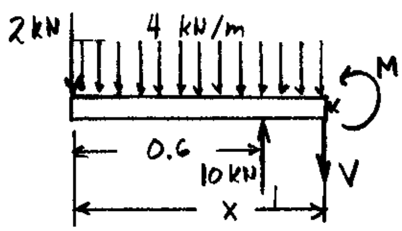
$\circlearrowleft^{+} \Sigma M_J = 0$

$2x + (4x)\left(\frac{x}{2}\right) + M = 0$

$M = -2x^2 - 2x$

At C $\quad M = -1.92 \text{ kN}\cdot\text{m}$

C to B $\quad 0.6 \text{ m} < x < 1.0 \text{ m}$



$+\uparrow \Sigma F_y = 0 \qquad -2 - 4x + 10 - V = 0$

$V = 8 - 4x$

$\circlearrowleft^{+} \Sigma M_K = 0 \qquad -2x + (4x)\left(\frac{x}{2}\right) - (10)(x - 0.6) + M = 0$

$$M = -2x^2 + 8x - 6$$

max $|M| = 1.92 \text{ kN}\cdot\text{m} = 1.92 \times 10^3 \text{ N}\cdot\text{m}$

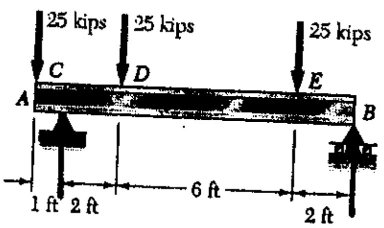
For rolled steel section S 100 × 11.5 $\qquad S = 49.6 \times 10^3 \text{ mm}^3 = 49.6 \times 10^{-6} \text{ m}^3$

Maximum normal stress $\qquad \sigma = \dfrac{|M|}{S} = \dfrac{1.92 \times 10^3}{49.6 \times 10^{-6}} = 38.7 \times 10^6 \text{ Pa}$

$= 38.7 \text{ MPa}$ ◄

PROBLEM 5.29

5.28 and 5.29 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



S12 × 35

SOLUTION

$\circlearrowleft \Sigma M_B = 0$

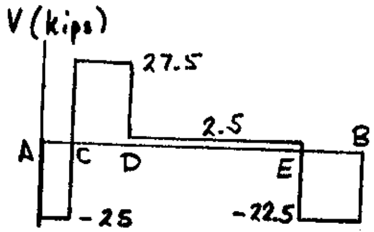
$(11)(25) - 10C + (8)(25) + (2)(25) = 0$

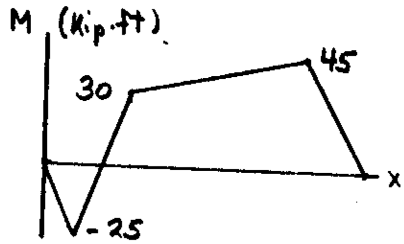
$C = 52.5$ kips

$\circlearrowleft \Sigma M_C = 0$

$(1)(25) - (2)(25) - (8)(25) + 10B = 0$

$B = 22.5$ kips



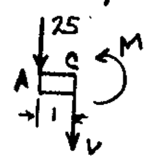


Shear

A to C^-	$V = -25$ kips
C^+ to D^-	$V = 27.5$ kips
D^+ to E^-	$V = 2.5$ kips
E^+ to B	$V = -22.5$ kips

Bending moments

At C

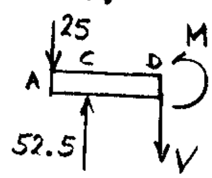


$\circlearrowleft \Sigma M_C = 0$

$(1)(25) + M = 0$

$M = -25$ kip·ft

At D

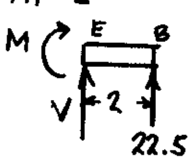


$\circlearrowleft \Sigma M_D = 0$

$(3)(25) - (2)(52.5) + M = 0$

$M = 30$ kip·ft

At E



$\circlearrowleft \Sigma M_E = 0$ $\quad -M + (2)(22.5) = 0 \quad M = 45$ kip·ft

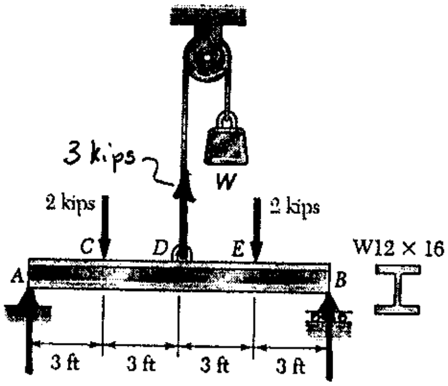
max $|M| = 45$ kip·ft $= 540$ kip·in

For S12 × 35 rolled steel section $\quad S = 38.2 \text{ in}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{540}{38.2} = 14.14$ ksi

PROBLEM 5.30

5.30 Knowing that $W = 3$ kips, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.



SOLUTION

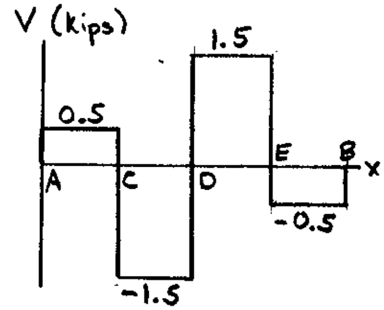
By symmetry $\quad A = B$

$+\uparrow \Sigma F_y = 0 \qquad A - 2 + 3 - 2 + B = 0$

$A = B = 0.5$ kip

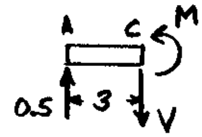
Shear

A to C^-	$V = 0.5$ kips
C^+ to D^-	$V = -1.5$ kips
D^+ to E^-	$V = 1.5$ kips
E^+ to B	$V = -0.5$ kips



Bending moment

At C

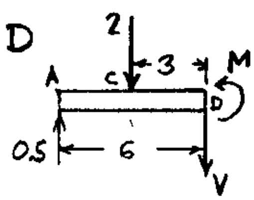


$\circlearrowleft \Sigma M_C = 0$

$-(3)(0.5) + M = 0$

$M = 1.5$ kip ft

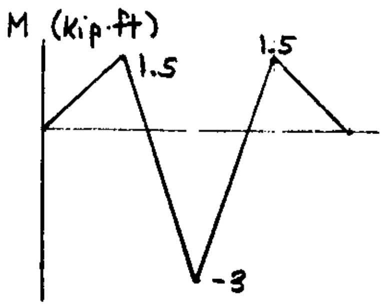
At D



$\circlearrowleft \Sigma M_D = 0$

$-(6)(0.5) + (3)(2) + M = 0$

$M = -3$ kip·ft



At E $\quad M = 1.5$ kip ft $\quad$ by symmetry

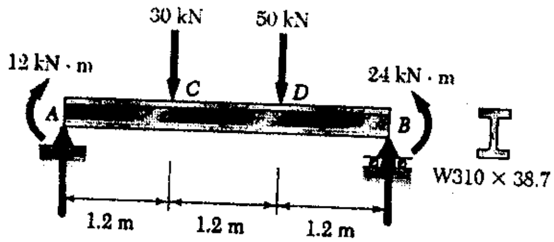
max $|M| = 3$ kip·ft $= 36$ kip·in

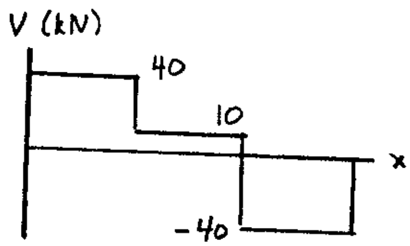
For rolled steel section $\quad$ W 12 × 16 $\quad S = 17.1\ \text{in}^3$

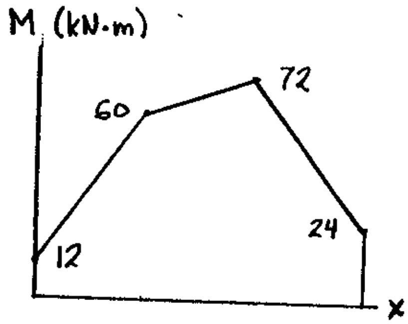
Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{36}{17.1} = 2.11$ ksi

PROBLEM 5.31

5.31* and *5.32 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.







SOLUTION

$\circlearrowleft^{+} \Sigma M_B = 0$

$-12 - 3.6A + (2.4)(30) + (1.2)(50) + 24 = 0$

$A = 40 \text{ kN}$

$\circlearrowleft^{+} \Sigma M_A = 0$

$-12 - (1.2)(30) - (2.4)(50) + 24 + 3.6B = 0$

$B = 40 \text{ kN}$

Shear

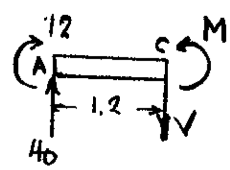
A to C^-	$V = 40$ kN
C^+ to D^-	$V = 10$ kN
D^+ to B	$V = -50$ kN

Bending moment

At A $M = 12$ kN·m

At B $M = 24$ kN·m

At C

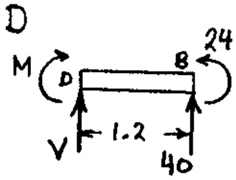


$\circlearrowleft^{+} \Sigma M_C = 0$

$-12 - (1.2)(40) + M = 0$

$M = 60$ kN·m

At D



$\circlearrowleft^{+} M_D = 0$

$-M + 24 + (1.2)(40) = 0$

$M = 72$ kN·m

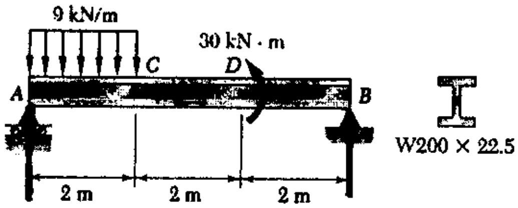
$\max |M| = 72 \text{ kN·m} = 72 \times 10^3 \text{ N·m}$

For rolled steel section W 310 × 38.7 $S = 549 \times 10^3 \text{ mm}^3 = 549 \times 10^{-6} \text{ m}^3$

Normal stress $\sigma = \dfrac{|M|}{S} = \dfrac{72 \times 10^3}{549 \times 10^{-6}} = 131.1 \times 10^6 \text{ Pa} = 131.1 \text{ MPa}$ ◄

PROBLEM 5.32

5.31* and *5.32 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending



SOLUTION

$\circlearrowleft + \Sigma M_B = 0$

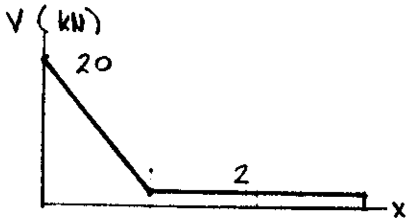
$-6A + (2)(9)(5) + 30 = 0$

$A = 20 \text{ kN}$

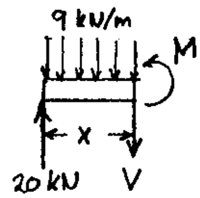
$\circlearrowleft + \Sigma M_A = 0$

$-(2)(9)(1) + 30 + 6B = 0$

$B = -2 \text{ kN}$ i.e $2 \text{ kN} \downarrow$



A to C $\quad 0 < x < 2 \text{ m}$



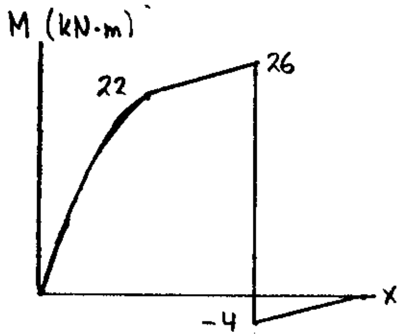
$+\uparrow \Sigma F_y = 0 \qquad 20 - 9x - V = 0$

$V = 20 - 9x$

$\circlearrowleft \Sigma M_J = 0$

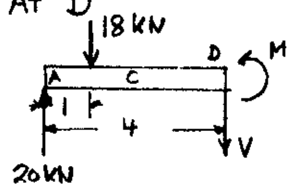
$-20x + (9x)\frac{x}{2} + M = 0$

$M = 20x - 4.5x^2$



At C $\qquad V = 2 \text{ kN} \qquad M = 22 \text{ kN·m}$

At D^-



$+\uparrow \Sigma F_y = 0$

$20 - 18 - V = 0$

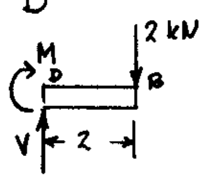
$V = 2 \text{ kN}$

$\circlearrowleft \Sigma M_D = 0$

$-(4)(20) + (3)(18) + M = 0$

$M = 26 \text{ kN·m}$

At D^+



$+\uparrow \Sigma F_y = 0 \qquad V - 2 = 0$

$V = 2 \text{ kN}$

$\circlearrowleft \Sigma M_D = 0$

$-M - (2)(2) = 0$

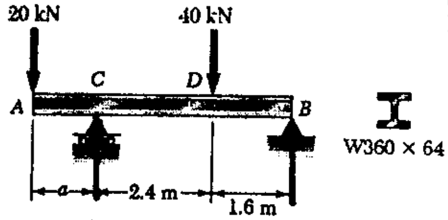
$M = -4 \text{ kN·m}$

$\max |M| = 26 \text{ kN·m} = 26 \times 10^3 \text{ N·m}$

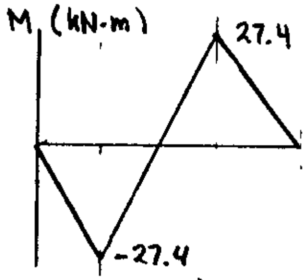
For rolled steel section W 200 × 22.5 $\qquad S = 194 \times 10^3 \text{ mm}^3 = 194 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{26 \times 10^3}{194 \times 10^{-6}} = 134.0 \times 10^6 \text{ Pa} = 134.0 \text{ MPa}$ ◄

PROBLEM 5.33



5.33 Determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending.
(*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)



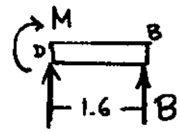
SOLUTION

Reaction at B $\quad \circlearrowleft \Sigma M_C = 0$

$$20a - (2.4)(40) + (4.0)B = 0$$

$$B = 24 - 5a$$

Bending moment at D

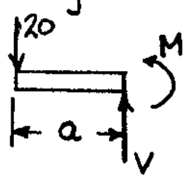


$$\circlearrowleft \Sigma M_D = 0$$

$$-M + 1.6B = 0$$

$$M_D = 1.6B = 38.4 - 8a$$

Bending moment at C



$$\circlearrowleft \Sigma M_C = 0$$

$$20a + M = 0$$

$$M_C = -20a$$

Equate $\quad -M_C = M_D$

$$20a = 38.4 - 8a$$

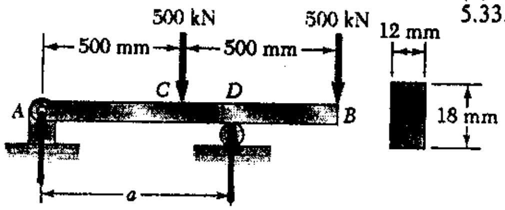
$$a = 1.3714 \text{ m}$$

$M_C = -27.429$ kN·m $\qquad M_D = 27.429$ kN·m

For W 360 × 64 rolled steel section $\quad S = 1030 \times 10^3 \text{ mm}^3 = 1030 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{27.429 \times 10^3}{1030 \times 10^{-6}} = 26.6 \times 10^6 \text{ Pa} = 26.6 \text{ MPa}$

PROBLEM 5.34



5.34 For the beam and loading shown, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.33.)

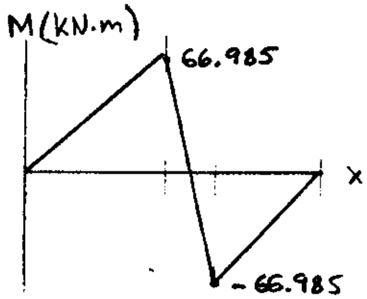
SOLUTION

Reaction at A $\circlearrowleft \Sigma M_D = 0$

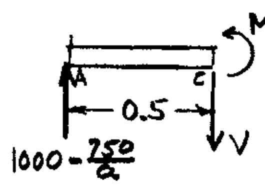
$$-Aa + (500)(a - 0.5) - 500(1 - a) = 0$$

$$Aa = 1000a - 750$$

$$A = 1000 - \frac{750}{a}$$



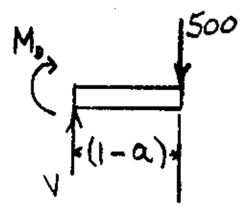
Bending moment at C $\circlearrowleft \Sigma M_C = 0$



$$-(0.5)\left(1000 - \frac{750}{a}\right) + M_C = 0$$

$$M_C = 500 - \frac{375}{a}$$

Bending moment at D $\circlearrowleft \Sigma M_D = 0$



$$-M_D - (500)(1-a) = 0$$

$$M_D = -500(1-a)$$

Equate $\quad -M_D = M_C \qquad 500(1-a) = 500 - \dfrac{375}{a}$

$$a = 0.86603 \text{ m} = 866.03 \text{ mm} \blacktriangleleft$$

$A = 133.98$ kN $\qquad M_C = 66.985$ kN·m $\qquad M_D = -66.985$ kN·m

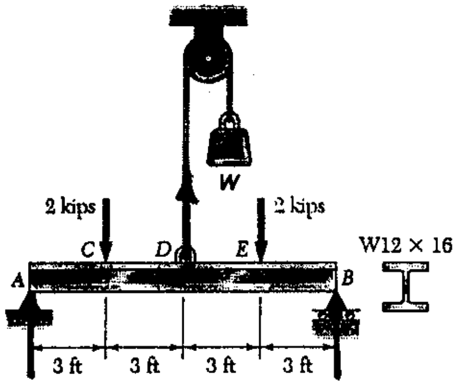
For rectangular cross section $\quad S = \frac{1}{6}bh^3 = \frac{1}{6}(12)(18)^3 = 11.664 \times 10^3 \text{ mm}^3$

$$= 11.664 \times 10^{-6} \text{ m}^3$$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{66.985 \times 10^3}{11.664 \times 10^{-6}} = 5.74 \times 10^6 \text{ Pa}$

$$= 5.74 \text{ MPa} \blacktriangleleft$$

PROBLEM 5.35



5.35 Determine (*a*) the magnitude of the counterweight W for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.33.)

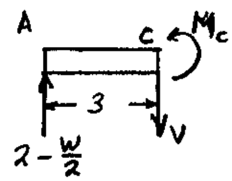
SOLUTION

By symmetry $\quad A = B$

$+\uparrow \Sigma F_y = 0 \qquad A - 2 + W - 2 + B = 0$

$$A = B = 2 - \frac{W}{2}$$

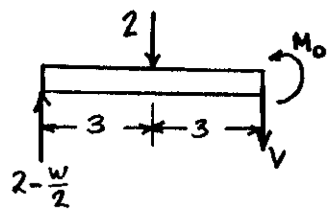
Bending moment at C $\quad \circlearrowleft \Sigma M_C = 0$



$$-(3)\left(2 - \frac{W}{2}\right) + M_C = 0$$

$$M_C = 6 - 1.5W$$

Bending moment at D $\quad \circlearrowleft \Sigma M_D = 0$



$$-(6)\left(2 - \frac{W}{2}\right) + (3)(2) + M_D = 0$$

$$M_D = 6 - 3W$$

Equate $\quad -M_D = M_C$

$3W - 6 = 6 - 1.5W \qquad\qquad W = 2.667$ kips ◄

$M_C = 2.0$ kip ft $\qquad\qquad M_D = -2.0$ kip ft

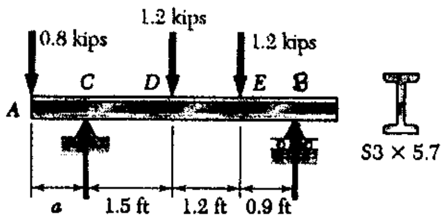
max $|M| = 2.0$ kip·ft $= 24$ kip·in

For W 12×16 rolled steel section $\quad S = 17.1$ in^3

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{24}{17.1} = 1.404$ ksi ◄

PROBLEM 5.36

5.36 For the beam and loading shown, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.33).



SOLUTION

$\circlearrowleft \Sigma M_C = 0$

$0.8a - (1.5)(1.2) - (2.7)(1.2) + (3.6)B = 0$

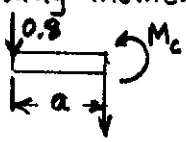
$B = 1.4 - 0.22222a$

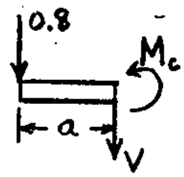
$\circlearrowleft \Sigma M_B = 0$

$(0.8)(3.6 + a) - 3.6C + (2.1)(1.2) + (0.9)(1.2) = 0$

$C = 1.8 + 0.22222a$

Bending moment at C



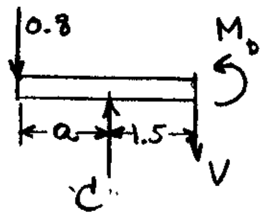


$\circlearrowleft \Sigma M_C = 0$

$M_C + (0.8)(a) = 0$

$M_C = -0.8a$

Bending moment at D

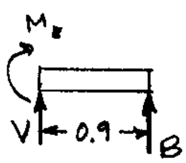


$\Sigma M_D = 0$

$M_D + (0.8)(a + 1.5) - 1.5C = 0$

$M_D = 1.5 - 0.46667a$

Bending moment at E



$\circlearrowleft \Sigma M_E = 0$

$-M_E + 0.9B = 0$

$M_E = 1.26 - 0.2a$

Assume $-M_C = M_E$ $\quad 0.8a = 1.26 - 0.2a$ $\quad a = 1.26$ ft ◄

$M_C = -1.008$ kip·ft $\quad M_E = 1.008$ kip ft $\quad M_D = 0.912$ kip·ft

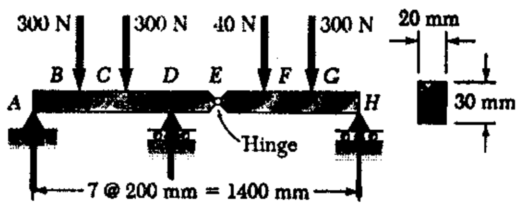
max $|M| = 1.008$ kip ft $= 12.096$ kip·in

For rolled steel section S 3×5.7 $\quad S = 1.68\ \text{in}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{12.096}{1.68} = 7.20$ ksi ◄

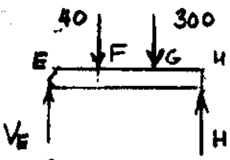
PROBLEM 5.37

5.37 and *5.38* Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending



SOLUTION

Free body EFGH



Note that $M_E = 0$ due to hinge.

$\circlearrowleft \Sigma M_E = 0$

$0.6\,H - (0.2)(40) - (0.4)(300) = 0$

$H = 213.33 \text{ N}$

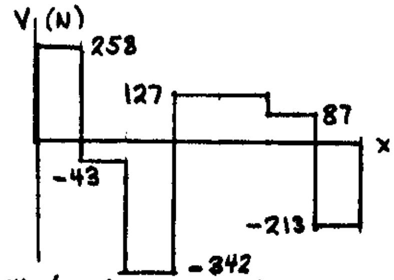
$\uparrow \Sigma F_y = 0 \qquad V_E - 40 - 300 + 213.33 = 0$

$V_E = 126.67 \text{ N}$

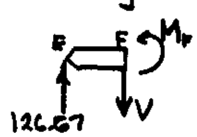
Shear: E to F $\quad V = 126.67 \text{ N·m}$

F to G $\quad V = 86.67 \text{ N·m}$

G to H $\quad V = -213.33 \text{ N·m}$



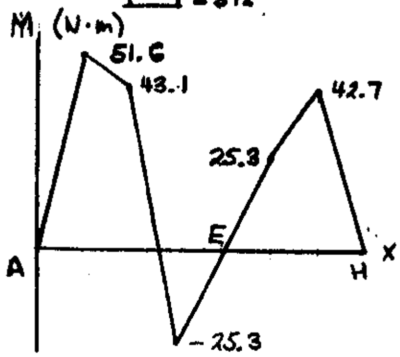
Bending moment at F



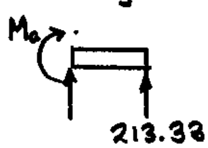
$\circlearrowleft \Sigma M_F = 0$

$M_F - (0.2)(126.67) = 0$

$M_F = 25.33 \text{ N·m}$



Bending moment at G

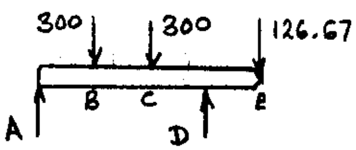


$\circlearrowleft \Sigma M_G = 0$

$-M_G + (0.2)(213.33) = 0$

$M_G = 42.67 \text{ N·m}$

Free body ABCDE

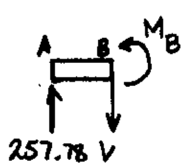


$+\circlearrowleft \Sigma M_B = 0 \qquad -0.6\,A + (0.4)(300) + (0.2)(300) - (0.2)(126.67) = 0$

$A = 257.78 \text{ N}$

$+\circlearrowleft \Sigma M_A = 0 \qquad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6\,D = 0$

$D = 468.89 \text{ N}$

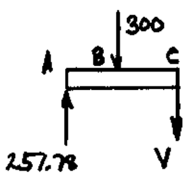


Bending moment at B

$\circlearrowleft \Sigma M_B = 0$

$-(0.2)(257.78) + M_B = 0$

$M_B = 51.56 \text{ N·m}$

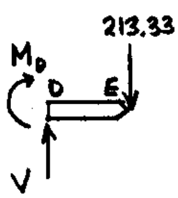


Bending moment at C

$+\circlearrowleft \Sigma M_C = 0$

$-(0.4)(257.78) + (0.2)(300) + M_C = 0$

$M_C = 43.11 \text{ N·m}$



Bending moment at D

$+\circlearrowleft \Sigma M_D = 0$

$-M_D - (0.2)(213.33) = 0$

$M_D = -25.33 \text{ N·m}$

$\max |M| = 51.56 \text{ N·m}$

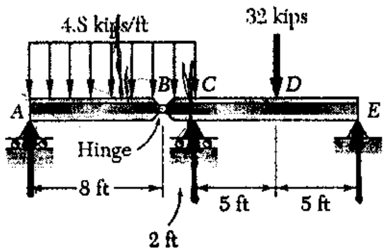
$S = \frac{1}{6} b h^2 = \frac{1}{6}(20)(30)^2 = 3 \times 10^3 \text{ mm}^3 = 3 \times 10^{-6} \text{ m}^3$

Normal stress

$\sigma = \dfrac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6 \text{ Pa} = 17.19 \text{ MPa}$ ◄

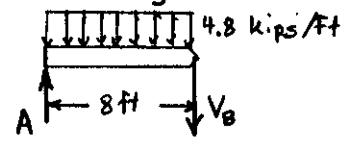
PROBLEM 5.38

5.37 and *5.38* Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending



SOLUTION

Free body AB



$\circlearrowleft \Sigma M_B = 0$

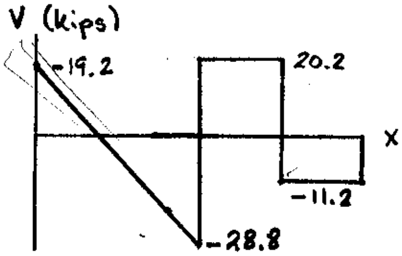
$(4.8)(8)(4) - 8A = 0$

$A = 19.2$ kips

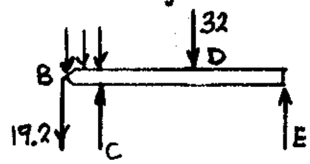
$\circlearrowleft M_A = 0$

$-(4.8)(8)(4) - 8V_B = 0$

$V_B = -19.2$ kips



Free body BCDE



$+\circlearrowleft \Sigma M_E = 0$

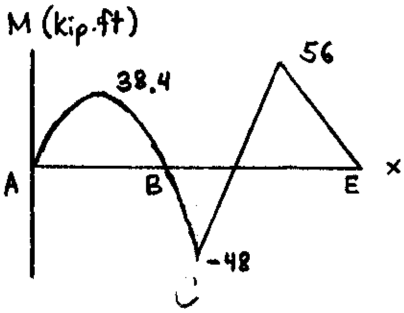
$(19.2)(12) + (4.8)(2)(11) - 10C + (32)(5) = 0$

$C = 49.2$ kips

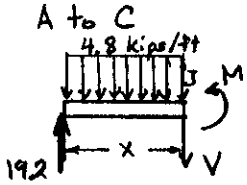
$+\circlearrowleft M_C = 0$

$(19.2)(2) + (4.8)(2)(1) - (32)(5) + 10E = 0$

$E = 11.2$ kips



A to C $\quad 0 < x < 10$ ft.



$+\uparrow \Sigma F_y = 0$

$19.2 - 4.8x - V = 0$

$V = 19.2 - 4.8x$ kips.

$\circlearrowleft M_J = 0$

$-19.2x + (4.8x)\frac{x}{2} + M = 0$

$M = 19.2x - 2.4x^2$ kip·ft

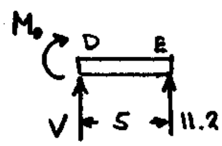
At C^- $\quad x = 10 \quad V = 19.2 - (4.8)(10) = -28.8$ kips

At C $\quad x = 10 \quad M_C = (19.2)(10) - (2.4)(10)^2 = -48$ kip·ft

C to D $\quad V = 19.2 - (4.8)(10) + 49.6 = 20.8$ kips.

D to E $\quad V = -11.2$ kips

Bending moment at D



$\circlearrowleft \Sigma M_D = 0$

$-M_D + (11.2)(5) = 0$

$M_D = 56$ kip·ft

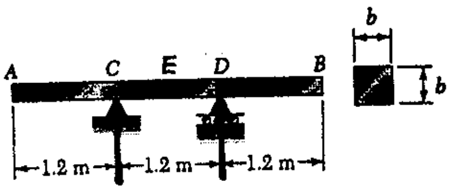
max $|M| = 56$ kip·ft $= 672$ kip·in

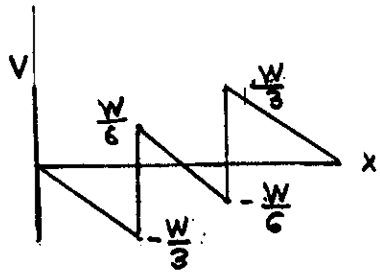
For W 12×40 rolled steel section $S = 51.9 \text{ in}^3$

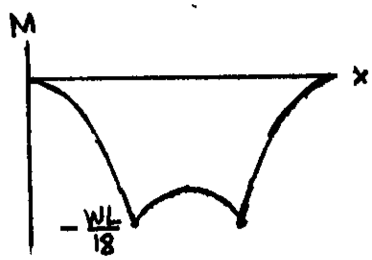
Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{672}{51.9} = 12.95$ ksi

PROBLEM 5.39

5.39 A solid steel bar has a square cross section of side b and is supported as shown. Knowing that for steel $\rho = 7860\ \text{kg/m}^3$, determine the dimension b of the bar for which the maximum normal stress due to bending is (*a*) 10 MPa, (*b*) 50 MPa.







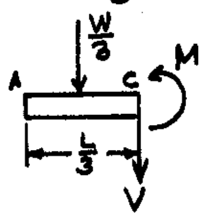
SOLUTION Weight density $\gamma = \rho g$

Let L = total length of beam

$$W = V\gamma = AL\rho g = b^2 L\rho g$$

Reactions at C and D $\quad C = D = \frac{W}{2}$

Bending moment at C

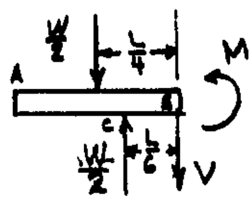


$$\circlearrowleft \Sigma M_C = 0$$

$$\left(\frac{L}{6}\right)\left(\frac{W}{3}\right) + M = 0$$

$$M = -\frac{WL}{18}$$

Bending moment at center of beam



$$\circlearrowleft \Sigma M_E = 0$$

$$\left(\frac{L}{4}\right)\left(\frac{W}{2}\right) - \left(\frac{L}{6}\right)\left(\frac{W}{2}\right) + M = 0$$

$$M = -\frac{WL}{24}$$

$$\max|M| = \frac{WL}{18} = \frac{b^2L^2\rho g}{18}$$

For a square section $\quad S = \frac{1}{6}b^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{b^2L^2\rho g/18}{b^3/6} = \frac{L^2\rho g}{3b}$

Solve for b $\quad b = \frac{L^2\rho g}{3\sigma}$

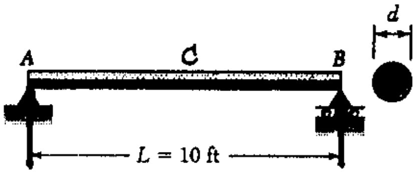
Data: $L = 3.6\ \text{m} \quad \rho = 7860\ \text{kg/m}^3 \quad g = 9.81\ \text{m/s}^2$

(a) $\sigma = 10\times10^6$ Pa $\qquad$ (b) $\sigma = 50\times10^6$ Pa

(a) $b = \frac{(3.6)^2(7860)(9.81)}{(3)(10\times10^6)} = 33.3\times10^{-3}\ \text{m} = 33.3\ \text{mm}$ ◄

(b) $b = \frac{(3.6)^2(7860)(9.81)}{(3)(50\times10^6)} = 6.66\times10^{-3}\ \text{m} = 6.66\ \text{mm}$ ◄

PROBLEM 5.40



5.40 A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 490$ lb/ft³, determine the smallest diameter d which can be used if the normal stress due to bending is not to exceed 4 ksi.

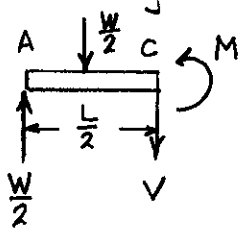
SOLUTION Let W = total weight

$$W = V\gamma = AL\gamma = \frac{\pi}{4} d^2 L \gamma$$

Reaction at A

$$A = \frac{1}{2} W$$

Bending moment at center of beam



$$+\circlearrowleft \Sigma M_C = 0$$

$$-\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32} d^2 L^2 \gamma$$

For circular cross section $(c = \frac{1}{2} d)$

$$I = \frac{\pi}{4} c^4, \qquad S = \frac{I}{c} = \frac{\pi}{4} c^3 = \frac{\pi}{32} d^3$$

Normal stress

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32} d^2 L^2 \gamma}{\frac{\pi}{32} d^3} = \frac{L^2 \gamma}{d}$$

Solving for d $\qquad d = \dfrac{L^2 \gamma}{\sigma}$

Data: $L = 10 \text{ ft} = (12)(10) = 120 \text{ in}$

$$\gamma = 490 \text{ lb/ft}^3 = \frac{490}{12^3} = 0.28356 \text{ lb/in}^3$$

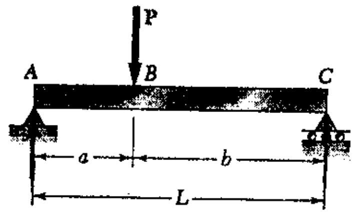
$$\sigma = 4 \text{ ksi} = 4000 \text{ lb/in}^2$$

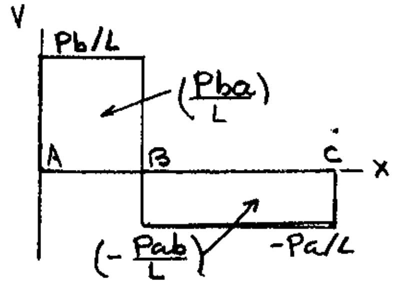
$$d = \frac{(120)^2 (0.28356)}{4000} = 1.021 \text{ in.}$$

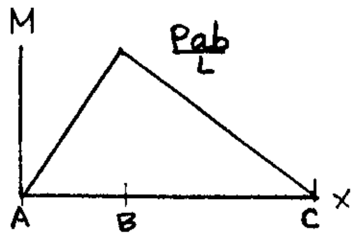
PROBLEM 5.41

5.41 Using the methods of Sec. 5.3, solve Prob. 5.1.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.







SOLUTION

$\circlearrowleft \Sigma M_C = 0 \qquad LA - bP = 0 \qquad A = \dfrac{Pb}{L}$

$\circlearrowleft \Sigma M_A = 0 \qquad LC - aP = 0 \qquad C = \dfrac{Pa}{L}$

At A^+ $\quad V = A = \dfrac{Pb}{L} \qquad M = 0$

A to B^- $\quad 0 < x < a$

$w = 0 \qquad \int_0^x w\,dx = 0$

$V_B - V_A = 0 \qquad V_B = \dfrac{Pb}{L}$

$M_B - M_A = \int_0^a V\,dx = \int_0^a \dfrac{Pb}{L}\,dx = \dfrac{Pba}{L}$

$M_B = \dfrac{Pba}{L}$

At B^+ $\quad V = A - P = \dfrac{Pb}{L} - P = -\dfrac{Pa}{L}$

B^+ to C $\quad a < x < L$

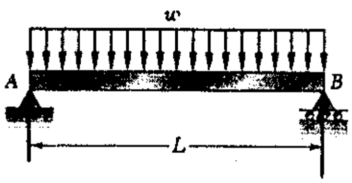
$w = 0 \qquad \int_a^x w\,dx = 0$

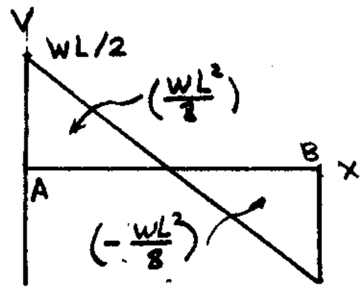
$V_C - V_B = 0 \qquad V = -\dfrac{Pa}{L}$

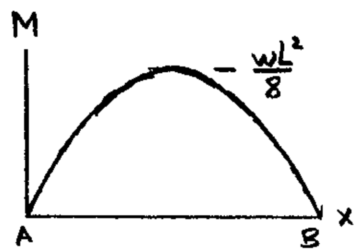
$M_C - M_B = \int_a^L V\,dx = -\dfrac{Pa}{L}(L-a) = -\dfrac{Pab}{L}$

$M_C = M_B - \dfrac{Pab}{L} = \dfrac{Pba}{L} - \dfrac{Pab}{L} = 0$

PROBLEM 5.42







5.42 Using the methods of Sec. 5.3, solve Prob. 5.2.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION

$\circlearrowleft \Sigma M_B = 0 \qquad -AL + wL\cdot\frac{L}{2} = 0 \qquad A = \frac{wL}{2}$

$\circlearrowleft \Sigma M_A = 0 \qquad BL - wL\cdot\frac{L}{2} = 0 \qquad B = \frac{wL}{2}$

$\frac{dV}{dx} = -w$

$V - V_A = -\int_0^x w\,dx = -wx$

$V = V_A - wx = A - wx = \frac{wL}{2} - wx$ ◄

$\frac{dM}{dx} = V$

$M - M_A = \int_0^x V\,dx = \int_0^x \left(\frac{wL}{2} - wx\right)dx$

$= \frac{wLx}{2} - \frac{wx^2}{2}$

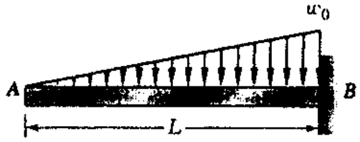
$M = M_A + \frac{wLx}{2} - \frac{wx^2}{2} = \frac{w}{2}(Lx - x^2)$ ◄

Maximum M occurs at $x = \frac{L}{2}$ where

$V = \frac{dM}{dx} = 0$

Max $M = \frac{wL^2}{8}$ ◄

PROBLEM 5.43



5.43 Using the methods of Sec. 5.3, solve Prob. 5.3.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION

$$w = w_0 \frac{x}{L}$$

$$V_A = 0, \qquad M_A = 0$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

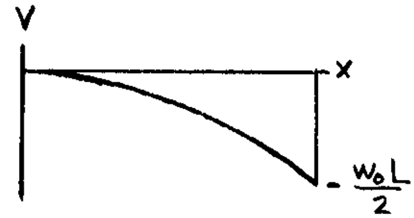
$$V - V_A = -\int_0^x \frac{w_0 x}{L} = -\frac{w_0 x^2}{2L}$$

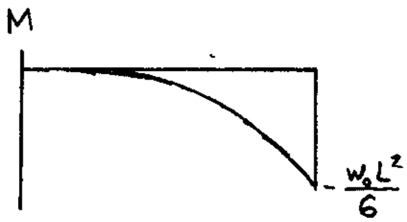
$$V = -\frac{w_0 x^2}{2L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L}$$

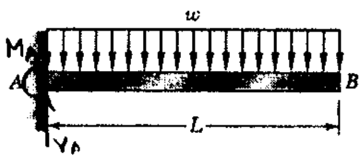
$$M - M_A = \int_0^x V\,dx = -\int_0^x \frac{w_0 x^2}{2L}\,dx$$

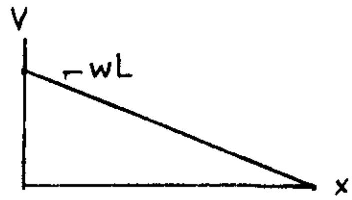
$$= -\frac{w_0 x^3}{6L}$$

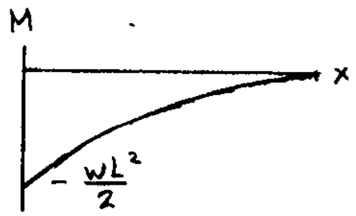




PROBLEM 5.44







5.44 Using the methods of Sec. 5.3, solve Prob. 5.4.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION

$+\uparrow \Sigma F_y = 0 \qquad V_A - wL = 0 \qquad V_A = wL$

$+\circlearrowleft \Sigma M_A = 0 \qquad -M - (wL)(\frac{L}{2}) = 0 \qquad M_A = -\frac{wL^2}{2}$

$$\frac{dV}{dx} = -w$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = wL - wx$$

$$\frac{dM}{dx} = V = wL - wx$$

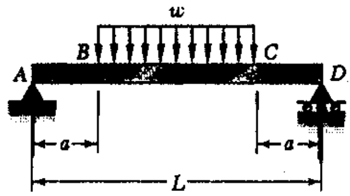
$$M - M_A = \int_0^x (wL - wx)\,dx = wLx - \frac{wx^2}{2}$$

$$M = -\frac{wL^2}{2} + wLx - \frac{wx^2}{2}$$

$\max |V| = wL$

$\max |M| = \frac{wL^2}{2}$

PROBLEM 5.45



5.45 Using the methods of Sec. 5.3, solve Prob. 5.5.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION

Reactions $\quad A = D = \frac{1}{2}w(L-2a)$

At A $\quad V_A = A = \frac{1}{2}w(L-2a), \quad M_A = 0$

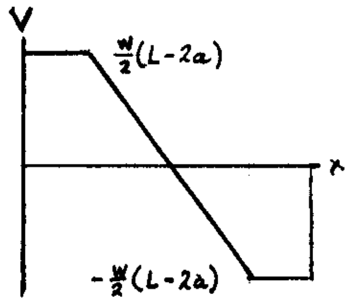
A to B $\quad 0 < x < a \qquad w = 0$

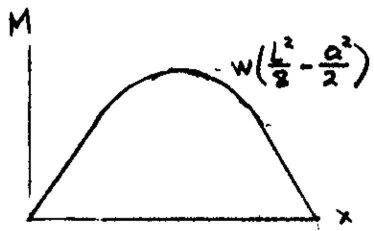
$$V_B - V_A = -\int_0^a w\,dx = 0$$

$$V_B = V_A = \frac{1}{2}w(L-2a)$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{1}{2}w(L-2a)\,dx$$

$$M_B = \frac{1}{2}w(L-2a)a$$





B to C $\quad a < x < L-a \qquad w = w$

$$V - V_B = -\int_a^x w\,dx = -w(x-a)$$

$$V = \frac{1}{2}w(L-2a) - w(x-a) = \frac{1}{2}w(L-2x)$$

$$\frac{dM}{dx} = V = \frac{1}{2}w(L-2x)$$

$$M - M_B = \int_a^x V\,dx = \frac{1}{2}w(Lx - x^2)\Big|_a^x$$

$$\ldots = \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$M = \frac{1}{2}w(L-2a)a + \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$= \frac{1}{2}w(Lx - x^2 - a^2)$$

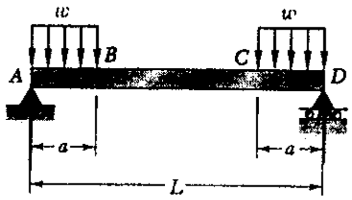
At C $\quad x = L-a \qquad V_C = -\frac{1}{2}w(L-2a) \qquad M_C = \frac{1}{2}(L-2a)a$

C to D $\quad V = V_C = -\frac{1}{2}w(L-2a)$

$$M_D = 0$$

At $x = \frac{L}{2} \qquad M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$

PROBLEM 5.46



5.46 Using the methods of Sec. 5.3, solve Prob. 5.6.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION

Reactions $\quad A = D = wa$

A to B $\quad 0 < x < a \qquad w = w$

$$V_A = A = wa, \qquad M_A = 0$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

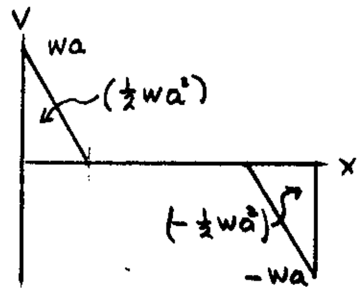
$$V = w(a-x) \qquad V_B = 0$$

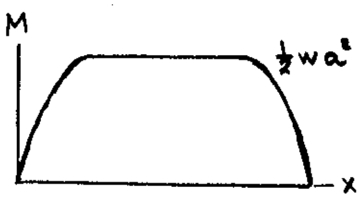
$$\frac{dM}{dx} = V = wa - wx$$

$$M - M_A = \int_0^x V\,dx = \int_0^x (wa - wx)\,dx$$

$$= wax - \tfrac{1}{2}wx^2$$

$$M_B = \tfrac{1}{2}wa^2 \quad \text{at } x = a.$$





B to C $\quad a < x < L - a \qquad V = 0$

$$\frac{dM}{dx} = V = 0$$

$$M - M_B = \int_a^x V\,dx = 0$$

$$M = M_B = \tfrac{1}{2}wa^2$$

C to D

$$V - V_C = -\int_{L-a}^{x} w\,dx = -w[x - (L-a)]$$

$$V = -w[x - (L-a)]$$

$$M - M_C = \int_{L-a}^{x} V\,dx = \int_{L-a}^{x} -w[x - (L-a)]\,dx$$

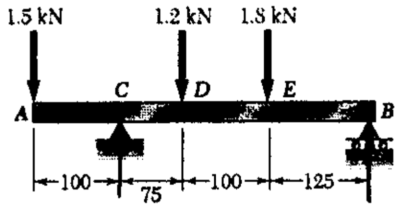
$$= -w\left[\frac{x^2}{2} - (L-a)x\right]\Big|_{L-a}^{x}$$

$$= -w\left[\frac{x^2}{2} - (L-a)x - \frac{(L-a)^2}{2} + (L-a)^2\right]$$

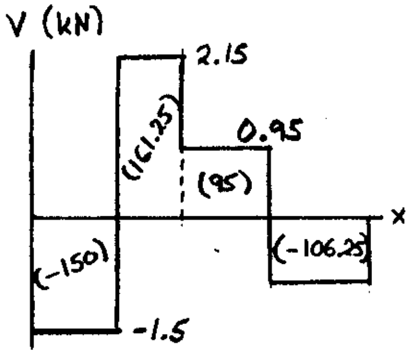
$$= -w\left[\frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2}\right]$$

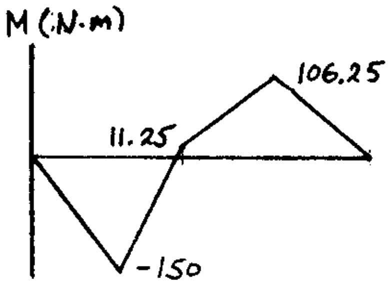
$$M = \tfrac{1}{2}wa^2 - w\left[\frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2}\right]$$

PROBLEM 5.47



Dimensions in mm





5.47 Using the methods of Sec. 5.3, solve Prob. 5.13.

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$\circlearrowleft + \Sigma M_B = 0$

$(400)(1.5) - 300\,C + (225)(1.2) + (125)(1.8) = 0$

$C = 3.65 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0 \qquad B = 0.85 \text{ kN}$

Shear:

A to C $\quad V = -1.5 \text{ kN}$

C to D $\quad V = -1.5 + 3.65 = 2.15 \text{ kN}$

D to E $\quad V = 2.15 - 1.2 = 0.95 \text{ kN}$

E to B $\quad V = 0.95 - 1.8 = -0.85 \text{ kN}$

Areas of shear diagram

A to C $\quad \int V dx = (-1.5)(100) = -150 \text{ N·m}$

C to D $\quad \int V dx = (2.15)(75) = 161.25 \text{ N·m}$

D to E $\quad \int V dx = (0.95)(100) = 95 \text{ N·m}$

E to B $\quad \int V dx = (-0.85)(125) = -106.25 \text{ N·m}$

Bending moments

$M_A = 0$

$M_C = M_A + \int_A^C V dx = 0 - 150 = -150 \text{ N·m}$

$M_D = M_C + \int_C^D V dx = -150 + 161.25 = 11.25 \text{ N·m}$

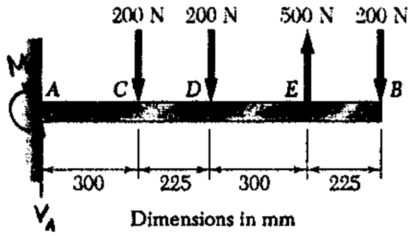
$M_E = M_D + \int_D^E V dx = 11.25 + 95 = 106.25 \text{ N·m}$

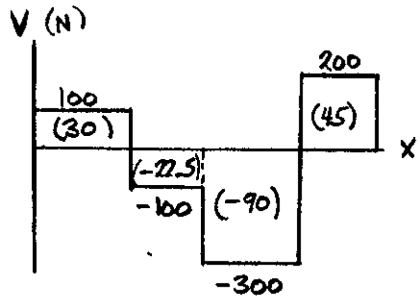
$M_B = M_E + \int_D^B V dx = 106.25 - 106.25 = 0$

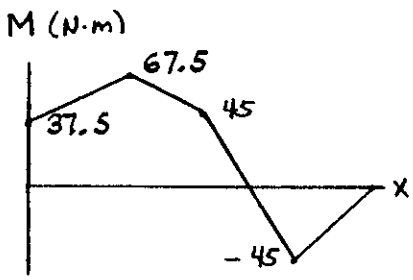
Maximum $|V| = 2.15 \text{ kN}$ ◄

Maximum $|M| = 150 \text{ N·m}$ ◄

PROBLEM 5.48







5.48 Using the methods of Sec. 5.3, solve Prob. 5.14.

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$$\circlearrowleft\!+ \ \Sigma M_A = 0$$

$$-M_A - (0.3)(200) - (0.525)(200) + (0.825)(500) - (1.05)(200) = 0$$

$$M_A = 37.5 \text{ N}\cdot\text{m}$$

$$+\uparrow \Sigma F_y = 0$$

$$V_A - 200 - 200 + 500 - 200 = 0$$

$$V_A = 100 \text{ N}$$

Shear

A to C $\quad V = 100 \text{ N}$

C to D $\quad V = 100 - 200 = -100 \text{ N}$

D to E $\quad V = -100 - 200 = -300 \text{ N}$

E to B $\quad V = -300 + 500 = 200 \text{ N}$

Areas under shear diagram

A to C $\quad \int V dx = (100)(0.3) = 30 \text{ N}\cdot\text{m}$

C to D $\quad \int V dx = (-100)(0.225) = -22.5 \text{ N}\cdot\text{m}$

D to E $\quad \int V dx = (-300)(0.3) = -90 \text{ N}\cdot\text{m}$

E to B $\quad \int V dx = (200)(0.225) = 45 \text{ N}\cdot\text{m}$

Bending moments

$$M_A = 37.5 \text{ N}\cdot\text{m}$$

$$M_C = M_A + \int_A^C V dx = 37.5 + 30 = 67.5 \text{ N}\cdot\text{m}$$

$$M_D = M_C + \int_C^D V dx = 67.5 - 22.5 = 45 \text{ N}\cdot\text{m}$$

$$M_E = M_D + \int_D^E V dx = 45 - 90 = -45 \text{ N}\cdot\text{m}$$

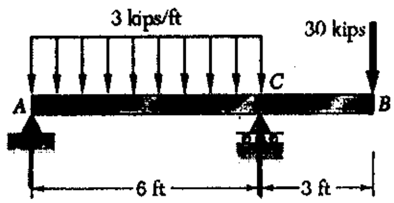
$$M_B = M_E + \int_E^D V dx = -45 + 45 = 0$$

Maximum $|V| = 300 \text{ N}$ ◄

Maximum $|M| = 67.5 \text{ N}\cdot\text{m}$ ◄

5.49 Using the methods of Sec. 5.3, solve Prob. 5.15.

PROBLEM 5.49



5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

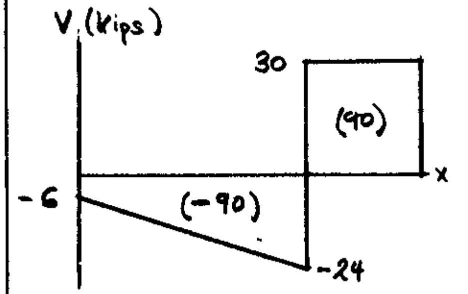
SOLUTION

$+\circlearrowleft \Sigma M_C = 0 \quad -6A + (3)(18) - (3)(30) = 0$

$A = -6$ kips i.e. 6 kips $\downarrow$

$+\circlearrowleft \Sigma M_A = 0 \quad 6C - (3)(18) - (9)(30) = 0$

$C = 54$ kips $\uparrow$



Shear

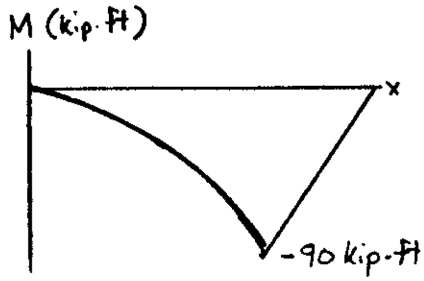
$V_A = -6$ Kips

A to C $\quad 0 < x < 6$ ft. $\quad w = -3$ kips/ft

$V_B - V_A = -\int_0^6 w\,dx = -\int_0^6 3\,dx = -18$ Kips

$V_B = -6 - 18 = -24$ kips

C to B $\quad V = -24 + 54 = 30$ kips.



