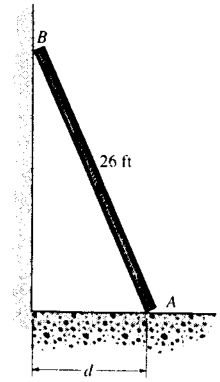
8-3. The uniform pole has a weight of 30 lb and a length of 26 ft. If it is placed against the smooth wall and on the rough floor in the position $d = 10$ ft, will it remain in this position when it is released? The coefficient of static friction is $\mu_s = 0.3$.



$\circlearrowleft + \Sigma M_A = 0;\quad 30(5) - N_B(24) = 0$

$N_B = 6.25$ lb

$\xrightarrow{+} \Sigma F_x = 0;\quad 6.25 - F_A = 0$

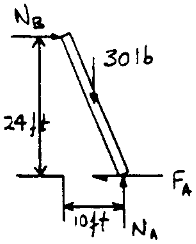
$F_A = 6.25$ lb

$+\uparrow \Sigma F_y = 0;\quad N_A - 30 = 0$

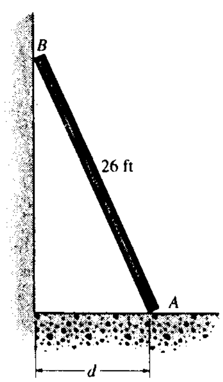
$N_A = 30$ lb

$(F_A)_{max} = 0.3(30) = 9\text{ lb} > 6.25\text{ lb}$

Yes, the pole will remain stationary. **Ans**



***8-4.** The uniform pole has a weight of 30 lb and a length of 26 ft. Determine the maximum distance d it can be placed from the smooth wall and not slip. The coefficient of static friction between the floor and the pole is $\mu_s = 0.3$.



$+\uparrow \Sigma F_y = 0;\quad N_A - 30 = 0$

$N_A = 30$ lb

$F_A = (F_A)_{max} = 0.3(30) = 9$ lb

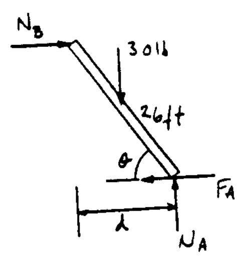
$\xrightarrow{+} \Sigma F_x = 0;\quad N_B - 9 = 0$

$N_B = 9$ lb

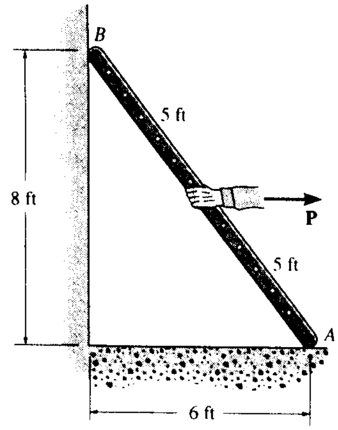
$\circlearrowleft + \Sigma M_A = 0;\quad 30(13\cos\theta) - 9(26\sin\theta) = 0$

$\theta = 59.04°$

$d = 26\cos 59.04° = 13.4$ ft **Ans**



8-5. The uniform 20-lb ladder rests on the rough floor for which the coefficient of static friction is $\mu_s = 0.8$ and against the smooth wall at B. Determine the horizontal force P the man must exert on the ladder in order to cause it to move.



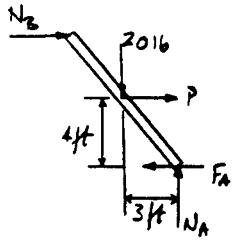
Assume that the ladder tips about A:

$N_B = 0;$

$\xrightarrow{+}\Sigma F_x = 0; \quad P - F_A = 0$

$+\uparrow \Sigma F_y = 0; \quad -20 + N_A = 0$

$N_A = 20 \text{ lb}$



$\circlearrowleft + \Sigma M_A = 0; \quad 20\,(3) - P\,(4) = 0$

$P = 15 \text{ lb}$

Thus

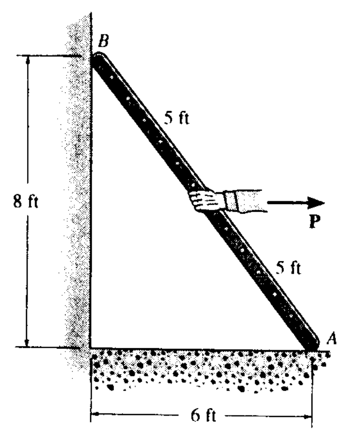
$F_A = 15 \text{ lb}$

$(F_A)_{max} = 0.8\,(20) = 16 \text{ lb} > 15 \text{ lb}$ OK

Ladder tips as assumed.

$P = 15 \text{ lb}$ **Ans**

8-6. The uniform 20-lb ladder rests on the rough floor for which the coefficient of static friction is $\mu_s = 0.4$ and against the smooth wall at B. Determine the horizontal force P the man must exert on the ladder in order to cause it to move.



Assume that the ladder slips at A:

$F_A = 0.4 N_A$

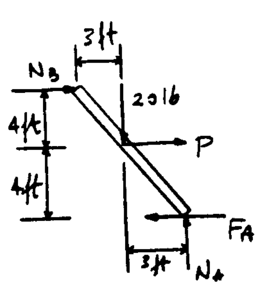
$+\uparrow \Sigma F_y = 0; \quad N_A - 20 = 0$

$N_A = 20 \text{ lb}$

$F_A = 0.4\,(20) = 8 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0; \quad P(4) - 20(3) + 20(6) - 8(8) = 0$

$P = 1 \text{ lb}$ **Ans**

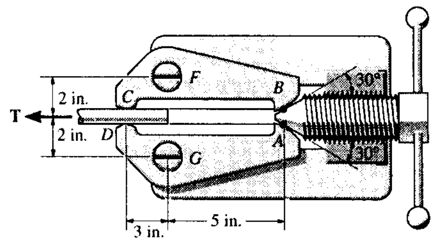


$\xrightarrow{+}\Sigma F_x = 0; \quad N_B + 1 - 8 = 0$

$N_B = 7 \text{ lb} > 0$ OK

The ladder will remain in contact with the wall.

