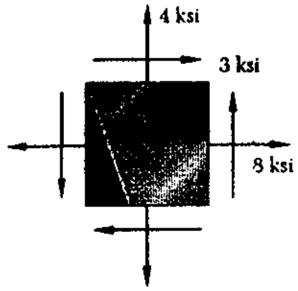
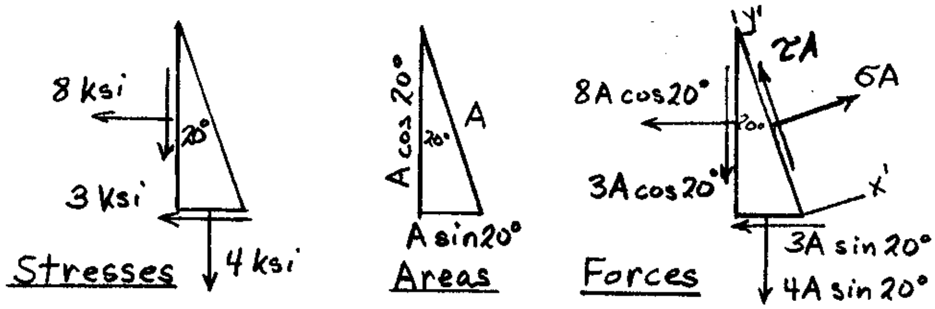
CHAPTER 7

PROBLEM 7.1

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



SOLUTION



$+\nearrow \Sigma F = 0$

$\sigma A - 8A\cos 20^\circ \cos 20^\circ - 3A\cos 20^\circ \sin 20^\circ - 3A\sin 20^\circ \cos 20^\circ - 4A\sin 20^\circ \sin 20^\circ = 0$

$\sigma = 8\cos^2 20^\circ + 3\cos 20^\circ \sin 20^\circ + 3\sin 20^\circ \cos 20^\circ + 4\sin^2 20^\circ = 9.46 \text{ ksi}$ ◄

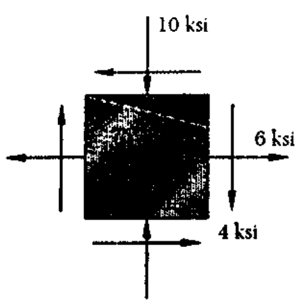
$+\nwarrow \Sigma F = 0$

$\tau A + 8A\cos 20^\circ \sin 20^\circ - 3A\cos 20^\circ \cos 20^\circ + 3A\sin 20^\circ \sin 20^\circ - 4A\sin 20^\circ \cos 20^\circ = 0$

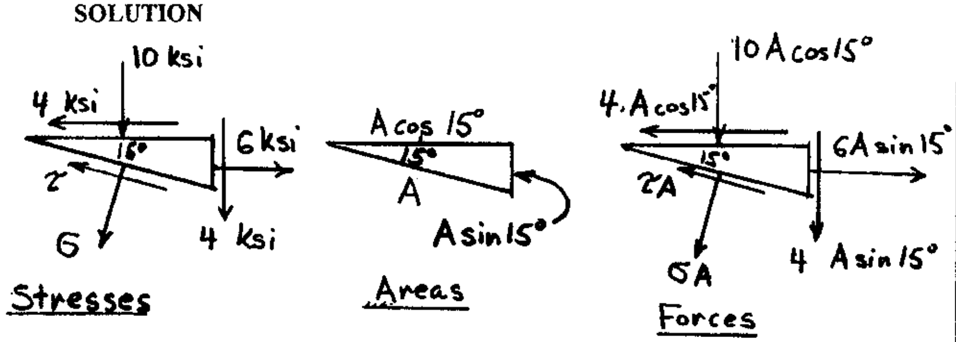
$\tau = -8\cos 20^\circ \sin 20^\circ + 3(\cos^2 20^\circ - \sin^2 20^\circ) + 4\sin 20^\circ \cos 20^\circ = 1.013 \text{ ksi}$ ↑ ◄

PROBLEM 7.2

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



SOLUTION



$+\swarrow \Sigma F = 0$

$\sigma A + 4A\cos 15^\circ \sin 15^\circ + 10A\cos 15^\circ \cos 15^\circ - 6A\sin 15^\circ \sin 15^\circ + 4A\sin 15^\circ \cos 15^\circ = 0$

$\sigma = -4\cos 15^\circ \sin 15^\circ - 10\cos^2 15^\circ + 6\sin^2 15^\circ - 4\sin 15^\circ \cos 15^\circ = 10.93 \text{ ksi}$ ◄

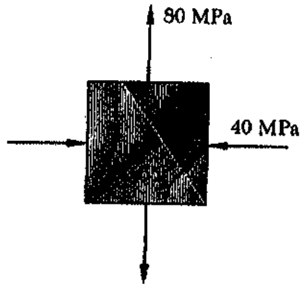
$+\nwarrow \Sigma F = 0$

$\tau A + 4A\cos 15^\circ \cos 15^\circ - 10A\cos 15^\circ \sin 15^\circ - 6A\sin 15^\circ \cos 15^\circ - 4A\sin 15^\circ \sin 15^\circ = 0$

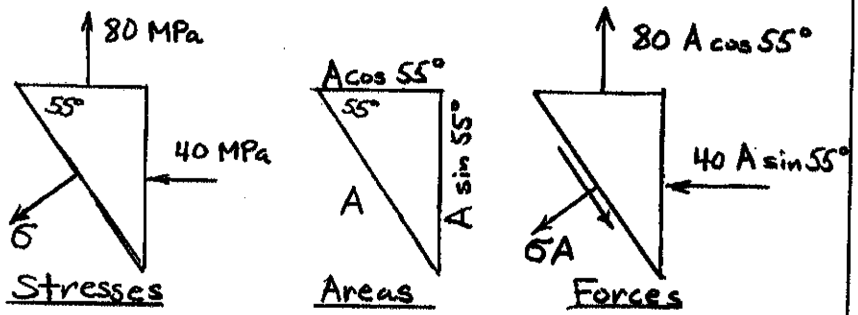
$\tau = -4(\cos^2 15^\circ - \sin^2 15^2) + (10+6)\cos 15^\circ \sin 15^\circ = 0.536 \text{ ksi}$ ↖ ◄

PROBLEM 7.3

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



SOLUTION



$+\swarrow \Sigma F = 0$

$\sigma A - 80 A \cos 55° \cos 55° + 40 A \sin 55° \sin 55° = 0$

$\sigma = 80 \cos^2 55° - 40 \sin^2 55° = -0.521 \text{ MPa}$ ◄

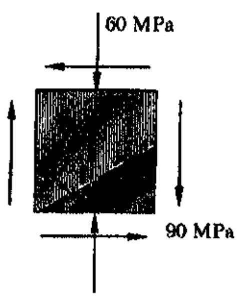
$+\searrow \Sigma F = 0$

$\tau A - 80 A \cos 55° \sin 55° - 40 A \sin 55° \cos 55°$

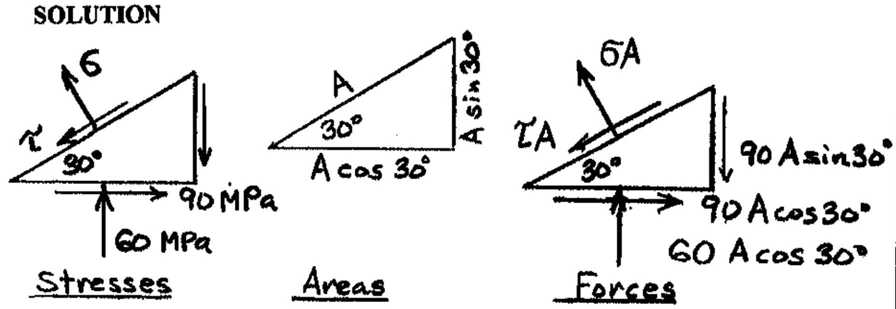
$\tau = 120 \cos 55° \sin 55° = 56.4 \text{ MPa} \searrow$ ◄

PROBLEM 7.4

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



SOLUTION



$+\nwarrow \Sigma F = 0$

$\sigma A - 90 A \sin 30° \cos 30° - 90 A \cos 30° \sin 30° + 60 A \cos 30° \cos 30° = 0$

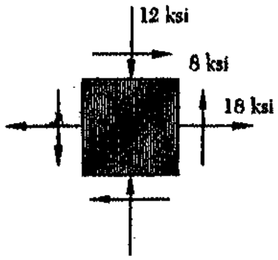
$\sigma = 180 \sin 30° \cos 30° - 60 \cos^2 30° = 32.9 \text{ MPa}$ ◄

$+\swarrow \Sigma F = 0$

$\tau A + 90 A \sin 30° \sin 30° - 90 A \cos 30° \cos 30° - 60 A \cos 30° \sin 30° = 0$

$\tau = 90(\cos^2 30° - \sin^2 30°) + 60 \cos 30° \sin 30° = 71.0 \text{ MPa} \swarrow$ ◄

PROBLEM 7.5



7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$\sigma_x = 18$ ksi $\qquad \sigma_y = -12$ ksi $\qquad \tau_{xy} = 8$ ksi

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(8)}{18+12} = 0.5333$

$2\theta_p = 28.07°$ $\qquad \theta_p = 14.04°,\ 104.04°$ ◄

(b) $\sigma_{max,min} = \dfrac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

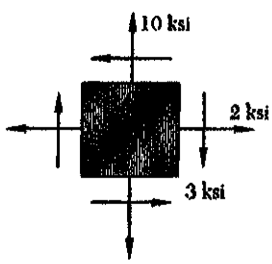
$= \dfrac{18-12}{2} \pm \sqrt{\left(\dfrac{18+12}{2}\right)^2 + (8)^2}$

$= 3 \pm 17$ ksi

$\sigma_{max} = 20$ ksi ◄

$\sigma_{min} = -14$ ksi ◄

PROBLEM 7.6



7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

SOLUTION

$\sigma_x = 2$ ksi $\qquad \sigma_y = 10$ ksi $\qquad \tau_{xy} = -3$ ksi

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(-3)}{2-10} = 0.750$

$2\theta_p = 36.87°$ $\qquad \theta_p = 18.43°,\ 108.43°$ ◄

(b) $\sigma_{max,min} = \dfrac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \dfrac{2+10}{2} \pm \sqrt{\left(\dfrac{2-10}{2}\right)^2 + (-3)^2}$

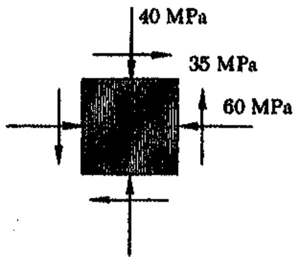
$= 6 \pm 5$ ksi

$\sigma_{max} = 11$ ksi

$\sigma_{min} = 1$ ksi

PROBLEM 7.7

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



SOLUTION

$\sigma_x = -60$ MPa $\quad \sigma_y = -40$ MPa $\quad \tau_{xy} = 35$ MPa

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(35)}{-60+40} = -3.50$

$2\theta_p = -74.05°$ $\qquad \theta_p = -37.03°,\ 52.97°$

(b) $\sigma_{max,min} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \dfrac{-60-40}{2} \pm \sqrt{\left(\dfrac{-60+40}{2}\right)^2 + (35)^2}$

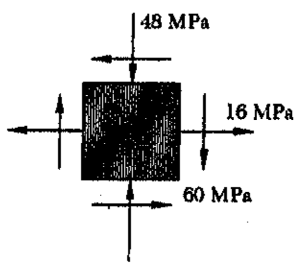
$= -50 \pm 36.4$ MPa

$\sigma_{max} = -13.60$ MPa

$\sigma_{min} = -86.4$ MPa

PROBLEM 7.8

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



SOLUTION

$\sigma_x = 16$ MPa $\quad \sigma_y = -48$ MPa $\quad \tau_{xy} = -60$ MPa

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(-60)}{16+48} = -1.875$

$2\theta_p = -61.93°$ $\qquad \theta_p = -30.96°,\ 59.04°$

(b) $\sigma_{max,min} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

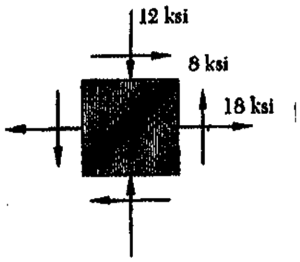
$= \dfrac{16-48}{2} \pm \sqrt{\left(\dfrac{16+48}{2}\right)^2 + (-60)^2}$

$= -16 \pm 68$

$\sigma_{max} = 52$ MPa

$\sigma_{min} = -84$ MPa

PROBLEM 7.9



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = 18$ ksi $\qquad \sigma_y = -12$ ksi $\qquad \tau_{xy} = 8$ ksi

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{18+12}{(2)(8)} = -1.875$

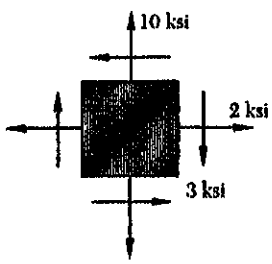
$2\theta_s = -61.93°$ $\qquad \theta_s = -30.96°,\ 59.04°$

(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{18+12}{2}\right)^2 + (8)^2} = 17$ ksi

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{18-12}{2} = 3$ ksi

PROBLEM 7.10



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = 2$ ksi $\qquad \sigma_y = 10$ ksi $\qquad \tau_{xy} = -3$ ksi

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{2-10}{(2)(-3)} = -1.3333$

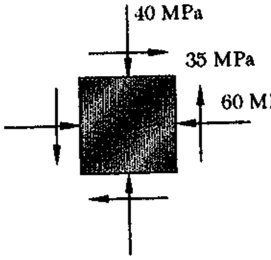
$2\theta_s = -53.13°$ $\qquad \theta_s = -26.57°,\ 63.43°$

(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{2-10}{2}\right)^2 + (-3)^2} = 5$ ksi

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{2+10}{2} = 6$ ksi

PROBLEM 7.11



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = -60$ MPa $\qquad \sigma_y = -40$ MPa $\qquad \tau_{xy} = 35$ MPa

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{-60+40}{(2)(35)} = 0.2857$

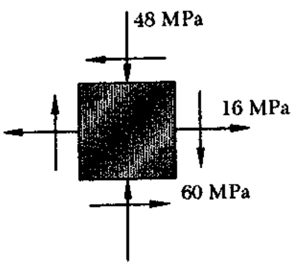
$2\theta_s = 15.95°$ $\qquad \theta_s = 7.97°,\ 97.97°$ ◄

(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{-60+40}{2}\right)^2 + (35)^2} = 36.4$ MPa ◄

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{-60-40}{2} = -50$ MPa ◄

PROBLEM 7.12



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = 16$ MPa $\qquad \sigma_y = -48$ MPa $\qquad \tau_{xy} = -60$ MPa

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{16+48}{(2)(-60)} = 0.5333$

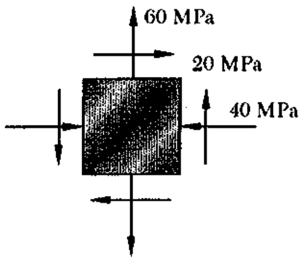
$2\theta_s = 28.07°$ $\qquad \theta_s = 14.04°,\ 104.04°$ ◄

(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{16+48}{2}\right)^2 + (-60)^2} = 68$ MPa ◄

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{16-48}{2} = -16$ MPa ◄

PROBLEM 7.13



7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$\sigma_x = -40\ \text{MPa} \qquad \sigma_y = 60\ \text{MPa} \qquad \tau_{xy} = 20\ \text{MPa}$

$\dfrac{\sigma_x + \sigma_y}{2} = 10\ \text{MPa} \qquad \dfrac{\sigma_x - \sigma_y}{2} = -50\ \text{MPa}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25^\circ \qquad 2\theta = -50^\circ$

$\sigma_{x'} = 10 - 50\cos(-50^\circ) + 20\sin(-50^\circ) = -37.5\ \text{MPa}$ ◄

$\tau_{x'y'} = +50\sin(-50^\circ) + 20\cos(-50^\circ) = -25.4\ \text{MPa}$ ◄

$\sigma_{y'} = 10 + 50\cos(-50^\circ) - 20\sin(-50^\circ) = 57.5\ \text{MPa}$ ◄

(b) $\theta = 10^\circ \qquad 2\theta = 20^\circ$

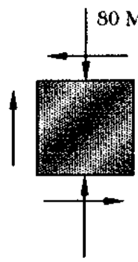
$\sigma_{x'} = 10 - 50\cos(20^\circ) + 20\sin(20^\circ) = -30.1\ \text{MPa}$ ◄

$\tau_{x'y'} = +50\sin(20^\circ) + 20\cos(20^\circ) = 35.9\ \text{MPa}$ ◄

$\sigma_{y'} = 10 + 50\cos(20^\circ) - 20\sin(20^\circ) = 50.1\ \text{MPa}$ ◄

PROBLEM 7.14

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



SOLUTION

$\sigma_x = 0 \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = -50 \text{ MPa}$

$\dfrac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \qquad \dfrac{\sigma_x - \sigma_y}{2} = 40 \text{ MPa}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25° \qquad 2\theta = -50°$

$\sigma_{x'} = -40 + 40\cos(-50°) - 50\sin(-50°) = 24.0 \text{ MPa}$ ◄

$\tau_{x'y'} = -40\sin(-50°) - 50\cos(-50°) = -1.5 \text{ MPa}$ ◄

$\sigma_{y'} = -40 - 40\cos(-50°) + 50\sin(-50°) = -104.0 \text{ MP}$ ◄

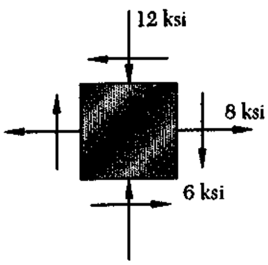
(b) $\theta = 10° \qquad 2\theta = 20°$

$\sigma_{x'} = -40 + 40\cos(20°) - 50\sin(20°) = -19.5 \text{ MPa}$ ◄

$\tau_{x'y'} = -40\sin(20°) - 50\cos(20°) = -60.7 \text{ MPa}$ ◄

$\sigma_{y'} = -40 - 40\cos(20°) + 50\sin(20°) = -60.5 \text{ MPa}$ ◄

PROBLEM 7.15



7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$\sigma_x = 8$ ksi $\qquad \sigma_y = -12$ ksi $\qquad \tau_{xy} = -6$ ksi

$\dfrac{\sigma_x + \sigma_y}{2} = -2$ ksi $\qquad \dfrac{\sigma_x - \sigma_y}{2} = 10$ ksi

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25°$ $\qquad 2\theta = -50°$

$\sigma_{x'} = -2 + 10\cos(-50°) - 6\sin(-50°) = 9.02$ ksi

$\tau_{x'y'} = -10\sin(-50°) - 6\cos(-50°) = 3.80$ ksi

$\sigma_{y'} = -2 - 10\cos(-50°) + 6\sin(-50°) = -13.02$ ksi

(b) $\theta = 10°$ $\qquad 2\theta = 20°$

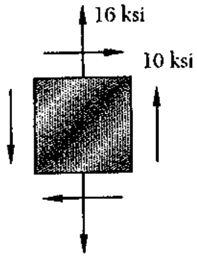
$\sigma_{x'} = -2 + 10\cos(20°) - 6\sin(20°) = 5.34$ ksi

$\tau_{x'y'} = -10\sin(20°) - 6\cos(20°) = -9.06$ ksi

$\sigma_{y'} = -2 - 10\cos(20°) + 6\sin(20°) = -9.34$ ksi

PROBLEM 7.16

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



SOLUTION

$$\sigma_x = 0 \qquad \sigma_y = 16 \text{ ksi} \qquad \tau_{xy} = 10 \text{ ksi}$$

$$\frac{\sigma_x + \sigma_y}{2} = 8 \text{ ksi} \qquad \frac{\sigma_x - \sigma_y}{2} = -8 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25^\circ \qquad 2\theta = -50^\circ$

$$\sigma_{x'} = 8 - 8\cos(-50^\circ) + 10\sin(-50^\circ) = -4.80 \text{ ksi}$$

$$\tau_{x'y'} = 8\sin(-50^\circ) + 10\cos(-50^\circ) = 0.30 \text{ ksi}$$

$$\sigma_{y'} = 8 + 8\cos(-50^\circ) - 10\sin(-50^\circ) = 20.80 \text{ ksi}$$

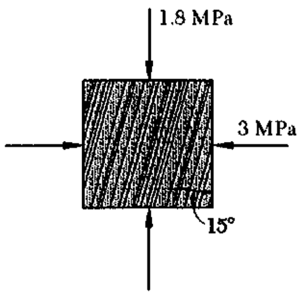
(b) $\theta = 10^\circ \qquad 2\theta = 20^\circ$

$$\sigma_{x'} = 8 - 8\cos(20^\circ) + 10\sin(20^\circ) = 3.90 \text{ ksi}$$

$$\tau_{x'y'} = 8\sin(20^\circ) + 10\cos(20^\circ) = 12.13 \text{ ksi}$$

$$\sigma_{y'} = 8 + 8\cos(20^\circ) - 10\cos(20^\circ) = 12.10 \text{ ksi}$$

PROBLEM 7.17



7.17 and *7.18* The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$\sigma_x = -3 \text{ MPa} \qquad \sigma_y = -1.8 \text{ MPa} \qquad \tau_{xy} = 0$

$\theta = -15^\circ \qquad 2\theta = -30^\circ$

(a) $\tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\sin 2\theta$

$= -\dfrac{-3 + 1.8}{2}\sin(-30^\circ) + 0$

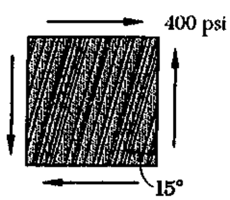
$= -0.300 \text{ MPa}$ ◄

(b) $\sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$= \dfrac{-3 - 1.8}{2} + \dfrac{-3 + 1.8}{2}\cos(-30^\circ) + 0$

$= -2.92 \text{ MPa}$ ◄

PROBLEM 7.18



7.17 and *7.18* The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 400 \text{ psi}$

$\theta = -15^\circ \qquad 2\theta = -30^\circ$

(a) $\tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$= -0 + 400\cos(-30^\circ)$

$= 346 \text{ psi}$ ◄

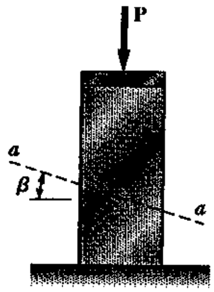
(b) $\sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$= 0 + 0 + 400\sin(-30^\circ)$

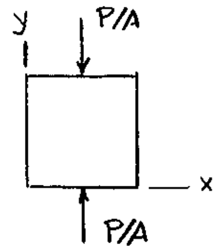
$= -200 \text{ psi}$ ◄

PROBLEM 7.19

7.19 The centric force **P** is applied to a short post as shown. Knowing that the stresses on plane *a-a* are $\sigma = -15$ ksi and $\tau = 5$ ksi, determine (*a*) the angle β that plane *a-a* forms with the horizontal, (*b*) the maximum compressive stress in the post.



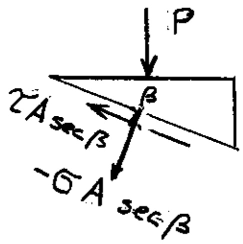
SOLUTION



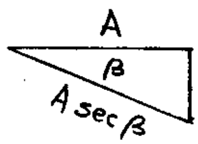
$\sigma_x = 0$

$\tau_{xy} = 0$

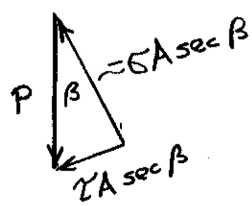
$\sigma_y = \sigma_{max\ comp.} = -\frac{P}{A}$



Forces



Areas



Force Triangle

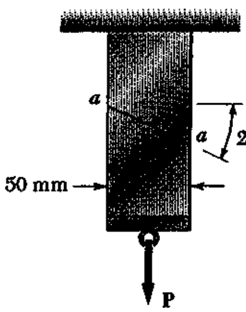
$\tan\beta = \frac{\tau \cancel{A \sec\beta}}{-\sigma \cancel{A \sec\beta}} = \frac{5}{15} = \frac{1}{3}$

(a) $\beta = \arctan\frac{1}{3} = 18.4°$ ◄

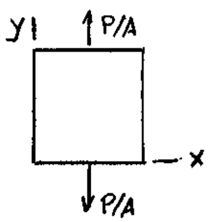
(b) $P = (-\sigma A \sec\beta)(\sec\beta)$ $\qquad \frac{P}{A} = \frac{-\sigma}{\cos^2\beta} = \frac{15}{\cos^2 18.4°} = 16.67$ ksi ◄

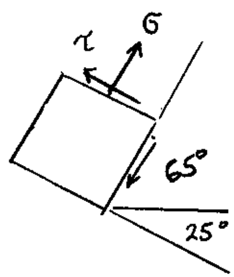
PROBLEM 7.20

7.20 Two members of uniform cross section 50 × 80 mm are glued together along plane *a-a*, which forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest axial load **P** that can be applied.



SOLUTION





For plane a-a $\theta = 65°$

$\sigma_x = 0 \qquad \tau_{xy} = 0 \qquad \sigma_y = \frac{P}{A}$

$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta = 0 + \frac{P}{A}\sin^2 65° + 0$$

$$P = \frac{A\sigma}{\sin^2 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(800\times10^3)}{\sin^2 65°} = 3.90\times10^3 \text{ N}$$

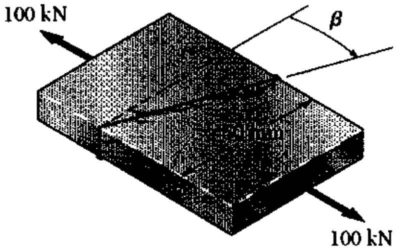
$$\tau = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) = \frac{P}{A}\sin 65°\cos 65° + 0$$

$$P = \frac{A\tau}{\sin 65°\cos 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(600\times10^3)}{\sin 65°\cos 65°} = 6.27\times10^3 \text{ N}$$

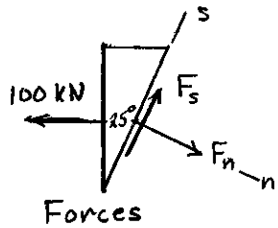
Allowable value of P is the smaller. $\quad P = 3.90\times10^3 \text{ N} = 3.90 \text{ kN}$ ◄

PROBLEM 7.21

7.21 Two steel plates of uniform cross section 10 × 80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25°$, determine (*a*) the in-plane shearing stress parallel to the weld, (*b*) the normal stress perpendicular to the weld.



SOLUTION



Area of weld

$$A_w = \frac{(10\times10^{-3})(80\times10^{-3})}{\cos 25°}$$

$$= 882.7 \quad 10^{-6}\ m^2$$

(a) $\Sigma F_s = 0 \qquad F_s - 100\ \sin 25° = 0 \qquad F_s = 42.26\ kN$

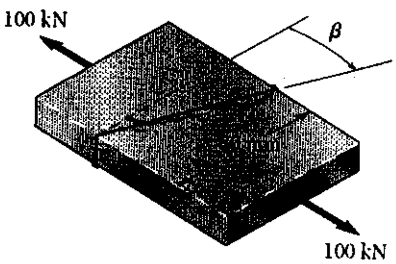
$$\tau_w = \frac{F_s}{A_w} = \frac{42.26\times10^3}{1882.7\times10^{-6}} = 47.9\times10^6\ Pa = 47.9\ MPa$$

(b) $\Sigma F_n = 0 \qquad F_N - 100\ \cos 25° = 0 \qquad F_n = 90.63\ kN$

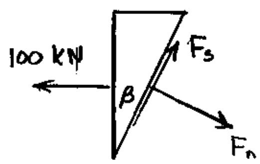
$$\sigma_w = \frac{F_n}{A_w} = \frac{90.63\times10^3}{882.7\times10^{-6}} = 102.7\times10^6\ Pa = 102.7\ MPa$$

PROBLEM 7.22

7.22 Two steel plates of uniform cross section 10 × 80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (*a*) the angle β, (*b*) the corresponding normal stress perpendicular to the weld.



SOLUTION



Area of weld

$$A_w = \frac{(10\times10^{-3})(80\times10^{-3})}{\cos\beta}$$

$$= \frac{800\times10^{-6}}{\cos\beta}\ m^2$$

(a) $\Sigma F_s = 0 \qquad F_s - 100\ \sin\beta = 0 \qquad F_s = 100\ \sin\beta\ kN = 100\times10^3\ \sin\beta\ N$

$$\tau_w = \frac{F_s}{A_w} \qquad 30\times10^6 = \frac{100\times10^3\ \sin\beta}{800\times10^{-6}/\cos\beta} = 125\times10^6\ \sin\beta\cos\beta$$

$$\sin\beta\cos\beta = \frac{1}{2}\sin 2\beta = \frac{30\times10^6}{125\times10^6} = 0.240 \qquad \beta = 14.34°$$

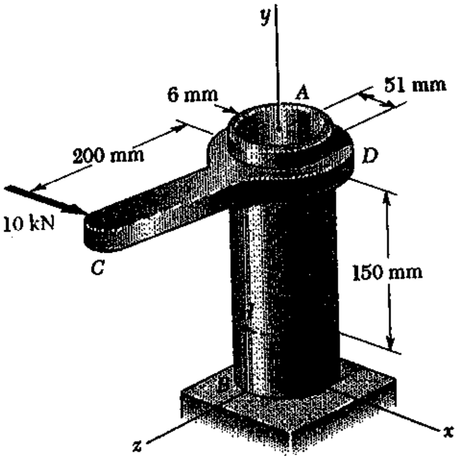
(b) $\Sigma F_n = 0 \qquad F_n - 100\ \cos\beta = 0 \qquad F_n = 100\ \cos 14.34° = 96.88\ kN$

$$A_w = \frac{800\times10^{-6}}{\cos 14.34} = 825.74\times10^{-6}\ m^2$$

$$\sigma = \frac{F_n}{A_w} = \frac{96.88\times10^3}{825.74\times10^{-6}} = 117.3\times10^6\ Pa = 117.3\ MPa$$

PROBLEM 7.23

7.23 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *H*.



SOLUTION

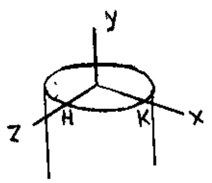
$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4$
$= 4.1855 \times 10^{-6} \text{ m}^4$

$I = \frac{1}{2}J = 2.0927 \text{ m}^4$

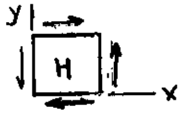
Force-couple system at center of tube in the plane containing points H and K.

$F_x = 10 \times 10^3 \text{ N}$
$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N·m}$
$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N·m}$



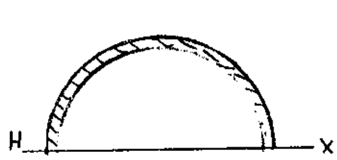
Torsion

$T = M_y = 2000 \text{ N·m}$
$c = r_o = 51 \times 10^{-3} \text{ m}$



$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \times 10^6 \text{ Pa}$

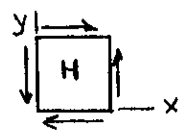
Transverse Shear



For semicircle $\qquad A = \frac{\pi}{2}r^2 \quad \bar{y} = \frac{4}{3\pi}r$
$Q = A\bar{y} = \frac{2}{3}r^3$

$Q = Q_o - Q_i = \frac{2}{3}r_o^3 - \frac{2}{3}r_i^3 = 27.684 \times 10^3 \text{ mm}^3$
$= 27.684 \times 10^{-6} \text{ m}^3$

$V = F_x = 10 \times 10^3 \text{ N} \qquad t = (2)(6 \text{ mm}) = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$



$\tau_{xy} = \frac{VQ}{It} = \frac{(10 \times 10^3)(27.684 \times 10^{-6})}{(2.0927 \times 10^{-6})(12 \times 10^{-3})} = 11.02 \times 10^6 \text{ Pa}$

Bending: Point H lies on neutral axis. $\sigma_y = 0$

Total stresses at point H: $\sigma_x = 0, \ \sigma_y = 0$
$\tau_{xy} = 24.37 \times 10^6 + 11.02 \times 10^6 = 35.39 \times 10^6 \text{ Pa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 35.39 \times 10^6 \text{ Pa}$

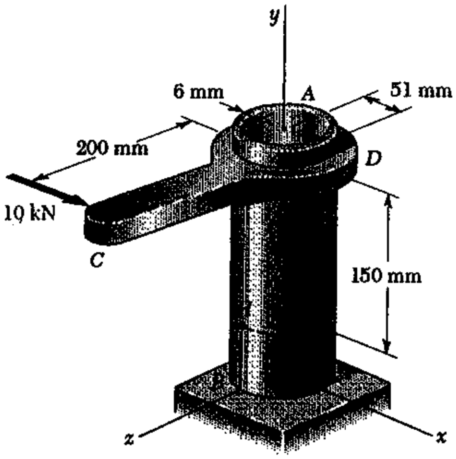
$\sigma_{max} = \sigma_{ave} + R = 35.39 \times 10^6 \text{ Pa} = 35.4 \text{ MPa}$

$\sigma_{min} = \sigma_{ave} - R = -35.39 \times 10^6 \text{ Pa} = -35.4 \text{ MPa}$

$\tau_{max} = R = 35.4 \text{ MPa}$

PROBLEM 7.24

7.24 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.



SOLUTION

$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$

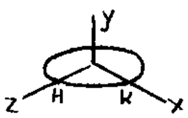
$I = \frac{1}{2}J = 2.0927 \times 10^{-6} \text{ m}^4$

Force-couple system at center of tube in the plane containing points H and K

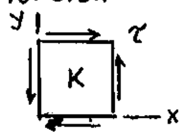
$F_x = 10 \text{ kN} = 10 \times 10^3 \text{ N}$

$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$

$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$



Torsion:



At point K, place local x-axis in negative global z-direction

$T = M_y = 2000 \text{ N}\cdot\text{m} \qquad c = r_o = 51 \times 10^{-3} \text{ m}$

$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \times 10^6 \text{ Pa} = 24.37 \text{ MPa}$

Transverse Shear: Stress due to transverse shear $V = F_x$ is zero at pt. K.

Bending: $|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56 \times 10^6 \text{ Pa} = 36.56 \text{ MPa}$

Point K lies on compression side of neutral axis: $\sigma_y = -36.56 \text{ MPa}$

Total stresses at point K $\quad \sigma_x = 0 \quad \sigma_y = -36.56 \text{ MPa}, \ \tau_{xy} = 24.37 \text{ MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$

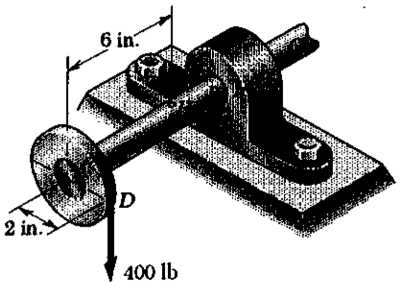
$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46 = +12.18 \text{ MPa}$ ◄

$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46 = -48.74 \text{ MPa}$ ◄

$\tau_{max} = R = 30.46 \text{ MPa}$ ◄

PROBLEM 7.25

7.25 A 400-lb vertical force is applied at D to a gear attached to the solid one-inch diameter shaft AB. Determine the principal stresses and the maximum shearing stress at point H located as shown on top of the shaft.



SOLUTION

Equivalent force-couple system at center of shaft in section at point H.

$V = 400$ lb. $\quad M = (400)(6) = 2400$ lb·in

$T = (400)(2) = 800$ lb·in.

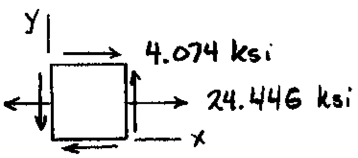
Shaft cross section. $\quad d = $ in $\quad c = \frac{1}{2}d = 0.5$ in

$J = \frac{\pi}{2}c^4 = 0.098175\ \text{in}^4 \quad I = \frac{1}{2}J = 0.049087\ \text{in}^4$

Torsion: $\quad \tau = \dfrac{Tc}{J} = \dfrac{(800)(0.5)}{0.098175} = 4.074\times10^3$ psi $= 4.074$ ksi

Bending: $\quad \sigma = \dfrac{Mc}{I} = \dfrac{(2400)(0.5)}{0.049087} = 24.446\times10^3$ psi $= 24.446$ ksi

Transverse shear: Stress at point H is zero.



$\sigma_x = 24.446$ ksi $\quad \sigma_y = 0 \quad \tau_{xy} = 4.074$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(12.223)^2 + (4.074)^2}$

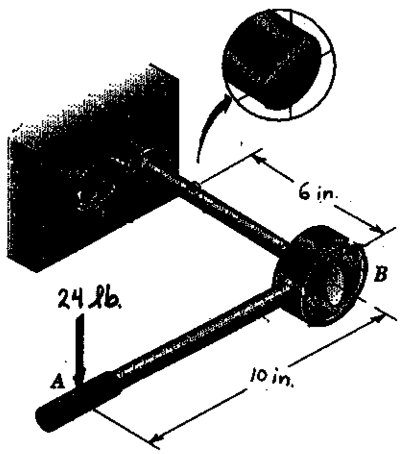
$= 12.884$ ksi

$\sigma_a = \sigma_{ave} + R = 25.107$ ksi

$\sigma_b = \sigma_{ave} - R = -0.661$ ksi

$\tau_{max} = R = 12.884$ ksi

PROBLEM 7.26



7.26 A mechanic uses a crowfoot wrench to loosen at bolt at E. Knowing that the mechanic applies a vertical 24-lb force at A, determine the principal stresses and the maximum shearing stress at point H located as shown on top of the $\frac{3}{4}$ - in. diameter shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point H.

$V = 24$ lb. $\quad M = (24)(6) = 144$ lb·in

$T = (24)(10) = 240$ lb·in

Shaft cross section: $d = 0.75$ in, $c = \frac{1}{2}d = 0.375$ in.

$J = \frac{\pi}{2}c^4 = 0.031063$ in^4 $\quad I = \frac{1}{2}J = 0.015532$ in^4

Torsion: $\tau = \frac{Tc}{J} = \frac{(240)(0.375)}{0.031063} = 2.897 \times 10^3$ psi $= 2.897$ ksi

Bending: $\sigma = \frac{Mc}{I} = \frac{(144)(0.375)}{0.015532} = 3.477 \times 10^3$ psi $= 3.477$ ksi

Transverse Shear: At point H stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 3.477$ ksi, $\sigma_y = 0$, $\tau_{xy} = 2.897$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738$

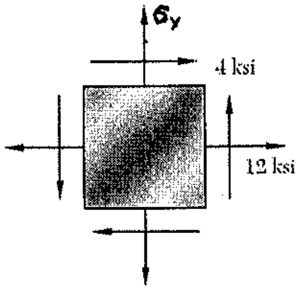
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{1.738^2 + 2.897^2} = 3.378$ ksi

$\sigma_a = \sigma_{ave} + R = 5.116$ ksi

$\sigma_b = \sigma_{ave} - R = -1.640$ ksi

$\tau_{max} = R = 3.378$ ksi

PROBLEM 7.27



7.27 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 15 ksi.

SOLUTION

$\sigma_x = 12$ ksi, $\sigma_y = ?$, $\tau_{xy} = 4$ ksi

Let $u = \dfrac{\sigma_x - \sigma_y}{2}$ $\qquad \sigma_y = \sigma_x - 2u$

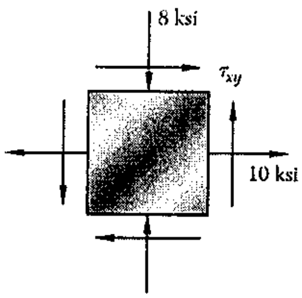
$R = \sqrt{u^2 + \tau_{xy}^2} = 15$ ksi

$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{15^2 - 4^2} = \pm 14.457$ ksi

$\sigma_y = \sigma_x - 2u = 12 \mp (2)(14.457) = 40.9$ ksi, -16.91 ksi

Largest value for σ_y is required. $\quad \sigma_y = 40.9$ ksi ◄

PROBLEM 7.28



7.28 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (*b*) the corresponding principal stresses.

SOLUTION

$\sigma_x = 10$ ksi, $\sigma_y = -8$ ksi, $\tau_{xy} = ?$

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{10-(-8)}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{9^2 + \tau_{xy}^2} = 12 \text{ ksi}$$

(a) $\tau_{xy} = \sqrt{12^2 - 9^2} = 7.94$ ksi ◄

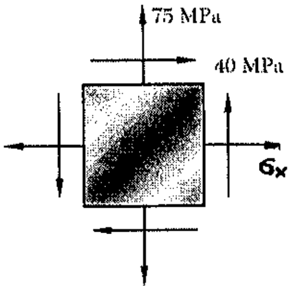
(b) $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1$ ksi

$\sigma_a = \sigma_{ave} + R = 1 + 12 = 13$ ksi ◄

$\sigma_b = \sigma_{ave} - R = 1 - 12 = -11$ ksi ◄

PROBLEM 7.29

7.29 **Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 50 MPa.**



SOLUTION

$\sigma_x = ?\ ,\quad \sigma_y = 75\ \text{MPa}\ ,\quad \tau_{xy} = 40\ \text{MPa}$

Let $\quad u = \dfrac{\sigma_x - \sigma_y}{2} \qquad \sigma_x = \sigma_y + 2u$

$R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{max} = 50\ \text{MPa}$

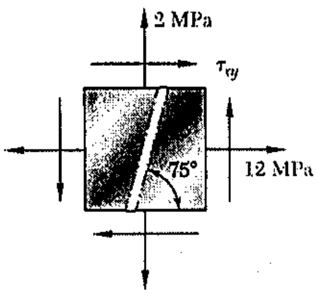
$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{50^2 - 40^2} = \pm 30\ \text{MPa}$

$\sigma_x = \sigma_y + 2u = 75 \pm (2)(30) = 135\ \text{MPa}\ ,\ 15\ \text{MPa}$

Allowable range $\quad 15\ \text{MPa} \le \sigma_x \le 135\ \text{MPa}$ ◄

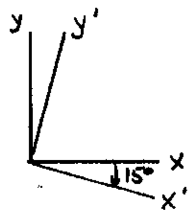
PROBLEM 7.30

7.30 **For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.**



SOLUTION

$\sigma_x = 12\ \text{MPa}\ ,\quad \sigma_y = 2\ \text{MPa}\ ,\quad \tau_{xy} = ?$



Since $\tau_{x'y'} = 0$, x'-direction is a principal direction.