Areas under shear diagram

A to C $\quad \int V dx = (\frac{1}{2})(-6-24)(6)$

$= -90$ kip ft.

C to B $\quad \int V\,dx = (3)(30) = 90$ kip·ft

Bending moments

$M_A = 0$

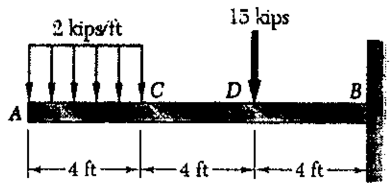
$M_C = M_A + \int V dx = 0 - 90 = -90$ kip·ft

$M_B = M_C + \int V dx = -90 + 90 = 0$

Maximum $|V| = 30$ Kips ◀

Maximum $|M| = 90$ kip·ft ◀

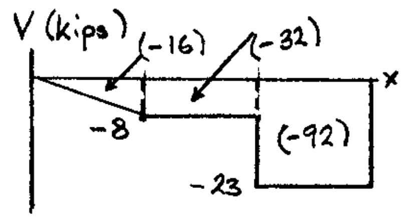
PROBLEM 5.50

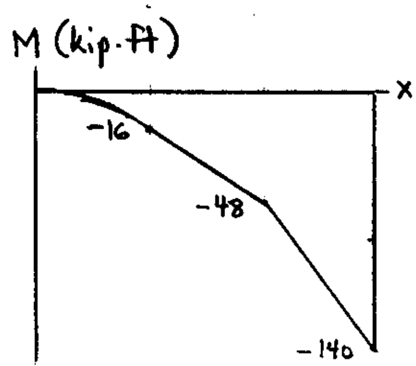


5.50 Using the methods of Sec. 5.3, solve Prob. 5.16.

5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION





Shear

$V_A = 0$

$V_B = V_A - \int_A^B w\,dx = 0 - (4)(2) = -8$ kips.

C to D $\quad V = -8$ kips

D to B $\quad V = -8 - 15 = -23$ kips

Areas under shear diagram

A to C $\quad \int V dx = (\frac{1}{2})(4)(-8) = -16$ kip·ft

C to D $\quad \int V dx = (4)(-8) = -32$ kip·ft.

D to B $\quad \int V dx = (4)(-23) = -92$ kip·ft

Bending moments

$M_A = 0$

$M_C = M_A + \int V\,dx = 0 - 16 = -16$ kip·ft.

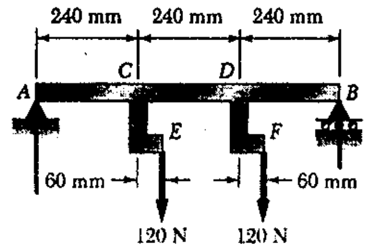
$M_D = M_C + \int V dx = -16 - 32 = -48$ kip·ft

$M_B = M_D + \int V dx = -48 - 92 = -140$ kip·ft

Maximum $|V| = 23$ kips ◀

Maximum $|M| = 140$ kip·ft ◀

PROBLEM 5.51



5.51* and *5.52 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

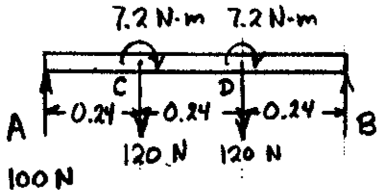
SOLUTION

$+\circlearrowleft \Sigma M_B = 0 \qquad -0.72A + (0.48)(120) + (0.24)(120) - 7.2 - 7.2 = 0$

$A = 100 \text{ N}$

$\circlearrowleft \Sigma M_A = 0 \qquad -(0.24)(120) - (0.48)(120) - 7.2 - 7.2 + 0.72B = 0$

$B = 140 \text{ N}$

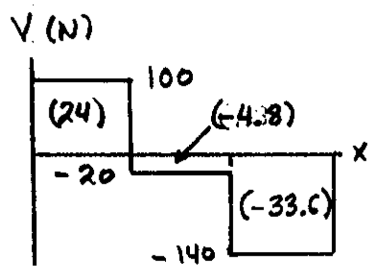


Shear

A to C $\quad V = 100 \text{ N}$

C to D $\quad V = 100 - 120 = -20 \text{ N}$

D to B $\quad V = -20 - 120 = -140 \text{ N}$

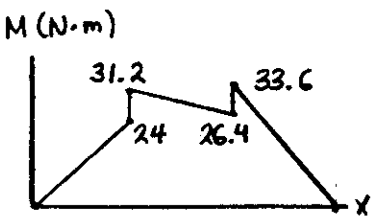


Areas under shear diagram

A to C $\quad \int V\,dx = (0.24)(100) = 24 \text{ N}\cdot\text{m}$

C to D $\quad \int V\,dx = (0.24)(-20) = -4.8 \text{ N}\cdot\text{m}$

D to B $\quad \int V\,dx = (0.24)(-140) = -33.6 \text{ N}\cdot\text{m}$



Bending moments

$M_A = 0$

$M_C^- = 0 + 24 = 24 \text{ N}\cdot\text{m}$

$M_C^+ = 24 + 7.2 = 31.2 \text{ N}\cdot\text{m}$

$M_D^- = 31.2 - 4.8 = 26.4 \text{ N}\cdot\text{m}$

$M_D^+ = 26.4 + 7.2 = 33.6 \text{ N}\cdot\text{m}$

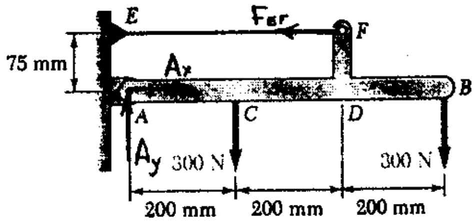
$M_B = 33.6 - 33.6 = 0$

Maximum $|V| = 140 \text{ N}$ ◄

Maximum $|M| = 33.6 \text{ N}\cdot\text{m}$ ◄

PROBLEM 5.52

5.51* and *5.52 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



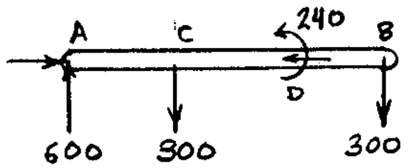
SOLUTION

$+\circlearrowleft \Sigma M_A = 0$

$0.075\, F_{EF} - (0.2)(300) - (0.6)(300) = 0$

$F_{EF} = 3.2 \times 10^3 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0 \qquad A_x - F_{EF} = 0 \qquad A_x = 3.2 \times 10^3 \text{ N}$

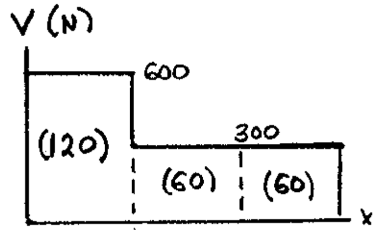


$+\uparrow \Sigma F_y = 0 \qquad A_y - 300 - 300 = 0$

$A_y = 600 \text{ N}$

Couple at D $\quad M_D = (0.075)(3.2 \times 10^3)$

$= 240 \text{ N}\cdot\text{m}$



Shear

A to C $\quad V = 600 \text{ N}$

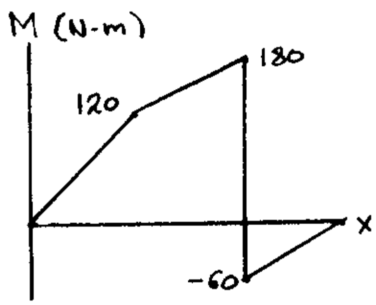
C to B $\quad V = 600 - 300 = 300 \text{ N}$

Areas under shear diagram

A to C $\quad \int V dx = (0.2)(600) = 120 \text{ N}\cdot\text{m}$

C to D $\quad \int V dx = (0.2)(300) = 60 \text{ N}\cdot\text{m}$

D to B $\quad \int V dx = (0.2)(300) = 60 \text{ N}\cdot\text{m}$



Bending moments

$M_A = 0$

$M_C = 0 + 120 = 120 \text{ N}\cdot\text{m}$

$M_D^- = 120 + 60 = 180 \text{ N}\cdot\text{m}$

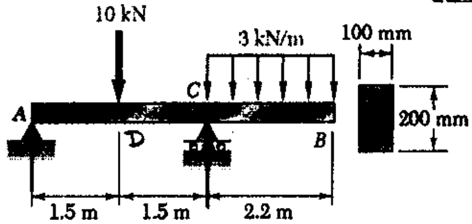
$M_D^+ = 180 - 240 = -60 \text{ N}\cdot\text{m}$

$M_B = -60 + 60 = 0$

Maximum $|V| = 600 \text{ N}$

Maximum $|M| = 180 \text{ N}\cdot\text{m}$

PROBLEM 5.53



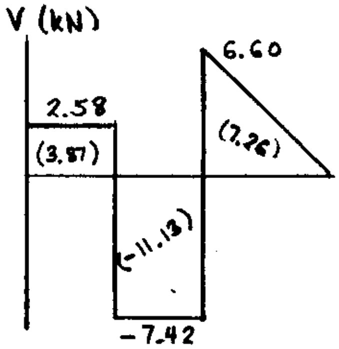
5.53 Using the methods of Sec. 5.3, solve Prob. 5.21.

5.21 For the beam and loading shown, determine the maximum normal stress on a transverse section at C.

SOLUTION

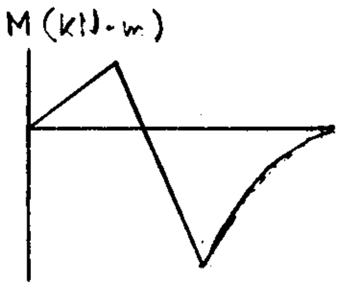
$$+\circlearrowleft \Sigma M_C = 0$$
$$-3A + (1.5)(10) - (1.1)(2.2)(3) = 0$$
$$A = 2.58 \text{ kN}$$

$$+\circlearrowleft \Sigma M_A = 0$$
$$-(1.5)(10) + 3C - (4.1)(2.2)(3) = 0$$
$$C = 14.02 \text{ kN}$$



Shear

A to D^-	$V = 2.58$ kN
D^+ to C^-	$V = 2.58 - 10 = -7.42$ kN
C^+	$V = -7.42 + 14.02 = 6.60$ kN
B	$V = 6.60 - (2.2)(3) = 0$



Areas under shear diagram

A to D	$\int V dx = (1.5)(2.58) = 3.87$ kN·m
D to C	$\int V dx = (1.5)(-7.42) = -11.13$ kN·m
C to B	$\int V dx = (\frac{1}{2})(2.2)(6.60) = 7.26$ kN·m

Bending moments

$$M_A = 0$$
$$M_D = 0 + 3.87 = 3.87 \text{ kN·m}$$
$$M_C = 3.87 - 11.13 = -7.26 \text{ kN·m}$$
$$M_B = 7.26 - 7.26 = 0$$

$$|M_C| = 7.26 \text{ kN·m} = 7.26 \times 10^3 \text{ N·m}$$

For rectangular cross section $S = \frac{1}{6}bh^2 = (\frac{1}{6})(100)(200)^2$

$$= 666.67 \times 10^3 \text{ mm}^3 = 666.67 \times 10^{-6} \text{ m}^2$$

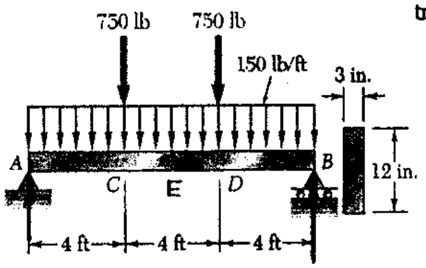
Normal stress $\sigma = \dfrac{|M_C|}{S} = \dfrac{7.26 \times 10^3}{666.67 \times 10^{-6}} = 10.89 \times 10^6$ Pa

$= 10.89$ MPa

PROBLEM 5.54

5.54 Using the methods of Sec. 5.3, solve Prob. 5.22.

5.22 For the beam and loading shown, determine the maximum normal stress on a transverse section at the center of the beam

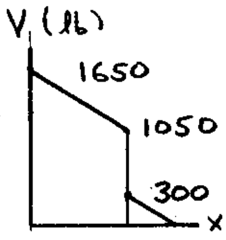


SOLUTION

Reactions : $C = A$ by symmetry

$+\uparrow \Sigma F_y = 0 \quad A + C - (2)(750) - (12)(150) = 0$

$A = C = 1650$ lb.



Shear :

$V_A = 1650$ lb.

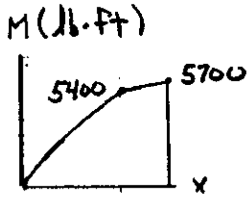
$V_{C^-} = 1650 - (4)(150) = 1050$ lb

$V_{C^+} = 1050 - 750 = 300$ lb

$V_E = 300 - (2)(150) = 0$

Areas under shear diagram

A to C $\quad \int V\,dx = (\frac{1}{2})(1650 + 1050)(4)$
$= 5400$ lb·ft



C to E $\quad \int V\,dx = (\frac{1}{2})(300)(2) = 300$ lb·ft.

Bending moments

$M_A = 0$

$M_C = 0 + 5400 = 5400$ lb·ft

$M_E = 5400 + 300 = 5700$ lb·ft

$M_E = 5700$ lb·ft $= 68.4 \times 10^3$ lb·in.

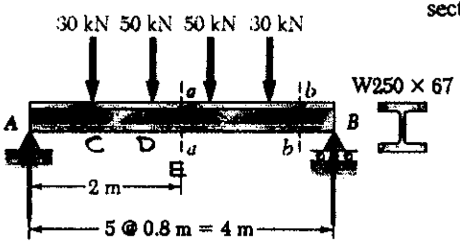
For rectangular cross section $\quad S = \frac{1}{6} b h^2 = (\frac{1}{6})(3)(12)^2 = 72$ in^3

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{68.4 \times 10^3}{72} = 950$ psi ◄

PROBLEM 5.55

5.55 Using the methods of Sec. 5.3, solve Prob. 5.23.

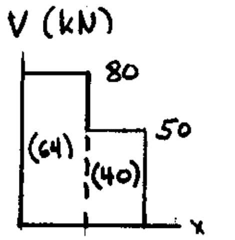
5.23 For the beam and loading shown, determine the maximum normal stress on section *a-a*.



SOLUTION

Reactions: By symmetry $A = B$

$+\uparrow \Sigma F_y = 0 \qquad A = B = 80\ \text{kN}$



Shear

A to C $\quad V = 80\ \text{kN}$

C to D $\quad V = 80 - 30 = 50\ \text{kN}$

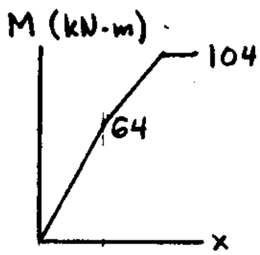
D to E $\quad V = 50 - 50 = 0$

Areas under shear diagram

A to C $\quad \int V dx = (80)(0.8) = 64\ \text{kN}\cdot\text{m}$

C to D $\quad \int V dx = (50)(0.8) = 40\ \text{kN}\cdot\text{m}$

D to E $\quad \int V dx = 0$



Bending moments

$M_A = 0$

$M_C = 0 + 64 = 64\ \text{kN}\cdot\text{m}$

$M_D = 64 + 40 = 104\ \text{kN}\cdot\text{m}$

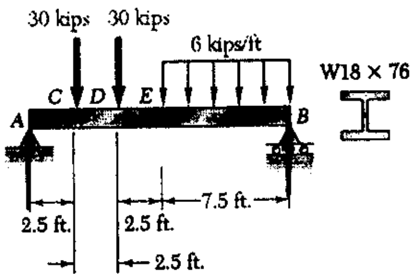
$M_E = 104 + 0 = 104\ \text{kN}\cdot\text{m}$

$M_E = 104\ \text{kN}\cdot\text{m} = 104 \times 10^3\ \text{N}\cdot\text{m}$

For W 250 × 67 $\quad S = 809 \times 10^3\ \text{mm}^3 = 809 \times 10^{-6}\ \text{m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{104 \times 10^3}{809 \times 10^{-6}} = 128.6 \times 10^6\ \text{Pa} = 128.6\ \text{MPa}$ ◄

PROBLEM 5.56



5.56 Using the methods of Sec. 5.3, solve Prob. 5.24.

5.24 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0$$

$$-15A + (12.5)(30) + (10)(30) + (6)(7.5)(3.75) = 0$$

$$A = 56.25 \text{ kips}$$

Shear A to C $\quad V = 56.25$ kips

Area under shear curve A to C. $\int V\,dx = (56.25)(2.5) = 140.625$ kip·ft

$M_A = 0$

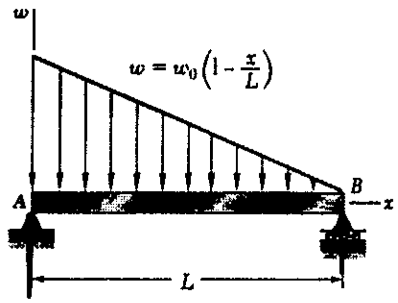
$M_C = 0 + 140.625 = 140.625$ kip·ft $= 1687.5$ kip·in

For W 18×76 rolled steel section $\quad S = 146 \text{ in}^3$

Normal stress $\quad \sigma = \frac{M}{S} = \frac{1687.5}{146} = 11.56$ ksi ◄

PROBLEM 5.57

5.57 and 5.58 Determine (*a*) the equations of the shear and bending-moment curves for the given beam and loading, *(b)* the maximum absolute value of the bending moment in the beam.



SOLUTION

$$w = w_0\left(1 - \frac{x}{L}\right)$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{w_0 x}{L}$$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$M = 0$ at $x = L$ $\qquad 0 = -\frac{w_0 L^2}{2} + \frac{w_0 L^2}{6} + C_1 L \quad \therefore \quad C_1 = \frac{w_0 L}{3}$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + \frac{w_0 L}{3}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + \frac{w_0 L x}{3}$$

M is maximum where $\frac{dM}{dx} = V = 0$

$$0 = -w_0 x_m + \frac{w_0 x_m^2}{2L} + \frac{w_0 L}{3}$$

$$\frac{1}{2}x_m^2 - L x_m + \frac{1}{3}L^2 = 0 \qquad x_m = \frac{L \pm \sqrt{L^2 - (4)(\frac{1}{2})(\frac{1}{3}L^2)}}{(2)(\frac{1}{2})}$$

$$= \left(1 \pm \frac{\sqrt{3}}{3}\right)L$$

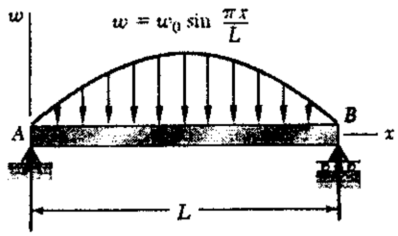
$$= \cancel{1.57735L},\ 0.42265\,L$$

$$M_{max} = \frac{-w_0(0.42265\,L)^2}{2} + \frac{w_0(0.42265\,L)^3}{6L} + \frac{w_0 L(0.42265L)}{3}$$

$$= 0.06415\ w_0 L^2$$

PROBLEM 5.58

5.57 and 5.58 Determine (*a*) the equations of the shear and bending-moment curves for the given beam and loading, (*b*) the maximum absolute value of the bending moment in the beam.



SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin\frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi}\cos\frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi x}{L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$M = 0$ at $x = L$ $\qquad 0 = 0 + C_1 L + 0 \qquad C_1 = 0$

$$V = \frac{w_0 L}{\pi}\cos\frac{\pi x}{L}$$

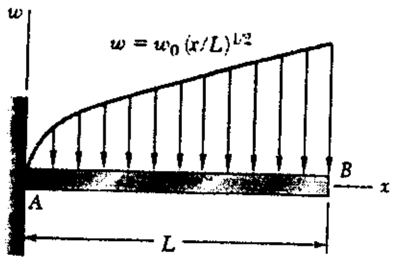
$$M = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = 0 \quad \text{at} \quad x = \frac{L}{2}$$

$$M_{max} = \frac{w_0 L^2}{\pi^2}\sin\frac{\pi}{2} = \frac{w_0 L^2}{\pi^2}$$

PROBLEM 5.59

5.59 Determine (*a*) the equations of the shear and bending-moment curves for the given beam and loading, (*b*) the maximum absolute value of the bending moment in the beam.



SOLUTION

$$\frac{dV}{dx} = -w = -w_0\left(\frac{x}{L}\right)^{\frac{1}{2}} = -\frac{w_0 x^{1/2}}{L^{1/2}}$$

$$V = -\frac{2}{3}\frac{w_0 x^{3/2}}{L^{1/2}} + C_1$$

$V = 0$ at $x = L$

$$0 = -\frac{2}{3}w_0 L + C_1 \qquad C_1 = \frac{2}{3}w_0 L$$

$$V = \frac{2}{3}w_0 L - \frac{2}{3}\frac{w_0 x^{3/2}}{L^{1/2}}$$ ◄

$$\frac{dM}{dx} = V \qquad M = C_2 + \frac{2}{3}w_0 L x - \frac{2}{3}\cdot\frac{2}{5}\frac{w_0 x^{5/2}}{L^{1/2}}$$

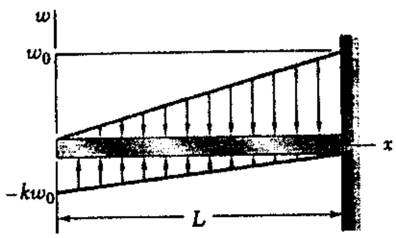
$M = 0$ at $x = L$ $\qquad 0 = C + \frac{2}{3}w_0 L^2 - \frac{4}{15}w_0 L^2 \qquad C_2 = -\frac{2}{5}w_0 L^2$

$$M = \frac{2}{3}w_0 L x - \frac{4}{15}\frac{w_0 x^{5/2}}{L^{1/2}} - \frac{2}{5}w_0 L^2$$ ◄

$|M|_{max}$ occurs at $x = 0$ $\qquad |M|_{max} = \frac{2}{5}w_0 L^2$ ◄

PROBLEM 5.60

5.60 For the beam and loading shown, determine the equations of the shear and bending-moment curves and the maximum absolute value of the bending moment in the beam, knowing that (*a*) $k = 1$, (*b*) $k = 0.5$.



SOLUTION

$$w = \frac{w_0 x}{L} - \frac{k w_0 (L-x)}{L} = (1+k)\frac{w_0 x}{L} - k w_0$$

$$\frac{dV}{dx} = -w = k w_0 - (1+k)\frac{w_0 x}{L}$$

$$V = k w_0 x - (1+k)\frac{w_0 x^2}{2L} + C_1$$

$V = 0$ at $x = 0$ $\qquad C_1 = 0$

$$\frac{dM}{dx} = V = k w_0 x - (1+k)\frac{w_0 x^2}{2L}$$

$$M = \frac{k w_0 x^2}{2} - (1+k)\frac{w_0 x^3}{6L} + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$$M = \frac{k w_0 x^2}{2} - \frac{(1+k) w_0 x^3}{6L}$$

(a) $k = 1$

$$V = w_0 x - \frac{w_0 x^2}{L}$$

$$M = \frac{w_0 x^2}{2} - \frac{w_0 x^3}{3L}$$

Maximum M occurs at $x = L$ $\qquad |M|_{max} = \frac{w_0 L^2}{6}$

(b) $k = \frac{1}{2}$

$$V = \frac{w_0 x}{2} - \frac{3 w_0 x^2}{4L}$$

$$M = \frac{w_0 x^2}{4} - \frac{w_0 x^3}{4L}$$

$V = 0$ at $x = \frac{2}{3}L$

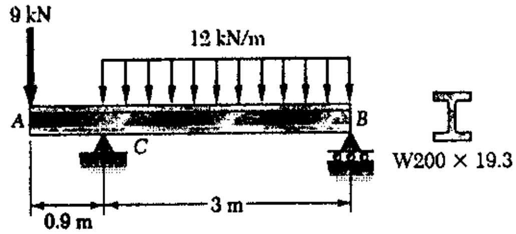
At $x = \frac{2}{3}L$ $\quad M = \frac{w_0(\frac{2}{3}L)^2}{4} - \frac{w_0(\frac{2}{3}L)^3}{4L} = \frac{w_0 L^2}{27} = 0.03704\, w_0 L^2$

At $x = L$ $\quad M = 0$

$$|M|_{max} = \frac{w_0 L^2}{27}$$

PROBLEM 5.61

5.61 and 5.62 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$+\circlearrowleft \Sigma M_C = 0$

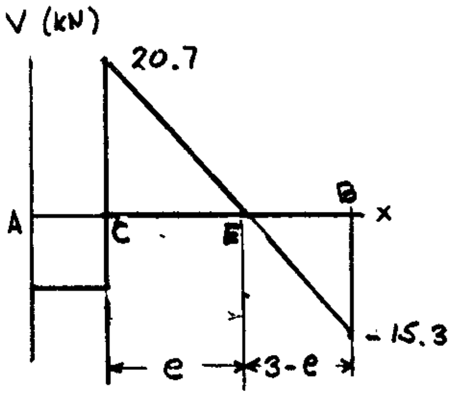
$(0.9)(9) - (1.5)(3)(12) + 3B = 0$

$B = 15.3$ kN

$+\circlearrowleft \Sigma M_B = 0$

$(3.9)(9) - 3C + (1.5)(3)(12) = 0$

$C = 29.7$ kN



Shear: A to C $\quad V = -9$ kN

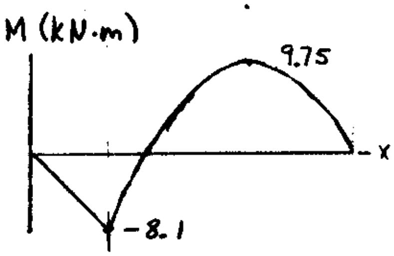
$C^+ \quad V = -9 + 29.7 = 20.7$ kN

B $\quad V = 20.7 - (3)(12) = -15.3$ kN

Locate point E where $V = 0$

$\frac{e}{20.7} = \frac{3-e}{15.3} \qquad 36e = (20.7)(3)$

$e = 1.725$ ft $\qquad 3 - e = 1.275$ ft



Areas under shear diagram

A to C $\quad \int V dx = (0.9)(9) = 8.1$ kN·m

C to E $\quad \int V dx = (\frac{1}{2})(1.725)(20.7)$
$= 17.85375$ kN·m

E to B $\quad \int V dx = (\frac{1}{2})(-1.275)(15.3)$
$= -9.75375$ kN·m

Bending moments

$M_A = 0$

$M_B = 0 - 8.1 = -8.1$ kN·m

$M_E = -8.1 + 17.85375 = 9.75375$ kN·m

$M_B = 9.75375 - 9.75375 = 0$

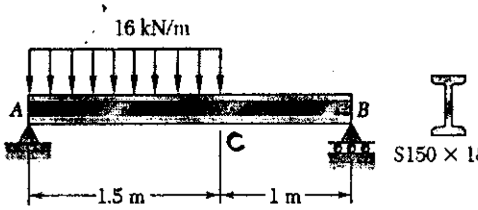
Maximum $|M| = 9.75375 \times 10^3$ N·m at point E

For W 200 × 19.3 rolled steel section $S = 164 \times 10^3\ mm^3 = 164 \times 10^{-6}\ m^3$

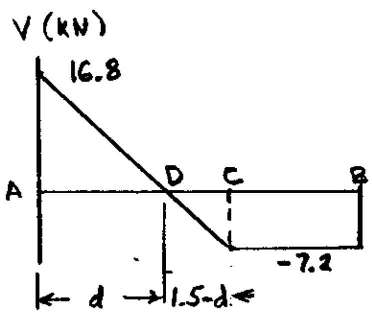
Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{9.75375 \times 10^3}{164 \times 10^{-6}} = 59.5 \times 10^6$ Pa $= 59.5$ MPa ◄

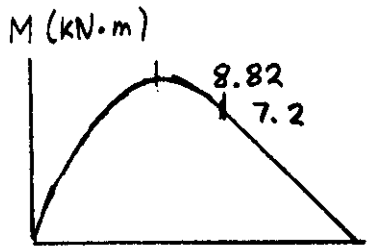
PROBLEM 5.62

5.61 and 5.62 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



S150 × 18.6





SOLUTION

$+\circlearrowleft \Sigma M_B = 0$

$-2.5A + (1.75)(1.5)(16) = 0$

$A = 16.8 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$

$-(0.75)(1.5)(16) + 2.5B = 0$

$B = 7.2 \text{ kN}$

Shear

$V_A = 16.8 \text{ kN}$

$V_C = 16.8 - (1.5)(16) = -7.2 \text{ kN}$

$V_B = -7.2 \text{ kN}$

Locate point D where $V = 0$

$\frac{d}{16.8} = \frac{1.5-d}{7.2} \qquad 24d = 25.2$

$d = 1.05 \text{ m} \qquad 1.5 - d = 0.45 \text{ m}$

Areas under shear diagram

A to D $\int V dx = (\frac{1}{2})(1.05)(16.8) = 8.82 \text{ kN}\cdot\text{m}$

D to C $\int V dx = (\frac{1}{2})(0.45)(-7.2) = -1.62 \text{ kN}\cdot\text{m}$

C to B $\int V dx = (1)(-7.2) = -7.2 \text{ kN}\cdot\text{m}$

Bending moments

$M_A = 0$

$M_D = 0 + 8.82 = 8.82 \text{ kN}\cdot\text{m}$

$M_C = 8.82 - 1.62 = 7.2 \text{ kN}\cdot\text{m}$

$M_B = 7.2 - 7.2 = 0$

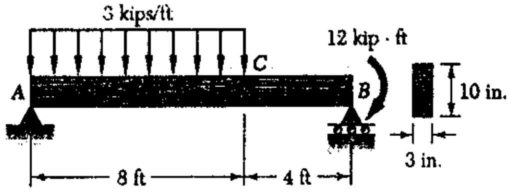
Maximum $|M| = 8.82 \text{ kN}\cdot\text{m} = 8.82 \times 10^3 \text{ N}\cdot\text{m}$

For S 150 × 18.6 rolled steel section $S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

Normal stress $\sigma = \frac{|M|}{S} = \frac{8.82 \times 10^3}{120 \times 10^{-6}} = 73.5 \times 10^6 \text{ Pa} = 73.5 \text{ MPa}$ ◄

PROBLEM 5.63

5.63 and 5.64 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$+\circlearrowleft \Sigma M_B = 0$

$-12A + (8)(8)(3) - 12 = 0$

$A = 15$ kips

$\circlearrowleft \Sigma M_A = 0$

$-(4)(8)(3) + 12B - 12 = 0$

$B = 9$ kips

Shear: $V_A = 15$ kips

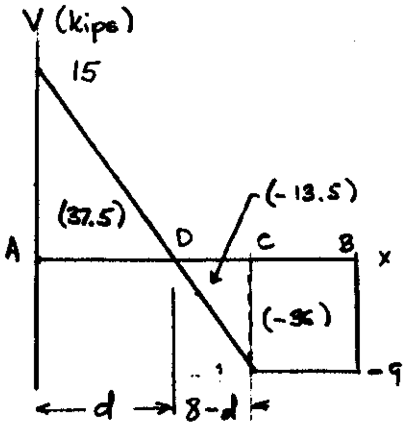
$V_C = 15 - (8)(3) = -9$ kips

C to B $\quad V = -9$ kips.

Locate point D where $V = 0$

$\frac{d}{15} = \frac{8-d}{9} \qquad 24d = 120$

$d = 5$ ft $\qquad 8 - d = 3$ ft

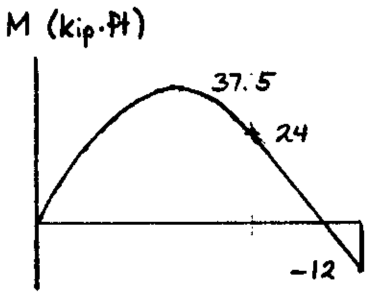


Areas under shear diagram

A to D $\quad \int V\,dx = (\frac{1}{2})(5)(15) = 37.5$ kip·ft

D to C $\quad \int V\,dx = (\frac{1}{2})(3)(-9) = -13.5$ kip·ft

C to B $\quad \int V\,dx = (4)(-9) = -36$ kip·ft



Bending moments: $\quad M_A = 0$

$M_D = 0 + 37.5 = 37.5$ kip·ft

$M_C = 37.5 - 13.5 = 24$ kip·ft

$M_B = 24 - 36 = -12$ kip·ft

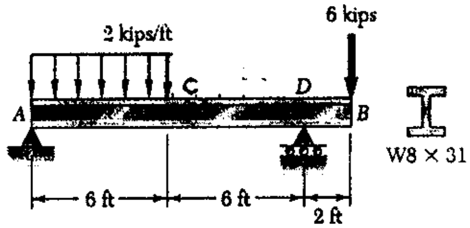
Maximum $|M| = 37.5$ kip·ft $= 450$ kip·in

For rectangular cross section $\quad S = \frac{1}{6}bh^2 = (\frac{1}{6})(3)(10)^2 = 50$ in^3

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{450}{50} = 9$ ksi

PROBLEM 5.64

5.63 and 5.64 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$+\circlearrowleft M_D = 0$

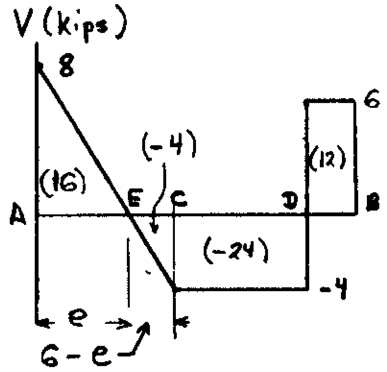
$-12A + (9)(6)(2) - (2)(6) = 0$

$A = 8$ kips

$+\circlearrowleft M_A = 0$

$-(3)(6)(2) + 12D - (14)(6) = 0$

$D = 10$ kips



Shear: $V_A = 8$ kips

$V_C = 8 - (6)(2) = -4$ kips

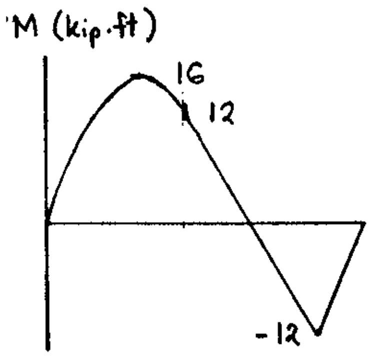
C to D $\quad V = -4$ kips

D to B $\quad V = -4 + 10 = 6$ kips.

Locate point E where $V = 0$

$$\frac{e}{8} = \frac{6-e}{4} \qquad 12e = 48$$

$$e = 4 \text{ ft} \qquad 6 - e = 3 \text{ ft}$$



Areas under shear diagram

A to E $\quad \int V\,dx = (\tfrac{1}{2})(4)(8) = 16$ kip·ft

E to C $\quad \int V\,dx = (\tfrac{1}{2})(2)(-4) = -4$ kip·ft

C to D $\quad \int V\,dx = (6)(-4) = -24$ kip·ft

D to B $\quad \int V\,dx = (2)(6) = 12$ kip·ft

Bending moments: $\quad M_A = 0$

$M_E = 0 + 16 = 16$ kip·ft

$M_C = 16 - 4 = 12$ kip·ft

$M_D = 12 - 24 = -12$ kip·ft

$M_B = -12 + 12 = 0$

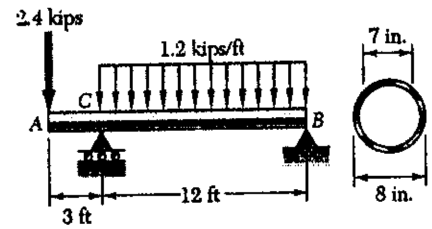
Maximum $|M| = 16$ kip·ft $= 192$ kip·in.

For W 8×31 rolled steel section $\quad S = 27.5 \text{ in}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{192}{27.5} = 6.98$ ksi ◄

PROBLEM 5.65

5.65 **and** *5.66* Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$+\circlearrowleft \Sigma M_C = 0$

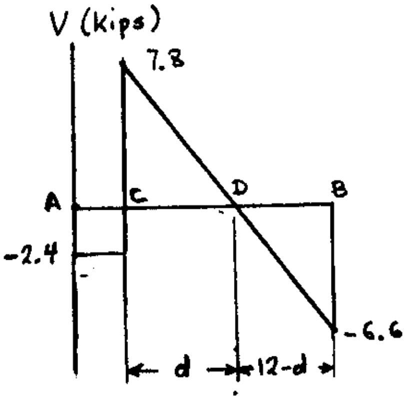
$(3)(2.4) - (6)(12)(1.2) + 12\,B = 0$

$B = 6.6 \text{ kips}$

$+\circlearrowleft \Sigma M_B = 0$

$(15)(2.4) + (6)(12)(1.2) - 12\,C = 0$

$C = 10.2 \text{ kips}$



Shear: A to C $\quad V = -2.4 \text{ kips}$

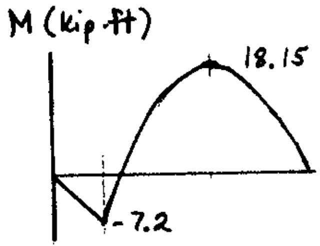
$C^+ \quad V = -2.4 + 10.2 = 7.8 \text{ kips}$

$B \quad V_B = 7.8 - (12)(1.2) = -6.6 \text{ kips}$

Locate point D where $V = 0$

$\frac{d}{7.8} = \frac{12-d}{6.6} \qquad 14.4\,d = 93.6$

$d = 6.5 \text{ ft.} \qquad 12 - d = 5.5 \text{ ft.}$



Areas under shear diagram

A to C $\quad \int V\,dx = (3)(-2.4) = -7.2 \text{ kip·ft}$

C to D $\quad \int V\,dx = (\frac{1}{2})(6.5)(7.8) = 25.35 \text{ kip·ft}$

D to B $\quad \int V\,dx = (\frac{1}{2})(5.5)(-6.6) = -18.15 \text{ kip·ft}$

Bending moments $\quad M_A = 0$

$M_C = 0 - 7.2 = -7.2 \text{ kip·ft}$

$M_D = -7.2 + 25.35 = 18.15 \text{ kip·ft}$

$M_B = 18.15 - 18.15 = 0$

Maximum $|M| = 18.15 \text{ kip·ft} = 217.8 \text{ kip·in}$

For pipe $\quad c_o = \frac{d_o}{2} = \frac{8}{2} = 4 \text{ in.} \qquad c_i = \frac{d_i}{2} = \frac{7}{2} = 3.5 \text{ in.}$

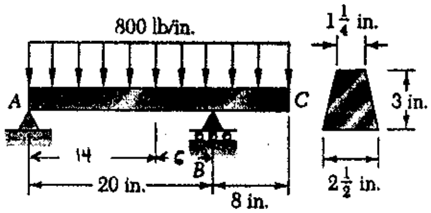
$I = \frac{\pi}{4}(c_o^4 - c_i^4) = \frac{\pi}{4}(4^4 - 3.5^4) = 82.20 \text{ in}^4$

$S = \frac{I}{c_o} = \frac{82.20}{4} = 20.80 \text{ in}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{217.8}{20.80} = 10.47 \text{ ksi}$ ◄

PROBLEM 5.66

5.65 and 5.66 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



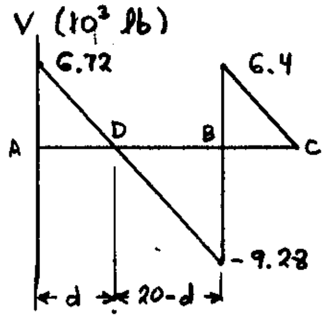
SOLUTION

$\circlearrowleft \Sigma M_B = 0 \qquad -20A + (6)(28)(800) = 0$

$A = 6.72 \times 10^3$ lb.

$\circlearrowleft \Sigma M_A = 0 \qquad 20B - (14)(28)(800) = 0$

$B = 15.68 \times 10^3$ lb.



Shear: $V_A = 6.72 \times 10^3$ lb.

B⁻ $\quad V_{B^-} = 6.72 \times 10^3 - (20)(800) = -9.28 \times 10^3$ lb.

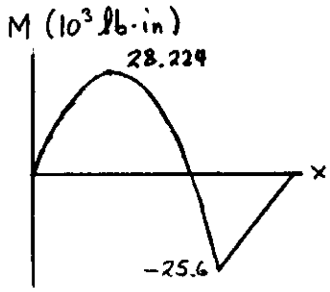
B⁺ $\quad V_{B^+} = -9.28 \times 10^3 + (15.68 \times 10^3) = 6.4 \times 10^3$ lb

C $\quad V_C = 6.4 \times 10^3 - (8)(800) = 0$

Locate point D where $V = 0$

$\frac{d}{6.72} = \frac{20-d}{9.28} \qquad 16d = 134.4$

$d = 8.4$ in $\qquad 20 - d = 11.6$ in.



Areas under shear diagram

A to D $\quad \int V dx = (\frac{1}{2})(8.4)(6.72 \times 10^3) = 28.224 \times 10^3$ lb·in

D to B $\quad \int V dx = (\frac{1}{2})(11.6)(-9.28 \times 10^3) = -53.824 \times 10^3$ lb·in

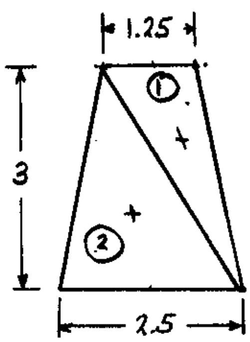
B to C $\quad \int V dx = (\frac{1}{2})(8)(6.4 \times 10^3) = 25.6 \times 10^3$ lb·in

Bending moments: $\quad M_A = 0$

$M_D = 0 + 28.224 \times 10^3 = 28.224 \times 10^3$ lb·in

$M_B = 28.224 \times 10^3 - 53.824 \times 10^3 = -25.6 \times 10^3$ lb·in.