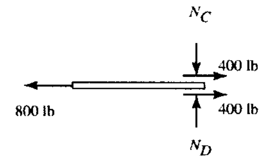
8-7. An axial force of $T = 800$ lb is applied to the bar. If the coefficient of static friction at the jaws C and D is $\mu_s = 0.5$, determine the smallest normal force that the screw at A must exert on the smooth surface of the links at B and C in order to hold the bar stationary. The links are pin-connected at F and G.

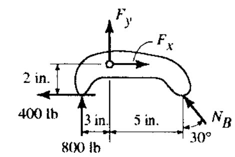


Require $F_C = \mu_s N_C$

$400 = 0.5\, N_C$

$N_C = 800$ lb **Ans**



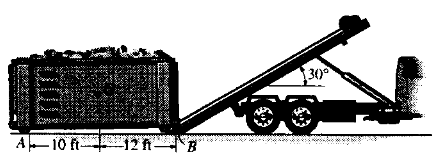


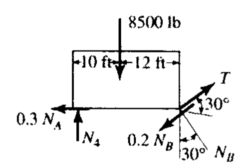
$\circlearrowleft + \Sigma M_F = 0; \quad -800(3) - 400(2) - (N_B \sin 30°)(2)$

$+ (N_B \cos 30°)(5) = 0$

$N_B = 961$ lb **Ans**

***8-8.** The winch on the truck is used to hoist the garbage bin onto the bed of the truck. If the loaded bin has a weight of 8500 lb and center of gravity at G, determine the force in the cable needed to begin the lift. The coefficients of static friction at A and B are $\mu_A = 0.3$ and $\mu_B = 0.2$, respectively. Neglect the height of the support at A.





$\circlearrowleft + \Sigma M_B = 0; \quad 8500(12) - N_A(22) = 0$

$N_A = 4636.364$ lb

$\xrightarrow{+} \Sigma F_x = 0; \quad T \cos 30°$

$- 0.2 N_B \cos 30° - N_B \sin 30° - 0.3(4636.364) = 0$

$T(0.86603) - 0.67321\, N_B = 1390.91$

$+\uparrow \Sigma F_y = 0; \quad 4636.364 - 8500 + T \sin 30° + N_B \cos 30°$

$- 0.2 N_B \sin 30° = 0$

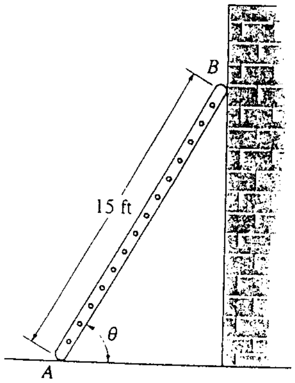
$T(0.5) + 0.766025\, N_B = 3863.636$

Solving;

$T = 3666.5$ lb $= 3.67$ kip **Ans**

$N_B = 2650.5$ lb

8-9. The 15-ft ladder has a uniform weight of 80 lb and rests against the smooth wall at B. If the coefficient of static friction at A is $\mu_A = 0.4$, determine if the ladder will slip. Take $\theta = 60°$.



$+\Sigma M_A = 0;\quad N_B(15\sin 60°) - 80(7.5)\cos 60° = 0$

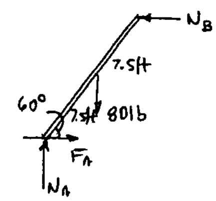
$N_B = 23.094$ lb

$\xrightarrow{+} \Sigma F_x = 0;\quad F_A = 23.094$ lb

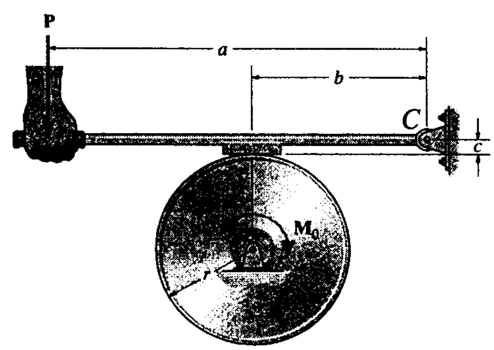
$+\uparrow \Sigma F_y = 0;\quad N_A = 80$ lb

$(F_A)_{max} = 0.4(80) = 32 \text{ lb} > 23.094 \text{ lb}$ **(O.K!)**

The ladder will not slip. **Ans**



8-10. The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment $\mathbf{M}_0$. If the coefficient of static friction between the wheel and the block is μ_s, determine the smallest force P that should be applied.



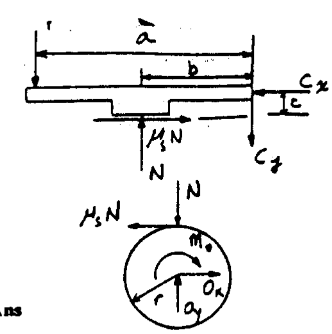
$\curvearrowleft + \Sigma M_C = 0;\quad Pa - Nb + \mu_s Nc = 0$

$$N = \frac{Pa}{(b - \mu_s c)}$$

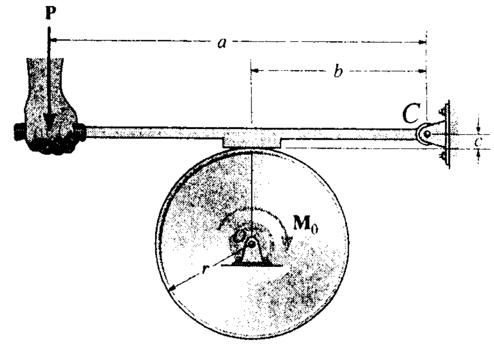
$\curvearrowleft + \Sigma M_O = 0;\quad \mu_s Nr - M_0 = 0$

$$\mu_s P\left(\frac{a}{b - \mu_s c}\right)r = M_0$$

$$P = \frac{M_0}{\mu_s ra}(b - \mu_s c) \quad \textbf{Ans}$$



8-11. Show that the brake in Prob. 8-10 is self locking, i.e., $P \leq 0$, provided $b/c \leq \mu_s$.