$\theta_p = -15°$

$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

(a) $\quad \tau_{xy} = \frac{1}{2}(\sigma_x - \sigma_y)\tan 2\theta_p = \frac{1}{2}(12 - 2)\tan(-30°) = -2.89\ \text{MPa}$ ◄

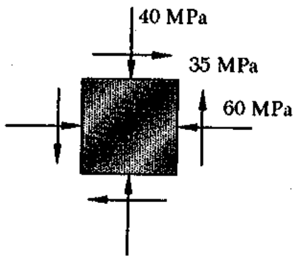
$R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5^2 + 2.89^2} = 5.7735\ \text{MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 7\ \text{MPa}$

(b) $\quad \sigma_a = \sigma_{ave} + R = 7 + 5.7735 = 12.77\ \text{MPa}$ ◄

$\sigma_b = \sigma_{ave} - R = 7 - 5.7735 = 1.226\ \text{MPa}$ ◄

PROBLEM 7.31



7.31 Solve Probs. 7.7 and 7.11, using Mohr's circle.

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = -60 \text{ MPa} \qquad \sigma_y = -40 \text{ MPa} \qquad \tau_{xy} = 35 \text{ MPa}$

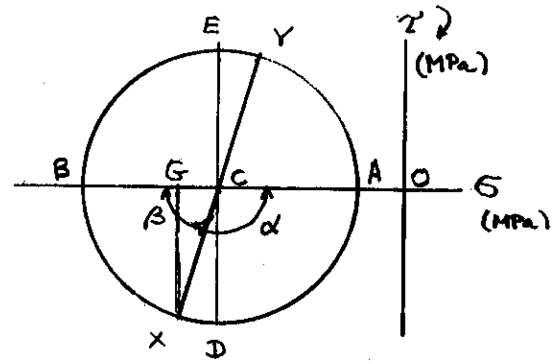
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -50 \text{ MPa}$

Points

$X: (\sigma_x, -\tau_{xy}) = (-60 \text{ MPa}, -35 \text{ MPa})$

$Y: (\sigma_y, \tau_{xy}) = (-40 \text{ MPa}, 35 \text{ MPa})$

$C: (\sigma_{ave}, 0) = (-50 \text{ MPa}, 0)$



$\tan\beta = \dfrac{GX}{CG} = \dfrac{35}{10} = 3.500$

$\beta = 74.05°$

$\theta_B = -\frac{1}{2}\beta = -37.03°$

$\alpha = 180° - \beta = 105.95°$

$\theta_A = \frac{1}{2}\alpha = 52.97°$

$R = \sqrt{\overline{CG}^2 + \overline{GX}^2} = \sqrt{10^2 + 35^2} = 36.4 \text{ MPa}$

$\sigma_{min} = \sigma_{ave} - R = -50 - 36.4 = -86.4 \text{ MPa}$

$\sigma_{max} = \sigma_{ave} + R = -50 + 36.4 = -13.6 \text{ MPa}$

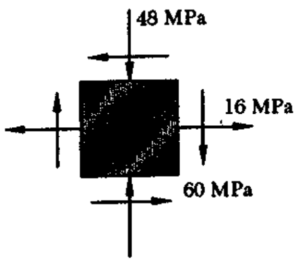
$\theta_D = \theta_B + 45° = 7.97°$

$\theta_E = \theta_A + 45° = 97.97°$

$\tau_{max} = R = 36.4 \text{ MPa}$

$\sigma' = \sigma_{ave} = -50 \text{ MPa}$

PROBLEM 7.32



7.32 Solve Probs. 7.8 and 7.12, using Mohr's circle.

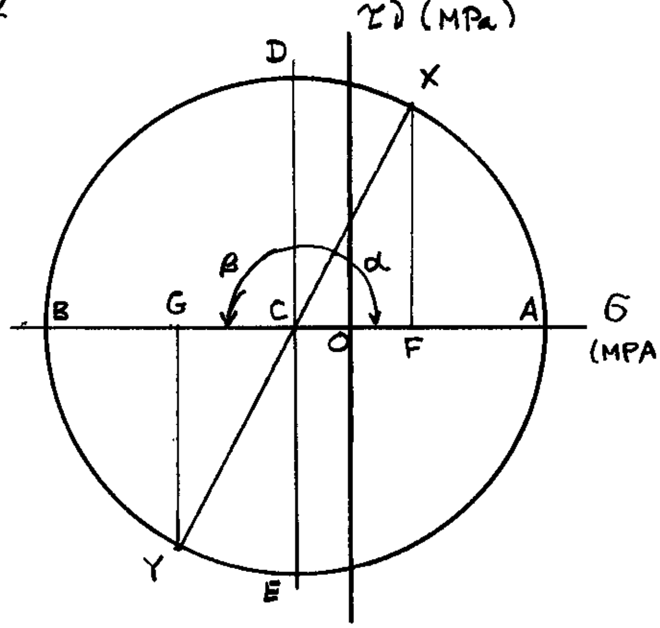
7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = 16\ \text{MPa} \qquad \sigma_y = -48\ \text{MPa} \qquad \tau_{xy} = -60\ \text{MPa}$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -16\ \text{MPa}$$



Points:

X: $(\sigma_x, -\tau_{xy}) = (16\,\text{MPa}, 60\,\text{MPa})$

Y: $(\sigma_y, \tau_{xy}) = (-48\,\text{MPa}, -60\,\text{MPa})$

C: $(\sigma_{ave}, 0) = (-16\,\text{MPa}, 0)$

$$\tan\alpha = \frac{FX}{CF} = \frac{60}{32} = 1.875$$

$\alpha = 61.93°$

$\theta_A = -\frac{1}{2}\alpha = -30.96°$ ◄

$\beta = 180° - \alpha = 118.07°$

$\theta_B = \frac{1}{2}\beta = 59.04°$ ◄

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{32^2 + 60^2} = 68\ \text{MPa}$

$\sigma_A = \sigma_{max} = \sigma_{ave} + R = -16 + 68 = 52\ \text{MPa}$ ◄

$\sigma_B = \sigma_{min} = \sigma_{ave} - R = -16 - 68 = -84\ \text{MPa}$ ◄

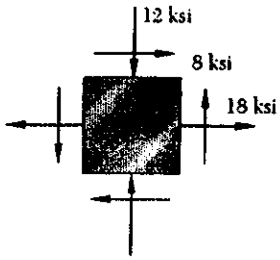
$\theta_D = \theta_A + 45° = 14.04°$ ◄

$\theta_E = \theta_B + 45° = 104.04°$ ◄

$\tau_{max} = R = 68\ \text{MPa}$ ◄

$\sigma' = \sigma_{ave} = -16\ \text{MPa}$ ◄

PROBLEM 7.33



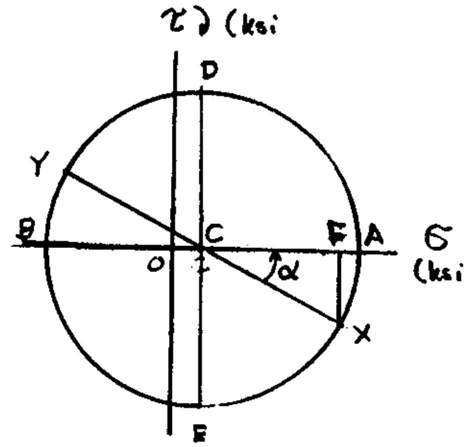
7.33 Solve Prob. 7.9, using Mohr's circle.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = 18$ ksi $\qquad \sigma_y = -12$ ksi $\qquad \tau_{xy} = 8$ ksi

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 3 \text{ ksi}$$



Points

X: $(\sigma_x, -\tau_{xy}) = (18 \text{ ksi}, -8 \text{ ksi})$

Y: $(\sigma_y, \tau_{xy}) = (-12 \text{ ksi}, 8 \text{ ksi})$

C: $(\sigma_{ave}, 0) = (3 \text{ ksi}, 0)$

$$\tan \alpha = \frac{FX}{CF} = \frac{8}{15} = 0.5333$$

$\alpha = 28.07°$

$\theta_A = \frac{1}{2}\alpha = 14.04°$

$\theta_D = \theta_A + 45° = 59.04°$ ◄

$\theta_E = \theta_A - 45° = -30.96°$ ◄

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{15^2 + 8^2} = 17$ ksi

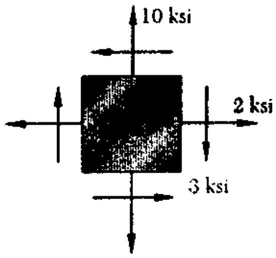
$\tau_{max} = R = 17$ ksi ◄

$\sigma' = \sigma_{ave} = 3$ ksi ◄

PROBLEM 7.34

7.34 Solve Prob. 7.10, using Mohr's circle.

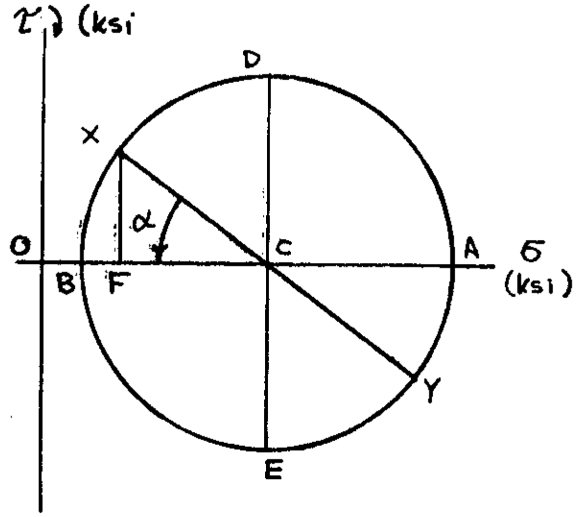
7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



SOLUTION

$\sigma_x = 2$ ksi $\quad \sigma_y = 10$ ksi $\quad \tau_{xy} = -3$ ksi

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{2+10}{2} = 6 \text{ ksi}$$



Points

X: $(\sigma_x, -\tau_{xy}) = (2 \text{ ksi}, 3 \text{ ksi})$

Y: $(\sigma_y, \tau_{xy}) = (10 \text{ ksi}, -3 \text{ ksi})$

C: $(\sigma_{ave}, 0) = (6 \text{ ksi}, 0)$

$$\tan \alpha = \frac{FX}{FC} = \frac{3}{4} = 0.75$$

$\alpha = 36.87°$

$\theta_B = \frac{1}{2}\alpha = 18.43°$

$\theta_D = \theta_B - 45° = -26.57°$ ◄

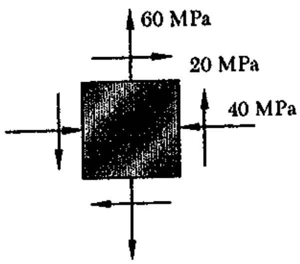
$\theta_E = \theta_B + 45° = 63.43°$ ◄

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{4^2 + 3^2} = 5$ ksi

$\tau_{max} = R = 5$ ksi ◄

$\sigma' = \sigma_{ave} = 6$ ksi ◄

PROBLEM 7.35



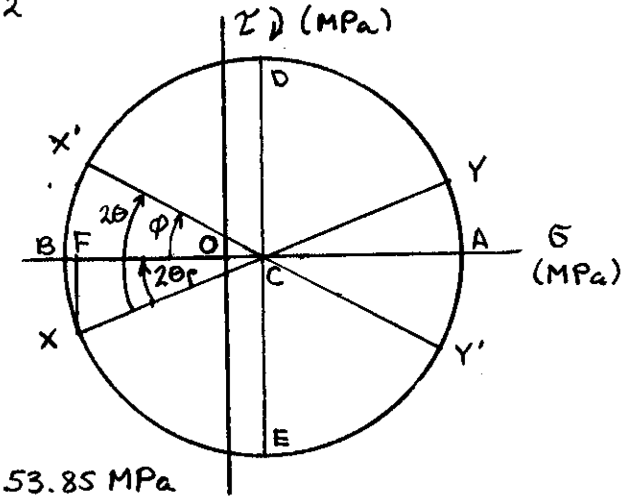
7.35 Solve Prob. 7.13, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$\sigma_x = -40 \text{ MPa} \qquad \sigma_y = 60 \text{ MPa} \qquad \tau_{xy} = 20 \text{ MPa}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 10 \text{ MPa}$



Points

X: (−40 MPa, −20 MPa)

Y: (60 MPa, 20 MPa)

C: (10 MPa, 0)

$\tan 2\theta_p = \dfrac{FX}{FC} = \dfrac{20}{50} = 0.4$

$2\theta_p = 21.80° \qquad \theta_p = 10.90°$

$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{50^2 + 20^2} = 53.85 \text{ MPa}$

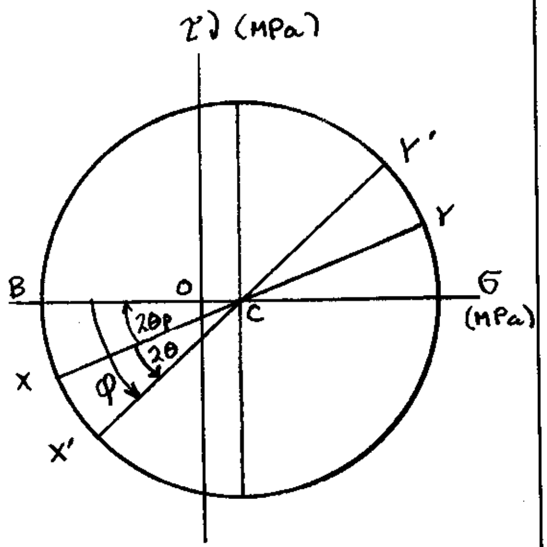
(a) $\theta = 25° \qquad 2\theta = 50°$

$\varphi = 2\theta - 2\theta_p = 50° - 21.80° = 28.20°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = -37.5 \text{ MPa}$ ◄

$\tau_{x'y'} = -R\sin\varphi = -25.4 \text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 57.5 \text{ MPa}$ ◄



(b) $\theta = 10° \qquad 2\theta = 20°$

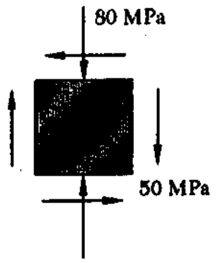
$\varphi = 2\theta_p + 2\theta = 21.80° + 20° = 41.80°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = -30.1 \text{ MPa}$ ◄

$\tau_{x'y'} = R\sin\varphi = 35.9 \text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 50.1 \text{ MPa}$ ◄

PROBLEM 7.36



7.36 **Solve Prob. 7.14, using Mohr's circle.**

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$\sigma_x = 0 \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = -50 \text{ MPa}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$

Points

X: (0, 50 MPa)

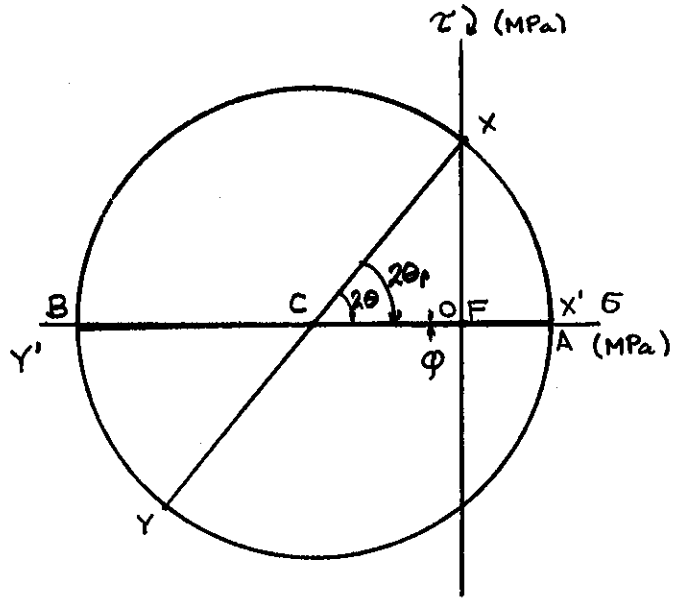
Y: (-80 MPa, -50 MPa)

C: (-40 MPa, 0)

$\tan 2\theta_p = \dfrac{FX}{CF} = \dfrac{50}{40} = 1.25$

$2\theta_p = 51.34°$

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{40^2 + 50^2}$
$= 64.03 \text{ MPa}$



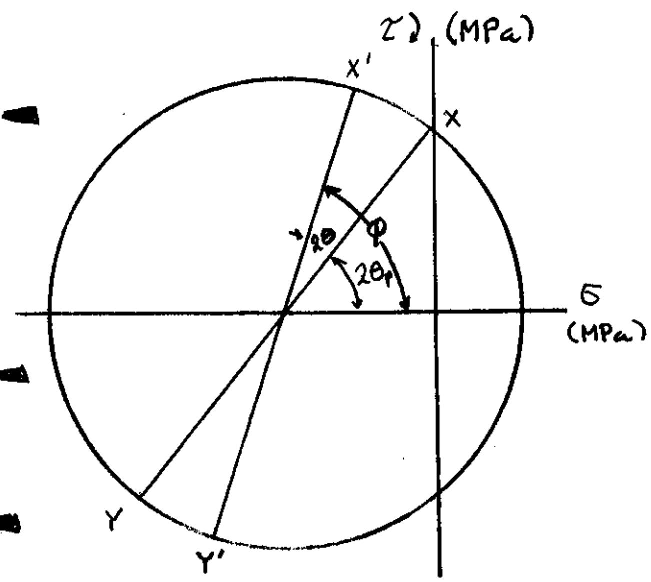
(a) $\theta = 25°$ ↻ $\quad 2\theta = 50°$ ↻

$\varphi = 51.34° - 50° = 1.34°$ ↺

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = 24.0 \text{ MPa}$ ◄

$\tau_{x'y'} = -R\sin\varphi = -1.5 \text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -104.0 \text{ MPa}$ ◄



(b) $\theta = 10°$ ↺ $\quad 2\theta = 20°$ ↺

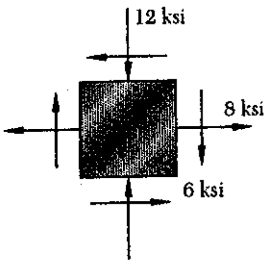
$\varphi = 51.34° + 20° = 71.34°$

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = -19.5 \text{ MPa}$ ◄

$\tau_{x'y'} = -R\sin\varphi = -60.7 \text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -60.5 \text{ MPa}$ ◄

PROBLEM 7.37



7.37 Solve Prob. 7.15, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$\sigma_x = 8$ ksi $\qquad \sigma_y = -12$ ksi $\qquad \tau_{xy} = -6$ ksi

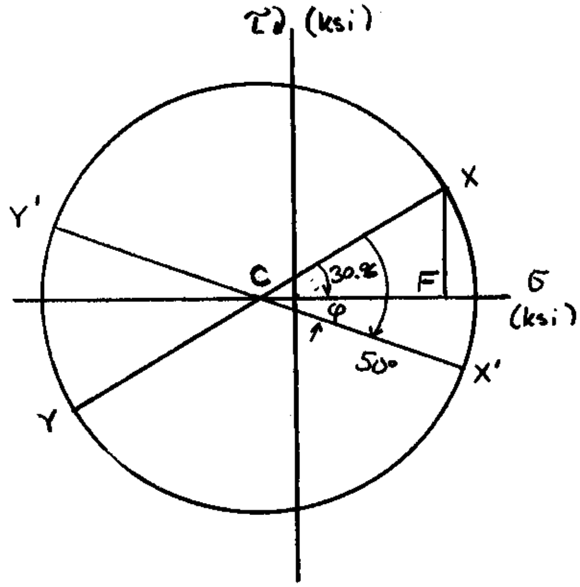
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -2 \text{ ksi}$$

Points

X: (8 ksi, 6 ksi)

Y: (-12 ksi, -6 ksi)

C: (-2 ksi, 0)



$$\tan 2\theta_p = \frac{FX}{CF} = \frac{6}{10} = 0.6$$

$$2\theta_p = 30.96°$$

$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{10^2 + 6^2} = 11.66 \text{ ksi}$$

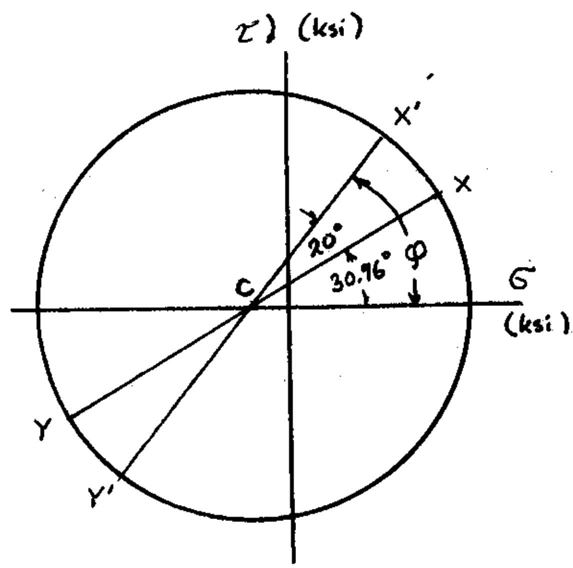
(a) $\theta = 25°$ ↻ $\qquad 2\theta = 50°$ ↻

$\varphi = 50° - 30.96° = 19.04°$

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = 9.02$ ksi ◄

$\tau_{x'y'} = R\sin\varphi = 3.80$ ksi ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -13.02$ ksi ◄



(b) $\theta = 10°$ ↺ $\qquad 2\theta = 20°$ ↺

$\varphi = 30.96° + 20° = 50.96°$

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = 5.34$ ksi ◄

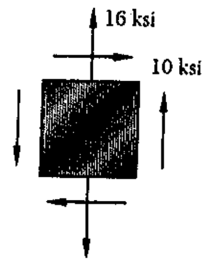
$\tau_{x'y'} = -R\sin\varphi = -9.06$ ksi ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -9.34$ ksi ◄

PROBLEM 7.38

7.38 Solve Prob. 7.16, using Mohr's circle.

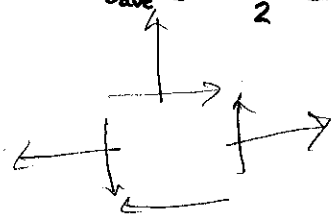
7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

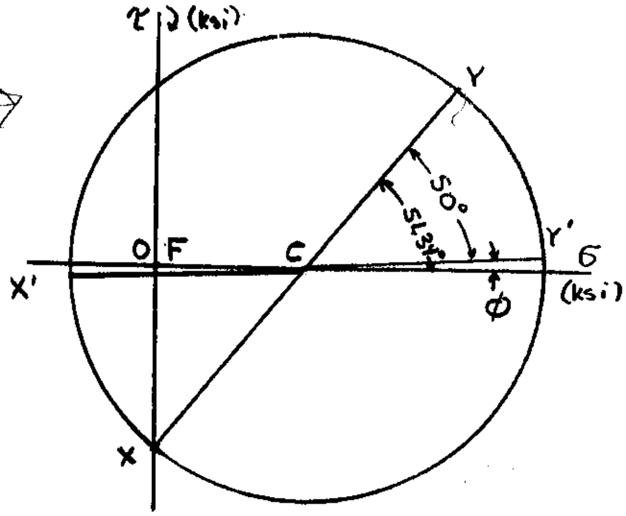


SOLUTION

$\sigma_x = 0 \qquad \sigma_y = 16 \text{ ksi} \qquad \tau_{xy} = 10 \text{ ksi}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 8 \text{ ksi}$





Points:
X: (0, −10 ksi)
Y: (16 ksi, 10 ksi)
C: (8 ksi, 0)

$\tan 2\theta_p = \dfrac{FX}{FC} = \dfrac{10}{8} = 1.25$

$2\theta_p = 51.34°$

$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{8^2 + 10^2}$
$= 12.81 \text{ ksi}$

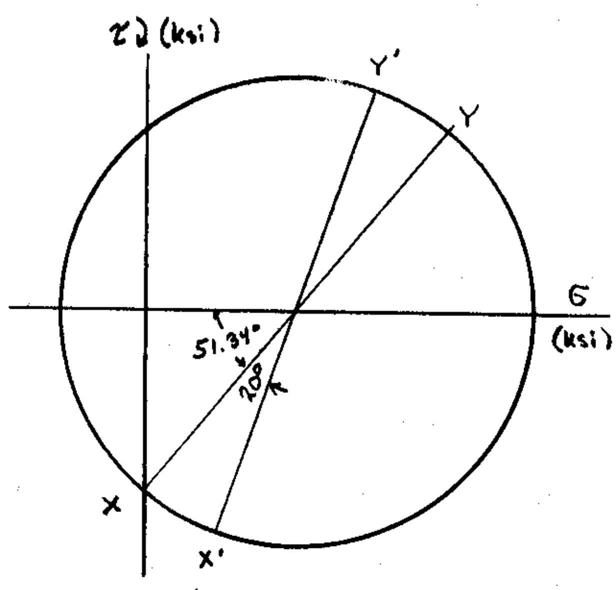
(a) $\theta = 25°$ ↻ $\quad 2\theta = 50°$ ↻

$\varphi = 51.34° - 50° = 1.34°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = -4.81 \text{ ksi}$ ◄

$\tau_{x'y'} = R\sin\varphi = 0.30 \text{ ksi}$ ◄

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 20.81 \text{ ksi}$ ◄



(b) $\theta = 10°$ ↺ $\quad 2\theta = 20°$ ↺

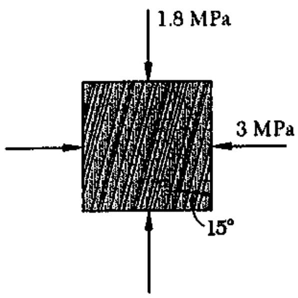
$\varphi = 51.34° + 20° = 71.34°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = 3.90 \text{ ksi}$ ◄

$\tau_{x'y'} = R\sin\varphi = 12.14 \text{ ksi}$ ◄

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 12.10 \text{ ksi}$ ◄

PROBLEM 7.39



7.39 Solve Prob. 7.17, using Mohr's circle.

7.17 and *7.18* The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$\sigma_x = -3\text{ MPa} \qquad \sigma_y = -1.8\text{ MPa} \qquad \tau_{xy} = 0$

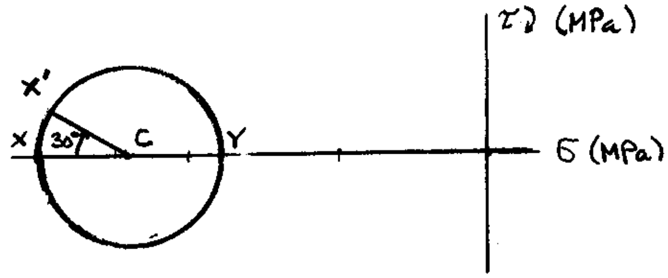
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -2.4\text{ MPa}$

Points

X: $(\sigma_x, -\tau_{xy}) = (-3\text{ Mpa}, 0)$

Y: $(\sigma_y, \tau_{xy}) = (-1.8\text{ MPa}, 0)$

C: $(\sigma_{ave}, 0) = (-2.4\text{ MPa}, 0)$



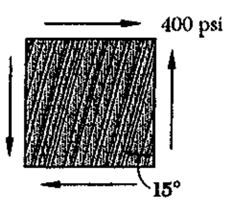
$\theta = -15° \qquad 2\theta = -30°$

$\overline{CX} = 0.6\text{ MPa} \qquad R = 0.6\text{ MPa}$

(a) $\tau_{x'y'} = -\overline{CX}'\sin 30 = -R\sin 30° = -0.6\sin 30° = -0.3\text{ MPa}$

(b) $\sigma_{x'} = \sigma_{ave} - \overline{CX}'\cos 30° = -2.4 - 0.6\cos 30° = -2.92\text{ MPa}$

PROBLEM 7.40



7.40 Solve Prob. 7.18, using Mohr's circle.

7.17 and *7.18* The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = 400\text{ psi}$

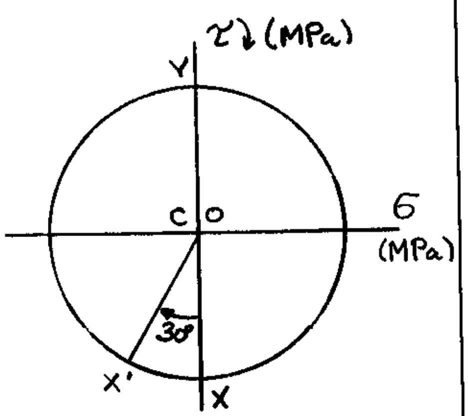
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 0$

Points

X: $(\sigma_x, -\tau_{xy}) = (0, -400\text{ psi})$

Y: $(\sigma_y, \tau_{xy}) = (0, 400\text{ psi})$

C: $(\sigma_{ave}, 0) = (0, 0)$



$\theta = -15° \qquad 2\theta = -30°$

$\overline{CX} = R = 400\text{ psi}$

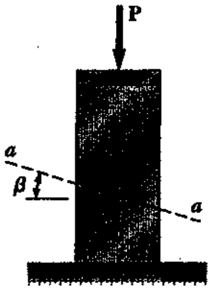
(a) $\tau_{x'y'} = R\cos 30° = 400\cos 30° = 346\text{ psi}$

(b) $\sigma_{x'} = \sigma_{ave} - R\sin 30° = -400\sin 30° = -200\text{ psi}$

PROBLEM 7.41

7.41 Solve Prob. 7.19, using Mohr's circle.

7.19 The centric force **P** is applied to a short post as shown. Knowing that the stresses on plane *a-a* are $\sigma = -15$ ksi and $\tau = 5$ ksi, determine (*a*) the angle β that plane *a-a* forms with the horizontal, (*b*) the maximum compressive stress in the post.

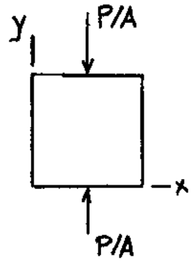


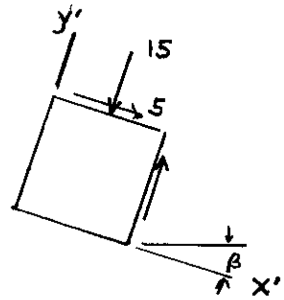
SOLUTION

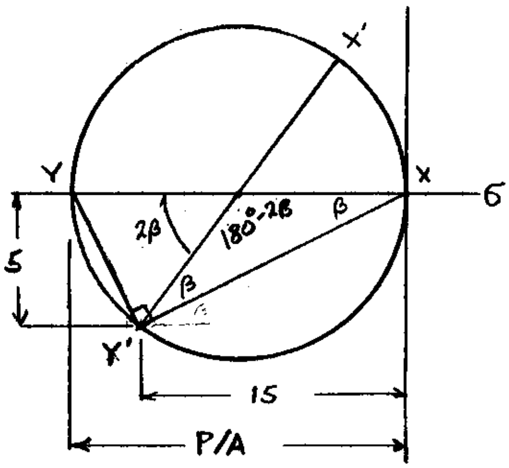
$\sigma_x = 0$

$\tau_{xy} = 0$

$\sigma_y = -P/A$







From the Mohr's circle

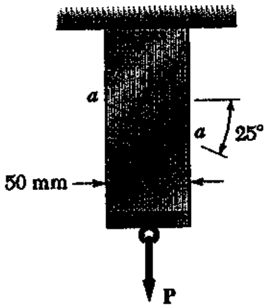
$\tan\beta = \frac{5}{15} = 0.3333$ $\qquad \beta = 18.4°$

$-\sigma = \frac{P}{2A} + \frac{P}{2A}\cos 2\beta$

$\frac{P}{A} = \frac{2(-\sigma)}{1+\cos 2\beta} = \frac{(2)(15)}{1+\cos 2\beta}$

$= 16.67$ ksi

PROBLEM 7.42



7.42 Solve Prob. 7.20, using Mohr's circle.

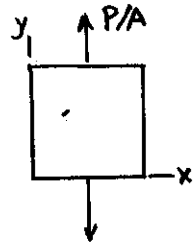
7.20 Two members of uniform cross section 50 × 80 mm are glued together along plane *a-a*, which forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest axial load **P** that can be applied.

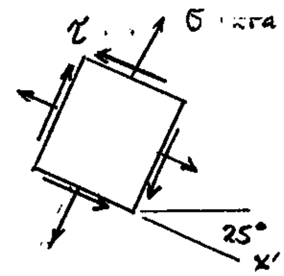
SOLUTION

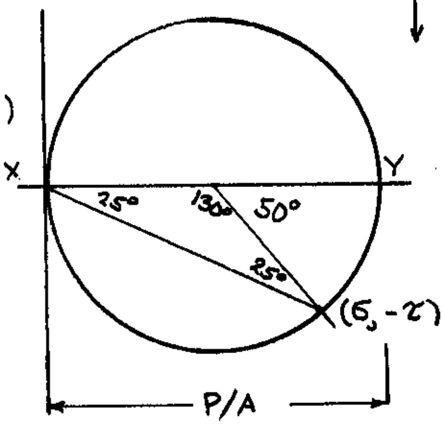
$\sigma_x = 0$

$\tau_{xy} = 0$

$\sigma_y = P/A$







$A = (50\times10^{-3})(80\times10^{-3})$
$= 4\times10^{-3}\ \text{m}^2$

$$\sigma = \frac{P}{2A}(1+\cos 50°)$$

$$P = \frac{2A\sigma}{1+\cos 50°}$$

$$P \le \frac{(2)(4\times10^{-3})(800\times10^{3})}{1+\cos 50°}$$

$$P \le 3.90\times10^{3}\ \text{N}$$

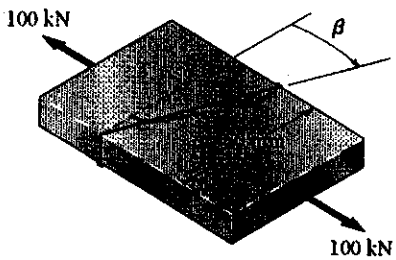
$$\tau = \frac{P}{2A}\sin 50° \qquad P = \frac{2A\tau}{\sin 50°} \le \frac{(2)(4\times10^{-3})(600\times10^{3})}{\sin 50°} = 6.27\times10^{3}\ \text{N}$$

Choosing the smaller value $P \le 3.90\times10^{3}\ \text{N} = 3.90\ \text{kN}$

PROBLEM 7.43

7.43 Solve Prob. 7.21, using Mohr's circle.

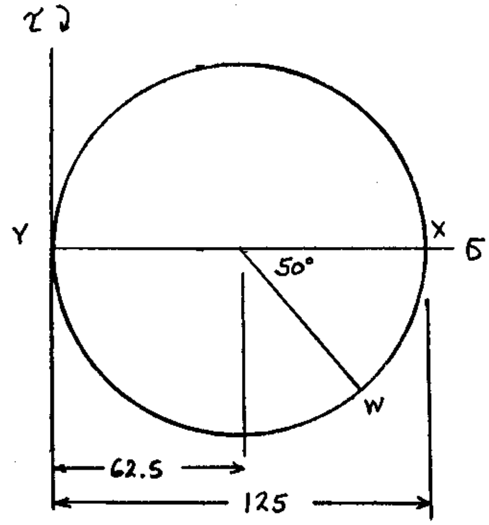
7.21 Two steel plates of uniform cross section 10 × 80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25°$, determine (*a*) the in-plane shearing stress parallel to the weld, (*b*) the normal stress perpendicular to the weld.



SOLUTION

$$\sigma_x = \frac{P}{A} = \frac{100 \times 10^3}{(10\times10^{-3})(80\times10^{-3})} = 125\times10^6 \text{ Pa} = 125 \text{ MPa}$$

$$\sigma_y = 0 \qquad \tau_{xy} = 0$$



From Mohr's circle

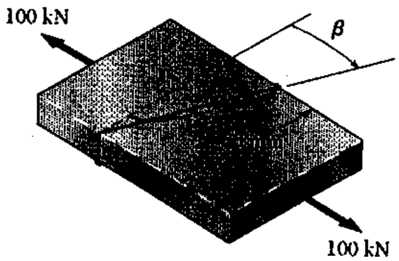
(a) $\tau_w = 62.5 \sin 50° = 47.9 \text{ MPa}$

(b) $\sigma_w = 62.5 + 62.5 \cos 50°$
$= 102.7 \text{ MPa}$

PROBLEM 7.44

7.44 Solve Prob. 7.22, using Mohr's circle.

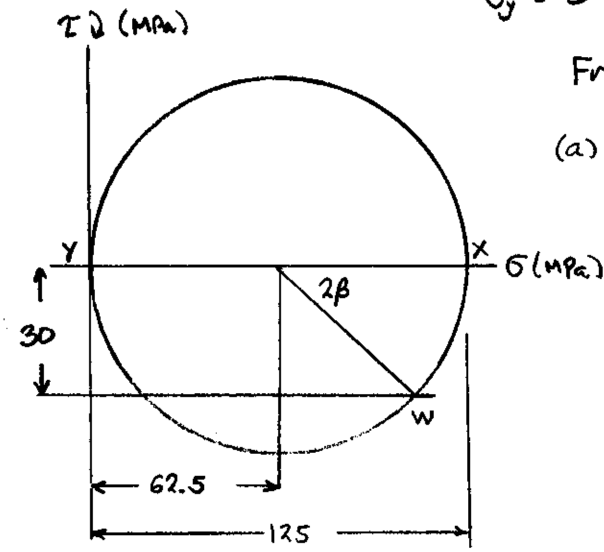
7.22 Two steel plates of uniform cross section 10 × 80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (*a*) the angle β, (*b*) the corresponding normal stress perpendicular to the weld.



SOLUTION

$$\sigma_x = \frac{P}{A} = \frac{100\times10^3}{(10\times10^{-3})(80\times10^{-3})} = 125\times10^6 \text{ Pa} = 125 \text{ MPa}$$

$\sigma_y = 0 \qquad \tau_{xy} = 0$



From Mohr's circle

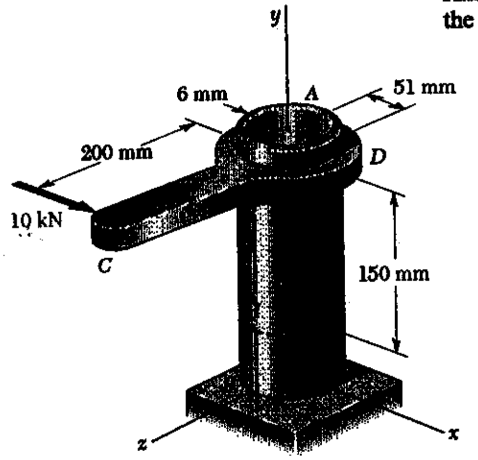
(a) $\sin 2\beta = \frac{30}{62.5} = 0.48 \qquad \beta = 14.3°$ ◄

(b) $\sigma = 62.5 + 62.5\cos 2\beta$
$= 117.3 \text{ MPa}$ ◄

PROBLEM 7.45

7.45 Solve Prob. 7.23, using Mohr's circle.

7.23 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point H.



SOLUTION

$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855\times10^6 \text{ mm}^4 = 4.1855\times10^{-6} \text{ m}^4$

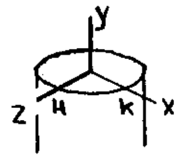
$I = \frac{1}{2}J = 2.0927\times10^{-6} \text{ m}^4$

Force-couple system at center of tube in the plane containing points H and K

$F_x = 10\times10^3 \text{ N}$

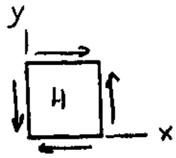
$M_y = (10\times10^3)(200\times10^{-3}) = 2000 \text{ N·m}$

$M_z = -(10\times10^3)(150\times10^{-3}) = -1500 \text{ N·m}$

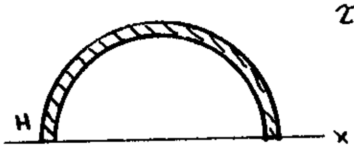


Torsion: $T = M_y = 2000 \text{ N-m}$

$c = r_o = 51\times10^{-3} \text{ m}$



$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51\times10^{-3})}{4.1855\times10^{-6}} = 24.37 \text{ MPa}$$



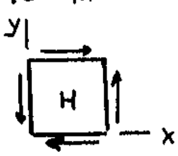
Transverse Shear:
For semicircle $\quad A = \frac{\pi}{2}r^2 \quad \bar{y} = \frac{4}{3\pi}r$

$Q = A\bar{y} = \frac{2}{3}r^3$

For pipe $\quad Q = Q_o - Q_i = \frac{2}{3}r_o^3 - \frac{2}{3}r_i^3 = 27.684\times10^3 \text{ mm}^3 = 27.684\times10^{-6} \text{ m}^3$

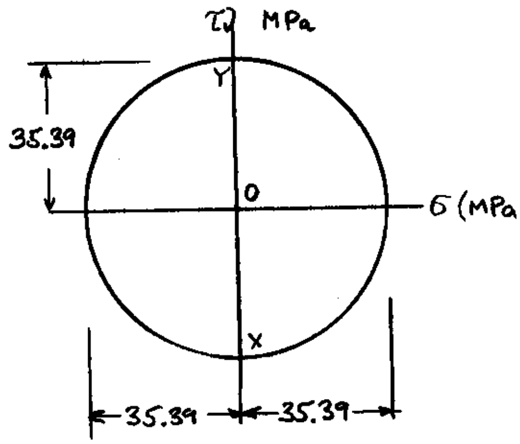
$V = F_x = 10\times10^3 \text{ N} \qquad t = (2)(6) = 12 \text{ mm} = 12\times10^{-3} \text{ m}$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(10\times10^3)(27.684\times10^{-6})}{(2.0927\times10^{-6})(12\times10^{-3})} = 11.02 \text{ MPa}$$



Bending: Point H lies on neutral axis $\quad \sigma_y = 0$

Total stresses at point H $\quad \sigma_x = 0, \sigma_y = 0 \qquad \tau_{xy} = 24.37 + 11.02 = 35.39 \text{ MPa}$



$\sigma_{ave} = 0$

$R = 35.39 \text{ MPa}$

$\sigma_{max} = \sigma_{ave} + R = 35.39 \text{ MPa}$

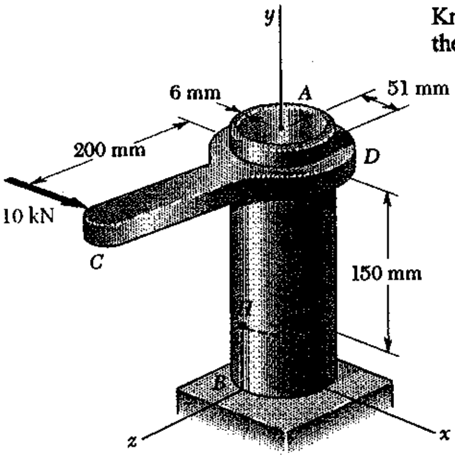
$\sigma_{min} = \sigma_{ave} - R = -35.39 \text{ MPa}$

$\tau_{max} = R = 35.39 \text{ MPa}$

PROBLEM 7.46

7.46 Solve Prob. 7.24, using Mohr's circle.

7.24 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.



SOLUTION

$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$

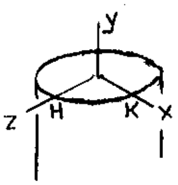
$I = \frac{1}{2} J = 2.0927 \times 10^{-6} \text{ m}^4$

Force-couple system at center of tube in the plane containing points H and K

$F_x = 10 \times 10^3 \text{ N}$

$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N·m}$

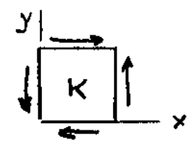
$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N·m}$



Torsion: $T = M_y = 2000 \text{ N·m}$

$c = r_o = 51 \times 10^{-3} \text{ m}$

$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \text{ MPa}$



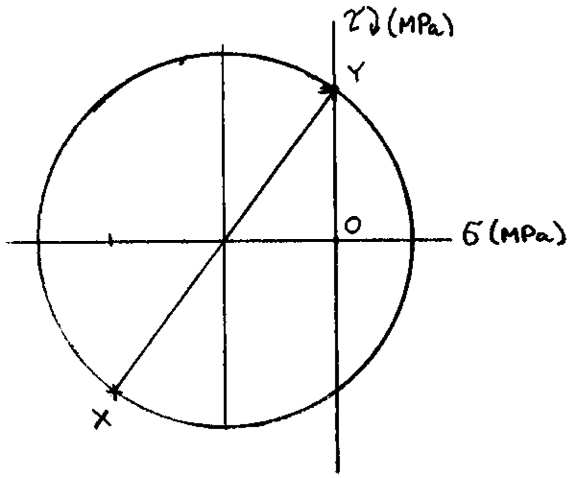
Note that local x-axis is taken along negative global z-direction.

Transverse Shear: Stress due to $V = F_x$ is zero at point K.

Bending: $|\sigma_y| = \frac{|M_z| c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56 \text{ MPa}$

Point K lies on compression side of neutral axis. $\sigma_y = -36.56 \text{ MPa}$

Total stresses at point K $\quad \sigma_x = 0, \ \sigma_y = -36.56 \text{ MPa}, \ \tau_{xy} = 24.37 \text{ MPa}$



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$

$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46 = 12.18 \text{ MPa}$

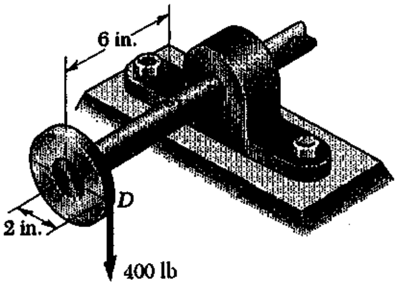
$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46 = -48.74 \text{ MPa}$

$\tau_{max} = R = 30.46 \text{ MPa}$

PROBLEM 7.47

7.47 Solve Prob. 7.25, using Mohr's circle.

7.25 A 400-lb vertical force is applied at D to a gear attached to the solid one-inch diameter shaft AB. Determine the principal stresses and the maximum shearing stress at point H located as shown on top of the shaft.



SOLUTION

Equivalent force-couple system at center of shaft in section at point H.

$V = 400$ lb. $\qquad M = (400)(6) = 2400$ lb·in

$T = (400)(2) = 800$ lb·in.

Shaft cross section $\qquad d = 1$ in $\quad c = \frac{1}{2}d = 0.5$ in.

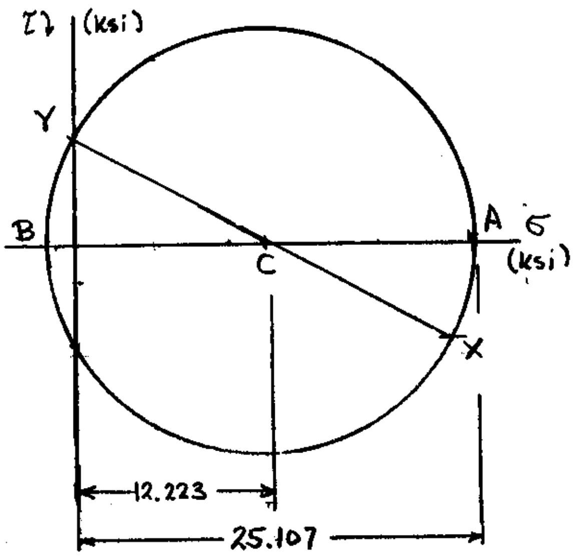
$J = \frac{\pi}{2}c^4 = 0.098175 \text{ in}^4 \qquad I = \frac{1}{2}J = 0.049087 \text{ in}^4$

Torsion: $\quad \tau = \dfrac{Tc}{J} = \dfrac{(800)(0.5)}{0.098175} = 4.074 \times 10^3 \text{ psi} = 4.074 \text{ ksi}$

Bending: $\quad \sigma = \dfrac{Mc}{I} = \dfrac{(2400)(0.5)}{0.049087} = 24.446 \times 10^3 \text{ psi} = 24.446 \text{ ksi}$

Transverse Shear: $\quad$ Stress at point H is zero.

Resultant stresses: $\quad \sigma_x = 24.446$ ksi, $\sigma_y = 0$, $\tau_{xy} = 4.074$ ksi



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(12.223)^2 + (4.074)^2} = 12.884$ ksi

$\sigma_a = \sigma_{ave} + R = 25.107$ ksi ◄

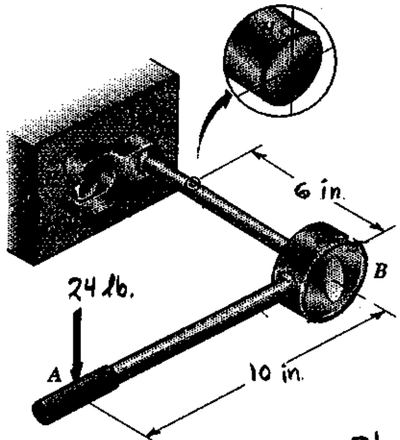
$\sigma_b = \sigma_{ave} - R = -0.661$ ksi ◄

$\tau_{max} = R = 12.88$ ksi ◄

PROBLEM 7.48

7.48 Solve Prob. 7.26, using Mohr's circle.

7.26 A mechanic uses a crowfoot wrench to loosen at bolt at E. Knowing that the mechanic applies a vertical 24-lb force at A, determine the principal stresses and the maximum shearing stress at point H located as shown on top of the $\frac{3}{4}$ - in. diameter shaft.



SOLUTION

Equivalent force-couple system at center of shaft in section at point H.

$V = 24$ lb. $\quad M = (24)(6) = 144$ lb·in

$T = (24)(10) = 240$ lb·in

Shaft cross section: $\quad d = 0.75$ in. $\quad c = \frac{1}{2}d = 0.375$ in

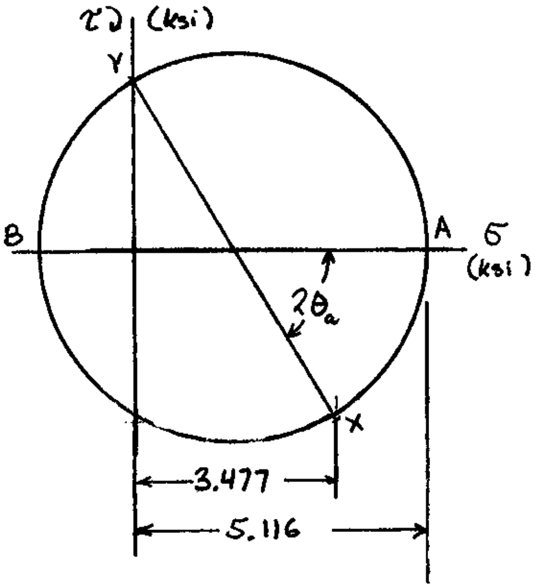
$J = \frac{\pi}{2}c^4 = 0.031063 \text{ in}^4 \qquad I = \frac{1}{2}J = 0.015532 \text{ in}^4$

Torsion: $\tau = \dfrac{Tc}{J} = \dfrac{(240)(0.375)}{0.031063} = 2.897 \times 10^3 \text{ psi} = 2.897$ ksi

Bending: $\sigma = \dfrac{Mc}{I} = \dfrac{(144)(0.375)}{0.015532} = 3.477 \times 10^3 \text{ psi} = 3.477$ ksi

Transverse Shear: At point H stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 3.477$ ksi, $\sigma_y = 0$, $\tau_{xy} = 2.897$ ksi



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{1.738^2 + 2.897^2} = 3.378$ ksi

$\sigma_a = \sigma_{ave} + R = 5.116$ ksi

$\sigma_b = \sigma_{ave} - R = -1.640$ ksi

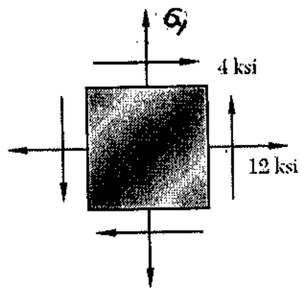
$\tau_{max} = R = 3.378$ ksi

The center of the Mohr's circle lies at point C

PROBLEM 7.49

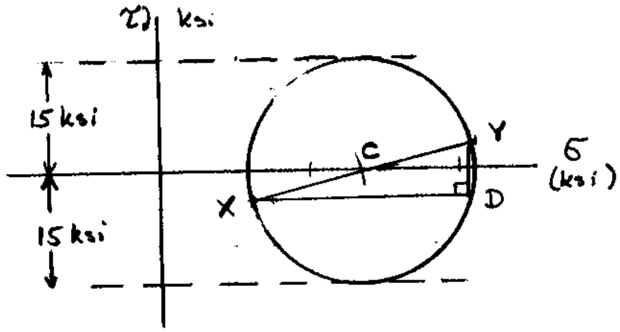
7.49 Solve Prob. 7.27, using Mohr's circle.

7.27 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 15 ksi.



SOLUTION

$\sigma_x = 12$ ksi , $\sigma_y = ?$, $\tau_{xy} = 4$ ksi



Given: $\tau_{max} = R = 15$ ksi

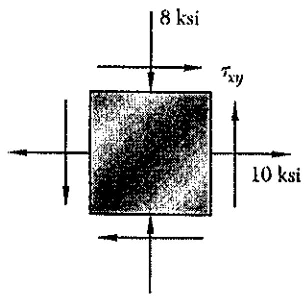
$\overline{XY} = 2R = 30$ ksi

$\overline{DY} = (2)(\tau_{xy}) = 8$ ksi

$\overline{XD} = \sqrt{\overline{XY}^2 - \overline{DY}^2} = \sqrt{30^2 - 8^2} = 28.9$ ksi

$\sigma_y = \sigma_x + \overline{XD} = 12 + 28.9 = 40.9$ ksi

PROBLEM 7.50

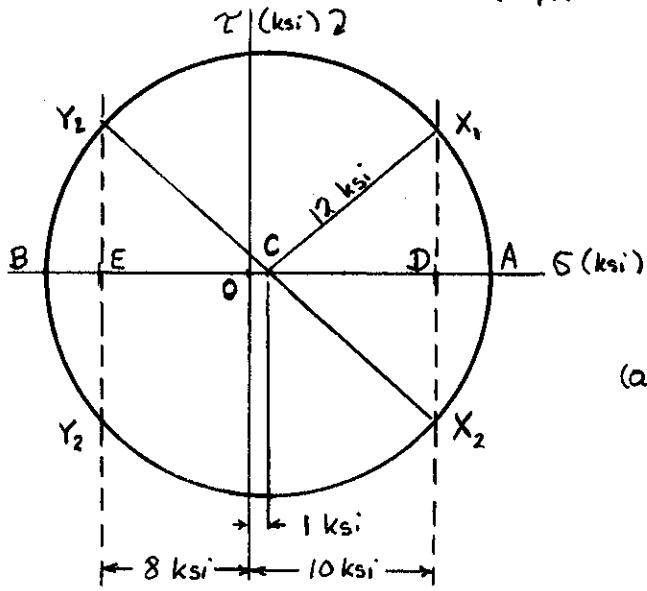


***7.50** Solve Prob. 7.28, using Mohr's circle.

7.28 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (*b*) the corresponding principal stresses.