$M_C = -25.6 \times 10^3 + 25.6 \times 10^3 = 0$



Maximum $|M| = 28.224 \times 10^3$ lb·in

Locate centroid of cross section

$\bar{Y} = \frac{7.5}{5.625} = 1.3333$ in. from bottom

For each triangle $\bar{I} = \frac{1}{36} b h^3$

Part	A, in²	$\bar{y}$, in	$A\bar{y}$, in³	d, in	Ad^2, in⁴	$\bar{I}$, in⁴
1	1.875	2	3.75	0.6667	0.8333	0.9375
2	3.75	1	3.75	0.3333	0.4167	1.875
Σ	5.625		7.5		1.25	2.8125

Moment of inertia

$I = \Sigma \bar{I} + \Sigma A d^2$

$= 1.25 + 2.8125 = 4.0625$ in⁴

Normal stress

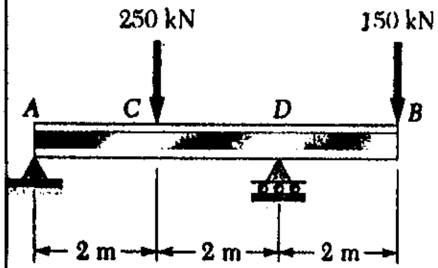
$\sigma = \frac{Mc}{I} = \frac{(28.224 \times 10^3)(1.6667)}{4.0625}$

$= 11.58 \times 10^3$ psi

$= 11.58$ ksi ◄

PROBLEM 5.67

5.67 and 5.68 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



W410 × 114

SOLUTION

$w = 0$

$\circlearrowleft \Sigma M_D = 0$

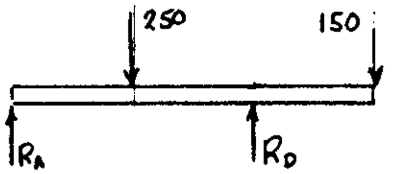
$-4R_A + (2)(250) - (2)(150) = 0$

$R_A = 50 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_A = 0$

$4R_D - (2)(250) - (6)(150) = 0$

$R_D = 350 \text{ kN} \uparrow$

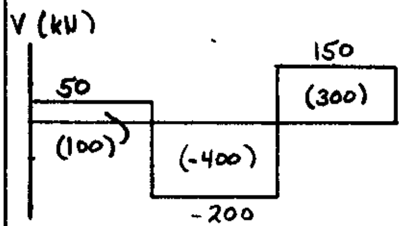


Shear: $V_A = 50$ kN

A to C $\quad V = 50$ kN

C to D $\quad V = 50 - 250 = -200$ kN

D to B $\quad V = -200 + 350 = 150$ kN

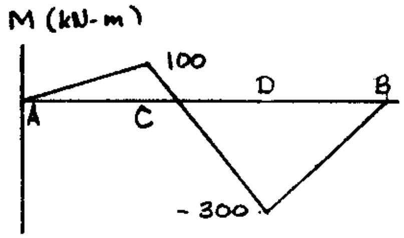


Areas of shear diagram

A to C $\quad \int V\,dx = (50)(2) = 100$ kN·m

C to D $\quad \int V\,dx = (-200)(2) = -400$ kN·m

D to B $\quad \int V\,dx = (150)(2) = 300$ kN·m



Bending moments: $\quad M_A = 0$

$M_C = M_A + \int V\,dx = 0 + 100 = 100$ kN·m

$M_D = M_C + \int V\,dx = 100 - 400 = -300$ kN·m

$M_B = M_D + \int V\,dx = -300 + 300 = 0$

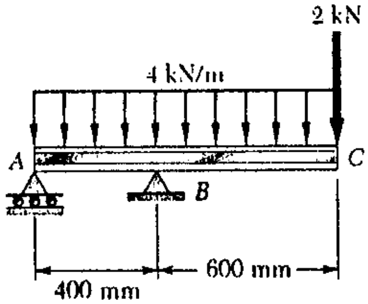
maximum $|M| = 300$ kN·m $= 300 \times 10^3$ N·m

For W410 × 114 rolled steel section $\quad S_x = 2200 \times 10^3 \text{ mm}^3 = 2200 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{300 \times 10^3}{2200 \times 10^{-6}} = 136.4 \times 10^6 \text{ Pa} = 136.4 \text{ MPa}$$

PROBLEM 5.68

5.67 and 5.68 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



S100 × 11.5

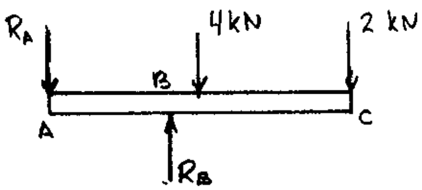
SOLUTION

$\circlearrowleft \Sigma M_B = 0 \quad (0.4)(R_A) - (0.1)(4) - (0.6)(2) = 0$

$R_A = 4 \text{ kN} \downarrow$

$\circlearrowleft \Sigma M_A = 0 \quad (0.4)(R_B) - (0.5)(4) - (1)(2) = 0$

$R_B = 10 \text{ kN} \uparrow$



A to B $\quad 0 < x < 0.4$ m

$\frac{dV}{dx} = -w = -4$ kN/m

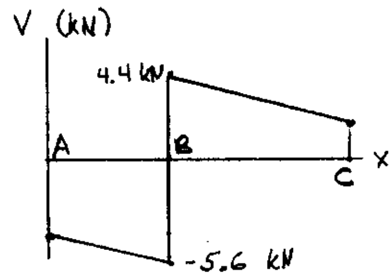
$V = -4x - 4$ kN

At $x = 0.4$ m $\quad V_B = -5.6$ kN

$\frac{dM}{dx} = -4x - 4$

$M = M_A - 2x^2 - 4x = 0 - 2x^2 - 4x$

At $x = 0.4$ m $\quad M_B = 1.92$ kN·m



B to C $\quad 0.4 \text{ m} < x < 1.0$ m

$\frac{dV}{dx} = -w = -4$ kN/m

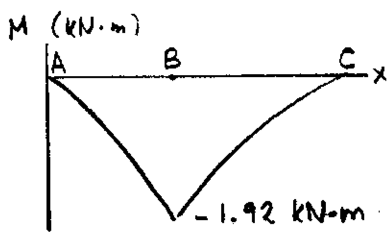
$V = 4.4 - 4(x - 0.4) = 6 - 4x$ kN

$\frac{dM}{dx} = 6 - 4x$

$M = 6x - 2x^2 + C_1$ kN·m

$M = 0$ at $x = 1 \quad \therefore \quad C_1 = 4$ kN·m

$M = 4 + 6x - 2x^2$ kN·m



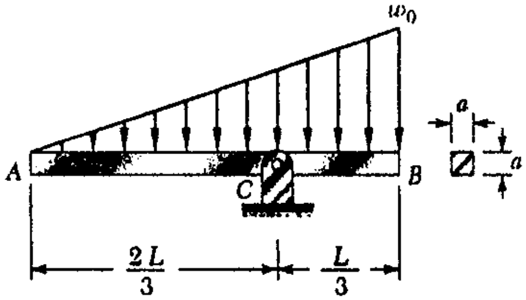
$|M|_{max} = 1.92 \text{ kN·m} = 1.92 \times 10^3 \text{ N·m}$

For S 100 × 11.5 rolled steel section $\quad S_x = 49.6 \times 10^3 \text{ mm}^3 = 49.6 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{1.92 \times 10^3}{49.6 \times 10^{-6}} = 38.7 \times 10^6 \text{ Pa} = 38.7 \text{ MPa} \blacktriangleleft$$

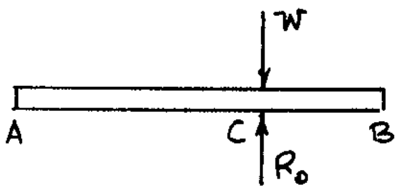
PROBLEM 5.69

5.69 Beam *AB*, of length L and square cross section of side a, is supported by a pivot at C and loaded as shown. (*a*) Check that the beam is in equilibrium. (*b*) Show that the maximum normal stress due to bending occurs at C and is equal to $w_0L^2/(1.5a)^3$.



SOLUTION

Replace distributed load by equivalent concentrated load at the centroid of the area of the load diagram.

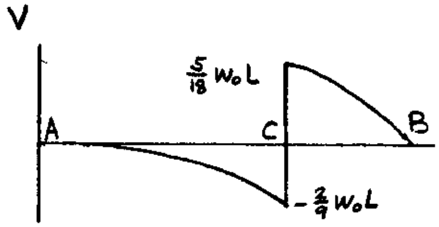


For the triangular distribution the centroid lies at $x = \frac{2L}{3}$.

$$W = \frac{1}{2} w_o L$$

$\Sigma F_y = 0 \qquad R_D - W = 0 \qquad R_D = \frac{1}{2} w_o L$

$\Sigma M_c = 0 \qquad 0 = 0 \qquad$ equilibrium



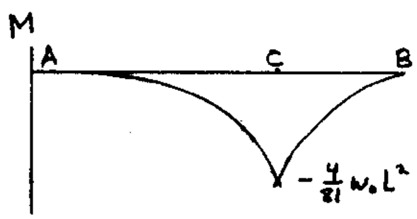
$V = 0, \; M = 0 \quad$ at $x = 0$

$0 < x < \frac{2L}{3}$

$$\frac{dV}{dx} = -w = -\frac{w_o x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_o x^2}{2L} + C_1 = -\frac{w_o x^2}{2L}$$

$$M = -\frac{w_o x^3}{6L} + C_2 = -\frac{w_o x^3}{6L}$$



Just to the left of C

$$V = -\frac{w_o (2L/3)^2}{2L} = -\frac{2}{9} w_o L$$

Just to the right of C

$$V = -\frac{2}{9} w_o L + R_D = \frac{5}{18} w_o L$$

Note sign change. Maximum $|M|$ occurs at C.

$$M_c = -\frac{w_o (2L/3)^3}{6L} = -\frac{4}{81} w_o L^2$$

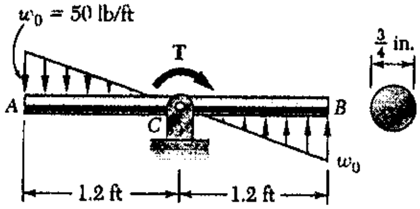
Maximum $|M| = \frac{4}{81} w_o L^2$

For square cross section $\qquad I = \frac{1}{12} a^4 \qquad c = \frac{1}{2} a$

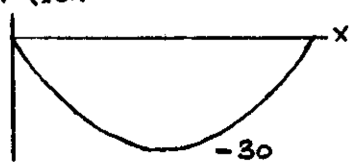
$$\sigma_m = \frac{|M|_{max} c}{I} = \frac{4}{81} \frac{w_o L^2 \, 6}{a^3} = \frac{8}{27} \frac{w_o L^2}{a^3} = \left(\frac{2}{3}\right)^3 \frac{w_o L^2}{a^3} = \frac{w_o L^2}{(1.5\,a)^3}$$ ◄

PROBLEM 5.70

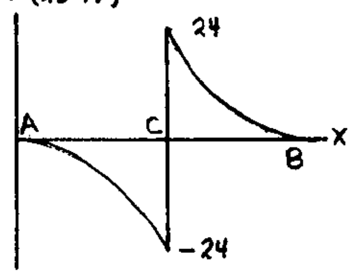
5.70 Knowing that rod *AB* is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.



V (lb.)



M (lb·ft)



SOLUTION

A to C $\quad 0 < x < 1.2$ ft

$$w = 50\left(1 - \frac{x}{1.2}\right) = 50 - 41.667x$$

$$\frac{dV}{dx} = -w = 41.667x - 50$$

$$V = V_A + \int_0^x (41.667x - 50)\,dx$$

$$= 0 + 20.833x^2 - 50x = \frac{dM}{dx}$$

$$M = M_A + \int_0^x V\,dx$$

$$= 0 + \int_0^x (20.833x^2 - 50x)\,dx$$

$$= 6.944x^3 - 25x^2$$

At $x = 1.2$ ft, $\quad V \;\; -30$ lb.
$\qquad M \cdot -24$ lb·in.

C to B $\quad$ Use symmetry conditions.

Maximum $|M| = 24$ lb·ft $= 288$ lb·in.

Cross section $\quad c = \frac{d}{2} = \left(\frac{1}{2}\right)(0.75) = 0.375$ in

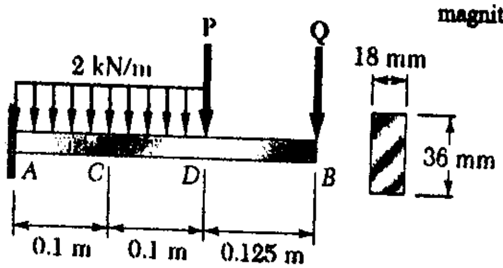
$$I = \frac{\pi}{4}c^4 = \left(\frac{\pi}{4}\right)(0.375) = 15.532 \times 10^{-3} \text{ in}^4$$

Normal stress $\quad \sigma = \frac{|M|c}{I} = \frac{(288)(0.375)}{15.532 \times 10^{-3}} = 6.95 \times 10^3$ psi

$= 6.95$ ksi

PROBLEM 5.71

*5.71 Beam *AB* supports a uniformly distributed load of 2 kN/m and two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending on the bottom edge of the beam is −56.9 MPa at *A* and −29.9 MPa at *C*. Draw the shear and bending-moment diagrams for the beam and determine the magnitudes of the loads **P** and **Q**.

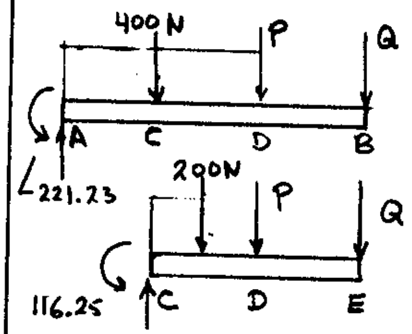


SOLUTION

$I = \frac{1}{12}(18)(36)^3 = 69.984 \times 10^3 \text{ mm}^4$

$c = \frac{1}{2}d = 18 \text{ mm}$

$S = \frac{I}{c} = 3.888 \times 10^3 \text{ mm}^3 = 3.888 \times 10^{-6} \text{ m}^3$



At A $\quad M_A = S\sigma_A$

$M_A = (3.888 \times 10^{-6})(-56.9) = -221.25 \text{ N·m}$

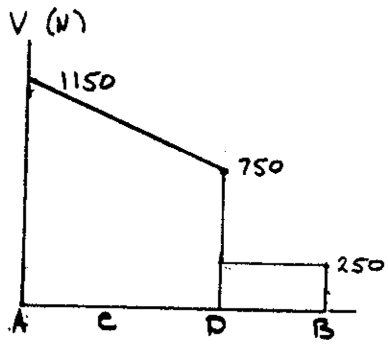
At C $\quad M_c = S\sigma_c$

$M_c = (3.888 \times 10^{-6})(-29.9) = -116.25 \text{ N·m}$

$\circlearrowleft \Sigma M_A = 0$

$221.23 - (0.1)(400) - 0.2P - 0.325Q = 0$

$0.2P + 0.325Q = 181.25 \qquad (1)$



$+\circlearrowleft M_c = 0$

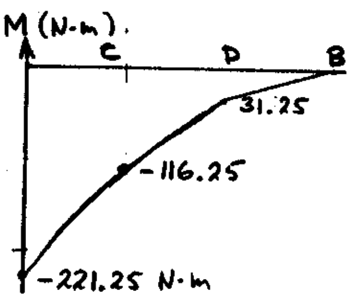
$116.25 - (0.05)(200) - 0.1P - 0.225Q = 0$

$0.1P + 0.225Q = 106.25 \qquad (2)$

Solving (1) and (2) simultaneously

$P = 500 \text{ N}$ ◄

$Q = 250 \text{ N}$ ◄



Reaction force at A

$R_A - 400 - 500 - 250 = 0$

$R_A = 1150 \text{ N·m}$

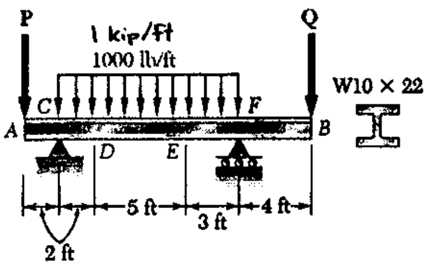
$V_A = 1150 \text{ N} \qquad V_D = 250$

$M_A = -221.25 \text{ N·m}$

$M_c = -116.25 \text{ N·m}$

$M_D = -31.25 \text{ N·m}$

PROBLEM 5.72



* 5.72 Beam *AB* supports a uniformly distributed load of 1000 lb/ft and two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending on the bottom edge of the lower flange of the W 10 × 22 rolled-steel beam is +2.07 ksi at *D* and +0.776 ksi at *E*. (*a*) Draw the shear and bending- moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending which occurs in the beam.

SOLUTION

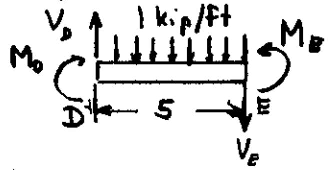
For W10×22 rolled steel section $S = 23.2 \text{ in}^3$

Bending moments at D and E $\quad M = S\sigma$

$M_D = (23.2)(2.07) = 48.0 \text{ kip·in} = 4.00 \text{ kip·ft}$

$M_E = (23.2)(0.776) = 18.0 \text{ kip·in} = 1.50 \text{ kip·ft}$

Using portion DE as a free body



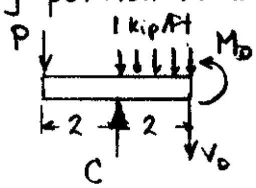
$+\circlearrowleft \Sigma M_E = 0 \qquad -M_D + M_E - 5V_D + (2.5)(5)(1) = 0$

$V_D = 2 \text{ kips}$

$+\circlearrowleft \Sigma M_D = \qquad -M_D + M_E - 5V_E - (2.5)(5)(1) = 0$

$V_E = -3 \text{ kips}$

Using portion ACD as a free body



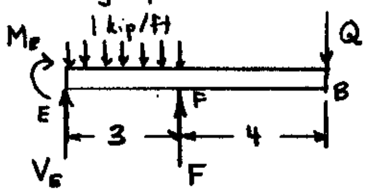
$+\circlearrowleft \Sigma M_C = 0 \qquad 2P + (1)(2)(1) + M_D - 2V_D = 0$

$P = 1 \text{ kip}$

$+\uparrow \Sigma F_y = 0 \qquad -P + C - (2)(1) - V_D = 0$

$C = 5 \text{ kips.}$

Using portion EFB as a free body



$+\circlearrowleft \Sigma M_F = 0 \qquad -4Q + (1.5)(3)(1) - 3V_E - M_E = 0$

$Q = 3 \text{ kips}$

$+\uparrow \Sigma F_y = 0 \qquad F + V_E - (3)(1) - Q = 0$

$F = 9 \text{ kips}$

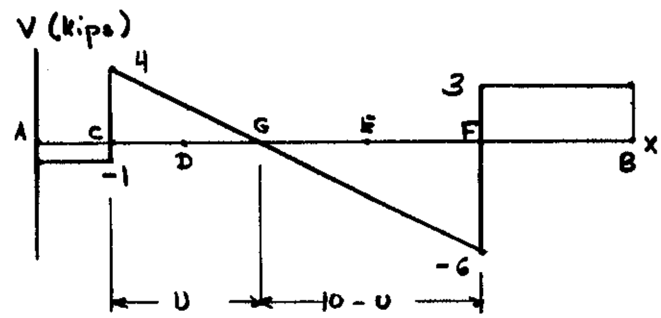
Shear: A to C $\quad V = -1 \text{ kips}$

$C^+ \quad V = -1 + 5 = 4 \text{ kips}$

$F^- \quad V = 4 - (10)(1) = -6 \text{ kips}$

$F^+ \quad V = -6 + 9 = 3 \text{ kips}$

F to B $\quad V = 3 \text{ kips.}$



Locate point G where V = 0

$\frac{U}{4} = \frac{10-U}{6} \qquad 10U = 40$

$U = 4 \text{ ft} \qquad 10 - U = 6 \text{ ft.}$

continued

Problem 5.72 continued

Areas under shear diagram

A to C $\quad \int V dx = (2)(-1) = -2$ kip·ft

C to G $\quad \int V dx = (\frac{1}{2})(4)(4) = 8$ kip ft

G to F $\quad \int V dx = (\frac{1}{2})(6)(-6) = -18$ kip·ft

F to B $\quad \int V dx = (4)(3) = 12$ kip ft

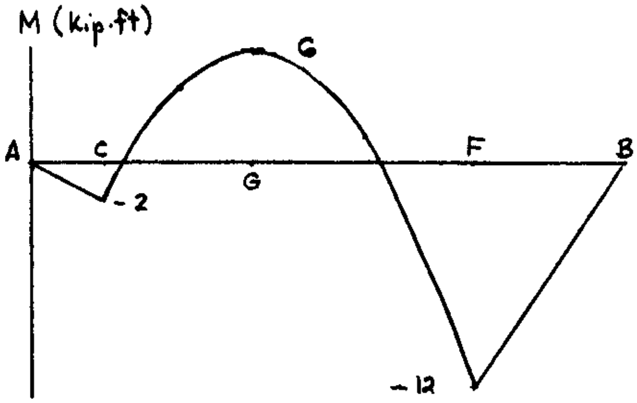
Bending moments $\quad M_A = 0$

$M_C = 0 - 2 = -2$ kip·ft

$M_G = -2 + 8 = 6$ kip ft

$M_F = 6 - 18 = -12$ kip·ft

$M_B = -12 + 12 = 0$

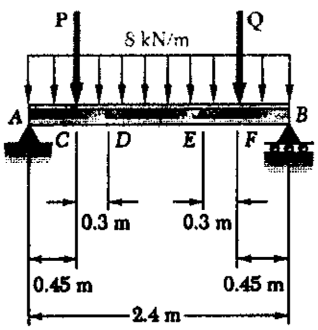


Maximum $|M| = 12$ kip·ft
$= 144$ kip·in.

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{144}{23.2} = 6.21$ ksi

PROBLEM 5.73

* *5.73* Beam *AB* supports a uniformly distributed load of 8 kN/m and two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending on the bottom edge of the lower flange of the W 200 ×52 rolled-steel beam is 100 MPa at *D* and 70 MPa at *E*. (*a*) Draw the shear and bending- moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending which occurs in the beam.



W200 × 52

SOLUTION

For W 200 × 52 rolled steel section

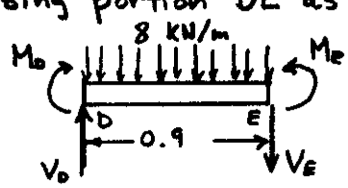
$S = 512 \times 10^3 \ mm^3 = 512 \times 10^{-6} \ m^3$

Bending moments at D and E $\qquad M = S\sigma$

$M_D = (512 \times 10^{-6})(100 \times 10^6) = 51.2 \times 10^3$ N·m

$M_E = (512 \times 10^{-6})(70 \times 10^6) = 35.84 \times 10^3$ N·m

Using portion DE as a free body



$+\circlearrowleft \Sigma M_E = 0 \qquad -0.9 V_D - M_D + M_E + (0.45)(0.9)(8) = 0$

$V_D = -13.467$ kN

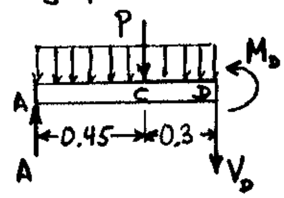
$+\uparrow \Sigma F_y = 0 \qquad V_D - V_E - (0.9)(8) = 0$

$V_E = -20.667$ kN

continued

Problem 5.73 continued

Using portion ACD as a free body



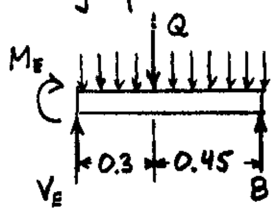
$+\circlearrowleft \Sigma M_A = 0 \qquad -0.45P - (0.375)(0.75)(8) - 0.75V_D + M_D = 0$

$P = 131.222$ kN

$+\uparrow \Sigma F_y = 0 \qquad A - P - V_D - (0.75)(8) = 0$

$A = 123.756$ kN

Using portion EFB as a free body



$+\circlearrowleft \Sigma M_B = 0 \qquad 0.45Q + (0.375)(0.75)(8) - 0.75V_E - M_E = 0$

$Q = 40.2$ kN

$+\uparrow \Sigma F_y = 0 \qquad V_E - Q - (0.75)(8) + B = 0$

$B = 66.867$ kN

Shear: $V_A = 123.756$ kN

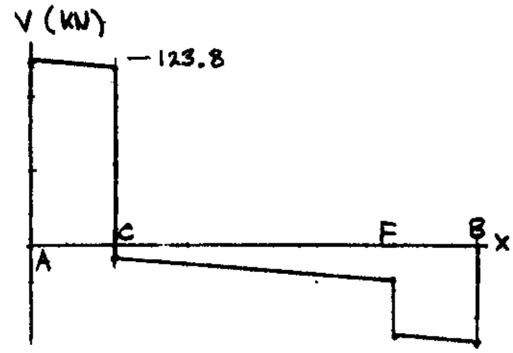
$V_{C^-} = 123.756 - (0.45)(8) = 120.155$ kN

$V_{C^+} = 120.155 - 131.222 = -11.067$ kN

$V_{F^-} = -11.067 - (1.5)(8) = -23.067$ kN

$V_{F^+} = -23.067 - 40.2 = -63.267$ kN

$V_B = -63.276 - (0.45)(8) = -66.867$ kN



Areas under shear diagram

A to C: $\frac{1}{2}(0.45)(123.756 + 120.155) = 54.88$ kN·m

C to F: $\frac{1}{2}(1.5)(-11.067 - 23.067) = -25.6$ kN·m

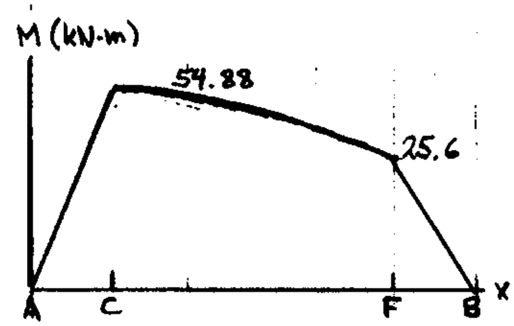
F to B: $\frac{1}{2}(0.45)(-63.267 - 66.867) = -29.28$ kN·m

Bending moments: $M_A = 0$

$M_C = 0 + 54.88 = 54.88$ kN·m

$M_F = 54.88 - 25.6 = 29.28$ kN·m

$M_B = 29.28 - 29.28 = 0$

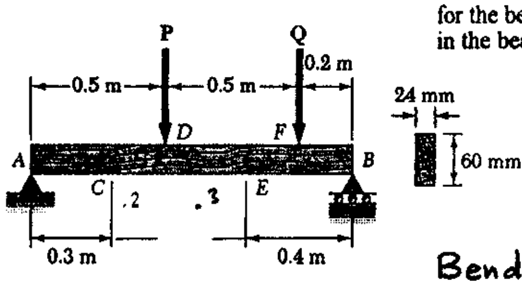


Maximum $|M| = 54.88$ kN·m $= 54.88 \times 10^3$ N·m

Normal stress $\qquad \sigma = \dfrac{|M|}{S} = \dfrac{54.88 \times 10^3}{512 \times 10^{-6}} = 107.2 \times 10^6$ Pa

$= 107.2$ MPa

PROBLEM 5.74



*5.74 Beam AB supports two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending on the bottom edge of the beam is +15 MPa at C and +22 MPa at E. (*a*) Draw the shear and beriding-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending which occurs in the beam.

SOLUTION

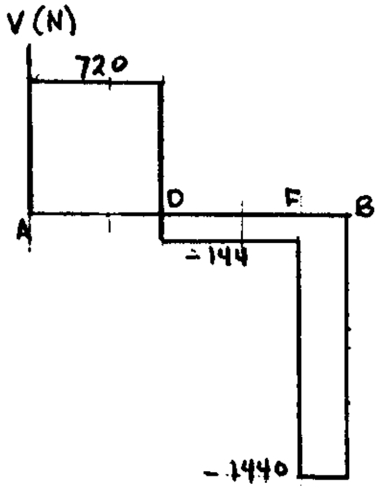
For rectangular cross section $S = \frac{1}{6}bh^2$

$S = (\frac{1}{6})(24)(60)^2 = 14.4 \times 10^3 \text{ mm}^3 = 14.4 \times 10^{-6} \text{ m}^3$

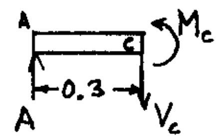
Bending moments at C and E $\quad M = S\sigma$

$M_C = (14.4 \times 10^{-6})(15 \times 10^6) = 216 \text{ N}\cdot\text{m}$

$M_E = (14.4 \times 10^{-6})(22 \times 10^6) = 316.8 \text{ N}\cdot\text{m}$



Using portion AC as a free body



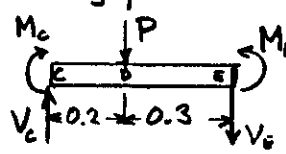
$\circlearrowright \Sigma M_C = 0 \quad -0.3A + M_C = 0$

$A = 720 \text{ N}$

$+\uparrow \Sigma F_y = 0 \quad A - V_C = 0$

$V_C = 720 \text{ N}$

Using portion CDE as a free body

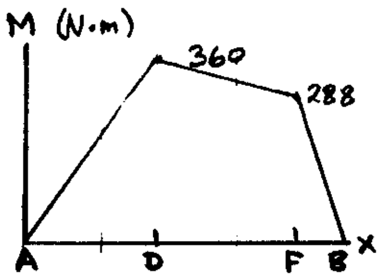


$\circlearrowright \Sigma M_E = 0$

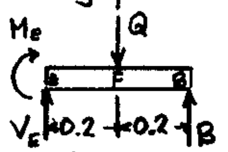
$0.3P - 0.5V_C - M_C + M_E = 0$

$P = 864 \text{ N}$

$+\uparrow \Sigma F_y = 0, \quad V_C - P - V_E = 0, \qquad V_E = -144 \text{ N}$



Using portion EFB as a free body



$\circlearrowright \Sigma M_B = 0$

$0.2Q - 0.4V_E - M_E = 0$

$Q = 1296 \text{ N}$

$+\uparrow \Sigma F_y = 0, \quad V_E - Q + B = 0, \qquad B = 1440 \text{ N}$

Areas under shear diagram

A to D $\quad (0.5)(720) = 360 \text{ N}\cdot\text{m}$

D to F $\quad (0.5)(-144) = -72 \text{ N}\cdot\text{m}$

F to B $\quad (0.2)(-1440) = -288 \text{ N}\cdot\text{m}$

Bending moments: $M_A = 0$

$M_D = 0 + 360 = 360 \text{ N}\cdot\text{m}$

$M_F = 360 - 72 = 288 \text{ N}\cdot\text{m}$

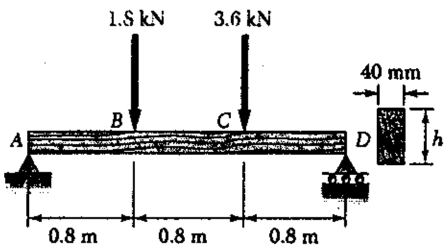
$M_B = 288 - 288 = 0$

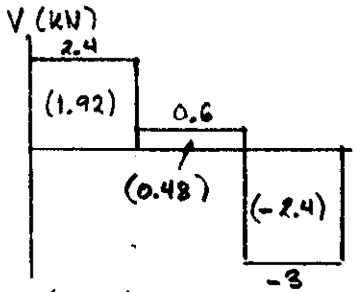
Maximum $|M| = 360 \text{ N}\cdot\text{m}$

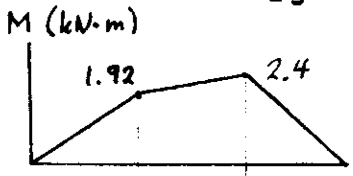
Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{360}{14.4 \times 10^{-6}} = 25 \times 10^6 \text{ Pa} = 25 \text{ MPa}$ ◄

PROBLEM 5.75

5.75 and 5.76 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.







SOLUTION

$+\circlearrowleft \Sigma M_D = 0$

$-2.4A + (1.6)(1.8) + (0.8)(3.6) = 0$

$A = 2.4 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$

$-(0.8)(1.8) - (1.6)(3.6) + 2.4D = 0$

$D = 3 \text{ kN}$

Construct shear and bending moment diagrams

$|M|_{max} = 2.4 \text{ kN}\cdot\text{m} = 2.4 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$

$S_{min} = \dfrac{|M|_{max}}{\sigma_{all}} = \dfrac{2.4 \times 10^3}{12 \times 10^6} = 200 \times 10^{-6} \text{ m}^3$

$= 200 \times 10^3 \text{ mm}^3$

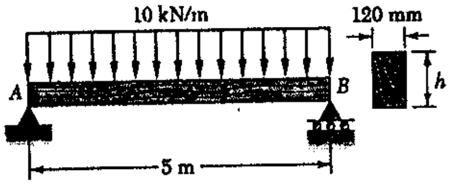
$S = \frac{1}{6} bh^2 = \frac{1}{6}(40)h^2 = 200 \times 10^3$

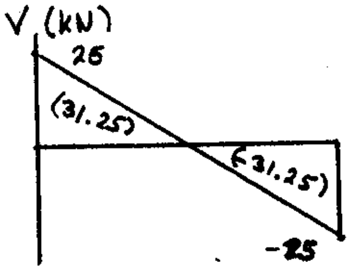
$h^2 = \dfrac{(6)(200 \times 10^3)}{40} = 30 \times 10^3 \text{ mm}^2$

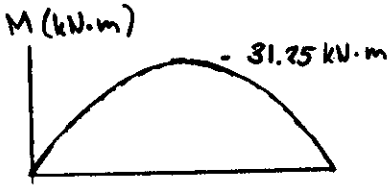
$h = 173.2 \text{ mm}$ ◄

PROBLEM 5.76

5.75 and 5.76 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.







SOLUTION

Reactions: $A = B$ by symmetry

$+\uparrow \Sigma F_y = 0 \qquad A + B - (5)(10) = 0$

$A = B = 25 \text{ kN}$

From bending moment diagram

$|M|_{max} = 31.25 \text{ kN·m} = 31.25 \times 10^3 \text{ N·m}$

$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$

$S_{min} = \dfrac{|M|_{max}}{\sigma_{all}} = \dfrac{31.25 \times 10^3}{12 \times 10^6} = 2.604 \times 10^{-3} \text{ m}^3$

$= 2.604 \times 10^6 \text{ mm}^3$

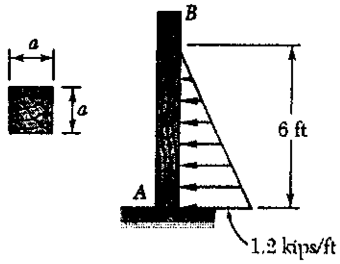
$S = \frac{1}{6} b h^2 = (\frac{1}{6})(120) h^2 = 2.604 \times 10^6$

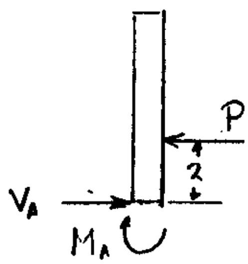
$h^2 = \dfrac{(6)(2.064 \times 10^6)}{120} = 130.21 \times 10^3 \text{ mm}^2$

$h = 361 \text{ mm}$

PROBLEM 5.77

5.77 and 5.78 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1.75 ksi.





SOLUTION

Equivalenent concentrated load

$$P = (\tfrac{1}{2})(6)(1.2) = 3.6 \text{ kips}$$

Bending moment at A

$$M_A = (2)(3.6) = 7.2 \text{ kip}\cdot\text{ft} = 86.4 \text{ kip}\cdot\text{in}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{86.4}{1.75} = 49.37 \text{ in}^3$$

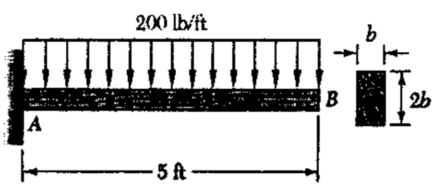
For a square section $S = \frac{1}{6} a^3$

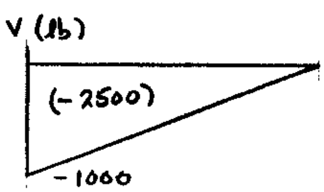
$$a = \sqrt[3]{6\,S}$$

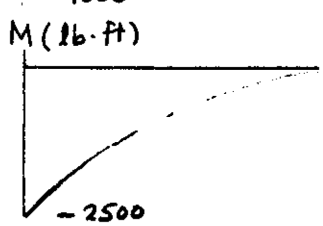
$$a_{min} = \sqrt[3]{(6)(49.37)} = 6.67 \text{ in.}$$

PROBLEM 5.78

5.77 and 5.78 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1.75 ksi.







SOLUTION

Construct shear and bending moment curves.

$$|M|_{max} = 2500 \text{ lb}\cdot\text{ft} = 2.5 \text{ kip}\cdot\text{ft}$$
$$= 30 \text{ kip}\cdot\text{in}$$

$$\sigma_{all} = 1.75 \text{ ksi}$$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{30}{1.75} = 17.143 \text{ in}^3$$

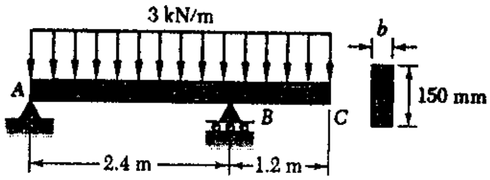
$$S = \tfrac{1}{6} b h^2 = \tfrac{1}{6} b (2b)^2 = \tfrac{2}{3} b^3 = 17.143$$

$$b^3 = \frac{(3)(17.143)}{2} = 25.7 \text{ in}^3$$

$$b = 2.95 \text{ in.}$$

PROBLEM 5.79

5.79* and *5.80 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 Mpa.



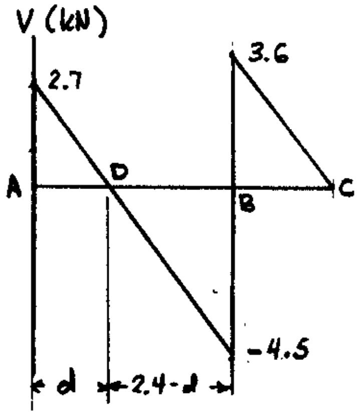
SOLUTION

$+\circlearrowleft M_B = 0$

$-2.4A + (0.6)(3.6)(3) = 0 \qquad A = 2.7 \text{ kN}$

$+\circlearrowleft M_A = 0$

$-(1.8)(3.6)(3) + 2.4B = 0 \qquad B = 8.1 \text{ kN}$



Shear: $V_A = 2.7$ kN

$V_B^- = 2.7 - (2.4)(3) = -4.5$ kN

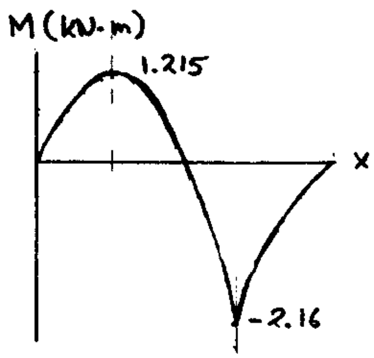
$V_B^+ = -4.5 + 8.1 = 3.6$ kN

$V_C = 3.6 - (1.2)(3) = 0$

Locate point D where V = 0

$\frac{d}{2.7} = \frac{2.4-d}{4.5} \qquad 7.2\,d = 6.48$

$d = 0.9 \text{ m} \qquad 2.4 - d = 1.5 \text{ m}$



Areas under shear curve

A to D $\quad \int V dx = (\frac{1}{2})(0.9)(2.7) = 1.215$ kN·m

D to B $\quad \int V dx = (\frac{1}{2})(1.5)(-4.5) = -3.375$ kN·m

B to C $\quad \int V dx = (\frac{1}{2})(1.2)(3.6) = 2.16$ kN·m

Bending moments: $\quad M_A = 0$

$M_D = 0 + 1.215 = 1.215$ kN·m

$M_B = 1.215 - 3.375 = -2.16$ kN·m

$M_C = -2.16 + 2.16 = 0$

Maximum $|M| = 2.16 \text{ kN·m} = 2.16 \times 10^3 \text{ N·m}$

$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{2.16 \times 10^3}{12 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$

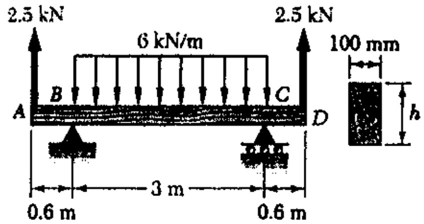
For rectangular section

$S = \frac{1}{6} b h^2 = \frac{1}{6} b (150)^2 = 180 \times 10^3$

$b = \frac{(6)(180 \times 10^3)}{150^2} = 48 \text{ mm}$

PROBLEM 5.80

5.79 and 5.80 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



SOLUTION

By symmetry $B = C$

$+\uparrow \Sigma F_y = 0 \qquad B + C + 2.5 + 2.5 - (3)(6) = 0$

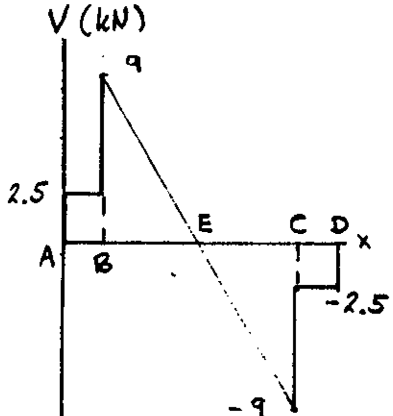
$B = C = 6.5 \text{ kN}$

Shear: A to B $\quad V = 2.5 \text{ kN}$

$V_{B^+} = 2.5 + 6.5 = 9 \text{ kN}$

$V_{C^-} = 9 - (3)(6) = -9 \text{ kN}$

C to D $\quad V = -9 + 6.5 = -2.5 \text{ kN}$



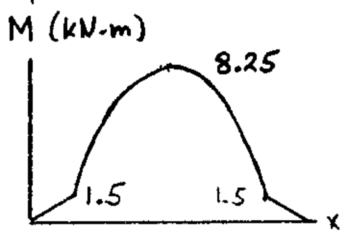
Areas under shear diagram

A to B $\quad \int V dx = (0.6)(2.5) = 1.5 \text{ kN}\cdot\text{m}$

B to E $\quad \int V dx = (\frac{1}{2})(1.5)(9) = 6.75 \text{ kN}\cdot\text{m}$

E to C $\quad \int V dx = -6.75 \text{ kN}\cdot\text{m}$

C to D $\quad \int V dx = -1.5 \text{ kN}\cdot\text{m}$



Bending moments $\quad M_A = 0$

$M_B = 0 + 1.5 = 1.5 \text{ kN}\cdot\text{m}$

$M_E = 1.5 + 6.75 = 8.25 \text{ kN}\cdot\text{m}$

$M_C = 8.25 - 6.75 = 1.5 \text{ kN}\cdot\text{m}$

$M_D = 1.5 - 1.5 = 0$

Maximum $|M| = 8.25 \text{ kN}\cdot\text{m} = 8.25 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{8.25 \times 10^3}{12 \times 10^6} = 687.5 \times 10^{-6} \text{ m}^3 = 687.5 \times 10^3 \text{ mm}^3$$

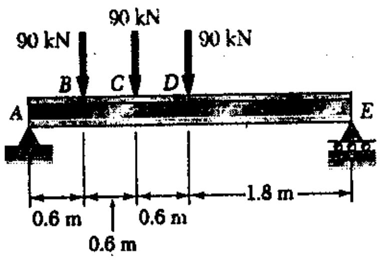
For a rectangular section $\quad S = \frac{1}{6} b h^2$

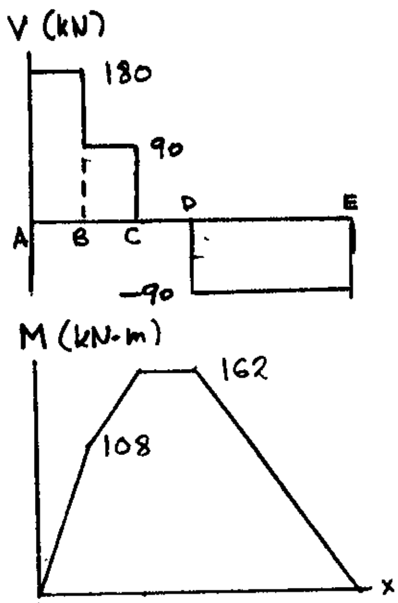
$687.5 \times 10^3 = (\frac{1}{6})(100) h^2 \qquad h^2 = \frac{(6)(687.5 \times 10^3)}{100} = 41.25 \times 10^3 \text{ mm}^2$

$h = 203 \text{ mm}$ ◄

PROBLEM 5.81

5.81 and 5.82 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric wide-flange beam to support the loading shown.





SOLUTION

$\circlearrowleft \Sigma M_E = 0 \qquad -3.6A + (3)(90) + (2.4)(90) + (1.8)(90) = 0$

$A = 180 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0 \qquad 3.6E - (1.8)(90) - (1.2)(90) - (0.6)(90) = 0$

$E = 90 \text{ kN}$

Shear:
- A to B $\quad V = 180$ kN
- B to C $\quad V = 180 - 90 = 90$ kN
- C to D $\quad V = 90 - 90 = 0$
- D to E $\quad V = 0 - 90 = -90$ kN.

Areas under shear diagram
- A to B $\quad \int V dx = (0.6)(180) = 108$ kN·m
- B to C $\quad \int V dx = (0.6)(90) = 54$ kN·m
- C to D $\quad \int V dx = 0$
- D to E $\quad \int V dx = (1.8)(-90) = -162$ kN·m

Bending moments $\quad M_A = 0$

$M_B = 0 + 108 = 108$ kN·m

$M_C = 108 + 54 = 162$ kN·m

$M_D = 162 + 0 = 162$ kN·m

$M_E = 162 - 162 = 0$

Maximum $|M| = 162$ kN·m $= 162 \times 10^3$ N·m

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{162 \times 10^3}{160 \times 10^6} = 1.0125 \times 10^{-3} \text{ m}^3 = 1012.5 \times 10^3 \text{ mm}^3$$

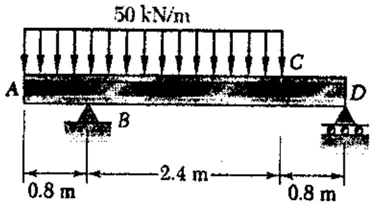
Shape	S (10^3 mm^3)
W 460 × 74	1460
W 410 × 60	1060
W 360 × 64	1030
W 310 × 74	1060

Lightest wide flange beam

W 410 × 60 @ 60 kg/m

PROBLEM 5.82

5.81 and 5.82 Knowing that the allowable normal stress for the steel used is 160 MPa,.select the most economical metric wide-flange beam to support the loading shown.



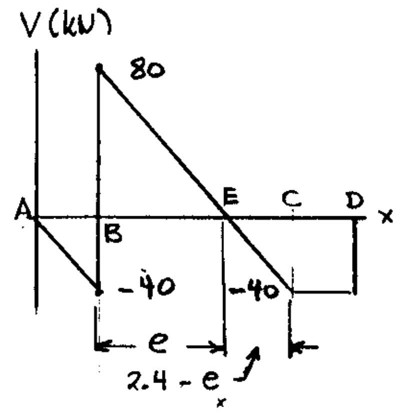
SOLUTION

$\circlearrowleft \Sigma M_D = 0 \quad -3.2B + (2.4)(3.2)(50) = 0$

$B = 120 \text{ kN}$

$+\circlearrowleft \Sigma M_B = 0 \quad 3.2D - (0.8)(3.2)(50) = 0$

$D = 40 \text{ kN}$



Shear: $V_A = 0$

$V_{B^-} = 0 - (0.8)(50) = -40 \text{ kN}$

$V_{B^+} = -40 + 120 = 80 \text{ kN}$

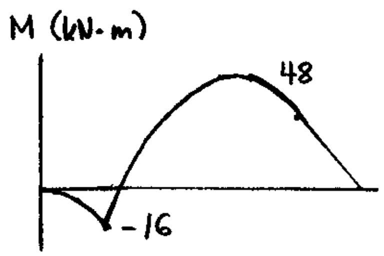
$V_C = 80 - (2.4)(50) = -40 \text{ kN}$

$V_D = -40 + 0 = -40 \text{ kN}$

Locate point E where $V = 0$

$\frac{e}{80} = \frac{2.4 - e}{40} \qquad 120e = 192$

$e = 1.6 \text{ m} \qquad 2.4 - e = 0.8 \text{ m}.$



Areas: A to B, $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN·m}$

B to E $\int V dx = (\frac{1}{2})(1.6)(80) = 64 \text{ kN·m}$

E to C $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN·m}$

C to D $\int V dx = (0.8)(-40) = -32 \text{ kN·m}$

Bending moments: $M_A = 0$

$M_B = 0 - 16 = -16 \text{ kN·m}$

$M_E = -16 + 64 = 48 \text{ kN·m}$

$M_C = 48 - 16 = 32 \text{ kN·m}$

$M_D = 32 - 32 = 0$

Maximum $|M|$: $48 \text{ kN·m} = 48 \times 10^3 \text{ N·m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{48 \times 10^3}{160 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$

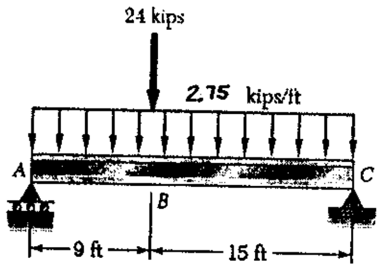
Shape	S (10^3 mm^3)
W 310 × 32.7	415
W 250 × 28.4	308
W 200 × 35.9	342

Lightest wide flange beam

W 250 × 28.4 @ 28.4 kg/m

PROBLEM 5.83

5.83 and 5.84 Knowing that the allowable normal stress for the steel used is 24 ksi,.select the most economical wide-flange beam to support the loading shown.