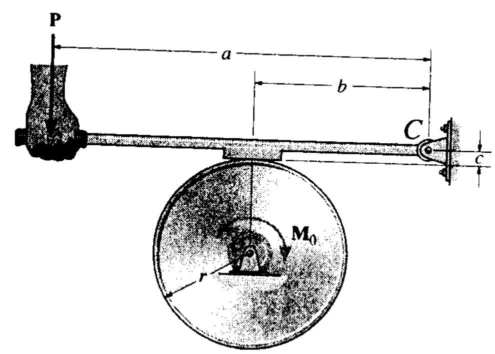


See solution to Prob. 8-10. Require $P \leq 0$. Then

$$b \leq \mu_s c$$

$$\mu_s \geq \frac{b}{c} \quad \textbf{Ans}$$

***8-12.** Solve Prob. 8-10 if the couple moment $\mathbf{M}_0$ is applied counterclockwise.



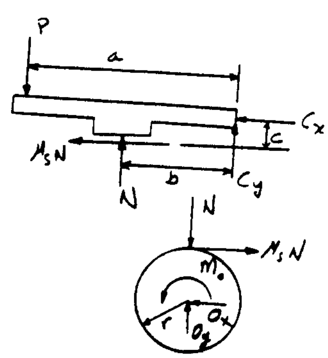
$\circlearrowleft + \Sigma M_C = 0;\quad Pa - Nb - \mu_s Nc = 0$

$$N = \frac{Pa}{(b + \mu_s c)}$$

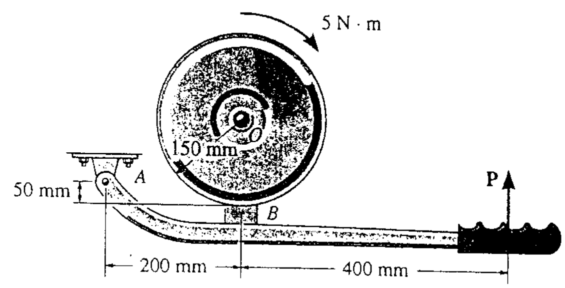
$\circlearrowleft + \Sigma M_O = 0;\quad \mu_s Nr - M_0 = 0$

$$\mu_s P\left(\frac{a}{b + \mu_s c}\right)r = M_0$$

$$P = \frac{M_0}{\mu_s ra}(b + \mu_s c) \quad \textbf{Ans}$$



8-13. The block brake consists of a pin-connected lever and friction block at B. The coefficient of static friction between the wheel and the lever is $\mu_s = 0.3$, and a torque of 5 N · m is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) $P = 30$ N, (b) $P = 70$ N.



To hold lever:

$\circlearrowleft + \Sigma M_O = 0; \quad F_B(0.15) - 5 = 0; \quad F_B = 33.333$ N

Require

$$N_B = \frac{33.333 \text{ N}}{0.3} = 111.1 \text{ N}$$

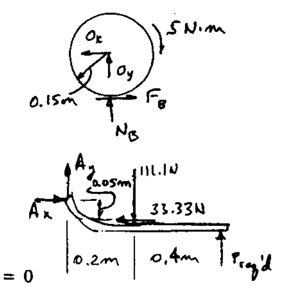
Lever,

$\circlearrowleft + \Sigma M_A = 0; \quad P_{Reqd.}(0.6) - 111.1(0.2) - 33.333(0.05) = 0$

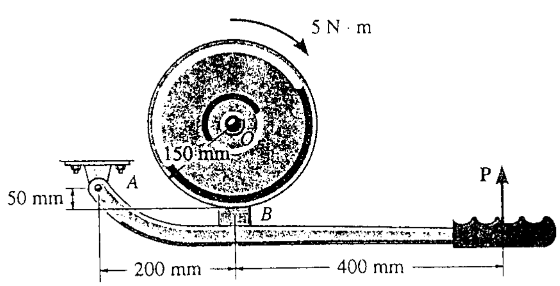
$P_{Reqd.} = 39.8$ N

a) $P = 30 \text{ N} < 39.8 \text{ N}$ No **Ans**

b) $P = 70 \text{ N} > 39.8 \text{ N}$ Yes **Ans**



8-14. Solve Prob. 8-13 if the 5-N · m torque is applied counter-clockwise.

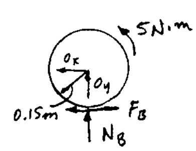


To hold lever:

$\circlearrowleft + \Sigma M_O = 0; \quad -F_B(0.15) + 5 = 0; \quad F_B = 33.333$ N

Require

$$N_B = \frac{33.333 \text{ N}}{0.3} = 111.1 \text{ N}$$



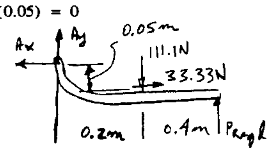
Lever,

$\circlearrowleft + \Sigma M_A = 0; \quad P_{Reqd.}(0.6) - 111.1(0.2) + 33.333(0.05) = 0$

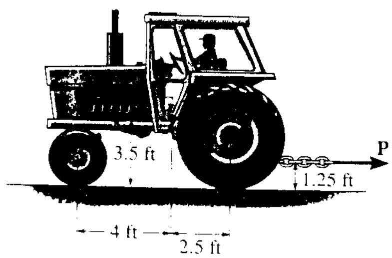
$P_{Reqd.} = 34.26$ N

a) $P = 30 \text{ N} < 34.26 \text{ N}$ No **Ans**

b) $P = 70 \text{ N} > 34.26 \text{ N}$ Yes **Ans**



8-15. The tractor has a weight of 4500 lb with center of gravity at G. The driving traction is developed at the rear wheels B, while the front wheels at A are free to roll. If the coefficient of static friction between the wheels at B and the ground is $\mu_s = 0.5$, determine if it is possible to pull at $P = 1200$ lb without causing the wheels at B to slip or the front wheels at A to lift off the ground.