SOLUTION

The center of the Mohr's circle lies at point C with coordinates $\left(\frac{\sigma_x+\sigma_y}{2}, 0\right) = \left(\frac{10-8}{2}, 0\right)$ $= (1.0 \text{ ksi}, 0)$. The radius of the circle is $\tau_{max(in\text{-}plane)} = 12$ ksi.



The stress point $(\sigma_x, -\tau_{xy})$ lie along the line X_1X_2 of th Mohr circle diagram. The extreme points with $R \leq 12$ ksi are X_1 and X_2.

(a) The largest allowable value of τ_{xy} is obtained from triangle CDX_1

$$\overline{DX_1}^2 = \overline{DX_2}^2 = \sqrt{\overline{CX_1}^2 - \overline{CD}^2}$$

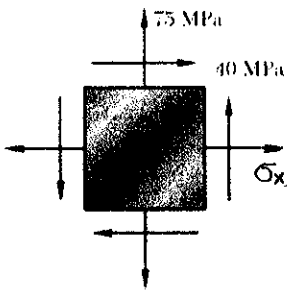
$$\tau_{xy}^2 = \sqrt{12^2 - 9^2} = 7.94 \text{ ksi}$$

(b) The principal stresses are

$$\sigma_a = 1 + 12 = 13 \text{ ksi}$$

$$\sigma_b = 1 - 12 = -11 \text{ ksi}$$

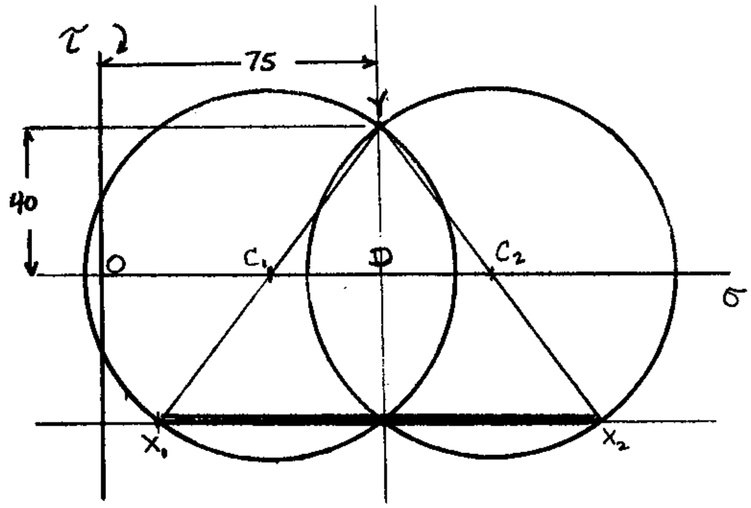
PROBLEM 7.51



7.51 Solve Prob. 7.29, using Mohr's circle.

7.29 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 50 MPa.

SOLUTION



For the Mohr's circle, point Y lies at (75 MPa, 40 MPa).

The radius of limiting circles is $R = 50$ MPa

Let C_1 be the location of the left most limiting circle and C_2 be that of the right most one.

$\overline{C_1Y} = 50$ MPa

$\overline{C_2Y} = 50$ MPa

Noting right triangles C_1DY and C_2DY

$\overline{C_1D}^2 + \overline{DY}^2 = \overline{C_1Y}^2 \qquad \overline{C_1D}^2 = 40^2 = 50^2 \qquad \overline{C_1D} = 30$

Coordinates of point C_1 are $(0, 75-30) = (0, 45 \text{ MPa})$

Likewise, coordinates of point C_2 are $= (0, 75+30) = (0, 105 \text{ MPa})$

Coordinates of point X_1 $\quad (45-30, -40) = (15 \text{ MPa}, -40 \text{ MPa})$

Coordinates of point X_2 $\quad (105+30, -40) = (135 \text{ MPa}, -40 \text{ MPa})$

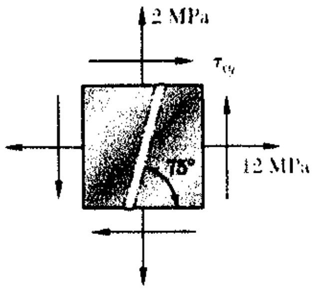
The point $(\sigma_x, -\tau_{xy})$ must lie on the line X_1X_2

Thus $\qquad 15 \text{ MPa} \leq \sigma_x \leq 135 \text{ MPa}$

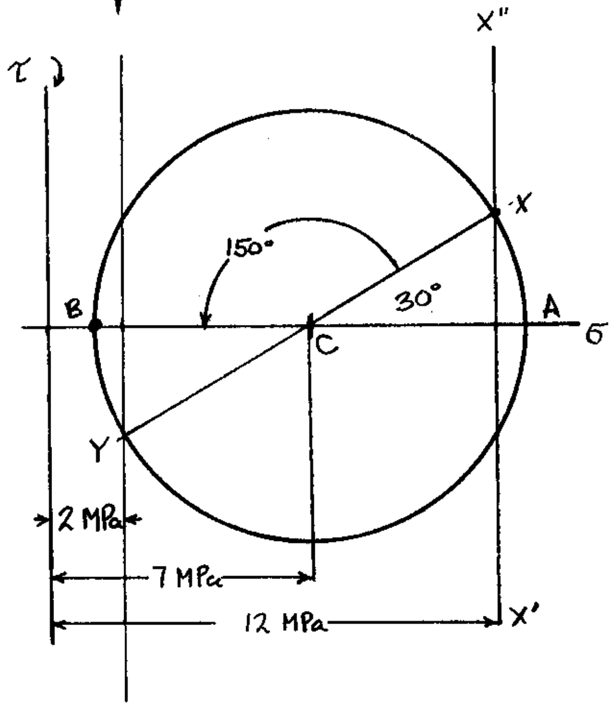
PROBLEM 7.52

7.52 Solve Prob. 7.30, using Mohr's circle.

7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.



SOLUTION



Point X of Mohr's circle must lie on X'X" so that $\sigma_x = 12$ MPa. Likewise, point Y lies on line Y'Y" so that $\sigma_y = 2$ MPa. The coordinates of C are

$$\frac{2+12}{2},\ 0 = (7\text{ MPa}, 0).$$

Counterclockwise rotation through 150° brings line CX to CB, where $\tau = 0$.

$$R = \frac{\sigma_x - \sigma_y}{2}\sec 30°$$

$$= \frac{12-2}{2}\sec 30°$$

$$= 5.77\text{ MPa}$$

$$\tau_{xy} = -\frac{\sigma_x - \sigma_y}{2}\tan 30°$$

$$= -\frac{12-2}{2}\tan 30°$$

$$= -2.89\text{ MPa}$$ ◄

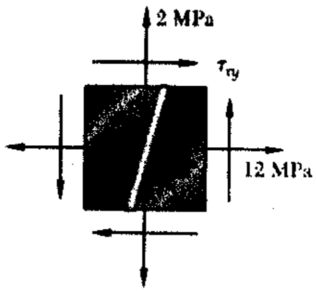
$$\sigma_A = \sigma_{ave} + R = 7 + 5.77 = 12.77\text{ MPa}$$ ◄

$$\sigma_B = \sigma_{ave} - R = 7 - 5.77 = 1.23\text{ MPa}$$ ◄

PROBLEM 7.53

7.53 Solve Prob. 7.30, using Mohr's circle and assuming that the weld forms an angle of 60° with the horizontal

7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

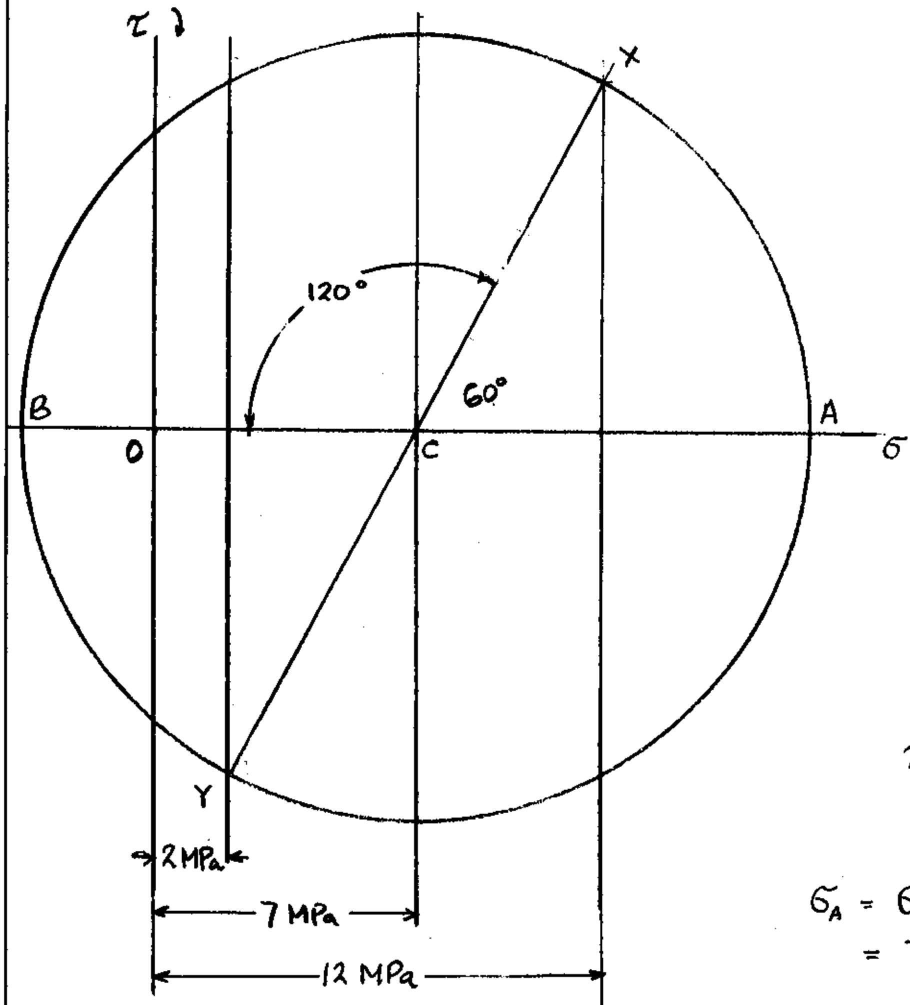


SOLUTION

Locate point C at $\sigma = \frac{12+2}{2} = 7$ MPa with $\tau = 0$.

Angle XCB = 120°

$$\frac{\sigma_x - \sigma_y}{2} = \frac{12-2}{2} = 5 \text{ MPa}$$



$R = 5 \sec 60° = 10$ MPa

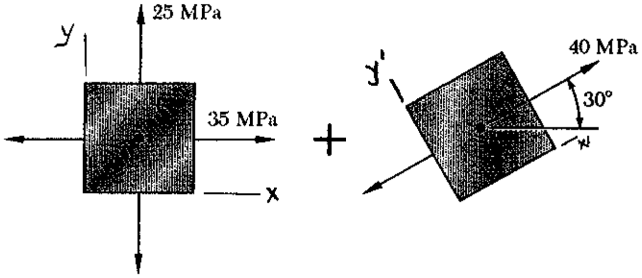
$\tau_{xy} = -5 \tan 60° = -8.66$ MPa ◄

$\sigma_A = \sigma_{ave} + R = 7 + 10 = 17$ MPa ◄

$\sigma_B = \sigma_{ave} - R = 7 - 10 = -3$ MPa ◄

PROBLEM 7.54

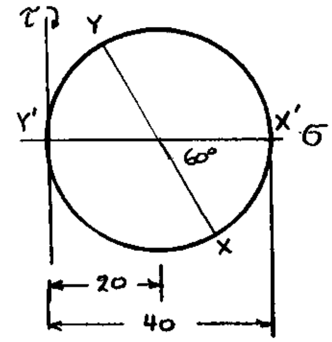
7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



SOLUTION

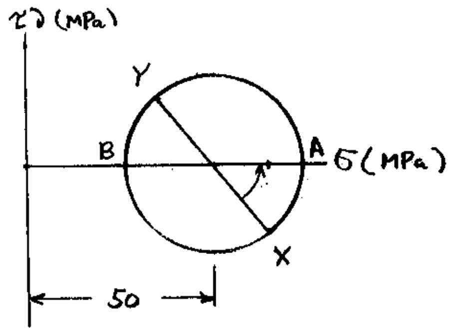
Mohr's circle for 2nd stress state

$\sigma_x = 20 + 20 \cos 60°$
$= 30$ MPa
$\sigma_y = 20 - 20 \cos 60°$
$= 10$ MPa
$\tau_{xy} = 20 \sin 60°$
$= 17.32$ MPa



Resultant stresses

$\sigma_x = 35 + 30 = 65$ MPa
$\sigma_y = 25 + 10 = 35$ MPa
$\tau_{xy} = 0 + 17.32 = 17.32$ MPa



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(65 + 35) = 50$ MPa

$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(17.32)}{65 - 35} = 1.1547$

$2\theta_p = 49.11°$ $\quad \theta_a = 24.6°$ $\quad \theta_b = 114.6°$

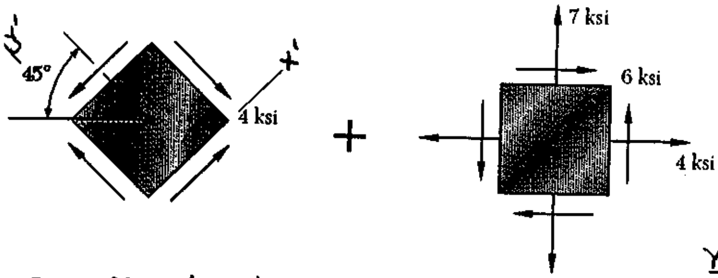
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 22.91$ MPa

$\sigma_a = \sigma_{ave} + R = 72.91$ MPa

$\sigma_b = \sigma_{ave} - R = 27.09$ MPa

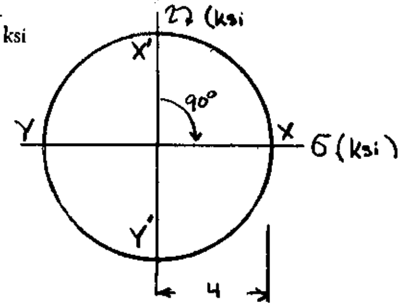
PROBLEM 7.55

7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



SOLUTION

Mohr's circle for 1st stress state.



$\sigma_x = 4$ ksi

$\sigma_y = -4$ ksi

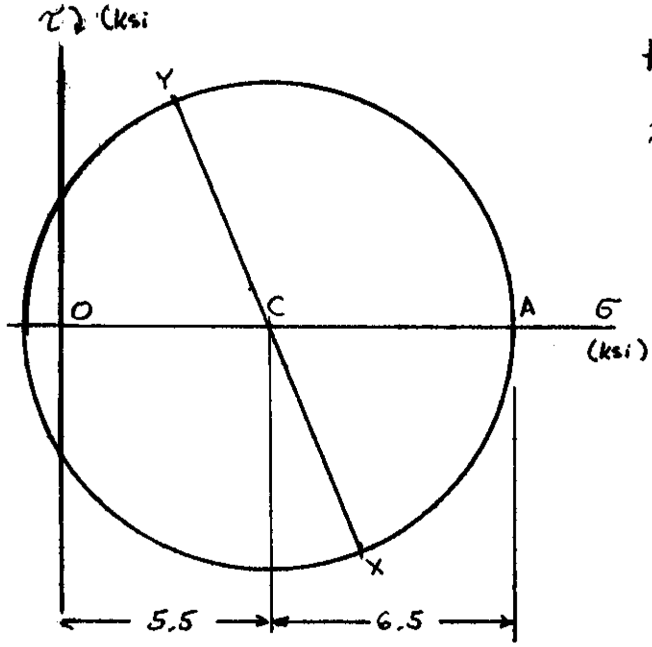
$\tau_{xy} = 0$

Resultant stresses

$\sigma_x = 4 + 4 = 8$ ksi

$\sigma_y = -4 + 7 = 3$ ksi

$\tau_{xy} = 6 + 0 = 6$ ksi



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.5$ ksi

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6)}{5} = 2.4$

$2\theta_p = 67.38°$ $\quad \theta_a = 33.69°$ ◄

$\theta_b = 123.69°$ ◄

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

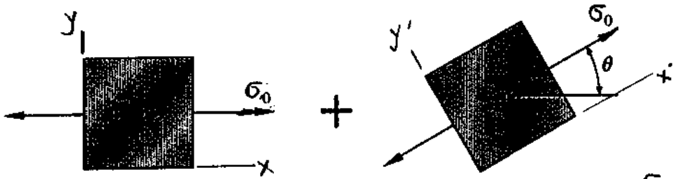
$= \sqrt{2.5^2 + 6^2} = 6.5$ ksi

$\sigma_a = \sigma_{ave} + R = 12$ ksi ◄

$\sigma_b = \sigma_{ave} - R = -1$ ksi ◄

PROBLEM 7.56

7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



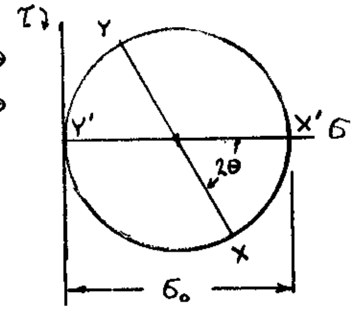
SOLUTION

Mohr's circle for 2nd stress state

$$\sigma_x = \tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0 \cos 2\theta$$

$$\sigma_y = \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0 \cos 2\theta$$

$$\tau_{xy} = \tfrac{1}{2}\sigma_0 \sin 2\theta$$



Resultant stresses

$$\sigma_x = \sigma_0 + \tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0 \cos 2\theta = \tfrac{3}{2}\sigma_0 + \tfrac{1}{2}\sigma_0 \cos 2\theta$$

$$\sigma_y = 0 + \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0 \cos 2\theta = \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0 \cos 2\theta$$

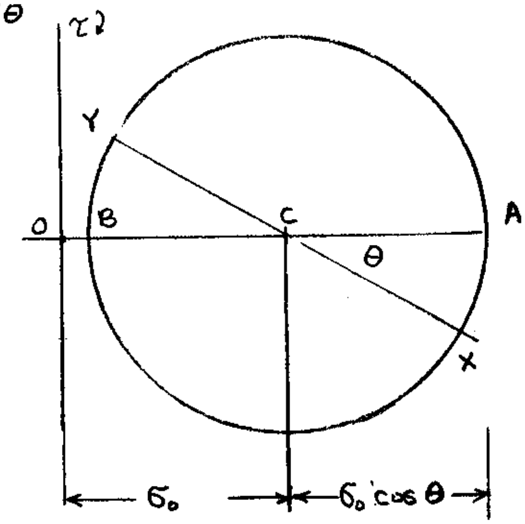
$$\tau_{xy} = 0 + \tfrac{1}{2}\sigma_0 \sin 2\theta = \tfrac{1}{2}\sigma_0 \sin 2\theta$$

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = \sigma_0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sigma_0 \sin 2\theta}{\sigma_0 + \sigma_0 \cos 2\theta}$$

$$= \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$$

$$\theta_p = \tfrac{1}{2}\theta$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} =$$

$$= \sqrt{\left(\tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0 \cos 2\theta\right)^2 + \left(\tfrac{1}{2}\sigma_0 \sin 2\theta\right)^2}$$

$$= \tfrac{1}{2}\sigma_0 \sqrt{1 + 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta}$$

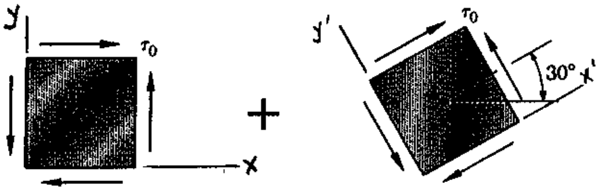
$$= \frac{\sqrt{2}}{2}\sigma_0 \sqrt{1 + \cos 2\theta} = \sigma_0 |\cos\theta|$$

$$\sigma_a = \sigma_{ave} + R = \sigma_0 + \sigma_0 \cos\theta \blacktriangleleft$$

$$\sigma_b = \sigma_{ave} - R = \sigma_0 - \sigma_0 \cos\theta \blacktriangleleft$$

PROBLEM 7.57

7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



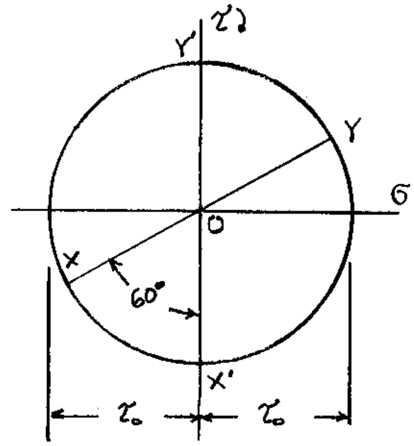
SOLUTION

Mohr's circle for 2nd state of stress

$\sigma_{x'} = 0$

$\sigma_{y'} = 0$

$\tau_{x'y'} = \tau_0$



$\sigma_x = -\tau_0 \sin 60° = -\frac{\sqrt{3}}{2}\tau_0$

$\sigma_y = \tau_0 \sin 60° = \frac{\sqrt{3}}{2}\tau_0$

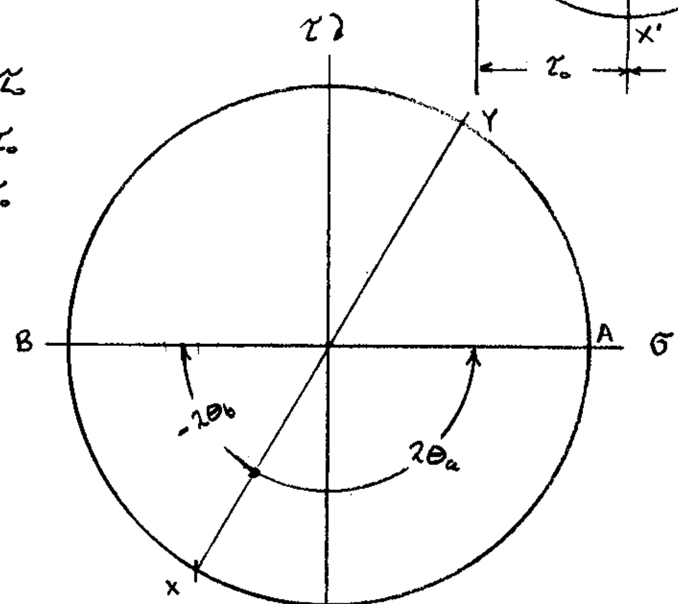
$\tau_{xy} = \tau_0 \cos 60° = \frac{1}{2}\tau_0$

Resultant stresses

$\sigma_x = 0 - \frac{\sqrt{3}}{2}\tau_0 = -\frac{\sqrt{3}}{2}\tau_0$

$\sigma_y = 0 + \frac{\sqrt{3}}{2}\tau_0 = \frac{\sqrt{3}}{2}\tau_0$

$\tau_{xy} = \tau_0 + \frac{1}{2}\tau_0 = \frac{3}{2}\tau_0$



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$

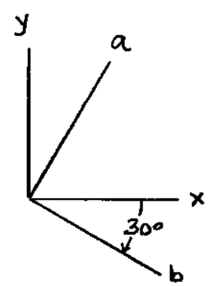
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{\sqrt{3}}{2}\tau_0\right)^2 + \left(\frac{3}{2}\tau_0\right)^2}$$

$$= \sqrt{3}\,\tau_0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\cdot\frac{3}{2}}{-\sqrt{3}} = -\sqrt{3}$$

$2\theta_p = -60°$ $\theta_b = -30°$ $\theta_a = 60°$ ◄

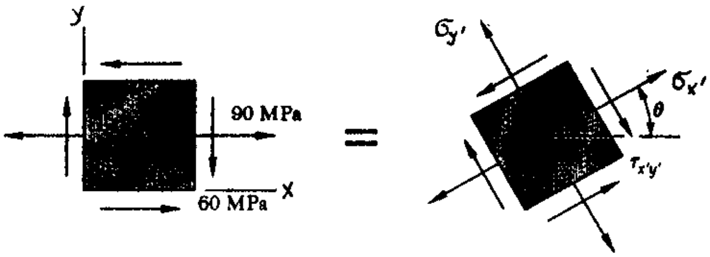


$\sigma_a = \sigma_{ave} + R = \sqrt{3}\,\tau_0$ ◄

$\sigma_b = \sigma_{ave} - R = -\sqrt{3}\,\tau_0$ ◄

PROBLEM 7.58

7.58 For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 100 MPa.



SOLUTION

$\sigma_x = 90$ MPa, $\sigma_y = 0$

$\tau_{xy} = -60$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45$ MPa

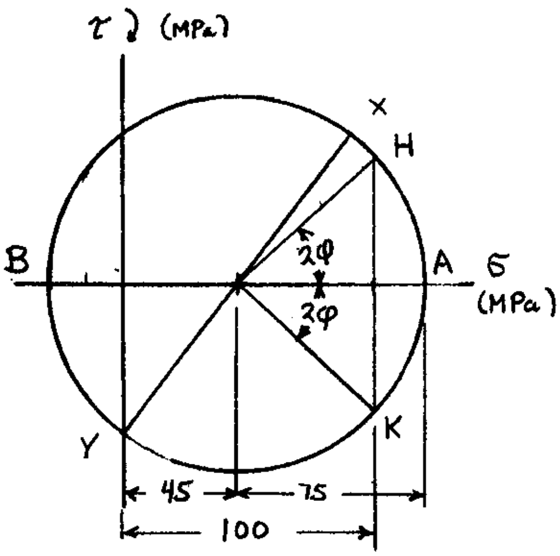
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{90} = -\frac{4}{3}$$

$2\theta_p = -53.13°$

$\theta_a = -26.565°$



$\sigma_{x'} \le 100$ MPa for states of stress corresponding to arc HBK of Mohr's circle. From the circle

$R \cos 2\varphi = 100 - 45 = 55$ MPa

$\cos 2\varphi = \frac{55}{75} = 0.73333$

$2\varphi = 42.833°$ $\qquad \varphi = 21.417°$

$\theta_H = \theta_a + \varphi = -26.565° + 21.417° = -5.15°$

$2\theta_K = 2\theta_H + 360° - 4\varphi = -10.297° + 360° - 85.666° = 264.037°$

$\theta_K = 132.02°$

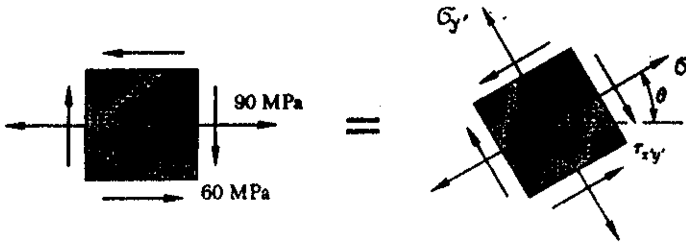
Permissibe range of θ is $\quad \theta_H \le \theta \le \theta_K$

$$-5.15° \le \theta \le 132.02°$$

Also $174.85° \le \theta \le 312.02°$

PROBLEM 7.59

7.59 For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 50 MPa.



SOLUTION

$\sigma_x = 90$ MPa, $\sigma_y = 0$

$\tau_{xy} = -60$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45$ MPa

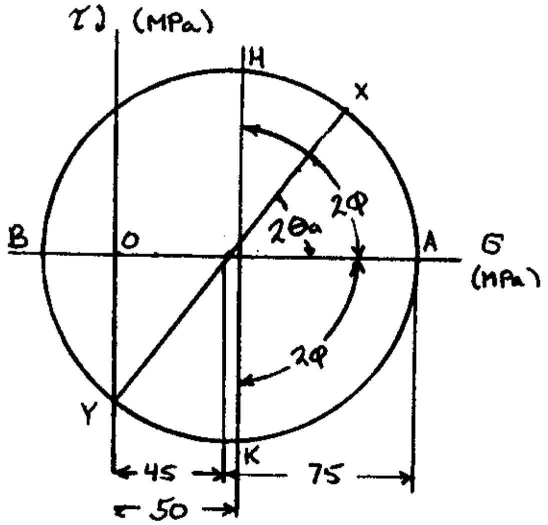
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{45^2 + 60^2} = 75$ MPa

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{90} = -\frac{4}{3}$

$2\theta_p = -53.13°$

$\theta_a = -26.565°$



$\sigma_{x'} \le 50$ MPa for states of stress corresponding to the arc HBK of Mohr's circle. From the circle

$R \cos 2\varphi = 50 - 45 = 5$ MPa

$\cos 2\varphi = \frac{5}{75} = 0.066667$

$2\varphi = 86.177°$ $\qquad \varphi = 43.089°$

$\theta_H = \theta_a + \varphi = -26.565° + 43.089° = 16.524°$

$2\theta_K = 2\theta_H + 360° - 4\varphi = 32.524° + 360° - 172.355° = 220.169°$

$\theta_K = 110.085°$

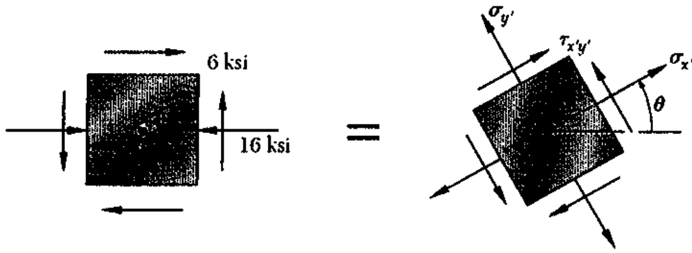
Permissible range of θ is $\theta_H \le \theta \le \theta_K$

$16.524° \le \theta \le 110.085°$

Also $196.524° \le \theta \le 290.085°$

PROBLEM 7.60

7.60 For the state of stress shown, determine the range of values of θ for which the magnitude of the shearing stress $\tau_{x'y'}$ is equal to or less than 8 ksi.



SOLUTION

$\sigma_x = -16$ ksi, $\sigma_y = 0$

$\tau_{xy} = 6$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -8$ ksi

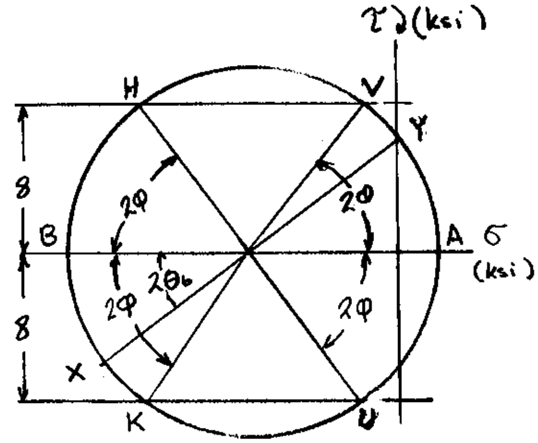
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-8)^2 + (6)^2} = 10$ ksi

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6)}{-16} = -0.75$

$2\theta_p = -36.870°$

$\theta_b = -18.435°$



$|\tau_{x'y'}| \le 8$ ksi for states of stress corresponding to arcs HBK and UAV of Mohr's circle. The angle φ is calculated from

$R \sin 2\varphi = 8$

$\sin 2\varphi = \frac{8}{10} = 0.8$

$2\varphi = 53.130°$ $\varphi = 26.565°$

$\theta_H = \theta_b - \varphi = -18.435° - 26.565° = -45°$

$\theta_K = \theta_b + \varphi = -18.435 + 26.565° = 8.13°$

$\theta_U = \theta_H + 90° = 45°$

$\theta_V = \theta_K + 90° = 98.13°$

Permissible ranges of θ $\theta_H \le \theta \le \theta_K$

$-45° \le \theta \le 8.13°$

$\theta_U \le \theta \le \theta_V$

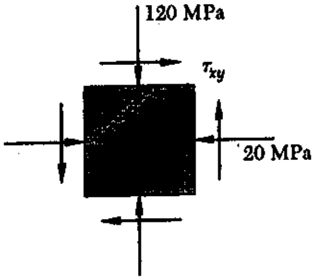
$45° \le \theta \le 98.13°$

Also $135° \le \theta \le 188.13°$

$225° \le \theta \le 278.13°$

PROBLEM 7.61

7.61 For the element shown, determine the range of values of τ_{xy} for which the maximum tensile stress is equal to or less than 60 MPa.



SOLUTION

$\sigma_x = -20$ MPa $\sigma_y = -120$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -70$ MPa

Set $\sigma_{max} = 60 \text{ MPa} = \sigma_{ave} + R$

$R = \sigma_{max} - \sigma_{ave} = 130$ MPa

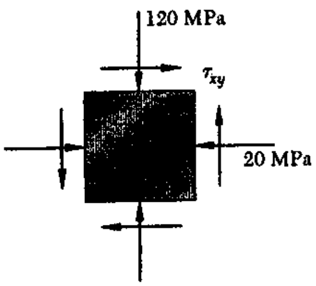
But $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{130^2 - 50^2} = 120$ MPa

Range of τ_{xy} $-120 \text{ MPa} \le \tau_{xy} \le 120$ MPa ◄

PROBLEM 7.62

7.62 For the element shown, determine the range of values of τ_{xy} for which the maximum in plane shearing stress is equal to or less than 150 MPa.



SOLUTION

$\sigma_x = -20$ MPa $\sigma_y = -120$ MPa

$\frac{1}{2}(\sigma_x - \sigma_y) = 50$ MPa

Set $\tau_{max(in\text{-}plane)} = R = 150$ MPa

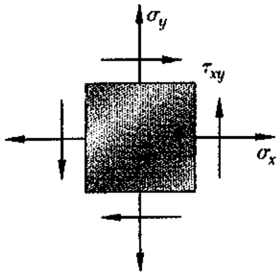
But $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{150^2 - 50^2} = 141.4$ MPa

Range of τ_{xy} $-141.4 \text{ MPa} \le \tau_{xy} \le 141.4$ MPa ◄

PROBLEM 7.63

7.63 For the state of stress shown it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 14$ ksi, $\sigma_y = 9$ ksi, and $\sigma_{min} = 5$ ksi. Determine (*a*) the orientation of the principal planes, (*b*) the principal stress σ_{max}, (*c*) the maximum in-plane shearing stress.



SOLUTION

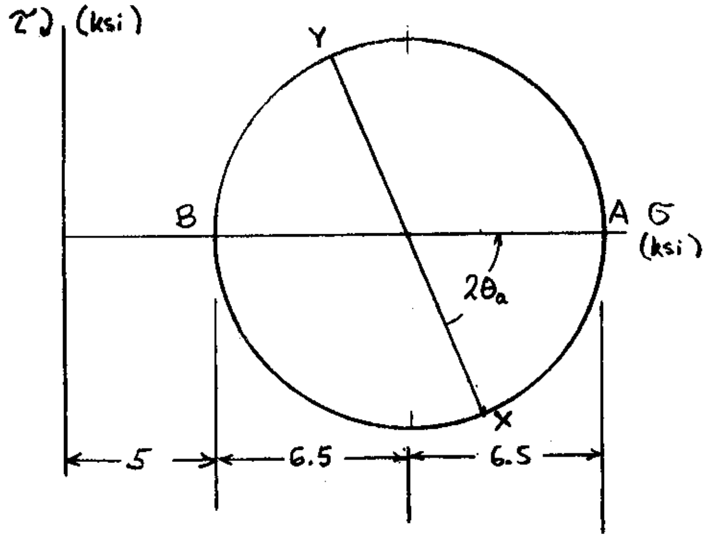
$\sigma_x = 14$ ksi, $\sigma_y = 9$ ksi, $\quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 11.5$ ksi

$\sigma_{min} = \sigma_{ave} - R \quad \therefore \quad R = \sigma_{ave} - \sigma_{min}$
$= 11.5 - 5 = 6.5$ ksi

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \pm\sqrt{6.5^2 - 2.5^2} = \pm 6 \text{ ksi}$$

But it is given that τ_{xy} is positive, thus $\tau_{xy} = +6$ ksi



(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(6)}{5} = 2.4$

$2\theta_p = 67.38°$

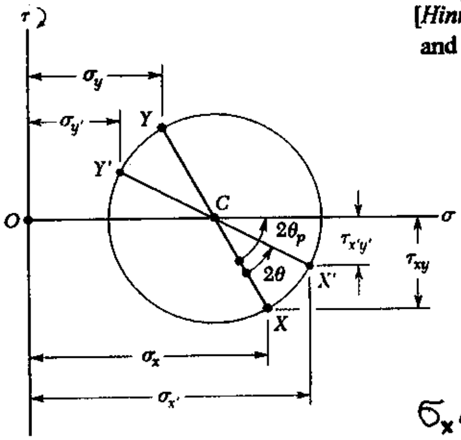
$\theta_a = 33.69°$

$\theta_b = 123.69°$

(b) $\sigma_{max} = \sigma_{ave} + R$
$= 18$ ksi

(c) $\tau_{max(in\text{-}plane)} = R$
$= 6.5$ ksi

PROBLEM 7.64



7.64 The Mohr circle shown corresponds to the state of stress given in Fig. *xxa* and *b*, page yyy. Noting that $\sigma_{x'} = OC + (CX')\cos(2\theta_p - 2\theta)$ and that $\tau_{x'y'} = (CX')\sin(2\theta_p - 2\theta)$, derive the expressions for $\sigma_{x'}$ and $\tau_{x'y'}$ given in Eqs. (7.5) and (7.6), respectively. [*Hint:* Use $\sin(A + B) = \sin A \cos B + \cos A \sin B$ and $\cos(A + B) = \cos A \cos B + \sin A \sin B$.]

SOLUTION

$$\overline{OC} = \tfrac{1}{2}(\sigma_x + \sigma_y) \qquad \overline{CX'} = \overline{CX}$$

$$\overline{CX'}\cos 2\theta_p = \overline{CX}\cos 2\theta_p = \frac{\sigma_x - \sigma_y}{2}$$

$$\overline{CX'}\sin 2\theta_p = \overline{CX}\sin 2\theta_p = \tau_{xy}$$

$$\sigma_{x'} = \overline{OC} + \overline{CX'}\cos(2\theta_p - 2\theta)$$

$$= \overline{OC} + \overline{CX'}(\cos 2\theta_p \cos 2\theta + \sin 2\theta_p \sin 2\theta)$$

$$= \overline{OC} + \overline{CX'}\cos 2\theta_p \cos 2\theta + \overline{CX'}\sin 2\theta_p \sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = \overline{CX'}\sin(2\theta_p - 2\theta) = \overline{CX'}(\sin 2\theta_p \cos 2\theta - \cos 2\theta_p \sin 2\theta)$$

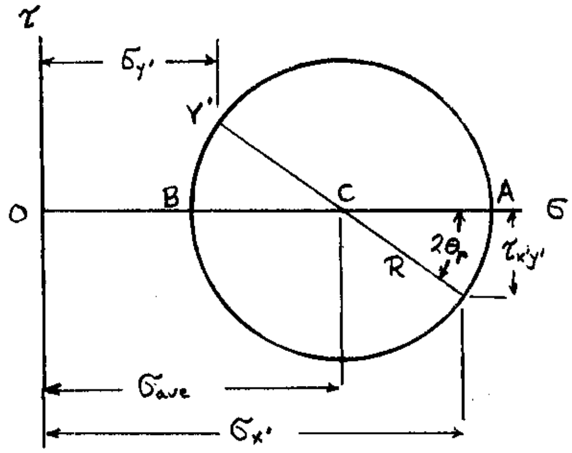
$$= \overline{CX'}\sin 2\theta_p \cos 2\theta - \overline{CX'}\cos 2\theta_p \sin 2\theta$$

$$= \tau_{xy}\cos 2\theta - \frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$

PROBLEM 7.65

7.65 (*a*) Prove that the expression $\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$, where $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$ are components of stress along the rectangular axes x' and y', is independent of the orientation of these axes. Also, show that the given expression represents the square of the tangent drawn from the origin of the coordinates to Mohr's circle. (*b*) Using the invariance property established in part *a*, express the shearing stress τ_{xy} in terms of σ_x, σ_y, and the principal stresses σ_{max} and σ_{min}.

SOLUTION



(a) From Mohr's circle

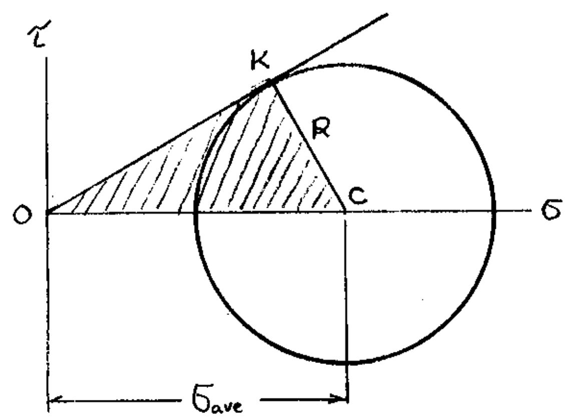
$$\tau_{x'y'} = R \sin 2\theta_p$$

$$\sigma_{x'} = \sigma_{ave} + R \cos 2\theta_p$$

$$\sigma_{y'} = \sigma_{ave} - R \cos 2\theta_p$$

$$\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$$
$$= \sigma_{ave}^2 - R^2\cos^2 2\theta_p - R^2 \sin^2 2\theta_p$$
$$= \sigma_{ave}^2 - R^2 ;$$ independent of θ_p.

Draw line $\overline{OK}$ from origin tangent to the circle at K. Triangle OCK is a right triangle



$$\overline{OC}^2 = \overline{OK}^2 + \overline{CK}^2$$

$$\overline{OK}^2 = \overline{OC}^2 - \overline{CK}^2$$
$$= \sigma_{ave}^2 - R^2$$
$$= \sigma_{x'}\sigma_{y'} - \tau_{x'y'}$$

(b) Applying above to σ_x, σ_y, and τ_{xy} and to σ_a, σ_b,

$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_a\sigma_b - \tau_{ab}^2 = \sigma_{ave}^2 - R^2$$

But $\tau_{ab} = 0$, $\sigma_a = \sigma_{max}$, $\sigma_b = \sigma_{min}$

$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_{max}\sigma_{min}$$

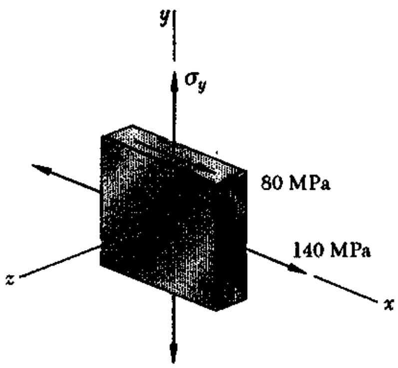
$$\tau_{xy}^2 = \sigma_x\sigma_y - \sigma_{max}\sigma_{min}$$

$$\tau_{xy} = \pm\sqrt{\sigma_x\sigma_y - \sigma_{max}\sigma_{min}}$$ ◄

The sign cannot be determined from above equations.