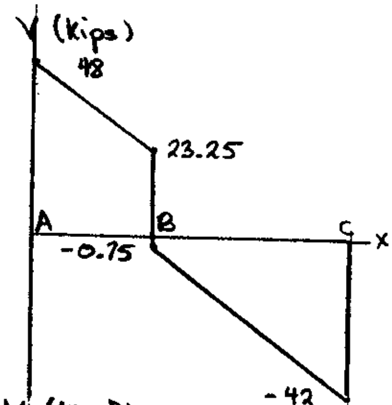
SOLUTION

$+\circlearrowleft \Sigma M_C = 0 \qquad -24A + (12)(24)(2.75) + (15)(24) = 0$

$A = 48 \text{ kips}$

$+\circlearrowleft \Sigma M_A = 0 \qquad 24C - (12)(24)(2.75) - (9)(24) = 0$

$C = 42 \text{ kips}$



Shear: $V_A = 48$

$V_{B^-} = 48 - (9)(2.75) = 23.25 \text{ kips}$

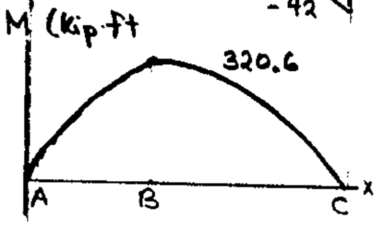
$V_{B^+} = 23.25 - 24 = -0.75 \text{ kips}$

$V_C = -0.75 - (15)(2.75) = -42 \text{ kips}$

Areas under shear diagram

A to B $\int V dx = (\frac{1}{2})(9)(48 + 23.25) = 320.6 \text{ kip}\cdot\text{ft}$

B to C $\int V dx = (\frac{1}{2})(15)(-0.75 - 42) = -320.6 \text{ kip ft}$



Bending moments: $M_A = 0$

$M_B = 0 + 320.6 = 320.6 \text{ kip ft}$

$M_C = 320.6 - 320.6 = 0$

Maximum $|M| = 320.6 \text{ kip ft} = 3848 \text{ kip}\cdot\text{in}$

$\sigma_{all} = 24 \text{ ksi}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3848}{24} = 160.3 \text{ in}^3$

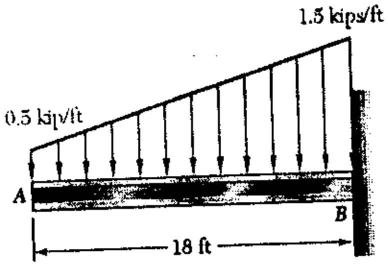
Shape	S (in³)
W 30 × 99	269
W 27 × 84	213
W 24 × 104	258
W 21 × 101	227
W 18 × 106	204

Lightest wide flange beam

W 27 × 84 @ 84 lb/ft

PROBLEM 5.84

5.83 and 5.84 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



SOLUTION

$$w = 0.5 + \frac{(1.5 - 0.5)x}{18} = 0.5 + 0.0555\ x$$

$$\frac{dV}{dx} = -w = -0.5 + 0.05556\ x$$

$$V = 0 - 0.5x - 0.02778\ x^2 = \frac{dM}{dx}$$

$$M = 0 - 0.25x^2 - 0.009259\ x^3$$

Maximum $|M|$ occurs at $x = 18$ ft.

$$|M|_{max} = (0.25)(18)^2 + (0.009259)(18)^3 = 135 \text{ kip}\cdot\text{ft} = 1620 \text{ kip}\cdot\text{in}$$

$$\sigma_{all} = 24 \text{ ksi} \qquad S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{1620}{24} = 67.5 \text{ in}^3$$

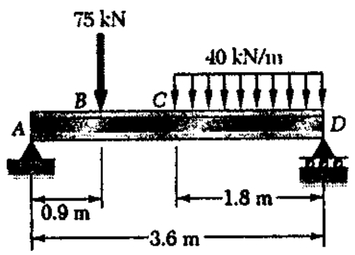
Shape	S (in^3)
W21 × 44	81.6
W18 × 50	88.9
W16 × 57	92.2
W14 × 53	77.8
W12 × 72	97.4
W10 × 68	75.7

Lightest wide flange beam

W18 × 50 @ 50 lb/ft ◄

PROBLEM 5.85

5.85 and 5.86 Knowing that the allowable normal stress for the steel used is 160 MPa,.select the most economical metric S-shape beam to support the loading shown.



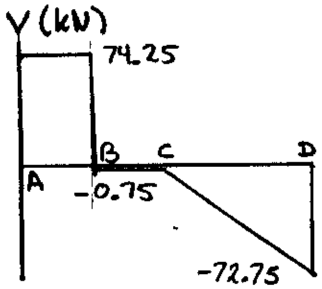
SOLUTION

$$+\circlearrowleft \Sigma M_D = 0 \qquad -3.6 A + (2.7)(75) + (0.9)(1.8)(40) = 0$$

$$A = 74.25 \text{ kN}$$

$$+\circlearrowleft \Sigma M_A = 0 \qquad 3.6 D - (0.9)(75) - (2.7)(1.8)(40) = 0$$

$$D = 72.75 \text{ kN}$$



Shear: A to B $\quad V = 74.25$ kN

B to C $\quad V = 74.25 - 75 = -0.75$ kN

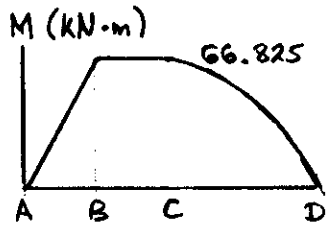
$V_D = -0.75 - (1.8)(40) = -72.75$ kN

Areas under shear diagram

A to B $\quad \int V dx = (0.9)(74.25) = 66.825$ kN·m

B to C $\quad \int V dx = (0.9)(-0.75) = -0.675$ kN·m

C to D $\quad (\frac{1}{2})(1.8)(-0.75 - 72.75) = -66.15$ kN·m



Bending moments: $\quad M_A = 0$

$M_B = 0 + 66.825 = 66.825$ kN·m

$M_C = 66.825 - 0.675 = 66.15$ kN·m

$M_D = 66.15 - 66.15 = 0$

Maximum $|M| = 66.825$ kN·m $= 66.825 \times 10^3$ N·m

$\sigma_{all} = 160$ MPa $= 160 \times 10^6$ Pa

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{66.825 \times 10^3}{160 \times 10^6} = 417.7 \times 10^{-6} \text{ m}^3 = 417.7 \times 10^3 \text{ mm}^3$$

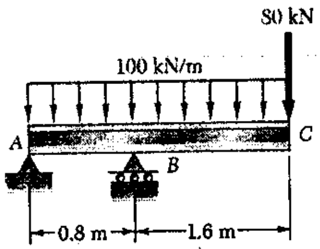
Shape	S (10^3 mm^3)
S 380 × 64	971
S 310 × 47.3	593
S 250 × 52	482

Lightest S-section

S 310 × 47.3 @ 47.3 kg/m

PROBLEM 5.86

5.85 and 5.86 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric S-shape beam to support the loading shown.



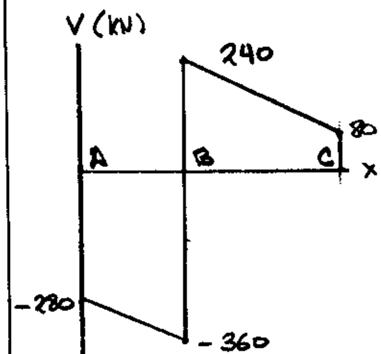
SOLUTION

$+\circlearrowleft \Sigma M_B = 0 \quad 0.8A - (0.4)(2.4)(100) - (1.6)(80) = 0$

$A = 280 \text{ kN} \downarrow$

$+\circlearrowleft \Sigma M_A = 0 \quad 0.8B - (1.2)(2.4)(100) - (2.4)(80) = 0$

$B = 600 \text{ kN} \uparrow$



Shear: $V_A = -280 \text{ kN}$

$V_{B^-} = -280 - (0.8)(100) = -360 \text{ kN}$

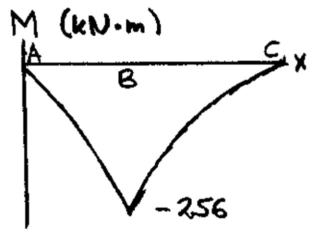
$V_{B^+} = -360 + 600 = 240 \text{ kN}$

$V_C = 240 - (1.6)(100) = 80 \text{ kN}$

Areas under shear diagram

A to B $\quad (\frac{1}{2})(0.8)(-280 - 360) = -256 \text{ kN}\cdot\text{m}$

B to C $\quad (\frac{1}{2})(1.6)(240 + 80) = 256 \text{ kN}\cdot\text{m}$



Bending moments: $\quad M_A = 0$

$M_B = 0 - 256 = -256 \text{ kN}\cdot\text{m}$

$M_C = -256 + 256 = 0$

Maximum $|M| = 256 \text{ kN}\cdot\text{m} = 256 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{256 \times 10^3}{160 \times 10^6} = 1.6 \times 10^{-3} \text{ m}^3 = 1600 \times 10^3 \text{ mm}^3$$

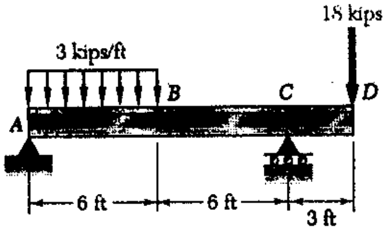
Shape	S (10^3 mm^3)
S 510 × 98.3	1950
S 460 × 104	1685

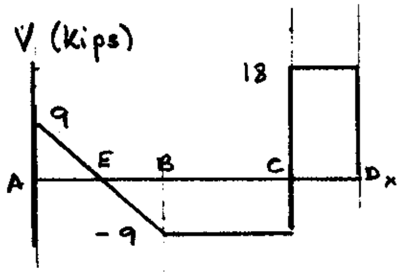
Lightest S-section

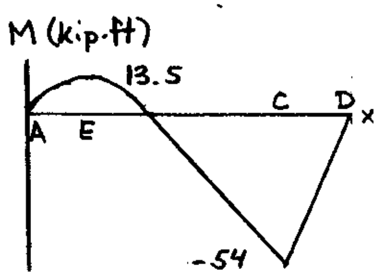
S 510 × 98.3 ◄

PROBLEM 5.87

5.87* and *5.88 Knowing that the allowable normal stress for the steel used is 24 ksi,.select the most economical S-shape beam to support the loading shown.







SOLUTION

$+\circlearrowleft \Sigma M_C = 0 \qquad -12A + (9)(6)(3) - (3)(18) = 0$

$A = 9 \text{ kips}$

$+\circlearrowleft \Sigma M_A = 0 \qquad 12C - (3)(6)(3) - (15)(18) = 0$

$C = 27 \text{ kips}$

Shear: $V_A = 9$ kips

B to C $\quad V = 9 - (6)(3) = -9$ kips

C to D $\quad V = -9 + 27 = 18$ kips

Areas: A to E $\quad (\frac{1}{2})(3)(9) = 13.5$ kip·ft

E to B $\quad (\frac{1}{2})(3)(-9) = -13.5$ kip·ft

B to C $\quad (6)(-9) = -54$ kip·ft

C to D $\quad (3)(18) = 54$ kip·ft

Bending moments: $M_A = 0$

$M_E = 0 + 13.5 = 13.5$ kip ft

$M_B = 13.5 - 13.5 = 0$

$M_C = 0 + 54 = 54$ kip·ft

$M_D = 54 - 54 = 0$

Maximum $|M| = 54$ kip·ft $= 648$ kip·in

$\sigma_{all} = 24$ ksi

$$S_{min} = \frac{648}{24} = 27 \text{ in}^3$$

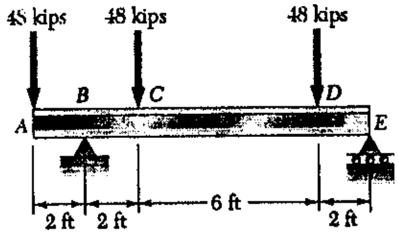
Shape	$S(\text{in}^3)$
S 12×31.8	36.4
S 10×35	29.4

Lightest S-shaped beam

S 12×31.8

PROBLEM 5.88

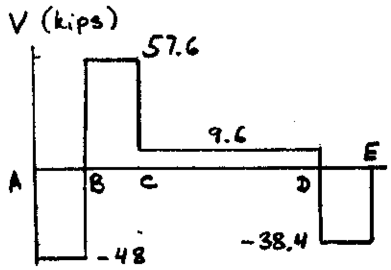
5.87* and *5.88 Knowing that the allowable normal stress for the steel used is 24 ksi,.select the most economical S-shape beam to support the loading shown.



SOLUTION

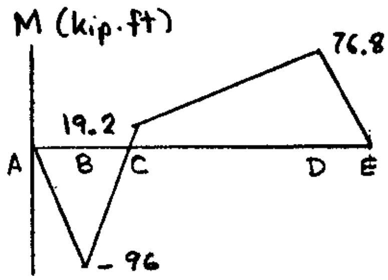
$$+\sum M_E = 0 \quad (12)(48) - 10B + (8)(48) + (2)(48) = 0$$
$$B = 105.6 \text{ kips}$$

$$+\sum M_B = 0 \quad (2)(48) - (2)(48) - (8)(48) + 10E = 0$$
$$E = 38.4 \text{ kips}$$



Shear: A to B $\quad V = -48$ kips
B to C $\quad V = -48 + 105.6 = 57.6$ kips
C to D $\quad V = 57.6 - 48 = 9.6$ kips
D to E $\quad V = 9.6 - 48 = -38.4$ kips

Areas: A to B $\quad (2)(-48) = -96$ kip·ft
B to C $\quad (2)(57.6) = 115.2$ kip·ft
C to D $\quad (6)(9.6) = 57.6$ kip·ft
D to E $\quad (2)(-38.4) = 76.8$ kip·ft



Bending moments: $M_A = 0$
$M_B = 0 - 96 = -96$ kip·ft
$M_C = -96 + 115.2 = 19.2$ kip·ft
$M_D = 19.2 + 57.2 = 76.8$ kip ft
$M_E = 76.8 - 76.8 = 0$

Maximum $|M| = 96$ kip·ft $= 1152$ kip·in

$\sigma_{all} = 24$ ksi

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{1152}{24} = 48 \text{ in}^3$$

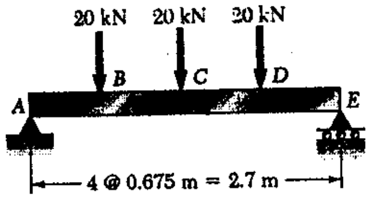
Shape	S (in³)
S 15 × 42.9	59.6
S 12 × 50	50.8

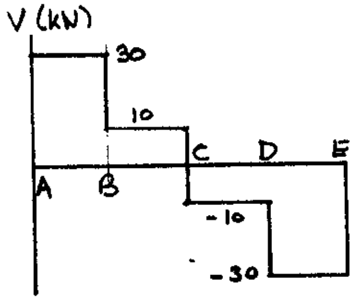
Lightest S-shaped beam

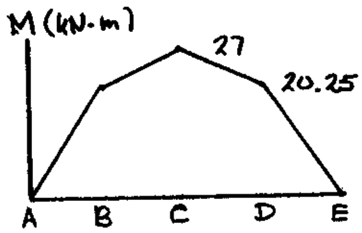
S 15 × 42.9

PROBLEM 5.89

5.89 Two metric rolled-steel channels are to be welded along their edges and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 150 MPa, determine the most economical channels that can be used.







SOLUTION

By symmetry $A = E$

$+\uparrow \Sigma F_y = 0 \quad A + E - 20 - 20 - 20 = 0$

$A = E = 30 \text{ kN}$

Shear:
- A to B $\quad V = 30 \text{ kN}$
- B to C $\quad V = 30 - 20 = 10 \text{ kN}$
- C to D $\quad V = 10 - 20 = -10 \text{ kN}$
- D to E $\quad V = -10 - 20 = -30 \text{ kN}$

Areas:
- A to B $\quad (0.675)(30) = 20.25 \text{ kN}\cdot\text{m}$
- B to C $\quad (0.675)(10) = 6.75 \text{ kN}\cdot\text{m}$
- C to D $\quad (0.675)(-10) = -6.75 \text{ kN}\cdot\text{m}$
- D to E $\quad (0.675)(-30) = -20.25 \text{ kN}\cdot\text{m}$

Bending moments: $M_A = 0$

$M_B = 0 + 20.25 = 20.25 \text{ kN}\cdot\text{m}$

$M_C = 20.25 + 6.75 = 27 \text{ kN}\cdot\text{m}$

$M_D = 27 - 6.75 = 20.25 \text{ kN}\cdot\text{m}$

$M_E = 20.25 - 20.25 = 0$

Maximum $|M| = 27 \text{ kN}\cdot\text{m} = 27\times10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 150 \text{ MPa} = 150\times10^6 \text{ Pa}$

For a section consisting of two channels

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{27\times10^3}{150\times10^6} = 180\times10^{-6} \text{ m}^3 = 180\times10^3 \text{ mm}^3$$

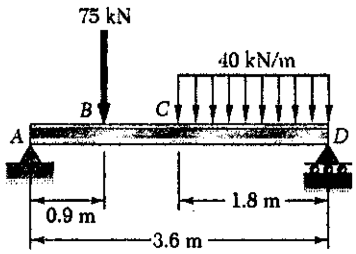
For each channel $\quad S_{min} = (\tfrac{1}{2})(180\times10^3) = 90\times10^3 \text{ mm}^3$

Shape	$S\ (10^3 \text{ mm}^3)$
C 180 × 14.6	99.2
C 150 × 19.3	93.6

Lightest channel section
C 180 × 14.6

PROBLEM 5.90

5.90 Two metric rolled-steel channels are to be welded back to back and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 190 MPa, determine the most economical channels that can be used.



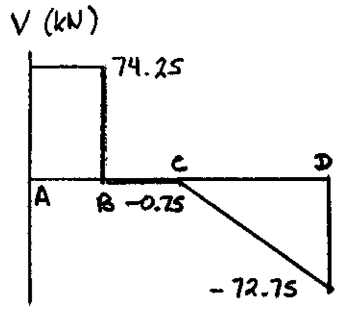
SOLUTION

$+\circlearrowleft \Sigma M_D = 0 \quad -3.6A + (2.7)(75) + (0.9)(1.8)(40) = 0$

$A = 74.25 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0 \quad 3.6D - (0.9)(75) - (2.7)(1.8)(40) = 0$

$D = 72.75 \text{ kN}$



Shear: A to B $\quad V = 74.25 \text{ kN}$

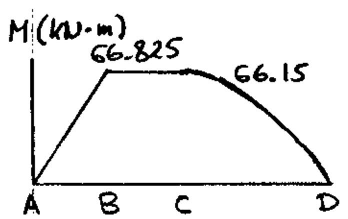
B to C $\quad V = 74.25 - 75 = -0.75 \text{ kN}$

$V_D = -0.75 - (1.8)(40) = -72.75 \text{ kN}$

Areas: A to B $\quad (0.9)(74.25) = 66.825 \text{ kN}\cdot\text{m}$

B to C $\quad (0.9)(-0.75) = -0.675 \text{ kN}\cdot\text{m}$

C to D $\quad (\frac{1}{2})(1.8)(-0.75 - 72.75) = -66.15$



Bending moments: $\quad M_A = 0$

$M_B = 0 + 66.825 = 66.825 \text{ kN}\cdot\text{m}$

$M_C = 66.825 - 0.675 = 66.15 \text{ kN}\cdot\text{m}$

$M_D = 66.15 - 66.15 = 0$

Maximum $|M| = 66.825 \text{ kN}\cdot\text{m} = 66.825 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 190 \text{ MPa} = 190 \times 10^6 \text{ Pa}$

For double channel $\quad S_{min} = \frac{|M|}{\sigma_{all}} = \frac{66.825 \times 10^3}{190 \times 10^6} = 351.7 \times 10^{-6} \text{ m}^3 = 351.7 \times 10^3 \text{ mm}^3$

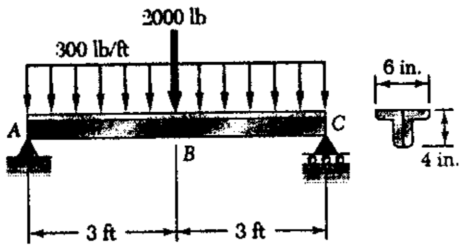
For each channel $\quad S_{min} = (\frac{1}{2})(351.7 \times 10^3) = 175.9 \times 10^3 \text{ mm}^3$

Shape	$S\ (10^3 \text{ mm}^3)$
C 230 × 22	185
C 200 × 27.9	179

Lightest channel section
C 230 × 22

PROBLEM 5.91

5.91 Two L 4 × 3 rolled-steel angles are bolted together to support the loading shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine the minimum angle thickness that can be used.



SOLUTION

By symmetry $A = C$

$+\Sigma F_y = 0 \quad A + C - 2000 - (6)(300) = 0$

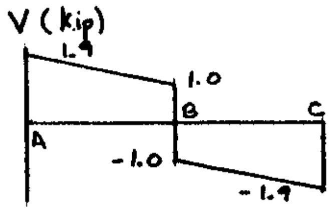
$A = C = 1900 \text{ lb.}$

Shear: $V_A = 1900 \text{ lb.} = 1.9 \text{ kips}$

$V_{B^-} = 1900 - (3)(300) = 1000 \text{ lb} = 1 \text{ kip}$

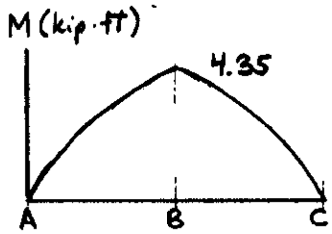
$V_{B^+} = 1000 - 2000 = -1000 \text{ lb} = -1 \text{ kip}$

$V_C = -1000 - (3)(300) = -1900 \text{ lb} = -1.9 \text{ kip}$



Areas: A to B $(\frac{1}{2})(3)(1.9 + 1) = 4.35$ kip·ft

B to C $(\frac{1}{2})(3)(-1 - 1.9) = -4.35$ kip·ft



Bending moments: $M_A = 0$

$M_B = 0 + 4.35 = 4.35$ kip·ft

$M_C = 4.35 - 4.35 = 0$

Maximum $|M| = 4.35$ kip·ft $= 52.2$ kip·in

$\sigma_{all} = 24$ ksi

For section consisting of two angles $S_{min} = \frac{|M|}{\sigma_{all}} = \frac{52.2}{24} = 2.175 \text{ in}^3$

For each angle $S_{min} = (\frac{1}{2})(2.175) = 1.0875 \text{ in}^3$

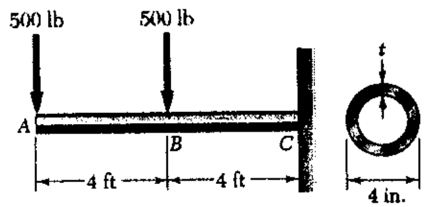
Angle section	S (in³)
L 4×3 × $\frac{1}{2}$	1.89
L 4×3 × $\frac{3}{8}$	1.46
L 4×3 × $\frac{1}{4}$	1.00

Smallest allowable thickness

$t = \frac{3}{8}$ in.

PROBLEM 5.92

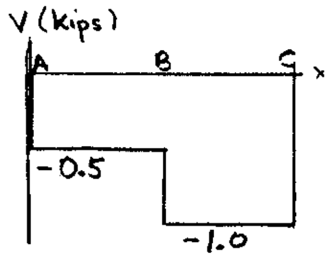
5.92 A steel pipe of 4-in. diameter is to support the loading shown. Knowing that the stock of pipes available have thicknesses varying from $\frac{1}{4}$ in. to 1 in. in $\frac{1}{8}$- in. increments, and that the allowable normal stress for the steel used is 24 ksi, determine the minimum wall thickness t that can be used.



SOLUTION

Shear: A to B $\quad V = -500 \text{ lb} = -0.5 \text{ kip}$

B to C $\quad V = -500 - 500 = -1000 \text{ lb} = -1.0 \text{ kip}$



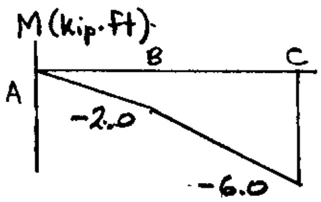
Areas: A to B $\quad (4)(-0.5) = -2.0 \text{ kip}\cdot\text{ft}$

B to C $\quad (4)(-1.0) = -4.0 \text{ kip}\cdot\text{ft}$

Bending moments: $\quad M_A = 0$

$M_B = 0 - 2.0 = -2.0 \text{ kip}\cdot\text{ft}$

$M_C = -2.0 - 4.0 = -6.0 \text{ kip}\cdot\text{ft}$



Maximum $|M| = 6.0 \text{ kip}\cdot\text{ft} = 72 \text{ kip}\cdot\text{in}$

$\sigma_{all} = 24 \text{ ksi}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{72}{24} = 3 \text{ in}^3$$

$I = \frac{\pi}{4}(c_2^4 - c_1^4) \qquad c = c_2 \qquad c_2 = \frac{1}{2}d = 2.0 \text{ in.}$

$$S = \frac{I}{c} = \frac{\pi}{4}\,\frac{c_2^4 - c_1^4}{c_2} = \frac{\pi}{4}\,\frac{2^4 - c_1^4}{2} = 3 \text{ in}^3$$

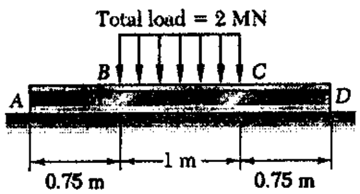
$c_1^4 = 2^4 - \frac{(4)(2)(3)}{\pi} = 8.3606 \text{ in}^4 \qquad c_1 = 1.7004 \text{ in.}$

$t_{min} = c_2 - c_1 = 2.0 - 1.7004 = 0.2996 \text{ in}$

Using $\frac{1}{8}$ in. increments for design $\quad t = \frac{3}{8}$ in.

PROBLEM 5.93

5.93 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa, select the most economical metric wide-flange beam to support the loading shown..

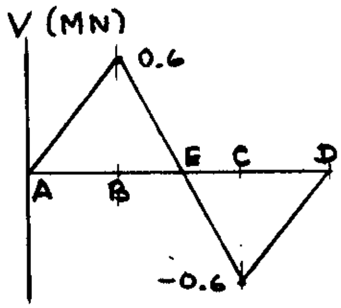


SOLUTION

Downward distributed load $w = \frac{2}{1.0} = 2$ MN/m

Upward distributed reaction $q = \frac{2}{2.5} = 0.8$ MN/m

Net distributed load over BC 1.2 MN/m



Shear: $V_A = 0$

$V_B = 0 + (0.75)(0.8) = 0.6$ MN

$V_C = 0.6 - (1.0)(1.2) = -0.6$ MN

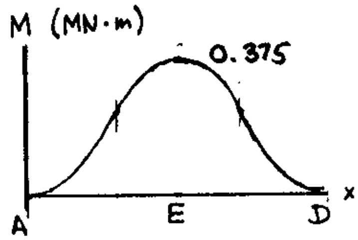
$V_D = -0.6 + (0.75)(0.8) = 0$

Areas: A to B $(\frac{1}{2})(0.75)(0.6) = 0.225$ MN·m

B to E $(\frac{1}{2})(0.5)(0.6) = 0.150$ MN·m

E to C $(\frac{1}{2})(0.5)(-0.6) = -0.150$ MN·m

C to D $(\frac{1}{2})(0.75)(-0.6) = -0.225$ MN·m



Bending moments: $M_A = 0$

$M_B = 0 + 0.225 = 0.225$ MN·m

$M_E = 0.225 + 0.150 = 0.375$ MN·m

$M_C = 0.375 - 0.150 = 0.225$ MN·m

$M_D = 0.225 - 0.225 = 0$

Maximum $|M| = 0.375$ MN·m $= 375 \times 10^3$ N·m

$\sigma_{all} = 170$ MPa $= 170 \times 10^6$ Pa

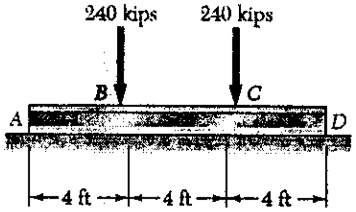
$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{375 \times 10^3}{170 \times 10^6} = 2.206 \times 10^{-3} \text{ m}^3 = 2206 \times 10^3 \text{ mm}^3$$

Shape	$S(10^3 mm^3)$
W 690 × 125	3510
W 610 × 101	2530
W 530 × 150	3720
W 460 × 113	2400

Lightest wide flange section

W 610 × 101

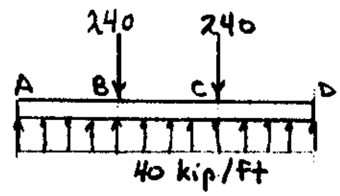
PROBLEM 5.94



5.94 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

SOLUTION

Distributed reaction $q = \frac{480}{12} = 40$ kip/ft



Shear: $V_A = 0$

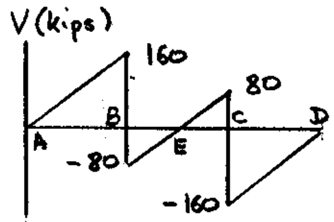
$V_{B^-} = 0 + (4)(40) = 160$ kips

$V_{B^+} = 160 - 240 = -80$ kips

$V_{C^-} = -80 + (4)(40) = 80$ kips

$V_{C^+} = 80 - 240 = -160$ kips

$V_D = -160 + (4)(40) = 0$

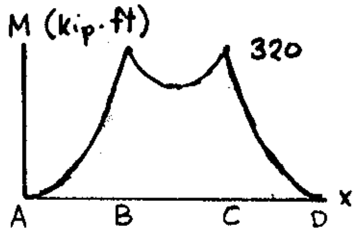


Areas: A to B $(\frac{1}{2})(4)(160) = 320$ kip·ft

B to E $(\frac{1}{2})(2)(-80) = -80$ kip·ft

E to C $(\frac{1}{2})(2)(80) = 80$ kip·ft

C to D $(\frac{1}{2})(4)(-160) = -320$ kip·ft



Bending moments: $M_A = 0$

$M_B = 0 + 320 = 320$ kip·ft

$M_E = 320 - 80 = 240$ kip·ft

$M_C = 240 + 80 = 320$ kip·ft

$M_D = 320 - 320 = 0$

Maximum $|M| = 320$ kip·ft $= 3840$ kip·in

$\sigma_{all} = 24$ ksi $\qquad S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3840}{24} = 160 \text{ in}^3$

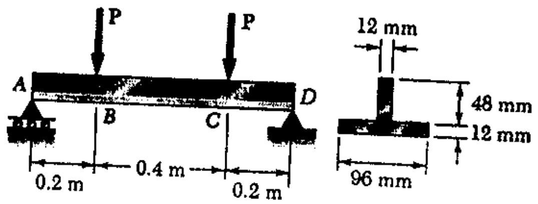
Shape	S (in³)
S 24 × 80	175
S 20 × 96	165

Lightest S-shaped section

S 24 × 80

PROBLEM 5.95

5.95 and 5.96 Determine the largest permissible value of **P** for the beam and loading shown, knowing that the allowable normal stress is +80 MPa in tension and − 140 MPa in compression.



SOLUTION

Reactions: $A = D = P$

Shear: A to B $V = P$

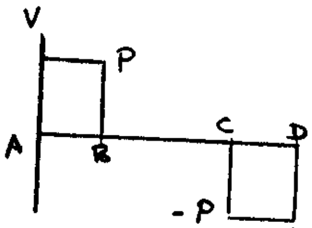
B to C $V = P - P = 0$

C to D $V = 0 - P = -P$

Areas: A to B $0.2P$

B to C 0

C to D $-0.2P$

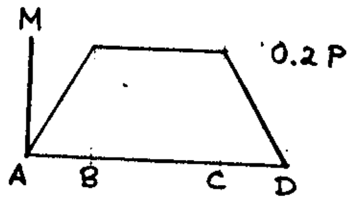


Bending moments: $M_A = 0$

$M_B = 0 + 0.2P = 0.2P$

$M_C = 0.2P + 0 = 0.2P$

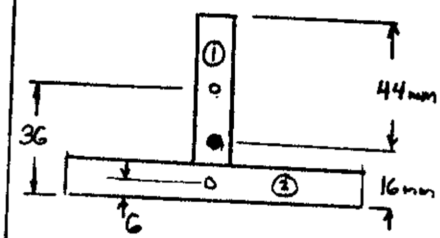
$M_D = 0.2P - 0.2P = 0$



Largest positive bending moment: $0.2P$

Largest negative bending moment: 0

Centroid and moment of inertia



Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)	d (mm)	Ad^2 (mm⁴)	$\bar{I}$ mm⁴
①	576	36	20736	20	230.4×10^3	110.6×10^3
②	1152	6	6912	10	115.2×10^3	13.8×10^3
Σ	1728		27648		345.6×10^3	124.4×10^3

$\bar{Y} = \frac{27648}{1728} = 16$ mm

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 470.0\times10^3 \text{ mm}^4$$

Top $c = 44$ mm $\quad S = \frac{I}{c} = \frac{470.0\times10^3}{44} = 10.68\times10^3 \text{ mm}^3 = 10.68\times10^{-6} \text{ m}^3$

Allowable pos. M. $\quad M = |\sigma|S = (140\times10^6)(10.68\times10^{-6}) = 1495$ N·m

Bot. $c = 16$ mm $\quad S = \frac{I}{c} = \frac{470.0\times10^3}{16} = 29.38\times10^3 \text{ mm}^3 = 29.38\times10^{-6} \text{ m}^3$

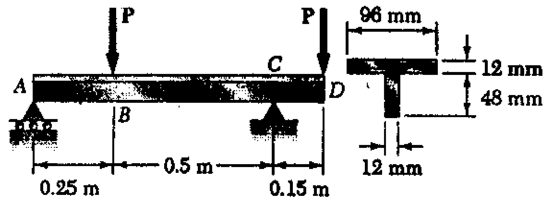
Allowable pos. M $\quad M = |\sigma_{ten}|S = (80\times10^6)(29.38\times10^{-6}) = 2350$ N·m

Smaller value $M = 1495$ N·m

Allowable value of P $\quad 0.2P = 1495 \qquad P = 7475 \text{ N} = 7.48 \text{ kN}$ ◄

PROBLEM 5.96

5.95 and 5.96 Determine the largest permissible value of P for the beam and loading shown, knowing that the allowable normal stress is +80 MPa in tension and - 140 MPa in compression



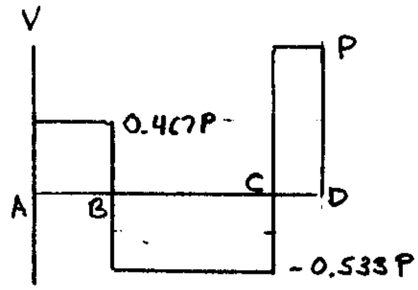
SOLUTION

$+\circlearrowleft \Sigma M_C = 0 \quad -0.75A + 0.5P - 0.15P = 0$

$A = 0.46667\,P$

$+\circlearrowleft \Sigma M_A = 0 \quad 0.75C - 0.25P - 0.9P = 0$

$C = 1.53333\,P$



Shear: A to B $\quad V = 0.46667\,P$

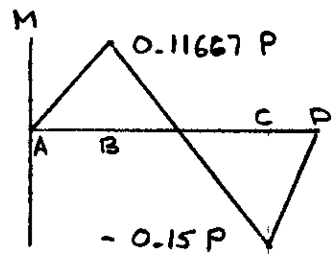
B to C $\quad V = 0.46667\,P - P = -0.53333\,P$

C to D $\quad V = -0.53333\,P + 1.53333\,P = P$

Areas: A to B $\quad (.25)(0.46667\,P) = 0.11667\,P$

B to C $\quad (0.5)(-.53333\,P) = -0.26667\,P$

C to D $\quad (0.15)\,P = 0.15\,P$



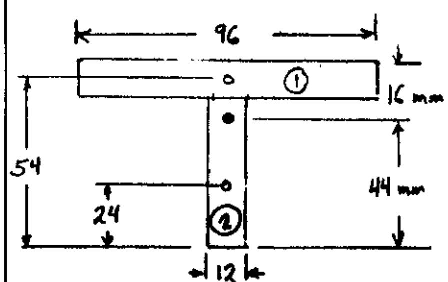
Bending moments: $\quad M_A = 0$

$M_B = 0 + 0.11667\,P = 0.11667\,P$

$M_C = 0.11667\,P - 0.26667\,P = -0.15\,P$

$M_D = -0.15\,P + 0.15\,P = 0$

Centroid and moment of inertia.



Part	$A\,(mm^2)$	$\bar{y}\,(mm)$	$A\bar{y}\,(mm^3)$	$d\,(mm)$	$Ad^2\,(mm^4)$	$\bar{I}\,(mm^4)$
①	1152	54	62208	10	115200	13824
②	576	24	13824	20	230400	110592
Σ	1728		76032		345600	124416

$\bar{Y} = \frac{76032}{1728} = 44\text{ mm}$

$I = \Sigma Ad^2 + \Sigma \bar{I} = 470016\text{ mm}^4$

Top: $y = 16\text{ mm} \quad \frac{I}{y} = \frac{470016}{16} = 29.376 \times 10^3\text{ mm}^3 = 29.376 \times 10^{-6}\text{ m}^3$

Bottom: $y = -44\text{ mm} \quad \frac{I}{y} = \frac{470016}{44} = 10.682 \times 10^3\text{ mm}^3 = -10.682 \times 10^{-6}\text{ m}^3$

Bending moment limits: $\quad M = -\frac{I\sigma}{y}$

Tension at B $\quad = (-10.682 \times 10^{-6})(80 \times 10^6) = 854.56\text{ N}\cdot\text{m} \quad \triangleleft$ B

Comp. at B $\quad -(29.376 \times 10^{-6})(-140 \times 10^6) = 4.1126 \times 10^3\text{ N}\cdot\text{m}$

Tension at C $\quad -(29.376 \times 10^{-6})(80 \times 10^6) = -2.35 \times 10^3\text{ N}\cdot\text{m}$

Comp. at C $\quad -(-10.682 \times 10^{-6})(-140 \times 10^6) = -1.4955 \times 10^3\text{ N}\cdot\text{m} \quad \triangleleft$ C

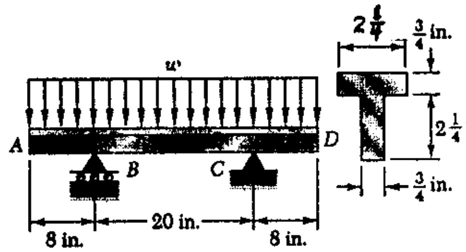
Allowable load: $\quad 0.11667\,P = 854.56 \qquad P = 7.32 \times 10^3\text{ N}$

$-0.15\,P = -1.4955 \times 10^3 \qquad P = 9.97 \times 10^3\text{ N}$

The smaller value is $\quad P = 7.32\text{ kN}$ ◄

PROBLEM 5.97

5.97 Determine the largest permissible uniformly distributed load *w* for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and - 19,5 ksi in compression.



SOLUTION

Reactions: $B + C - 36w = 0$ $B = C = 18w$

Shear: $V_A = 0$

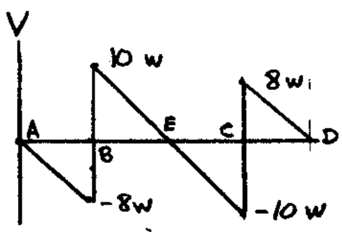
$V_{B^-} = 0 - 8w = -8w$

$V_{B^+} = -8w + 18w = 10w$

$V_{C^-} = 10w - 20w = -10w$

$V_{C^+} = -10w + 18w = 8w$

$V_D = 8w - 8w = 0$



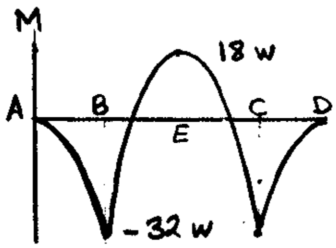
Areas: A to B $(\frac{1}{2})(8)(-8w) = -32w$

B to E $(\frac{1}{2})(10)(10w) = 50w$

Bending moments: $M_A = 0$

$M_B = 0 - 32w = -32w$

$M_E = -32w + 50w = 18w$

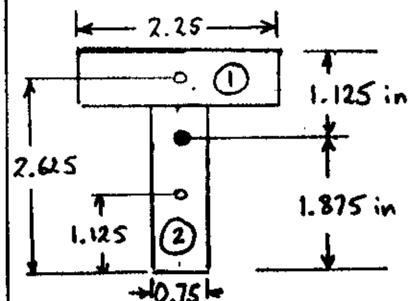


Centroid and moment of inertia

Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d in²	Ad² (in⁴)	$\bar{I}$ (in⁴)
①	1.6875	2.625	4.4297	0.75	0.9492	0.0791
②	1.6875	1.125	1.8984	0.75	0.9492	0.7119
Σ	3.375		6.3281		1.8984	0.7910

$\bar{Y} = \frac{6.3281}{3.375} = 1.875$ in

$I = \Sigma Ad^2 + \Sigma \bar{I} = 2.6894$ in⁴



Top: $y = 1.125$ $I/y = 2.3906$ in³

Botton: $y = -1.875$ $I/y = -1.4343$ in³

Bending moment limits $M = -\sigma I/y$

Tension at B and C	$-(12)(2.3906) =$	-28.687 kip·in
Comp. at B and C	$-(-19.5)(-1.4343) =$	-27.969 kip·in
Tension at E	$-(12)(-1.4343) =$	17.212 kip·in
Compression at E	$-(-19.5)(2.3906) =$	46.6 kip·in

Allowable load w

B & C $-32w = -27.969$ $w = 0.874$ kip/in

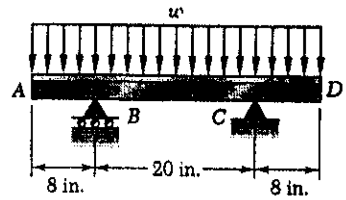
E $18w = 17.212$ $w = 0.956$ kip/in

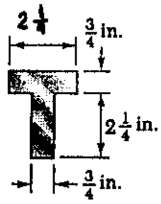
Smallest $w = 0.874$ kip/in $= 10.49$ kip/ft.

PROBLEM 5.98

5.97 Determine the largest permissible uniformly distributed load w for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and −19,5 ksi in compression.

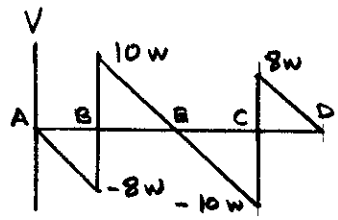
5.98 Solve Prob. 5.97, assuming that the cross section of the beam is reversed, with the flange of the beam resting on the supports at B and C.





SOLUTION

Reactions $\quad B + C - 36w = 0 \qquad B = C = 18w$



Shear: $V_A = 0$

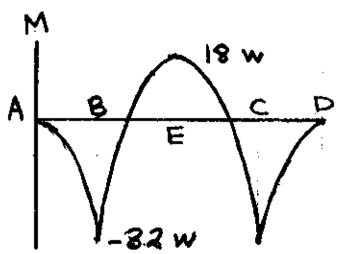
$V_{B^-} = 0 - 8w = -8w$

$V_{B^+} = -8w + 18w = 10w$

$V_{C^-} = 10w - 20w = -10w$

$V_{C^+} = -10w + 18w = 8w$

$V_D = 8w - 8w = 0$



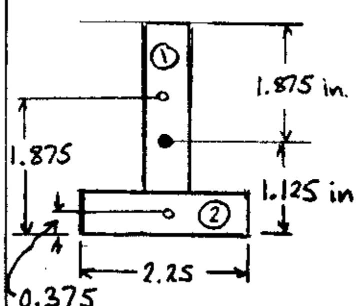
Areas: $\quad$ A to B $\quad (\tfrac{1}{2})(8)(-8w) = -32w$

B to E $\quad (\tfrac{1}{2})(10)(10w) = 50w$

Bending moments: $\quad M_A = 0$

$M_B = 0 - 32w = -32w$

$M_E = -32w + 50w = 18w$



Centroid and moment of inertia

Part	$A\,(in^2)$	$\bar{y}\,(in)$	$A\bar{y}\,(in^3)$	$d\,(in)$	$Ad^2\,(in^4)$	$\bar{I}\,(in^4)$
1	1.6875	1.875	3.1641	0.75	0.9492	0.7119
2	1.6875	0.375	0.6328	0.75	0.9492	0.0791
Σ	3.375		3.7969		1.8984	0.7910

$\bar{Y} = \dfrac{3.7969}{3.375} = 1.125$ in.

$I = \Sigma Ad^2 + \Sigma\bar{I} = 2.6894\ in^4$

Top: $y = 1.875$ in $\qquad I/y = 1.4343\ in^3$

Bottom $\quad y = -1.125 \qquad -I/y = -2.3906\ in^3$

Bending moment limits $\qquad M = -\sigma I/y$

Tension at B and C $\quad -(12)(1.4343) = -17.212$ kip-in ←

Comp. at B and C $\quad -(-19.5)(-2.3906) = -46.6$ kip-in

Tension at E $\quad -(12)(-2.3906) = 28.687$ kip-in

Compression at E $\quad -(-19.5)(1.4343) = 27.969$ kip-in ←

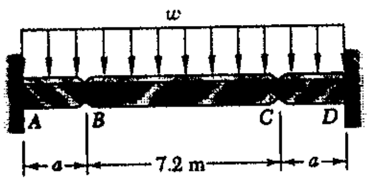
Allowable load w: $\quad$ B & C $\quad -32w = -17.212 \qquad w = 0.539$ kip/in

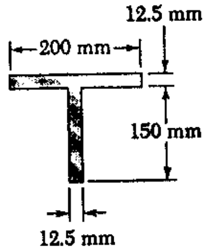
E $\quad 18w = 27.969 \qquad w = 1.554$ kip/in

Smallest $\qquad w = 0.539$ kip/in $= 6.45$ kip/ft ◄

PROBLEM 5.99

5.99 Beams *AB*, *BC*, and *CD* have the cross section shown and are pin-connected at *B* and *C*. Knowing that the allowable normal stress is +110 MPa in tension and -150 MPa in compression, determine (*a*) the largest permissible value of *w* if beam *BC* is not to be overstressed, (*b*) the corresponding maximum distance *a* for which the cantilever beams *AB* and *CD* are not overstressed.