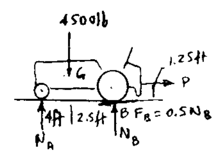
Slipping:

$\circlearrowleft + \Sigma M_A = 0; \quad -4500(4) - P(1.25) + N_B(6.5) = 0$

$\xrightarrow{+} \Sigma F_x = 0; \quad P = 0.5 N_B$

$P = 1531.9$ lb

$N_B = 3063.8$ lb



Tipping ($N_A = 0$)

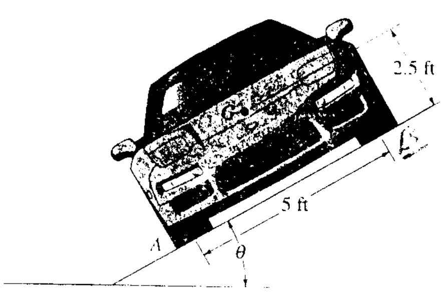
$\circlearrowleft + \Sigma M_B = 0; \quad -P(1.25) + 4500(2.5) = 0$

$P = 9000$ lb

Since $P_{Req'd} = 1200 \text{ lb} < 1531.9 \text{ lb}$

It is possible to pull the load without slipping or tipping. **Ans**

***8-16.** The car has a mass of 1.6 Mg and center of mass at G. If the coefficient of static friction between the shoulder of the road and the tires is $\mu_s = 0.4$, determine the greatest slope θ the shoulder can have without causing the car to slip or tip over if the car travels along the shoulder at constant velocity.

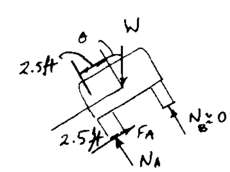


Tipping:

$\circlearrowleft + \Sigma M_A = 0; \quad -W\cos\theta(2.5) + W\sin\theta(2.5) = 0$

$\tan\theta = 1$

$\theta = 45°$



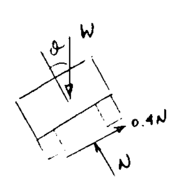
Slipping:

$\Sigma F_x = 0; \quad 0.4N - W\sin\theta = 0$

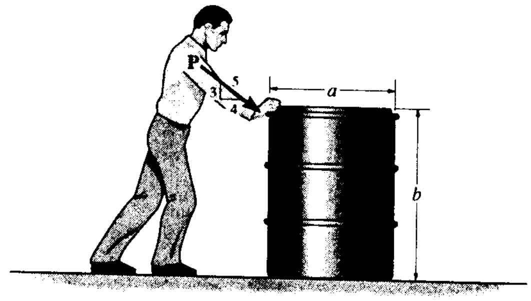
$\Sigma F_y = 0; \quad N - W\cos\theta = 0$

$\tan\theta = 0.4$

$\theta = 21.8°$ **Ans** **(car slips)**



8-17. The drum has a weight of 100 lb and rests on the floor for which the coefficient of static friction is $\mu_s = 0.6$. If $a = 2$ ft and $b = 3$ ft, determine the smallest magnitude of the force **P** that will cause impending motion of the drum.



Assume that the drum tips :

$x = 1$ ft

$\circlearrowleft + \Sigma M_O = 0; \quad 100(1) + P\left(\frac{3}{5}\right)(2) - P\left(\frac{4}{5}\right)(3) = 0$

$P = 83.3$ lb

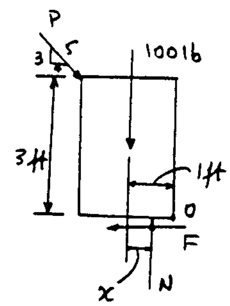
$\xrightarrow{+} \Sigma F_x = 0; \quad -F + 83.3\left(\frac{4}{5}\right) = 0$

$F = 66.7$ lb

$+\uparrow \Sigma F_y = 0; \quad N - 100 - 83.3\left(\frac{3}{5}\right) = 0$

$N = 150$ lb

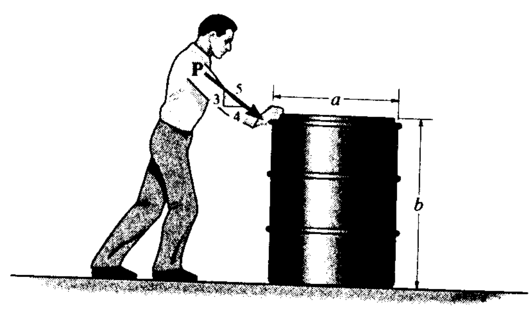
$F_{max} = 0.6(150) = 90 \text{ lb} > 66.7$ OK



Drum tips as assumed.

$P = 83.3$ lb **Ans**

8-18. The drum has a weight of 100 lb and rests on the floor for which the coefficient of static friction is $\mu_s = 0.5$. If $a = 3$ ft and $b = 4$ ft, determine the smallest magnitude of the force **P** that will cause impending motion of the drum.