PROBLEM 7.66

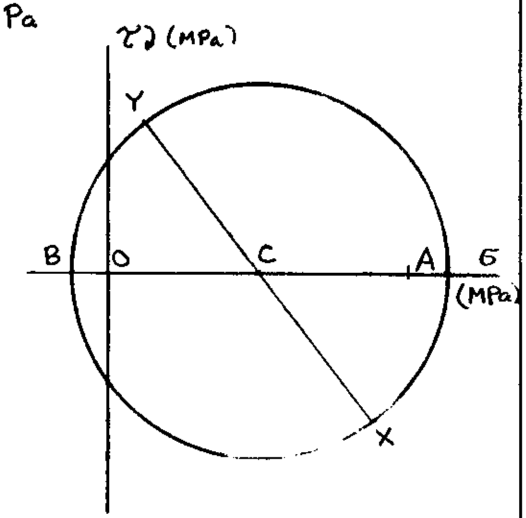
7.66 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 20$ MPa, (*b*) $\sigma_y = 140$ MPa. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)



SOLUTION

(a) $\sigma_x = 140$ MPa, $\sigma_y = 20$ MPa

$\tau_{xy} = 80$ MPa



$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 80 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{60^2 + 80^2} = 100 \text{ MPa}$$

$\sigma_a = \sigma_{ave} + R = 80 + 100 = 180$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 80 - 100 = -20$ MPa (min)

$\sigma_c = 0$

$\tau_{max(inplane)} = \tfrac{1}{2}(\sigma_a - \sigma_b) = 100$ MPa

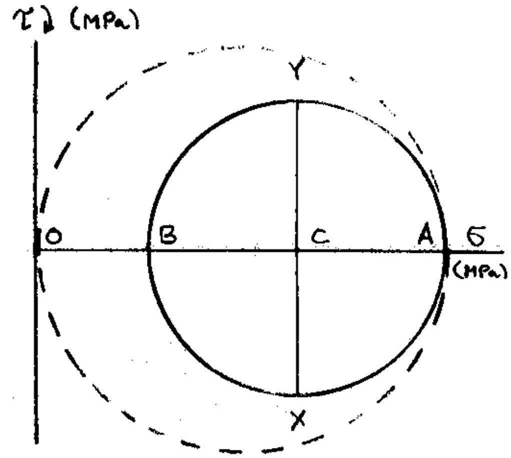
$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = 100$ MPa ◄

(b) $\sigma_x = 140$ MPa, $\sigma_y = 140$ MPa

$\tau_{xy} = 80$ MPa

$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 140$ MPa

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + 80^2} = 80 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 220$ MPa (max)

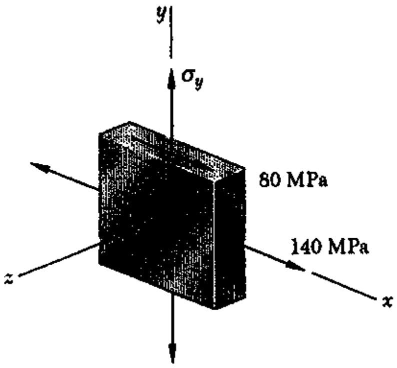
$\sigma_b = \sigma_{ave} - R = 60$ MPa

$\sigma_c = 0$ (min)

$\tau_{max(in\text{-}plane)} = \tfrac{1}{2}(\sigma_a - \sigma_b) = 80$ MPa

$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = 110$ MPa ◄

PROBLEM 7.67

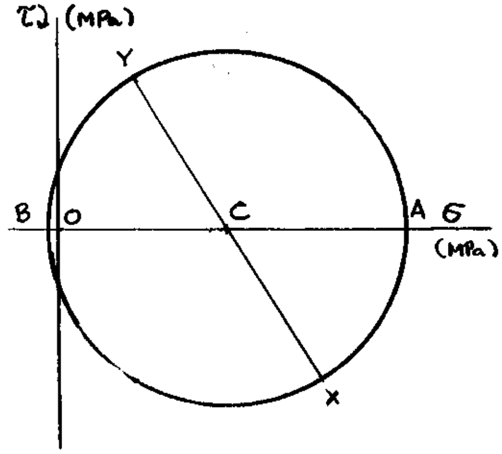


7.67 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 40$ MPa, (*b*) $\sigma_y = 120$ MPa. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(a) $\sigma_x = 140$ MPa $\quad \sigma_y = 40$ MPa $\quad \tau_{xy} = 80$ MPa

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y)$$
$$= 90 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$
$$= \sqrt{50^2 + 80^2}$$
$$= 94.34 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 184.34$ MPa $\quad$ (max)

$\sigma_b = \sigma_{ave} - R = -4.34$ MPa $\quad$ (min)

$\sigma_c = 0$

$\tau_{max(in\text{-}plane)} = \tfrac{1}{2}(\sigma_a - \sigma_b) = R = 94.34$ MPa

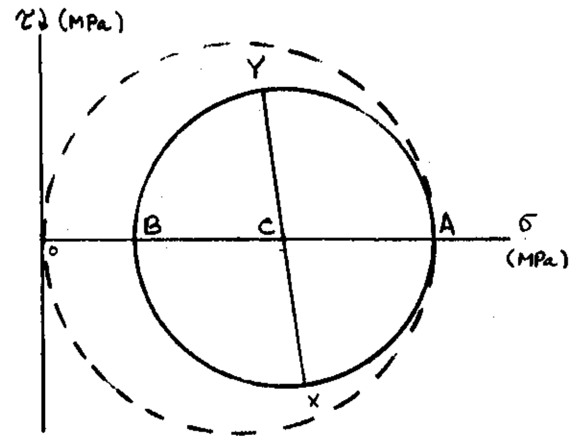
$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = \tfrac{1}{2}(\sigma_a - \sigma_b) = 94.34$ MPa ◀

(b) $\sigma_x = 140$ MPa, $\sigma_y = 120$ MPa, $\tau_{xy} = 80$ MPa

$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 130$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{10^2 + 80^2} = 80.62$ MPa



$\sigma_a = \sigma_{ave} + R = 210.62$ MPa $\quad$ (max)

$\sigma_b = \sigma_{ave} - R = 49.38$ MPa

$\sigma_c = 0 \quad$ (min)

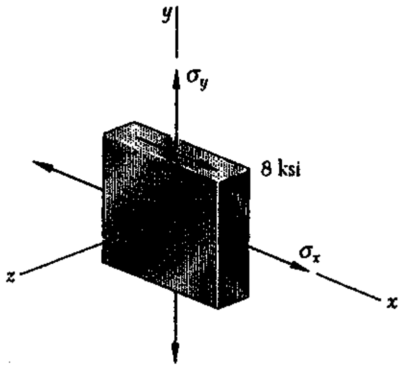
$\sigma_{max} = \sigma_a = 210.62$ MPa

$\sigma_{min} = \sigma_c = 0$

$\tau_{max(in\text{-}plane)} = R = 86.62$ MPa

$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = 105.31$ MPa ◀

PROBLEM 7.68



7.68 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 6$ ksi and $\sigma_y = 18$ ksi, (*b*) $\sigma_x = 14$ ksi and $\sigma_y = 2$ ksi. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(a) $\sigma_x = 6$ ksi $\quad \sigma_y = 18$ ksi $\quad \tau_{xy} = 8$ ksi

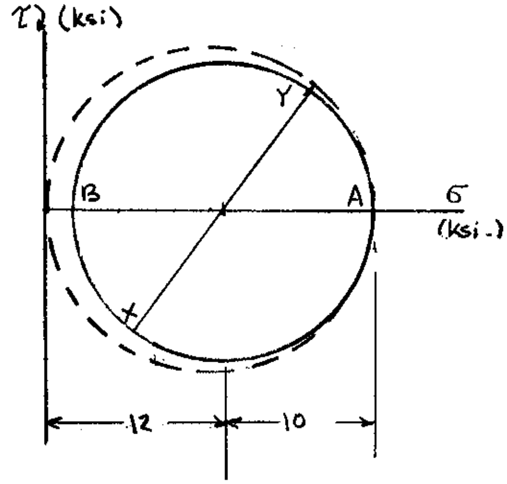
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 12$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{6^2 + 8^2}$

$= 10$ ksi



$\sigma_a = \sigma_{ave} + R = 12 + 10 = 22$ ksi (max)

$\sigma_b = \sigma_{ave} - R = 12 - 10 = 2$ ksi

$\sigma_c = 0$ (min)

$\tau_{max(in\text{-}plane)} = R = 10$ ksi

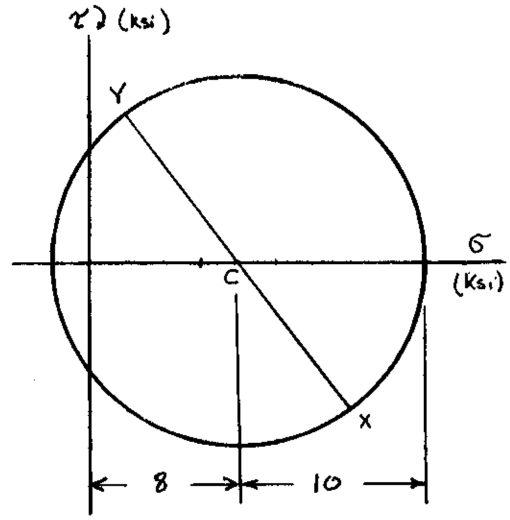
$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 11$ ksi ◄

(b) $\sigma_x = 14$ ksi $\quad \sigma_y = 2$ ksi $\quad \tau_{xy} = 8$ ksi

$\sigma_a = \frac{1}{2}(\sigma_x + \sigma_y) = 8$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{6^2 + 8^2} = 10$ ksi



$\sigma_a = \sigma_{ave} + R = 18$ ksi (max)

$\sigma_b = \sigma_{ave} - R = -2$ ksi (min)

$\sigma_c = 0$

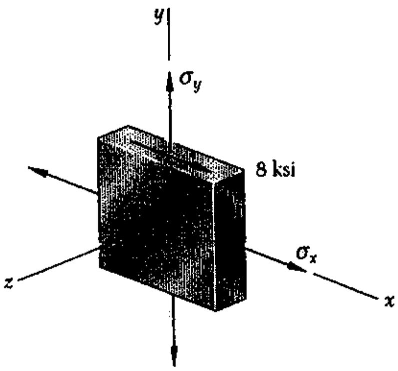
$\sigma_{max} = 18$ ksi

$\sigma_{min} = -2$ ksi

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 10$ ksi ◄

PROBLEM 7.69

7.69 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 0$ and $\sigma_y = 12$ ksi, (*b*) $\sigma_x = 21$ ksi and $\sigma_y = 9$ ksi. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)

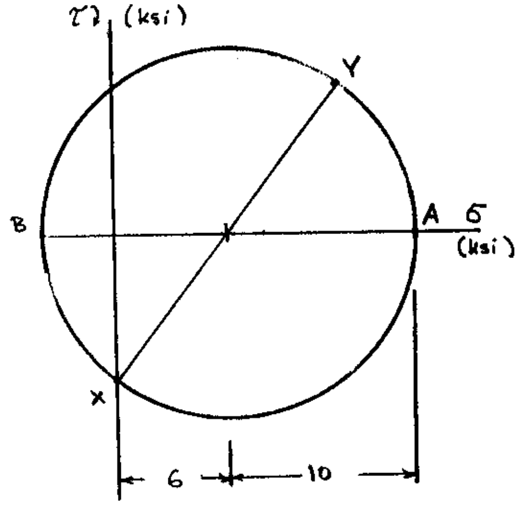


SOLUTION

(a) $\sigma_x = 0$, $\sigma_y = 12$ ksi, $\tau_{xy} = 8$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$
$= 6$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
$= \sqrt{(-6)^2 + 8^2}$
$= 10$ ksi



$\sigma_a = \sigma_{ave} + R = 16$ ksi (max)

$\sigma_b = \sigma_{ave} - R = -4$ ksi (min)

$\sigma_c = 0$

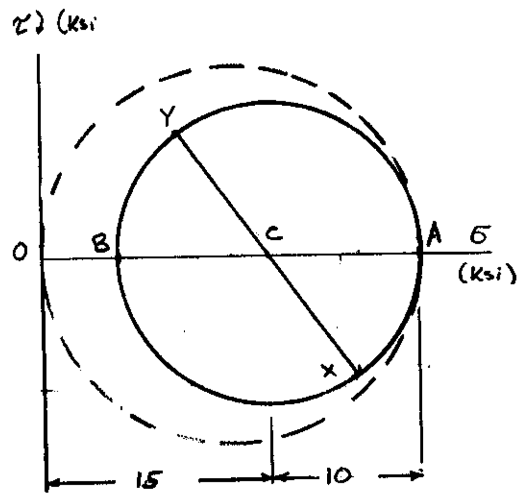
$\sigma_{max} = 16$ ksi $\quad \sigma_{min} = -4$ ksi

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 10$ ksi ◄

(b) $\sigma_x = 21$ ksi $\quad \sigma_y = 9$ ksi $\quad \tau_{xy} = 8$ ksi

$\sigma_{ave} = 15$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
$= \sqrt{(-6)^2 + 8^2} = 10$ ksi



$\sigma_a = \sigma_{ave} + R = 25$ ksi (max)

$\sigma_b = \sigma_{ave} - R = 5$ ksi

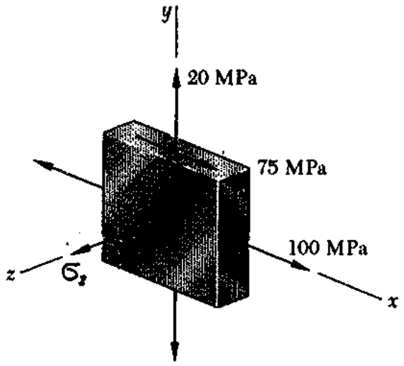
$\sigma_c = 0$ (min)

$\sigma_{max} = 25$ ksi, $\quad \sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 12.5$ ksi ◄

PROBLEM 7.70

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +45$ MPa, (*c*) $\sigma_z = -45$ MPa.



SOLUTION

$\sigma_x = 100$ MPa, $\sigma_y = 20$ MPa, $\tau_{xy} = 75$ MPa

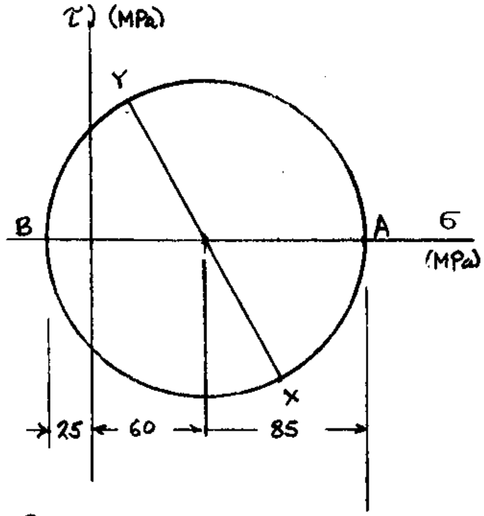
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 60$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{40^2 + 75^2}$

$= 85$ MPa



$\sigma_a = \sigma_{ave} + R = 145$ MPa

$\sigma_b = \sigma_{ave} - R = -25$ MPa

(a) $\sigma_z = 0$, $\sigma_a = 145$ MPa, $\sigma_b = -25$ MPa

$\sigma_{max} = 145$ MPa, $\sigma_{min} = -25$ MPa, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 85$ MPa ◄

(b) $\sigma_z = +45$ MPa, $\sigma_a = 145$ MPa, $\sigma_b = -25$ MPa

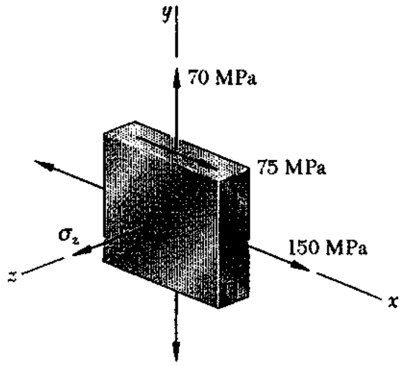
$\sigma_{max} = 145$ MPa, $\sigma_{min} = -25$ MPa, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 85$ MPa ◄

(c) $\sigma_z = -45$ MPa, $\sigma_a = 145$ MPa, $\sigma_b = -25$ MPa

$\sigma_{max} = 145$ MPa, $\sigma_{min} = -45$ MPa $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 95$ MPa ◄

PROBLEM 7.71

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +45$ MPa, (*c*) $\sigma_z = -45$ MPa.



SOLUTION

$\sigma_x = 150$ MPa, $\sigma_y = 70$ MPa, $\tau_{xy} = 75$ MPa

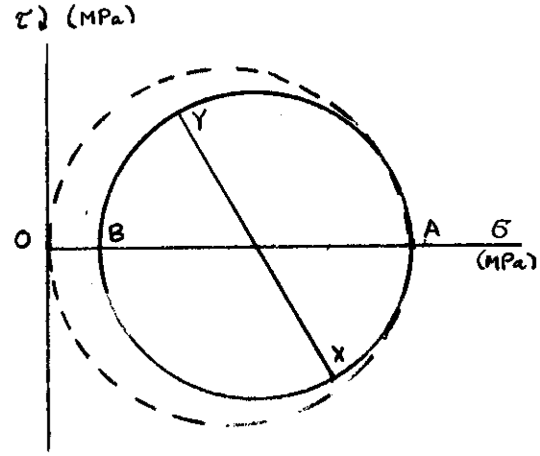
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 110$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{40^2 + 75^2}$

$= 85$ MPa



$\sigma_a = \sigma_{ave} + R = 195$ MPa

$\sigma_b = \sigma_{ave} - R = 25$ MPa

(a) $\sigma_z = 0$, $\sigma_a = 195$ MPa, $\sigma_b = 25$ MPa

$\sigma_{max} = 195$ MPa, $\sigma_{min} = 0$, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 97.5$ MPa ◄

(b) $\sigma_z = +45$ MPa, $\sigma_a = 195$ MPa, $\sigma_b = 25$ MPa

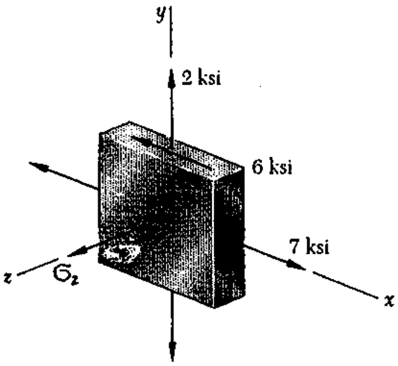
$\sigma_{max} = 195$ MPa, $\sigma_{min} = 25$ MPa, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 85$ MPa ◄

(c) $\sigma_z = -45$ MPa, $\sigma_a = 195$ MPa, $\sigma_b = 25$ MPa

$\sigma_{max} = 195$ MPa, $\sigma_{min} = -45$ MPa, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 120$ MPa ◄

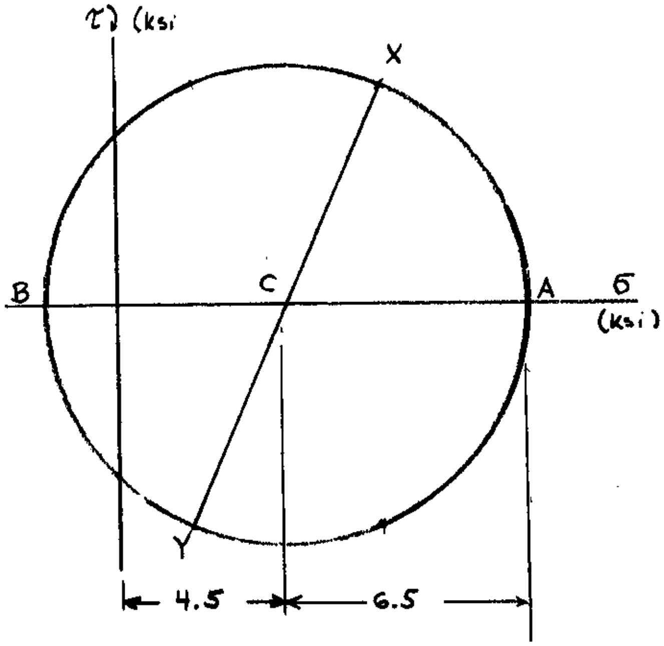
PROBLEM 7.72

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = +4$ ksi, (*b*) $\sigma_z = -4$ ksi, (c) $\sigma_z = 0$.



SOLUTION

$\sigma_x = 7$ ksi , $\sigma_y = 2$ ksi , $\tau_{xy} = -6$ ksi



$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 4.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{2.5^2 + (-6)^2} = 6.5 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 11 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -2 \text{ ksi}$$

(a) $\sigma_z = 4$ ksi , $\sigma_a = 11$ ksi , $\sigma_b = -2$ ksi

$\sigma_{max} = 11$ ksi , $\sigma_{min} = -2$ ksi , $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5$ ksi ◄

(b) $\sigma_z = -4$ ksi , $\sigma_a = 11$ ksi , $\sigma_b = -2$ ksi

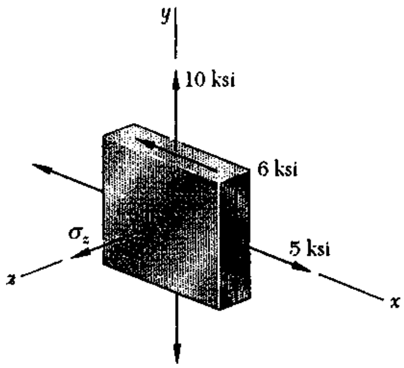
$\sigma_{max} = 11$ ksi , $\sigma_{min} = -4$ ksi , $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7.5$ ksi ◄

(c) $\sigma_z = 0$, $\sigma_a = 11$ ksi , $\sigma_b = -2$ ksi

$\sigma_{max} = 11$ ksi , $\sigma_{min} = -2$ ksi , $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5$ ksi ◄

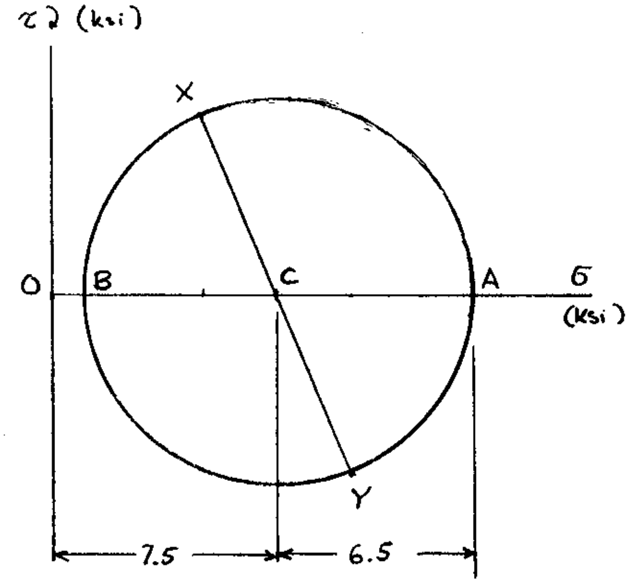
PROBLEM 7.73

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = +4$ ksi, (*b*) $\sigma_z = -4$ ksi, (c) $\sigma_z = 0$.



SOLUTION

$\sigma_x = 5$ ksi, $\sigma_y = 10$ ksi, $\tau_{xy} = -6$ ksi



$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 7.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-2.5)^2 + (-6)^2} = 6.5 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 14 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = 1 \text{ ksi}$$

(a) $\sigma_z = +4$ ksi, $\sigma_a = 14$ ksi, $\sigma_b = 1$ ksi

$\sigma_{max} = 14$ ksi, $\sigma_{min} = 1$ ksi, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5$ ksi ◄

(b) $\sigma_z = -4$ ksi, $\sigma_a = 14$ ksi, $\sigma_b = 1$ ksi

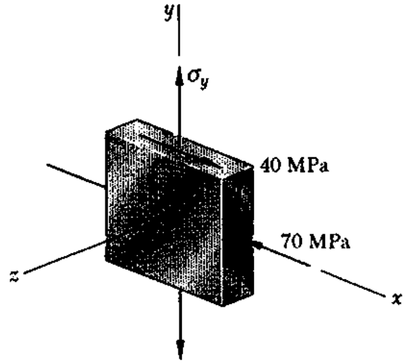
$\sigma_{max} = 14$ ksi, $\sigma_{min} = -4$ ksi, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 9$ ksi ◄

(c) $\sigma_z = 0$, $\sigma_a = 14$ ksi, $\sigma_b = 1$ ksi

$\sigma_{max} = 14$ ksi, $\sigma_{min} = 0$, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7$ ksi ◄

PROBLEM 7.74

7.74 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 75 MPa.



SOLUTION

$\sigma_x = -70$ MPa, $\tau_{xy} = 40$ MPa

Let $u = \frac{\sigma_y - \sigma_x}{2}$ $\qquad \sigma_y = 2u + \sigma_x$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$

$R = \sqrt{u^2 + \tau_{xy}^2} \qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$

Case 1 $\qquad \tau_{max} = R = 75$ MPa, $\quad u = \pm\sqrt{75^2 - 40^2} = \pm 63.44$ MPa

(1a) $u = +63.44$ MPa $\qquad \sigma_y = 2u + \sigma_x = 56.88$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.56$ MPa

$\sigma_a = \sigma_{ave} + R = 68.44$ MPa, $\quad \sigma_b = \sigma_{ave} - R = -81.56$ MPa

$\sigma_c = 0 \qquad \sigma_{max} = 68.44$ MPa, $\sigma_{min} = -81.56$ MPa $\quad \tau_{max} = 75$ MPa

(1b) $u = -63.44$ MPa $\qquad \sigma_y = 2u + \sigma_x = -196.88$ MPa (reject)

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -133.44$ MPa $\qquad \sigma_a = \sigma_{ave} + R = -58.44$ MPa

$\sigma_b = \sigma_{ave} - R = -208.44$ MPa, $\quad \sigma_c = 0, \quad \sigma_{max} = 0$

$\sigma_{min} = -208.44$ MPa, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 104.22$ MPa $\neq 75$ MPa

Case (2) $\quad$ Assume $\quad \sigma_{max} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 75$ MPa

$\sigma_{min} = -150$ MPa $= \sigma_b$

$\sigma_b = \sigma_{ave} - R = \sigma_x + u - \sqrt{u^2 + \tau_{xy}^2}$

$\sqrt{u^2 + \tau_{xy}^2} = \sigma_x + u - \sigma_b$

$\cancel{u^2} + \tau_{xy}^2 = (\sigma_x - \sigma_b)^2 + 2(\sigma_x - \sigma_b)u + \cancel{u^2}$

$2u = \frac{\tau_{xy}^2 - (\sigma_x - \sigma_b)^2}{\sigma_x - \sigma_b} \qquad \frac{(40)^2 - (-70 + 150)^2}{-70 + 150} = -160$ MPa

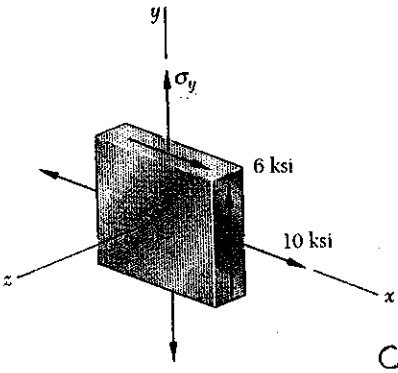
$u = -30$ MPa $\qquad \sigma_y = 2u + \sigma_x = -130$ MPa

$R = \sqrt{u^2 + \tau_{xy}^2} = 50$ MPa

$\sigma_a = \sigma_b + 2R = -150 + 100 = -50$ MPa $\qquad$ O.K.

PROBLEM 7.75

7.75 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 7.5 ksi.



SOLUTION

$\sigma_x = 10$ ksi , $\tau_{xy} = 6$ ksi , $\tau_{max} = 7.5$ ksi

Let $u = \dfrac{\sigma_y - \sigma_x}{2}$ $\qquad \sigma_y = 2u + \sigma_x$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$

$R = \sqrt{u^2 + \tau_{xy}^2}$ $\qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$

Case 1 $\quad \tau_{max} = R = 7.5$ ksi , $u = \pm 4.5$ ksi

(1a) $u = +4.5$ ksi $\qquad \sigma_y = 2u + \sigma_x = 19$ ksi $\qquad$ reject

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 14.5$ ksi , $\sigma_a = \sigma_{ave} + R = 22$ ksi, $\sigma_b = \sigma_{ave} - R = 7$ ksi

$\sigma_{max} = 22$ ksi , $\sigma_{min} = 0$, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 11$ ksi $\neq 7.5$ ksi

(1b) $u = -4.5$ ksi $\qquad \sigma_y = 2u + \sigma_x = 1$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.5$ ksi, $\sigma_a = \sigma_{ave} + R = 13$ ksi, $\sigma_b = \sigma_{ave} - R = -2$ ksi

$\sigma_{max} = 13$ ksi , $\sigma_{min} = -2$ ksi , $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7.5$ ksi $\quad$ O.K.

Case 2 $\quad$ Assume $\sigma_{min} = 0$ $\qquad \sigma_{max} = 2\tau_{max} = 15$ ksi $= \sigma_a$

$\sigma_a = \sigma_{ave} + R = \sigma_x + u + \sqrt{u^2 + \tau_{xy}^2}$

$\sigma_a - \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$

$(\sigma_a - \sigma_x - u)^2 = u^2 + \tau_{xy}^2$

$(\sigma_a - \sigma_x)^2 - 2(\sigma_a - \sigma_x)u + \cancel{u^2} = \cancel{u^2} + \tau_{xy}^2$

$2u = \dfrac{(\sigma_a - \sigma_x)^2 - \tau_{xy}^2}{\sigma_a - \sigma_x} = \dfrac{(15-10)^2 - 6^2}{15 - 10} = -2.2$ ksi

$u = -1.1$ ksi $\qquad \sigma_y = 2u + \sigma_x = 7.8$ ksi

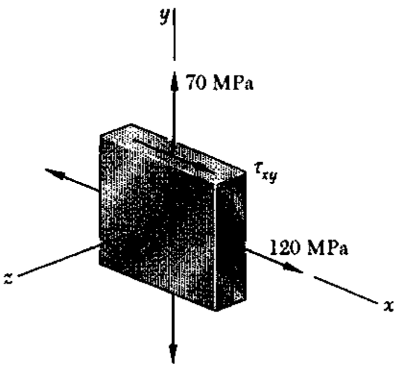
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 8.9$ ksi $\qquad R = \sqrt{u^2 + \tau_{xy}^2} = 6.1$ ksi

$\sigma_a = \sigma_{ave} + R = 15$ ksi ✓ $\qquad \sigma_b = \sigma_{ave} - R = 2.8$ ksi

$\sigma_{max} = 15$ ksi , $\sigma_{min} = 0$ $\qquad \tau_{max} = 7.5$ ksi ✓

PROBLEM 7.76

7.76 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is 80 MPa.



SOLUTION

$\sigma_x = 120$ MPa $\quad \sigma_y = 70$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 95$ MPa

$\frac{\sigma_x - \sigma_y}{2} = \frac{120 - 70}{2} = 25$ MPa

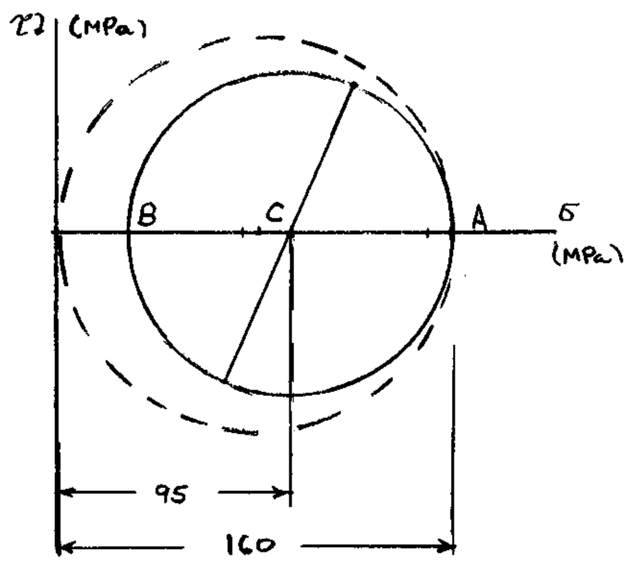
Assume $\sigma_{min} = 0 \qquad \sigma_{max} = 2\tau_{max} = 160$ MPa

$\sigma_a = \sigma_{max} = \sigma_{ave} + R \qquad R = \sigma_{max} - \sigma_{ave} = 160 - 95 = 65$ MPa

$R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \qquad \tau_{xy}^2 = R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 = 65^2 - 25^2 = 60^2$

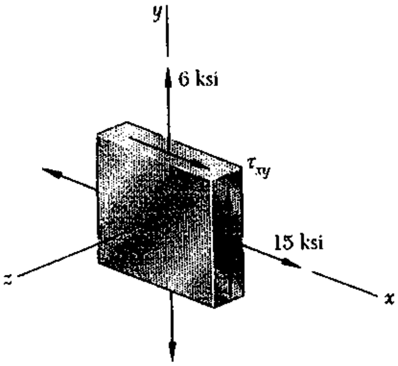
$\tau_{xy} = \pm 60$ ksi

$\sigma_b = \sigma_a - 2R = 160 - 130 = 30$ MPa ≥ 0 O.K.



PROBLEM 7.77

7.77 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 9 ksi, (*b*) 12 ksi.



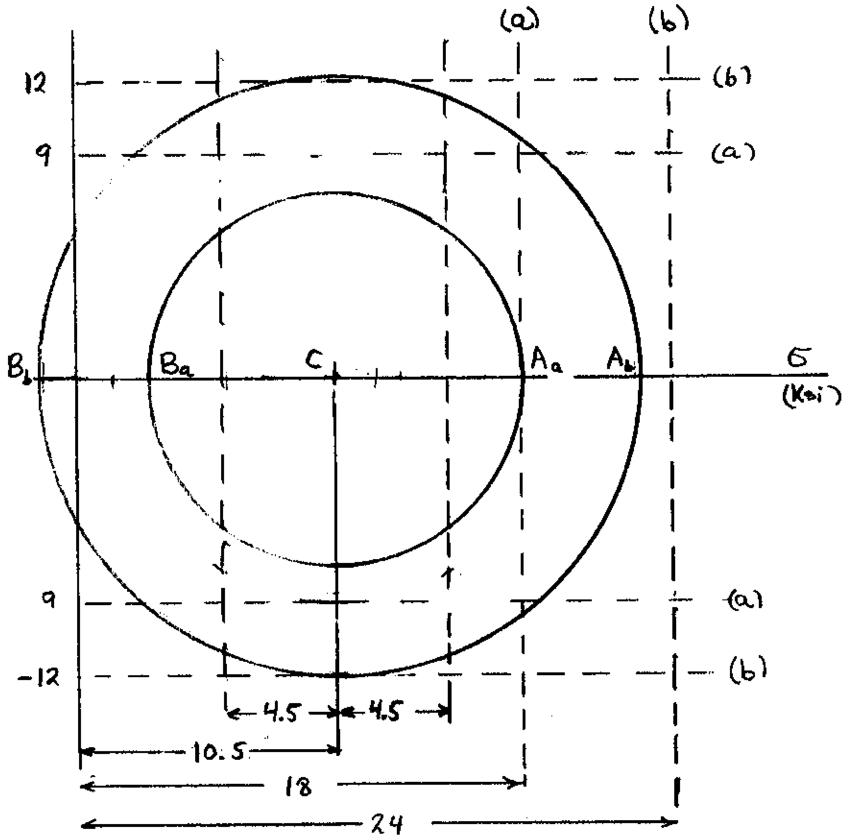
SOLUTION

$\sigma_x = 15$ ksi $\quad \sigma_y = 6$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 10.5$ ksi

$U = \dfrac{\sigma_x - \sigma_y}{2} = 4.5$ ksi

τ (ksi)



(a) For $\tau_{max} = 9$ ksi

Center of Mohrs circle lies at point C. Lines marked (a) show the limits on τ_{max}. Limit on σ_{max} is. $\sigma_{max} = 2\tau_{max} = 18$ ksi. For the Mohr's cirle $\sigma_a = \sigma_{max}$ corresponds to point A_a.

$R = \sigma_a - \sigma_{ave}$
$= 18 - 10.5 = 7.5$ ksi

$R = \sqrt{U^2 + \tau_{xy}^2}$

$\tau_{xy} = \pm\sqrt{R^2 - U^2}$

$= \pm\sqrt{7.5^2 - 4.5^2}$

$= \pm 6$ ksi

(b) For $\tau_{max} = 12$ ksi.

Center of Mohr's circle lies at point C. $\quad R = 12$ ksi

$\tau_{xy} = \pm\sqrt{R^2 - U^2} = \pm 11.24$ ksi

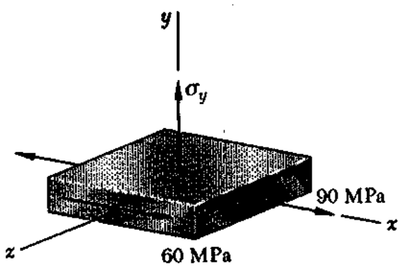
Checking $\quad \sigma_a = 10.5 + 12 = 22.5$ ksi $\quad \sigma_b = 10.5 - 12 = -1.5$ ksi

$\sigma_c = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 12$ ksi $\quad$ O.K

PROBLEM 7.78

7.78 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 80 MPa.



SOLUTION

$\sigma_x = 90$ MPa $\qquad \sigma_z = 0 \qquad \tau_{xz} = 60$ MPa

Mohr's circle for stresses in zx-plane

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 45$ MPa

$R = \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{xz}^2} = \sqrt{45^2 + 60^2} = 75$ MPa

$\sigma_a = \sigma_{ave} + R = 120$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -30$ MPa

Assume $\sigma_{max} = \sigma_a = 120$ MPa

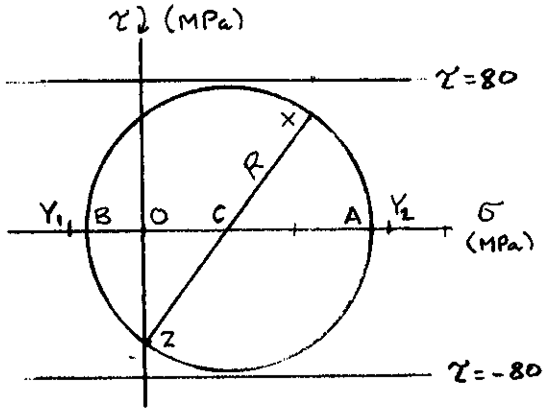
$\sigma_y = \sigma_{min} = \sigma_{max} - 2\tau_{max}$

$= 120 - (2)(80) = -40$ MPa ◄

Assume $\sigma_{min} = \sigma_b = -30$ MPa

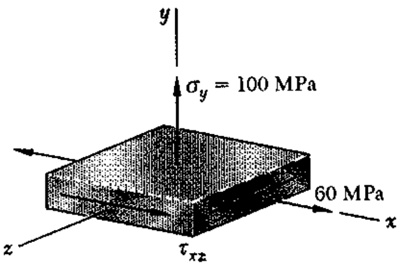
$\sigma_y = \sigma_{max} = \sigma_{min} + 2\tau_{max}$

$= -30 + (2)(8) = 130$ MPa ◄



PROBLEM 7.79

7.79 For the state of stress shown, determine the range of values of τ_{xy} for which the maximum shearing stress is equal to or less than 60 MPa.



SOLUTION

$\sigma_x = 60$ MPa, $\sigma_z = 0$, $\sigma_y = 100$ MPa

For Mohr's circle of stresses in zx-plane

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 30$ MPa

$U = \frac{\sigma_x - \sigma_z}{2} = 30$

Assume $\sigma_{max} = \sigma_y = 100$ MPa

$\sigma_{min} = \sigma_b = \sigma_{max} - 2\tau_{max}$

$= 100 - (2)(60) = -20$ MPa

$R = \sigma_{ave} - \sigma_b$

$= 30 - (-20) = 50$ MPa

$\sigma_a = \sigma_{ave} + R$

$= 30 + 50 = 80 \text{ MPa} < \sigma_y$

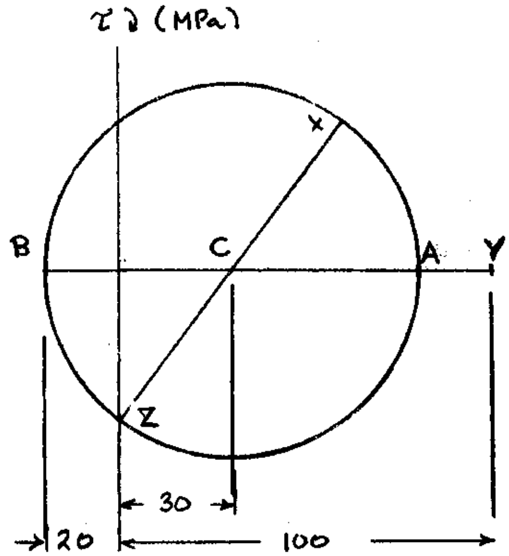
o.k.

$R = \sqrt{U^2 + \tau_{xz}^2}$

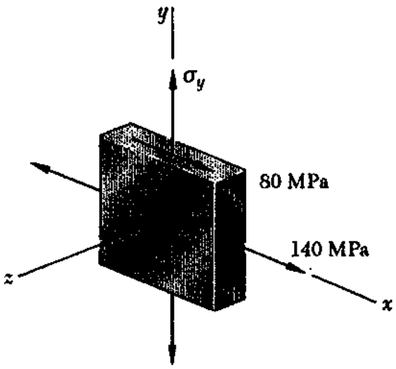
$\tau_{xz} = \pm\sqrt{R^2 - U^2}$

$= \pm\sqrt{50^2 - 30^2} = \pm 40$ MPa

$-40 \text{ MPa} \leq \tau_{xz} \leq 40$ MPa



PROBLEM 7.80



*7.80 For the state of stress of Prob. 6.66, determine (*a*) the value of σ_y for which the maximum shearing stress is as small as possible, (*b*) the corresponding value of the shearing stress.

SOLUTION

Let $u = \dfrac{\sigma_x - \sigma_y}{2}$ $\qquad \sigma_y = \sigma_x - 2u$

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = \sigma_x - u$$

$$R = \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{ave} - R = \sigma_x - u - \sqrt{u^2 + \tau_{xy}^2}$$

Assume τ_{max} is the in-plane shearing stress $\qquad \tau_{max} = R$

Then $\tau_{max\,(in\text{-}plane)}$ is minimum if $u = 0$

$$\sigma_y = \sigma_x - 2u = \sigma_x = 140 \text{ MPa}, \qquad \sigma_{ave} = \sigma_x - u = 140 \text{ MPa}$$

$$R = |\tau_{xy}| = 80 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 140 + 80 = 220 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 140 - 80 = 60 \text{ MPa}$$

$$\sigma_{max} = 220 \text{ MPa}, \quad \sigma_{min} = 0, \quad \tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = 110 \text{ MPa}$$

Assumption is incorrect.

Assume $\sigma_{max} = \sigma_a = \sigma_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$

$$\sigma_{min} = 0 \qquad \tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = \tfrac{1}{2}\sigma_a$$

$$\frac{d\sigma_a}{du} = -1 + \frac{u}{\sqrt{u^2 + \tau_{xy}^2}} \neq 0 \qquad \text{(no minimum)}$$

Optimum value for u occurs when $\tau_{max\,(out\text{-}of\text{-}plane)} = \tau_{max\,(in\text{-}plane)}$

$$\therefore \tfrac{1}{2}(\sigma_a + R) = R \quad \text{or} \quad \sigma_a = R \quad \text{or} \quad \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

$$(\sigma_x - u)^2 = \sigma_x^2 - 2u\sigma_x + \cancel{u^2} = \cancel{u^2} + \tau_{xy}^2$$

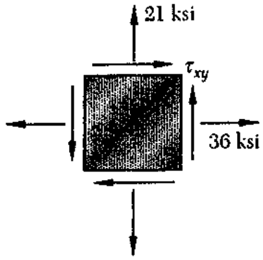
$$2u = \frac{\sigma_x^2 - \tau_{xy}^2}{\sigma_x} = \frac{140^2 - 80^2}{140} = 94.3 \text{ MPa} \qquad u = 47.14 \text{ MPa}$$

$$\sigma_y = \sigma_x - 2u = 140 - 94.3 = 45.7 \text{ MPa} \quad \blacktriangleleft$$

$$R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{max} = 92.9 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 7.81

7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 45$ ksi. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\tau_{xy} = 9$ ksi, (*b*) $\tau_{xy} = 18$ ksi, (*c*) $\tau_{xy} = 20$ ksi. If yield does not occur, determine the corresponding factor of safety.



SOLUTION

$\sigma_x = 36$ ksi $\qquad \sigma_y = 21$ ksi $\qquad \sigma_z = 0$

For stresses in xy-plane $\quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.5$ ksi

$\frac{\sigma_x - \sigma_y}{2} = 7.5$ ksi

(a) $\tau_{xy} = 9$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (9)^2} = 11.715$ ksi

$\sigma_a = \sigma_{ave} + R = 40.215$ ksi, $\qquad \sigma_b = \sigma_{ave} - R = 16.875$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 34.977$ ksi < 45 ksi (No yielding)

$F.S. = \frac{45}{34.977} = 1.287$

(b) $\tau_{xy} = 18$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (18)^2} = 19.5$ ksi

$\sigma_a = \sigma_{ave} + R = 48$ ksi, $\qquad \sigma_b = \sigma_{ave} - R = 9$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 44.193$ ksi < 45 ksi (No yielding)

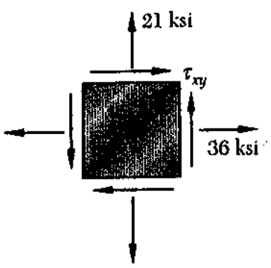
$F.S. = \frac{45}{44.193} = 1.018$

(c) $\tau_{xy} = 20$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (20)^2} = 21.36$ ksi

$\sigma_a = \sigma_{ave} + R = 49.86$, $\qquad \sigma_b = \sigma_{ave} - R = 7.14$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 46.732$ ksi > 45 ksi (Yielding occurs)

PROBLEM 7.82



7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 45$ ksi. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\tau_{xy} = 9$ ksi, (*b*) $\tau_{xy} = 18$ ksi, (*c*) $\tau_{xy} = 20$ ksi. If yield does not occur, determine the corresponding factor of safety.

7.82 Solve Prob. 7.81, using the maximum-shearing-stress criterion.

SOLUTION

$\sigma_x = 36$ ksi $\quad \sigma_y = 21$ ksi $\quad \sigma_z = 0$

For stresses in xy-plane $\quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.5$ ksi

$$\frac{\sigma_x - \sigma_y}{2} = 7.5 \text{ ksi}$$

(a) $\tau_{xy} = 9$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 11.715$ ksi

$\sigma_a = \sigma_{ave} + R = 40.215$ ksi, $\quad \sigma_b = \sigma_{ave} - R = 16.875$ ksi

$\sigma_{max} = 34.977$ ksi, $\quad \sigma_{min} = 0$

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 40.215$ ksi < 45 ksi $\quad$ (No yielding)

$$\text{F.S.} = \frac{45}{40.215} = 1.119 \quad \blacktriangleleft$$

(b) $\tau_{xy} = 18$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 19.5$ ksi

$\sigma_a = \sigma_{ave} + R = 48$ ksi, $\quad \sigma_b = \sigma_{ave} - R = 9$ ksi

$\sigma_{max} = 48$ ksi $\quad \sigma_{min} = 0$

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 48$ ksi > 45 ksi $\quad$ (Yielding occurs)

(c) $\tau_{xy} = 20$ ksi $\quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 21.36$ ksi

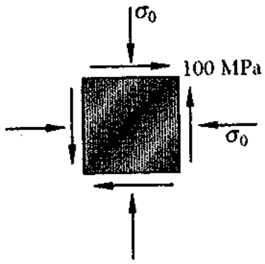
$\sigma_a = \sigma_{ave} + R = 49.86$ ksi $\quad \sigma_b = \sigma_{ave} - R = 7.14$ ksi

$\sigma_{max} = 49.86$ ksi $\quad \sigma_{min} = 0$

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 49.86$ ksi > 45 ksi $\quad$ (Yielding occurs)

PROBLEM 7.83

7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-shearing-stress criterion, determine whether yield occurs when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.



SOLUTION

$\sigma_{ave} = -\sigma_0$ $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100$ MPa

(a) $\sigma_0 = 200$ MPa, $\quad \sigma_{ave} = -200$ MPa

$\sigma_a = \sigma_{ave} + R = -100$ MPa $\qquad \sigma_b = \sigma_{ave} - R = -300$ MPa

$\sigma_{max} = 0$, $\quad \sigma_{min} = -300$ MPa

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 300$ MPa < 325 MPa (No yielding)

$$F.S. = \frac{325}{300} = 1.083$$

(b) $\sigma_0 = 240$ MPa, $\quad \sigma_{ave} = -240$ MPa

$\sigma_a = \sigma_{ave} + R = -140$ MPa , $\qquad \sigma_b = \sigma_{ave} - R = -340$ MPa

$\sigma_{max} = 0$, $\quad \sigma_{min} = -340$ MPa

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 340$ MPa > 325 MPa (Yielding occurs)

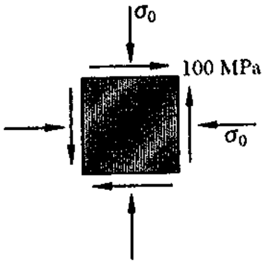
(c) $\sigma_0 = 280$ MPa, $\quad \sigma_{ave} = -280$ MPa

$\sigma_a = \sigma_{ave} + R = -180$ MPa , $\qquad \sigma_b = \sigma_{ave} - R = -380$ MPa

$\sigma_{max} = 0$, $\quad \sigma_{min} = -380$ MPa

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 380$ MPa > 325 MPa (Yielding occurs)

PROBLEM 7.84



7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-shearing-stress criterion, determine whether yield occurs when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.

7.84 Solve Prob. 7.83, using the maximum-distortion-energy criterion.

SOLUTION

$\sigma_{ave} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$

(a) $\sigma_0 = 200$ MPa $\qquad \sigma_{ave} = -200$ MPa

$\sigma_a = \sigma_{ave} + R = -100$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -300$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 264.56 \text{ MPa} < 325 \text{ MPa}$ (No yielding)

$F.S. = \frac{325}{264.56} = 1.228$ ◄

(b) $\sigma_0 = 240$ MPa $\qquad \sigma_{ave} = -240$ MPa

$\sigma_a = \sigma_{ave} + R = -140$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -340$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 295.97 \text{ MPa} < 325 \text{ MPa}$ (No yielding)

$F.S. = \frac{325}{295.97} = 1.098$ ◄

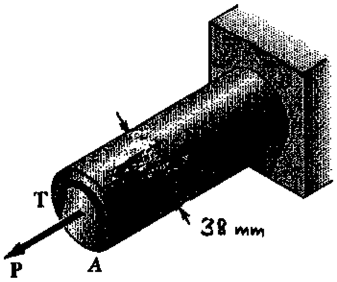
(c) $\sigma_0 = 280$ MPa $\qquad \sigma_{ave} = -280$ MPa

$\sigma_a = \sigma_{ave} + R = -180$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -380$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 329.24 \text{ MPa} > 325 \text{ MPa}$ (Yielding occurs)

PROBLEM 7.85

7.85 The 38-mm-diameter shaft *AB* is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 240$ kN.



SOLUTION

$P = 240 \times 10^3$ N

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(38)^2 = 1.1341 \times 10^3 \text{ mm}^2 = 1.1341 \times 10^{-3} \text{ m}^2$

$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.1341 \times 10^{-3}} = 211.6 \times 10^6 \text{ Pa} = 211.6 \text{ MPa}$

$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y$

$4\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \qquad \tau_{xy} = \frac{1}{2}\sqrt{\sigma_Y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{250^2 - 211.6^2}$

$= 66.568 \text{ MPa} = 65.568 \times 10^6 \text{ Pa}$

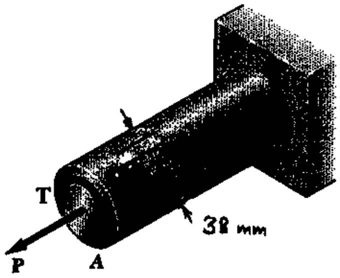
From torsion $\quad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{38}{2}\right)^4 = 204.71 \times 10^3 \text{ mm}^4 = 204.71 \times 10^{-9} \text{ m}^4$

$c = \frac{1}{2}d = 19 \times 10^{-3}$ m

$T = \frac{(204.71 \times 10^{-9})(65.568 \times 10^6)}{19 \times 10^{-3}} = 717 \text{ N}\cdot\text{m}$

PROBLEM 7.86



7.85 The 38-mm-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 240$ kN.

7.86 Solve Prob. 7.85, using the maximum-distortion-energy criterion.

SOLUTION

$$P = 240 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(38)^2 = 1.1341 \times 10^3 \text{ mm}^2 = 1.1341 \times 10^{-3} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.1341 \times 10^{-3}} = 211.6 \times 10^6 \text{ Pa} = 211.6 \text{ MPa}$$

$$\sigma_y = 0 \qquad \sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = \tfrac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\tfrac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R = \tfrac{1}{2}\sigma_x + \sqrt{\tfrac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{ave} - R = \tfrac{1}{2}\sigma_x - \sqrt{\tfrac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\begin{aligned}\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = {} & \tfrac{1}{4}\sigma_x^2 + \sigma_x\sqrt{\tfrac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \tfrac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ & + \tfrac{1}{4}\sigma_x^2 - \sigma_x\sqrt{\tfrac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \tfrac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ & - \tfrac{1}{4}\sigma_x^2 + \tfrac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ = {} & \sigma_x^2 + 3\tau_{xy}^2 = \sigma_Y^2\end{aligned}$$

$$\tau_{xy}^2 = \tfrac{1}{3}(\sigma_Y^2 - \sigma_x^2)$$

$$\tau_{xy} = \frac{1}{\sqrt{3}}\sqrt{250^2 - 211.6^2} = 76.867 \text{ MPa} = 76.867 \times 10^6 \text{ Pa}$$

From torsion $\qquad \tau_{xy} = \dfrac{Tc}{J} \qquad T = \dfrac{J\tau_{xy}}{c}$

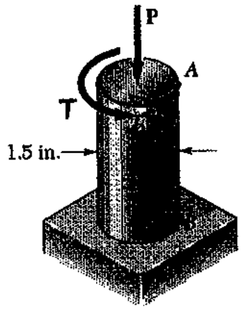
$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{38}{2}\right)^4 = 204.71 \times 10^3 \text{ mm}^4 = 204.71 \times 10^{-9} \text{ m}^4$$

$$c = \tfrac{1}{2}d = 19 \times 10^{-3} \text{ m}$$

$$T = \frac{(204.71 \times 10^{-9})(76.876 \times 10^6)}{19 \times 10^{-3}} = 828 \text{ N}\cdot\text{m}$$

PROBLEM 7.87

7.87 The 1.5-in-diameter shaft *AB* is made of a grade of steel for which the yield strength is $\sigma_Y = 42$ ksi.. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 60$ kips.