SOLUTION

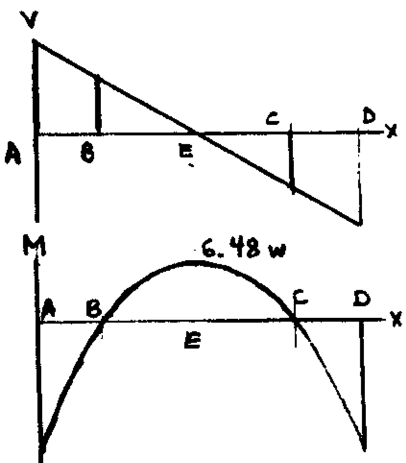
(a) $M_B = M_C = 0$

$V_B = -V_C = (\frac{1}{2})(7.2)w = 3.6\,w$

Area B to E of shear diagram

$(\frac{1}{2})(3.6)(3.6w) = 6.48\,w$

$M_E = 0 + 6.48\,w = 6.48\,w$

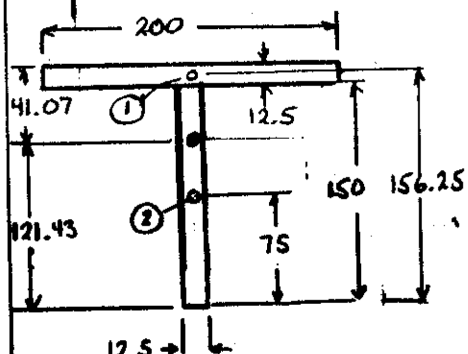


Centroid and moment of inertia

Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)	d (mm)	Ad^2 (mm⁴)	$\bar{I}$ (mm⁴)
1	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
2	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$\bar{Y} = \frac{531250}{4375} = 121.43$ mm

$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6$ mm⁴



Location	y (mm)	I/y (10^3 mm³) ← also (10^{-6} m³)
top	41.07	258.6
bottom	-121.43	-87.47

Bending moment limits $\quad M = -\sigma I/y$

Tension at E: $\quad -(110\times10^6)(-87.47\times10^{-6}) = 9.622\times10^3$ N·m

Comp. at E: $\quad -(-150\times10^{-6})(258.6\times10^{-6}) = 38.8\times10^3$ N·m

Tension at A & D: $\quad -(110\times10^6)(258.6\times10^{-6}) = -28.45\times10^3$ N·m

Comp. at A & D: $\quad -(-150\times10^6)(-87.47\times10^{-6}) = -13.121\times10^3$ N·m

Allowable load w $\qquad 6.48\,w = 9.622\times10^3 \qquad w = 1.485\times10^3$ N/m

$= 1.485$ kN/m ◄

Shear at A $\qquad V_A = (a + 3.6)\,w$

Area A to B of shear diagram $\qquad \frac{1}{2}a(V_A + V_B) = \frac{1}{2}a(a + 7.2)w$

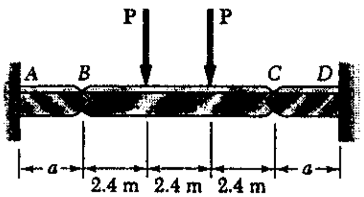
Bending moment at A (also D) $\qquad M_A = -\frac{1}{2}a(a + 7.2)\,w$

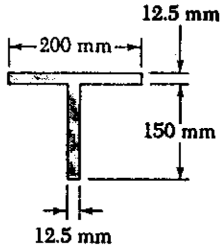
$-\frac{1}{2}a(a + 7.2)(1.485\times10^3) = -13.121\times10^3$

$\frac{1}{2}a^2 + 3.6\,a - 8.837 = 0 \qquad a = 1.935$ m ◄

PROBLEM 5.100

5.100 Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and -150 MPa in compression, determine (*a*) the largest permissible value of P if beam BC is not to be overstressed, (*b*) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed





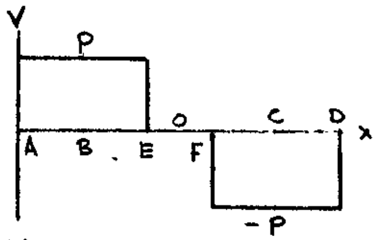
SOLUTION

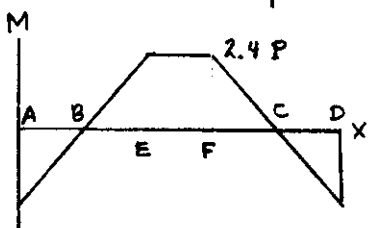
(a) $M_B = M_C = 0$

$V_B = -V_C = P$

Area B to E of shear diagram.
$2.4\ P$

$M_E = 0 + 2.4\ P = 2.4\ P = M_F$



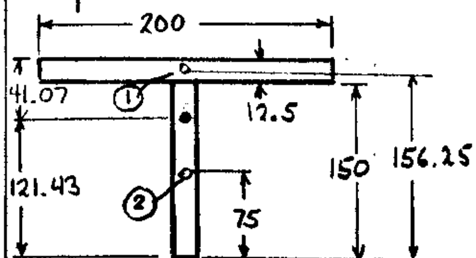


Centroid and moment of inertia

Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)	d (mm)	Ad^2 (mm⁴)	$\bar{I}$ (mm⁴)
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$$\bar{Y} = \frac{531250}{4375} = 121.43\ \text{mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6\ \text{mm}^4$$



Location	y (mm)	I/y (10^3 mm³) ← also (10^{-6} m³)
top	41.07	258.6
bottom	-121.43	-87.47

Bending moment limits $\quad M = -\sigma I/y$

Tension at E & F: $-(110\times10^6)(-87.47\times10^{-6}) = 9.622\times10^3$ N·m

Comp. at E & F: $-(-150\times10^6)(258.6\times10^{-6}) = 38.8\times10^3$ N·m

Tension at A & D: $-(110\times10^6)(258.6\times10^{-6}) = -28.45\times10^3$ N·m

Comp. at A & D: $-(-150\times10^6)(-87.47\times10^{-6}) = -13.121\times10^3$ N·m

Allowable load P $\qquad 2.4\ P = 9.622\times10^3 \qquad P = 4.01\times10^3$ N $= 4.01$ kN

Shear at A $\qquad V_A = P$

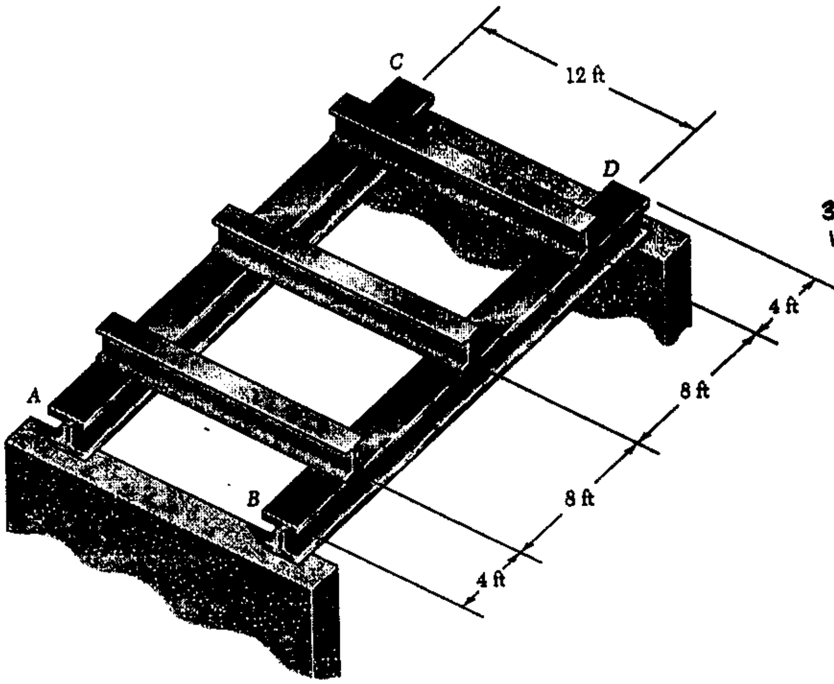
Area A to B of shear diagram $\qquad aV_A = aP$

Bending moment at A $\qquad M_A = -aP = -4.01\times10^3\ a$

Distance a $\qquad -4.01\times10^3\ a = -13.121\times10^3 \qquad a = 3.27$ m

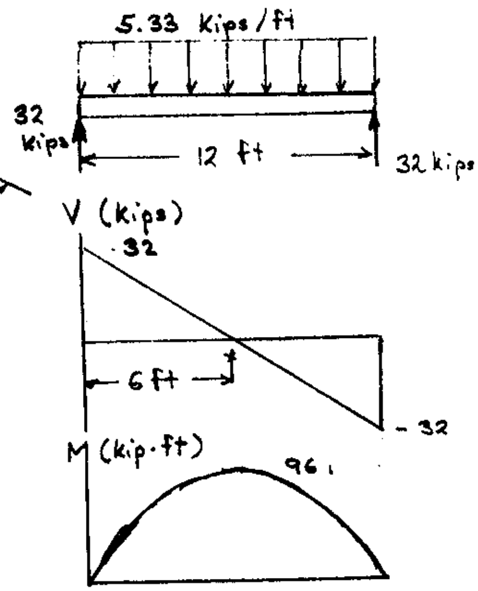
PROBLEM 5.101

5.101 Each of the three rolled-steel beams shown (numbered 1, 2, and 3) is to carry a 64-kip load uniformly distributed over the beam. Each of these beams has a 12-ft span and is to be supported by the two 24-ft rolled-steel girders *AC* and *BD*. Knowing that the allowable normal stress for the steel used is 24 ksi, select (*a*) the most economical S-shape for the three beams, (*b*) the most economical W-shape for the two girders.



SOLUTION

Beams 1, 2, and 3



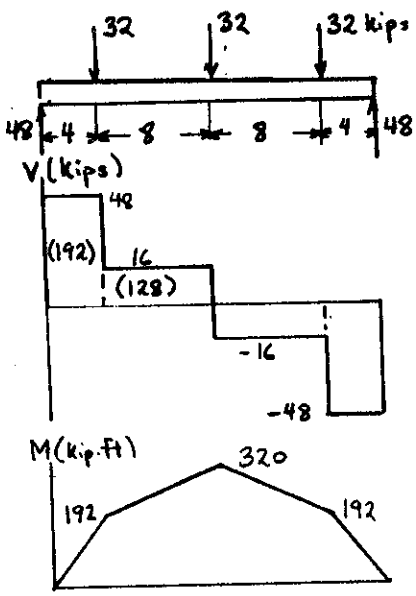
Maximum $M = (\frac{1}{2})(6)(32) = 96$ kip·ft $= 1152$ kip·in

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{1152}{24} = 48 \text{ in}^3$

Shape	S (in³)
S 15 × 42.9	59.6
S 12 × 50	50.8

(a) Use S 15 × 42.9 ◄

Beams AC and BC



Areas under shear diagram

(4)(48) = 192 kip·ft

(8)(16) = 128 kip·ft

Maximum $M = 192 + 128 = 320$ kip·ft $= 3840$ kip·in

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3840}{24} = 160 \text{ in}^3$

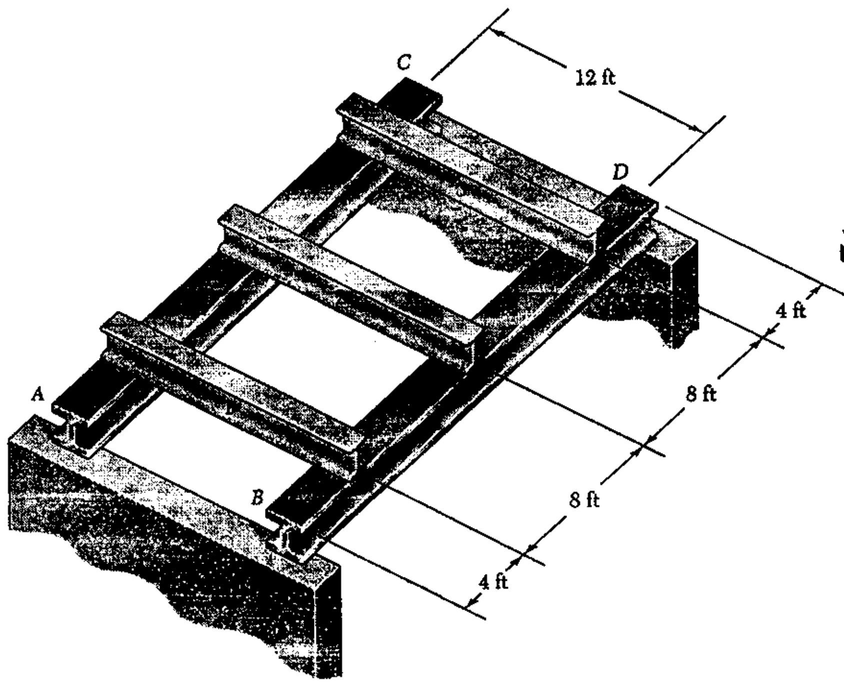
Shape	S (in³)
W 30 × 99	269
W 27 × 84	213
W 24 × 104	258
W 21 × 101	227
W 18 × 106	204

(b) Use W 27 × 84 ◄

PROBLEM 5.102

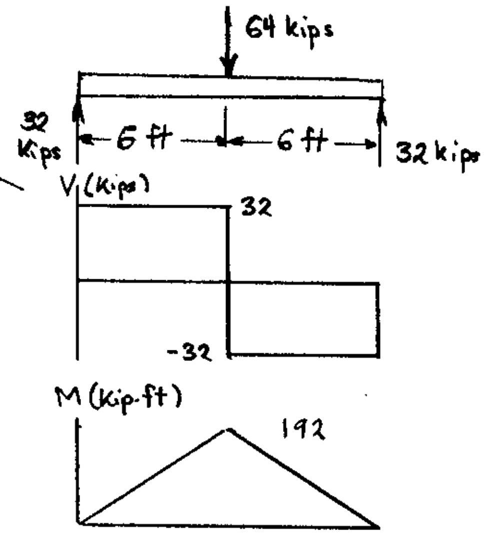
5.101 Each of the three rolled-steel beams shown (numbered 1, 2, and 3) is to carry a 64-kip load uniformly distributed over the beam. Each of these beams has a 12-ft span and is to be supported by the two 24-ft rolled-steel girders *AC* and *BD*. Knowing that the allowable normal stress for the steel used is 24 ksi, select (*a*) the most economical S-shape for the three beams, (*b*) the most economical W-shape for the two girders.

5.102 Solve Prob. 5.101, assuming that the 64-kip distributed loads are replaced by 64-kip concentrated loads applied at the midpoints of the three beams.



SOLUTION

Beams 1, 2, and 3



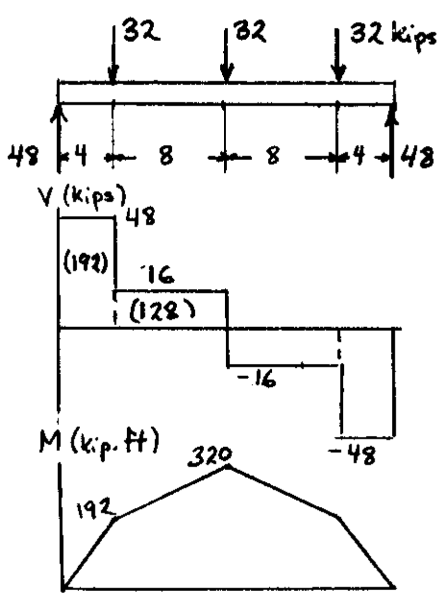
Maximum $M = (6)(32) = 192$ kip·ft $= 2304$ kip·in

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{2304}{24} = 96 \text{ in}^3$$

Shape	S (in³)
S20 × 66	119
S18 × 70	103

(a) Use S20 × 66 ◀

Beams AC and BD



Areas under shear diagram

$(4)(48) = 192$ kip ft

$(8)(16) = 128$ kip ft

Maximum $M = 192 + 128 = 320$ kip·ft $= 3840$ kip·in.

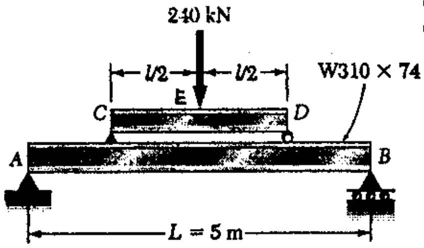
$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3840}{24} = 160 \text{ in}^3$$

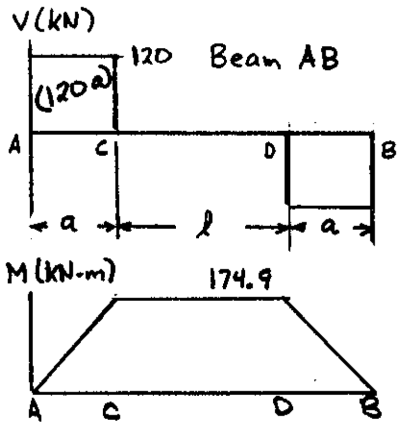
Shape	S (in³)
W 30 × 99	269
W 27 × 84	213
W 24 × 104	258
W 21 × 101	227
W 18 × 106	204

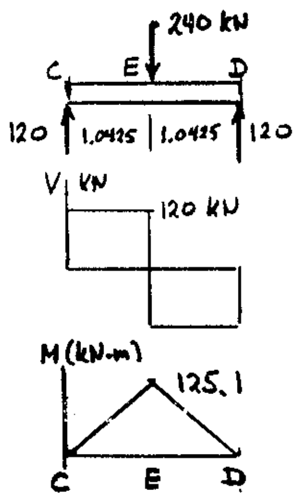
(b) Use W 27 × 84 ◀

PROBLEM 5.103

5.103 A 240-kN load is to be supported at the center of the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (*a*) the smallest allowable length *l* of beam *CD* if the W 310 × 74 beam *AB* is not to be overstressed, (*b*) the W shape which should be used for beam *CD*. Neglect the weight of both beams.







SOLUTION

For rolled steel section W 310 × 74 of beam AB

$S = 1060 \times 10^3 \ mm^3 = 1060 \times 10^{-6} \ m^3$

$\sigma_{all} = 165 \ MPa = 165 \times 10^6 \ Pa$

Allowable bending moment

$$M_{all} = S\sigma_{all} = (1060 \times 10^{-6})(165 \times 10^6) = 174.9 \times 10^3 \ N\cdot m$$
$$= 174.9 \ kN\cdot m$$

(a) Beam AB

Area A to C of shear diagram $= 120 \ a$

Bending moment at C $\quad 120 \ a$

$120 \ a = 174.9 \quad = 1.4575 \ m$

Geometry: $2a + l = 5 \qquad l = 5 - 2a = 2.085 \ m$ ◄

(b) Beam CD (midpoint E)

Area C to E of shear diagram

$= (1.0425)(120) = 125.1 \ kN\cdot m$

Bending moment at E

$M = 125.1 \ kN\cdot m = 125.1 \times 10^3 \ N\cdot m$

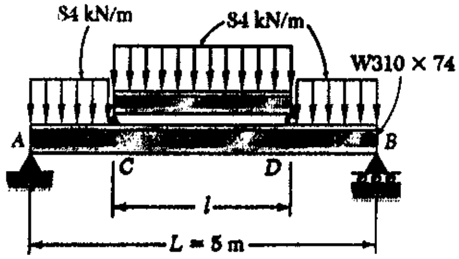
$$S_{min} = \frac{M}{\sigma_{all}} = \frac{125.1 \times 10^3}{165 \times 10^6} = 758.2 \times 10^{-6} \ m^3$$
$$= 758.2 \times 10^3 \ mm^3$$

Shape	S ($10^3 \ mm^3$)
W 460 × 52	942
W 410 × 46.1	774 ◄
W 360 × 57.8	899
W 310 × 60	851
W 250 × 67	809
W 200 × 86	853

Answer
W 410 × 46.1 ◄

PROBLEM 5.104

5.104 A uniformly distributed load of 84 kN/m is to be supported over the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (*a*) the smallest allowable length *l* of beam *CD* if the W 310 × 74 beam *AB* is not to be overstressed, (*b*) the W shape which should be used for beam *CD*. Neglect the weight of both beams.



SOLUTION

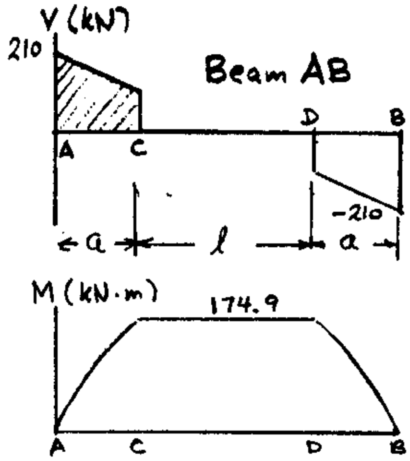
For rolled steel section W310×74 of beam AB

$S = 1060 \times 10^3\ \text{mm}^3 = 1060 \times 10^{-6}\ \text{m}^3$

$\sigma_{all} = 165\ \text{MPa} = 165 \times 10^6\ \text{Pa}$

Allowable bending moment

$M_{all} = S\,\sigma_{all} = (1060 \times 10^{-6})(165 \times 10^6) = 174.9 \times 10^3\ \text{N}\cdot\text{m}$
$= 174.9\ \text{kN}\cdot\text{m}$



By symmetry reactions A and B are equal.

$+\uparrow \Sigma F_y = 0 \qquad A + B - (5)(84) = 0$

$A = B = 210\ \text{kN}$

By symmetry, reaction at C and D are equal.

$+\uparrow \Sigma F_y = 0 \qquad C + D - 84\,l = 0$

$C = D = 42\,l$

Geometry $\quad a = \frac{1}{2}(5 - l)$

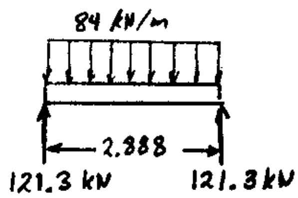
Beam AB: Area A to C of shear diagram

$\frac{1}{2}(a)(A + C) = \frac{1}{2}\cdot\frac{1}{2}(5 - l)(210 + 42\,l)$
$= \frac{1}{4}(1050 - 42\,l^2)$

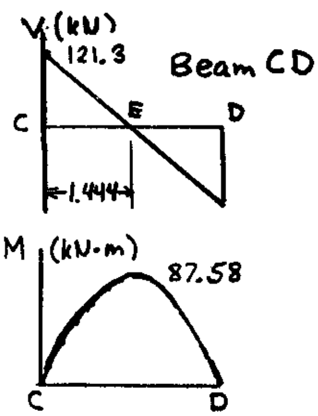
Bending moment at C $\quad \frac{1}{4}(1050 - 42\,l^2)$

$\frac{1}{4}(1050 - 42\,l^2) = 174.9 \qquad l^2 = 8.3429\ \text{m}^2$

(a) $\qquad l = 2.888\ \text{m}$ ◄



$C = D = 42\,l = 121.3\ \text{kN}$



Beam CD (midpoint E)
Area C to E of shear diagram
$\frac{1}{2}(1.444)(121.3) = 87.58\ \text{kN}\cdot\text{m}$

Bending moment at E $\qquad M = 87.58\ \text{kN}\cdot\text{m} = 87.58 \times 10^3\ \text{N}\cdot\text{m}$

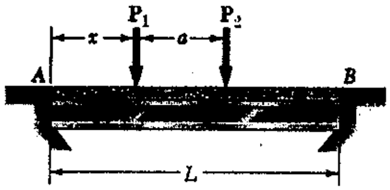
$\sigma_{all} = 165 \times 10^6\ \text{Pa}$

$S_{min} = \frac{M}{\sigma_{all}} = \frac{87.58 \times 10^3}{165 \times 10^6} = 531 \times 10^{-6}\ \text{m}^3 = 531 \times 10^3\ \text{mm}^3$

Shape	S ($10^3\ \text{mm}^3$)
W 460 × 52	942
W 410 × 38.8	637
W 360 × 39	578
W 310 × 38.7	549
W 250 × 44.8	535
W 200 × 59	582

(b) Use W 310 × 38.7 ◄

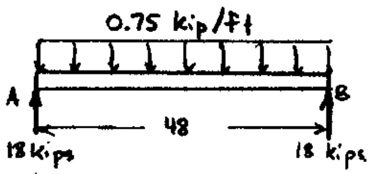
PROBLEM 5.105

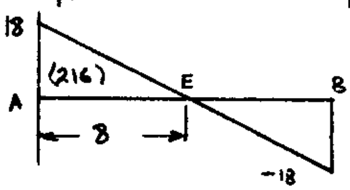


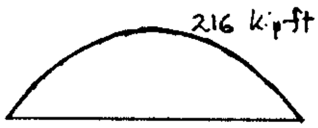
***5.105** A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kip/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using *LRFD* with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint.* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

$L = 48$ ft
$a = 14$ ft
$P_1 = 24$ kips
$P_2 = 6$ kips
$w = 0.75$ kip/ft

SOLUTION







Dead load: $R_A = R_B = (\frac{1}{2})(48)(0.75) = 18$ kips

Area A to E of shear diagram $(\frac{1}{2})(8)(18) = 216$ kip·ft

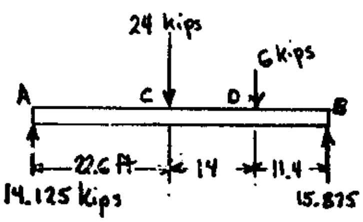
$M_{max} = 216$ kip·ft $= 2592$ kip·in. at point E

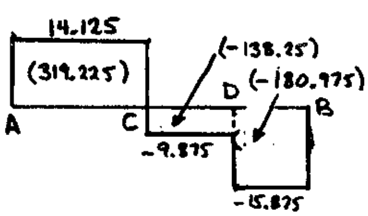
Live load: $u = \dfrac{aP_2}{2(P_1+P_2)} = \dfrac{(14)(6)}{(2)(30)} = 1.4$ ft

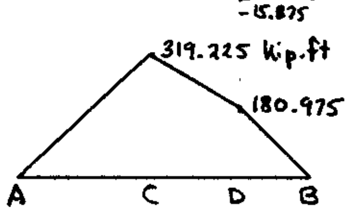
$x = \frac{L}{2} - u = 24 - 1.4 = 22.6$ ft
$x + a = 22.6 + 14 = 36.6$ ft
$L - x - a = 48 - 36.6 = 11.4$ ft.

$\Sigma M_B = 0 \quad -48R_A + (25.4)(24) + (11.4)(6) = 0$
$R_A = 14.125$ kips

Shear: A to C $\quad V = 14.125$ kips
C to D $\quad V = 14.125 - 24 = -9.875$ kips
D to B $\quad V = -15.875$ kips







Area A to C $(22.6)(14.125) = 319.225$ kip·ft

Bending moment: $M_C = 319.225$ kip·ft $= 3831$ kip·in.

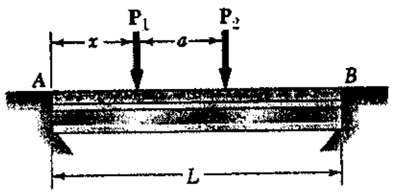
Design: $\gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_{all} S_{min}$

$$S_{min} = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_{all}} = \frac{(1.25)(2592) + (1.75)(3831)}{(0.9)(60)}$$
$$= 184.2 \text{ in}^3$$

Shape	S (in³)
W 30×99	269
W 27×84	213 ←
W 24×104	258
W 21×101	227
W 18×106	204

W27×84 ◄

PROBLEM 5.106



$L = 48$ ft
$a = 14$ ft
$P_1 = 24$ kips
$P_2 = 6$ kips
$w = 0.75$ kip/ft

*5.105 A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kip/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using *LRFD* with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint.* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

*5.106 Assuming that the front and rear axle loads remain in the same ratio as for the truck of Prob. 5.105, determine how much heavier a truck could safely cross the bridge designed in that problem.

SOLUTION

See solution to PROBLEM 5.105 for calculation of the following:

$M_D = 2592$ kip·in $\quad M_L = 3831$ kip·in.

For rolled steel section W27×84 $\quad S = 213$ in^3

Allowable live load moment M_L^*

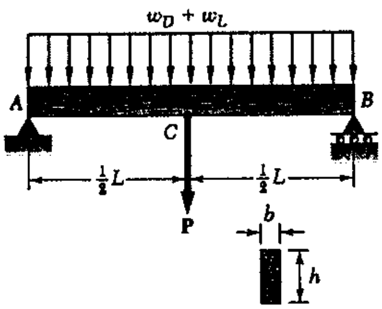
$$\gamma_D M_D + \gamma_L M_L^* = \varphi M_U = \varphi \sigma_U S$$

$$M_L^* = \frac{\varphi \sigma_U S - \gamma_D M_D}{\gamma_L} = \frac{(0.9)(60)(213) - (1.25)(2592)}{1.75} = 4721 \text{ kip·in}$$

Ratio $\dfrac{M_L^*}{M_L} = \dfrac{4721}{3831} = 1.232 = 1 + 0.232$

Increase $\quad$ 23.2 % ◄

PROBLEM 5.107

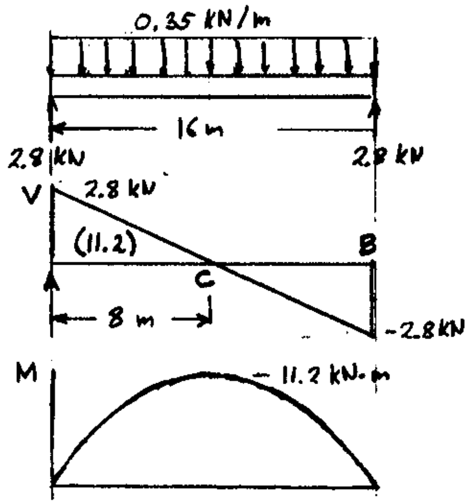


*5.107 A roof structure consisting of plywood and roofing material is supported by several timber beams of length L = 16 m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load w_D = 350 N/m. The live loads consist of the snow load, represented by a uniformly distributed load w_L = 600 N/m, and a 6-kN concentrated load **P** applied at the midpoint *C* of each beam. Knowing that the ultimate strength for the timber used is σ_U = 50 MPa and that the width of the beams is b = 75 mm, determine the minimum allowable depth h of the beams, using *LRFD* with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

SOLUTION

$L = 16$ m, $W_D = 350$ N/m $= 0.35$ kN/m

$W_L = 600$ N/m $= 0.6$ kN/m, $P = 6$ kN



Dead load: $R_A = (\frac{1}{2})(16)(0.35) = 2.8$ kN

Area A to C of shear diagram

$(\frac{1}{2})(8)(2.8) = 11.2$ kN·m

Bending moment at C: 11.2 kN·m $= 11.2 \times 10^3$ N·m

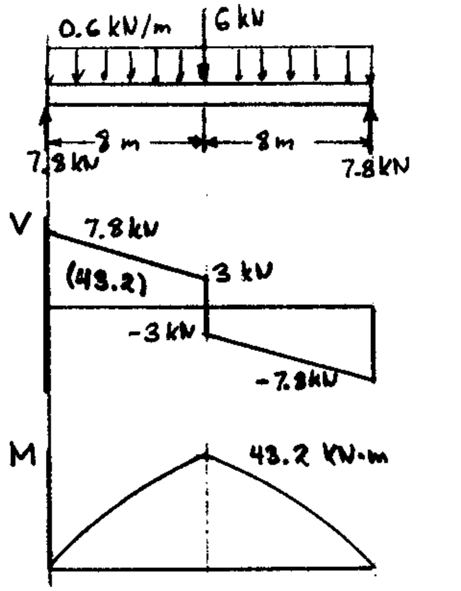
Live load: $R_A = \frac{1}{2}[(16)(0.6) + 6] = 7.8$ kN

Shear at C^- $\quad V = 7.8 - (8)(0.6) = 3$ kN

Area A to C of shear diagram

$(\frac{1}{2})(8)(7.8 + 3) = 43.2$ kN·m

Bending moment at C: 43.2 kN·m $= 43.2 \times 10^3$ N·m



Design $\quad \gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_U S$

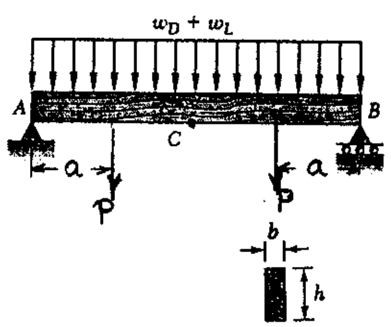
$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_U} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(43.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$

$$= 1.8347 \times 10^{-3}\ \text{m}^3 = 1.8347 \times 10^6\ \text{mm}^3$$

For a rectangular section $\quad S = \frac{1}{6} b h^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.8347 \times 10^6)}{75}} = 383\ \text{mm}$$

PROBLEM 5.108

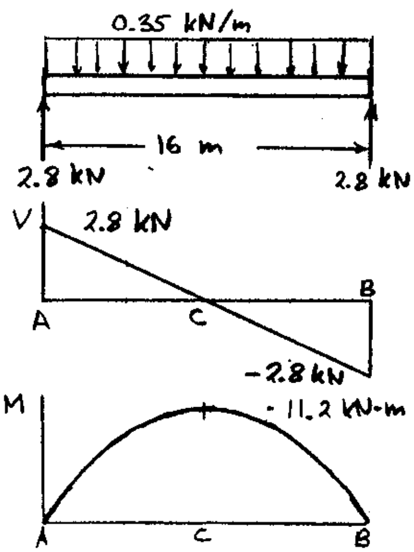


*5.107 A roof structure consisting of plywood and roofing material is supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live loads consist of the snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beams is $b = 75$ mm, determine the minimum allowable depth h of the beams, using *LRFD* with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.

*5.108 Solve Prob. 5.107, assuming that the 6-kN concentrated loads are replaced by 3-kN concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ applied at a distance of 4 m from each end of the beams.

SOLUTION

$L = 16\ m, \quad a = 4\ m, \quad W_D = 350\ N/m = 0.35\ kN/m$
$W_L = 600\ N/m = 0.6\ kN/m \quad P = 3\ kN$



Dead load: $R_A = (\frac{1}{2})(16)(0.35) = 2.8\ kN$

Area A to C of shear diagram
$(\frac{1}{2})(8)(2.8) = 11.2\ kN\cdot m$

Bending moment at C: $11.2\ kN\cdot m = 11.2 \times 10^3\ N\cdot m$

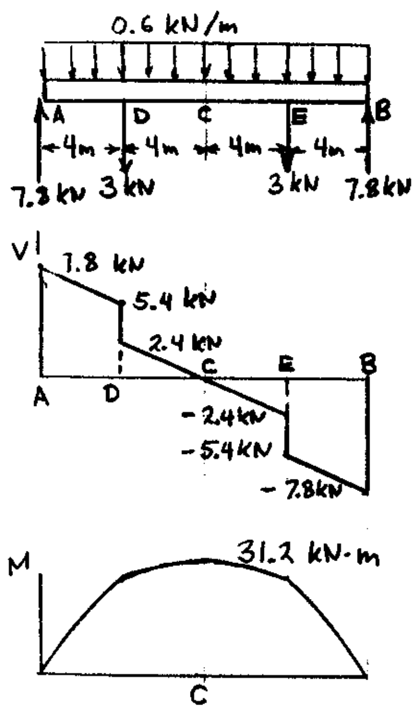
Live load: $R_A = \frac{1}{2}[(16)(0.6) + 3 + 3] = 7.8\ kN$

Shear at D^- $\quad 7.8 - (4)(0.6) = 5.4\ kN$
Shear at D^+ $\quad 5.4 - 3 = 2.4\ kN$

Area A to D $\quad (\frac{1}{2})(4)(7.8 + 5.4) = 26.4\ kN\cdot m$

Area D to C $\quad (\frac{1}{2})(4)(2.4) = 4.8\ kN\cdot m$

Bending moment at C $= 26.4 + 4.8 = 31.2\ kN\cdot m$
$= 31.2 \times 10^3\ N\cdot m$



Design $\quad \gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_U S$

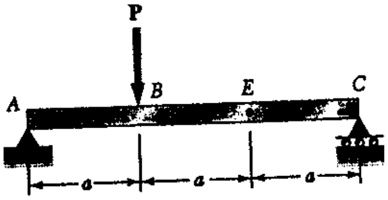
$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_U} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(31.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$

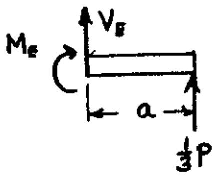
$$= 1.408 \times 10^{-3}\ m^3 = 1.408 \times 10^6\ mm^3$$

For a rectangular section $\quad S = \frac{1}{6} b h^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.408 \times 10^6)}{75}} = 336\ mm$$

PROBLEM 5.109





5.109 through 5.111 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of beam to the right of *E*.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0 \qquad -3aA + 2aP = 0 \qquad A = \frac{2}{3}P$

$V = \frac{2}{3}P - P\langle x-a\rangle^0$

$M = \frac{2}{3}Px - P\langle x-a\rangle^1$

At point $E \qquad x = 2a$

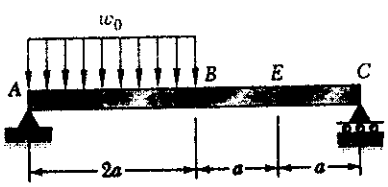
$M_E = \frac{2}{3}P(2a) - Pa = \frac{1}{3}Pa$ ◄

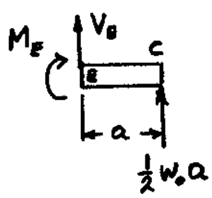
$\Sigma M_A = 0 \qquad 3aC - aP = 0 \qquad C = \frac{1}{3}P$

$+\circlearrowleft \Sigma M_E = 0 \qquad -M_E + (a)(\frac{1}{3}P) = 0$

$M_E = \frac{1}{3}Pa$ ◄

PROBLEM 5.110





5.109 through 5.111 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of beam to the right of *E*.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0 \qquad -4aA + (3a)(2aw_0) = 0 \qquad A = \frac{3}{2}w_0a$

$w = w_0 - w_0\langle x-2a\rangle^0 = -\frac{dV}{dx}$

$V = -w_0x + w_0\langle x-2a\rangle^1 + \frac{3}{2}w_0a = \frac{dM}{dx}$

$M = -\frac{1}{2}w_0x^2 + \frac{1}{2}w_0\langle x-2a\rangle^2 + \frac{3}{2}w_0ax + 0$

At point $E \qquad x = 3a$

$M_E = -\frac{1}{2}w_0(3a)^2 + \frac{1}{2}w_0a^2 + \frac{3}{2}w_0a(3a)$

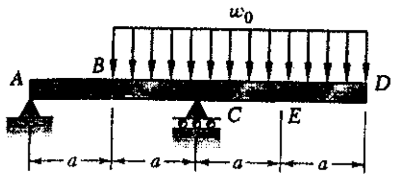
$= \frac{1}{2}w_0a^2$ ◄

$+\circlearrowleft \Sigma M_A = 0 \qquad 4aC - (a)(2aw_0) = 0 \qquad C = \frac{1}{2}w_0a$

$+\circlearrowleft \Sigma M_E = 0 \qquad -M_E + (a)(\frac{1}{2}w_0a) = 0$

$M_E = \frac{1}{2}w_0a^2$ ◄

PROBLEM 5.111

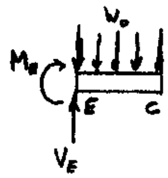


5.109 through 5.111 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of beam to the right of *E*.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0 \qquad -2aA - (\frac{a}{2})(3a\,w_0) = 0 \qquad A = -\frac{3}{4}w_0 a$

$+\circlearrowleft \Sigma M_A = 0 \qquad 2aC + (\frac{5a}{2})(3a\,w_0) = 0 \qquad C = \frac{15}{4}w_0 a$



$w = w_0\langle x-a\rangle^0 = -\frac{dV}{dx}$

$V = -w_0\langle x-a\rangle^1 - \frac{3}{4}w_0 a + \frac{15}{4}w_0 a\langle x-2a\rangle^0 = \frac{dM}{dx}$

$M = -\frac{1}{2}w_0\langle x-a\rangle^2 - \frac{3}{4}w_0 a x + \frac{15}{4}w_0 a\langle x-2a\rangle^1 + 0$

At point $E \qquad x = 3a$

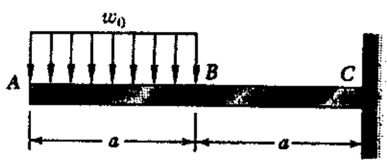
$M_E = -\frac{1}{2}w_0(2a)^2 - \frac{3}{4}w_0 a(3a) + \frac{15}{4}w_0 a(a)$

$= -\frac{1}{2}w_0 a^2$

Check: $+\Sigma M_E = 0 \qquad -M_E - \frac{a}{2}(w_0 a) = 0$

$M_E = -\frac{1}{2}w_0 a^2$

PROBLEM 5.112



5.112 through 5.114 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

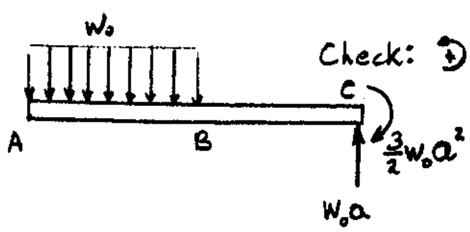
$w = w_0 - w_0\langle x-a\rangle^0 = -\frac{dV}{dx}$

$V = -w_0 x + w_0\langle x-a\rangle^1 = \frac{dM}{dx}$

$M = -\frac{1}{2}w_0 x^2 + \frac{1}{2}w_0\langle x-a\rangle^2$

At point $C \qquad x = 2a$

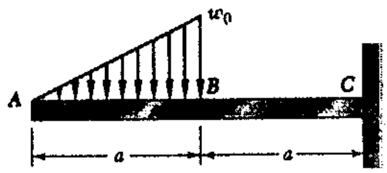
$M_C = -\frac{1}{2}w_0(2a)^2 + \frac{1}{2}w_0 a^2 = -\frac{3}{2}w_0 a^2$



Check: $\circlearrowleft \Sigma M_C = 0 \qquad (\frac{3a}{2})(w_0 a) + M_C = 0$

$M_C = -\frac{3}{2}w_0 a^2$

PROBLEM 5.113



5.112 through 5.114 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = \frac{w_0 x}{a} - w_0\langle x-a\rangle^0 - \frac{w_0}{a}\langle x-a\rangle^1 = -\frac{dV}{dx}$$

$$V = -\frac{w_0 x^2}{2a} + w_0\langle x-a\rangle^1 + \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$

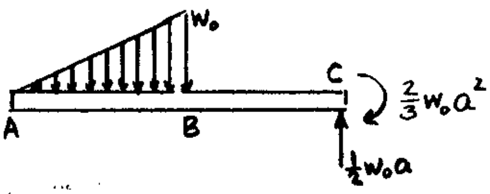
$$M = -\frac{w_0 x^3}{6a} + \frac{w_0}{2}\langle x-a\rangle^2 + \frac{w_0}{6a}\langle x-a\rangle^3$$

At point C $x = 2a$

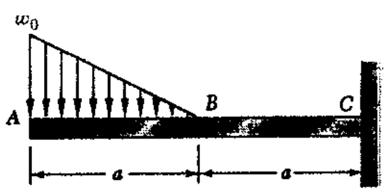
$$M_c = -\frac{w_0(2a)^3}{6a} + \frac{w_0 a^2}{2} + \frac{w_0 a^3}{6a} = -\frac{2}{3}w_0 a^2$$

Check: $+\circlearrowleft \Sigma M_c = 0$ $\quad \left(\frac{4a}{3}\right)\left(\frac{1}{2}w_0 a\right) + M_c = 0$

$$M_c = -\frac{2}{3}w_0 a^2$$



PROBLEM 5.114



5.112 through 5.114 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for M to determine the bending moment at point C and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

$$w = w_0 - \frac{w_0 x}{a} + \frac{w_0}{a}\langle x-a\rangle^1 = -\frac{dV}{dx}$$

$$V = -w_0 x + \frac{w_0 x^2}{2a} - \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6a} - \frac{w_0}{6a}\langle x-a\rangle^3$$

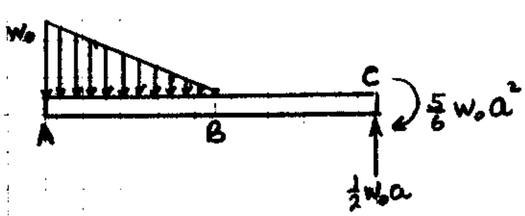
At point C $x = 2a$

$$M_c = -\frac{w_0(2a)^2}{2} + \frac{w_0(2a)^3}{6a} - \frac{w_0 a^3}{6a} = -\frac{5}{6}w_0 a^2$$

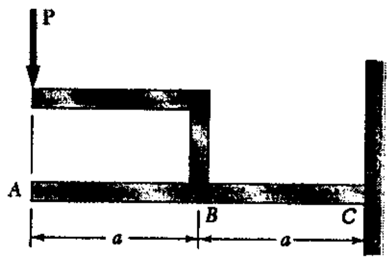
Check:

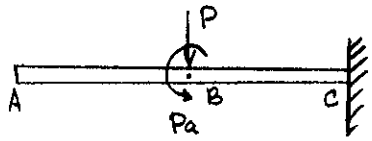
$+\circlearrowleft \Sigma M_c = 0$ $\quad \left(\frac{5}{3}a\right)\left(\frac{1}{2}w_0 a\right) + M_c = 0$

$$M_c = -\frac{5}{6}w_0 a^2$$



PROBLEM 5.115





5.115 and **5.116** (*a*) Using singularity functions, write the equations defining the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *B*.

SOLUTION

$$V = -P\langle x-a\rangle^0$$

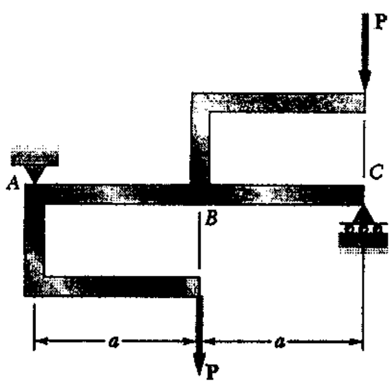
$$\frac{dM}{dx} = -P\langle x-a\rangle^0$$

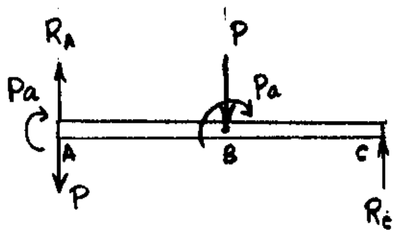
$$M = -P\langle x-a\rangle^1 - Pa\langle x-a\rangle^0$$

Just to the right of B $\quad x = a^+$

$$M = -0 - Pa = -Pa$$

PROBLEM 5.116





5.115 and **5.116** (*a*) Using singularity functions, write the equations defining the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *B*.

SOLUTION

$$\Sigma M_C = 0 \qquad (2a)P + aP - 2(Pa) - 2aR_A = 0$$

$$R_A = \tfrac{1}{2}P$$

$$V = (R_A - P) - P\langle x-a\rangle^0$$

$$= -\tfrac{1}{2}P - P\langle x-a\rangle^0$$

$$\frac{dM}{dx} = -\tfrac{1}{2}P - P\langle x-a\rangle^0$$

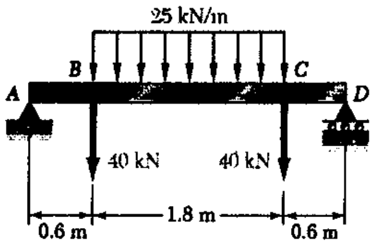
$$M = -\tfrac{1}{2}Px - P\langle x-a\rangle^1 + Pa + Pa\langle x-a\rangle^0$$

Just to the right of point B $\quad x = a^+$

$$M = -\tfrac{1}{2}Pa - 0 + Pa + Pa = \tfrac{3}{2}Pa$$

PROBLEM 5.117

5.117 through 5.120 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



SOLUTION

By symmetry $R_A = R_D$

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_D - 40 - (1.8)(25) - 40 = 0$

$R_A = R_D = 62.5 \text{ kN}$

$$w = 25\langle x-0.6\rangle^0 - 25\langle x-2.4\rangle^0 = -\frac{dV}{dx}$$

$$V = 62.5 - 25\langle x-0.6\rangle^1 + 25\langle x-2.4\rangle^1 - 40\langle x-0.6\rangle^0 - 40\langle x-2.4\rangle^0 \text{ kN}$$

$$M = 62.5x - 12.5\langle x-0.6\rangle^2 + 12.5\langle x-2.4\rangle^2 - 40\langle x-0.6\rangle^1 - 40\langle x-2.4\rangle^1 \text{ kN}\cdot\text{m}$$

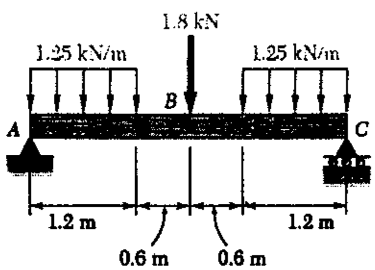
Locate point where $V = 0$. Assume $0.6 < x^* < 1.8$

$0 = 62.5 - 25(x^* - 0.6) + 0 - 40 - 0 \qquad x^* = 1.5 \text{ m}$

$M = (62.5)(1.5) - (12.5)(0.9)^2 + 0 - (40)(0.9) - 0 = 47.625 \text{ kN}\cdot\text{m}$

PROBLEM 5.118

5.117 through 5.120 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



SOLUTION

By symmetry $R_A = R_C$

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_C - (1.2)(1.25) - 1.8 - (1.2)(1.25) = 0$

$R_A = R_C = 2.4 \text{ kN}$

$$w = 1.25 - 1.25\langle x-1.2\rangle^0 + 1.25\langle x-2.4\rangle^0 = -\frac{dV}{dx}$$

$$V = -1.25x + 1.25\langle x-1.2\rangle^1 - 1.25\langle x-2.4\rangle^1 + 2.4 - 1.8\langle x-1.8\rangle^0 \text{ kN}$$

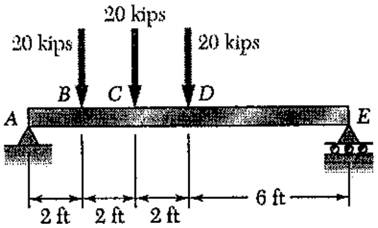
$$M = -0.625x^2 + 0.625\langle x-1.2\rangle^2 - 0.625\langle x-2.4\rangle^2 + 2.4x - 1.8\langle x-1.8\rangle^1 \text{ kN}\cdot\text{m}$$

M_{max} occurs at $x = 1.8$ m

$M_{max} = -(0.625)(1.8)^2 + (0.625)(0.6)^2 + 0 + (2.4)(1.8) - 0 = 2.52 \text{ kN}\cdot\text{m}$

PROBLEM 5.119

5.117 through 5.120 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



SOLUTION

$+\Sigma M_E = 0 \quad -12A + (10)(20) + (8)(20) + (6)(20) = 0$

$A = 40$ kips.