Assume that the drum slips :

$F = 0.5N$

$\xrightarrow{+} \Sigma F_x = 0; \quad -0.5N + P\left(\frac{4}{5}\right) = 0$

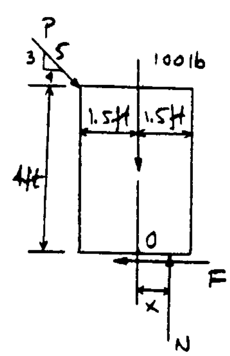
$+\uparrow \Sigma F_y = 0; \quad -P\left(\frac{3}{5}\right) - 100 + N = 0$

$P = 100$ lb

$N = 160$ lb

$\circlearrowleft + \Sigma M_O = 0; \quad 160(x) + 100\left(\frac{3}{5}\right)(1.5) - 100\left(\frac{4}{5}\right)(4) = 0$

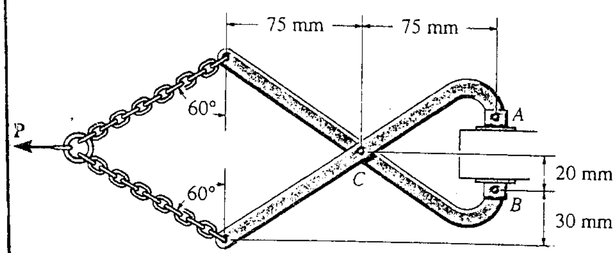
$x = 1.44 \text{ ft} < 1.5 \text{ ft}$ OK

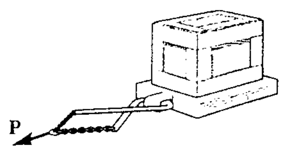


Drum slips as assumed.

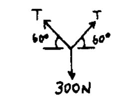
$P = 100$ lb **Ans**

8-19. The coefficient of static friction between the shoes at A and B of the tongs and the pallet is $\mu_s' = 0.5$, and between the pallet and the floor $\mu_s = 0.4$. If a horizontal towing force of $P = 300$ N is applied to the tongs, determine the largest mass that can be towed.





Chain:



$+\uparrow \Sigma F_y = 0;\quad 2T\sin 60° - 300 = 0$

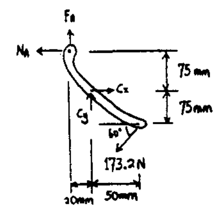
$T = 173.2$ N

Tongs:

$\circlearrowleft + \Sigma M_C = 0;\quad -173.2\cos 60°(75) - 173.2\sin 60°(50) + N_A(75) - F_A(20) = 0$

$F = \mu N;\quad F_A = 0.5 N_A$

$F_A = 107.7$ N



Crate:

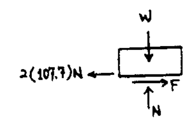
$\xrightarrow{+} \Sigma F_x = 0;\quad F = 2(107.7) = 215.3$ N

$F = \mu N;\quad F = 0.4N$

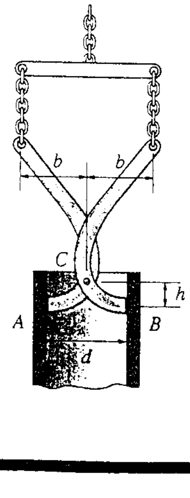
$N = 538.3$ N

$+\uparrow \Sigma F_y = 0;\quad W = 538.3$ N

$m = \dfrac{538.3}{9.81} = 54.9$ kg **Ans**



***8-20.** The pipe is hoisted using the tongs. If the coefficient of static friction at A and B is μ_s, determine the smallest dimension b so that any pipe of inner diameter d can be lifted.

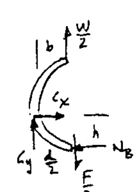


Require:

$F_B = \dfrac{W}{2} \le \mu_s N_B$

$\circlearrowleft + \Sigma M_C = 0;\quad -\dfrac{W}{2}\left(\dfrac{d}{2}\right) - N_A(h) + b\left(\dfrac{W}{2}\right) = 0$

$N_B = \dfrac{W}{2h}\left(b - \dfrac{d}{2}\right)$



Thus,

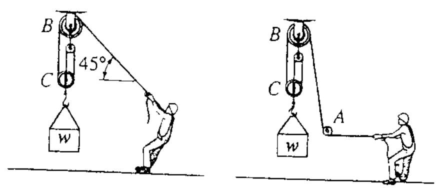
$\dfrac{W}{2} \le \dfrac{\mu_s W}{2h}\left(b - \dfrac{d}{2}\right)$

$h \le \left(b - \dfrac{d}{2}\right)\mu_s$

$b \ge \dfrac{h}{\mu_s} + \dfrac{d}{2}$

$b = \dfrac{h}{\mu_s} + \dfrac{d}{2}$ **Ans**

8-21. Determine the maximum weight W the man can lift with constant velocity using the pulley system, without and then with the "leading block" or pulley at A. The man has a weight of 200 lb and the coefficient of static friction between his feet and the ground is $\mu_s = 0.6$.



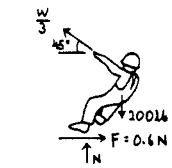
(a) (b)

(a)

$+\uparrow \Sigma F_y = 0;\quad \frac{W}{3}\sin 45° + N - 200 = 0$

$\xrightarrow{+} \Sigma F_x = 0;\quad -\frac{W}{3}\cos 45° + 0.6N = 0$

$W = 318 \text{ lb}$ **Ans**

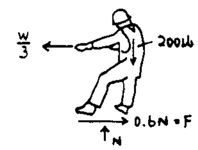


(b)

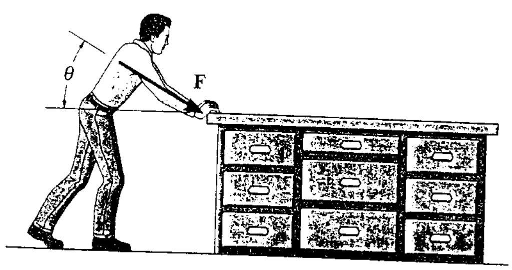
$+\uparrow \Sigma F_y = 0;\quad N = 200 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad 0.6(200) = \frac{W}{3}$

$W = 360 \text{ lb}$ **Ans**



8-22. The uniform dresser has a weight of 90 lb and rests on a tile floor for which $\mu_s = 0.25$. If the man pushes on it in the horizontal direction $\theta = 0°$, determine the smallest magnitude of force **F** needed to move the dresser. Also, if the man has a weight of 150 lb, determine the smallest coefficient of static friction between his shoes and the floor so that he does not slip.