SOLUTION

$P = 60$ kips $\qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.7671 \text{ in}^2$

$$\sigma_x = -\frac{P}{A} = -\frac{60}{1.7671} = -33.953 \text{ ksi}$$

$$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y$$

$$4\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \qquad \tau_{xy} = \frac{1}{2}\sqrt{\sigma_Y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{42^2 - 33.953^2}$$

$$= 12.361 \text{ ksi}$$

From torsion $\qquad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

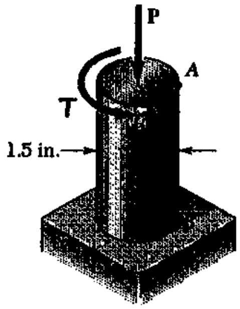
$$c = \frac{1}{2}d = 0.75 \text{ in} \qquad J = \frac{\pi}{2}c^4 = 0.49701 \text{ in}^4$$

$$T = \frac{(0.49701)(12.361)}{0.75} = 8.19 \text{ kip}\cdot\text{in}$$

PROBLEM 7.88

7.87 The 1.5-in-diameter shaft *AB* is made of a grade of steel for which the yield strength is $\sigma_Y = 42$ ksi.. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 60$ kips.

7.88 Solve Prob. 7.87, using the maximum-distortion-energy criterion.



SOLUTION

$P = 60$ kips $\quad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.7671 \text{ in}^2$

$\sigma_x = -\frac{P}{A} = -\frac{60}{1.7671} = -33.953$ ksi

$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$\sigma_a = \sigma_{ave} + R \qquad \sigma_b = \sigma_{ave} - R$

$\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = (\sigma_{ave} + R)^2 + (\sigma_{ave} - R)^2 - (\sigma_{ave} + R)(\sigma_{ave} - R)$

$= \sigma_{ave}^2 + \cancel{2\sigma_{ave}R} + R^2 + \sigma_{ave}^2 - \cancel{2\sigma_{ave}R} + R^2 - \sigma_{ave}^2 + R^2$

$= \sigma_{ave}^2 + 3R^2$

$= \frac{1}{4}\sigma_x^2 + 3\left(\frac{1}{4}\sigma_x^2 + \tau_{xy}^2\right) = \sigma_x^2 + 3\tau_{xy}^2 = \sigma_Y^2$

$3\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \qquad \tau_{xy} = \frac{1}{\sqrt{3}}(\sigma_Y^2 - \sigma_x^2) = \frac{1}{\sqrt{3}}\sqrt{42^2 - 33.953^2}$

$= 14.273$ ksi

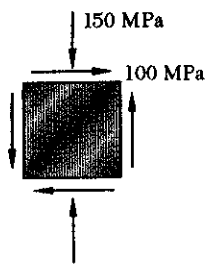
From torsion $\qquad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

$c = \frac{1}{2}d = 0.75$ in. $\qquad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701 \text{ in}^4$

$T = \frac{(0.49701)(14.273)}{0.75} = 9.46$ kip·in ◄

PROBLEM 7.89

7.89 and 7.90 The state of plane stress shown is expected to occur in a cast-iron machine base. Knowing that for the grade of cast iron used $\sigma_{UT} = 160$ MPa and $\sigma_{UC} = 320$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.



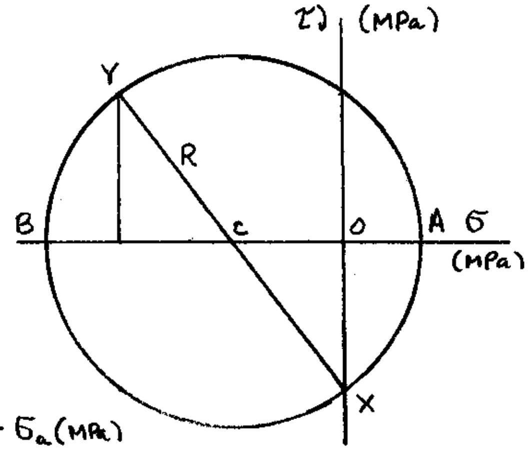
SOLUTION

$\sigma_x = 0 \qquad \sigma_y = -150 \text{ MPa} \qquad \tau_{xy} = 100 \text{ MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -75 \text{ MPa}$

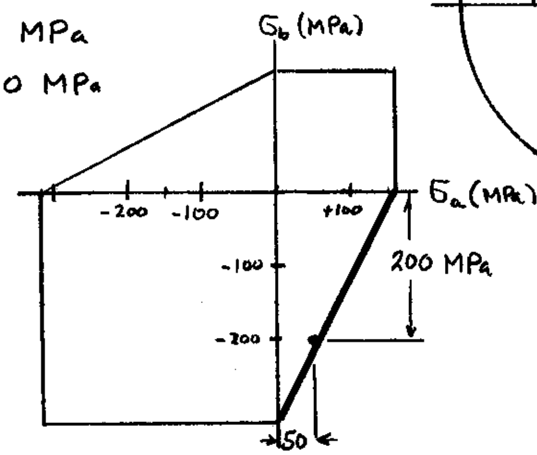
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{75^2 + 100^2} = 125 \text{ MPa}$



$\sigma_a = \sigma_{ave} + R = 50 \text{ MPa}$

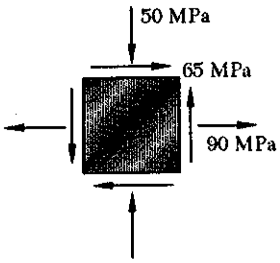
$\sigma_b = \sigma_{ave} - R = -200 \text{ MPa}$



Equation of the 4th quandrant boundary is $\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

$\frac{50}{160} - \frac{(-200)}{320} = 0.9375 < 1$, No rupture.

PROBLEM 7.90



7.89 and 7.90 The state of plane stress shown is expected to occur in a cast-iron machine base. Knowing that for the grade of cast iron used $\sigma_{UT} = 160$ MPa and $\sigma_{UC} = 320$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.

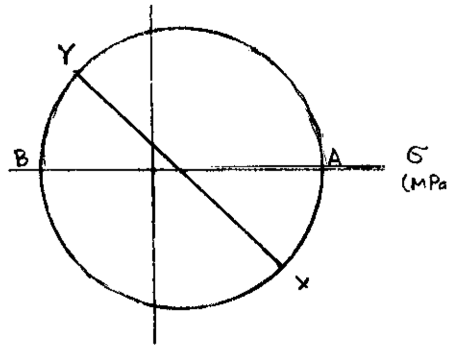
SOLUTION

$\sigma_x = 90$ MPa , $\sigma_y = -50$ MPa , $\tau_{xy} = 65$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 20$ MPa

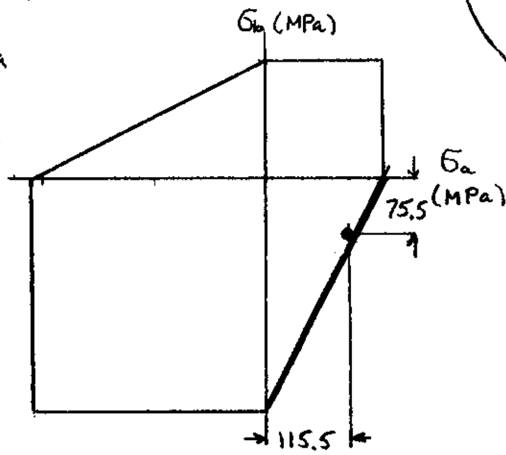
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{70^2 + 65^2} = 95.5$ MPa



$\sigma_a = \sigma_{ave} + R = 115.5$ MPa

$\sigma_b = \sigma_{ave} - R = -75.5$ MPa

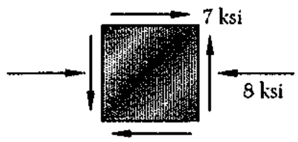


Equation of 4th quadrant boundary $\quad \frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

$\frac{115.5}{160} - \frac{(-75.5)}{320} = 0.958 < 1$ No rupture

PROBLEM 7.91

7.91 and 7.92 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 30$ ksi and using Mohr's criterion, determine whether rupture of the component will occur.



SOLUTION

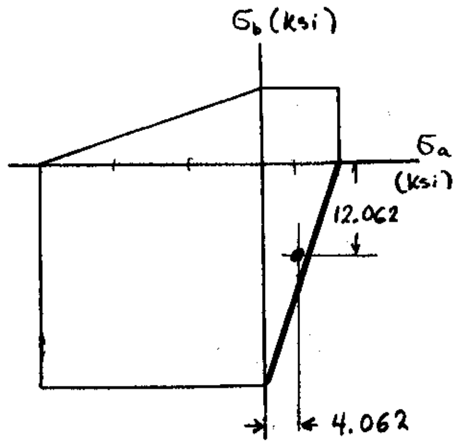
$$\sigma_x = -8 \text{ ksi}, \quad \sigma_y = 0, \quad \tau_{xy} = 7 \text{ ksi}$$

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = -4 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + 7^2} = 8.062 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = -4 + 8.062 = 4.062 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -4 - 8.062 = -12.062 \text{ ksi}$$



Equation of 4 th quadrant of boundary

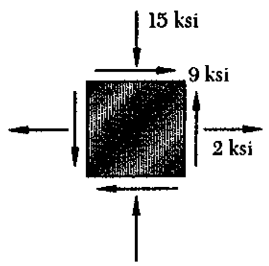
$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{4.062}{10} - \frac{(-12.062)}{30} = 0.808 < 1$$

(No rupture)

PROBLEM 7.92

7.91 and 7.92 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 30$ ksi and using Mohr's criterion, determine whether rupture of the component will occur.



SOLUTION

$\sigma_x = 2$ ksi $\qquad \sigma_y = -15$ ksi $\qquad \tau_{xy} = 9$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.5$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{8.5^2 + 9^2} = 12.379$ ksi

$\sigma_a = \sigma_{ave} + R = 5.879$ ksi

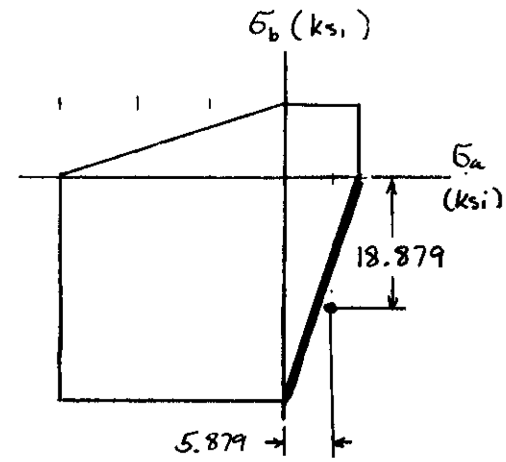
$\sigma_b = \sigma_{ave} - R = -18.879$ ksi

Equation of 4th quadrant of boundary

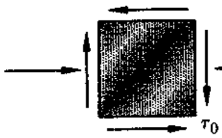
$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{5.879}{10} - \frac{(-18.879)}{30} = 1.217 > 1$$

Rupture will occur.



PROBLEM 7.93



7.93 The state of plane stress shown will occur at a critical point in a cast pipe made of an aluminum alloy for which $\sigma_{UT} = 75$ MPa and $\sigma_{UC} = 150$ MPa. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.

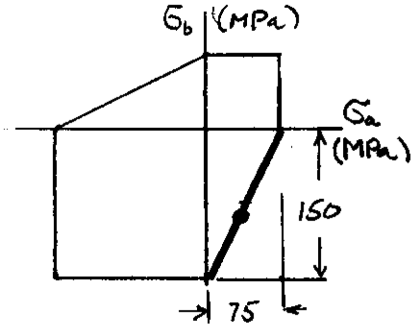
SOLUTION

$\sigma_x = -80$ MPa $\qquad \sigma_y = 0 \qquad \tau_{xy} = -\tau_0$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -40$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + \tau_0^2}$ MPa

$\sigma_a = \sigma_{ave} + R, \qquad \sigma_b = \sigma_{ave} - R, \qquad \tau_0 = \pm\sqrt{R^2 - 40^2}$



Since $|\sigma_{ave}| < R$, stress point lies in 4th quandrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

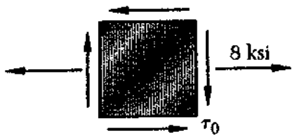
$$\frac{-40 + R}{75} - \frac{-40 - R}{150} = 1$$

$$\frac{R}{75} + \frac{R}{150} = 1 + \frac{40}{75} - \frac{40}{150} = 1.2667$$

$R = 63.33$ MPa, $\qquad \tau_0 = \pm\sqrt{63.33^2 - 40^2} = \pm 49.1$ MPa ◄

PROBLEM 7.94

7.94 The state of plane stress shown will occur in an aluminum casting that is made of an alloy for which $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 25$ ksi. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.



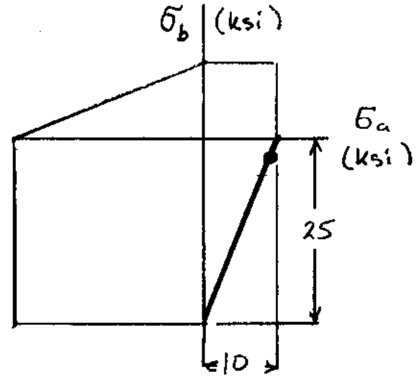
SOLUTION

$\sigma_x = 8$ ksi , $\sigma_y = 0$, $\tau_{xy} = \tau_o$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 4$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + \tau_o^2}$, $\tau_o = \pm\sqrt{R^2 - 4^2}$

$\sigma_a = \sigma_{ave} + R = (4 + R)$ ksi $\quad \sigma_b = \sigma_{ave} - R = (4 - R)$ ksi



Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

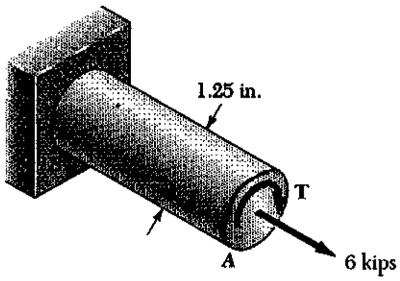
$$\frac{4+R}{10} - \frac{4-R}{25} = 1$$

$$\left(\frac{1}{10} + \frac{1}{25}\right)R = 1 - \frac{4}{10} + \frac{4}{25}$$

$R = 5.429$ ksi $\quad \tau_o = \pm\sqrt{5.429^2 - 4^2} = \pm 3.67$ ksi ◀

PROBLEM 7.95

7.95 The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 8$ ksi and $\sigma_{UC} = 16$ ksi. Using Mohr's criterion, determine the magnitude of the torque **T** for which rupture should be expected.



SOLUTION

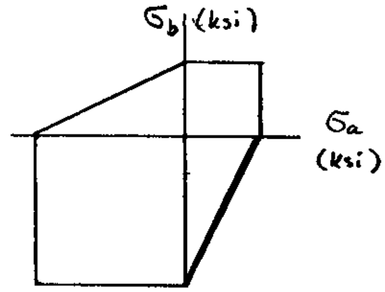
$P = 6$ kips, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.25)^2 = 1.2272$ in^2

$\sigma_x = \frac{P}{A} = 4.889$ ksi, $\sigma_y = 0$, $\tau_{xy} = \frac{Tc}{J}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 2.4446$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5.976 + \tau_{xy}^2}$ ksi, $\quad \tau_{xy} = \pm\sqrt{R^2 - 5.976}$ ksi

$\sigma_a = \sigma_a + R = 2.4446 + R$ ksi, $\quad \sigma_b = 2.4446 - R$ ksi



Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{2.4446 + R}{8} - \frac{2.4446 - R}{16} = 1$$

$$\left(\frac{1}{8} + \frac{1}{16}\right)R = 1 - \frac{2.4446}{8} + \frac{2.4446}{16}$$

$R = 4.5185$ ksi $\qquad \tau_{xy} = \pm\sqrt{4.5185^2 - 5.976} = 3.80$ ksi

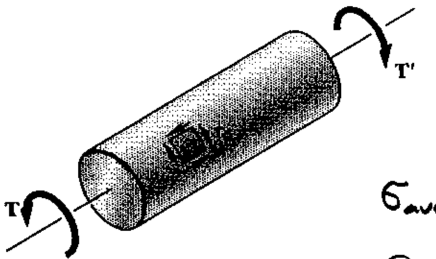
For torsion: $\quad c = \frac{1}{2}d = 0.625$ in

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.625)^4 = 0.23968 \text{ in}^4$$

$$T = \frac{J\tau_{xy}}{c} = \frac{(0.23968)(3.80)}{0.625} = 1.457 \text{ kip}\cdot\text{in} \quad \blacktriangleleft$$

PROBLEM 7.96

7.96 The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 175$ MPa. Knowing that the magnitude T of the applied torques is slowly increased and using Mohr's criterion, determine the shearing stress τ_0 which should be expected at rupture.



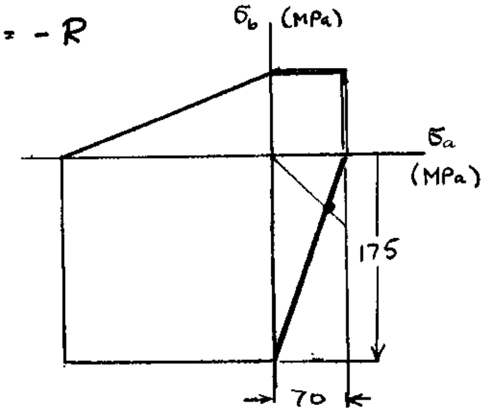
SOLUTION

$\sigma_x = 0$, $\quad \sigma_y = 0 \qquad \tau_{xy} = -\tau_0$

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + \tau_{xy}^2} = |\tau_{xy}|$$

$\sigma_a = \sigma_{ave} + R = R \qquad \sigma_b = \sigma_{ave} - R = -R$



Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of boundary of 4th quadrant is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

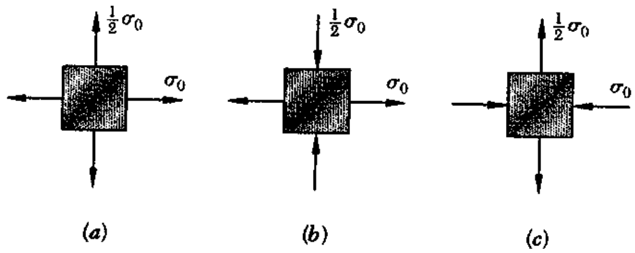
$$\frac{R}{70} - \frac{-R}{175} = 1$$

$$\left(\frac{1}{70} + \frac{1}{175}\right) R = 1$$

$$R = 50 \text{ MPa} \qquad |\tau_{xy}| = 50 \text{ MPa}$$

PROBLEM 7.97

7.97 A machine component is made of a grade of cast iron for which $\sigma_{UT} = 8$ ksi and $\sigma_{UC} = 20$ ksi. For each of the states of plane stress shown, and using Mohr's criterion, determine the normal stress σ_0 at which rupture of the component should be expected.

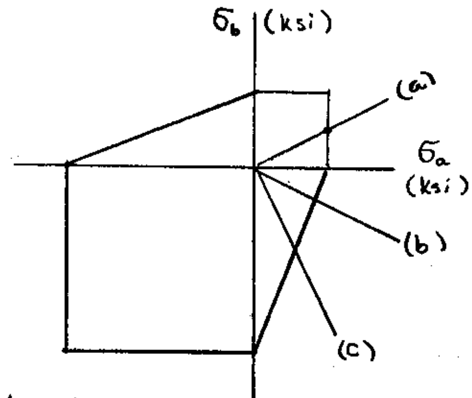


SOLUTION

(a) $\sigma_a = \sigma_0$, $\sigma_b = \frac{1}{2}\sigma_0$

Stress point lies in 1st quadrant.

$\sigma_a = \sigma_0 = \sigma_{UT} = 8$ ksi ◄



(b) $\sigma_a = \sigma_0$, $\sigma_b = -\frac{1}{2}\sigma_0$

Stress point lies in 4th quadrant.
Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{\sigma_0}{8} - \frac{-\frac{1}{2}\sigma_0}{20} = 1 \qquad \sigma_0 = 6.67 \text{ ksi} \blacktriangleleft$$

(c) $\sigma_a = \frac{1}{2}\sigma_0$, $\sigma_b = -\sigma_0$, 4th quadrant

$$\frac{\frac{1}{2}\sigma_0}{8} - \frac{-\sigma_0}{20} = 1 \qquad \sigma_0 = 8.89 \text{ ksi} \blacktriangleleft$$

PROBLEM 7.98

7.98 Determine the normal stress in a basketball of 9.5-in. diameter and 0.125-in. wall thickness that is inflated to a gage pressure of 9 psi.

SOLUTION

$$r = \tfrac{1}{2}d - t = (\tfrac{1}{2})(9.5) - 0.125 = 4.625 \text{ in}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(9)(4.625)}{(2)(0.125)} = 166.5 \text{ psi} \blacktriangleleft$$

PROBLEM 7.99

7.99 A spherical gas container made of steel has an 18-ft diameter and a wall thickness of $\frac{3}{8}$ in. Knowing that the internal pressure is 60 psi, determine the maximum normal stress and the maximum shearing stress in the container.

SOLUTION

$d = 18 \text{ ft} = 216 \text{ in} \qquad r = \frac{1}{2}d - t = 107.625 \text{ in.}$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(60)(107.625)}{(2)(0.375)} = 8610 \text{ psi} = 8.61 \text{ ksi}$$

$$\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}\sigma_1 = 4.31 \text{ ksi}$$

PROBLEM 7.100

7.100 The maximum gage pressure is known to be 8 MPa in a spherical steel pressure vessel having a 250-mm diameter and a 6-mm wall thickness. Knowing that the ultimate stress in the steel used is $\sigma_U = 400$ MPa, determine the factor of safety with respect to tensile failure.

SOLUTION

$p = 8 \text{ MPa} = 8 \times 10^6 \text{ Pa} \qquad t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$

$r = \frac{1}{2}d - t = \frac{1}{2}(250) - 6 = 119 \text{ mm} = 0.119 \text{ m}$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(8\times10^6)(0.119)}{(2)(6\times10^{-3})} = 79.33\times10^6 \text{ Pa} = 79.33 \text{ MPa}$$

$$F.S. = \frac{\sigma_U}{\sigma_1} = \frac{400}{79.33} = 5.04$$

PROBLEM 7.101

7.101 A spherical pressure vessel of 900-mm outside diameter is to be fabricated from a steel having an ultimate stress $\sigma_U = 400$ MPa. Knowing that a factor of safety of 4 is desired and that the gage pressure can reach 3.5 MPa, determine the smallest wall thickness that should be used.

SOLUTION

$p = 3.5 \text{ MPa}, \qquad r = \frac{1}{2}d - t = (\frac{1}{2})(900) - t = 450 - t \text{ mm}$

$$\sigma_1 = \sigma_2 = \frac{\sigma_U}{F.S.} = \frac{400}{4} = 100 \text{ MPa}$$

$$\sigma_1 = \frac{pr}{2t} \quad \therefore \quad t = \frac{pr}{2\sigma_1} = \frac{(3.5)(450-t)}{(2)(100)} = 7.875 - 0.0175\,t$$

$$1.0175\,t = 7.875 \qquad t = 7.74 \text{ mm}$$

PROBLEM 7.102

7.102 A spherical gas container having a diameter of 5 m and a wall thickness of 24 mm is made of a steel for which $E = 200$ GPa and $\nu = 0.29$. Knowing that the gage pressure in the container is increased from zero to 1.8 MPa, determine (*a*) the maximum normal stress in the container, (*b*) the increase in the diameter of the container.

SOLUTION

$p = 1.8 \text{ MPa} \qquad r = \frac{1}{2}d - t = \frac{1}{2}(5) - 24\times10^{-3} = 2.476 \text{ m}$

(a) $\sigma_1 = \sigma_2 = \dfrac{pr}{2t} = \dfrac{(1.8)(2.476)}{(2)(24\times10^{-3})} = 92.85 \text{ MPa}$

$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1 = \dfrac{1-0.29}{200\times10^9}(92.85\times10^6) = 329.6\ \mu$

$\Delta d = d\,\varepsilon_1 = (5)(329.6\times10^{-6}) = 1.648\times10^{-3} \text{ m} = 1.648 \text{ mm}$

PROBLEM 7.103

7.103 A spherical pressure vessel is 3 m in diameter and has a wall thickness of 12 mm. Knowing that for the steel used $\sigma_{all} = 80$ MPa, $E = 200$ GPa and $\nu = 0.29$, determine (*a*) the allowable gage pressure, (*b*) the corresponding increase in the diameter of the vessel.

SOLUTION

$r = \frac{1}{2}d - t = \frac{1}{2}(3000) - 12 = 1488 \text{ mm}$

$\sigma_1 = \sigma_2 = \sigma_{all} = 8 \text{MPa}$

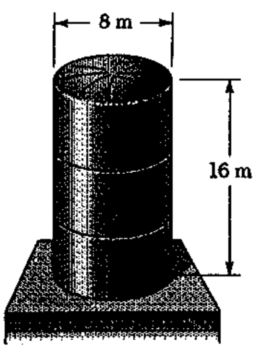
(a) $\sigma_1 = \sigma_2 = \dfrac{pr}{2t} \qquad p = \dfrac{2t\sigma_1}{r} = \dfrac{(2)(12)(80)}{1488} = 1.290 \text{ MPa}$

$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1 = \dfrac{1-0.29}{200\times10^9}(8\times10^6) = 28.4\ \mu$

(b) $\Delta d = d\,\varepsilon_1 = (3000)(28.4\times10^{-6}) = 85.2\times10^{-3} \text{ mm} = 0.0852 \text{ mm}$

PROBLEM 7.104

7.104 When filled to capacity, the unpressurized storage tank shown contains water to a height of 15.5 m above its base. Knowing that the lower portion of the tank has a wall thickness of 16 mm, determine the maximum normal stress and the maximum shearing stress in the tank. (Density of water = 1000 kg/m³)



SOLUTION

$p = \rho g h = (1000)(9.81)(15.5) = 152.06\times10^3 \text{ Pa}$

$r = \frac{1}{2}d - t = \frac{1}{2}(8) - 16\times10^{-3} = 3.984 \text{ m}$

$\sigma_1 = \dfrac{pr}{t} = \dfrac{(152.06\times10^3)(3.984)}{16\times10^{-3}} = 37.9\times10^6 \text{ Pa}$

$= 37.9 \text{ MPa}$

$\tau_{max\,(out\ of\ plane)} = \frac{1}{2}\sigma_1 = 18.93 \text{ MPa}$

PROBLEM 7.105

7.105 Determine the largest internal pressure that can be applied to a cylindrical tank of 5.5-ft diameter and $\frac{5}{8}$ - in. wall thickness if the ultimate normal stress of the steel used is 65 ksi and a factor of safety of 5.0 is desired.

SOLUTION

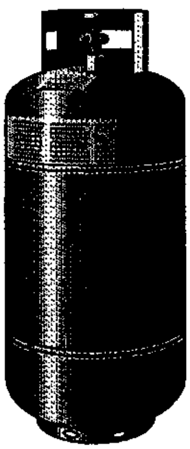
$\sigma_1 = \frac{\sigma_U}{F.S.} = \frac{65}{5.0} = 13$ ksi $\qquad d = 5.5$ ft $= 66$ in

$r = \frac{1}{2}d - t = \frac{1}{2}(66) - 0.625 = 32.375$ in.

$\sigma_1 = \frac{pr}{t} \qquad p = \frac{\sigma_1 t}{r} = \frac{(13)(0.625)}{32.375} = 0.251$ ksi $= 251$ psi ◄

PROBLEM 7.106

7.106 The storage tank shown contains liquified propane under a pressure of 210 psi at a temperature of 100° F. Knowing that the tank has a diameter of 12.6 in. and a wall thickness of 0.11 in, determine the maximum normal stress and the maximum shearing stress in the tank.



SOLUTION

$p = 210$ psi, $\qquad r = \frac{1}{2}d - t = \frac{1}{2}(12.6) - 0.11 = 6.19$ in.

$\sigma_1 = \frac{pr}{t} = \frac{(210)(6.19)}{0.11} = 11.82 \times 10^3$ psi $= 11.82$ ksi ◄

$\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}\sigma_1 = 5.91$ ksi ◄

PROBLEM 7.107

7.107 The bulk storage tank shown in Fig. 7.49 has an outer diameter of 3.3 m and a wall thickness of 18 mm. At a time when the internal pressure of the tank is 1.5 MPa, determine the maximum normal stress and the maximum shearing stress in the tank.

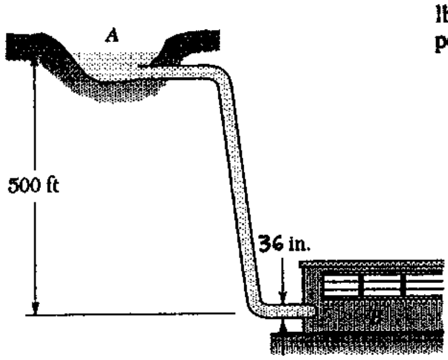
SOLUTION

$d = 3.3$ m, $\quad t = 18 \times 10^{-3}$ m $\qquad r = \frac{1}{2}d - t = 1.632$ m

$p = 1.5$ MPa $\qquad \sigma_1 = \frac{pr}{t} = \frac{(1.5 \times 10^6)(1.632)}{18 \times 10^{-3}} = 136 \times 10^6$ Pa $= 136$ MPa ◄

$\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}\sigma_1 = 68$ MPa ◄

PROBLEM 7.108



7.108 A 36-in.-diameter penstock has a 0.5-in wall thickness and connects a reservoir at A with a generating station at B. Knowing that the specific weight of water is 62.4 lb/ft^3, determine the maximum normal stress and the maximum shearing stress in the penstock under static conditions.

SOLUTION

$r = \frac{1}{2}d - t = \frac{1}{2}(30) - 0.5 = 1.5 \text{ in.}$

$p = \gamma h = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2 \times 10^3 \text{ lb/ft}^2$

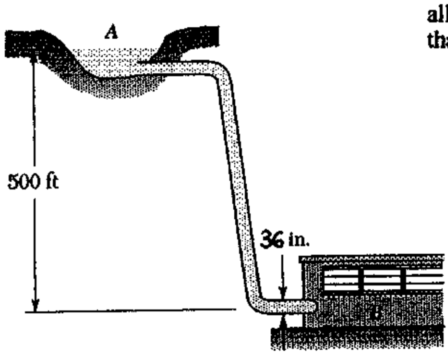
$= 216.67 \text{ psi}$

$\sigma_1 = \frac{pr}{t} = \frac{(216.67)(17.5)}{0.5} = 7583 \text{ psi}$

$= 7.58 \text{ ksi}$ ◄

$\tau_{max\,(\text{out-of-plane})} = \frac{1}{2}\sigma_1 = 3.79 \text{ ksi}$ ◄

PROBLEM 7.109



7.109 A 36-in.-diameter steel penstock connects a reservoir at A with a generating station at B. Knowing that the specific weight of water is 62.4 lb/ft^3 and that the allowable normal stress in the steel is 12.5 ksi, determine the smallest wall thickness that can be used for the penstock.

SOLUTION

$p = \gamma h = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2 \times 10^3 \text{ lb/ft}^2$

$= 216.67 \text{ psi}$

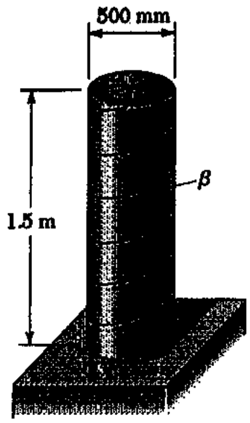
$\sigma_1 = 12.5 \text{ ksi} = 12.5 \times 10^3 \text{ psi}$

$r = \frac{1}{2}d - t = 18 - t$

$\sigma_1 = \frac{pr}{t}, \quad \frac{r}{t} = \frac{\sigma_1}{p}, \qquad \frac{18 - t}{t} = \frac{12.5 \times 10^3}{216.67} = 57.692$

$\frac{18}{t} = 58.692 \qquad t = 0.307 \text{ in}$ ◄

PROBLEM 7.110

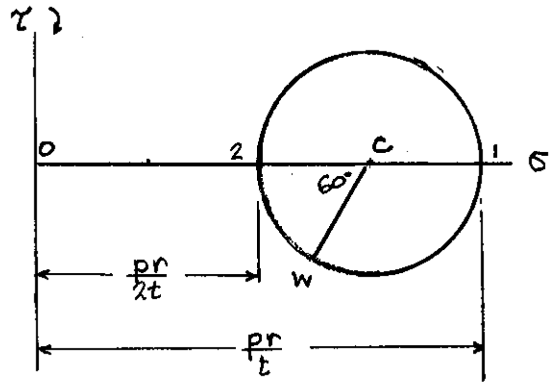


7.110 The cylindrical portion of the compressed air tank shown is fabricated of 6-mm-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Knowing that the allowable stress normal to the weld is 75 MPa, determine the largest gage pressure that can be used in the tank.

SOLUTION

$r = \frac{1}{2}d - t = \frac{1}{2}(500) - 6 = 244 \text{ mm}$

$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{1}{2}\frac{pr}{t}$



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$

$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

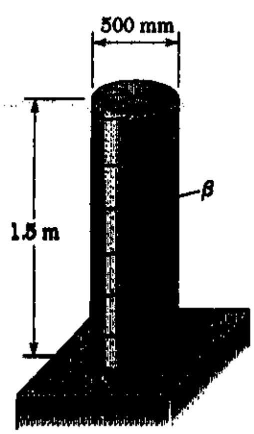
$\sigma_w = \sigma_{ave} - R\cos 60°$

$= \frac{5}{8}\frac{pr}{t}$

$p = \frac{8}{5}\frac{\sigma_w t}{r}$

$p = \frac{8}{5}\frac{(75)(6)}{244} = 2.95 \text{ MPa}$ ◄

PROBLEM 7.111

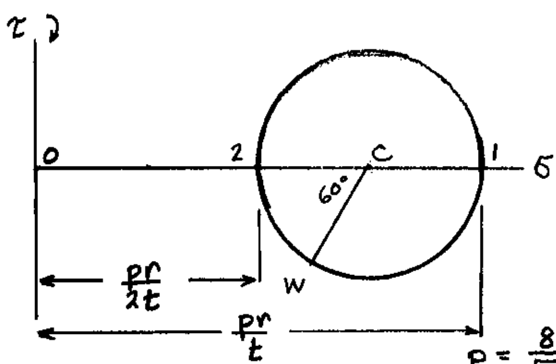


7.111 The cylindrical portion of the compressed air tank shown is fabricated of 6-mm-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Determine the gage pressure that will cause a shearing stress parallel to the weld of 30 MPa.

SOLUTION

$r = \frac{1}{2}d - t = \frac{1}{2}(500) - 6 = 244 \text{ mm}$

$\sigma_1 = \frac{pr}{t}, \qquad \sigma_2 = \frac{1}{2}\frac{pr}{t}$



$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

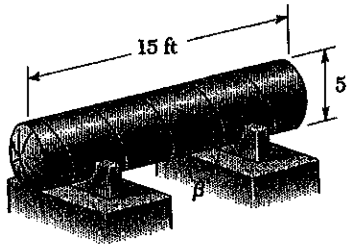
$\tau_w = R\sin 60°$

$= \frac{\sqrt{3}}{8}\frac{pr}{t}$

$p = \frac{8}{\sqrt{3}}\frac{\tau_w t}{R}$

$p = \frac{8}{\sqrt{3}}\frac{(30)(6)}{244} = 3.41 \text{ MPa}$ ◄

PROBLEM 7.112

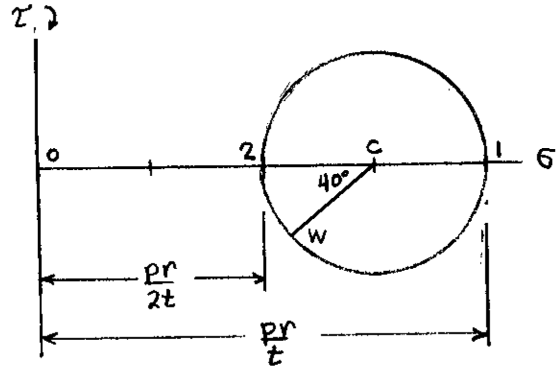


7.112 The pressure tank shown has a $\frac{3}{8}$-in. wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 85 psi, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$d = 5 \text{ ft} = 60 \text{ in.} \qquad r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625 \text{ in.}$

$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715 \text{ psi}$



$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5 \text{ psi}$

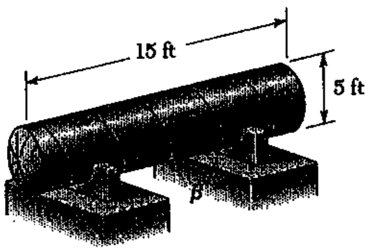
$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 5036.25 \text{ psi}$

$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75 \text{ psi}$

(a) $\sigma_w = \sigma_{ave} - R\cos 40° = 3750 \text{ psi}$

(b) $\tau_w = R\sin 40° = 1079 \text{ psi}$

PROBLEM 7.113



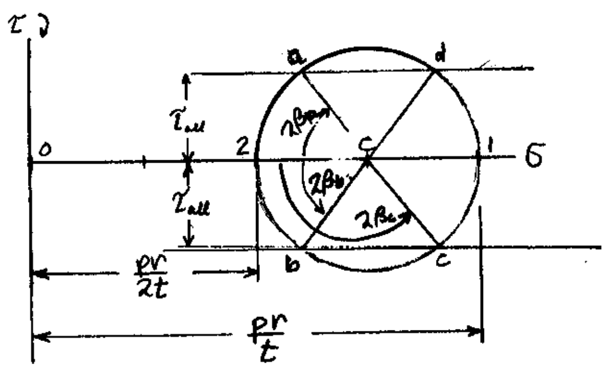
7.113 The pressure tank shown has a $\frac{3}{8}$-in. wall thickness and butt-welded seams forming an angle β with a transverse plane. Determine the range of values of β that can be used if the shearing stress parallel to the weld is not to exceed 1350 psi when the gage pressure is 85 psi.

SOLUTION

$d = 5 \text{ ft} = 60 \text{ in} \qquad r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625 \text{ in.}$

$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715 \text{ psi}$

$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5 \text{ psi}$



$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75$

$\tau_w = R\sin 2\beta = \tau_{all}$

$\sin 2\beta_a = \frac{\tau_w}{R} = \frac{1350}{1678.75} = 0.80417$

$2\beta_a = -53.53°$

$2\beta_b = +53.53°$

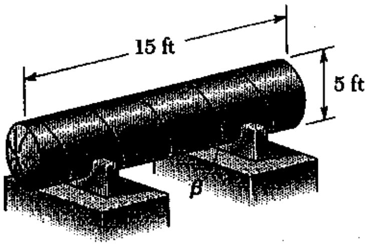
$2\beta_c = -53.53° + 180° = 126.47°$

$2\beta_d = 53.53° + 180° = 233.53°$

$\left.\begin{array}{l}\beta_a = -26.8° \\ \beta_b = 26.8°\end{array}\right\} \quad -26.8° \le \beta \le 26.8°$

$\left.\begin{array}{l}\beta_c = 63.2° \\ \beta_d = 116.8°\end{array}\right\} \quad 63.2° \le \beta \le 116.8°$

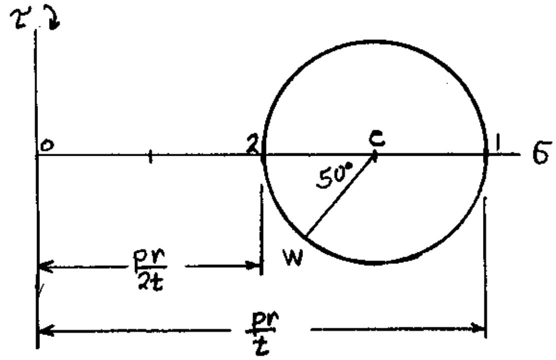
PROBLEM 7.114



7.114 The pressure tank shown has a $\frac{3}{8}$-in. wall thickness and butt-welded seams forming an angle $\beta = 25°$ with a transverse plane. Determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 18 ksi and the allowable shearing stress parallel to the weld is 10 ksi.

SOLUTION

$d = 5\text{ ft} = 60\text{ in.} \qquad r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625\text{ in.}$



$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{ave} - R\cos 50°$$

$$= \left(\frac{3}{4} - \frac{1}{4}\cos 50°\right)\frac{pr}{t}$$

$$= 0.5893\frac{pr}{t}$$

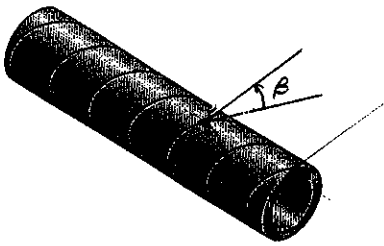
$$p = \frac{\sigma_w t}{0.5893\, r} = \frac{(18)(0.375)}{(0.5893)(29.625)} = 0.387\text{ ksi} = 387\text{ psi}$$

$$\tau_w = R\sin 50° = 0.19151\frac{pr}{t}$$

$$p = \frac{\tau_w t}{0.19151\, r} = \frac{(10)(0.375)}{(0.19151)(29.625)} = 0.661\text{ ksi} = 661\text{ psi}$$

Allowable gage pressure is the smaller value $\qquad p = 387\text{ psi}$ ◄

PROBLEM 7.115



7.115 The pipe shown was fabricated by welding strips of plate along a helix forming an angle β with a transverse plane. Determine the largest value of β that can be used if the normal stress perpendicular to the weld is not to be larger than 85 percent of the maximum stress in the pipe.

SOLUTION

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

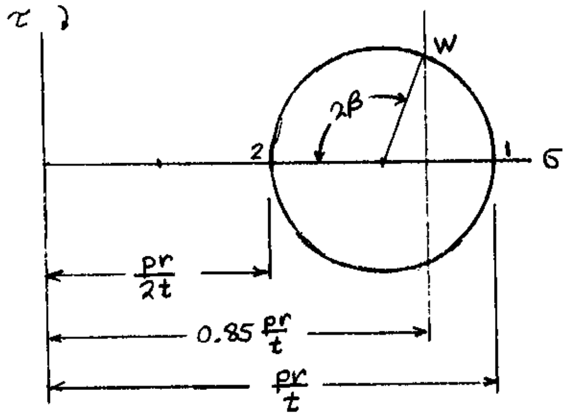
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_W = \sigma_{ave} - R\cos 2\beta$$

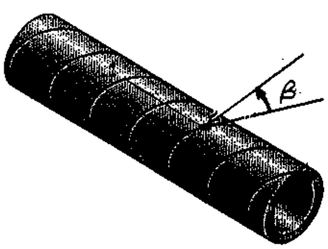
$$0.85\frac{pr}{t} = \left(\frac{3}{4} - \frac{1}{4}\cos 2\beta\right)\frac{pr}{t}$$

$$\cos 2\beta = -4\left(0.85 - \frac{3}{4}\right) = -0.4$$

$$2\beta = 113.6° \qquad \beta = 56.8°$$



PROBLEM 7.116



7.116 The pipe shown has a diameter of 600 mm and was fabricated by welding strips of 10-mm-thick plate along a helix forming an angle β = 25° with a transverse plane. Knowing that the ultimate normal stress perpendicular to the weld is 450 MPa and that a factor of safety of 6.0 is desired, determine the largest allowable gage pressure that can be used.

SOLUTION

$$t = 10 \text{ mm} \qquad r = \frac{1}{2}d - t = 300 - 10 = 290 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

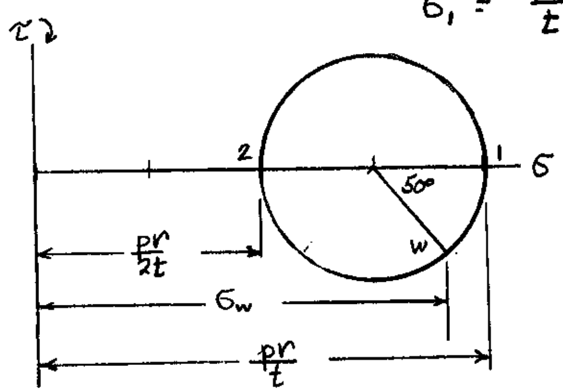
$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_W = \sigma_{ave} + R\cos 50° = 0.9107\frac{pr}{t}$$

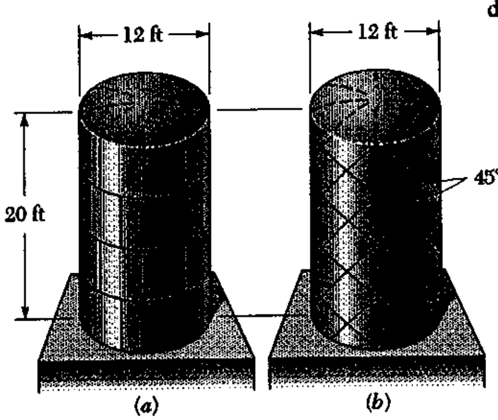
$$\sigma_{W,all} = \frac{\sigma_U}{F.S} = \frac{450}{6} = 75 \text{ MPa}$$

$$0.9107\frac{pr}{t} = 75$$

$$p = \frac{(75)(10)}{(0.9107)(290)} = 2.84 \text{ MPa}$$



PROBLEM 7.117



7.117 Square plates, each of 0.5-in. thickness, can be bent and welded together in either of the two ways shown to form the cylindrical portion of a compressed-air tank. Knowing that the allowable normal stress perpendicular to the weld is 12 ksi, determine the largest allowable gage pressure in each case.