$V = 40 - 20\langle x-2\rangle^0 - 20\langle x-4\rangle^0 - 20\langle x-6\rangle^0$ kips

$M = 40x - 20\langle x-2\rangle^1 - 20\langle x-4\rangle^1 - 20\langle x-6\rangle^1$ kip·ft

Values of V

A to B $\quad V = 40$ kip

B to C $\quad V = 40 - 20 = 20$ kips

C to D $\quad V = 40 - 20 - 20 = 0$

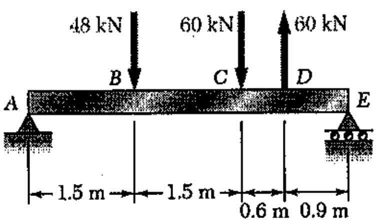
D to E $\quad V = 40 - 20 - 20 - 20 = -20$ kip

Bending moment is constant and maximum over C to D.

At C $\quad x = 4$ ft $\quad M = (40)(4) - (20)(2) - 0 - 0 = 120$ kip·ft

PROBLEM 5.120

5.117 through 5.120 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



SOLUTION

$+\circlearrowleft\Sigma M_E = 0 \quad -4.5R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$

$R_A = 40$ kN

$V = 40 - 48\langle x-1.5\rangle^0 - 60\langle x-3.0\rangle^0 + 60\langle x-3.6\rangle^0$ kN

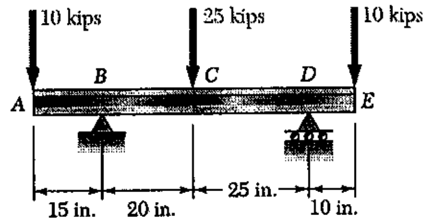
$M = 40x - 48\langle x-1.5\rangle^1 - 60\langle x-3.0\rangle^1 + 60\langle x-3.6\rangle^1$ kN·m

Pt.	x (m)	M (kN·m)
A	0	0
B	1.5	(40)(1.5) = 60 kN·m
C	3.0	(40)(3.0) − (48)(1.5) = 48 kN·m
D	3.6	(40)(3.6) − (48)(2.1) − (60)(0.6) = 7.2 kN·m
E	4.5	(40)(4.5) − (48)(3.0) − (60)(1.5) + (60)(0.9) = 0

$M_{max} = 60$ kN·m

PROBLEM 5.121

5.121 and ***5.122*** (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.




S6 × 12.5

SOLUTION

$+\circlearrowleft \Sigma M_D = 0$

$(60)(10) + 45 R_B + (25)(25) - (10)(10) = 0$

$R_B = 25$ kips

$+\circlearrowleft \Sigma M_B = 0$

$(15)(10) - (20)(25) + 45 R_D - (55)(10) = 0$

$R_D = 20$ kips

$V = -10 + 25\langle x-15\rangle^0 - 25\langle x-35\rangle^0 + 20\langle x-60\rangle^0$ kips ◄

$M = -10x + 25\langle x-15\rangle^1 - 25\langle x-35\rangle^1 + 20\langle x-60\rangle^1$ kip·in ◄

Pt	x (ft)	M (kip·in)
B	15	$-(10)(15) = -150$ kip·in
C	35	$-(10)(35) + (25)(20) = 150$ kip·in
D	60	$-(10)(60) + (25)(45) - (25)(25) = -100$ kip·in
E	70	$-(10)(70) + (25)(55) - (25)(35) + (20)(10) = 0$ checks

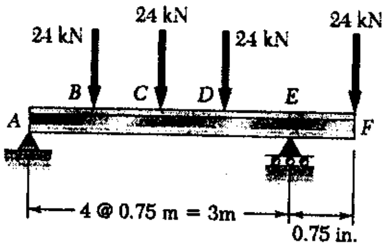
Maximum $|M| = 150$ kip·in

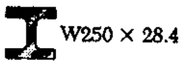
For S6×12.5 rolled steel section $S = 7.37$ in³

Normal stress $\sigma = \dfrac{|M|}{S} = \dfrac{150}{7.37} = 20.35$ ksi ◄

PROBLEM 5.122

5.121 and *5.122* (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.





SOLUTION

$+\circlearrowleft \Sigma M_E = 0$

$-3R_A + (2.25)(24) - (1.5)(24) - (0.75)(24) + (0.75)(24) = 0$

$R_A = 30$ kips

$+\circlearrowleft \Sigma M_A = 0 \quad -(0.75)(24) - (1.5)(24) - (2.25)(24) + 3R_E - (3.75)(24) = 0$

$R_D = 66$ kips

$V = 30 - 24\langle x-0.75\rangle^0 - 24\langle x-1.5\rangle^0 - 24\langle x-2.25\rangle^0 + 66\langle x-3\rangle^0$ kN

$M = 30x - 24\langle x-0.75\rangle^1 - 24\langle x-1.5\rangle^1 - 24\langle x-2.25\rangle^1 + 66\langle x-3\rangle^1$ kN·m

Pt	x (m)	M (kN·m)
B	0.75	$(30)(0.75) = 22.5$ kN·m
C	1.5	$(30)(1.5) - (24)(0.75) = 27$ kN·m
D	2.25	$(30)(2.25) - (24)(1.5) - (24)(0.75) = 13.5$ kN·m
E	3.0	$(30)(3.0) - (24)(2.25) - (24)(1.5) - (24)(0.75) = -18$ kN·m
F	3.75	$(30)(3.75) - (24)(3.0) - (24)(2.25) - (24)(1.5) + (66)(0.75) = 0$ ✓

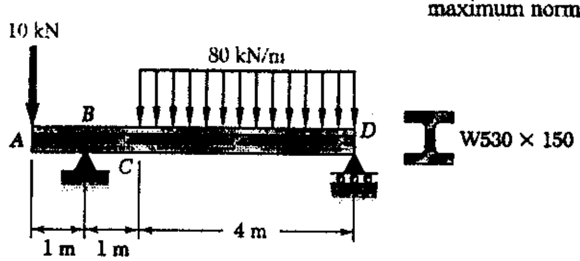
Maximum $|M| = 27$ kN·m $= 27 \times 10^3$ N·m

For rolled steel section W250 × 28.4 $\quad S = 308 \times 10^3\ mm^3 = 308 \times 10^{-6}\ m^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{27\times10^3}{308\times10^{-6}} = 87.7\times10^6$ Pa $= 87.7$ MPa

PROBLEM 5.123

5.123 and 5.124 (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



SOLUTION

$+\circlearrowleft M_D = 0$

$(6)(10) - 5R_B + (2)(4)(80) = 0$

$R_B = 140 \text{ kN}$

$$w = 80\langle x-2\rangle^0 \text{ kN/m} = -dV/dx$$

$$V = -10 + 140\langle x-1\rangle^0 - 80\langle x-2\rangle^1 \text{ kN}$$

A to B $\quad V = -10 \text{ kN}$

B to C $\quad V = -10 + 140 = 130 \text{ kN}$

D $\quad (x = 6) \quad V = -10 + 140 - 80(4) = -190 \text{ kN}$

V changes sign at B and at point E $(x = x_E)$ between C and D.

$$V = 0 = -10 + 140\langle x_E - 1\rangle^0 - 80\langle x_E - 2\rangle^1$$

$$= -10 + 140 - 80(x_E - 2) \qquad x_E = 3.625 \text{ m}$$

$$M = -10x + 140\langle x-1\rangle^1 - 40\langle x-2\rangle^2 \text{ kN}\cdot\text{m}$$

At pt. B $\quad x = 1 \quad M_B = -(10)(1) = -10 \text{ kN}\cdot\text{m}$

At pt. E $\quad x = 3.625$

$$M_E = -(10)(3.625) + (140)(2.625) - (40)(1.625)^2 = 225.6 \text{ kN}\cdot\text{m}$$

$$|M|_{max} = 225.6 \text{ kN}\cdot\text{m} \quad \text{at} \quad x = 3.625 \text{ m}$$

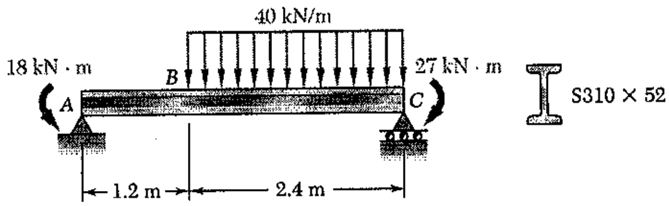
For W 530 × 150 $\quad S = 3720 \times 10^3 \text{ mm}^3 = 3720 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{225.6 \times 10^3}{3720 \times 10^{-6}} = 60.6 \times 10^6 \text{ Pa}$

$= 60.6 \text{ MPa}$

PROBLEM 5.124

5.123 and 5.124 (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



SOLUTION

$\curvearrowleft + \; M_C = 0$

$18 - 3.6 R_A + (1.2)(2.4)(40) - 27 = 0$

$R_A = 29.5 \text{ kN}$

$V = 29.5 - 40\langle x - 1.2\rangle^1 \text{ kN}$

Point D $\quad V = 0 \quad\quad 29.5 - 40(x_D - 1.2) = 0$

$x_D = 1.9375 \text{ m}$

$M = -18 + 29.5x - 20\langle x - 1.2\rangle^2 \text{ kN-m}$

$M_A = -18 \text{ kN-m}$

$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278 \text{ kN-m}$

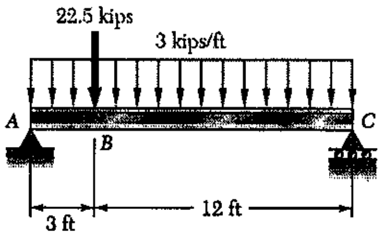
$M_E = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27 \text{ kN-m}$

Maximum $|M| = 28.278 \text{ kN-m}$ at $x = 1.9375 \text{ m}$ ◄ ◄

For S 310 × 52 rolled steel section $\quad S = 625 \times 10^3 \text{ mm}^3 = 625 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{28.278 \times 10^3}{625 \times 10^{-6}} = 45.2 \times 10^6 \text{ Pa} = 45.2 \text{ MPa}$ ◄

PROBLEM 5.125



5.125 and 5.126 A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam (*b*) Knowing that the allowable stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that should be selected..

SOLUTION

$$\circlearrowleft \Sigma M_C = 0 \quad -15 R_A + (7.5)(15)(3) + (12)(22.5) = 0$$

$$R_A = 40.5 \text{ kips.}$$

$$w = 3 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 40.5 - 3x - 22.5\langle x-3\rangle^0 \text{ kips}$$

Location of point D where $V = 0$. Assume $3 < x_D < 12$

$$0 = 40.5 - 3x_D - 22.5 \qquad x_D = 6 \text{ ft}$$

$$M = 40.5x - 1.5x^2 - 22.5\langle x-3\rangle^1 \text{ kip·ft}$$

At point D $(x = 6 \text{ ft})$ $\quad M = (40.5)(6) - (1.5)(6)^2 - (22.5)(3)$

$$= 121.5 \text{ kip·ft} = 1458 \text{ kip·in.}$$

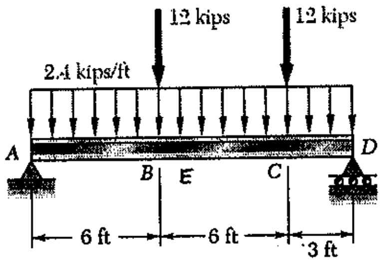
Maximum $|M| = 121.5$ kip·ft at $x = 6$ ft.

$$S_{min} = \frac{M}{\sigma_{all}} = \frac{1458}{24} = 60.75 \text{ in}^3$$

Shape	S (in³)
W 21 × 44	81.6
W 18 × 50	88.9
W 16 × 40	64.7 ←
W 14 × 43	62.7
W 12 × 50	64.7
W 10 × 68	75.7

Answer W16 × 40

PROBLEM 5.126



5.125 and 5.126 A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam (*b*) Knowing that the allowable stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that should be selected..

SOLUTION

$$\circlearrowleft \Sigma M_C = 0$$

$$-15 R_A + (7.5)(15)(2.4) - (9)(12) - (3)(12) = 0$$

$$R_A = 27.6 \text{ kips}$$

$$w = 2.4 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 27.6 - 2.4x - 12\langle x-6\rangle^0 - 12\langle x-12\rangle^0 \text{ kips}$$

$$V_B^- = 27.6 - (2.4)(6) = 13.2 \text{ Kips}$$
$$V_B^+ = 27.6 - (2.4)(6) - 12 = 1.2 \text{ kips}$$
$$V_C^- = 27.6 - (2.4)(12) - 12 = -13.2 \text{ kips}$$

Point where $V = 0$ lies between B and C.

Locate point E where $V = 0$

$$0 = 27.6 - 2.4x_E - 12 - 0 \qquad x_E = 6.5 \text{ ft.}$$

$$M = 27.6x - 1.2x^2 - 12\langle x-6\rangle^1 - 12\langle x-12\rangle^1 \text{ kip-ft}$$

At point E $(x = 6.5 \text{ ft})$ $\quad M = (27.6)(6.5) - (1.2)(6.5)^2 - (12)(0.5) - 0$

$$= 122.7 \text{ kip-ft} = 1472.4 \text{ kip-in.}$$

Maximum $|M|$ $\quad$ 122.7 kip-ft at $x = 6.5$ ft.

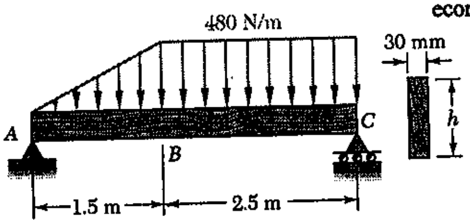
$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{1472.4}{24} = 61.35 \text{ in}^3$$

Shape	S (in³)
W21 × 44	81.6
W18 × 50	88.9
W16 × 40	64.7 ←
W14 × 43	62.7
W12 × 50	64.7
W10 × 68	75.7

Answer: W16 × 40

PROBLEM 5.127

5.127 **and** *5.128*. A timber beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with a 12-MPa allowable stress and a rectangular cross section of 30-mm width and depth *h* varying from 80 to160 mm in 10-mm increments, determine the most economical cross section that can be used.



SOLUTION

$480 \text{ N/m} = 0.48 \text{ kN/m}$

$+\circlearrowleft \Sigma M_C = 0$

$-4 R_A + (3)(\tfrac{1}{2})(1.5)(0.48) + (1.25)(2.5)(0.48) = 0$

$R_A = 0.645 \text{ kN}$

$$w = \frac{0.48}{1.5} x - \frac{0.48}{1.5} \langle x - 1.5 \rangle^1 = 0.32\,x - 0.32 \langle x - 1.5 \rangle^1 \text{ kN/m} = -\frac{dV}{dx}$$

$$V = 0.645 - 0.16\,x^2 + 0.16 \langle x - 1.5 \rangle^2 \quad \text{kN}$$

Locate point D where $V = 0$. Assume $1.5 \text{ m} < x_D < 4 \text{ m}$

$$0 = 0.645 - 0.16\,x_D^2 + 0.16(x_D - 1.5)^2$$
$$= 0.645 - \cancel{0.16\,x_D^2} + \cancel{0.16\,x_D^2} - 0.48\,x_D + 0.36$$

$x_D = 2.09375 \text{ m}$ ◄

$$M = 0.645\,x - 0.05333\,x^3 + 0.05333 \langle x - 1.5 \rangle^3 \quad \text{kN-m}$$

At point D $\quad M_D = (0.645)(2.09375) - (0.05333)(2.09375)^3 + (0.05333)(0.59375)^3$

$= 0.87211 \text{ kN}\cdot\text{m}$ ◄

$$S_{min} = \frac{M_D}{\sigma_{all}} = \frac{0.87211 \times 10^3}{12 \times 10^6} = 72.6758 \times 10^{-6} \text{ m}^3 = 72.6758 \times 10^3 \text{ mm}^3$$

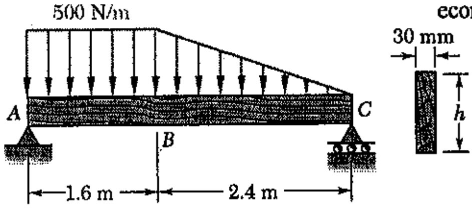
For a rectangular cross section $\quad S = \frac{1}{6} b h^2 \qquad h = \sqrt{\frac{6S}{b}}$

$$h_{min} = \sqrt{\frac{(6)(72.6758 \times 10^3)}{30}} = 120.56 \text{ mm}$$

At next larger 10-mm increment $\quad h = 130 \text{ mm}$ ◄

PROBLEM 5.128

5.127 **and** *5.128*. A timber beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with a 12-MPa allowable stress and a rectangular cross section of 30-mm width and depth *h* varying from 80 to 160 mm in 10-mm increments, determine the most economical cross section that can be used.



SOLUTION

$500\ \text{N/m} = 0.5\ \text{kN/m}$

$+\circlearrowleft \Sigma M_C = 0$

$-4R_A + (3.2)(1.6)(0.5) + (1.6)(\frac{1}{2})(2.4)(0.5) = 0$

$R_A = 0.880\ \text{kN}$

$$w = 0.5 - \frac{0.5}{2.4}\langle x-1.6\rangle^1 = 0.5 - 0.20833\langle x-1.6\rangle^1\ \text{kN/m} = -\frac{dV}{dx}$$

$$V = 0.880 - 0.5x + 0.104167\langle x-1.6\rangle^2\ \text{kN}$$

$V_A = 0.880\ \text{kN}$

$V_B = 0.880 - (0.5)(1.6) = 0.080\ \text{kN}$

$V_C = 0.880 - (0.5)(4) + (0.104167)(2.4)^2 = -0.520\ \text{kN}$ } Sign change

Locate point D where $V = 0$

$$0 = 0.880 - 0.5x_D + 0.104167(x_D - 1.6)^2$$

$$0.104167x_D^2 - 0.83333x_D + 1.14667 = 0$$

$$x_D = \frac{0.83333 \pm \sqrt{(0.83333)^2 - (4)(0.104167)(1.14667)}}{(2)(0.104167)}$$

$$= 4.0 \pm 2.2342 = \cancel{6.2342},\ 1.7658\ \text{m}$$

$$M = 0.880x - 0.25x^2 + 0.347222\langle x-1.6\rangle^3\ \text{kN-m}$$

$$M_D = (0.880)(1.7658) - (0.25)(1.7658)^2 + (0.34722)(0.1658)^3 = 0.776\ \text{kN}\cdot\text{m}$$

$M_{max} = 0.776\ \text{kN}\cdot\text{m}$ at $x = 1.7658\ \text{m}$

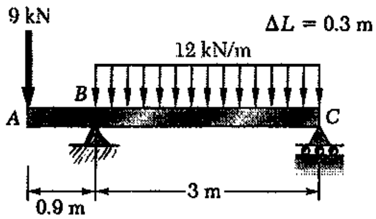
$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{0.776\times 10^3}{12\times 10^6} = 64.66\times 10^{-6}\ \text{m}^3 = 64.66\times 10^3\ \text{mm}^3$$

For a rectangular cross section $S = \frac{1}{6}bh^2$ $\quad h = \frac{6S}{b}$

$$h_{min} = \sqrt{\frac{(6)(64.66\times 10^3)}{30}} = 113.7\ \text{mm}$$

At next higher 10-mm increment $\quad h = 120\ \text{mm}$

PROBLEM 5.129



5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increments ΔL, starting at point A and ending at the right-hand support.

SOLUTION

$+\circlearrowleft M_C = 0 \quad (3.9)(9) - 3R_B + (1.5)(3.0)(12) = 0$

$R_B = 29.7 \text{ kN}$

$w = 12\langle x - 0.9\rangle^0$

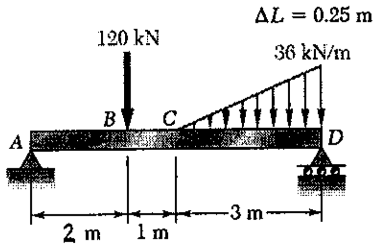
$V = -9 + 29.7\langle x-0.9\rangle^0 - 12\langle x - 0.9\rangle^1 \text{ kN}$ ◀

$M = -9x + 29.7\langle x-0.9\rangle^1 - 6\langle x-0.9\rangle^2 \text{ kN-m}$ ◀

x m	V kN	M kN·m
0.0	-9.0	0.00
0.3	-9.0	-2.70
0.6	-9.0	-5.40
0.9	20.7	-8.10
1.2	17.1	-2.43
1.5	13.5	2.16
1.8	9.9	5.67
2.1	6.3	8.10
2.4	2.7	9.45
2.7	-0.9	9.72
3.0	-4.5	8.91
3.3	-8.1	7.02
3.6	-11.7	4.05
3.9	-15.3	0.00

PROBLEM 5.130

5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increments ΔL, starting at point A and ending at the right-hand support.



SOLUTION

$+\circlearrowleft \Sigma M_D = 0$

$-6R_A + (4)(120) + (1)(\frac{1}{2})(3)(36) = 0$

$R_A = 89$ kN

$w = \frac{36}{3}\langle x-3\rangle^1 = 12\langle x-3\rangle^1$

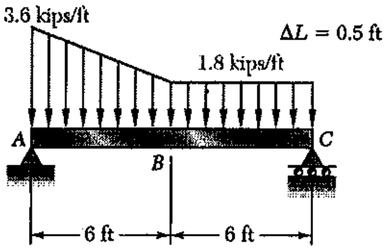
$V = 89 - 120\langle x-2\rangle^0 - 6\langle x-3\rangle^2$ kN ◄

$M = 89x - 120\langle x-2\rangle^1 - 2\langle x-3\rangle^3$ kN·m ◄

x m	V kN	M kN·m
0.0	89.0	0.0
0.3	89.0	22.3
0.5	89.0	44.5
0.8	89.0	66.8
1.0	89.0	89.0
1.3	89.0	111.3
1.5	89.0	133.5
1.8	89.0	155.8
2.0	-31.0	178.0
2.3	-31.0	170.3
2.5	-31.0	162.5
2.8	-31.0	154.8
3.0	-31.0	147.0
3.3	-31.4	139.2
3.5	-32.5	131.3
3.8	-34.4	122.9
4.0	-37.0	114.0
4.3	-40.4	104.3
4.5	-44.5	93.8
4.8	-49.4	82.0
5.0	-55.0	69.0
5.3	-61.4	54.5
5.5	-68.5	38.3
5.8	-76.4	20.2
6.0	-85.0	-0.0

PROBLEM 5.131

5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increments ΔL, starting at point A and ending at the right-hand support.



SOLUTION

$+\circlearrowleft \Sigma M_C = 0$

$-12 R_A + (6)(12)(1.8) + (10)(\frac{1}{2})(6)(1.8) = 0$

$R_A = 15.3$ kips.

$w = 3.6 - \frac{1.8}{6} x + \frac{1.8}{6} \langle x-6 \rangle^1$

$= 3.6 - 0.3x + 0.3\langle x-6 \rangle^1$

$V = 15.3 - 3.6x + 0.15x^2 - 0.15\langle x-6 \rangle^2$ kips ◄

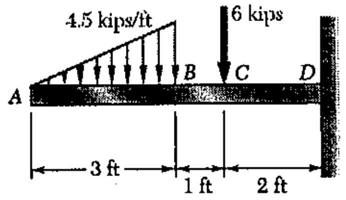
$M = 15.3x - 1.8x^2 + 0.05x^3 - 0.05\langle x-6 \rangle^3$ kip·ft ◄

x ft	V kips	M kip·ft
0.0	15.30	0.0
0.5	13.54	7.2
1.0	11.85	13.6
1.5	10.24	19.1
2.0	8.70	23.8
2.5	7.24	27.8
3.0	5.85	31.1
3.5	4.54	33.6
4.0	3.30	35.6
4.5	2.14	37.0
5.0	1.05	37.8
5.5	0.04	38.0
6.0	-0.90	37.8
6.5	-1.80	37.1
7.0	-2.70	36.0
7.5	-3.60	34.4
8.0	-4.50	32.4
8.5	-5.40	29.9
9.0	-6.30	27.0
9.5	-7.20	23.6
10.0	-8.10	19.8
10.5	-9.00	15.5
11.0	-9.90	10.8
11.5	-10.80	5.6
12.0	-11.70	0.0

PROBLEM 5.132

5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increments ΔL, starting at point A and ending at the right-hand support.

$\Delta L = 0.5$ ft



SOLUTION

$$w = \frac{4.5}{3}x - \frac{4.5}{3}\langle x-3\rangle^1 - 4.5\langle x-3\rangle^0$$

$$= 1.5x - 1.5\langle x-3\rangle^1 - 4.5\langle x-3\rangle^0$$

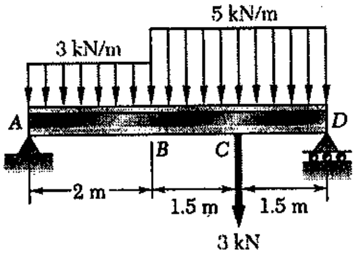
$$V = -0.75x^2 + 0.75\langle x-3\rangle^2 + 4.5\langle x-3\rangle^1 - 6\langle x-4\rangle^0 \quad \text{kips}$$

$$M = -0.25x^3 + 0.25\langle x-3\rangle^3 + 2.25\langle x-3\rangle^2 - 6\langle x-4\rangle^1 \quad \text{kip}\cdot\text{ft}$$

x ft	V kips	M kip·ft
0.0	0.00	0.00
0.5	-0.19	-0.03
1.0	-0.75	-0.25
1.5	-1.69	-0.84
2.0	-3.00	-2.00
2.5	-4.69	-3.91
3.0	-6.75	-6.75
3.5	-6.75	-10.13
4.0	-12.75	-13.50
4.5	-12.75	-19.88
5.0	-12.75	-26.25
5.5	-12.75	-32.63
6.0	-12.75	-39.00

PROBLEM 5.133

5.133 and 5.134 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam..



W200 × 22.5
$L = 5$ m
$\Delta L = 0.25$ m

SOLUTION

$+\circlearrowleft \Sigma M_D = 0$

$-5R_A + (4.0)(2.0)(3) + (1.5)(3)(5) + (1.5)(3) = 0$

$R_A = 10.2$ kN

$w = 3 + 2\langle x-2\rangle^0$ kN/m $= -\frac{dV}{dx}$

$V = 10.2 - 3x - 2\langle x-2\rangle^1 - 3\langle x-3.5\rangle^0$ kN

$M = 10.2x - 1.5x^2 - \langle x-2\rangle^2 - 3\langle x-3.5\rangle^1$ kN·m

x m	V kN	M kN·m	sigma MPa
0.00	10.20	0.00	0.0
0.25	9.45	2.46	12.7
0.50	8.70	4.72	24.4
0.75	7.95	6.81	35.1
1.00	7.20	8.70	44.8
1.25	6.45	10.41	53.6
1.50	5.70	11.92	61.5
1.75	4.95	13.26	68.3
2.00	4.20	14.40	74.2
2.25	2.95	15.29	78.8
2.50	1.70	15.88	81.8
2.75	0.45	16.14	83.2
3.00	-0.80	16.10	83.0
3.25	-2.05	15.74	81.2
3.50	-6.30	15.07	77.7
3.75	-7.55	13.34	68.8
4.00	-8.80	11.30	58.2
4.25	-10.05	8.94	46.1
4.50	-11.30	6.27	32.3
4.75	-12.55	3.29	17.0
5.00	-13.80	-0.00	-0.0
2.83	0.05	16.164	83.3
2.84	0.00	16.164	83.3 ←
2.85	-0.05	16.164	83.3

For rolled steel section
W 200 × 22.5

$S = 194 \times 10^3$ mm^3

$S = 194 \times 10^3$ mm$^3 = 194 \times 10^{-6}$ m^3

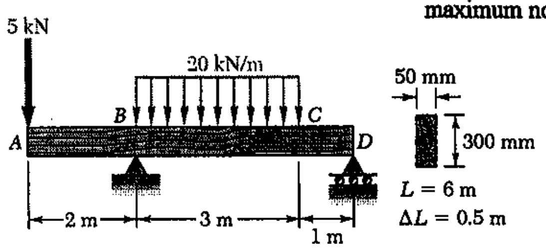
$\sigma_{max} = \frac{M_{max}}{S} = \frac{16.164 \times 10^3}{194 \times 10^{-6}}$

$= 83.3 \times 10^6$ Pa

$= 83.3$ MPa

PROBLEM 5.134

5.133 and 5.134 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam..



$L = 6$ m
$\Delta L = 0.5$ m

SOLUTION

$+\circlearrowleft \Sigma M_D = 0$

$-4R_B + (6)(5) + (2.5)(3)(20) = 0$

$R_B = 45$ kN

$w = 20\langle x-2\rangle^0 - 20\langle x-5\rangle^0$ kN/m $= -\frac{dV}{dx}$

$V = -5 + 45\langle x-2\rangle^0 - 20\langle x-2\rangle^1 + 20\langle x-5\rangle^1$ kN

$M = -5x + 45\langle x-2\rangle^1 - 10\langle x-2\rangle^2 + 10\langle x-5\rangle^2$ kN·m

x m	V kN	M kN·m	sigma MPa
0.00	-5	0.00	0.0
0.50	-5	-2.50	-3.3
1.00	-5	-5.00	-6.7
1.50	-5	-7.50	-10.0
2.00	40	-10.00	-13.3
2.50	30	7.50	10.0
3.00	20	20.00	26.7
3.50	10	27.50	36.7
4.00	0	30.00	40.0 ←
4.50	-10	27.50	36.7
5.00	-20	20.00	26.7
5.50	-20	10.00	13.3
6.00	-20	0.00	0.0

Maximum $|M| = 30$ kN·m at $x = 4.0$ m

For rectangular cross section

$S = \frac{1}{6}bh^2 = (\frac{1}{6})(50)(300)^2$

$= 750 \times 10^3$ mm^3

$= 750 \times 10^{-6}$ m^3

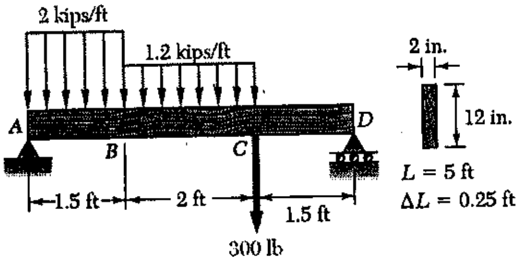
$\sigma_{max} = \frac{M_{max}}{S} = \frac{30 \times 10^3}{750 \times 10^{-6}}$

$= 40 \times 10^6$ Pa $= 40$ MPa

$\sigma = \frac{|M|}{S}$

PROBLEM 5.135

5.135* and *5.136 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam..



$L = 5$ ft
$\Delta L = 0.25$ ft

SOLUTION

300 lb = 0.3 kips

$+\circlearrowleft \Sigma M_D = 0$

$-5R_A + (4.25)(1.5)(2) + (2.5)(2)(1.2) + (1.5)(0.3) = 0$

$R_A = 3.84$ kips.

$$w = 2 - 0.8\langle x-1.5\rangle^0 - 1.2\langle x-3.5\rangle^0 \text{ kip/ft}$$

$$V = 3.84 - 2x + 0.8\langle x-1.5\rangle^1 + 1.2\langle x-3.5\rangle^1 - 0.3\langle x-3.5\rangle^0 \text{ kips}$$

$$M = 3.84x - x^2 + 0.4\langle x-1.5\rangle^2 + 0.6\langle x-3.5\rangle^2 - 0.3\langle x-3.5\rangle^1 \text{ kip·ft}$$

x ft	V kips	M kip·ft	sigma ksi
0.00	3.84	0.00	0.000
0.25	3.34	0.90	0.224
0.50	2.84	1.67	0.417
0.75	2.34	2.32	0.579
1.00	1.84	2.84	0.710
1.25	1.34	3.24	0.809
1.50	0.84	3.51	0.877
1.75	0.54	3.68	0.921
2.00	0.24	3.78	0.945
2.25	-0.06	3.80	0.951
2.50	-0.36	3.75	0.937
2.75	-0.66	3.62	0.906
3.00	-0.96	3.42	0.855
3.25	-1.26	3.14	0.786
3.50	-1.86	2.79	0.697
3.75	-1.86	2.32	0.581
4.00	-1.86	1.86	0.465
4.25	-1.86	1.39	0.349
4.50	-1.86	0.93	0.232
4.75	-1.86	0.46	0.116
5.00	-1.86	-0.00	-.000
2.10	0.12	3.80	0.949
2.20	0.00	3.80	0.951
2.30	-0.12	3.80	0.949

Maximum $|M| = 3.804$ kip·ft
$= 45.648$ kip·in
at $x = 2.20$ ft

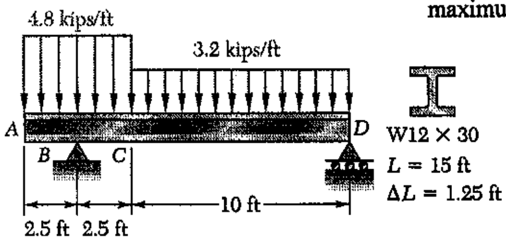
Rectangular section
2 in × 12 in.

$S = \frac{1}{6}bh^2 = (\frac{1}{6})(2)(12)^2$
$= 48 \text{ in}^3$

$\sigma = \frac{M}{S} = \frac{45.648}{48}$
$= 0.951$ ksi

PROBLEM 5.136

5.135* and *5.136 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam..



SOLUTION

$+\circlearrowleft \Sigma M_D = 0$

$-12.5 R_B + (12.5)(5.0)(4.8) + (5)(10)(3.2) = 0$

$R_B = 36.8$ kips.

$w = 4.8 - 1.6\langle x-5\rangle^0$ kips/ft

$V = -4.8x + 36.8\langle x-2.5\rangle^0 + 1.6\langle x-5\rangle^1$ kips

$M = -2.4x^2 + 36.8\langle x-2.5\rangle^1 + 0.8\langle x-5\rangle^2$ kip-ft

x ft	V kips	M kip·ft	sigma ksi
0.00	0.0	0.00	0.00
1.25	-6.0	-3.75	-1.17
2.50	24.8	-15.00	-4.66
3.75	18.8	12.25	3.81
5.00	12.8	32.00	9.95
6.25	8.8	45.50	14.15
7.50	4.8	54.00	16.79
8.75	0.8	57.50	17.88
10.00	-3.2	56.00	17.41
11.25	-7.2	49.50	15.39
12.50	-11.2	38.00	11.81
13.75	-15.2	21.50	6.68
15.00	-19.2	0.00	0.00
8.90	0.32	57.58	17.90
9.00	-0.00	57.60	17.91 ←
9.10	-0.32	57.58	17.90

Maximum $M = 57.6$ kip·ft
$= 691.2$ kip·in
at $x = 9.0$ ft.

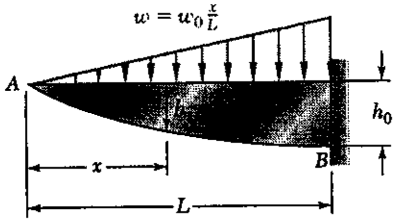
For rolled steel section W 12×30

$S = 38.6$ in^3

Maximum normal stress

$$\sigma = \frac{M}{S} = \frac{691.2}{38.6} = 17.91 \text{ ksi}$$

PROBLEM 5.137



5.137 and 5.138 The cantilever beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (b) Determine the smallest allowable value of h_0 if L =750 mm, b = 30 mm, w_0 = 300 kN/m, and σ_{all} = 250 MPa.

SOLUTION

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V = -\frac{w_0 x^2}{2L} = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6L} \qquad |M| = \frac{w_0 x^3}{6L}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w_0 x^3}{6L\sigma_{all}}$$

For a rectangular cross section $S = \frac{1}{6} b h^2$

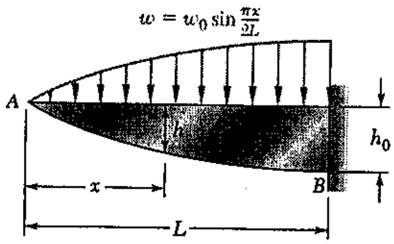
Equating $\frac{1}{6} b h^2 = \frac{w_0 x^3}{6L\sigma_{all}}$ $\qquad h = \sqrt{\frac{w_0 x^3}{\sigma_{all} b L}}$

At $x = L$ $\qquad h = h_0 = \sqrt{\frac{w_0 L^2}{\sigma_{all} b}}$ $\qquad \therefore \quad h = h_0 \left(\frac{x}{L}\right)^{3/2}$ ◄

Data: $L = 750$ mm $= 0.75$ m, $b = 30$ mm $= 0.030$ m
$w_0 = 300$ kN/m $= 300 \times 10^3$ N/m, $\sigma_{all} = 250$ MPa $= 250 \times 10^6$ Pa

$$h_0 = \sqrt{\frac{(300\times 10^3)(0.75)^2}{(250\times 10^6)(0.030)}} = 150 \times 10^{-3} \text{ m} = 150 \text{ mm}$$ ◄

PROBLEM 5.138



5.137 and 5.138 The cantilever beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (b) Determine the smallest allowable value of h_0 if L =750 mm, b = 30 mm, w_0 = 300 kN/m, and σ_{all} = 250 Mpa.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin\frac{\pi x}{2L}$$

$$V = \frac{2w_0L}{\pi}\cos\frac{\pi x}{2L} + C_1$$

$$V = 0 \text{ at } x = 0 \quad \longrightarrow \quad C_1 = -\frac{2w_0L}{\pi}$$

$$\frac{dM}{dx} = V = -\frac{2w_0L}{\pi}\left(1 - \cos\frac{\pi x}{2L}\right)$$

$$M = -\frac{2w_0L}{\pi}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right) \qquad |M| = \frac{2w_0L}{\pi}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{L}\right)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{2w_0L}{\pi\sigma_{all}}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)$$

For a rectangular cross section $\quad S = \frac{1}{6}bh^2$

Equating $\quad \frac{1}{6}bh^2 = \frac{2w_0L}{\pi\sigma_{all}}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)$

$$h = \left\{\frac{12w_0L}{\pi\sigma_{all}b}\left(x - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right)\right\}^{1/2}$$

At $x = L \qquad h = h_0 = \left\{\frac{12w_0L^2}{\pi\sigma_{all}b}\left(1 - \frac{2}{\pi}\right)\right\}^{1/2} = 1.178\sqrt{\frac{w_0L^2}{\sigma_{all}b}}$

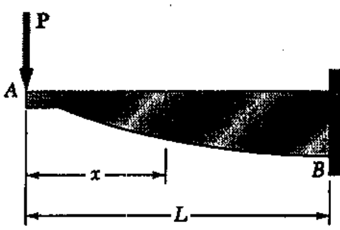
(a) $\quad h = h_0\left[\left(\frac{x}{L} - \frac{2L}{\pi}\sin\frac{\pi x}{2L}\right) / \left(1 - \frac{2}{\pi}\right)\right]^{1/2}$ ◄

Data: $\quad L = 750$ mm $= 0.75$ m, $\quad b = 30$ mm $= 0.030$ m
$w_0 = 300$ kN/m $= 300\times10^3$ N/m, $\quad \sigma_{all} = 250$ MPa $= 250\times10^6$ Pa

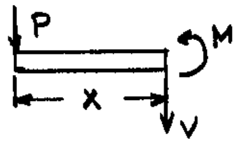
(b) $\quad h_0 = 1.178\sqrt{\frac{(300\times10^3)(0.75)^2}{(250\times10^6)(0.030)}} = 176.7\times10^{-3}$ m $= 176.7$ mm ◄

PROBLEM 5.139

5.139 and 5.140 The beam *AB*, consisting of a cast-iron plate of uniform thickness *b* and length *L*, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0. (*b*) Determine the maximum allowable load if L = 36 in., h_0 = 12 in., b = 1.25 in., and σ_{all} = 36 ksi.



SOLUTION



$$V = -P$$

$$M = -Px \qquad |M| = Px$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{P}{\sigma_{all}} x$$

For a rectangular cross section $\quad S = \frac{1}{6} bh^2$

Equating $\quad \frac{1}{6} bh^2 = \frac{Px}{\sigma_{all}} \qquad h = \left(\frac{6Px}{\sigma_{all} b}\right)^{\frac{1}{2}}$

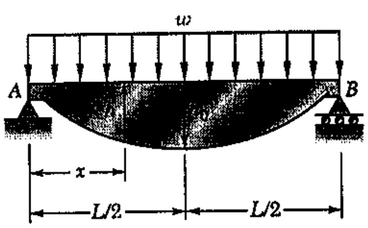
At $x = L \qquad h = h_0 = \left\{\frac{6PL}{\sigma_{all} b}\right\}^{1/2}$

$$h = h_0 \frac{x}{L}, \quad ◄$$

Solving for P $\qquad P = \frac{\sigma_{all} b h_0^2}{6L} = \frac{(36)(1.25)(12)^2}{(6)(36)} = 30$ kips ◄

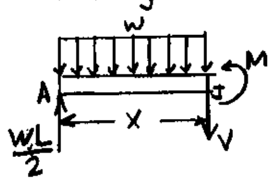
PROBLEM 5.140

5.139 and 5.140 The beam *AB*, consisting of a cast-iron plate of uniform thickness *b* and length *L*, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0. (*b*) Determine the maximum allowable load if L = 36 in., h_0 = 12 in., b = 1.25 in., and σ_{all} = 36 ksi.



SOLUTION

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B - wL = 0 \qquad R_A = R_B = \frac{wL}{2}$



$$\circlearrowleft \Sigma M_J = 0$$

$$\frac{wL}{2} x - wx \frac{x}{2} + M = 0$$

$$M = \frac{w}{2} x (L - x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$$

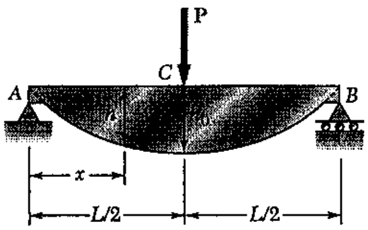
For a rectangular cross section $\quad S = \frac{1}{6} bh^2$

Equating $\quad \frac{1}{6} bh^2 = \frac{wx(L-x)}{2\sigma_{all}} \qquad h = \left\{\frac{3wx(L-x)}{\sigma_{all} b}\right\}^{1/2}$ ◄

At $x = \frac{L}{2} \qquad h = h_0 = \left\{\frac{3wL^2}{4\sigma_{all} b}\right\}^{1/2} \qquad h = h_0\left[\frac{x}{L}\left(1 - \frac{x}{L}\right)\right]^{\frac{1}{2}}$ ◄

Solving for w $\qquad w = \frac{4\sigma_{all} b h_0^2}{3L^2} = \frac{(4)(36)(1.25)(12)^2}{(3)(36)} = 6.67$ kip/in ◄

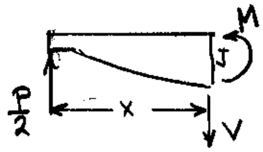
PROBLEM 5.141



5.141* and *5.142 The beam AB, consisting of a cast-aluminum plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (b) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.

SOLUTION

$R_A = R_B = \frac{P}{2}$



$+\circlearrowleft \sum M_J = 0$

$-\frac{P}{2}x + M = 0$

$M = \frac{Px}{2} \qquad (0 < x < \frac{L}{2})$

$S = \frac{M}{\sigma_{all}} = \frac{Px}{2\sigma_{all}}$

For a rectangular cross section $\quad S = \frac{1}{6}bh^2$

Equating $\quad \frac{1}{6}bh^2 = \frac{Px}{2\sigma_{all}} \qquad h = \sqrt{\frac{3Px}{\sigma_{all} b}}$

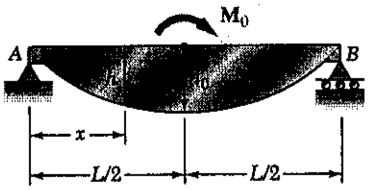
(a) At $x = \frac{L}{2} \qquad h = h_o = \sqrt{\frac{3PL}{2\sigma_{all} b}} \qquad h = h_o\sqrt{\frac{2x}{L}}, \quad 0 < x < \frac{L}{2}$ ◀

For $x > \frac{L}{2}$ replace x by $L - x$

(b) Solving for P $\quad P = \frac{2\sigma_{all} b h_o^2}{3L} = \frac{(2)(72\times10^6)(0.025)(0.200)^2}{(3)(0.8)} = 60\times10^3$ N

$= 60$ kN ◀

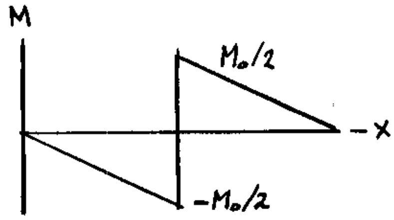
PROBLEM 5.142

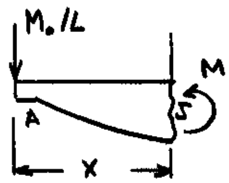


5.141 **and** *5.142* The beam *AB*, consisting of a cast-aluminum plate of uniform thickness *b* and length *L*, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0 for portion *AC* of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ Mpa.

SOLUTION

$R_A = M_0/L \downarrow \qquad R_B = M_0/L \uparrow$





$\circlearrowright \Sigma M_J = 0$

$\frac{M_0}{L}x + M = 0$

$M = -\frac{M_0 x}{L} \quad (0 < x < \frac{L}{2})$

For $x > \frac{L}{2}$ $\quad M = \frac{M_0(L-x)}{L} \quad (\frac{L}{2} < x < L)$

$S = \frac{|M|}{\sigma_{all}} = \frac{M_0 x}{\sigma_{all} L}$ for $(0 < x \frac{L}{2})$ $\quad$ For $x > \frac{L}{2}$ replace x by $L - x$.

For a rectangular cross section $\quad S = \frac{1}{6}bh^2$

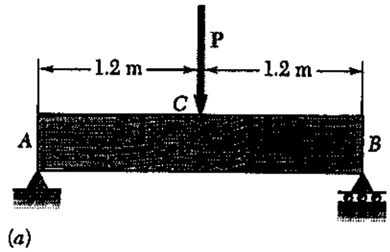
Equating $\frac{1}{6}bh^2 = \frac{M_0 x}{\sigma_{all} L} \qquad h = \sqrt{\frac{6 M_0 x}{\sigma_{all} b L}}$

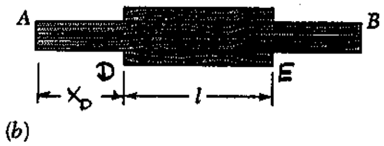
At $x = \frac{L}{2} \quad h = h_0 = \sqrt{\frac{3M_0}{\sigma_{all} b}} \qquad h = h_0\sqrt{2x/L}$ ◄

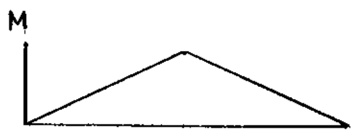
Solving for $M_0 \quad M_0 = \frac{\sigma_{all} b h_0^2}{3} = \frac{(72\times10^6)(0.025)(0.200)^2}{3} = 24\times10^3$ N·m

$= 24$ kN·m ◄

PROBLEM 5.143



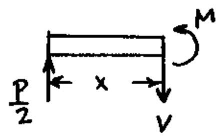




5.143 and 5.144 A preliminary design based on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50- mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{P}{2}$$



$0 < x < \frac{L}{2}$

$\circlearrowleft \Sigma M_J = 0 \qquad -\frac{P}{2}x + M = 0$

$M = \frac{Px}{2} \quad$ or $\quad M = \frac{M_{max} x}{1.2}$

Bending moment diagram is two straight lines.

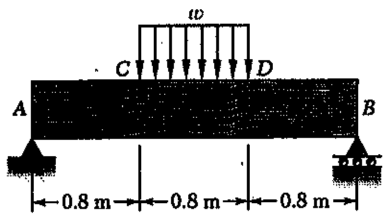
At C $\qquad S_C = \frac{1}{6} b h_c^2 \qquad M_C = M_{max}$

At D $\qquad S_D = \frac{1}{6} b h_D^2 \qquad M_D = \frac{M_{max} x_D}{1.2}$

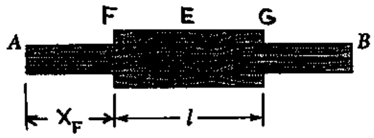
$$\frac{S_D}{S_C} = \frac{h_D^2}{h_c^2} = \left(\frac{100\text{ mm}}{200\text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_D}{M_C} = \frac{x_D}{1.2} \qquad x_D = 0.3\text{ m}$$

$$\frac{l}{2} = 1.2 - x_D = 0.9 \qquad l = 1.800\text{ m}$$

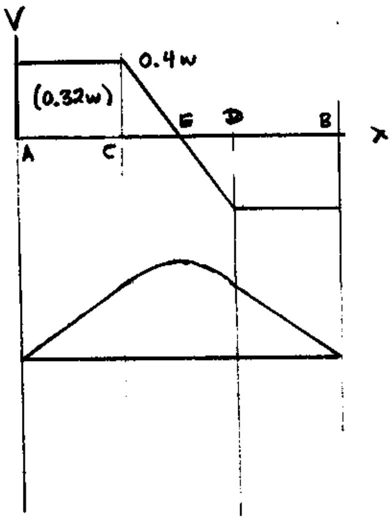
PROBLEM 5.144



(a)



(b)



5.143 and 5.144 A preliminary design based on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50- mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{0.8\,w}{2} = 0.4\,w$$

Shear: A to C $\quad V = 0.4\,w$

D to B $\quad V = -0.4\,w$

Areas: A to C $\quad (0.8)(0.4)w = 0.32\,w$

C to E $\quad (\tfrac{1}{2})(0.4)(0.4)w = 0.08\,w$

Bending moments.