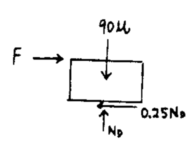
Dresser :

$+\uparrow \Sigma F_y = 0;\quad N_D - 90 = 0$

$N_D = 90 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad F - 0.25(90) = 0$

$F = 22.5 \text{ lb}$ **Ans**



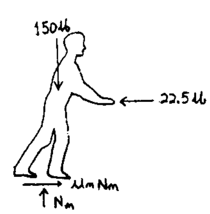
Man :

$+\uparrow \Sigma F_y = 0;\quad N_m - 150 = 0$

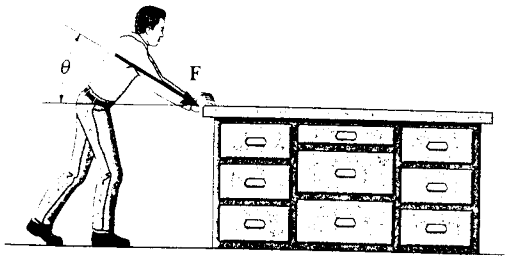
$N_m = 150 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad -22.5 + \mu_m(150) = 0$

$\mu_m = 0.15$ **Ans**



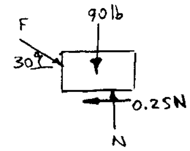
8-23. The uniform dresser has a weight of 90 lb and rests on a tile floor for which $\mu_s = 0.25$. If the man pushes on it in the direction $\theta = 30°$, determine the smallest magnitude of force **F** needed to move the dresser. Also, if the man has a weight of 150 lb, determine the smallest coefficient of static friction between his shoes and the floor so that he does not slip.



Dresser:

$+\uparrow \Sigma F_y = 0; \quad N - 90 - F\sin 30° = 0$

$\xrightarrow{+} \Sigma F_x = 0; \quad F\cos 30° - 0.25N = 0$



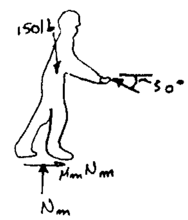
$N = 105.1 \text{ lb}$

$F = 30.363 \text{ lb} = 30.4 \text{ lb}$ **Ans**

Man:

$+\uparrow \Sigma F_y = 0; \quad N_m - 150 + 30.363\sin 30° = 0$

$\xrightarrow{+} \Sigma F_x = 0; \quad F_m - 30.363\cos 30° = 0$

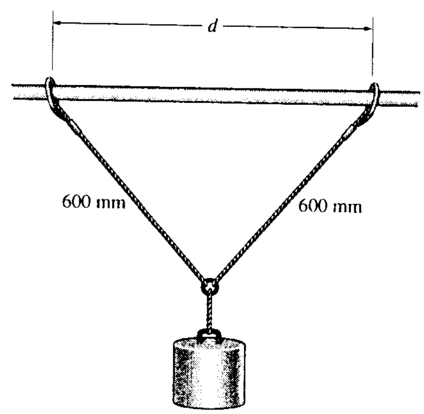


$N_m = 134.82 \text{ lb}$

$F_m = 26.295 \text{ lb}$

$$\mu_m = \frac{F_m}{N_m} = \frac{26.295}{134.82} = 0.195$$ **Ans**

***8-24.** The 5-kg cylinder is suspended from two equal-length cords. The end of each cord is attached to a ring of negligible mass, which passes along a horizontal shaft. If the coefficient of static friction between each ring and the shaft is $\mu_s = 0.5$, determine the greatest distance d by which the rings can be separated and still support the cylinder.

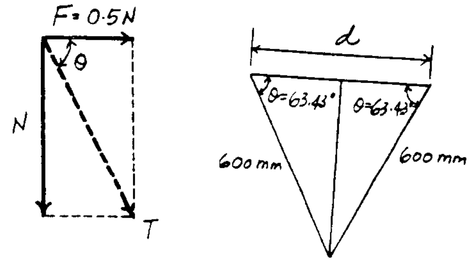


Friction : **When the ring is on the verge to sliding along the rod, slipping will have to occur. Hence, $F = \mu N = 0.5N$. From the force diagram (T is the tension developed by the cord)**

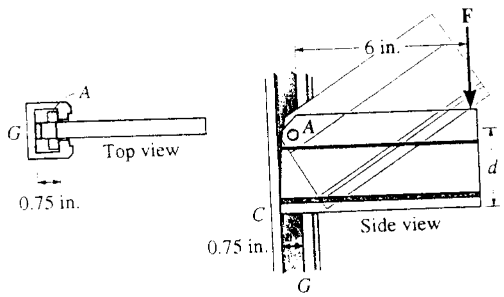
$$\tan\theta = \frac{N}{0.5N} = 2 \qquad \theta = 63.43°$$

Geometry :

$d = 2(600\cos 63.43°) = 537 \text{ mm}$ **Ans**



8-25. The board can be adjusted vertically by tilting it up and sliding the smooth pin A along the vertical guide G. When placed horizontally, the bottom C then bears along the edge of the guide, where $\mu_s = 0.4$. Determine the largest dimension d which will support any applied force $\mathbf{F}$ without causing the board to slip downward.

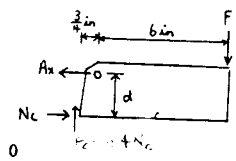


$+\uparrow \Sigma F_y = 0;\qquad 0.4N_C - F = 0$

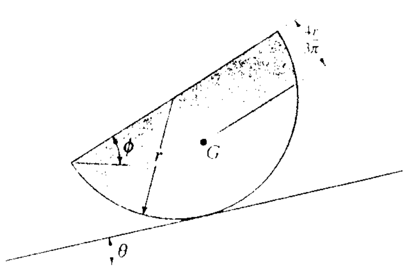
$\circlearrowleft + \Sigma M_A = 0;\qquad -F(6) + d(N_C) - 0.4N_C(0.75) = 0$

Thus, $\qquad -0.4N_C(6) + d(N_C) - 0.4N_C(0.75) = 0$

$d = 2.70$ in. **Ans**



8-26. The homogeneous semicylinder has a mass m and mass center at G. Determine the largest angle θ of the inclined plane upon which it rests so that it does not slip down the plane. The coefficient of static friction between the plane and the cylinder is $\mu_s = 0.3$. Also, what is the angle ϕ for this case?