SOLUTION

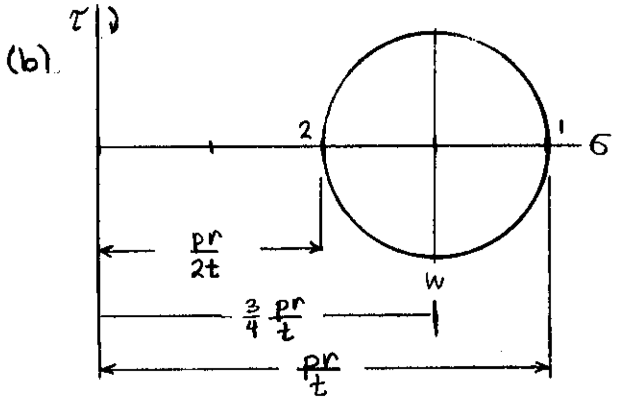
$d = 12 \text{ ft} = 144 \text{ in} \qquad r = \frac{1}{2}d - t = 71.5 \text{ in.}$

$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$

(a) $\sigma_1 = 12 \text{ ksi}$

$$p = \frac{\sigma_1 t}{r} = \frac{(12)(0.5)}{71.5} = 0.0839 \text{ ksi} = 83.9 \text{ psi}$$

(b)



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$

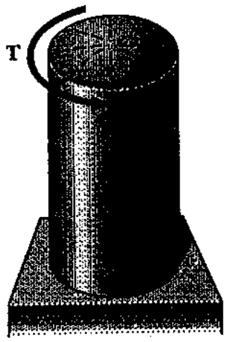
$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

$\beta = \pm 45°$

$\sigma_w = \sigma_{ave} + R\cos\beta = \frac{3}{4}\frac{pr}{t}$

$$p = \frac{4}{3}\frac{\sigma_w t}{r} = \frac{4}{3}\cdot\frac{(12)(0.5)}{71.5} = 0.1119 \text{ ksi} = 111.9 \text{ psi}$$

PROBLEM 7.118



7.118 A torque of magnitude $T = 12$ kN · m is applied to the end of a tank containing compressed air under a pressure of 8 MPa. Knowing that the tank has a 180-mm inside diameter and a 12-mm wall thickness, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$d = 180$ mm $\quad r = \frac{1}{2}d = 90$ mm $\quad t = 12$ mm

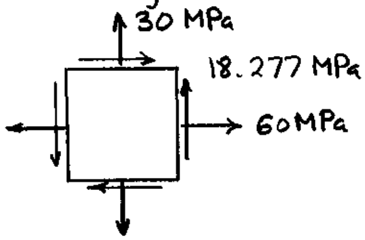
Torsion: $c_1 = 90$ mm $\quad c_2 = 90 + 12 = 102$ mm

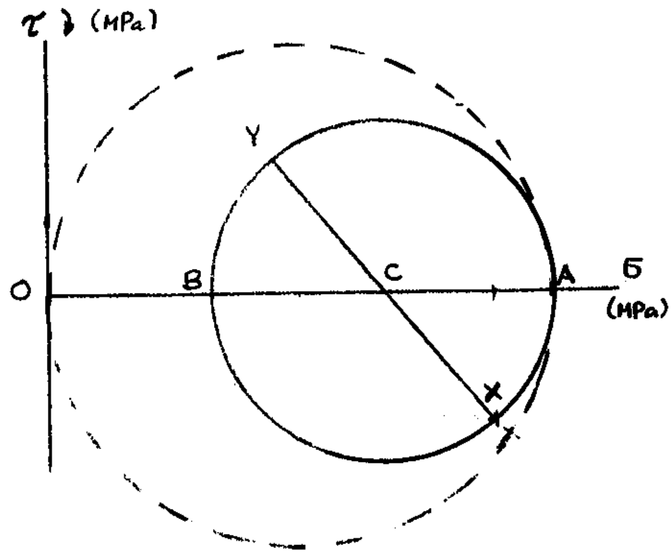
$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(12 \times 10^3)(102 \times 10^{-3})}{66.968 \times 10^{-6}} = 18.277 \text{ MPa}$$

Pressure: $\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60$ MPa $\qquad \sigma_2 = \frac{pr}{2t} = 30$ MPa

Summary of stresses $\qquad \sigma_x = 60$ MPa, $\sigma_y = 30$ MPa, $\tau_{xy} = 18.277$ MPa





$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.64$ MPa

$\sigma_a = \sigma_{ave} + R = 68.64$ MPa

$\sigma_b = \sigma_{ave} - R = 21.36$ MPa

$\sigma_c \approx 0$

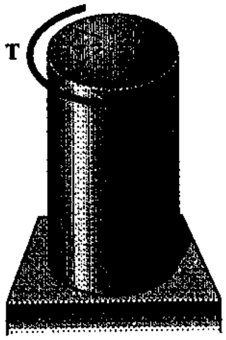
$\sigma_{max} = 68.64$ MPa

$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 34.32$ MPa

PROBLEM 7.119

7.119 The tank shown has a 180-mm inside diameter and a 12-mm wall thickness. Knowing that the tank contains compressed air under a pressure of 8 MPa, determine the magnitude T of the applied torque for which the maximum normal stress in the tank is 75 MPa.



SOLUTION

$r = \frac{1}{2}d = (\frac{1}{2})(180) = 90$ mm $\qquad t = 12$ mm

$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60$ MPa $\qquad \sigma_2 = \frac{pr}{2t} = 30$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_y) = 45$ MPa

$\sigma_{max} = 75$ MPa $\qquad R = \sigma_{max} - \sigma_{ave} = 30$ MPa

$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{xy}^2} = \sqrt{15^2 + \tau_{xy}^2}$

$\tau_{xy} = \sqrt{R^2 - 15^2} = \sqrt{30^2 - 15^2} = 25.98$ MPa

$= 25.98 \times 10^6$ Pa

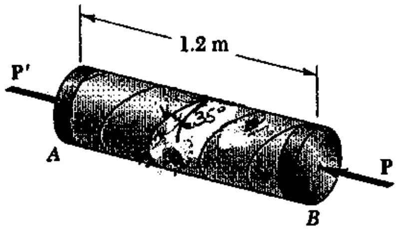
Torsion: $c_1 = 90$ mm $\qquad c_2 = 90 + 12 = 102$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$

$\tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c} = \frac{(66.968 \times 10^{-6})(25.98 \times 10^6)}{102 \times 10^{-3}} = 17.06 \times 10^3 \text{ N}\cdot\text{m}$

$= 17.06 \text{ kN}\cdot\text{m}$

PROBLEM 7.120



7.120 A pressure vessel of 250-mm inside diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe *AB* and is with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 45-kN centric axial forces **P** and **P'** are applied to the end plates.. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$r = \frac{1}{2}d = 125 \text{ mm} \qquad t = 6 \text{ mm}$

$\sigma_1 = \frac{pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$

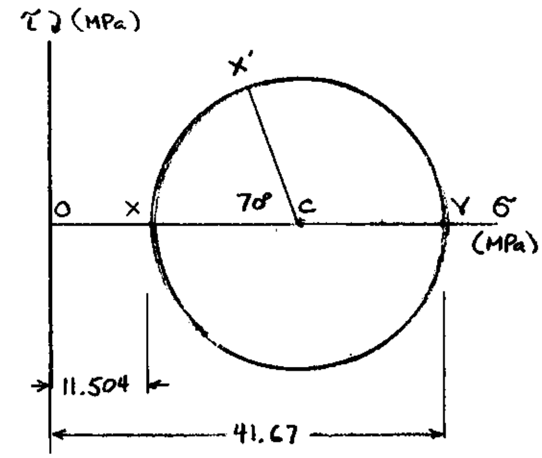
$\sigma_2 = \frac{pr}{2t} = \frac{(2)(125)}{(2)(6)} = 20.83 \text{ MPa}$

$r_o = r + t = 125 + 6 = 131 \text{ mm} \qquad A = \pi(r_o^2 - r^2) = 4.825 \times 10^3 \text{ mm}^2 = 4.825 \times 10^{-3} \text{ m}^2$

$\sigma = -\frac{P}{A} = -\frac{45 \times 10^3}{4.825 \times 10^{-3}} = -9.326 \times 10^6 \text{ Pa} = -9.326 \text{ MPa}$

Total stresses: longitudinal $\sigma_x = 20.83 - 9.326 = 11.504 \text{ MPa}$

Circumferential $\sigma_y = 41.67 \text{ MPa}$



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 26.585 \text{ MPa}$

$R = \frac{\sigma_x - \sigma_y}{2} = 15.081$

(a) $\sigma_{x'} = \sigma_{ave} + R\cos 70°$

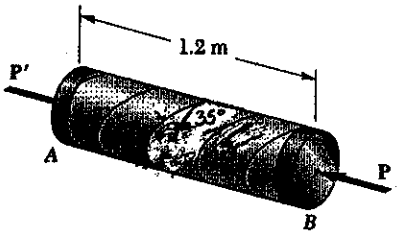
$= 26.585 - 15.081 \cos 70°$

$= 21.4 \text{ MPa}$ ◄

(b) $\tau_{x'y'} = R \sin 70° = 15.081 \sin 70°$

$= 14.17 \text{ MPa}$ ◄

PROBLEM 7.121



7.120 A pressure vessel of 250-mm inside diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe AB and is with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 45-kN centric axial forces **P** and **P'** are applied to the end plates.. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

7.121 Solve Prob. 7.120, assuming that the magnitude P of the two forces is increased to 120 kN.

SOLUTION

$r = \frac{1}{2}d = 125 \text{ mm} \qquad t = 6 \text{ mm}$

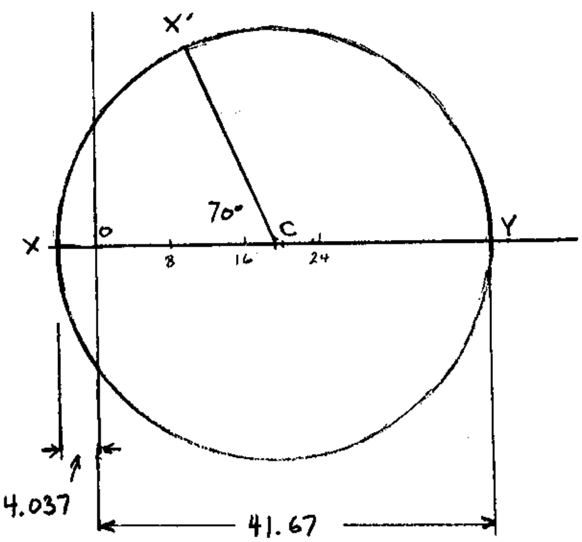
$$\sigma_1 = \frac{pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa} \qquad \sigma_2 = \frac{pr}{2t} = 20.833 \text{ MPa}$$

$$r_o = r + t = 125 + 6 = 131 \text{ mm} \qquad A = \pi(r_o^2 - r^2) = 4.825 \times 10^3 \text{ mm}^2 = 4.825 \times 10^{-3} \text{ m}^2$$

$$\sigma = -\frac{P}{A} = -\frac{120 \times 10^3}{4.825 \times 10^{-3}} = -24.870 \times 10^6 \text{ Pa} = -24.870 \text{ MPa}$$

Total stresses: Longitudinal $\sigma_x = 20.833 - 24.870 = -4.037$ MPa

Circumferential $\sigma_y = 41.67$ MPa



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 18.815 \text{ MPa}$$

$$R = \left|\frac{\sigma_x - \sigma_y}{2}\right| = 22.852 \text{ MPa}$$

(a) $\sigma_{x'} = \sigma_{ave} - R\cos 70°$

$= 18.815 - 22.852 \cos 70°$

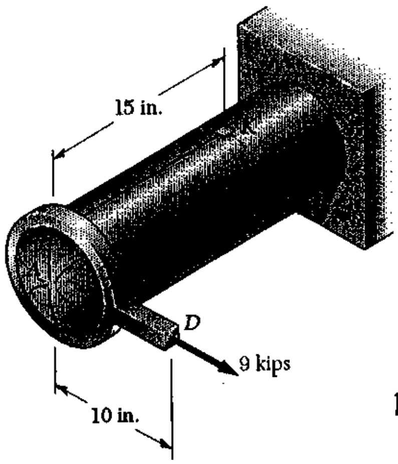
$= 11.00$ MPa

(b) $\tau_{x'y'} = R \sin 70° = 22.852 \sin 70°$

$= 21.5$ MPa

PROBLEM 7.122

7.122 The cylindrical tank *AB* has an 8-in. inside diameter and a 0.32-in. wall thickness. Knowing that the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shearing stress at point *K*.



SOLUTION

$r_i = \frac{d_i}{2} = 4 \text{ in} \qquad r_o = r_i + t = 4.32 \text{ in.}$

$\sigma_1 = \frac{p r_i}{t} = \frac{(600)(4)}{0.32} = 7500 \text{ psi} = 7.50 \text{ ksi}$

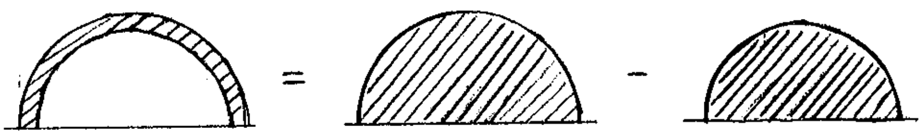
$\sigma_2 = \frac{1}{2}\sigma_1 = 3.75 \text{ ksi}$

Torsion: No applied torque

Bending: Point K lies on neutral axis.

Transverse shear: $V = 9$ kips

For semicircle
$A = \frac{\pi}{2} r^2$
$\bar{y} = \frac{4r}{3\pi}$
$Q = \frac{2}{3} r^3$



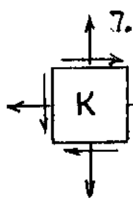
$Q = Q_o - Q_i = \frac{2}{3} r_o^3 - \frac{2}{3} r_i^3 = \frac{2}{3}(4.32^3 - 4^3) = 11.081 \text{ in}^3$

$t = (2)(0.32) = 0.64 \text{ in}$

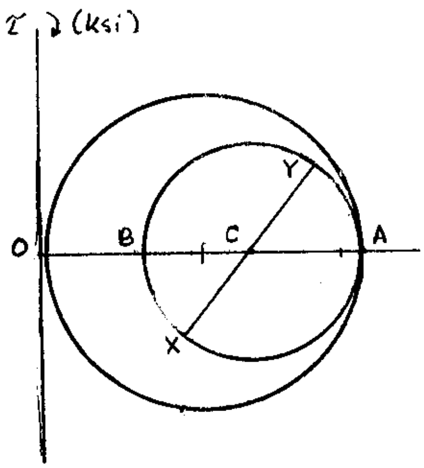
$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(4.32^4 - 4^4) = 72.481 \text{ in}^4$

$\tau = \frac{VQ}{It} = \frac{(9)(11.081)}{(72.481)(0.64)} = 2.15 \text{ ksi}$

Summary of stresses:



Longitudinal $\sigma_x = \sigma_1 = 3.75$ ksi
Circumferential $\sigma_y = \sigma_2 = 7.50$ ksi
Shear $\tau_{xy} = 2.15$ ksi



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.625 \text{ ksi}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 2.853 \text{ ksi}$

$\sigma_a = \sigma_{ave} + R = 8.48 \text{ ksi}$

$\sigma_b = \sigma_{ave} - R = 2.77 \text{ ksi}$

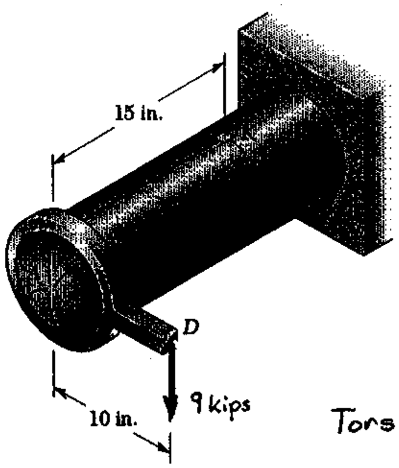
$\sigma_z = 0$

$\sigma_{max} = 8.48 \text{ ksi}$ ◄

$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 4.24 \text{ ksi}$ ◄

PROBLEM 7.123



7.122 The cylindrical tank *AB* has an 8-in. inside diameter and a 0.32-in. wall thickness. Knowing that the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shearing stress at point *K*.

7.123 Solve Prob. 7.122, assuming that the 9-kip force applied at point *D* is directed vertically downward.

SOLUTION

$r_i = \frac{d_i}{2} = 4 \text{ in.} \qquad r_o = r_i + t = 4.32 \text{ in}$

$\sigma_1 = \frac{p r_i}{t} = \frac{(600)(4)}{0.32} = 7500 \text{ psi} = 7.50 \text{ ksi}$

$\sigma_2 = \frac{1}{2}\sigma_1 = 3.75 \text{ ksi}$

Torsion: $J = \frac{\pi}{2}(r_o^4 - r_i^4) = 144.96 \text{ in}^4 \qquad c = r_o = 4.32 \text{ in}$

$T = (9)(10) = 90 \text{ kip·in}$

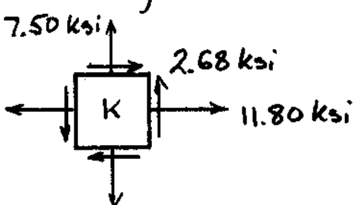
$\tau = \frac{Tc}{J} = \frac{(90)(4.32)}{144.96} = 2.68 \text{ ksi}$

Bending: $I = \frac{1}{2}J = 72.48 \text{ in}^4 \qquad c = r_o = 4.32 \text{ in}$

$M = (9)(15) = 135 \text{ kip·in} \qquad \sigma_m = \frac{Mc}{I} = \frac{(135)(4.32)}{72.48} = 8.05 \text{ ksi}$

Transverse shear: At point K, $VQ/It = 0$

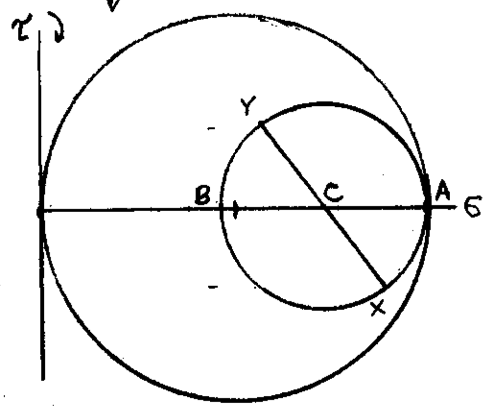
Summary of stresses:



Longitudinal $\sigma_x = \sigma_1 = 3.75 + 8.05 = 11.80 \text{ ksi}$

Circumferential $\sigma_y = \sigma_2 = 7.50 \text{ ksi}$

Shear $\tau_{xy} = 2.68 \text{ ksi}$



$\sigma_{ave} = \frac{1}{2}(11.80 + 7.50) = 9.65 \text{ ksi}$

$R = \sqrt{\left(\frac{11.80 - 7.50}{2}\right)^2 + (2.68)^2} = 3.44 \text{ ksi}$

$\sigma_a = \sigma_{ave} + R = 13.09 \text{ ksi}$

$\sigma_b = \sigma_{ave} - R = 6.21 \text{ ksi}$

$\sigma_z = 0$

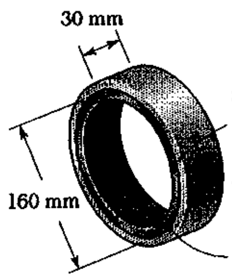
$\sigma_{max} = 13.09 \text{ ksi}$ ◄

$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.54 \text{ ksi}$ ◄

PROBLEM 7.124

7.124 A brass ring of 160-mm outside diameter fits exactly inside a steel ring of 160-mm inside diameter when the temperature of both rings is 5° C. Knowing that the temperature of the rings is then raised to 55° C, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.



STEEL
$t_s = 4$ mm
$E_s = 210$ GPa
$\alpha_s = 12 \times 10^{-6}/°C$

BRASS
$t_b = 4$ mm
$E_b = 105$ GPa
$\alpha_b = 20 \times 10^{-6}/°C$

SOLUTION

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p, $\quad \sigma_s = \dfrac{pr}{t_s}$ (1)

Corresponding strain $\quad \varepsilon_{sp} = \dfrac{\sigma_s}{E_s} = \dfrac{pr}{E_s t_s}$

Strain due to temperature change $\quad \varepsilon_{sT} = \alpha_s \Delta T$

Total strain $\quad \varepsilon_s = \dfrac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring: External pressure p, $\quad \sigma_b = -\dfrac{pr}{t_b}$

Corresponding strains $\quad \varepsilon_{bp} = -\dfrac{pr}{E_b t_b}$, $\quad \varepsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b $\quad \dfrac{pr}{E_s t_s} + \alpha_s \Delta T = -\dfrac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T \qquad (2)$$

Data $\quad \Delta T = 55°C - 5° = 50°C$

$r = \frac{1}{2} d = 80$ mm

From eq. (2) $\quad \left\{ \dfrac{80 \times 10^{-3}}{(210 \times 10^9)(4 \times 10^{-3})} + \dfrac{80 \times 10^{-3}}{(105 \times 10^9)(4 \times 10^{-3})} \right\} p = (8 \times 10^{-6})(50)$

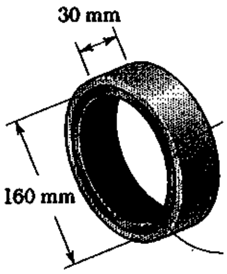
$285.71 \times 10^{-12}\, p = 400 \times 10^{-6}$, $\quad p = 1.4 \times 10^6$ Pa

From eq. (1) $\quad \sigma_s = \dfrac{pr}{t_s} = \dfrac{(1.4 \times 10^6)(80 \times 10^{-3})}{4 \times 10^{-3}} = 28 \times 10^6$ Pa

(a) $\quad \sigma_s = 28.0$ MPa

(b) $\quad p = 1.400$ MPa

PROBLEM 7.125



STEEL
$t_s = 4$ mm
$E_s = 210$ GPa
$\alpha_s = 12 \times 10^{-6}/°C$

BRASS
$t_b = 4$ mm
$E_b = 105$ GPa
$\alpha_b = 20 \times 10^{-6}/°C$

7.124 A brass ring of 160-mm outside diameter fits exactly inside a steel ring of 160-mm inside diameter when the temperature of both rings is 5° C. Knowing that the temperature of the rings is then raised to 55° C, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.

7.125 Solve Prob. 7.124, assuming that the thickness of the brass ring is $t_b = 6$ mm.

SOLUTION

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p, $\sigma_s = \frac{pr}{t_s}$ (1)

Corresponding strain $\varepsilon_{sp} = \frac{\sigma_s}{E_s} = \frac{pr}{E_s t_s}$

Strain due to temperature change $\varepsilon_{sT} = \alpha_s \Delta T$

Total strain $\varepsilon_s = \frac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring: External pressure p, $\sigma_b = -\frac{pr}{t_b}$

Corresponding strains $\varepsilon_{bp} = -\frac{pr}{E_b t_b}$, $\varepsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b $\quad \frac{pr}{E_s t_s} + \alpha_s \Delta T = -\frac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s)\Delta T \qquad (2)$$

Data: $\Delta T = 55°C - 5°C = 50°C$ $\quad t_b = 6 \text{ mm} = 6 \times 10^{-3}$ m

$r = \frac{1}{2} d = 80$ mm

From eq. (2) $\left\{ \frac{(80 \times 10^{-3})}{(210 \times 10^{9})(4 \times 10^{-3})} + \frac{(80 \times 10^{-3})}{(105 \times 10^{9})(6 \times 10^{-3})} \right\} p = (8 \times 10^{-6})(50)$

$222.22 \times 10^{-12}\, p = 400 \times 10^{-6}$; $\quad p = 1.8 \times 10^{6}$ Pa

From eq. (1) $\quad \sigma_s = \frac{pr}{t_s} = \frac{(1.8 \times 10^{6})(80 \times 10^{-3})}{4 \times 10^{-3}} = 36 \times 10^{6}$ Pa

$\sigma_s = 36.0$ MPa ◄

$p = 1.800$ MPa ◄

PROBLEM 7.126

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

$\varepsilon_x = -720\mu, \quad \varepsilon_y = 0, \quad \gamma_{xy} = +300\mu, \quad \theta = -30°$

SOLUTION

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -360\,\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -360\,\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left\{-360 - 360\cos(-60°) + \frac{300}{2}\sin(-60°)\right\}\mu = -670\,\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left\{-360 - (-360)\cos(-60°) - \frac{300}{2}\sin(-60°)\right\}\mu = -50\,\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$
$$= \left\{-(-720 - 0)\sin(-60°) + 300\cos(-60°)\right\}\mu = -474\,\mu$$

PROBLEM 7.127

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

$\varepsilon_x = 0 \quad \varepsilon_y = +320\,\mu \quad \gamma_{xy} = -100\,\mu \quad \theta = 30°$

SOLUTION

$$\frac{\varepsilon_x + \varepsilon_y}{2} = 160\,\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -160\,\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left\{160 - 160\cos 60° - \frac{100}{2}\sin 60°\right\}\mu = +36.7\,\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= \left\{160 + 160\cos 60° + \frac{100}{2}\sin 60°\right\}\mu = +283\,\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$
$$= \left\{-(0 - 320)\sin 60° - 100\cos 60°\right\}\mu = +227\,\mu$$

PROBLEM 7.128

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x and y' rotated through the given angle θ.

$\varepsilon_x = -800\,\mu,\ \varepsilon_y = +450\,\mu,\ \gamma_{xy} = +200\,\mu,\ \theta = -25°$

SOLUTION

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -175\,\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -625\,\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-175 - 625\cos(-50°) + \frac{200}{2}\sin(-50°)\right\}\mu = -653\,\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-175 + 625\cos(-50°) - \frac{200}{2}\sin(-50°)\right\}\mu = +303\,\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \left\{-(-800 - 450)\sin(-50°) + 200\cos(-50°)\right\}\mu = -829\,\mu$$

PROBLEM 7.129

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

$\varepsilon_x = -500\,\mu,\ \varepsilon_y = +250\,\mu,\ \gamma_{xy} = 0,\ \theta = 15°$

SOLUTION

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -125\,\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -375\,\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \{-125 - 375\cos 30° + 0\}\mu = -450\,\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \{-125 + 375\cos 30° - 0\}\mu = +200\,\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \{-(-500 - 250)\sin 30° + 0\}\mu = +375\,\mu$$

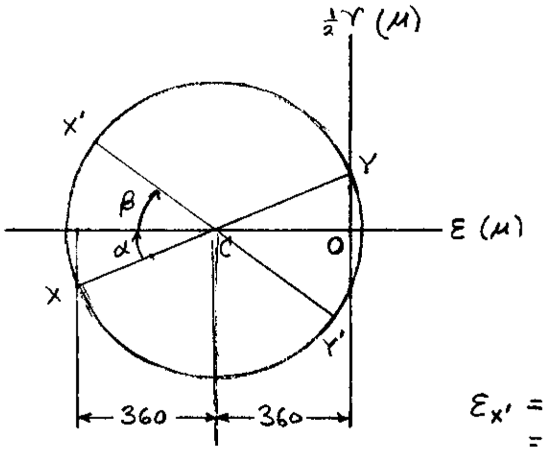
Brass ring: External pressure p, τ_b

PROBLEM 7.130

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$\varepsilon_x = -720\mu, \quad \varepsilon_y = 0, \quad \gamma_{xy} = +300\mu, \quad \theta = -30°$

SOLUTION



Plotted points

X: $(-720\mu, -150\mu)$

Y: $(0, 150\mu)$

C: $(-360\mu, 0)$

$\tan\alpha = \dfrac{150\mu}{360\mu} \qquad \alpha = 22.62°$

$R = \sqrt{(360\mu)^2 + (150\mu)^2} = 390\mu$

$\beta = 2\theta - \alpha = 60° - 22.62° = 37.38°$

$\varepsilon_{x'} = \varepsilon_{ave} - R\cos\beta = -360\mu - 390\mu\cos 37.38°$
$= -670\mu$ ◄

$\varepsilon_{y'} = \varepsilon_{ave} + R\cos\beta = -360\mu + 390\mu\cos 37.38°$
$= -50\mu$ ◄

$\dfrac{\gamma_{x'y'}}{2} = -R\sin\beta = -390\mu\sin 37.38°$

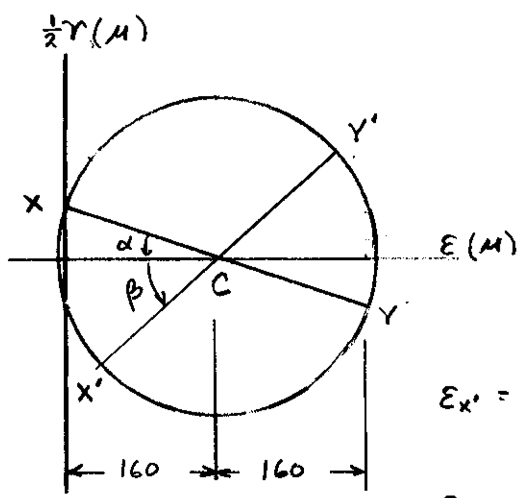
$\gamma_{x'y'} = -474\mu$ ◄

PROBLEM 7.131

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$\varepsilon_x = 0 \quad \varepsilon_y = +320\mu \quad \gamma_{xy} = -100\mu \quad \theta = 30°$

SOLUTION



Plotted points

X: $(0, 50\mu)$

Y: $(320\mu, -50\mu)$

C: $(160\mu, 0)$

$\tan\alpha = \dfrac{50}{160} \qquad \alpha = 17.35°$

$R = \sqrt{(160\mu)^2 + (50\mu)^2} = 167.63\mu$

$\beta = 2\theta - \alpha = 60° - 17.35° = 42.65°$

$\varepsilon_{x'} = \varepsilon_{ave} - R\cos\beta = 160\mu - 167.63\mu\cos 42.65°$
$= -36.7\mu$ ◄

$\varepsilon_{y'} = \varepsilon_{ave} + R\cos\beta = 160\mu + 167.63\mu\cos 42.65°$
$= 283\mu$ ◄

$\dfrac{\gamma_{x'y'}}{2} = R\sin\beta = 167.63\mu\sin 42.65°$

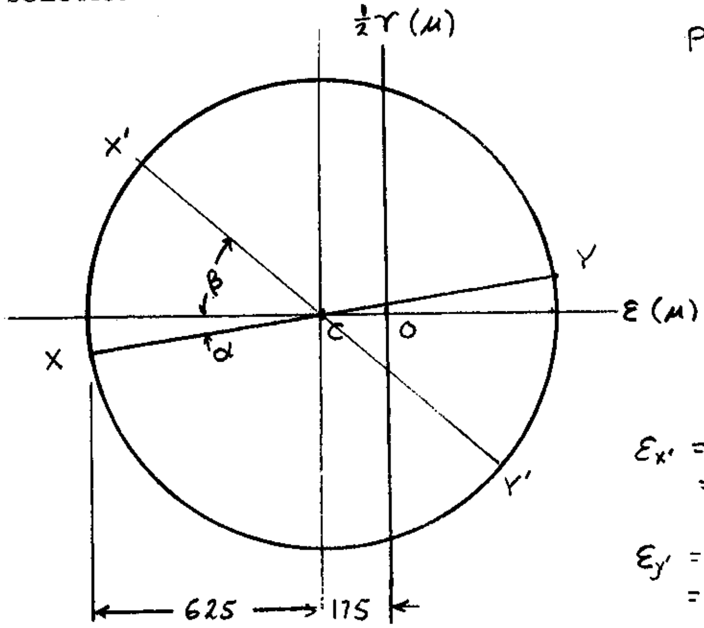
$\gamma_{x'y'} = 227\mu$ ◄

PROBLEM 7.132

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$\varepsilon_x = -800\,\mu \quad \varepsilon_y = 450\,\mu \quad \gamma_{xy} = +200\,\mu \quad \theta = -25°$

SOLUTION



Plotted points
X: $(-800\,\mu, -100\,\mu)$
Y: $(+450\,\mu, +100\,\mu)$
C: $(-175\,\mu, 0)$

$\tan\alpha = \frac{100}{625} \qquad \alpha = 9.09°$

$R = \sqrt{(625\,\mu)^2 + (100\,\mu)^2} = 632.95\,\mu$

$\beta = 2\theta - \alpha = 50° - 9.09° = 40.91°$

$\varepsilon_{x'} = \varepsilon_{ave} - R\cos\beta = -175\,\mu - 632.95\,\mu\cos 40.91°$
$= -653\,\mu$ ◄

$\varepsilon_{y'} = \varepsilon_{ave} + R\cos\beta = -175$
$= +303\,\mu$ ◄

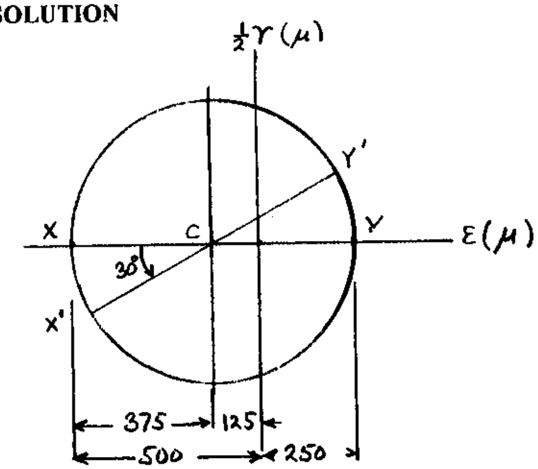
$\frac{\gamma_{x'y'}}{2} = -R\sin\beta = -632.95\,\mu\sin 40.91° \qquad \gamma_{x'y'} = -829\,\mu$ ◄

PROBLEM 7.133

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$\varepsilon_x = -500\,\mu, \quad \varepsilon_y = +250\,\mu, \quad \gamma_{xy} = 0, \quad \theta = 15°$

SOLUTION



Plotted points
X: $(-500\,\mu, 0)$
Y: $(+250\,\mu, 0)$
C: $(-125\,\mu, 0)$

$R = 375\,\mu$

$\varepsilon_{x'} = \varepsilon_{ave} - R\cos 2\theta = -125 - 375\cos 30°$
$= -450\,\mu$ ◄

$\varepsilon_{y'} = \varepsilon_{ave} + R\cos 2\theta = -125 + 375\cos 30°$
$= 200\,\mu$ ◄

$\frac{1}{2}\gamma_{xy} = R\sin 2\theta = 375\sin 30°$

$\gamma_{x'y'} = 375\,\mu$ ◄

PROBLEM 7.134

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $v = \frac{1}{3}$)

$\varepsilon_x = +160\,\mu \qquad \varepsilon_y = -480\,\mu \qquad \gamma_{xy} = -600\,\mu$

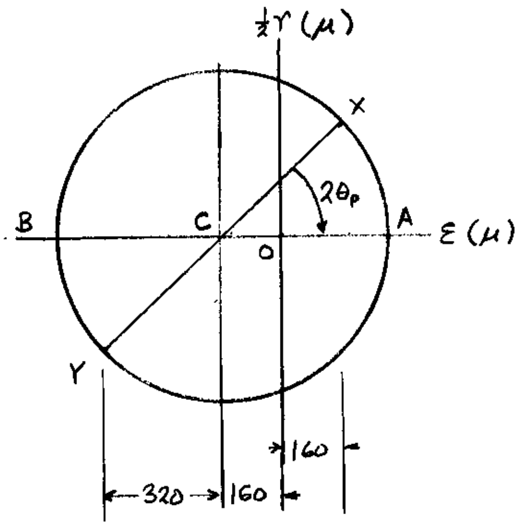
SOLUTION

(a) For Mohr's circle of strain, plot points

X: $(160\,\mu,\ 300\,\mu)$

Y: $(-480\,\mu,\ -300\,\mu)$

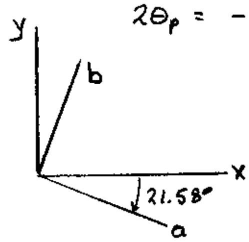
C: $(-160\,\mu,\ 0)$



(a) $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{-300}{320} = -0.9375$

$2\theta_p = -43.15° \qquad \theta_p = -21.58°$

and $-21.58 + 90 = 68.42°$



$\theta_a = -21.58°$ ◄

$\theta_b = 68.42°$ ◄

$R = \sqrt{(320\,\mu)^2 + (300\,\mu)^2} = 438.6\,\mu$

$\varepsilon_a = \varepsilon_{ave} + R = -160\,\mu + 438.6\,\mu = +278.6\,\mu$ ◄

$\varepsilon_b = \varepsilon_{ave} - R = -160\,\mu - 438.6\,\mu = -598.6\,\mu$ ◄

(b) $\frac{1}{2}\gamma_{(max,\ in\text{-}plane)} = R \qquad \gamma_{(max,\ in\text{-}plane)} = 2R = 877\,\mu$ ◄

(c) $\varepsilon_c = -\dfrac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\dfrac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\dfrac{1/3}{2/3}(160\,\mu - 480\,\mu)$

$= 160\,\mu$

$\varepsilon_{max} = 278.6\,\mu \qquad \varepsilon_{min} = -598.6\,\mu$

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 278.6\,\mu + 598.6\,\mu = 877\,\mu$ ◄

PROBLEM 7.135

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

$\varepsilon_x = -260\mu \qquad \varepsilon_y = -60\mu \qquad \gamma_{xy} = +480\mu$

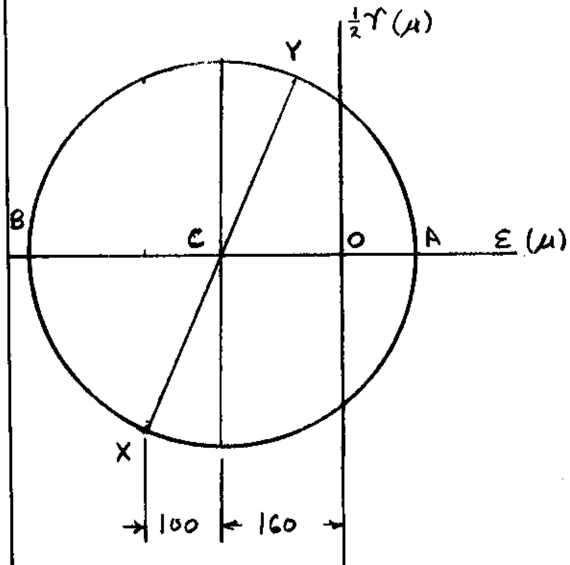
SOLUTION

For Mohr's circle of strain plot points

X: $(-260\mu,\ -240\mu)$

Y: $(-60\mu,\ 240\mu)$

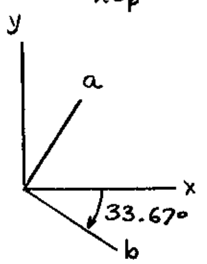
C: $(-160\mu,\ 0)$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{480}{-260+60} = -2.4$$

$2\theta_p = -67.38°$ $\qquad \theta_b = -33.67°$

$\theta_a = 56.31°$



$R = \sqrt{(100\mu)^2 + (240\mu)^2}$

$R = 260\mu$

(a) $\varepsilon_a = \varepsilon_{ave} + R = -160\mu + 260\mu = 100\mu$

$\varepsilon_b = \varepsilon_{ave} - R = -160\mu - 260\mu = -420\mu$

(b) $\frac{1}{2}\gamma_{max\,(in\text{-}plane)} = R \qquad \gamma_{max\,(in\text{-}plane)} = 2R = 520\mu$

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-260-60)$$
$$= 160\mu$$

$\varepsilon_{max} = 160\mu \qquad \varepsilon_{min} = -420\mu$

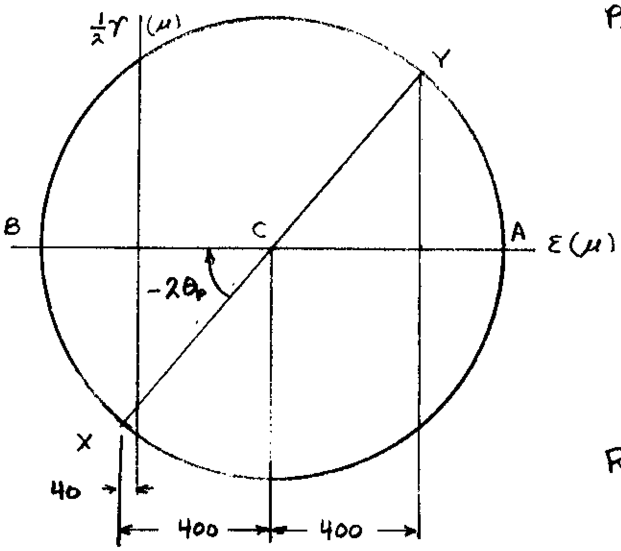
(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 160\mu + 420\mu = 580\mu$

PROBLEM 7.136

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

$\varepsilon_x = -40\,\mu \qquad \varepsilon_y = 760\,\mu \qquad \gamma_{xy} = +960\,\mu$

SOLUTION



Plotted points

X: $(-40\,\mu, -480\,\mu)$

Y: $(760\,\mu, +480\,\mu)$

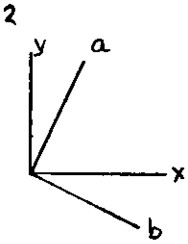
C: $(360\,\mu, 0)$

$\tan 2\theta_p = \frac{-480}{400} = -1.2$

$2\theta_p = -50.19°$

$\theta_b = -25.10°$ ◄

$\theta_a = 64.90°$ ◄



$R = \sqrt{(400\,\mu)^2 + (480\,\mu)^2}$

$= 624.8\,\mu$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 360\,\mu + 624.8\,\mu = 985\,\mu$ ◄

$\varepsilon_b = \varepsilon_{ave} - R = 360\,\mu - 624.8\,\mu = -265\,\mu$ ◄

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 1250\,\mu$ ◄

$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-40\,\mu + 760\,\mu)$

$= -360\,\mu$ ◄

$\varepsilon_{max} = 985\,\mu \qquad \varepsilon_{min} = -265\,\mu$

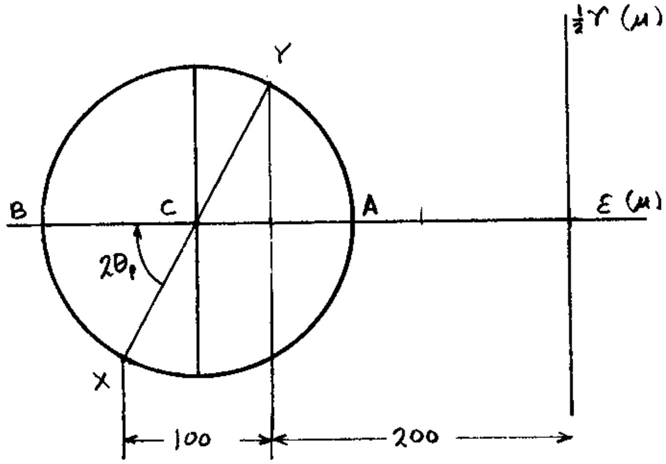
(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 985 + 360 = 1345\,\mu$ ◄

PROBLEM 7.137

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

$\varepsilon_x = -300\mu \qquad \varepsilon_y = -200\mu \qquad \gamma_{xy} = +175\mu$

SOLUTION



Plotted points.

X: $(-300\mu, -87.5\mu)$

Y: $(-200\mu, +87.5\mu)$

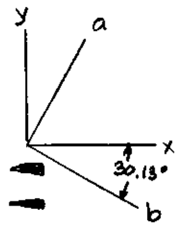
C: $(-250\mu, 0)$

$\tan 2\theta_p = -\dfrac{87.5}{50}$

$2\theta_p = -60.26$

$\theta_b = -30.13°$

$\theta_a = 59.87°$



$R = \sqrt{(50\mu)^2 + (87.5\mu)^2}$

$= 100.8$

(a) $\varepsilon_a = \varepsilon_{ave} + R = -250\mu + 100.8\mu = -149.2\ \mu$

$\varepsilon_b = \varepsilon_{ave} - R = -250\mu - 100.8\mu = -351\ \mu$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 201.6\ \mu$

$\varepsilon_c = -\dfrac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\dfrac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\dfrac{1/3}{2/3}(-300\mu - 200\mu)$

$= +250\ \mu$

$\varepsilon_{max} = 250\mu \qquad \varepsilon_{min} = -351\mu$

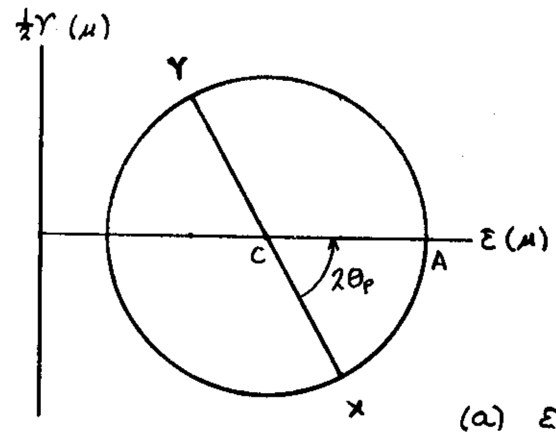
(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 250\mu + 351\mu = 601\ \mu$

PROBLEM 7.140

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$\varepsilon_x = +400\,\mu \quad \varepsilon_y = +200\,\mu \quad \gamma_{xy} = +375\,\mu$

SOLUTION



Plotted points

X: $(+400\mu,\ -187.5\mu)$

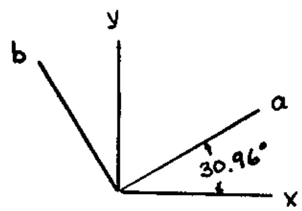
Y: $(+200\mu,\ +187.5\mu)$

C: $(+300\mu,\ 0)$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{375}{400-200} = 1.875$$

$2\theta_p = 61.93°$ $\quad \theta_a = 30.96°$, $\theta_b = 120.96°$

$$R = \sqrt{(100\mu)^2 + (187.5\mu)^2} = 212.5\,\mu$$



(a) $\varepsilon_a = \varepsilon_{ave} + R = 300\mu + 212.5\mu = 512.5\,\mu$

$\varepsilon_b = \varepsilon_{ave} - R = 300\mu - 212.5\mu = 87.5\,\mu$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 425\,\mu$

(c) $\varepsilon_c = 0 \quad \varepsilon_{max} = 512.5\,\mu \quad \varepsilon_{min} = 0$

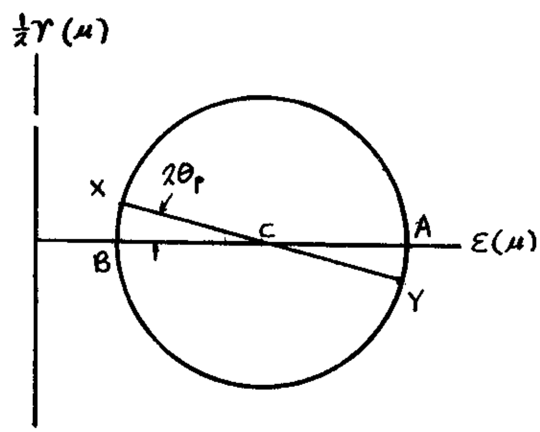
$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 512.5\,\mu$

PROBLEM 7.141

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$\varepsilon_x = +60\,\mu \quad \varepsilon_y = +240\,\mu \quad \gamma_{xy} = -50\,\mu$

SOLUTION



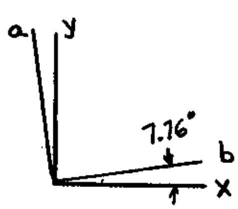
Plotted points X: $(60\mu,\ 25\mu)$

Y: $(240\mu,\ -25\mu)$ $\quad$ C: $(150\mu,\ 0)$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-50}{60-240} = 0.277778$$

$2\theta_p = 15.52°$ $\quad \theta_b = 7.76°$ $\quad \theta_a = 97.76°$

$$R = \sqrt{(90\mu)^2 + (25\mu)^2} = 93.4\ \mu$$



(a) $\varepsilon_a = \varepsilon_{ave} + R = 150\mu + 93.4\,\mu = 243.4\,\mu$

$\varepsilon_b = \varepsilon_{ave} - R = 150\,\mu - 93.4\,\mu = 56.6\,\mu$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 186.8\,\mu$

(c) $\varepsilon_c = 0 \quad \varepsilon_{max} = 243.4\,\mu \quad \varepsilon_{min} = 0$

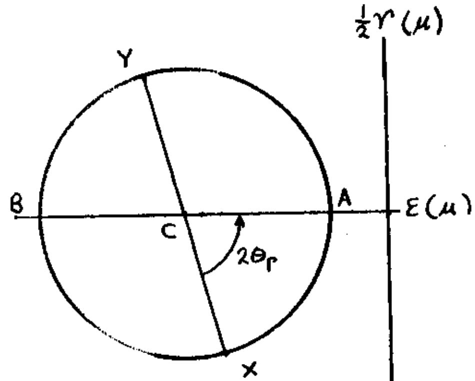
$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 243.4$

PROBLEM 7.138

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$\varepsilon_x = -90\mu \qquad \varepsilon_y = -130\mu \qquad \gamma_{xy} = +150\mu$

SOLUTION

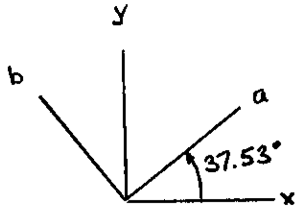


Plot points
X: $(-90\mu, -75\mu)$ Y: $(-130\mu, +75\mu)$
C: $(-110\mu, 0)$

(a) $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{150}{40} = 3.75$

$2\theta_p = 75.07°$ $\qquad \theta_a = 37.53°$
$\theta_b = 127.53°$

$R = \sqrt{(20\mu)^2 + (75\mu)^2} = 77.6\mu$



$\varepsilon_a = \varepsilon_{ave} + R = -110\mu + 77.6\mu = -32.4\mu$
$\varepsilon_b = \varepsilon_{ave} - R = -110\mu - 77.6\mu = -187.6\mu$

(b) $\gamma_{max(inplane)} = 2R = 155.2\mu$

(c) $\varepsilon_c = 0 \qquad \varepsilon_{max} = 0, \quad \varepsilon_{min} = -187.6\mu$

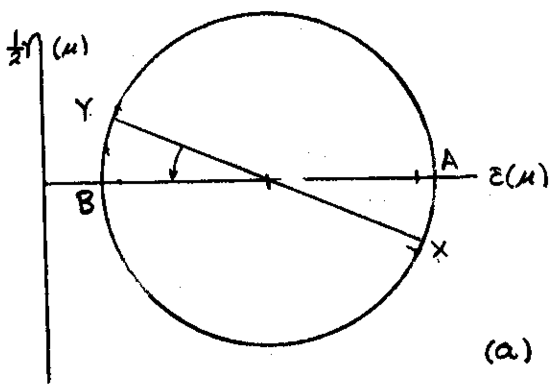
$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 0 + 187.6\mu = 187.6\mu$

PROBLEM 7.139

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$\varepsilon_x = +375\mu \qquad \varepsilon_y = +75\mu \qquad \gamma_{xy} = +125\mu$

SOLUTION



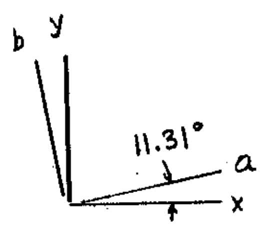
X: $(375\mu, -62.5\mu)$ Y: $(75\mu, 62.5\mu)$
C: $(225\mu, 0)$

$\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{125}{375 - 75} = 22.62°$

$\theta_a = 11.31° \qquad \theta_b = 101.31°$

$R = \sqrt{(150\mu)^2 + (62.5\mu)^2} = 162.5\mu$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 225\mu + 162.5\mu = 387.5\mu$
$\varepsilon_b = \varepsilon_{ave} - R = 225\mu - 162.5\mu = 62.5\mu$



(b) $\gamma_{max(in\text{-}plane)} = 2R = 325\mu$

(c) $\varepsilon_c = 0 \qquad \varepsilon_{max} = 387.5\mu \qquad \varepsilon_{min} = 0$

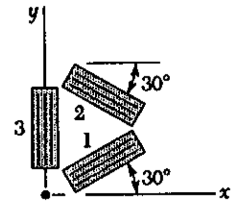
$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 387.5\mu$

PROBLEM 7.142

7.142 The strains determined by use of the rosette shown during the test of a rocker arm are:

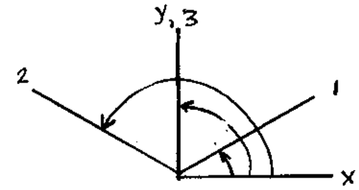
$\epsilon_1 = +600\ \mu \qquad \epsilon_2 = +450\ \mu \qquad \epsilon_3 = -75\ \mu$

Determine (*a*) the in-plane principal strains, (*b*) the in-plane maximum shearing strain..