At C $\quad M_C = 0.40\,w$

A to C $\quad M = 0.40\,wx$

At C $\quad S_c = \frac{1}{6}bh_c^2 \qquad M_c = M_{max} = 0.40\,w$

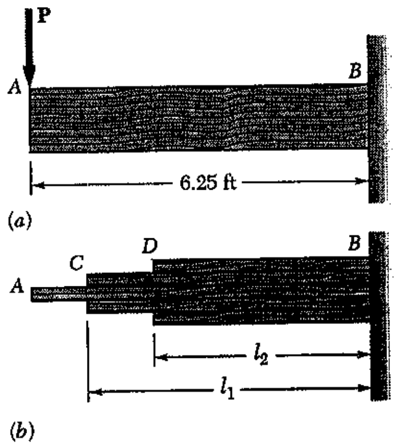
At F $\quad S_F = \frac{1}{6}bh_F^2 \qquad M_F = 0.40\,wx_F$

$$\frac{S_F}{S_c} = \frac{h_F^2}{h_c^2} = \left(\frac{100\text{ mm}}{200\text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_F}{M_c} = \frac{0.40\,wx_F}{0.40\,w}$$

$$x_F = 0.25\text{ m} \qquad \frac{l}{2} = 1.2 - x_F = 0.95\text{ m}$$

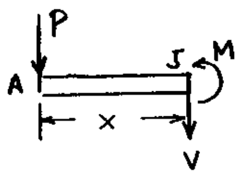
$$l = 1.900\text{ m}$$

PROBLEM 5.145



5.145 and 5.146 A preliminary design based on the use of a cantilever prismatic beam indicated that a beam with a rectangular cross section 2 in. wide and10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2 × 2- in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION



$$+\circlearrowleft \Sigma M_J = 0$$

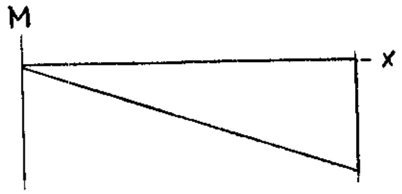
$$Px + M = 0 \qquad M = -Px$$

$$|M| = Px$$

At B $\quad |M|_B = M_{max}$

At C $\quad |M|_C = M_{max}\, x_C / 6.25$

At D $\quad |M|_D = M_{max}\, x_D / 6.25$



$$S_B = \frac{1}{6} b h^2 = \frac{1}{6} \cdot b (5b)^2 = \frac{25}{6} b^3$$

A to C $\quad S_C = \frac{1}{6} \cdot b (b)^2 = \frac{1}{6} b^3$

C to D $\quad S_D = \frac{1}{6} b (3b)^2 = \frac{9}{6} b^3$

$$\frac{|M|_C}{|M|_B} = \frac{x_C}{6.25} = \frac{S_C}{S_B} = \frac{1}{25}$$

$$x_C = \frac{(1)(6.25)}{25} = 0.25 \text{ ft}$$

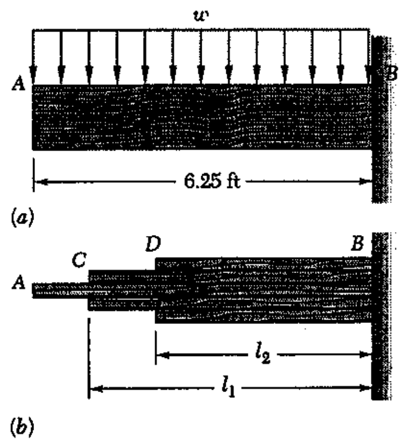
$$l_1 = 6.25 - 0.25 = 6.00 \text{ ft}$$

$$\frac{|M|_D}{|M|_B} = \frac{x_D}{6.25} = \frac{S_D}{S_B} = \frac{9}{25}$$

$$x_D = \frac{(9)(6.25)}{25} = 2.25 \text{ ft}$$

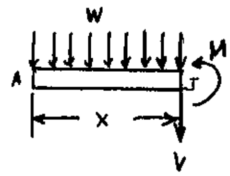
$$l_2 = 6.25 - 2.25 = 4.00 \text{ ft}$$

PROBLEM 5.146



5.145 and 5.146 A preliminary design based on the use of a cantilever prismatic beam indicated that a beam with a rectangular cross section 2 in. wide and10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2 × 2- in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION



$+\circlearrowleft \Sigma M_J = 0 \qquad wx\frac{x}{2} + M = 0$

$M = -\frac{wx^2}{2} \qquad |M| = \frac{wx^2}{2}$

At B $\quad |M|_B = |M|_{max}$

At C $\quad |M|_C = |M|_{max}(x_C/6.25)^2$

At D $\quad |M|_D = |M|_{max}(x_D/6.25)^2$

At B $\quad S_B = \frac{1}{6}bh^2 = \frac{1}{6}b(5b)^2 = \frac{25}{6}b^3$

A to C $\quad S_C = \frac{1}{6}bh^2 = \frac{1}{6}b(b)^2 = \frac{1}{6}b^3$

C to D $\quad S_D = \frac{1}{6}bh^2 = \frac{1}{6}b(3b)^2 = \frac{9}{6}b^3$

$$\frac{|M|_C}{|M|_B} = \left(\frac{x_C}{6.25}\right)^2 = \frac{S_C}{S_B} = \frac{1}{25}$$

$$x_C = \frac{6.25}{\sqrt{25}} = 1.25 \text{ ft}$$

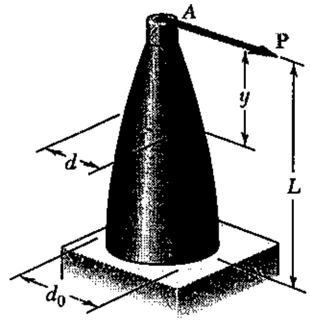
$$l_1 = 6.25 - 1.25 \text{ ft} = 5.00 \text{ ft} \blacktriangleleft$$

$$\frac{|M|_D}{|M|_B} = \left(\frac{x_D}{6.25}\right)^2 = \frac{S_D}{S_B} = \frac{9}{25}$$

$$x_D = \frac{6.25\sqrt{9}}{\sqrt{25}} = 3.75 \text{ ft}$$

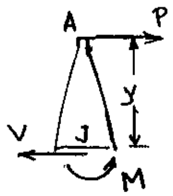
$$l_2 = 6.25 - 3.75 \text{ ft} = 2.50 \text{ ft} \blacktriangleleft$$

PROBLEM 5.147



5.147 A cantilevered machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support a horizontal concentrated load **P** as shown. (*a*) Knowing that the machine element is to be of constant strength, express d in terms of y, L, and d_0. (*b*) Determine the maximum allowable value of P if $L = 300$ mm, $d_0 = 60$ mm, and $\sigma_{all} = 72$ Mpa.

SOLUTION



$+\circlearrowleft \Sigma M_J = 0 \qquad M - Py = 0$

$M = Py$

$S = \dfrac{|M|}{\sigma_{all}} = \dfrac{Py}{\sigma_{all}}$

For a solid circular cross section $\quad c = \dfrac{d}{2} \qquad I = \dfrac{\pi}{4}c^4$

$S = \dfrac{I}{c} = \dfrac{\pi}{4}c^3 = \dfrac{\pi d^3}{32}$

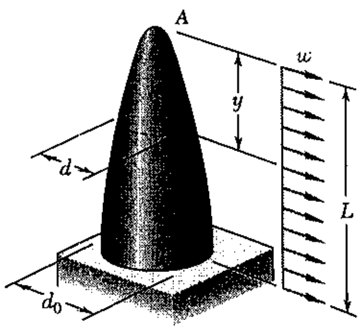
Equating $\quad \dfrac{\pi d^3}{32} = \dfrac{Py}{\sigma_{all}} \qquad d = \left(\dfrac{32\,Py}{\pi \sigma_{all}}\right)^{1/3}$

At $y = L \quad d = d_0 = \left(\dfrac{32\,PL}{\pi\sigma_{all}}\right)^{1/3} \qquad \dfrac{d}{d_0} = \left(\dfrac{y}{L}\right)^{1/3}$ ◄

Solving for P $\quad P = \dfrac{\pi d_0^3 \sigma_{all}}{32\,L} = \dfrac{\pi(0.060)^3(72\times10^6)}{(32)(0.300)} = 5.09\times10^3$ N

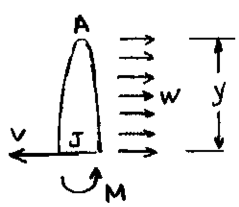
$= 5.09$ kN ◄

PROBLEM 5.148



5.148 A cantilevered machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support a horizontal distributed load w as shown. (*a*) Knowing that the machine element is to be of constant strength, express d in terms of y, L, and d_0. (*b*) Determine the smallest allowable value of d_0 if $L = 300$ mm, $w = 20$ kN/m, and $\sigma_{all} = 72$ MPa.

SOLUTION



$+\circlearrowleft \Sigma M_J = 0$

$M - \dfrac{y}{2}wy = 0 \qquad M = \dfrac{wy^2}{2}$

$S = \dfrac{|M|}{\sigma_{all}} = \dfrac{wy^2}{2\sigma_{all}}$

For a solid circular cross section $\quad c = \dfrac{d}{2}$

$I = \dfrac{\pi}{4}c^4 \qquad S = \dfrac{I}{c} = \dfrac{\pi c^3}{4} = \dfrac{\pi d^3}{32}$

Equating $\quad \dfrac{\pi d^3}{32} = \dfrac{wy^2}{2\sigma_{all}} \qquad d = \left(\dfrac{16\,wy^2}{\pi\sigma_{all}}\right)^{1/3}$

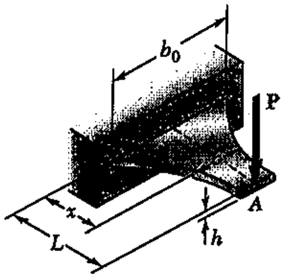
At $x = L \quad d = d_0 = \left(\dfrac{16\,wL^2}{\pi\sigma_{all}}\right)^{1/3} \qquad d = d_0\left(\dfrac{y}{L}\right)^{2/3}$ ◄

Using the data $\quad d_0 = \left\{\dfrac{(16)(20\times10^3)(0.300)^2}{\pi\,(72\times10^6)}\right\}^{1/3} = 50.3\times10^{-3}$ m

$= 50.3$ mm ◄

PROBLEM 5.149

5.149 A cantilever beam *AB* consisting of a steel plate of uniform depth *h* and variable width *b* is to support a concentrated load **P** at point *A*. (*a*) Knowing that the beam is to be of constant strength, express *b* in terms of *x*, *L*, and b_0. (*b*) Determine the smallest allowable value of *h* if
$L = 12$ in., $b_0 = 15$ in., $P = 3.2$ kips, and $\sigma_{all} = 24$ ksi.

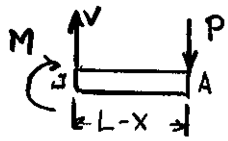


SOLUTION

$\circlearrowleft \Sigma M_J = 0 \qquad -M - P(L-x) = 0 \qquad M = -P(L-x)$

$|M| = P(L-x)$

$S = \dfrac{|M|}{\sigma_{all}} = \dfrac{P(L-x)}{\sigma_{all}}$



For a rectangular cross section $\quad S = \frac{1}{6} b h^2$

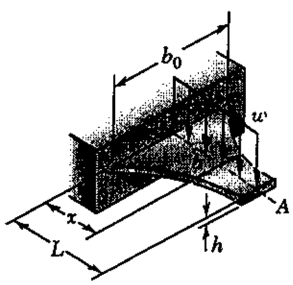
Equating $\quad \frac{1}{6} b h^2 = \dfrac{P(L-x)}{\sigma_{all}} \qquad b = \dfrac{6P(L-x)}{\sigma_{all} h^2}$

At $x = 0 \qquad b = b_0 = \dfrac{6PL}{\sigma_{all} h^2} \qquad b = b_0\left(1 - \dfrac{x}{L}\right)$ ◄

Solving for $h \qquad h = \sqrt{\dfrac{6PL}{\sigma_{all} b_0}} = \sqrt{\dfrac{(6)(3.2)(12)}{(24)(15)}} = 0.800$ in ◄

PROBLEM 5.150

5.150 A cantilever beam *AB* consisting of a steel plate of uniform depth *h* and variable width *b* is to support a distributed load *w* along its center line *AB*. (*a*) Knowing that the beam is to be of constant strength, express *b* in terms of *x*, *L*, and b_0. (*b*) Determine the maximum allowable value of *w* if $L = 15$ in., $b_0 = 18$ in., $h = 0.75$ in., and $\sigma_{all} = 24$ ksi

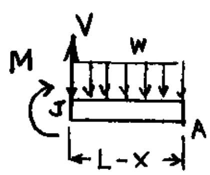


SOLUTION

$\circlearrowleft \Sigma M_J = 0 \qquad -M - w(L-x)\dfrac{L-x}{2} = 0$

$M = -\dfrac{w(L-x)^2}{2} \qquad |M| = \dfrac{w(L-x)^2}{2}$

$S = \dfrac{|M|}{\sigma_{all}} = \dfrac{w(L-x)^2}{2\sigma_{all}}$



For a rectangular cross section $\quad S = \frac{1}{6} b h^2$

$\frac{1}{6} b h^2 = \dfrac{w(L-x)^2}{2\sigma_{all}} \qquad b = \dfrac{3w(L-x)^2}{\sigma_{all} h^2}$

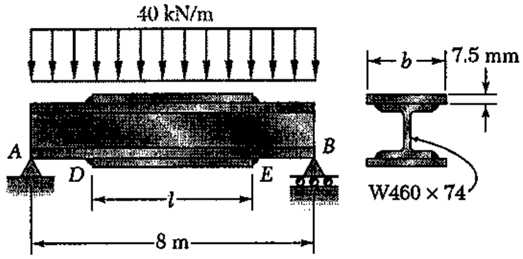
At $x = 0 \qquad b = b_0 = \dfrac{3wL^2}{\sigma_{all} h^2} \qquad b = b_0\left(1 - \dfrac{x}{L}\right)^2$ ◄

Solving for $w \qquad w = \dfrac{\sigma_{all} b_0 h^2}{3L^2} = \dfrac{(24)(18)(0.75)^2}{(3)(15)^2} = 0.360$ kip/in

$= 360$ lb/in ◄

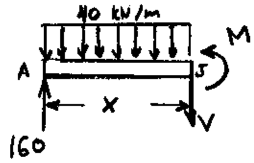
PROBLEM 5.151

5.151 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $l = 5$ m and $b = 200$ mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.



SOLUTION

$R_A = R_B = 160 \text{ kN}$



$+\circlearrowleft \Sigma M_J = 0$

$-160x + (40x)\frac{x}{2} + M = 0$

$M = 160x - 20x^2 \text{ kN}\cdot\text{m}$

At center of beam $x = 4\text{m}$ $\quad M_c = 320 \text{ kN}\cdot\text{m}$

At D $x = \frac{1}{2}(8 - l) = 1.5 \text{ m}$ $\quad M_D = 195 \text{ kN}\cdot\text{m}$

At center of beam

$$I = I_{beam} + 2I_{plate}$$
$$= 333 \times 10^6 + 2\left\{(200)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(200)(7.5)^3\right\}$$
$$= 494.8 \times 10^6 \text{ mm}^4$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm} \qquad S = \frac{I}{c} = 2097 \times 10^3 \text{ mm}^3 = 2097 \times 10^{-6} \text{ m}^3$$

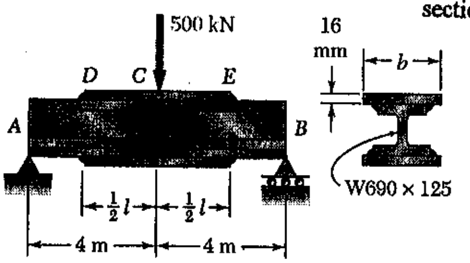
Normal stress $\quad \sigma = \frac{M}{S} = \frac{320 \times 10^3}{2097 \times 10^{-6}} = 152.6 \times 10^6 \text{ Pa} = 152.6 \text{ MPa}$ ◄

At D $\quad S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \frac{M}{S} = \frac{195 \times 10^3}{1460 \times 10^{-6}} = 133.6 \times 10^6 \text{ Pa} = 133.6 \text{ MPa}$ ◄

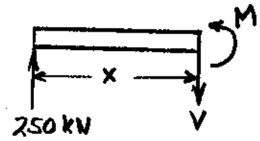
PROBLEM 5.152

5.152 Assuming that the length and width of the cover plates used with the beam of Sample Prob. 5.12 are, respectively, $l = 4$ m and $b = 285$ mm, and recalling that the thickness of each plate is 16 mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.



SOLUTION

$R_A = R_B = 250$ kN



$\circlearrowleft \Sigma M_J = 0$

$-250x + M = 0$

$M = 250x$ kN·m

At center of beam $x = 4$ m $\quad M_c = (250)(4) = 1000$ kN·m

At D $\quad x = \frac{1}{2}(8 - l) = \frac{1}{2}(8 - 4) = 2$ m $\quad M_D = 500$ kN·m

At center of beam $\quad I = I_{beam} + 2I_{plate}$

$$= 1190 \times 10^6 + 2\{(285)(16)(\tfrac{678}{2} + \tfrac{16}{2})^2 + \tfrac{1}{12}(285)(16)^3\}$$

$$= 2288 \times 10^6 \text{ mm}^4$$

$c = \frac{678}{2} + 16 = 355$ mm $\quad S = \frac{I}{c} = 6445 \times 10^3$ mm^3 $= 6445 \times 10^{-6}$ m^3

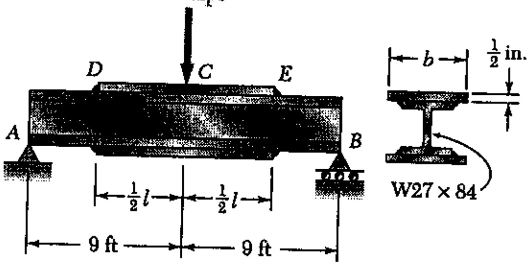
Normal stress $\quad \sigma = \frac{M}{S} = \frac{1000 \times 10^3}{6445 \times 10^{-6}} = 155.2 \times 10^6$ Pa $= 155.2$ MPa ◄

At D $\quad S = 3510 \times 10^3$ mm^3 $= 3510 \times 10^{-6}$ m^3

Normal stress $\quad \sigma = \frac{M}{S} = \frac{500 \times 10^3}{3510 \times 10^{-6}} = 142.4 \times 10^6$ Pa $= 142.4$ MPa ◄

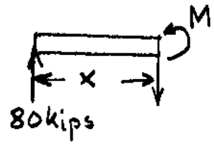
PROBLEM 5.153

5.153 Two cover plates, each $\frac{1}{2}$-in. thick, are welded to a W27 × 84 beam as shown. Knowing that $l = 10$ ft and $b = 10.5$ in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.



SOLUTION

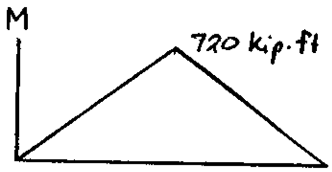
$R_A = R_B = 80$ kips



$+\circlearrowleft \Sigma M_J = 0$

$-80x + M = 0$

$M = 80x$ kip·ft



At C $\quad x = 9$ ft $\quad M_C = 720$ kip·ft $= 8640$ kip·in

At D $\quad x = 9 - 5 = 4$ ft

$M_D = (80)(4) = 320$ kip·ft $= 3840$ kip·in.

At center of beam $\quad I = I_{beam} + 2 I_{plate}$

$$I = 2850 + 2\left\{(10.5)(0.500)\left(\frac{26.71}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(10.5)(0.500)^3\right\}$$

$$= 4794 \text{ in}^3$$

$$c = \frac{26.71}{2} + 0.500 = 13.855 \text{ in}$$

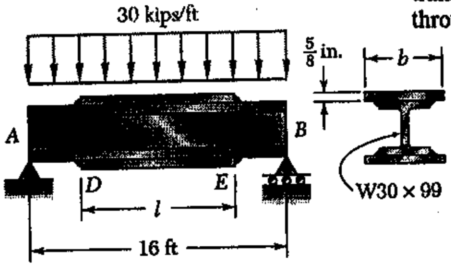
Normal stress $\quad \sigma = \dfrac{Mc}{I} = \dfrac{(8640)(13.855)}{4794} = 25.0$ ksi ◄

At point D $\quad S = 213$ in^3

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{3840}{213} = 18.03$ ksi ◄

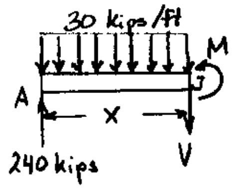
PROBLEM 5.154

5.154 Two cover plates, each $\frac{5}{8}$ -in. thick, are welded to a W30 × 99 beam as shown. Knowing that l = 9 ft and b = 12 in., determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



SOLUTION

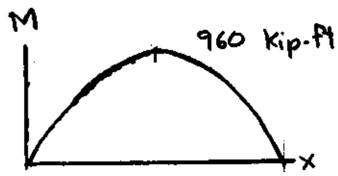
$R_A = R_B = 240$ kips



$\circlearrowleft \Sigma M_J = 0$

$-240x + 30x\frac{x}{2} + M = 0$

$M = 240x - 15x^2 \quad$ kip·ft



At center of beam $\quad x = 8$ ft

$M_c = 960$ kip·ft $= 11520$ kip·in

At point D, $x = \frac{1}{2}(16 - 9) = 3.5$ ft

$M_D = 656.25$ kip·ft $= 7875$ kip·in.

At center of beam $\quad I = I_{beam} + 2I_{plate}$

$$I = 3990 + 2\left\{(12)(0.625)\left(\frac{29.65}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(12)(0.625)^3\right\} = 7428 \text{ in}$$

$$c = \frac{29.65}{2} + 0.625 = 15.45 \text{ in}$$

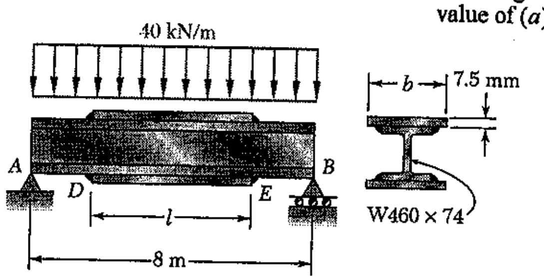
Normal stress $\quad \sigma = \frac{Mc}{I} = \frac{(11520)(15.45)}{7428} = 24.0$ ksi ◄

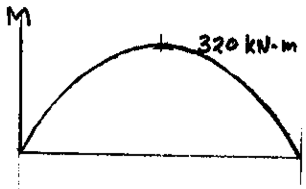
At point D $\quad S = 269$ in³

Normal stress $\quad \sigma = \frac{M}{S} = \frac{7875}{269} = 29.3$ ksi ◄

PROBLEM 5.155

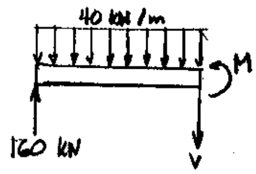
5.155 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $\sigma_{all} = 150$ MPa for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.





SOLUTION

$R_A = R_B = 160$ kN



$+\circlearrowleft \Sigma M_J = 0$

$-160x + (40x)\left(\frac{x}{2}\right) + M = 0$

$M = 160x - 20x^2$ kN·m

For W 460×74 rolled steel beam

$S = 1460 \times 10^3 \text{ mm}^3 = 1460 \times 10^{-6} \text{ m}^3$

Allowable bending moment

$M_{all} = \sigma_{all} S = (150 \times 10^6)(1460 \times 10^{-6})$

$= 219 \times 10^3 \text{ N·m} = 219 \text{ kN·m}$

To locate points D and E, set $M = M_{all}$

$160x - 20x^2 = 219 \qquad 20x^2 - 160x + 219 = 0$

$$x = \frac{160 \pm \sqrt{160^2 - (4)(20)(219)}}{(2)(20)} = 1.753 \text{ m}, \quad 6.247 \text{ m}$$

$x_D = 1.753$ ft. $\quad x_E = 6.247$ ft $\quad l = x_E - x_D = 4.49$ m ◄

At center of beam $\quad M = 320 \text{ kN·m} = 320 \times 10^3 \text{ N·m}$

$$S = \frac{M}{\sigma_{all}} = \frac{320 \times 10^3}{150 \times 10^6} = 2133 \times 10^{-6} \text{ m}^3 = 2133 \times 10^3 \text{ mm}^3$$

$c = \frac{457}{2} + 7.5 = 236 \text{ mm}^4$

Required moment of inertia $\quad I = Sc = 503.4 \times 10^6 \text{ mm}^4$

But $\quad I = I_{beam} + 2I_{plate}$

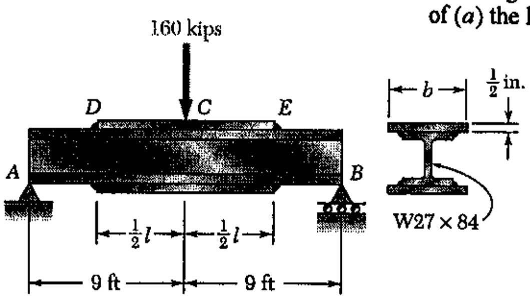
$$503.4 \times 10^6 = 333 \times 10^6 + 2\left\{(b)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(b)(7.5)^3\right\}$$

$$= 333 \times 10^6 + 809.2 \times 10^3\, b$$

$b = 211$ mm ◄

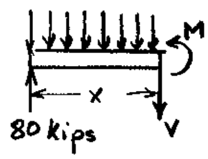
PROBLEM 5.156

5.156 Two cover plates, each $\frac{1}{2}$-in. thick, are welded to a W27 × 84 beam as shown. Knowing that $\sigma_{all} = 24$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.



SOLUTION

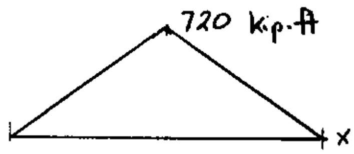
$R_A = R_B = 80$ kips



$+\circlearrowleft \Sigma M_J = 0$

$-80x + M = 0$

$M = 80x$ kip·ft



At D $\quad S = 213$ in^3

Allowable bending moment

$M_{all} = \sigma_{all} S = (24)(213) = 5112$ kip·in

$= 426$ kip.

Set $M_D = M_{all}$ $\quad 80x_D = 426 \quad x_D = 5.325$ ft.

$l = 18 - 2x_D = 7.35$ ft

At center of beam $\quad M = (80)(9) = 720$ kip·ft $= 8640$ kip·in.

$$S = \frac{M}{\sigma_{all}} = \frac{8640}{24} = 360 \text{ in}^3$$

$$c = \frac{26.71}{2} + 0.500 = 13.855 \text{ in}$$

Required moment of inertia $\quad I = Sc = 4987.8$ in^4

But $\quad I = I_{beam} + 2I_{plate}$

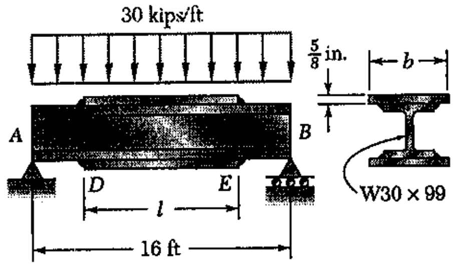
$$4987.8 = 2850 + 2\left\{(b)(0.500)\left(\frac{26.71}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(b)(0.500)^3\right\}$$

$$= 2850 + 185.12\, b$$

$b = 11.55$ in.

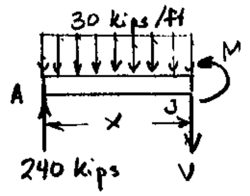
PROBLEM 5.157

5.157 Two cover plates, each $\frac{5}{8}$-in. thick, are welded to a W30 × 99 beam as shown. Knowing that $\sigma_{all} = 22$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.



SOLUTION

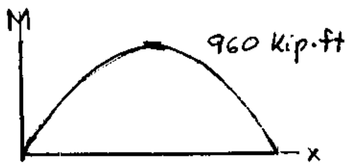
$R_A = R_B = 240$ kips



$\circlearrowleft \Sigma M_J = 0$

$-240x + 30x \frac{x}{2} + M = 0$

$M = 240x - 15x^2$ kip·ft



For W 30 × 99 rolled steel section

$S = 269 \text{ in}^3$

Allowable bending moment

$M_{all} = \sigma_{all} S = (22)(269) = 5918$ kip·in

$= 493.167$ kip·ft

To locate points D and E, set $M = M_{all}$

$240x - 15x^2 = 493.167 \qquad 15x^2 - 240x + 493.167 = 0$

$$x = \frac{240 \pm \sqrt{(240)^2 - (4)(15)(493.167)}}{(2)(15)} = 2.42 \text{ ft}, \ 13.58 \text{ ft}$$

$l = x_E - x_D = 13.58 - 2.42 = 11.16$ ft ◄

Center of beam $\qquad M = 960$ kip·ft $= 11520$ kip·in

$$S = \frac{M}{\sigma_{all}} = \frac{11520}{22} = 523.64 \text{ in}^3$$

$$c = \frac{29.65}{2} + 0.625 = 15.45 \text{ in}$$

Required moment of inertia $\qquad I = Sc = 8090 \text{ in}^4$

But $\quad I = I_{beam} + 2 I_{plate}$

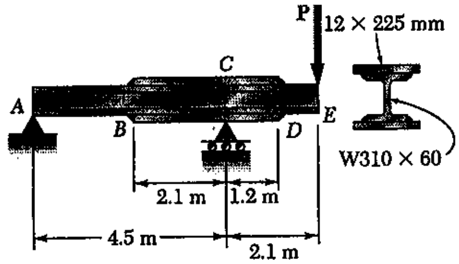
$$8090 = 3990 + 2\left\{(b)(0.625)\left(\frac{29.65}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(b)(0.625)^3\right\}$$

$$= 3990 + 286.47\, b$$

$b = 14.31$ in ◄

PROBLEM 5.158

be applied at end E of the beam shown. Neglect the weights of the beam and of the plates.



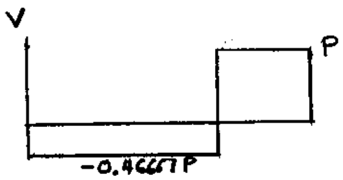
SOLUTION

$+\circlearrowleft \Sigma M_C = 0 \quad -4.5 R_A - 2.1 P = 0$

$R_A = -0.46667 P \quad \text{ie } 0.46667 P \downarrow$

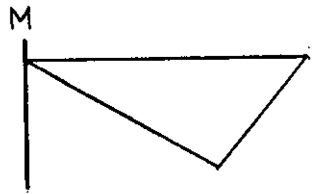
$+\circlearrowleft \Sigma M_A = 0 \quad 4.5 R_C - 6.6 P = 0$

$R_C = 1.46667 P$



Shear: A to C $\quad V = -0.46667 P$

C to E $\quad V = P$



Bending moments:

$M_C = -(4.5)(0.46667 P) = -2.10 P \text{ kN}\cdot\text{m}$

$M_B = \frac{2.4}{4.5} M_C = -1.12 P \text{ kN}\cdot\text{m}$

$M_D = \frac{0.9}{2.1} M_C = -0.9 P \text{ kN}\cdot\text{m}$

At B and D $\quad S = 851 \times 10^3 \text{ mm}^3 = 851 \times 10^{-6} \text{ m}^3$

$\sigma_{all} = \frac{|M|}{S} = \frac{1.120 P_{all}}{851 \times 10^{-6}} = 165 \times 10^6 \quad$ at B

$P_{all} = 125.4 \text{ kN}$

At C $\quad I = I_{beam} + 2 I_{plate}$

$= 129 \times 10^6 + 2\left\{(225)(12)\left(\frac{310}{2} + \frac{12}{2}\right)^2 + \frac{1}{12}(225)(12)^3\right\}$

$= 269 \times 10^6 \text{ mm}^3$

$c = \frac{310}{2} + 12 = 167 \text{ mm} \qquad S = \frac{I}{c} = 1611 \times 10^3 \text{ mm}^3$

$= 1611 \times 10^{-6} \text{ m}^3$

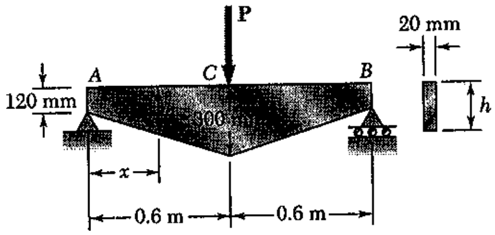
$\sigma_{all} = \frac{|M|}{S} = \frac{2.10 P}{1611 \times 10^{-6}} = 165 \times 10^6$

$P_{all} = 126.6 \text{ kN}$

Allowable load is the smaller value $\quad P = 125.4 \text{ kN}$

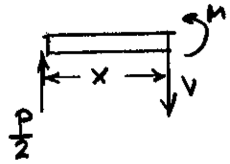
PROBLEM 5.159

5.159 For the tapered beam shown, and knowing that P = 150 kN, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the corresponding value of the normal stress.



SOLUTION

$R_A = R_B = \frac{P}{2}$



$\circlearrowleft \Sigma M_J = 0 \qquad -\frac{Px}{2} + M = 0$

$M = \frac{Px}{2} \qquad (0 < x < \frac{L}{2})$

For a tapered rectangular beam $\quad h = a + kx \qquad (0 < x < \frac{L}{2})$

$$S = \frac{1}{6} b h^2 = \frac{1}{6} b (a + kx)^2$$

Bending stress $\quad \sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3P}{b} \frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b} \frac{(a+kx)^2 - x \cdot 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b} \cdot \frac{a - kx}{(a+kx)^3} = 0 \qquad x_m = \frac{a}{k}$$

Data: $\quad a = 120$ mm, $\quad k = \frac{300 - 120}{0.6} = 300$ mm/m

$$x_m = \frac{120}{300} = 0.400 \text{ m}$$ ◄

$$M_m = \frac{Px_m}{2} = \frac{(150)(0.400)}{2} = 30 \text{ kN} \cdot \text{m}$$

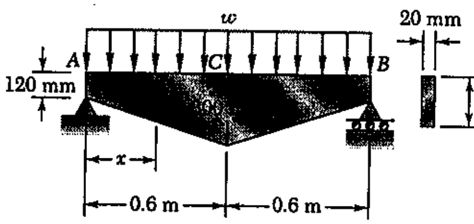
$$h_m = a + kx_m = 120 + (300)(0.400) = 240 \text{ mm}$$

$$S_m = \frac{1}{6} b h_m^2 = (\tfrac{1}{6})(20)(240)^2 = 192 \times 10^3 \text{ mm}^3 = 192 \times 10^{-6} \text{ m}^3$$

$$\sigma_m = \frac{M_m}{S_m} = \frac{30 \times 10^3}{192 \times 10^{-6}} = 156.3 \times 10^6 \text{ Pa} = 156.3 \text{ MPa}$$ ◄

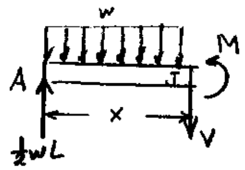
PROBLEM 5.160

5.160 For the tapered beam shown, and knowing that $w = 160$ kN/m, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the corresponding value of the normal stress.



SOLUTION

$R_A = R_B = \frac{1}{2}wL$

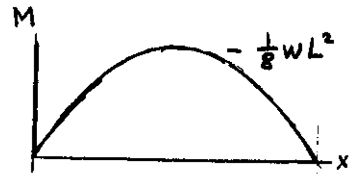


$\circlearrowleft \Sigma M_J = 0$

$-\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2}x(L - x)$



where $w = 160$ kN/m and $L = 1.2$ m.

For the tapered beam $h = a + kx$

$a = 120$ mm $\qquad k = \frac{300 - 120}{0.6} = 300$ mm/m

For a rectangular cross section $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a + kx)^2$

Bending stress $\sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a + kx)^2(L - 2x) - (Lx - x^2)2(a + kx)k}{(a + kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - 2ax - kLx}{(a + kx)^3}\right\} = 0$$

$$x_m = \frac{aL}{2a + kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.24 \text{ m}$$ ◄

$h_m = a + kx_m = 120 + (300)(0.24) = 192$ mm

$S_m = \frac{1}{6}bh_m^2 = \frac{1}{6}(20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$

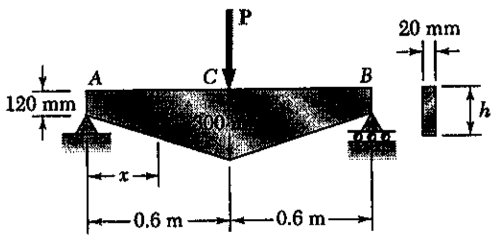
$M_m = \frac{w}{2}x_m(L - x_m) = \frac{160 \times 10^3}{2}(0.24)(0.96) = 18.432 \times 10^3$ N·m

Maximum bending stress $\sigma_m = \frac{M_m}{S_m} = \frac{18.432 \times 10^3}{122.88 \times 10^{-6}} = 150 \times 10^6$ Pa

$= 150$ MPa ◄

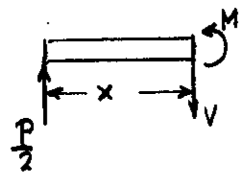
PROBLEM 5.161

5.161 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest concentrated load **P** that can be applied, knowing that $\sigma_{all} = 140$ MPa.



SOLUTION

$$R_A = R_B = \frac{P}{2}$$



$$\circlearrowleft \Sigma M_J = 0$$

$$-\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \qquad (0 < x < \tfrac{L}{2})$$

For a tapered beam $\quad h = a + kx$

For rectangular cross section $\quad S = \frac{1}{6} b h^2 = \frac{1}{6} b (a + kx)^2$

Bending stress $\quad \sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3P}{b}\frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b}\cdot\frac{(a+kx)^2 - x\cdot 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b}\,\frac{a-kx}{(a+kx)^3} = 0 \qquad x_m = \frac{a}{k}$$

Then $\quad M_m = \frac{Px_m}{2} = \frac{Pa}{2k}$

$$h_m = a + kx_m = 2a$$

$$S_m = \frac{1}{6} b h_m^2 = \frac{2}{3} b a^2$$

Data: $\quad a = 120$ mm $\quad k = \frac{300 - 120}{0.6} = 300$ mm/m, $\quad b = 20$ mm

$$x_m = \frac{120 \text{ mm}}{300 \text{ mm/m}} = 0.400 \text{ m} \quad \blacktriangleleft$$

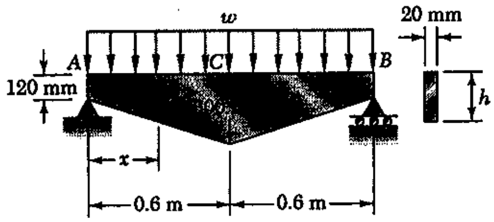
$$S_m = \frac{2}{3}(20)(120)^2 = 192 \times 10^3 \text{ mm}^3 = 192 \times 10^{-6} \text{ m}^3$$

$$M_m = \sigma_{all} S_m = (140 \times 10^6)(192 \times 10^{-6}) = 26.88 \times 10^3 \text{ N}\cdot\text{m}$$

$$P = \frac{2M_m}{x_m} = \frac{(2)(26.88 \times 10^3)}{0.400} = 134.4 \times 10^3 \text{ N} = 134.4 \text{ kN} \quad \blacktriangleleft$$

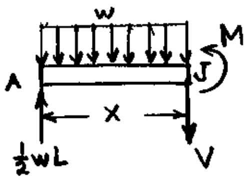
PROBLEM 5.162

5.162 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load w that can be applied, knowing that $\sigma_{all} = 140$ MPa.



SOLUTION

$R_A = R_B = \frac{1}{2} wL$ $\qquad L = 1.2$ m

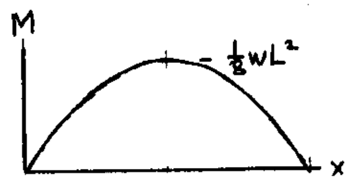


$\circlearrowleft \Sigma M_J = 0$

$-\frac{1}{2}wL + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2} x(L-x)$



For the tapered beam $\quad h = a + kx$

$a = 120$ mm $\qquad k = \frac{300-120}{0.6} = 300$ mm/m

For rectangular cross section $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress $\quad \sigma = \frac{M}{S} = \frac{3w}{b}\frac{Lx - x^2}{(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a+kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx-x^2)2(a+kx)k}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - \cancel{2kx^2} - 2kLx + \cancel{2kx^2}}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a+kL)x}{(a+kx)^3}\right\} = 0$$

$$x_m = \frac{aL}{2a+kL} = \frac{(120)(1.2)}{(2)(120)+(300)(1.2)} = 0.24 \text{ m}$$

$h_m = a + kx_m = 120 + (300)(0.24) = 192$ mm

$S_m = \frac{1}{6}bh_m^2 = \frac{1}{6}(20)(192)^2 = 122.88\times10^3 \text{ mm}^3 = 122.88\times10^{-6} \text{ m}^3$

Allowable value of M_m $\quad M_m = S_m\sigma_{all} = (122.88\times10^{-6})(140\times10^6)$

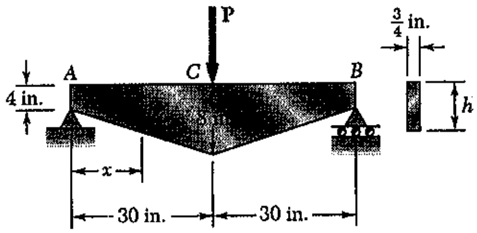
$= 17.2032\times10^3$ N·m

Allowable value of w $\quad w = \frac{2M_m}{x_m(L-x_m)} = \frac{(2)(17.2032\times10^3)}{(0.24)(0.96)}$

$= 149.3\times10^3$ N/m $= 149.3$ kN/m

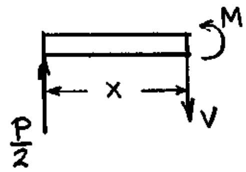
PROBLEM 5.163

5.163 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest concentrated load **P** that can be applied, knowing that $\sigma_{all} = 24$ ksi.



SOLUTION

$$R_A = R_B = \frac{P}{2}$$



$$\circlearrowleft \Sigma M_J = 0$$

$$-\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \qquad \left(0 < x < \frac{L}{2}\right)$$

For a tapered beam $\quad h = a + kx$

For a rectangular cross section $\quad S = \frac{1}{6} b h^2 = \frac{1}{6} b (a + kx)^2$

Bending stress $\quad \sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3P}{b}\frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b}\,\frac{(a+kx)^2 - x \cdot 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b}\,\frac{a - kx}{(a+kx)^3} = 0 \qquad x_m = \frac{a}{k}$$

Data: $\quad a = 4$ in., $\quad k = \frac{8-4}{30} = 0.13333$ in/in

$$x_m = \frac{4}{0.13333} = 30 \text{ in.}$$

$$h_m = a + kx_m = 8 \text{ in}$$

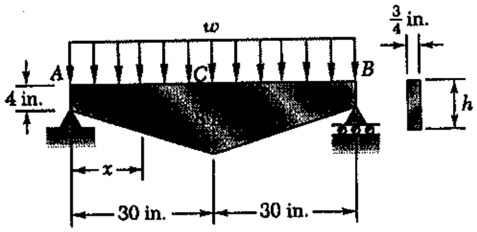
$$S_m = \frac{1}{6} b h_m^2 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(8)^2 = 8 \text{ in}^3$$

$$M_m = \sigma_{all} S_m = (24)(8) = 192 \text{ kip}\cdot\text{in}$$

$$P = \frac{2M_m}{x_m} = \frac{(2)(192)}{30} = 12.8 \text{ kips}$$

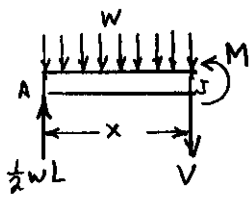
PROBLEM 5.164

5.164 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load *w* that can be applied, knowing that $\sigma_{all} = 24$ ksi



SOLUTION

$R_A = R_B = \frac{1}{2} wL \qquad L = 60$ in.

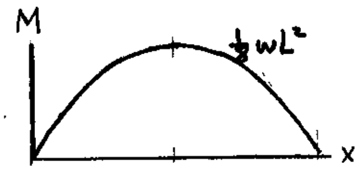


$\circlearrowleft \Sigma M_J = 0$

$-\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2}x(L-x)$



For the tapered beam $\qquad h = a + kx$

$a = 4$ in $\qquad k = \frac{8-4}{30} = \frac{2}{15}$ in./in.

For a rectangular cross section $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress $\quad \sigma = \frac{M}{S} = \frac{3w}{b}\cdot\frac{Lx - x^2}{(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a+kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x)-(Lx-x^2)2(a+kx)k}{(a+kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a + kL)x}{(a+kx)^3}\right\} = 0$$

$$x_m = \frac{aL}{2a + kL} = \frac{(4)(60)}{(2)(4)+(\frac{2}{15})(60)} = 15 \text{ in.}$$

$h_m = a + kx_m = 4 + (\frac{2}{15})(15) = 6.00$ in.

$S_m = \frac{1}{6}bh_m^2 = (\frac{1}{6})(\frac{3}{4})(6.00)^2 = 4.50 \text{ in}^3$

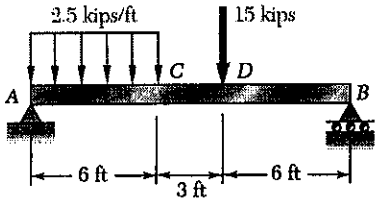
Allowable value of $M_m = S_m\sigma_{all} = (4.50)(24) = 180.0$ kip·in

Allowable value of $w \qquad w = \frac{2M_m}{x_m(L-x_m)} = \frac{(2)(108.0)}{(15)(45)} = 0.320$ kip/in

$= 320$ lb/in.

PROBLEM 5.165

5.165 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



SOLUTION

$+\circlearrowleft \Sigma M_B = 0$

$-15 R_A + (12)(6)(2.5) + (6)(15) = 0$

$R_A = 18$ kips

$+\circlearrowleft \Sigma M_A = 0$

$15 R_B - (3)(6)(2.5) - (9)(15) = 0$

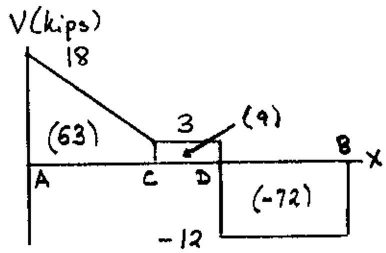
$R_B = 12$ kips

Shear: $V_A = 18$ kips

$V_C = 18 - (6)(2.5) = 3$ kips

C to D $V = 3$ kips

D to B $V = 3 - 15 = -12$ kips

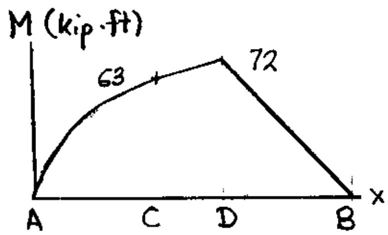


Areas under shear diagram

A to C $\int V dx = (\frac{1}{2})(6)(18+3) = 63$ kip·ft

C to D $\int V dx = (3)(3) = 9$ kip·ft

D to B $\int V dx = (6)(-12) = -72$ kip·ft



Bending moments: $M_A = 0$

$M_C = 0 + 63 = 63$ kip·ft

$M_D = 63 + 9 = 72$ kip·ft

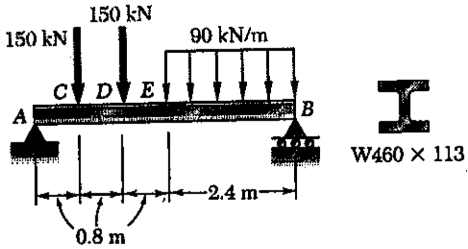
$M_B = 72 - 72 = 0$

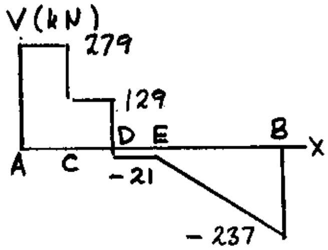
$|V|_{max} = 18$ kips ◄

$|M|_{max} = 72$ kip·ft ◄

PROBLEM 5.166

5.166 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *D*.