The semi cylinder is a two - force member :

Since $F = \mu N$

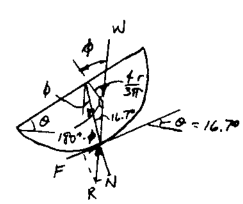
$\tan\theta = \dfrac{\mu N}{N} = \mu$

$\theta = \tan^{-1} 0.3 = 16.7°$ **Ans**

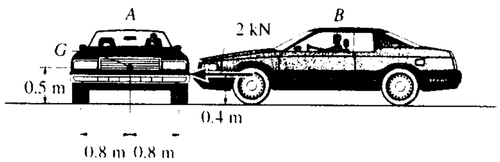
$\dfrac{r}{\sin(180° - \phi)} = \dfrac{\frac{4r}{3\pi}}{\sin 16.7°}$

$0.6771 = \sin\phi$

$\phi = 42.6°$ **Ans**



8-27. Car A has a mass of 1.4 Mg and mass center at G. If car B exerts a horizontal force on A of 2 kN, determine if this force is great enough to move car A. The coefficients of static and kinetic friction between the tires and the road are $\mu_s = 0.5$ and $\mu_k = 0.35$. Assume B's bumper is smooth.



Slipping :

$\xrightarrow{+} \Sigma F_x = 0; \quad F - 2 = 0$

$F = 2$ kN

$+\uparrow \Sigma F_y = 0; \quad N_A = 13.734$ kN

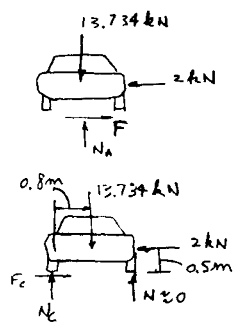
$F_{max} = 0.5(13.734) = 6.867 \text{ kN} > 2 \text{ kN}$

Tipping :

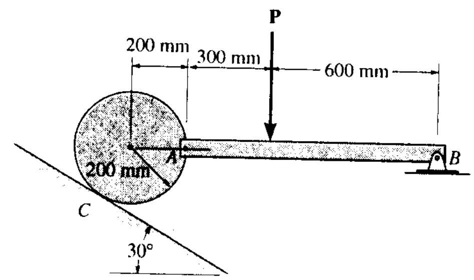
$\curvearrowleft + \Sigma M_C = 0; \quad 2(0.5) < 13.734(0.8)$

$1 < 10.99$

Therefore car A will not move. **Ans**



***8-28.** A 35-kg disk rests on an inclined surface for which $\mu_s = 0.2$. Determine the maximum vertical force **P** that may be applied to link AB without causing the disk to slip at C.



Equations of Equilibrium : From FBD (a),

$$\curvearrowleft + \Sigma M_B = 0; \quad P(600) - A_y(900) = 0 \quad A_y = 0.6667P$$

From FBD (b),

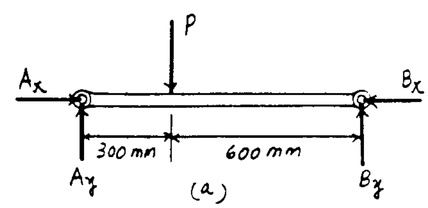
$$+\uparrow \Sigma F_y = 0 \quad N_C \sin 60° - F_C \sin 30° - 0.6667P - 343.35 = 0 \quad [1]$$

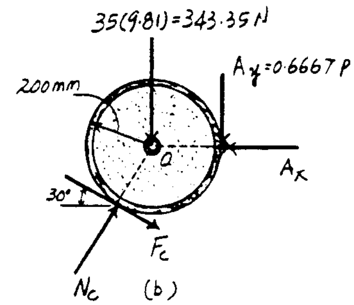
$$\curvearrowleft + \Sigma M_O = 0; \quad F_C(200) - 0.6667P(200) = 0 \quad [2]$$

Friction : If the disk is on the verge of moving, slipping would have to occur at point C. Hence, $F_C = \mu_s N_C = 0.2N_C$. Substituting this value into Eqs.[1] and [2] and solving, we have

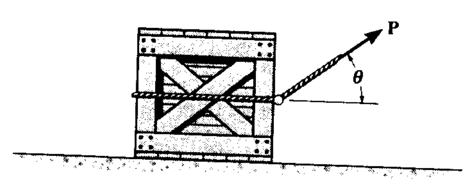
$P = 182$ N **Ans**

$N_C = 606.60$ N





8-29. The crate has a W and the coefficient of static friction at the surface is $\mu_s = 0.3$. Determine the orientation of the cord and the smallest possible force **P** that has to be applied to the cord so that the crate is on the verge of moving.