SOLUTION

$\theta_1 = 30°$
$\theta_2 = 150°$
$\theta_3 = 90°$



$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1\cos\theta_1 = \varepsilon_1$$
$$0.75\,\varepsilon_x + 0.25\,\varepsilon_y + 0.43301\,\gamma_{xy} = 600\mu \quad (1)$$
$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2\cos\theta_2 = \varepsilon_2$$
$$0.75\,\varepsilon_x + 0.25\,\varepsilon_y - 0.43301\,\gamma_{xy} = 450\mu \quad (2)$$
$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3\cos\theta_3 = \varepsilon_3$$
$$0 + \varepsilon_y \quad 0 = -75\mu \quad (3)$$

Solving (1), (2), and (3) simultaneously

$$\varepsilon_x = 725\,\mu, \quad \varepsilon_y = -75\,\mu, \quad \gamma_{xy} = 173.21\,\mu$$

$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = 325\,\mu$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{725 + 75}{2}\right)^2 + \left(\frac{173.21}{2}\right)^2} = 409.3\,\mu$$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 734\,\mu$

$\varepsilon_b = \varepsilon_{ave} - R = -84.3\,\mu$

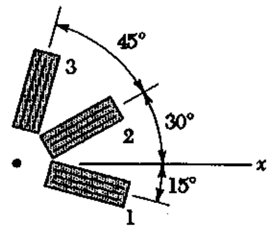
(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 819\,\mu$

PROBLEM 7.143

7.143 Determine the strain ϵ_x, knowing that the following strains have been determined by use of the rosette shown:

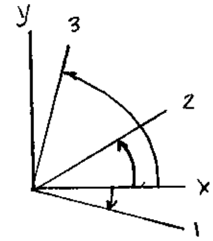
$\epsilon_1 = +720 \times 10^{-6}$ in./in. $\qquad \epsilon_2 = -180 \times 10^{-6}$ in./in.

$\epsilon_3 = +120 \times 10^{-6}$ in./in.



SOLUTION

$$\theta_1 = -15^\circ$$
$$\theta_2 = 30^\circ$$
$$\theta_3 = 75^\circ$$



$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1 = \varepsilon_1$$

$$0.9330\,\varepsilon_x + 0.06699\,\varepsilon_y - 0.25\,\gamma_{xy} = 720 \times 10^{-6} \qquad (1)$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2 = \varepsilon_2$$

$$0.75\,\varepsilon_x + 0.25\,\varepsilon_y + 0.4330\,\gamma_{xy} = -180 \times 10^{-6} \qquad (2)$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3 = \varepsilon_3$$

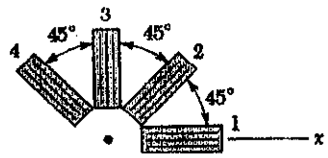
$$0.06699\,\varepsilon_x + 0.9330\,\varepsilon_y + 0.25\,\gamma_{xy} = 120 \times 10^{-6} \qquad (3)$$

Solving (1), (2), and (3) simultaneously

$\varepsilon_x = 380 \times 10^{-6}$ in/in, $\quad \varepsilon_y = 460 \times 10^{-6}$ in/in, $\quad \gamma_{xy} = -1339 \times 10^{-6}$ in/in

$$\varepsilon_x = 380 \times 10^{-6} \text{ in/in.}$$

PROBLEM 7.144



7.144 The rosette shown has been used to determine the following strains at a point on the surface of a crane hook:

$\epsilon_1 = +420\ \mu \qquad \epsilon_2 = -45\ \mu \qquad \epsilon_4 = +165\ \mu$

(*a*) What should be the reading of gage 3? (*b*) Determine the principal strains and the maximum in-plane shearing strain.

SOLUTION

(a) Gages 2 and 4 are 90° apart $\qquad \varepsilon_{ave} = \frac{1}{2}(\varepsilon_2 + \varepsilon_4)$

$$\varepsilon_{ave} = \frac{1}{2}(-45\mu + 165\mu) = 60\mu$$

Gages 1 and 3 are also 90° apart $\qquad \varepsilon_{ave} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$

$$\varepsilon_3 = 2\varepsilon_{ave} - \varepsilon_1 = (2)(60\mu) - 420\mu = -300\mu$$

(b) $\varepsilon_x = \varepsilon_1 = 420\mu \qquad \varepsilon_y = \varepsilon_3 = -300\mu$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_x - \varepsilon_1 = (2)(-45\mu) - 420\mu + 300\mu$$

$$= -210\mu$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{420\mu + 300\mu}{2}\right)^2 + \left(\frac{-210\mu}{2}\right)^2}$$

$$= 375\mu$$

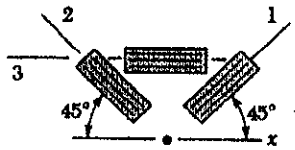
$$\varepsilon_a = \varepsilon_{ave} + R = 60\mu + 375\mu = 435\mu$$

$$\varepsilon_b = \varepsilon_{ave} - R = 60\mu - 375\mu = -315\mu$$

$$\gamma_{max\,(in\text{-}plane)} = 2R = 750\mu$$

PROBLEM 7.145

7.145 Determine the largest in-plane normal strain, knowing that the following strains have been obtained by use of the rosette shown:
$\epsilon_1 = -50 \times 10^{-6}$ in./in. $\epsilon_2 = +360 \times 10^{-6}$ in./in.
$\epsilon_3 = +315 \times 10^{-6}$ in./in



SOLUTION

$\theta_1 = 45° \qquad \theta_2 = -45° \qquad \theta_3 = 0$

$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1\cos\theta_1 = \varepsilon_1$$
$$0.5\,\varepsilon_x + 0.5\,\varepsilon_y + 0.5\,\gamma_{xy} = -50\times10^{-6} \qquad (1)$$
$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2\cos\theta_2 = \varepsilon_2$$
$$0.5\,\varepsilon_x + 0.5\,\varepsilon_y - 0.5\,\gamma_{xy} = 360\times10^{-6} \qquad (2)$$
$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3\cos\theta_3 = \varepsilon_3$$
$$\varepsilon_x + 0 + 0 = 315\times10^{-6} \qquad (3)$$

From (3) $\qquad \varepsilon_x = 315\times10^{-6}$ in/in.

Eq. (1) − Eq. (2) $\qquad \gamma_{xy} = -50\times10^{-6} - 360\times10^{-6} = -410\times10^{-6}$ in/in

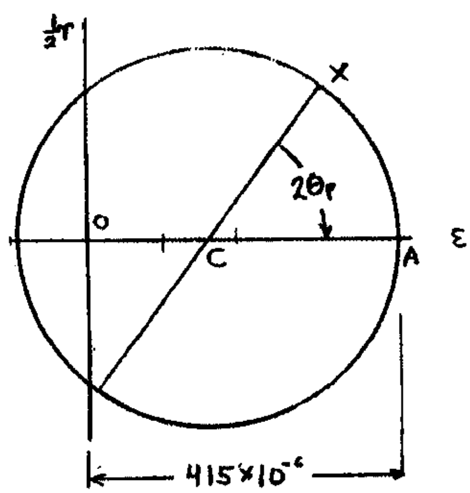
Eq (1) + Eq (2) $\qquad \varepsilon_x + \varepsilon_y = \varepsilon_1 + \varepsilon_2$

$$\varepsilon_y = \varepsilon_1 + \varepsilon_2 - \varepsilon_x = -50\times10^{-6} + 360\times10^{-6} - 315\times10^{-6} = -5\times10^{-6} \text{ in/in.}$$

$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = 155\times10^{-6} \text{ in/in}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{315\times10^{-6} + 5\times10^{-6}}{2}\right)^2 + \left(\frac{-410\times10^{-6}}{2}\right)^2}$$
$$= 260\times10^{-6} \text{ in/in.}$$

$$\varepsilon_{max} = \varepsilon_{ave} + R = 415\times10^{-6} \text{ in/in}$$



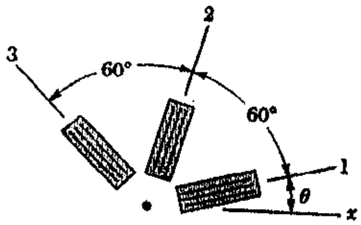
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -1.323$$
$$2\theta_p = -52.0°$$
$$\theta_p = -26.0°$$

PROBLEM 7.146

7.146 Show that the sum of the three strain measurements made with a 60° rosette is independent of the orientation of the rosette and equal to

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 3\epsilon_{ave}$$

where ϵ_{ave} is the abscissa of the center of the corresponding Mohr's circle for strain.



SOLUTION

$$\varepsilon_1 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (1)$$

$$\varepsilon_2 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 120°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 120°)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 120° \cos 2\theta - \sin 120° \sin 2\theta)$$
$$+ \frac{\gamma_{xy}}{2}(\cos 120° \sin 2\theta + \sin 120° \cos 2\theta)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta\right)$$
$$+ \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta + \frac{\sqrt{3}}{2}\cos 2\theta\right) \qquad (2)$$

$$\varepsilon_3 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 240°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 240°)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 240° \cos 2\theta - \sin 240° \sin 2\theta)$$
$$+ \frac{\gamma_{xy}}{2}(\cos 240° \sin 2\theta + \sin 240° \cos 2\theta)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta + \frac{\sqrt{3}}{2}\sin 2\theta\right)$$
$$+ \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta - \frac{\sqrt{3}}{2}\cos 2\theta\right) \qquad (3)$$

Adding (1), (2), and (3)

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{ave} + 0 + 0$$

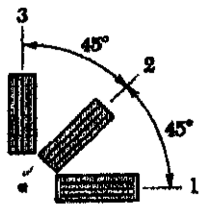
$$3\varepsilon_{ave} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

PROBLEM 7.147

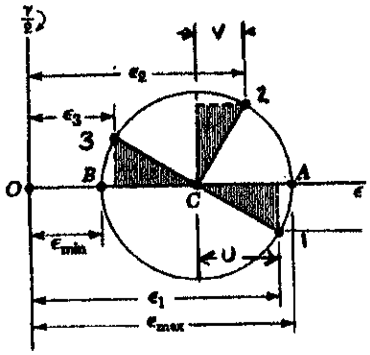
7.147 Using a 45° rosette, the strains ε_1, ε_2, and ε_3 have been determined at a given point. Using Mohr's circle, show that the principal strains are

$$\varepsilon_{\max,\min} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3) \pm \frac{1}{\sqrt{2}}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]^{1/2}$$

(*Hint:* The shaded triangles are congruent.)



SOLUTION



Since gage directions 1 and 3 are 90° apart

$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

Let $U = \varepsilon_1 - \varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_1 - \varepsilon_3)$

$$V = \varepsilon_2 - \varepsilon_{ave} = \varepsilon_2 - \tfrac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

$$R^2 = U^2 + V^2$$

$$= \tfrac{1}{4}(\varepsilon_1 - \varepsilon_3)^2 + \varepsilon_2^2 - \varepsilon_2(\varepsilon_1 + \varepsilon_3) + \tfrac{1}{4}(\varepsilon_1 + \varepsilon_3)^2$$

$$= \tfrac{1}{4}\varepsilon_1^2 - \cancel{\tfrac{1}{2}\varepsilon_1\varepsilon_3} + \tfrac{1}{4}\varepsilon_3^2 + \varepsilon_2^2 - \varepsilon_2\varepsilon_1 - \varepsilon_2\varepsilon_3 + \tfrac{1}{4}\varepsilon_1^2 + \cancel{\tfrac{1}{2}\varepsilon_1\varepsilon_3} + \tfrac{1}{4}\varepsilon_3^2$$

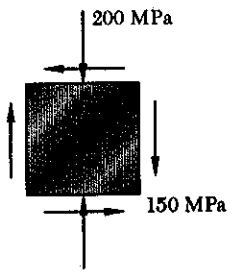
$$= \tfrac{1}{2}\varepsilon_1^2 - \varepsilon_2\varepsilon_1 + \varepsilon_2^2 - \varepsilon_2\varepsilon_3 + \tfrac{1}{2}\varepsilon_3^2$$

$$= \tfrac{1}{2}(\varepsilon_1 - \varepsilon_2)^2 + \tfrac{1}{2}(\varepsilon_2 - \varepsilon_3)^2$$

$$R = \frac{1}{\sqrt{2}}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2]$$

$\varepsilon_{max,min} = \varepsilon_{ave} \pm R$ gives the required formula.

PROBLEM 7.148



7.148 The given state of plane stress is known to exist on the surface of a machine component. Knowing that $E = 200$ GPa and $G = 77$ GPa, determine the direction and magnitude of the three principal strains (*a*) by determining the corresponding state of strain [use Eq. (2.43), page 94, and Eq. (2.38) page 91] and then using Mohr's circle for strain, (*b*) by using Mohr's circle for stress to determine the principal planes and principal stresses and then determining the corresponding strains.

SOLUTION

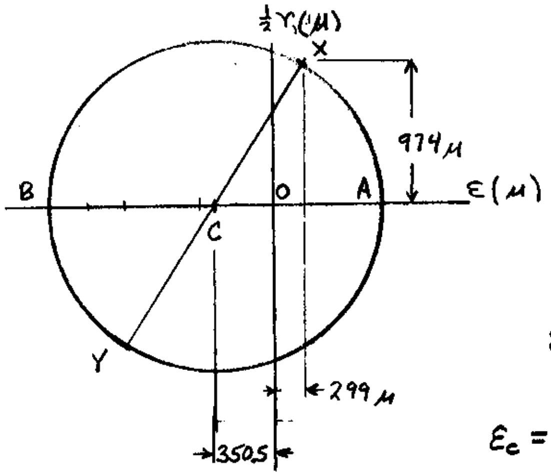
(a) $\sigma_x = 0$, $\sigma_y = -200 \times 10^6$ Pa, $\tau_{xy} = -150 \times 10^6$ Pa
$E = 200 \times 10^9$ Pa $\quad G = 77 \times 10^9$ Pa

$$G = \frac{E}{2(1+\nu)} \qquad \nu = \frac{E}{2G} - 1 = 0.2987$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{200 \times 10^{9}}\left[0 + (0.2987)(200 \times 10^6)\right] = 299\,\mu$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{200 \times 10^{9}}\left[(-200 \times 10^6) - 0\right] = -1000\,\mu$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-150 \times 10^6}{77 \times 10^9} = -1948\,\mu \qquad \frac{\gamma_{xy}}{2} = -974\,\mu$$



$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = -350.5\,\mu$$

$$\varepsilon_x - \varepsilon_y = 1299\,\mu$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-1948}{1299} = -1.4996$$

$$2\theta_a = -56.3° \qquad \theta_a = -28.15°$$

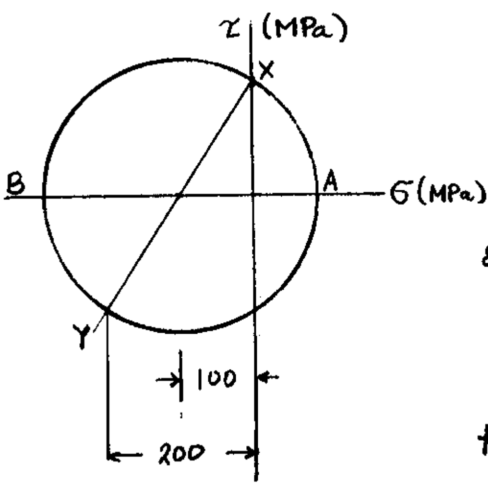
$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 1171\,\mu$$

$$\varepsilon_a = \varepsilon_{ave} + R = 820\,\mu$$

$$\varepsilon_b = \varepsilon_{ave} - R = -1521\,\mu$$

$$\varepsilon_c = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{(0.2987)(0 - 200 \times 10^6)}{200 \times 10^9} = -299\,\mu$$

(b)



$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 100 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0 + 200}{2}\right)^2 + 150^2} = 180.28 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 80.3 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -280.3 \text{ MPa}$$

$$\varepsilon_a = \frac{1}{E}(\sigma_a - \nu\sigma_b) = \frac{1}{200 \times 10^9}\left[80.3 \times 10^6 - (0.2987)(-280.3 \times 10^6)\right] = 820 \times 10^{-6} = 820\,\mu.$$

$$\tan 2\theta_a = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.5 \qquad 2\theta_a = -56.3° \qquad \theta_a = -28.15°$$

PROBLEM 7.149

7.149 The following state of strain has been determined on the surface of a cast-iron machine element:

$\epsilon_1 = -720 \times 10^{-6}$ in./in. $\qquad \epsilon_2 = -400 \times 10^{-6}$ in./in.

$\gamma = +660 \times 10^{-6}$ rad

Knowing that $E = 10 \times 10^6$ psi and $G = 4 \times 10^6$ psi, determine the principal planes and the principal stresses (*a*) by determining the corresponding state of plane stress [use Eq. 2.36, page 94 Eq. 2.43; page 91; and the first two equations of Prob. 2.75, page ww] and then using Mohr's circle for stress, (*b*) by using Mohr's circle for strain to determine the orientation and magnitude of the principal strains and then determining the corresponding stresses.

SOLUTION

$$G = \frac{E}{2(1+\nu)} \qquad \nu = \frac{E}{2G} - 1 = \frac{10}{(2)(4)} - 1 = 0.25$$

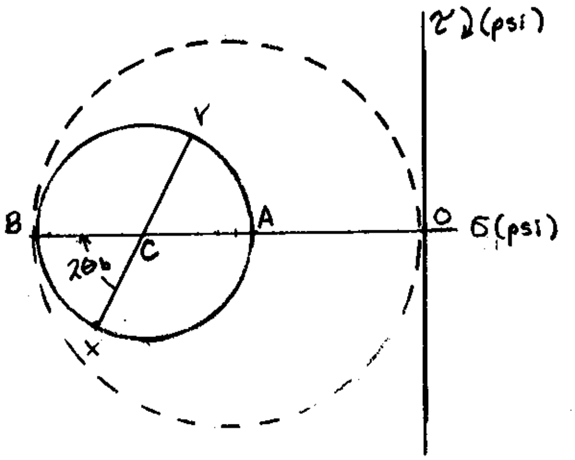
Note that the 3rd principal stress $\sigma_c = 0$

$$\frac{E}{1-\nu^2} = \frac{10 \times 10^6}{1 - 0.25^2} = 10.667 \times 10^6 \text{ psi}$$

(a) $$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) = 10.667 \times 10^6 \left[-720 \times 10^{-6} + (0.25)(-400 \times 10^{-6})\right] = -8746.7 \text{ psi}$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1) = 10.667 \times 10^6 \left[-400 \times 10^{-6} + (0.25)(-720 \times 10^{-6})\right] = -6186.7 \text{ psi}$$

$$\tau = G\gamma = (4 \times 10^6)(660 \times 10^{-6}) = 2640 \text{ psi}$$



$$\sigma_{ave} = \tfrac{1}{2}(\sigma_1 + \sigma_2) = -7466.7 \text{ psi}$$

$$\tan 2\theta_b = \frac{2\tau}{\sigma_1 - \sigma_2} = -2.0625$$

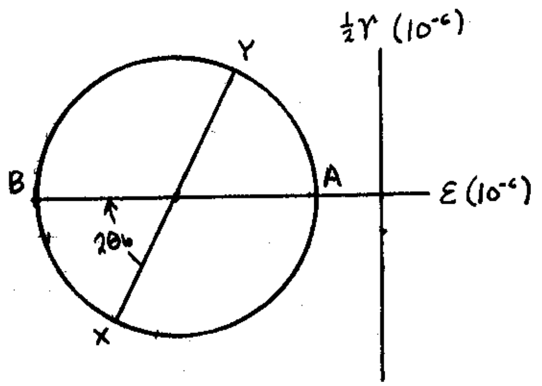
$2\theta_b = -64.1^\circ \qquad \theta_b = -32.1^\circ \qquad \theta_a = 57.9^\circ$

$$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} = 2934 \text{ psi}$$

$$\sigma_a = \sigma_{ave} + R = -4533 \text{ psi}$$

$$\sigma_b = \sigma_{ave} - R = -10400 \text{ psi}$$

(b) $$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_1 + \varepsilon_2) = -560 \times 10^{-6}$$



$$\tan 2\theta_b = \frac{\gamma}{\varepsilon_1 - \varepsilon_2} = \frac{660}{-720+400} = -2.0625$$

$2\theta_b = -64.1^\circ \qquad \theta_b = -32.1^\circ \qquad \theta_a = 57.9^\circ$

$$R = \sqrt{\left(\frac{\varepsilon_1 - \varepsilon_2}{2}\right)^2 + \left(\frac{\gamma}{2}\right)^2} = 366.74 \times 10^{-6}$$

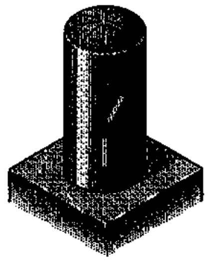
$$\varepsilon_a = \varepsilon_{ave} + R = -193.26 \times 10^{-6}$$

$$\varepsilon_b = \varepsilon_{ave} - R = -926.74 \times 10^{-6}$$

$$\sigma_a = \frac{E}{1-\nu^2}(\varepsilon_a + \nu\varepsilon_b) = -4533 \text{ psi}$$

$$\sigma_b = \frac{E}{1-\nu^2}(\varepsilon_b + \nu\varepsilon_a) = -10400 \text{ psi}$$

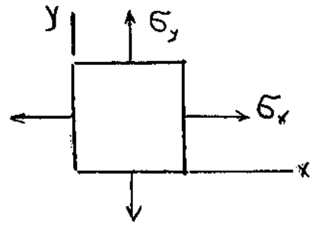
PROBLEM 7.150



7.150 A single strain gage forming an angle $\beta = 30°$ with the vertical is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is $\frac{3}{8}$ in. thick, has a 36-in. inside diameter, and is made of a steel with $E = 29 \times 10^6$ psi and $\nu = 0.30$. Determine the pressure in the tank corresponding to a gage reading of 220×10^{-6} in./in.

SOLUTION

Stresses in the tank wall



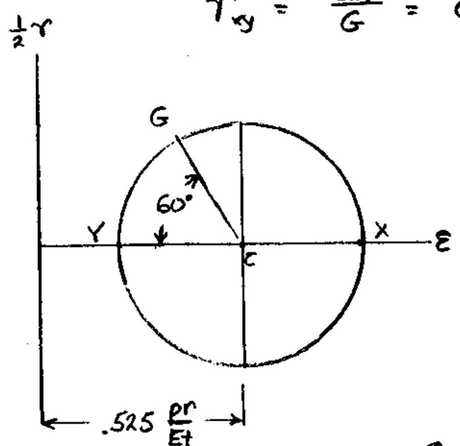
$$\sigma_x = \frac{pr}{t} \qquad \sigma_y = \frac{pr}{2t} \qquad \tau_{xy} = 0$$

Strains

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{E}\left(\frac{pr}{t} - \nu\frac{pr}{2t}\right) = \frac{pr}{Et}\left(1 - \frac{\nu}{2}\right) = 0.85\frac{pr}{Et}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{E}\left(\frac{pr}{2t} - \nu\frac{pr}{t}\right) = \frac{pr}{Et}\left(\frac{1}{2} - \nu\right) = 0.20\frac{pr}{Et}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



$$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{pr}{Et}$$

$$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325$$

$$\varepsilon_g = \varepsilon_{ave} - R\cos 60°$$

$$= .525\frac{pr}{Et} - 0.325\frac{pr}{Et}\cos 60°$$

$$= 0.3625\frac{pr}{Et}$$

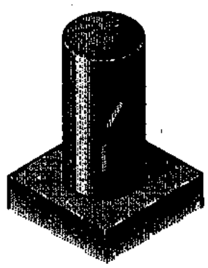
Solving for p $\qquad p = \dfrac{Et\,\varepsilon_g}{0.3625\,r}$

$$p = \frac{(29\times10^6)(\frac{3}{8})(220\times10^{-6})}{(0.3625)(36/2)} = 367 \text{ psi}$$

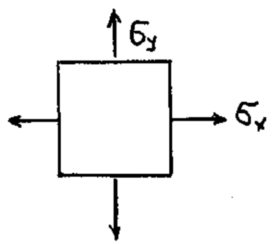
PROBLEM 7.151

7.150 A single strain gage forming an angle $\beta = 30°$ with the vertical is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is $\frac{3}{8}$ in. thick, has a 36-in. inside diameter, and is made of a steel with $E = 29 \times 10^6$ psi and $\nu = 0.30$. Determine the pressure in the tank corresponding to a gage reading of 220×10^{-6} in./in.

7.151 Solve Prob. 7.150, assuming that the gage forms an angle $\beta = 60°$ with the vertical.



SOLUTION



Stresses: $\sigma_x = \dfrac{pr}{t} \qquad \sigma_y = \dfrac{pr}{2t} \qquad \tau_{xy} = 0$

Strains: $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{E}\left(\frac{pr}{t} - \nu\frac{pr}{2t}\right)$

$= \frac{pr}{Et}\left(1 - \frac{\nu}{2}\right) = 0.85\,\frac{pr}{Et}$

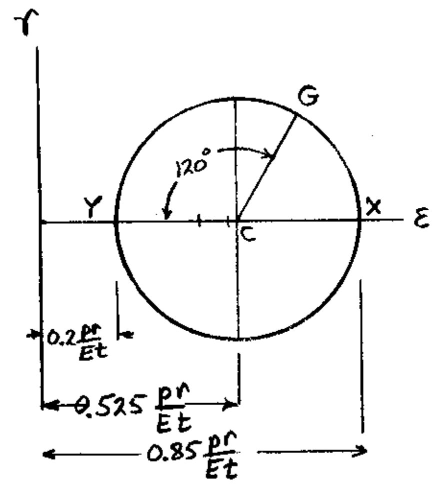
$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{E}\left(\frac{pr}{2t} - \nu\frac{pr}{t}\right)$

$= \left(\frac{1}{2} - \nu\right)\frac{pr}{Et} = 0.20\,\frac{pr}{Et}$

$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$

$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\,\frac{pr}{Et}$

$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\,\frac{pr}{Et}$



$\varepsilon_g = \varepsilon_{ave} + R\cos 60°$

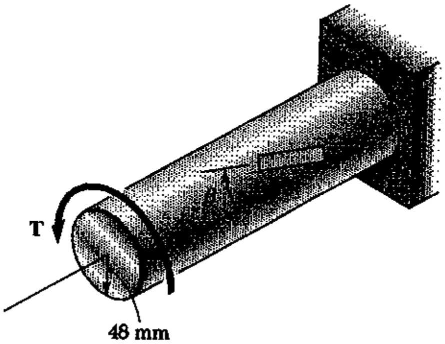
$= 0.525\,\frac{pr}{Et} + 0.325\,\frac{pr}{Et}\cos 60°$

$= 0.6875\,\frac{pr}{Et}$

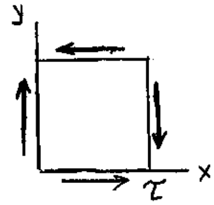
Solving for p $\qquad p = \dfrac{Et\varepsilon_g}{0.6875\,r} = \dfrac{(29\times10^6)(\frac{3}{8})(220\times10^{-6})}{(0.6875)(36/2)} = 193.3 \text{ psi}$ ◄

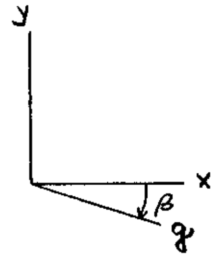
PROBLEM 7.152

7.152 A single strain gage is cemented to a solid 96-mm-diameter aluminum shaft at an angle $\beta = 20°$ with a line parallel to the axis of the shaft. Knowing that $G = 27$ GPa, determine the torque **T** corresponding to a gage reading of 400 μ.



SOLUTION





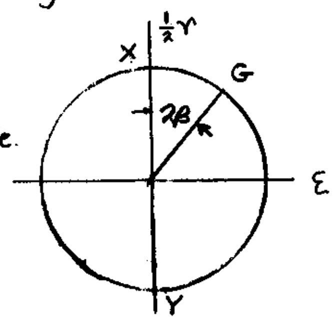
$\tau = \dfrac{Tc}{J}$ $\qquad J = \dfrac{\pi}{2}c^2$ $\qquad \gamma = \dfrac{\tau}{G}$

$\sigma_x = \sigma_y = 0$ $\qquad \varepsilon_x = \varepsilon_y = 0$

Sketch Mohr's circle for strain.

Gage direction is β clockwise from x

Point G is 2β clockwise from X on Mohr's circle.



$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0$

$R = \frac{1}{2}\gamma_{xy}$

$\varepsilon_g = \varepsilon_{ave} + R\sin 2\beta = \frac{1}{2}\gamma_{xy}\sin 2\beta = \dfrac{\tau_{xy}}{2G}\sin 2\beta$

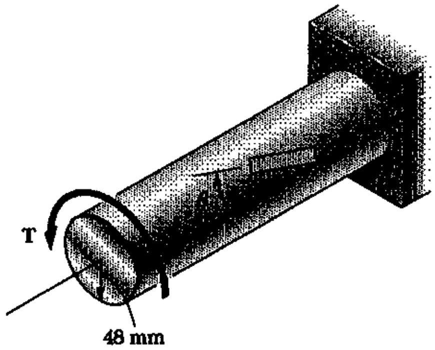
$= \dfrac{Tc}{2GJ}\sin 2\beta =$

Solving for T $\quad T = \dfrac{2GJ\varepsilon_g}{c\sin 2\beta} = \dfrac{\pi G c^3 \varepsilon_g}{\sin 2\beta}$

$T = \dfrac{\pi(27\times10^9)(48\times10^{-3})^3(400\times10^{-6})}{\sin 40°} = 5.84\times10^3$ N·m

$= 5.84$ kN·m

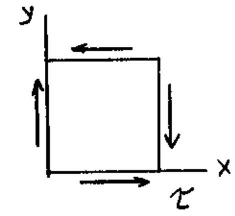
PROBLEM 7.153

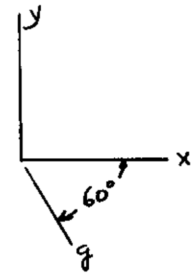


7.152 A single strain gage is cemented to a solid 96-mm-diameter aluminum shaft at an angle $\beta = 20°$ with a line parallel to the axis of the shaft. Knowing that $G = 27$ GPa, determine the torque **T** corresponding to a gage reading of 400 μ.

7.153 Solve Prob. 7.152, assuming that the gage forms an angle $\beta = 60°$ with a line parallel to the axis of the shaft.

SOLUTION

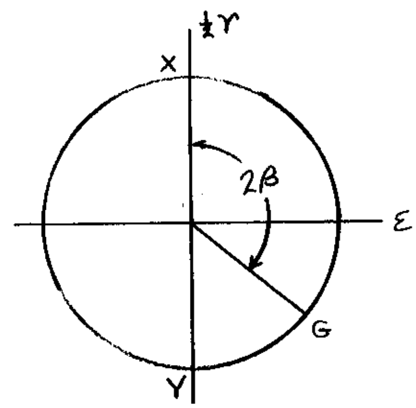




$\tau = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4 \qquad \gamma = \frac{\tau}{G}$

$\sigma_x = \sigma_y = 0 \qquad \varepsilon_x = \varepsilon_y = 0$

Sketch Mohr's circle for strain.
Gage direction g is $\beta = 60°$ clockwise from x.
Point G is $2\beta = 120°$ clockwise from point X on Mohr's circle.



$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0$

$R = \frac{1}{2}\gamma$

$\varepsilon_g = \varepsilon_{ave} + R\sin 2\beta = \frac{1}{2}\gamma \sin 2\beta = \frac{\tau}{2G}\sin 2\beta$

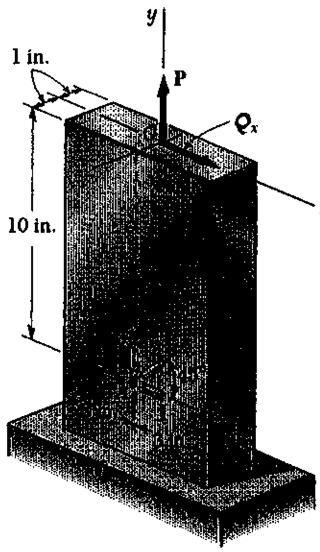
$= \frac{Tc}{2GJ}\sin 2\beta$

Solving for T $\quad T = \frac{2GJ\varepsilon_g}{c\sin 2\beta} = \frac{\pi G c^3 \varepsilon_g}{\sin 2\beta}$

$T = \frac{\pi(27\times 10^9)(48\times 10^{-3})^3(400\times 10^{-6})}{\sin 120°} = 4.33\times 10^3$ N·m

$= 4.33$ kN·m ◄

PROBLEM 7.154



7.154 A centric axial force **P** and a horizontal force $\mathbf{Q}_x$ are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:

$\epsilon_1 = -75 \times 10^{-6}$ in./in. $\quad \epsilon_2 = +300 \times 10^{-6}$ in./in. $\quad \epsilon_3 = +250 \times 10^{-6}$ in./in.

Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the magnitudes of **P** and $\mathbf{Q}_x$.

SOLUTION

$$\varepsilon_x = \varepsilon_1 = -75 \times 10^{-6} \qquad \varepsilon_y = \varepsilon_3 = 250 \times 10^{-6}$$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 425 \times 10^{-6}$$

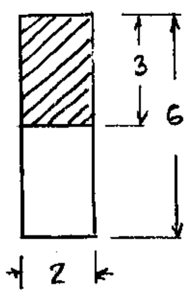
$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{29}{1-0.3^2}\left[-75 + (0.3)(250)\right] = 0$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{29}{1-0.3^2}\left[250 + (0.3)(-75)\right] = 7.25 \times 10^3 \text{ psi}$$

$$\frac{P}{A} = \sigma_y \qquad P = A\sigma_y = (2)(6)(7.25 \times 10^3) = 87.0 \times 10^3 \text{ lb} = 87.0 \text{ kips}$$

$$G = \frac{E}{2(1+\nu)} = \frac{29 \times 10^6}{(2)(1.3)} = 11.154 \times 10^6 \text{ psi}$$

$$\tau_{xy} = G\gamma_{xy} = (11.154)(425) = 4.740 \times 10^3 \text{ psi}$$



$$I = \frac{1}{12}bh^3 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$

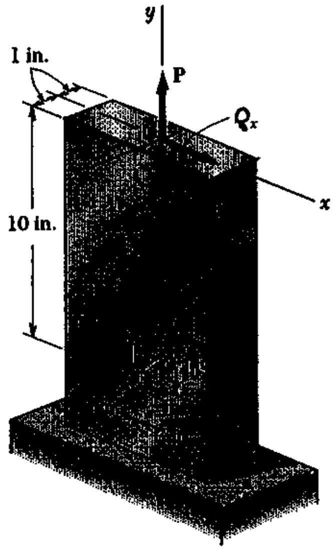
$$Q = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3 \qquad t = 2 \text{ in}$$

$$\tau_{xy} = \frac{VQ}{It}$$

$$V = \frac{It\tau_{xy}}{Q} = \frac{(36)(2)(4.74 \times 10^3)}{9} = 37.9 \times 10^3 \text{ lb.}$$

$$Q_x = V = 37.9 \times 10^3 \text{ lb} = 37.9 \text{ kips}$$

PROBLEM 7.155



7.154 A centric axial force **P** and a horizontal force $\mathbf{Q}_x$ are both applied at point *C* of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point *A* indicates the following strains:

$\epsilon_1 = -75 \times 10^{-6}$ in./in. $\quad \epsilon_2 = +300 \times 10^{-6}$ in./in. $\quad \epsilon_3 = +250 \times 10^{-6}$ in./in.

Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the magnitudes of **P** and $\mathbf{Q}_x$.

7.155 Solve Prob. 7.154, assuming that the rosette at point *A* indicates the following strains:

$\epsilon_1 = -60 \times 10^{-6}$ in./in. $\quad \epsilon_2 = +410 \times 10^{-6}$ in./in. $\quad \epsilon_3 = +200 \times 10^{-6}$ in./in.

SOLUTION

$$\varepsilon_x = \varepsilon_1 = -60 \times 10^{-6} \qquad \varepsilon_y = \varepsilon_3 = 200 \times 10^{-6}$$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 680 \times 10^{-6}$$

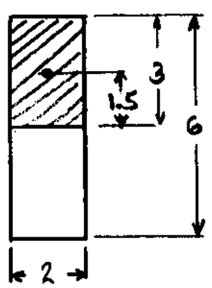
$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{29}{1-0.3^2}\left[-60 + (0.3)(200)\right] = 0$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{29}{1-0.3^2}\left[200 + (0.3)(-60)\right] = 5.800 \times 10^3 \text{ psi}$$

$$\frac{P}{A} = \sigma_y \qquad P = A\sigma_y = (2)(6)(5.800 \times 10^3) = 69.6 \times 10^3 \text{ lb} = 69.6 \text{ kips}$$

$$G = \frac{E}{2(1+\nu)} = \frac{29 \times 10^6}{(2)(1.3)} = 11.154 \times 10^6 \text{ psi}$$

$$\tau_{xy} = G\gamma_{xy} = (11.154)(680) = 7.585 \times 10^3 \text{ psi}$$



$$I = \frac{1}{12}bh^3 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$

$$Q = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3 \qquad t = 2 \text{ in.}$$

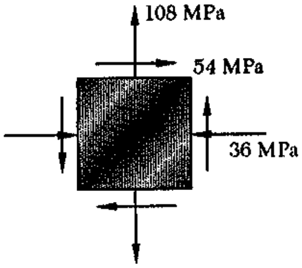
$$\tau_{xy} = \frac{VQ}{It}$$

$$V = \frac{It\tau_{xy}}{Q} = \frac{(36)(2)(7.585 \times 10^3)}{9} = 60.7 \times 10^3 \text{ lb.}$$

$$Q_x = V = 60.7 \times 10^3 \text{ lb.} = 60.7 \text{ kips}$$

PROBLEM 7.156

7.156 The state of stress shown occurs in a steel member made of a grade of steel with a tensile yield strength of 270 MPa. Determine the factor of safety with respect to yield strength, using (*a*) the maximum-shearing-stress criterion, (*b*) the maximum distortion-strength criterion.



SOLUTION

$\sigma_x = -36 \text{ MPa}, \quad \sigma_y = 108 \text{ MPa}, \quad \tau_{xy} = 54 \text{ MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 36 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 90 \text{ MPa}$

$\sigma_a = \sigma_{ave} + R = 126 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -54 \text{ MPa}, \quad \sigma_z = 0$

(a) $\sigma_{max} = 126 \text{ MPa}, \quad \sigma_{min} = -54 \text{ MPa}$

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 180 \text{ MPa} < 270 \text{ MPa}$ (No yielding)

$F.S. = \frac{\sigma_Y}{2\tau_{max}} = \frac{270}{180} = 1.500$

(b) $\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 159.99 \text{ MPa} < 270 \text{ MPa}$ (No yielding)

$F.S. = \frac{\sigma_Y}{\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b}} = \frac{270}{155.99} = 1.688$

PROBLEM 7.157

7.157 A spherical pressure tank has 1.2-m outer diameter and a uniform wall thickness of 10 mm. Knowing that the gage pressure is 1.25 MPa in the tank, determine (*a*) the maximum normal stress, (*b*) the maximum shearing stress, (*c*) the normal strain on the surface of the tank. (Use $E = 200$ GPa and $\nu = 0.30$.)

SOLUTION

$t = 10 \times 10^{-3} \text{ m}, \quad r = \frac{1}{2}d - t = \frac{1}{2}(1.2) - 10\times10^{-3} = 0.590 \text{ m}, \quad p = 1.25 \text{ MPa}$

For a spherical tank under internal pressure

$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(1.25)(0.590)}{(2)(10\times10^{-3})} = 36.9 \text{ MPa}$

$\sigma_3 \approx 0$

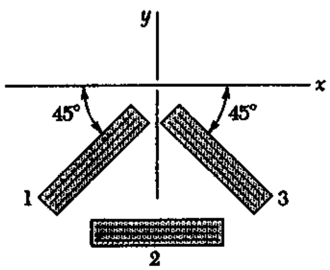
(a) $\sigma_{max} = 36.9 \text{ MPa}$

(b) $\sigma_{min} = 0 \qquad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 18.44 \text{ MPa}$

(c) $\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3) = \frac{1}{200\times10^9}\left[36.9\times10^6 - (0.3)(36.9\times10^6) - 0\right]$

$= 129 \times 10^{-6} = 129\,\mu$

PROBLEM 7.158

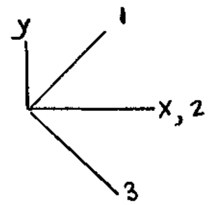


7.158 The strains determined by the use of a rosette attached as shown to the surface of a structural member are:

$\epsilon_1 = 220 \times 10^{-6}$ in./in. $\quad \epsilon_2 = 425 \times 10^{-6}$ in./in. $\quad \epsilon_3 = 480 \times 10^{-6}$ in./in.

Determine (*a*) the orientation and magnitude of the principal strains in the plane of the rosette, (*b*) the maximum in-plane shearing strain.

SOLUTION



$\theta_1 = 45°$

$\theta_2 = 0$

$\theta_3 = -45°$

$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1 = \varepsilon_1$$

$$\tfrac{1}{2}\varepsilon_x + \tfrac{1}{2}\varepsilon_y + \tfrac{1}{2}\gamma_{xy} = 220 \times 10^{-6} \text{ in/in} \qquad (1)$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2 = \varepsilon_2$$

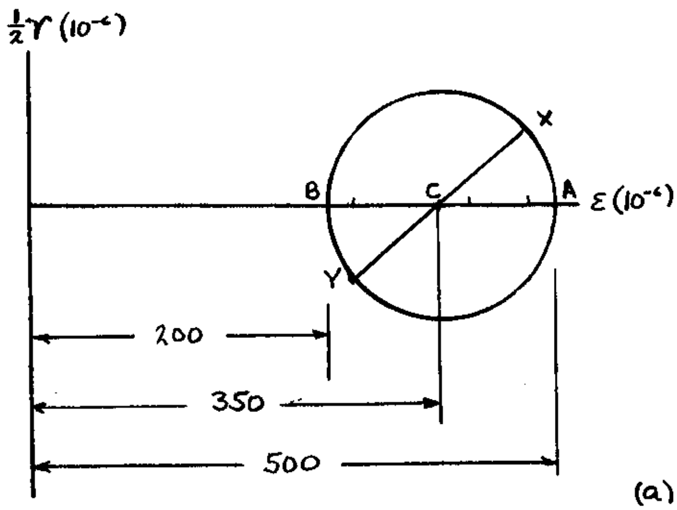
$$\varepsilon_x + 0 + 0 = 425 \times 10^{-6} \text{ in/in} \qquad (2)$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3 = \varepsilon_3$$

$$\tfrac{1}{2}\varepsilon_x + \tfrac{1}{2}\varepsilon_y - \tfrac{1}{2}\gamma_{xy} = 480 \times 10^{-6} \text{ in/in} \qquad (3)$$

Solving (1), (2) and (3) simultaneously gives

$$\varepsilon_x = 425 \times 10^{-6} \text{ in/in}, \quad \varepsilon_y = 275 \times 10^{-6} \text{ in/in}, \quad \gamma_{xy} = -260 \times 10^{-6} \text{ in/in.}$$



$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = 350 \times 10^{-6} \text{ in/in}$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-260}{425 - 275} = -1.7333$$

$$2\theta_a = -60° \qquad \theta_a = -30° \quad \blacktriangleleft$$

$$\theta_b = 60° \quad \blacktriangleleft$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 150 \times 10^{-6} \text{ in/in}$$

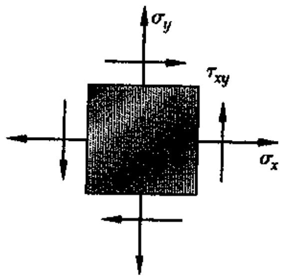
(a) $\varepsilon_a = \varepsilon_{ave} + R = 500 \times 10^{-6}$ in/in $\blacktriangleleft$

$\varepsilon_b = \varepsilon_{ave} - R = 200 \times 10^{-6}$ in/in $\blacktriangleleft$

(b) $\gamma_{max\,(in\ plane)} = 2R = 300 \times 10^{-6}$ in/in $\blacktriangleleft$

PROBLEM 7.159

7.159 For a state of plane stress it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 5$ ksi, $\sigma_y = 12$ ksi, and $\sigma_{max} = 18$ ksi. Determine (*a*) the orientation of the principal planes, (*b*) the maximum in-plane shearing stress.