SOLUTION

$\circlearrowleft \Sigma M_B = 0$

$-4.8 R_A + (4.0)(150) + (3.2)(150) + (1.2)(2.4)(90) = 0$

$R_A = 279 \text{ kN}$

$\circlearrowleft \Sigma M_A = 0$

$4.8 R_B - (0.8)(150) - (1.6)(150) - (3.6)(2.4)(90) = 0$

$R_B = 237 \text{ kN}$

Shear:
- A to C $\quad V = 279$
- C to D $\quad V = 279 - 150 = 129 \text{ kN}$
- D to E $\quad V = 129 - 150 = -21 \text{ kN}$
- $V_E = -21 \text{ kN}$
- $V_B = -21 - (2.4)(90) = -237 \text{ kN}$

Areas under shear diagram
- A to C $\quad \int V dx = (0.8)(279) = 223.2 \text{ kN}\cdot\text{m}$
- C to D $\quad \int V dx = (0.8)(129) = 103.2 \text{ kN}\cdot\text{m}$

Maximum bending moment occurs at point D where V changes sign.

$$M_D = M_A + \int_A^D V dx$$

$$= 0 + 223.2 + 103.2 = 326.4 \text{ kN}\cdot\text{m}$$

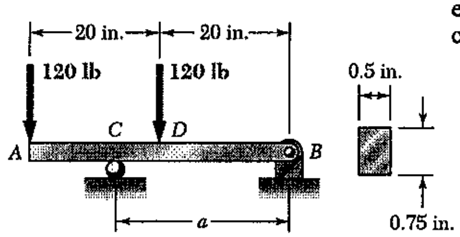
For rolled steel section W 460 × 113

$S = 2400 \times 10^3 \text{ mm}^3 = 2400 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{326.4 \times 10^3}{2400 \times 10^{-6}} = 136.0 \times 10^6 \text{ Pa}$

$= 136.0 \text{ MPa}$ ◄

PROBLEM 5.167



5.167 Determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

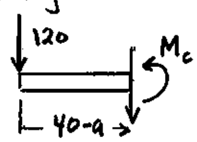
SOLUTION

$$\circlearrowleft \Sigma M_B = 0$$

$$-R_C\,a + (40)(120) + (20)(120) = 0$$

$$R_C = \frac{7200}{a}$$

Bending moment at C

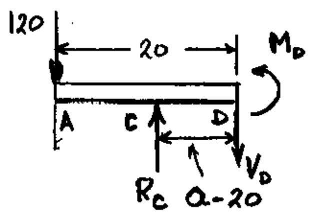


$$\circlearrowleft \Sigma M_C = 0$$

$$M_C + 120(40-a) = 0$$

$$M_C = -4800 + 120\,a$$

Bending moment at D



$$\circlearrowleft \Sigma M_D = 0$$

$$M_D + (20)(120) - R_c(a-20) = 0$$

$$M_D = R_C(a-20) - 2400$$

$$= R_C a - 20\,R_C - 2400$$

$$= 7200 - \frac{(20)(7200)}{a} - 2400$$

$$= 4800 - \frac{144000}{a}$$

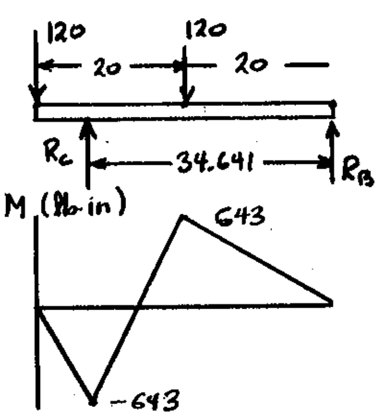
Equate $-M_C = M_D$

$$4800 - 120\,a = 4800 - \frac{144000}{a}$$

$$120\,a^2 = 144000 \qquad a = 34.641 \text{ in.} \blacktriangleleft$$

$$M_C = -4800 + (120)(34.641) = -643.08 \text{ lb·in}$$

$$M_D = 4800 - \frac{144000}{34.641} = 643.08 \text{ lb·in} \checkmark$$



For rectangular section $S = \frac{1}{6}bh^2$

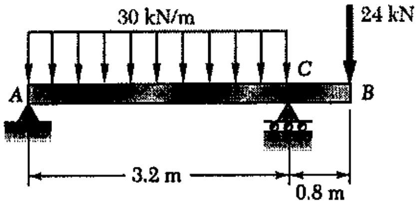
$$S = \left(\tfrac{1}{6}\right)(0.5)(0.75)^2 = 0.046875 \text{ in}^3$$

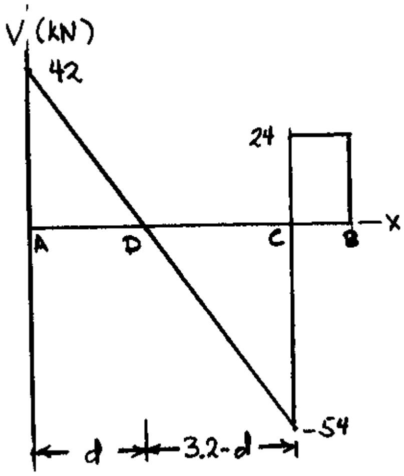
Maximum normal stress

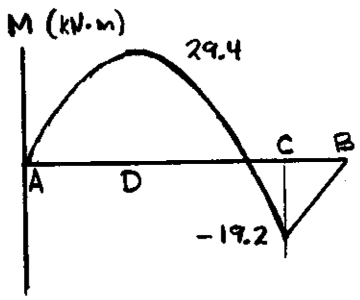
$$\sigma_{max} = \frac{M_{max}}{S} = \frac{643.08}{0.046875} = 13.72\times10^3 \text{ psi}$$

$$= 13.72 \text{ ksi} \blacktriangleleft$$

PROBLEM 5.168







5.168 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0$

$-3.2 R_A + (1.6)(3.2)(30) - (0.8)(24) = 0$

$R_A = 42 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$

$3.2 R_C - (1.6)(3.2)(30) - (4.0)(24) = 0$

$R_C = 78 \text{ kN}$

Shear: $V_A = 42 \text{ kN}$

$V_{C^-} = 42 - (3.2)(30) = -54 \text{ kN}$

$V_{C^+} = -54 + 78 = 24 \text{ kN}$

C to D $\quad V = 24 \text{ kN}.$

Locate point D where $V = 0$

$$\frac{d}{42} = \frac{3.2 - d}{54} \qquad 96d = 134.4$$

$d = 1.4 \text{ m} \qquad 3.2 - d = 1.8 \text{ m}$

Areas under shear diagram

A to D $\quad \int V dx = (\frac{1}{2})(1.4)(42) = 29.4 \text{ kN·m}$

D to C $\quad \int V dx = (\frac{1}{2})(-1.8)(54) = -48.6 \text{ kN·m}$

C to B $\quad \int V dx = (0.8)(24) = 19.2 \text{ kN·m}$

Bending moments: $\quad M_A = 0$

$M_D = 0 + 29.4 = 29.4 \text{ kN·m}$

$M_C = 29.4 - 48.6 = -19.2 \text{ kN·m}$

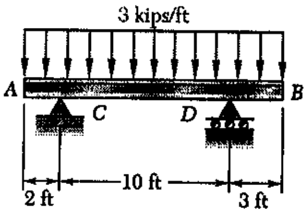
$M_B = 19.2 - 19.2 = 0 \quad$ checks

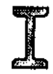
Maximum $|V| = 54 \text{ kN}$

Maximum $|M| = 29.4 \text{ kN-m}$

PROBLEM 5.169

5.169 For the beam and loading shown, determine (*a*) the maximum value of the bending moment, (*b*) the maximum normal stress due to bending.





S10 × 25.4

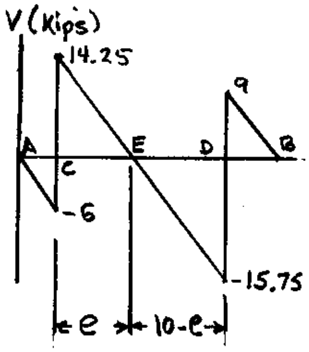
SOLUTION

$+\circlearrowleft \Sigma M_D = 0 \qquad -10R_C + (4.5)(15)(3) = 0$

$R_C = 20.25 \text{ kips}$

$+\circlearrowleft \Sigma M_C = 0 \qquad 10R_D - (5.5)(15)(3) = 0$

$R_D = 24.75 \text{ kips}$



Shear:

$V_A = 0$

$V_{C^-} = 0 - (2)(3) = -6 \text{ kips}$

$V_{C^+} = -6 + 20.25 = 14.25 \text{ kips}$

$V_{D^-} = 14.25 - (10)(3) = -15.75 \text{ kips}$

$V_{D^+} = -15.75 + 24.75 = 9 \text{ kips}$

$V_B = 9 - (3)(3) = 0$ checks

Locate point E where $V = 0$

$$\frac{e}{14.25} = \frac{10-e}{15.75} \qquad 30e = 142.5$$

$e = 4.75 \text{ ft.} \qquad 10 - e = 5.25 \text{ ft}$

Areas under shear diagram

A to C $\quad \int V\,dx = (\tfrac{1}{2})(2)(-6) = -6 \text{ kip·ft}$

C to E $\quad \int V\,dx = (\tfrac{1}{2})(4.75)(14.25) = 33.84 \text{ kip·ft}$

E to D $\quad \int V\,dx = (\tfrac{1}{2})(5.25)(15.75) = -41.34 \text{ kip·ft}$

D to B $\quad \int V\,dx = (\tfrac{1}{2})(3)(9) = 13.5 \text{ kip·ft}$

Bending moments: $\quad M_A = 0$

$M_C = 0 - 6 = -6 \text{ kip·ft}$

$M_E = -6 + 33.84 = 27.84 \text{ kip·ft}$

$M_D = 27.84 - 41.34 = -13.5 \text{ kip·ft}$

$M_B = -13.5 + 13.5 = 0$ checks

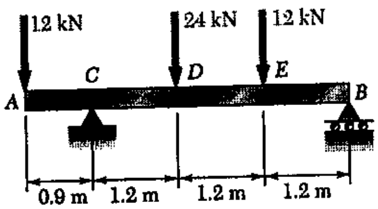
Maximum $|M| = 27.84 \text{ kip·ft}$ ◄

$= 334.1 \text{ kip·in}$

For rolled steel section S10 × 25.4 $\qquad S = 24.7 \text{ in}^3$

Maximum normal stress $\qquad \sigma_{max} = \dfrac{M_{max}}{S} = \dfrac{334.1}{24.7} = 13.53 \text{ ksi}$ ◄

PROBLEM 5.170



5.170 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$+\circlearrowleft \Sigma M_B = 0$

$(4.5)(12) - 3.6 R_c + (2.4)(24) + (1.2)(12) = 0$

$R_c = 35 \text{ kN}$

$V = -12 + 35\langle x-0.9\rangle^0 - 24\langle x-2.1\rangle^0 - 12\langle x-3.3\rangle^0 \text{ kN}$ ◄

$M = -12x + 35\langle x-0.9\rangle^1 - 24\langle x-2.1\rangle^1 - 12\langle x-3.3\rangle^1 \text{ kN·m}$ ◄

Pt. C $(x = 0.9 \text{ m})$ $M_C = -(12)(0.9) = -10.8 \text{ kN·m}$

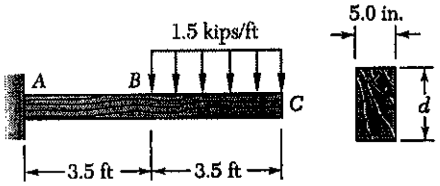
Pt D $(x = 2.1 \text{ m})$ $M_D = -(12)(2.1) + 35(1.2) = 16.8 \text{ kN·m}$

Pt E $(x = 3.3 \text{ m})$ $M_E = -(12)(3.3) + (35)(2.4) - (24)(1.2)$
$= 15.6 \text{ kN·m}$ ◄

Maximum $|M| = 16.8 \text{ kN·m}$ ◄

PROBLEM 5.171

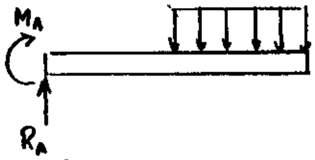
5.171 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1.75 ksi.



SOLUTION

$+\uparrow \Sigma F_y = 0 \qquad R_A - (3.5)(1.5) = 0$

$R_A = 5.25 \text{ kN}$

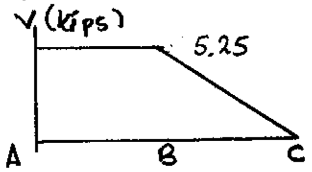


$+\circlearrowleft \Sigma M_A = 0 \qquad -M_A - (5.25)(3.5)(1.5) = 0$

$M_A = -27.56 \text{ kN}$

Shear: A to B $\quad V = 5.25$ kips

$V_C = 5.25 - (3.5)(1.5) = 0$ checks



Areas of shear diagram

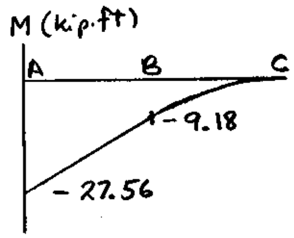
A to B $\quad \int V dx = (3.5)(5.25) = 18.38$ kip·ft

B to C $\quad \int V dx = (\frac{1}{2})(3.5)(5.25) = 9.19$ kip ft

Bending moments $\quad M_A = -27.56$ kip·ft

$M_B = -27.56 + 18.38 = -9.18$

$M_C = -9.18 + 9.19 \approx 0$ checks.



Maximum $|M| = 27.56$ kip·ft $= 330.7$ kip·in

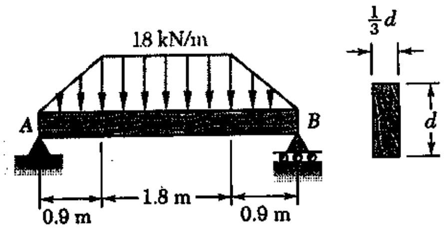
$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{330.7}{1.75} = 189.0 \text{ in}^3$$

For a rectangular cross section $\quad S = \frac{1}{6} b d^2 \qquad d = \sqrt{\frac{6S}{b}}$

$$d = \sqrt{\frac{(6)(189.0)}{5}} = 15.06 \text{ in.}$$

PROBLEM 5.172

5.172 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



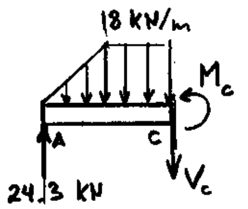
SOLUTION

By symmetry $R_A = R_B$ $\qquad +\sum F_y = 0$

$$R_A - (\tfrac{1}{2})(0.9)(18) - (1.8)(18) - (\tfrac{1}{2})(0.9)(18) + R_B = 0$$

$$R_A = R_B = 24.3 \text{ kN}$$

Maximum bending moment occurs at the center



$$+\circlearrowleft \sum M_c = 0$$

$$-(1.8)(24.3) + (1.2)(\tfrac{1}{2})(0.9)(18) + (0.45)(0.9)(18) + M_c = 0$$

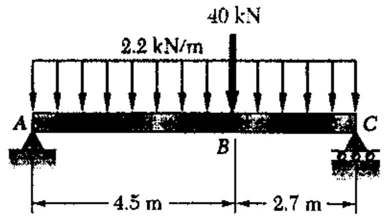
$$M_c = 26.73 \text{ kN}\cdot\text{m} = 26.73 \times 10^3 \text{ N/m}$$

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{26.73 \times 10^3}{12 \times 10^6} = 2.2275 \times 10^{-3} \text{ m}^3 = 2227.5 \times 10^3 \text{ mm}^3$$

For a rectangular section $\quad S = \frac{1}{6}bd^2 = \frac{1}{6}(\frac{1}{3}d)d^2 = \frac{1}{18}d^3$

$$\frac{1}{18}d^3 = 2227.5 \times 10^3 \qquad d^3 = 40.09 \times 10^6 \qquad d = 342 \text{ mm}$$

PROBLEM 5.173



5.173 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric wide-flange beam to support the loading shown.

SOLUTION

$\circlearrowleft \Sigma M_C = 0$

$-7.2 R_A + (3.6)(7.2)(2.2) + (2.7)(40) = 0$

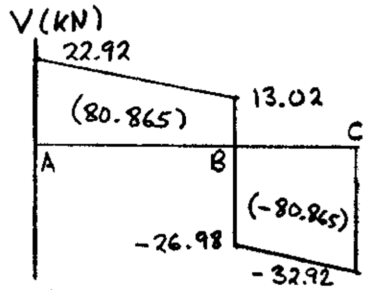
$R_A = 22.92 \text{ kN}$

Shear: $V_A = 22.92 \text{ kN}$

$V_{B^-} = 22.92 - (4.5)(2.2) = 13.02 \text{ kN}$

$V_{B^+} = 13.02 - 40 = -26.98 \text{ kN}$

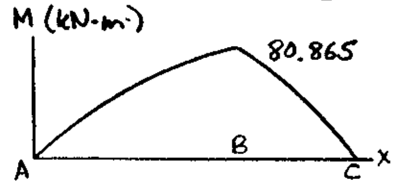
$V_C = -26.98 - (2.7)(2.2) = -32.92 \text{ kN}$



Areas under shear diagram

A to B $\int V\,dx = (\frac{1}{2})(4.5)(22.92 + 13.02) = 80.865 \text{ kN}\cdot\text{m}$

B to C $\int V\,dx = (\frac{1}{2})(2.7)(-26.98 - 32.92) = -80.865 \text{ kN}\cdot\text{m}$



Bending moments: $M_A = 0$

$M_B = 0 + 80.865 = 80.865 \text{ kN-m}$

$M_C = 80.865 - 80.865 = 0$ checks

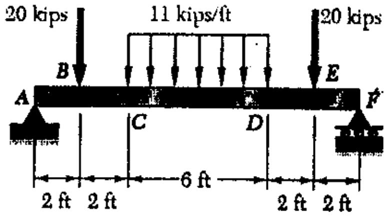
$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{80.865 \times 10^3}{160 \times 10^6} = 505 \times 10^{-6} \text{ m}^3 = 505 \times 10^3 \text{ mm}^3$$

Shape	$S\ (10^3 \text{mm}^3)$
W 410 × 38.8	637
W 360 × 39	578
W 310 × 38.7	549 ←
W 250 × 44.8	535
W 200 × 52	512

Use W 310 × 38.7 ◄

PROBLEM 5.174

5.174 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



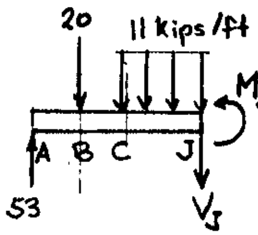
SOLUTION

By symmetry $R_A = R_F$

$+\uparrow \Sigma F_y = 0 \quad R_A - 20 - (6)(11) - 20 + R_F = 0$

$R_A = R_F = 50 \text{ kips.}$

Maximum bending moment occurs at center of beam.



$\circlearrowleft \Sigma M_J = 0 \quad -(7)(53) + (5)(20) + (1.5)(3)(11) + M_J = 0$

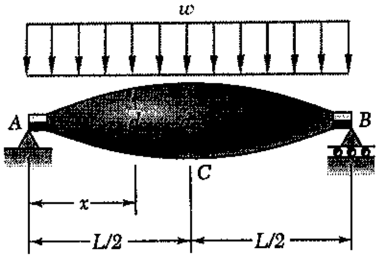
$M_J = 221.5 \text{ kip}\cdot\text{ft} = 2658 \text{ kip}\cdot\text{in}$

$S_{min} = \dfrac{M_{max}}{\sigma_{all}} = \dfrac{2658}{24} = 110.75 \text{ in}^3$

Shape	S (in^3)
W 24 × 68	154
W 21 × 62	127
W 18 × 76	146
W 16 × 77	134
W 14 × 82	123
W 12 × 96	131

Use W 21 × 62

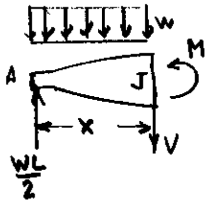
PROBLEM 5.175



5.175 A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support a distributed load w as shown. (*a*) Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0. (*b*) Determine the smallest allowable value of d_0 if $L = 250$ mm, $w = 30$ kN/m, and $\sigma_{all} = 72$ MPa.

SOLUTION

$$R_A = R_B = \frac{wL}{2}$$



$$+\circlearrowleft \Sigma M_J = 0$$

$$-\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$$

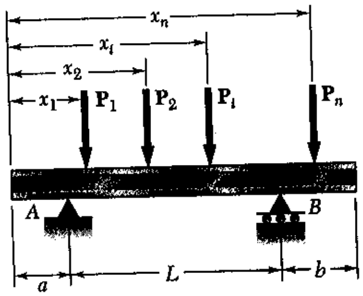
For a solid circular cross section $c = \frac{d}{2}$ $\quad I = \frac{\pi}{4}c^3$ $\quad S = \frac{I}{c} = \frac{\pi d^3}{32}$

Equating $\quad \frac{\pi d^3}{32} = \frac{wx(L-x)}{2\sigma_{all}} \qquad d = \left\{\frac{16\,wx(L-x)}{\pi\sigma_{all}}\right\}^{1/3}$

At $x = \frac{L}{2} \qquad d = d_0 = \left\{\frac{4wL^2}{\pi\sigma_{all}}\right\}^{1/3} \qquad d = d_0\left\{4\frac{x}{L}\left(1-\frac{x}{L}\right)\right\}^{1/3}$ ◄

Using the data $\quad d_0 = \frac{(4)(30\times10^3)(0.250)^2}{\pi(72\times10^6)} = 32.1\times10^{-3}\text{ m} = 32.1\text{ mm}$ ◄

PROBLEM 5.C1



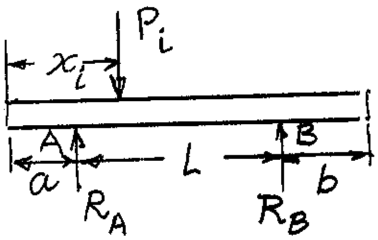
5.C1 Several concentrated loads P_i $(i = 1, 2, \ldots, n)$ can be applied to a beam as shown. Write a computer program that can be used to calculate the shear, bending moment, and normal stress at any point of the beam for a given loading of the beam and a given value of its section modulus. Use this program to solve Probs. 5.23, 5.27, and 5.29. (*Hint:* Maximum values will occur at a support or under a load.)

SOLUTION

REACTIONS AT A AND B

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - \sum_i P_i (x_i - a)$$

$$R_B = (1/L) \sum_i P_i (x_i - a)$$



$$R_A = \sum_i P_i - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text.)

WE DEFINE: IF $x \ge a$ THEN STPA=1 ELSE STPA = 0

IF $x \ge a+L$ THEN STPB=1 ELSE STPB=0

IF $x \ge x_i$ THEN STP(I)=1 ELSE STP(I)=0

$$V = R_A \, STPA + R_B \, STPB - \sum P_i \, STP(I)$$

$$M = R_A (x-a) STPA + R_B (x-a-L) STPB - \sum_i P_i (x - x_i) STP(I)$$

$\sigma = M/S$, where S is obtained from Appendix C.

PROGRAM OUTPUTS

Prob. 5.23

RA=80.0 kN RB=80.0 kN

X m	V kN	M kN.m	Sigma MPa
2.00	0.00	104.00	128.55

Prob. 5.27

R1 = 44.0 kN R2 = 16.0 kN

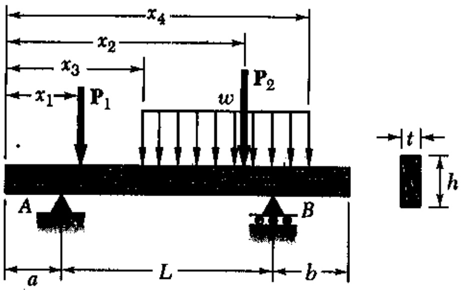
X m	V kN	M kN.m	Sigma MPa
0.00	-20.00	0.00	0.00
1.60	24.00	-32.00	-31.07
4.00	-16.00	25.60	24.85
5.60	-16.00	0.00	0.00

Prob. 5.29

R1 = 52.5 kips R2 = 22.5 kips

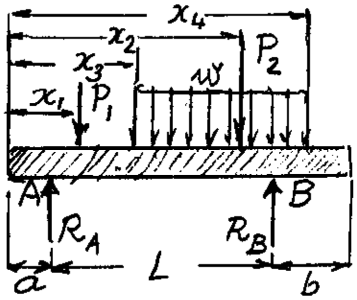
X ft	V kips	M kip.ft	Sigma ksi
0.00	-25.00	0.00	0.00
1.00	27.50	-25.00	-7.85
3.00	2.50	30.00	9.42
9.00	-22.50	45.00	14.14
11.00	0.00	0.00	0.00

PROBLEM 5.C2



5.C2 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress in the beam will not exceed a given allowable value σ_{all}. Write a computer program that can be used to calculate at given intervals ΔL the shear, the bending moment, and the smallest acceptable value of the unknown dimension. Apply this program to solve the following problems, using the intervals ΔL indicated: (*a*) Prob. 5.75 ($\Delta L = 0.1$ m), (*b*) Prob. 5.79 ($\Delta L = 0.2$ m), (*c*) Prob. 5.80 ($\Delta L = 0.3$ m),

SOLUTION



REACTIONS AT A AND B

$$+\circlearrowleft \sum M_A = 0: \; R_B L - P_1(x_1 - a) - P_2(x_2 - a) - w(x_4 - x_3)\left(\frac{x_4 + x_3}{2} - a\right) = 0$$

$$R_B = \frac{1}{L}\left[P_1(x_1 - a) + P_2(x_2 - a) + \frac{1}{2}w(x_4 - x_3)(x_4 + x_3 - 2a)\right]$$

$$R_A = P_1 + P_2 + w(x_4 - x_3) - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text)

SET $n = (a + b + L)/\Delta L$

FOR $i = 0$ TO n: $x = (\Delta L)i$

WE DEFINE: IF $x \geq a$ THEN STPA = 1 ELSE STPA = 0

IF $x \geq a + L$, THEN STPB = 1 ELSE, STB = 0

IF $x \geq x_1$ THEN STP1 = 1 ELSE STP1 = 0

IF $x \geq x_2$ THEN STP2 = 1 ELSE STP2 = 0

IF $x \geq x_3$ THEN STP3 = 1 ELSE STP3 = 0

IF $x \geq x_4$ THEN STP4 = 1 ELSE STP4 = 0

$$V = R_A\,STPA + R_B\,STPB - P_1\,STP1 - P_2\,STP2 - w(x - x_3)\,STP3 + w(x - x_4)\,STP4$$

$$M = R_A(x - a)\,STPA + R_B(x - a - L)\,STPB - P_1(x - x_1)\,STP1 - P_2(x - x_2)\,STP2 - \frac{1}{2}w(x - x_3)^2\,STP3 + \frac{1}{2}w(x - x_4)^2\,STP4$$

$$S_{min} = |M|/\sigma_{all}$$

IF UNKNOWN DIMENSION IS h:

From $S = \frac{1}{6}th^2$, we have $h = \sqrt{6S/t}$

IF UNKNOWN DIMENSION IS t:

From $S = \frac{1}{6}th^2$, we have $t = 6S/h^2$

(CONTINUED)

PROBLEM 5.C2 CONTINUED

PROGRAM OUTPUTS

Prob. 5.75

RA = 2.40 kN RB = 3.00 kN

X m	V kN	M kN.m	H mm
0.00	2.40	0.000	0.00
0.10	2.40	0.240	54.77
0.20	2.40	0.480	77.46
0.30	2.40	0.720	94.87
0.40	2.40	0.960	109.54
0.50	2.40	1.200	122.47
0.60	2.40	1.440	134.16
0.70	2.40	1.680	144.91
0.80	0.60	1.920	154.92
0.90	0.60	1.980	157.32
1.00	0.60	2.040	159.69
1.10	0.60	2.100	162.02
1.20	0.60	2.160	164.32
1.30	0.60	2.220	166.58
1.40	0.60	2.280	168.82
1.50	0.60	2.340	171.03
1.60	-3.00	2.400	173.21
1.70	-3.00	2.100	162.02
1.80	-3.00	1.800	150.00
1.90	-3.00	1.500	136.93
2.00	-3.00	1.200	122.47
2.10	-3.00	0.900	106.07
2.20	-3.00	0.600	86.60
2.30	-3.00	0.300	61.24
2.40	0.00	0.000	0.05

Prob. 5.79

RA = 2.70 kN RB = 8.10 kN

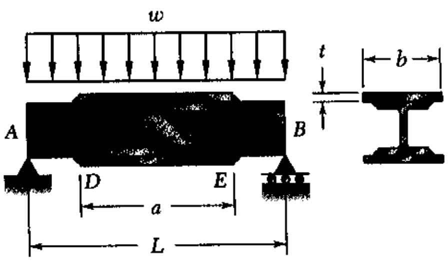
X m	V kN	M kN.m	T mm
0.00	2.70	0.000	0.00
0.20	2.10	0.480	10.67
0.40	1.50	0.840	18.67
0.60	0.90	1.080	24.00
0.80	0.30	1.200	26.67
1.00	-0.30	1.200	26.67
1.20	-0.90	1.080	24.00
1.40	-1.50	0.840	18.67
1.60	-2.10	0.480	10.67
1.80	-2.70	0.000	0.00
2.00	-3.30	-0.600	13.33
2.20	-3.90	-1.320	29.33
2.40	3.60	-2.160	48.00
2.60	3.00	-1.500	33.33
2.80	2.40	-0.960	21.33
3.00	1.80	-0.540	12.00
3.20	1.20	-0.240	5.33
3.40	0.60	-0.060	1.33
3.60	0.00	-0.000	0.00

Prob. 5.80

RA = 6.50 kN RB = 6.50 kN

X m	V kN	M kN.m	H mm
0.00	2.50	0.000	0.00
0.30	2.50	0.750	61.24
0.60	9.00	1.500	86.60
0.90	7.20	3.930	140.18
1.20	5.40	5.820	170.59
1.50	3.60	7.170	189.34
1.80	1.80	7.980	199.75
2.10	-0.00	8.250	203.10
2.40	-1.80	7.980	199.75
2.70	-3.60	7.170	189.34
3.00	-5.40	5.820	170.59
3.30	-7.20	3.930	140.18
3.60	-2.50	1.500	86.60
3.90	-2.50	0.750	61.24
4.20	0.00	0.000	0.06

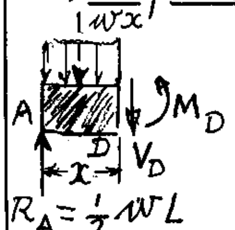
PROBLEM 5.C3



5.C3 Two cover plates, each of thickness t, are to be welded to a wide-flange beam of length L, which is to support a uniformly distributed load w. Denoting by σ_{all} the allowable normal stress in the beam and in the plates, by d the depth of the beam, and by I_b and S_b, respectively, the moment of inertia and the section modulus of the cross section of the unreinforced beam about a horizontal centroidal axis, write a computer program that can be used to calculate the required value of (*a*) the length a of the plates, (*b*) the width b of the plates. Use this program to solve Probs. 5.155 and 5.157.

SOLUTION

(a) Required Length of Plates



FB = AD: $+\circlearrowright \Sigma M_D = 0$: $M_D + wx\left(\frac{x}{2}\right) - R_A x = 0$

But: $R_A = \frac{1}{2} wL$ and $M_D = S\sigma_{all}$. Divide by $\frac{1}{2}w$:

$x^2 - Lx + (2S\sigma_{all}/w) = 0$. Set $k = \frac{2S\sigma_{all}}{w}$:

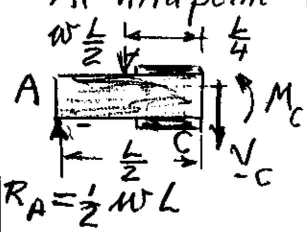
$x^2 - Lx + k = 0$

Solving the quadratic: $x = \frac{L - \sqrt{L^2 - 4k}}{2}$

Compute x and $a = L - 2x$ ◄

(b) Required Width of Plates

At midpoint C of beam:

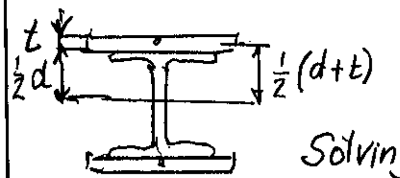


FB = AC: $+\circlearrowright \Sigma M_C = 0$: $M_C + \frac{wL}{2}\frac{L}{4} - \frac{wL}{2}\frac{L}{2} = 0$

Compute $M_C = \frac{1}{8} wL^2$

Compute: $c = t + \frac{1}{2}d$

From $\sigma_{all} = \frac{M_C c}{I}$ compute $I = \frac{M_C c}{\sigma_{all}}$



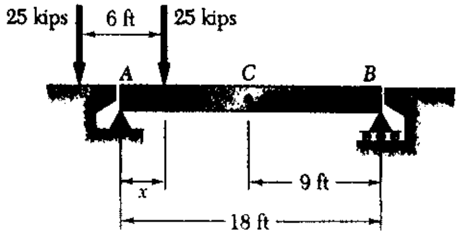
But $I = I_{beam} + I_{plates} = I_b + 2\left[\frac{1}{12}bt^3 + bt\left(\frac{d+t}{2}\right)^2\right]$

Solving for b: $b = \frac{6(I - I_b)}{t\left[t^2 + 3(d+t)^2\right]}$ ◄

PROGRAM OUTPUTS

PROB. 5.155: W460×74, $\sigma_{all} = 150$ MPa, $w = 40$ kN/m, $L = 8$ m, $t = 7.5$ mm, $d = 457$ mm, $I_b = 333\times10^6$ mm^4, $S = 1460\times10^3$ mm^3	PROB. 5.157: W30×99, $\sigma_{all} = 22$ ksi, $w = 30$ kips/ft, $L = 16$ ft, $t = 5/8$ in., $d = 29.65$ in., $I_b = 3990$ in^4, $S = 269$ in^3
Prob. 5.155 a = 4.49 m b = 211 mm	Prob. 5.157 a = 11.16 ft b = 14.31 in.

PROBLEM 5.C4



5.C4 Two 25-kip loads are maintained 6 ft apart as they are moved slowly across the 18-ft beam AB. Write a computer program and use it to calculate the bending moment under each load and at the midpoint C of the beam for values of x from 0 to 24 ft at intervals $\Delta x = 1.5$ ft.

SOLUTION

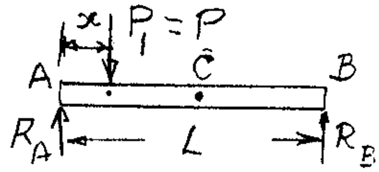
NOTATION: Length of beam $= L = 18$ ft

Loads: $P_1 = P_2 = P = 25$ kips

Distance between loads $= d = 6$ ft

We note that $d < L/2$

(1)



(1) FROM $x = 0$ TO $x = d$:

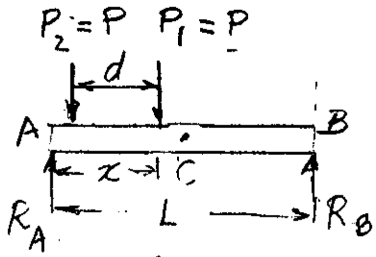
$+\circlearrowright \sum M_B = 0$: $P(L-x) - R_A L = 0$

$R_A = P(L-x)/L$

Under P_1: $M_1 = R_A x$

At C: $M_C = R_A(\frac{L}{2}) - P(\frac{L}{2} - x)$

(2 A)



(2) FROM $x = d$ TO $x = L$:

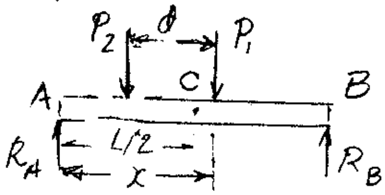
$+\circlearrowright \sum M_B = 0$: $P(L-x) + P(L-x+d) - R_A L = 0$

$R_A = P(2L - 2x + d)/L$

Under P_1: $M_1 = R_A x - Pd$

Under P_2: $M_2 = R_A(x-d)$

(2B)

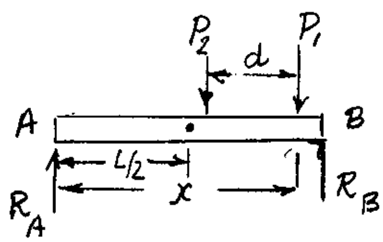


(2A) FROM $x = d$ TO $x = L/2$:

$M_C = R_A(\frac{L}{2}) - P(\frac{L}{2} - x) - P(\frac{L}{2} - x + d)$

$= R_A(L/2) - P(L - 2x + d)$

(2C)



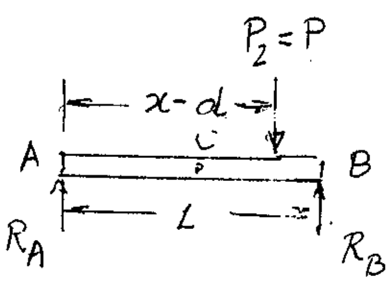
(2B) FROM $x = L/2$ TO $x = L/2 + d$:

$M_C = R_A(L/2) - P(\frac{L}{2} - x + d)$

(2C) FROM $x = L/2 + d$ TO $x = L$:

$M_C = R_A L/2$

(3)



(3) FROM $x = L$ TO $x = L + d$:

$+\circlearrowright \sum M_B = 0$: $P(L - x + d) - R_A L = 0$

$R_A = P(L - x + d)/L$

Under P_2: $M_2 = R_A(x - d)$

At C: $M_C = R_A(L/2)$

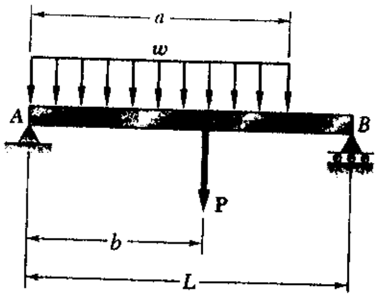
(CONTINUED)

PROGRAM OUTPUT

P = 25 kips, L = 18 ft, D = 6 ft

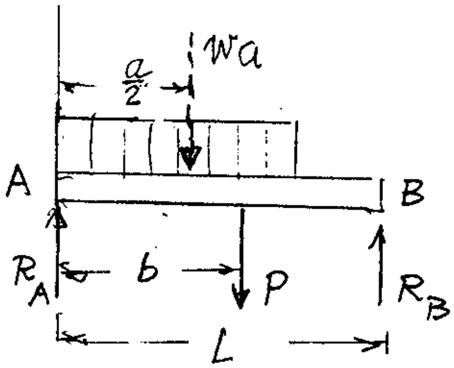
x ft	MC kip.ft	M1 kip.ft	M2 kip.ft
0.0	0.00	0.00	0.00
1.5	18.75	34.38	0.00
3.0	37.50	62.50	0.00
4.5	56.25	84.38	0.00
6.0	75.00	100.00	0.00
7.5	112.50	131.25	56.25
9.0	150.00	150.00	100.00
10.5	150.00	156.25	131.25
12.0	150.00	150.00	150.00
13.5	150.00	131.25	156.25
15.0	150.00	100.00	150.00
16.5	112.50	56.25	131.25
18.0	75.00	0.00	100.00
19.5	56.25	0.00	84.38
21.0	37.50	0.00	62.50
22.5	18.75	0.00	34.38
24.0	0.00	0.00	0.00

PROBLEM 5.C5



5.C5 Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.2$ ft to the beam and loading of (*a*) Prob. 5.83, (*b*) Prob. 5.125.

SOLUTION



REACTIONS AT A AND B
USING FB DIAGRAM OF BEAM:

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - Pb - wa(a/2) = 0$$

$$R_B = (1/L)(Pb + \tfrac{1}{2} w a^2)$$

$$R_A = P + wa - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text).

SET $n = L/\Delta L$. FOR $i = 0$ TO n: $x = (\Delta L)\, i$

WE DEFINE: IF $x \geq a$ THEN STPA=1 ELSE STPA=0

IF $x \geq b$ THEN STPB=1 ELSE STPB=0.

$$V = R_A - wx + w(x-a)\,STPA - P\,STPB$$

$$M = R_A x - \tfrac{1}{2} w x^2 + \tfrac{1}{2} w (x-a)^2 STPA - P(x-b)\,STPB$$

LOCATE AND PRINT (x, V) AND (x, M)

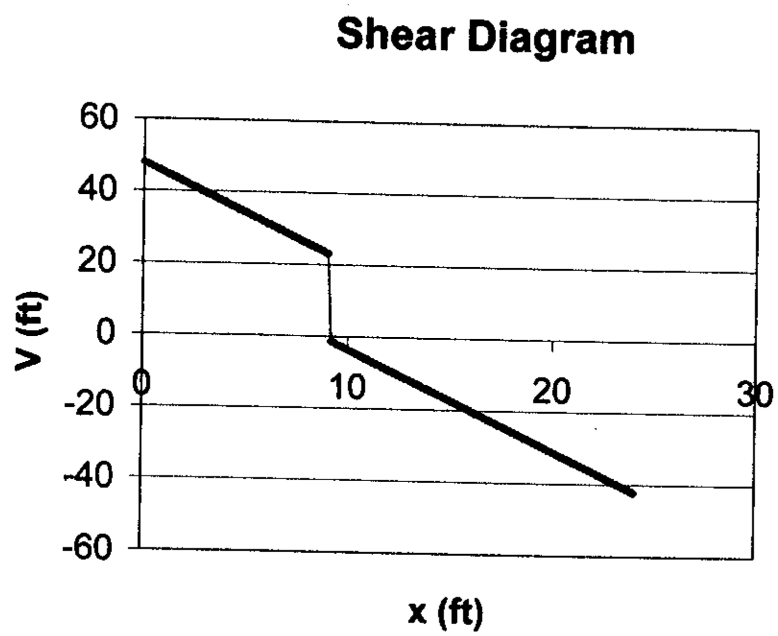
SEE NEXT PAGES FOR PROGRAM OUTPUTS

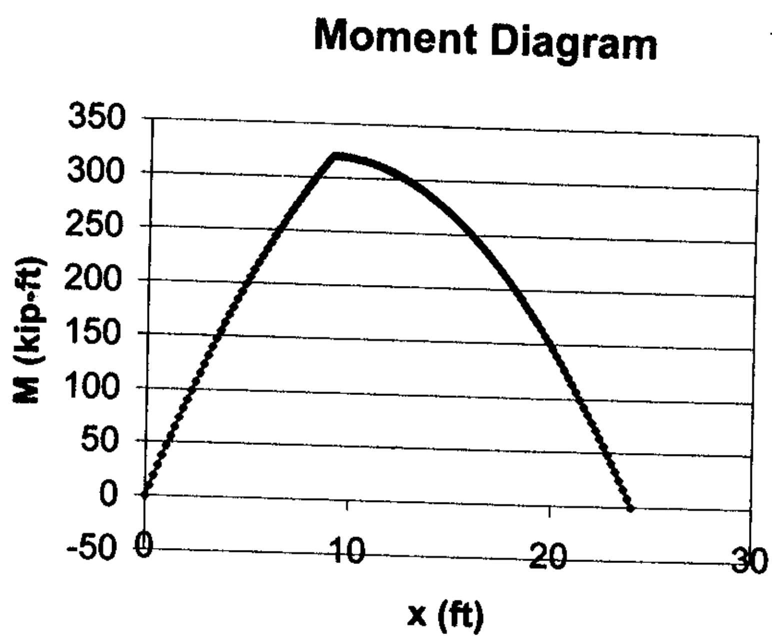
(CONTINUED)

PROGRAM OUTPUT FOR P5.83

```
PROBLEM 5.83

RA =    48.00 kips   RB =     42.00 kips
```



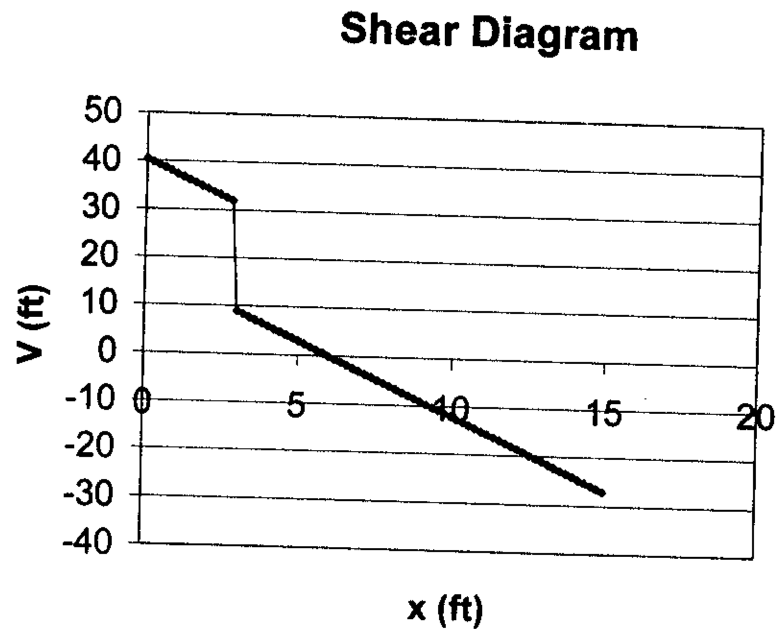


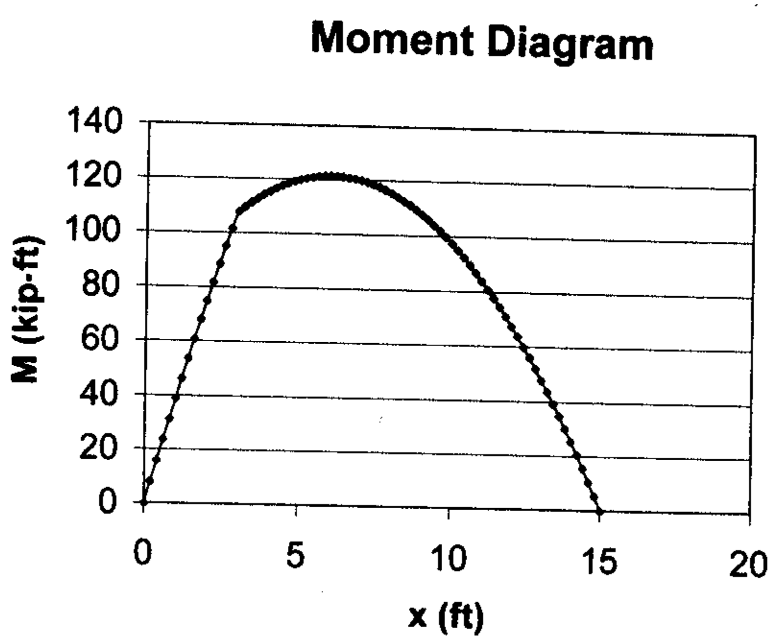
(CONTINUED)

PROGRAM OUTPUT FOR P5,125

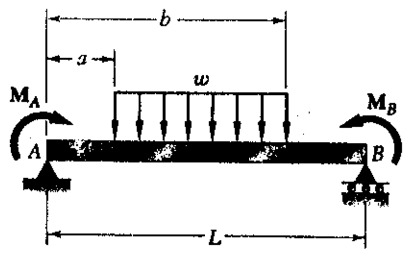
```
PROBLEM 5.125

RA =    40.50 kips   RB =    27.00 kips
```





PROBLEM 5.C6



5.C6 Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.025$ m to the beam and loading of Prob. 5.124.

SOLUTION

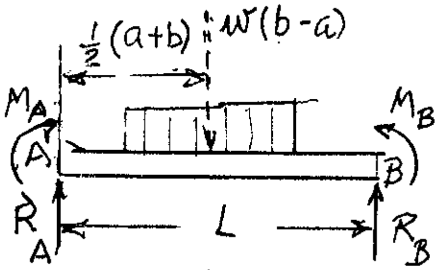
<u>REACTIONS AT A AND B</u>

$+\circlearrowleft \sum M_A = 0:$

$$R_B L + M_B - M_A - w(b-a)\tfrac{1}{2}(a+b) = 0$$

$$R_B = (1/L)\left[M_A - M_B + \tfrac{1}{2} w (b^2 - a^2)\right]$$

$$R_A = w(b-a) - R_B$$



<u>WE USE STEP FUNCTIONS</u> (See bottom of page 348 of text)

SET $n = L/\Delta L$, FOR $i = 0$ TO n: $x = (\Delta L) i$

WE DEFINE: IF $x \geq a$ THEN STPA = 1 ELSE STPA = 0

IF $x \geq b$ THEN STPB = 1 ELSE STPB = 0

$$V = R_A - w(x-a)\,STPA + w(x-b)\,STPB$$

$$M = M_A + R_A x - \tfrac{1}{2} w (x-a)^2\,STPA + \tfrac{1}{2} w (x-b)^2\,STPB$$

LOCATE AND PRINT (x, V) AND (x, M)

PROGRAM OUTPUT ON NEXT PAGE

(CONTINUED)

PROBLEM 5.C6 CONTINUED

PROGRAM OUTPUT

```
PROBLEM 5.124
RA =    29.50 kips  RB =     66.50 kips
```