SOLUTION

(a) $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(5 + 12) = 8.5$ ksi

$R = \sigma_{max} - \sigma_{ave} = 18 - 8.5 = 9.5$ ksi

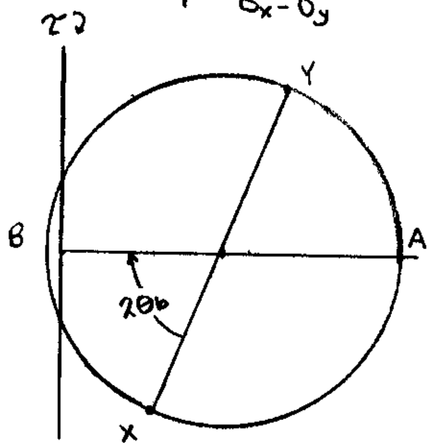
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{9.5^2 - \left(\frac{5-12}{2}\right)^2}$$

$\pm\ 8.83$ ksi

In the sketch τ_{xy} is shown positive; hence $\tau_{xy} = +8.83$ ksi

$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = -2.523 \qquad 2\theta_p = -68.4°$



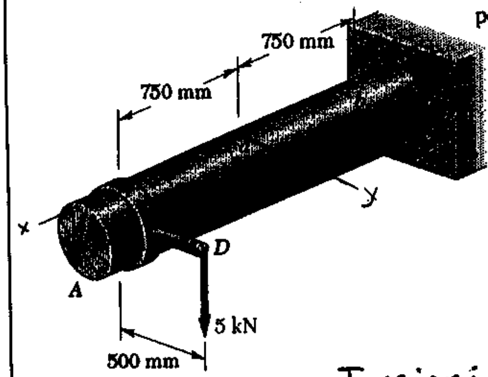
$\theta_b = -34.2°, \quad \theta_a = 55.8°$

$\sigma_a = \sigma_{ave} + R = \sigma_{max} = 18$ ksi

$\sigma_b = \sigma_{ave} - R = \sigma_{min} = -1$ ksi

(b) $\tau_{max(in\text{-}plane)} = R = 9.5$ ksi

PROBLEM 7.160



7.160 The compressed-air tank *AB* has an inside diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure in the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at points *a* and *b* on the top of the tank.

SOLUTION

$r = \frac{1}{2}d = 225 \text{ mm} \qquad t = 6 \text{ mm}$

$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45 \text{ MPa}$

$\sigma_2 = \frac{pr}{2t} = 22.5 \text{ MPa}$

Torsion: $c_1 = 225 \text{ mm}, \quad c_2 = 225 + 6 = 231 \text{ mm}$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 446.9 \times 10^6 \text{ mm}^4 = 446.9 \times 10^{-6} \text{ m}^4$

$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500 \text{ N·m}$

$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6 \text{ Pa} = 1.292 \text{ MPa}$

Transverse shear: $\tau = 0$ at points a and b.

Bending: $I = \frac{1}{2}J = 223.45 \times 10^{-6} \text{ m}^4, \quad c = 231 \times 10^{-3} \text{ m.}$

Point a

$M = (5 \times 10^3)(750 \times 10^{-3}) = 3750 \text{ N·m}$

$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.88 \text{ MPa}$

Total stresses: (MPa)

Longitudinal $\sigma_x = 22.5 + 3.88 = 26.38$

Circumferential $\sigma_y = 45$

Shear $\tau_{xy} = 1.292$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 35.69 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 9.40 \text{ MPa}$

$\sigma_{max} = \sigma_{ave} + R = 45.1 \text{ MPa}$ ◄

$\tau_{max\,(in\text{-}plane)} = R = 9.40 \text{ MPa}$ ◄

Point b

$M = (5 \times 10^3)(2 \times 750 \times 10^{-3}) = 7500 \text{ N·m}$

$\sigma = \frac{Mc}{I} = 7.75 \text{ MPa}$

Total stresses (MPa)

$\sigma_x = 22.5 + 7.75 = 30.25$

$\sigma_y = 45$

$\tau_{xy} = 1.292$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 37.625 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.487 \text{ MPa}$

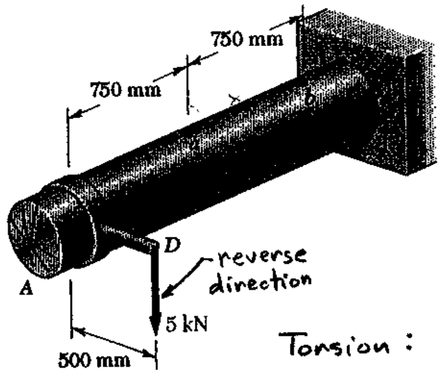
$\sigma_{max} = \sigma_{ave} + R = 45.1 \text{ MPa}$ ◄

$\tau_{max\,(in\text{-}plane)} = R = 7.49 \text{ MPa}$ ◄

PROBLEM 7.161

7.160 The compressed-air tank *AB* has an inside diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure in the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at points *a* and *b* on the top of the tank.

7.161 Solve Prob. 7.160, assuming that the 5-kN force applied at *D* is directed vertically upward.



SOLUTION

$r = \frac{1}{2}d = 225$ mm $\qquad t = 6$ mm

$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45$ MPa

$\sigma_2 = \frac{pr}{2t} = 22.5$ MPa

Torsion: $c_1 = 225$ mm $\qquad c_2 = 225 + 6 = 231$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 446.9 \times 10^6\ \text{mm}^4 = 446.9 \times 10^{-6}\ \text{m}^4$

$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500$ N·m

$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6$ Pa $= 1.292$ MPa

Transverse shear: $\tau = 0$ at points a and b.

Bending: $I = \frac{1}{2}J = 223.45 \times 10^{-6}\ \text{m}^4$, $\qquad c = 231 \times 10^{-3}$ m

Point a

$M = (5 \times 10^3)(750 \times 10^{-3}) = 3750$ N·m

$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.88$ MPa

Total stresses (MPa)

Longitudinal $\sigma_x = 22.5 - 3.88 = 18.62$ MPa

Circumferential $\sigma_y = 45$ MPa

Shear $\tau_{xy} = -1.292$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 31.81$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 13.25$ MPa

$\sigma_{max} = \sigma_{ave} + R = 45.1$ MPa ◄

$\tau_{max(in\text{-}plane)} = R = 13.25$ MPa ◄

Point b

$M = (5 \times 10^3)(2 \times 750 \times 10^{-3}) = 7500$ N·m

$\sigma = \frac{Mc}{I} = 7.75$ MPa

Total stresses (MPa)

$\sigma_x = 22.5 - 7.75 = 14.75$

$\sigma_y = 45$

$\tau_{xy} = -1.292$

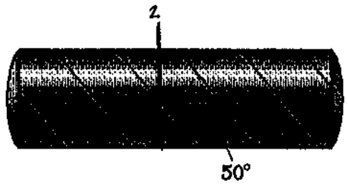
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 29.875$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 15.18$ MPa

$\sigma_{max} = \sigma_{ave} + R = 45.1$ MPa ◄

$\tau_{max(in\text{-}plane)} = R = 15.18$ MPa ◄

PROBLEM 7.162

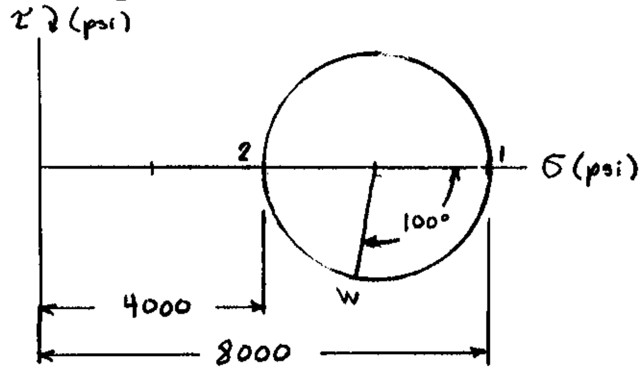


7.162 The steel pressure tank shown has a 30-in. inside diameter and a $\frac{3}{8}$-in. wall thickness. Knowing that the butt-welded seams form an angle of 50° with the longitudinal axis of the tank and that the gage pressure in the tank is 200 psi, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$r = \frac{1}{2}d = 15$ in.

$\sigma_1 = \frac{pr}{t} = \frac{(200)(15)}{0.375} = 8000$ psi $\qquad \sigma_2 = \frac{1}{2}\sigma_1 = 4000$ psi



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 6000$ psi

$R = \frac{\sigma_1 - \sigma_2}{2} = 2000$ psi

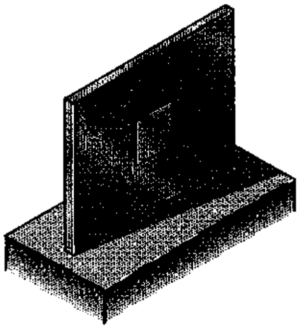
(a) $\sigma_w = \sigma_{ave} + R\cos 100°$

$= 5652$ psi ◄

(b) $\tau_w = R\sin 100°$

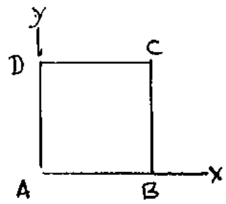
$= 1970$ psi ◄

PROBLEM 7.163



7.163 A square *ABCD* of 2.4-in. side is scribed on the surface of a thin plate while the plate is unloaded. After the plate is loaded, the lengths of sides *AB* and *AD* are observed to have increased, respectively, by 540×10^{-6} in. and 900×10^{-6} in., while the angle *DAB* is observed to have decreased by 360×10^{-6} rad. Knowing that $\nu = \frac{1}{3}$, determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain.

SOLUTION



$$\varepsilon_x = \frac{\Delta l_x}{\Delta x} = \frac{\Delta \overline{AB}}{\overline{AB}} = \frac{540\times10^{-6}}{2.4} = 225\times10^{-6}$$

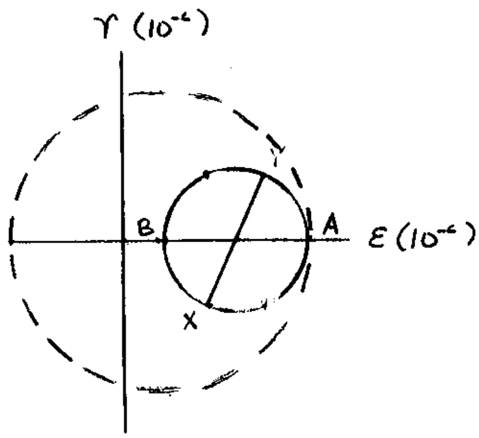
$$\varepsilon_y = \frac{\Delta l_y}{\Delta y} = \frac{\Delta \overline{AD}}{\overline{AD}} = \frac{900\times10^{-6}}{2.4} = 375\times10^{-6}$$

γ_{xy} = decrease in right angle DAB $= 360\times10^{-6}$ rad $= 360\times10^{-6}$

$$\varepsilon_{ave} = \frac{1}{2}\varepsilon_x + \varepsilon_y = 300\times10^{-6}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{360}{225-375} = -2.4$$

$$2\theta_p = -67.38^\circ \qquad \theta_p = -33.7^\circ$$



$$R = \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 195\times10^{-6}$$

$$\varepsilon_a = \varepsilon_{ave} + R = 495\times10^{-6} \blacktriangleleft$$

$$\varepsilon_b = \varepsilon_{ave} - R = 105\times10^{-6} \blacktriangleleft$$

(b) $\gamma_{max\,(in\text{-}plane)} = \varepsilon_a - \varepsilon_b = 390\times10^{-6} \blacktriangleleft$

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b)$$

$$= -\frac{(1/3)}{(2/3)}(495\times10^{-6} + 105\times10^{-6}) = -300\times10^{-6}$$

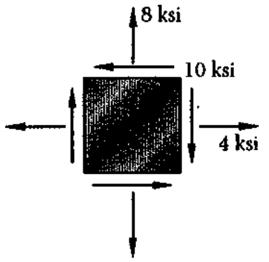
$$\varepsilon_{max} = 495\times10^{-6} \qquad \varepsilon_{min} = -300\times10^{-6}$$

(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 795\times10^{-6} \blacktriangleleft$

$$\left.\begin{aligned} \varepsilon_{ave} &= \tfrac{1}{2}(\varepsilon_{max} + \varepsilon_{min}) = 97.5\times10^{-6} \\ R &= \tfrac{1}{2}\gamma_{max} = 397.5\times10^{-6} \end{aligned}\right\} \text{For dotted Mohr's circle}$$

PROBLEM 7.164

7.164 For the state of plane stress shown, determine (*a*) the principal planes, (*b*) the principal stresses, (*c*) the maximum shearing stress.



SOLUTION

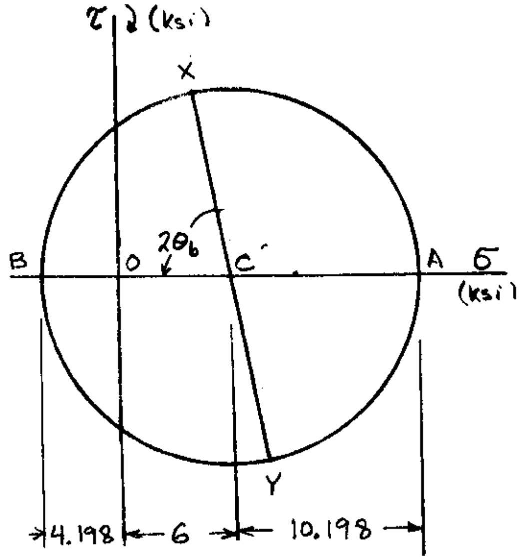
$\sigma_x = 4$ ksi, $\sigma_y = 8$ ksi, $\tau_{xy} = -10$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 6$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 10.198$ ksi



(a) $\tan 2\theta_b = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = 5.00$

$2\theta_b = 78.69°$ $\quad \theta_b = 39.345°$ ◄

$\theta_a = \theta_b - 90° = -50.655°$ ◄

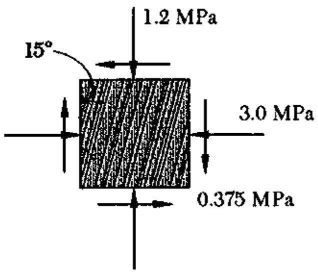
(b) $\sigma_a = \sigma_{ave} + R = 16.198$ ksi ◄

$\sigma_b = \sigma_{ave} - R = -4.198$ ksi ◄

(c) $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(\sigma_a - \sigma_b) = R = 10.198$ ksi ◄

PROBLEM 7.165

7.165 The grain a wooden member forms an angle of 15° with the vertical. For the state of plane stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



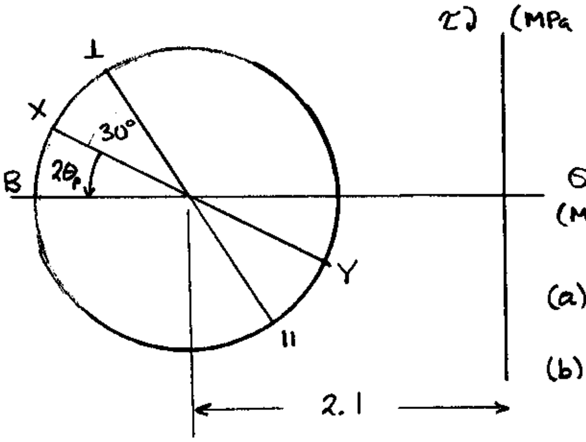
SOLUTION

$\sigma_x = -3.0$ MPa , $\sigma_y = -1.2$ MPa

$\tau_{xy} = -0.375$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -2.10$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 0.975$ MPa



$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.41667$

$2\theta_p = 22.62°$

$2\theta_p + 30° = 52.62°$

(a) $\tau_{\perp(\text{in-plane})} = R \sin 52.5° = 0.775$ MPa ◄

(b) $\sigma_{\perp} = \sigma_{ave} - R\cos 52.62°$

$= -2.10 - 0.592 = -2.692$ MPa ◄

PROBLEM 7.166

7.166 A cylindrical steel pressure tank has a 26-in. inside diameter and a uniform $\frac{1}{4}$-in. wall thickness. Knowing that the ultimate stress of the steel used is 65 ksi, determine the maximum allowable gage pressure if a factor of safety of 5.0 must be maintained.

SOLUTION

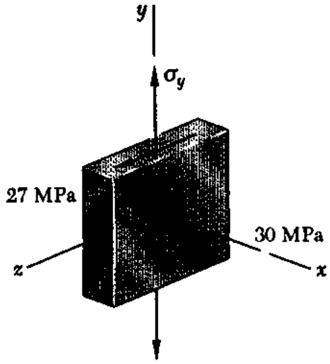
$r = \frac{1}{2}d = \frac{1}{2}(26) = 13$ in. $\qquad t = 0.25$ in

$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{65}{5} = 13$ ksi $\qquad \sigma_1 = \frac{pr}{t}$

$p = \frac{\sigma_1 t}{r} = \frac{(13)(0.25)}{13} = 0.25$ ksi $= 250$ psi ◄

PROBLEM 7.167

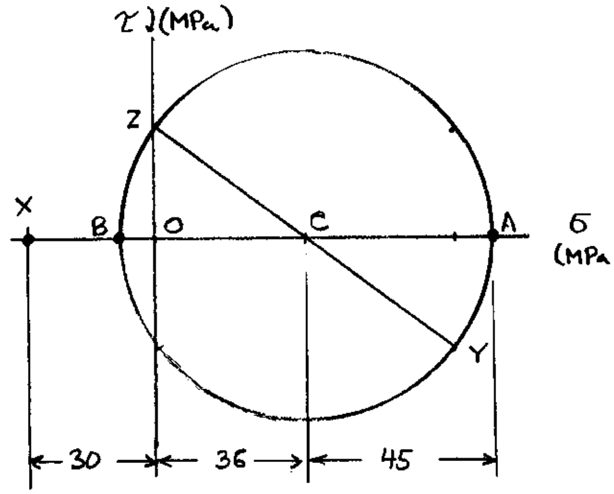
7.167 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = +72$ MPa, (*b*) $\sigma_y = -72$ MPa,



SOLUTION

$\sigma_x = -30$ MPa $\qquad \tau_{yz} = 27$ MPa , $\sigma_z = 0$

(a) $\sigma_y = +72$ MPa



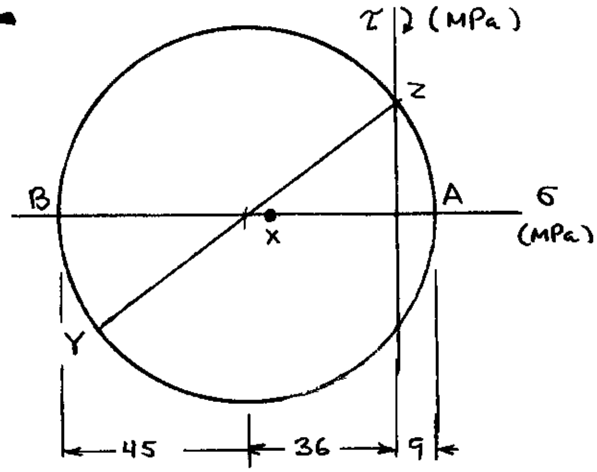
$\sigma_{ave} = \frac{1}{2}(\sigma_y + \sigma_z) = 36$ MPa

$R = \sqrt{\left(\frac{\sigma_y - \sigma_z}{2}\right)^2 + \tau_{yz}^2} = 45$ MPa

$\sigma_{max} = \sigma_a = \sigma_{ave} + R = 81$ MPa

$\sigma_{min} = \sigma_x = -30$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 55.5$ MPa ◄



(b) $\sigma_y = -72$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_y + \sigma_z) = -36$ MPa

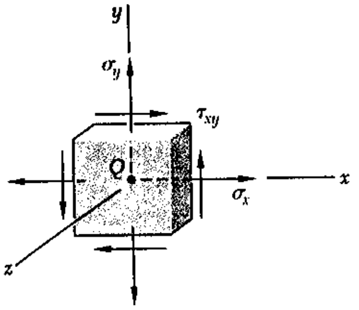
$R = \sqrt{\left(\frac{\sigma_y - \sigma_z}{2}\right)^2 + \tau_{yz}^2} = 45$ MPa

$\sigma_{max} = \sigma_a = \sigma_{ave} + R = 9$ MPa

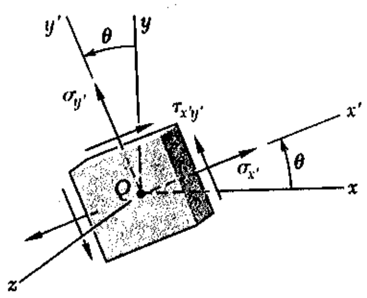
$\sigma_{min} = \sigma_b = \sigma_{ave} - R = -81$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 45$ MPa ◄

PROBLEM 7.C1



(a)



(b)

7.C1 A state of plane stress is defined by the stress components σ_x, σ_y, and τ_{xy} associated with the element shown in Fig. P7.C1*a*. (*a*) Write a computer program that can be used to calculate the stress components $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$ associated with the element after it has rotated through an angle θ about the *z* axis (Fig. P7.C1*b*). (*b*) Use this program to solve Probs. 7.13 through 7.16.

SOLUTION PROGRAM FOLLOWING EQUATIONS

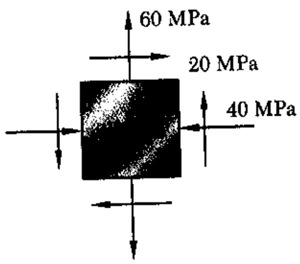
EQ (7.5), p 427: $\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

EQ (7.7), p 427: $\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

EQ (7.6), p 427: $\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

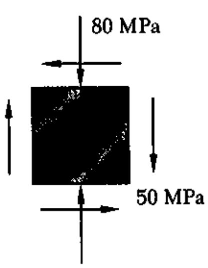
ENTER σ_x; σ_y, τ_{xy} AND θ

PRINT VALUES OBTAINED FOR $\sigma_{x'}$, $\sigma_{y'}$, AND $\tau_{x'y'}$




```
Problem 7.13a

  Sigma x = -40 MPa
  Sigma y =  60 MPa
  Tau xy  =  20 MPa

Rotation of element
(+ counterclockwise)
  theta = -25 degrees

  Sigma x' = -37.46 MPa
  Sigma y' =  57.46 MPa
  Tau x'y' = -25.45 MPa
```

```
Problem 7.13b

  Sigma x = -40 MPa
  Sigma y =  60 MPa
  Tau xy  =  20 MPa

Rotation of element
(+ counterclockwise)
  theta =  10 degrees

  Sigma x' = -30.14 MPa
  Sigma y' =  50.14 MPa
  Tau x'y' =  35.89 MPa
```



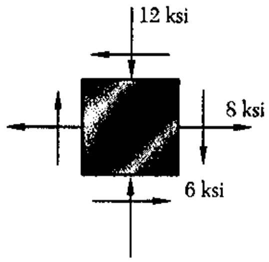

```
Problem 7.14a

  Sigma x =   0 MPa
  Sigma y = -80 MPa
  Tau xy  = -50 MPa

Rotation of element
(+ counterclockwise)
  theta = -25 degrees

  Sigma x' =   24.01 MPa
  Sigma y' = -104.01 MPa
  Tau x'y' =   -1.50 MPa
```

```
Problem 7.14b

  Sigma x =   0 MPa
  Sigma y = -80 MPa
  Tau xy  = -50 MPa

Rotation of element
(+ counterclockwise)
  theta =  10 degrees

  Sigma x' = -19.51 MPa
  Sigma y' = -60.49 MPa
  Tau x'y' = -60.67 MPa
```

CONTINUED

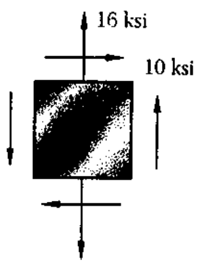
PROBLEM 7.C1 - CONTINUED

PROGRAM OUTPUT




```
Problem 7.15a

  Sigma x =    8 ksi
  Sigma y = -12 ksi
  Tau xy  =   -6 ksi

Rotation of element
(+ counterclockwise)
  theta = -25 degrees

  Sigma x' =    9.02 ksi
  Sigma y' = -13.02 ksi
  Tau x'y' =    3.80 ksi
```

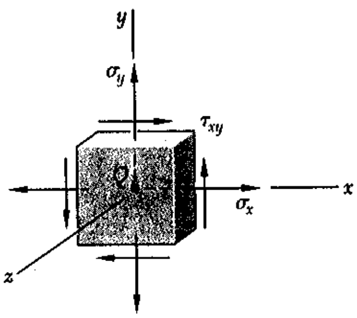
```
Problem 7.15b

  Sigma x =    8 ksi
  Sigma y = -12 ksi
  Tau xy  =   -6 ksi

Rotation of element
(+ counterclockwise)
  theta =  10 degrees

  Sigma x' =    5.34 ksi
  Sigma y' =   -9.34 ksi
  Tau x'y' =   -9.06 ksi
```




```
Problem 7.16a

  Sigma x =    0 ksi
  Sigma y =   16 ksi
  Tau xy  =   10 ksi

Rotation of element
(+ counterclockwise)
  theta = -25 degrees

  Sigma x' =  -4.80 ksi
  Sigma y' =  20.80 ksi
  Tau x'y' =   0.30 ksi
```

```
Problem 7.16b

  Sigma x =    0 ksi
  Sigma y =   16 ksi
  Tau xy  =   10 ksi

Rotation of element
(+ counterclockwise)
  theta =  10 degrees

  Sigma x' =    3.90 ksi
  Sigma y' =   12.10 ksi
  Tau x'y' =   12.13 ksi
```

PROBLEM 7.C2



7.C2 A state of plane stress is defined by the stress components σ_x, σ_y, and τ_{xy} associated with the element shown in Fig. P7.C1*a*. (*a*) Write a computer program that can be used to determine the principal axes, the principal stresses, the maximum in-plane shearing stress, and the maximum shearing stress. (*b*) Use this program to solve Probs. 7.7, 7.11, 7.66, and 7.67.

SOLUTION PROGRAM FOLLOWING EQUATIONS

EQ. (7.10) $\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2}$; $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

EQ. (7.14) $\sigma_{max} = \sigma_{ave} + R$

$\sigma_{min} = \sigma_{ave} - R$

EQ. (7.12) $\theta_p = \tan^{-1} \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

EQ. (7.15) $\theta_s = \tan^{-1} - \dfrac{\sigma_x - \sigma_y}{2\tau_{xy}}$

SHEARING STRESS IF $\sigma_{max} > 0$ and $\sigma_{min} < 0$:

THEN $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = R$

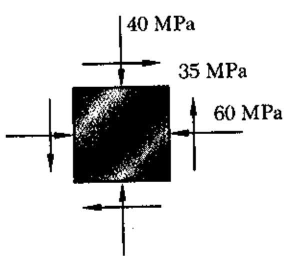
IF $\sigma_{max} > 0$ and $\sigma_{min} > 0$:

THEN $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}\sigma_{max}$

IF $\sigma_{max} < 0$ and $\sigma_{min} < 0$:

THEN $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}|\sigma_{min}|$

PROGRAM OUTPUT




```
Problems 7.7 AND 7.11

  Sigma x = -60.00 MPa
  Sigma y = -40.00 MPa
  Tau xy  =  35.00 MPa

Angle between xy axes and principal axes
           ( + counterclockwise )
      Theta p = -37.03 deg.  and  52.97 deg.
  Sigma max = -13.60 MPa
  Sigma min = -86.40 MPa
Angle between xy axis and planes of maximum in-plane shearing stress
           ( + counterclockwise )
      Theta s =   7.97 deg.  and  97.97 deg.
  Tau max (in plane) =  36.40 MPa
  Tau max    =  43.20 MPa
```

CONTINUED

PROBLEM 7.C2 - CONTINUED

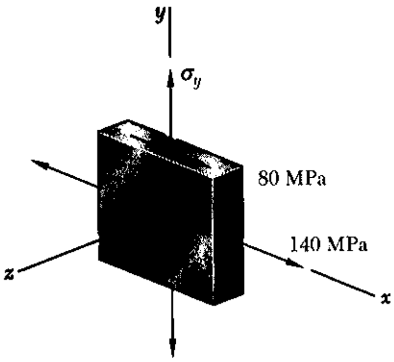


Fig. P7.66 and P7.67

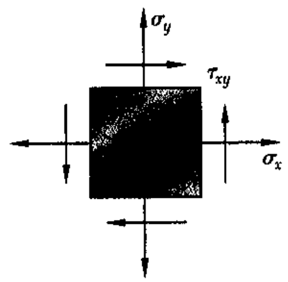
```
Problem 7.66a:  Sigma x = 140.00 MPa
                Sigma y =  20.00 MPa
                Tau xy  =  80.00 MPa

Angle between xy axes and principal axes
           ( + counterclockwise )
     Theta p =  26.57 deg.  and 116.57 deg.
  Sigma max = 180.00 MPa
  Sigma min = -20.00 MPa
Angle between xy axis and planes of maximum in-plane
    in-plane shearing stress ( + counterclockwise )
     Theta s =  71.57 deg.  and 161.57 deg.
  Tau max (in-plane)      = 100.00 MPa
  Tau max (out-of-plane)  = 100.00 MPa
```

```
Problem 7.66b:  Sigma x = 140.00 MPa
                Sigma y = 140.00 MPa
                Tau xy  =  80.00 MPa

Angle between xy axes and principal axes
           ( + counterclockwise )
     Theta p =  45.00 deg.  and 135.00 deg.
  Sigma max = 220.00 MPa
  Sigma min =  60.00 MPa
Angle between xy axis and planes of maximum in-plane
    in-plane shearing stress ( + counterclockwise )
     Theta s =  90.00 deg.  and 180.00 deg.
  Tau max (in-plane)      =  80.00 MPa
  Tau max (out-of-plane)  = 110.00 MPa
```

```
Problem 7.67a:  Sigma x = 140.00 MPa
                Sigma y =  40.00 MPa
                Tau xy  =  80.00 MPa

Angle between xy axes and principal axes
           ( + counterclockwise )
     Theta p =  29.00 deg.  and 119.00 deg.
  Sigma max = 184.34 MPa
  Sigma min =  -4.34 MPa
Angle between xy axis and planes of maximum in-plane
    in-plane shearing stress ( + counterclockwise )
     Theta s =  74.00 deg.  and 164.00 deg.
  Tau max (in-plane)      =  94.34 MPa
  Tau max (out-of-plane)  =  94.34 MPa
```

```
Problem 7.67b:  Sigma x = 140.00 MPa
                Sigma y = 120.00 MPa
                Tau xy  =  80.00 MPa

Angle between xy axes and principal axes
           ( + counterclockwise )
     Theta p =  41.44 deg.  and 131.44 deg.
  Sigma max = 210.62 MPa
  Sigma min =  49.38 MPa
Angle between xy axis and planes of maximum in-plane
    in-plane shearing stress ( + counterclockwise )
     Theta s =  86.44 deg.  and 176.44 deg.
  Tau max (in-plane)      =  80.62 MPa
  Tau max (out-of-plane)  = 105.31 MPa
```

PROBLEM 7.C3



7.C3 (*a*) Write a computer program that, for a given state of plane stress and a given yield strength of a ductile material, can be used to determine whether the material will yield. The program should use both the maximum-shearing-strength criterion and the maximum-distortion-energy criterion. It should also print the values of the principal stresses and, if the material does not yield, calculate the factor of safety. (*b*) Use this program to solve Probs. 7.81 through 7.84.

SOLUTION

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \quad ; \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R$$

MAXIMUM-SHEARING-STRESS CRITERION $\quad \tau_Y = \frac{1}{2}\sigma_Y$

IF σ_a AND σ_B HAVE SAME SIGN, $\tau_{max} = \frac{1}{2}\sigma_a$

IF $\tau_{max} > \tau_Y$, YIELDING OCCURS

IF $\tau_{max} < \tau_Y$, NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\tau_Y}{\tau_{max}}$$

MAXIMUM-DISTORTION-ENERGY CRITERION

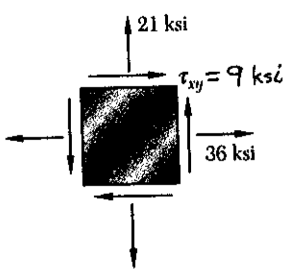
COMPUTE $\quad \text{RADICAL} = \sqrt{\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2}$

IF RADICAL $> \sigma_Y$, YIELDING OCCURS

IF RADICAL $< \sigma_Y$, NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\sigma_Y}{\text{RADICAL}}$$

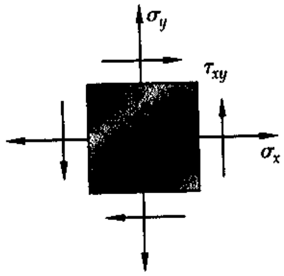
PROGRAM OUTPUT




```
Problems 7.81a and 7.82a     Sigma x =  36.00 ksi
                             Sigma y =  21.00 ksi
                             Tau xy  =   9.00 ksi
        Sigmax =  40.22 ksi
        Sigmin =  16.78 ksi
    Using the maximum-shearing-stress criterion:
      Material will not yield
      F.S. =  1.119
    Using the maximum-distortion-energy criterion:
      Material will not yield
      F.S. =  1.286
```

CONTINUED

PROBLEM 7.C4



7.C4 (*a*) Write a computer program based on Mohr's fracture criterion for brittle materials that, for a given state of plane stress and given values of the ultimate strength of the material in tension and in compression, can be used to determine whether rupture will occur. The program should also print the values of the principal stresses. (*b*) Use this program to solve Probs. 7.91 and 7.92 and to check the answers given for Probs. 7.93 and 7.94.

SOLUTION

<u>PRINCIPAL STRESSES</u>

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R$$

<u>MOHR'S FRACTURE CRITERION</u>

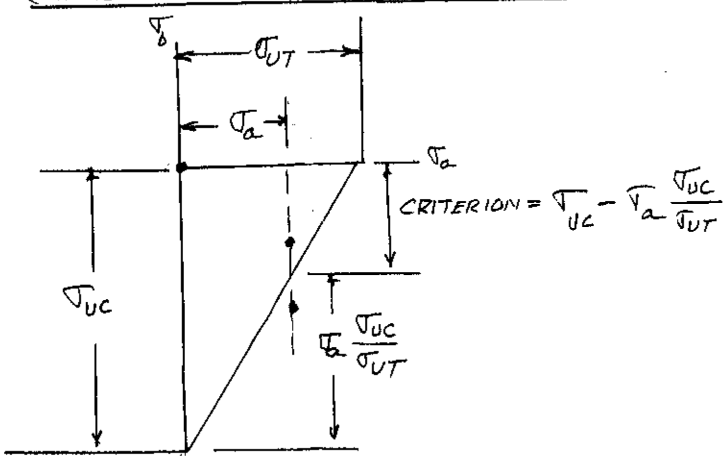
IF σ_a AND σ_b HAVE SAME SIGN, AND

$\sigma_a < \sigma_{UT}$ AND $\sigma_b < \sigma_{UC}$, NO FAILURE

$\sigma_a > \sigma_{UT}$ OR $\sigma_b > \sigma_{UC}$, FAILURE

IF $\sigma_a > 0$ AND $\sigma_b < 0$:

<u>CONSIDER FOURTH QUADRANT OF FIG. 7.47</u>



FOR NO RUPTURE TO OCCUR:
POINT (σ_a, σ_b) MUST LIE WITHIN MOHR'S ENVELOPE (FIG. 7.47)

IF $\sigma_b >$ CRITERION,
THEN <u>RUPTURE OCCURS</u>

IF $\sigma_b <$ CRITERION,
THEN <u>NO RUPTURE OCCURS</u>

<u>PROGRAM OUTPUT</u>

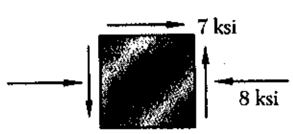
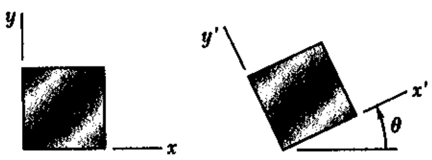


Fig. P7.91

```
Problem 7.91      Sigma x =  -8.00 ksi
                  Sigma y =   0.00 ksi
                   Tau xy =   7.00 ksi
Ultimate strength in tension      = 10 ksi
Ultimate strength in compression  = 30 ksi

  Sigma max = Sigma a    4.06 ksi
  Sigma min = Sigma b  -12.06 ksi
Rupture will not occur
```

CONTINUED

PROBLEM 7.C5



7.C5 A state of plane strain is defined by the strain components ϵ_x, ϵ_y, and γ_{xy} associated with the x and y axes. (*a*) Write a computer program that can be used to calculate the strain components $\epsilon_{x'}$, $\epsilon_{y'}$, and $\gamma_{x'y'}$ associated with the frame of reference $x'y'$ obtained by rotating the x and y axes through an angle θ. (*b*) Use this program to solve Probs. 7.126 through 7.129.

SOLUTION PROGRAM FOLLOWING EQUATIONS

EQ (7.44) $\varepsilon_x' = \dfrac{\varepsilon_x + \varepsilon_y}{2} + \dfrac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta$

EQ (7.45) $\varepsilon_y' = \dfrac{\varepsilon_x + \varepsilon_y}{2} - \dfrac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta$

EQ. (7.46) $\gamma_{xy}' = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$

ENTER ε_x, ε_y, γ_{xy}, AND θ

PRINT VALUES OBTAINED FOR ε_x', ε_y', AND γ_{xy}'

PROGRAM OUTPUT

```
Problem 7.126           Epsilon x = -720 micro meters
                        Epsilon y =    0 micro meters
                        Gamma xy  =  300 micro radians
Rotation of element,    in degrees   ( + counterclockwise )
                        Theta = -30 degrees

          Epsilon x'= -669.90 micro meters
          Epsilon y'=  -50.10 micro meters
          Gamma x'y'= -473.54 micro radians

Problem 7.127           Epsilon x =    0 micro meters
                        Epsilon y =  320 micro meters
                        Gamma xy  = -100 micro radians
Rotation of element,    in degrees   ( + counterclockwise )
                        Theta =  30 degrees

          Epsilon x'=   36.70 micro meters
          Epsilon y'=  283.30 micro meters
          Gamma x'y'=  227.13 micro radians

Problem 7.128           Epsilon x = -800 micro meters
                        Epsilon y =  450 micro meters
                        Gamma xy  =  200 micro radians
Rotation of element,    in degrees   ( + counterclockwise )
                        Theta = -25 degrees

          Epsilon x'= -653.35 micro meters
          Epsilon y'=  303.35 micro meters
          Gamma x'y'= -829.00 micro radians

Problem 7.129           Epsilon x = -500 micro meters
                        Epsilon y =  250 micro meters
                        Gamma xy  =    0 micro radians
Rotation of element,    in degrees   ( + counterclockwise )
                        Theta =  15 degrees

          Epsilon x'= -449.76 micro meters
          Epsilon y'=  199.76 micro meters
          Gamma x'y'=  375.00 micro radians
```

PROBLEM 7.C6

7.C6 A state of strain is defined by the strain components ϵ_x, ϵ_y, and γ_{xy} associated with the x and y axes. (a) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (b) Use this program to solve Probs. 7.134 through 7.137.

SOLUTION PROGRAM FOLLOWING EQUATIONS

EQ(7.50) $\quad \varepsilon_{ave} = \dfrac{\varepsilon_x + \varepsilon_y}{2} \qquad R = \sqrt{\left(\dfrac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\dfrac{\gamma_{xy}}{2}\right)^2}$

EQ.(7.51) $\quad \varepsilon_{max} = \varepsilon_{ave} + R \qquad \varepsilon_{min} = \varepsilon_{ave} - R$

EQ(7.52) $\quad \theta_p = \tan^{-1} \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{max(in\text{-}plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

LET $\varepsilon_a = \varepsilon_{max}$

$\varepsilon_b = \varepsilon_{min}$

CALCULATE $\varepsilon_c = -\dfrac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b)$

IF $\varepsilon_a > \varepsilon_b > \varepsilon_c$: $\gamma_{out\text{-}of\text{-}plane} = \varepsilon_a - \varepsilon_c$

IF $\varepsilon_a > \varepsilon_c > \varepsilon_b$: $\gamma_{out\text{-}of\text{-}plane} = \varepsilon_a - \varepsilon_b = 2R$

IF $\varepsilon_c > \varepsilon_a > \varepsilon_b$: $\gamma_{out\text{-}of\text{-}plane} = \varepsilon_c - \varepsilon_b$

PROGRAM PRINTOUT

```
Problem 7.134      Epsilon x =  160 micro meters
                   Epsilon y = -480 micro meters
                   Gamma xy  = -600 micro radians
                   nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
        Theta p    =  -21.58 degrees
        Epsilon a =  278.63 micro meters
        Epsilon b = -598.63 micro meters
        Epsilon c =  159.98 micro meters

        Gamma max (in plane) =  877.27 micro radians
        Gamma max =  877.27 micro radians
```

CONTINUED

PROBLEM 7.C6 - CONTINUED

```
Problem 7.135      Epsilon x = -260 micro meters
                   Epsilon y =  -60 micro meters
                   Gamma xy  =  480 micro radians
                   nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p    =  -33.69 degrees
       Epsilon a =  100.00 micro meters
       Epsilon b = -420.00 micro meters
       Epsilon c =  159.98 micro meters

       Gamma max (in plane) =  520.00 micro radians
       Gamma max =  579.98 micro radians


Problem 7.136      Epsilon x =  -40 micro meters
                   Epsilon y =  760 micro meters
                   Gamma xy  =  960 micro radians
                   nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p    =  -25.10 degrees
       Epsilon a =  984.82 micro meters
       Epsilon b = -264.82 micro meters
       Epsilon c = -359.95 micro meters

       Gamma max (in plane) = 1249.64 micro radians
       Gamma max = 1344.77 micro radians


Problem 7.137      Epsilon x = -300 micro meters
                   Epsilon y = -200 micro meters
                   Gamma xy  =  175 micro radians
                   nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p    =  -30.13 degrees
       Epsilon a = -149.22 micro meters
       Epsilon b = -350.78 micro meters
       Epsilon c =  250.00 micro meters

       Gamma max (in plane) =  201.56 micro radians
       Gamma max =  600.77 micro radians
```

PROBLEM 7.C7

7.C7 A state of plane strain is defined by the strain components ϵ_x, ϵ_y, and γ_{xy} measured at a point. (*a*) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (*b*) Use this program to solve Probs. 7.138 through 7.141.

SOLUTION

PROGRAM FOLLOWING EQUATIONS

EQ(7.50) $\quad \varepsilon_{ave} = \dfrac{\varepsilon_x + \varepsilon_y}{2} \qquad R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\dfrac{\gamma_{xy}}{2}\right)^2}$

EQ(7.51) $\quad \varepsilon_{max} = \varepsilon_{ave} + R \qquad \varepsilon_{min} = \varepsilon_{ave} - R$

EQ (7.52) $\quad \theta_p = \tan^{-1} \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{xy\,(in\text{-}plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

LET $\quad \varepsilon_a = \varepsilon_{max}$

$\varepsilon_b = \varepsilon_{min}$

$\varepsilon_c = 0$ (PLAIN STRAIN)

IF $\varepsilon_a > \varepsilon_b > \varepsilon_c$: $\gamma_{OUT\text{-}OF\text{-}PLANE} = \varepsilon_a - \varepsilon_c$

IF $\varepsilon_a > \varepsilon_c > \varepsilon_b$: $\gamma_{OUT\text{-}OF\text{-}PLANE} = \varepsilon_a - \varepsilon_b = 2R$

IF $\varepsilon_c > \varepsilon_a > \varepsilon_b$: $\gamma_{OUT\text{-}OF\text{-}PLANE} = \varepsilon_c - \varepsilon_b$

PROGRAM PRINTOUT

```
Problem 7.138        Epsilon x =  -90
                     Epsilon y = -130
                     Gamma xy  =  150

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =     37.53 and -52.47 degrees
       Epsilon a =    -32.38 micro meters at   37.53 degrees
       Epsilon b =   -187.62 micro meters at  -52.47 degrees
       Epsilon c =      0.00 micro meters

       Gamma max (in plane) =  155.24 micro radians
       Gamma max =  187.62 micro radians
```

CONTINUED

PROBLEM 7.C7 - CONTINUED

```
Problem 7.139        Epsilon x =  375
                     Epsilon y =   75
                     Gamma xy  =  125

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =    11.31 and -78.69 degrees
       Epsilon a =   387.50 micro meters at   11.31 degrees
       Epsilon b =    62.50 micro meters at  -78.69 degrees
       Epsilon c =     0.00 micro meters

       Gamma max (in plane) =  325.00 micro radians
       Gamma max =  387.50 micro radians


Problem 7.140        Epsilon x =  400
                     Epsilon y =  200
                     Gamma xy  =  375

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =    30.96 and -59.04 degrees
       Epsilon a =   512.50 micro meters at   30.96 degrees
       Epsilon b =    87.50 micro meters at  -59.04 degrees
       Epsilon c =     0.00 micro meters

       Gamma max (in plane) =  425.00 micro radians
       Gamma max =  512.50 micro radians


Problem 7.141        Epsilon x =   60
                     Epsilon y =  240
                     Gamma xy  =  -50

Angle between xy axes and principal axes (+ = counterclockwise)
       Theta p   =     7.76 and -82.24 degrees
       Epsilon a =   243.41 micro meters at    7.76 degrees
       Epsilon b =    56.59 micro meters at   97.76 degrees
       Epsilon c =     0.00 micro meters

       Gamma max (in plane) =  186.82 micro radians
       Gamma max =  243.41 micro radians
```

PROBLEM 7.C8

7.C8 A rosette consisting of three gages forming, respectively, angles θ_1, θ_2, and θ_3 with the x axis is attached to the free surface of a machine component made of a material with a given Poisson's ratio ν. (*a*) Write a computer program that, for given readings ϵ_1, ϵ_2, and ϵ_3 of the gages, can be used to calculate the strain components associated with the x and y axes and to determine the orientation and magnitude of the three principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (*b*) Use this program to solve Probs. 7.142 through 7.145.

SOLUTION

FOR $n = 1$ TO 3, ENTER θ_n and ϵ_n

ENTER: NU = ν

SOLVE EQS. (7.60) FOR ϵ_x, ϵ_y, AND γ_{xy} USING METHOD OF DETERMINATES OR ANY OTHER METHOD,

ENTER $\epsilon_{ave} = \dfrac{\epsilon_x + \epsilon_y}{2}$; $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \gamma_{xy}^2}$

$$\epsilon_a = \epsilon_{max} = \epsilon_{ave} + R$$

$$\epsilon_b = \epsilon_{min} = \epsilon_{avg} - R$$

$$\epsilon_c = -\frac{\nu}{1-\nu}(\epsilon_a + \epsilon_b)$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{max\,(in\text{-}plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN, AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN,

IF $\epsilon_c < \epsilon_b$: $\gamma_{OUT\text{-}OF\text{-}PLANE} = \epsilon_a - \epsilon_c$

IF $\epsilon_c > \epsilon_a$: $\gamma_{OUT\text{-}OF\text{-}PLANE} = \epsilon_c - \epsilon_b$

OTHERWISE: $\gamma_{OUT\text{-}OF\text{-}PLANE} = 2R$

PROGRAM OUTPUT

Problem 7.142

Gage	theta degrees	epsilon micro meters
1	30	600
2	-30	450
3	90	-75

```
Epsilon x =   725.000 micro meters
Epsilon y =   -75.000 micro meters
Gamma xy  =   173.205 micro radians

Epsilon a =   734.268 micro meters
Epsilon b =   -84.268 micro meters
Gamma max (in plane) =  818.535 micro radians
```

CONTINUED

Problem 7.143

Gage	theta degrees	epsilon in./in.
1	-15	720
2	30	-180
3	75	120

Epsilon x = 379.808 in./in.
Epsilon y = 460.192 in./in.
Gamma xy = -1339.230 micro radians

Epsilon a = 1090.820 in./in.
Epsilon b = -250.820 in./in.
Gamma max (in plane) = 1341.641 micro radians

Problem 7.144

OBSERVE THAT GAGE 3 IS ORIENTATED ALONG THE v AXIS. THEREFORE
ENTER θ_4 AND ϵ_4 AS θ_3 AND ϵ_3,
THE VALUE OF ϵ_y THAT IS OBTAINED
IS ALSO THE EXPECTED READING OF GAGE 3.

Gage	theta degrees	epsilon micro meters
1	0	420
2	45	-45
4 → ~~3~~	135	165

Epsilon x = 420.000 micro meters
Epsilon y = -300.000 micro meters
Gamma xy = -210.000 micro radians

Epsilon a = 435.000 micro meters
Epsilon b = -315.000 micro meters
Gamma max (in plane) = 750.000 micro radians

Problem 7.145

Gage	theta degrees	epsilon in./in.
1	45	-50
2	-45	360
3	0	315

Epsilon x = 315.000 in./in.
Epsilon y = -5.000 in./in.
Gamma xy = -410.000 micro radians

Epsilon a = 415.048 in./in.
Epsilon b = -105.048 in./in.
Gamma max (in plane) = 520.096 micro radians