Equations of Equilibrium :

$$+\uparrow \Sigma F_y = 0; \quad N + P\sin\theta - W = 0 \quad [1]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P\cos\theta - F = 0 \quad [2]$$

Friction : If the crate is on the verge of moving, slipping will have to occur. Hence, $F = \mu_s N = 0.3N$. Substituting this value into Eqs.[1] and [2] and solving, we have

$$P = \frac{0.3W}{\cos\theta + 0.3\sin\theta} \qquad N = \frac{W\cos\theta}{\cos\theta + 0.3\sin\theta}$$

In order to obtain the minimum P, $\frac{dP}{d\theta} = 0$.

$$\frac{dP}{d\theta} = 0.3W\left[\frac{\sin\theta - 0.3\cos\theta}{(\cos\theta + 0.3\sin\theta)^2}\right] = 0$$

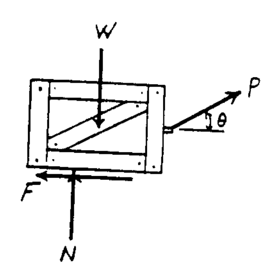
$$\sin\theta - 0.3\cos\theta = 0$$

$$\theta = 16.70° = 16.7° \quad \textbf{Ans}$$

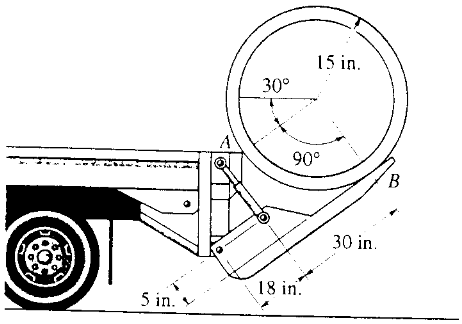
$$\frac{d^2P}{d\theta^2} = 0.3W\left[\frac{(\cos\theta + 0.3\sin\theta)^2 + 2(\sin\theta - 0.3\cos\theta)^2}{(\cos\theta + 0.3\sin\theta)^3}\right]$$

At $\theta = 16.70°$, $\frac{d^2P}{d\theta^2} = 0.2873W > 0$. Thus, $\theta = 16.70°$ will result in a minimun P.

$$P = \frac{0.3W}{\cos 16.70° + 0.3\sin 16.70°} = 0.287W \quad \textbf{Ans}$$



8-30. The 800-lb concrete pipe is being lowered from the truck bed when it is in the position shown. If the coefficient of static friction at the points of support A and B is $\mu_s = 0.4$, determine where it begins to slip first: at A or B, or both at A and B.

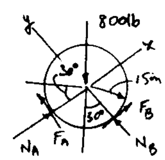


$\Sigma F_x = 0;\quad N_A + F_B - 800\sin 30° = 0$

$\Sigma F_y = 0;\quad F_A + N_B - 800\cos 30° = 0$

$\circlearrowleft +\Sigma M_O = 0;\quad F_B(15) - F_A(15) = 0$

$F_A = F_B$



Assume slipping at A:

$F_A = 0.4\,N_A$

Thus,

$N_A = 285.71$ lb

$N_B = 578.53$ lb

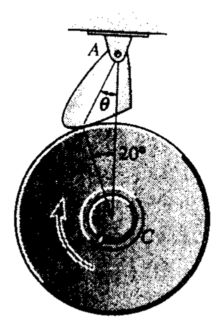
$F_A = F_B = 114.29$ lb

At B:

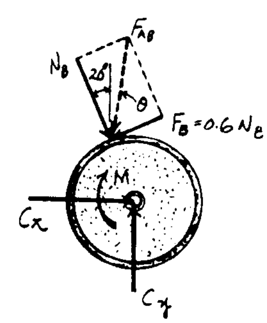
$(F_B)_{max} = 0.4\,N_B = 0.4(578.53) = 231.4 \text{ lb} > 114.29 \text{ lb}$ **(O.K!)**

Thus, slipping occurs at A. **Ans**

8-31. The friction pawl is pinned at A and rests against the wheel at B. It allows freedom of movement when the wheel is rotating counterclockwise about C. Clockwise rotation is prevented due to friction of the pawl which tends to bind the wheel. If $(\mu_s)_B = 0.6$, determine the design angle θ which will prevent clockwise motion for any value of applied moment M. *Hint:* Neglect the weight of the pawl so that it becomes a two-force member.



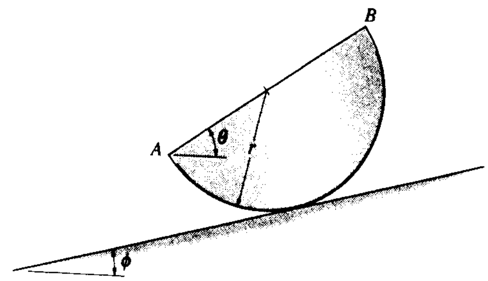
Friction : **When the wheel is on the verge of rotating, slipping would have to occur. Hence, $F_B = \mu N_B = 0.6N_B$. From the force diagram (F_{AB} is the force developed in the two force member AB)**



$$\tan(20° + \theta) = \frac{0.6N_B}{N_B} = 0.6$$

$$\theta = 11.0°$$ **Ans**

***8-32.** The semicylinder of mass m and radius r lies on the rough inclined plane for which $\phi = 10°$ and the coefficient of static friction is $\mu_s = 0.3$. Determine if the semicylinder slides down the plane, and if not, find the angle of tip θ of its base AB.

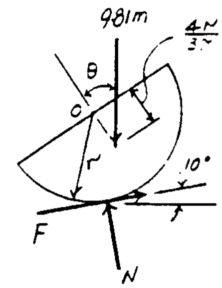


Equations of Equilibrium :

$\curvearrowleft + \Sigma M_O = 0;\quad F(r) - 9.81m\sin\theta\left(\frac{4r}{3\pi}\right) = 0 \qquad [1]$

$\xrightarrow{+} \Sigma F_x = 0;\quad F\cos 10° - N\sin 10° = 0 \qquad [2]$

$+\uparrow \Sigma F_y = 0\quad F\sin 10° + N\cos 10° - 9.81m = 0 \qquad [3]$

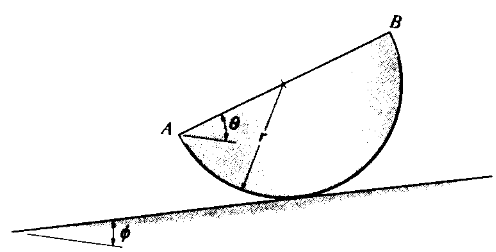


Solving Eqs.[1], [2] and [3] yields

$$N = 9.661m \quad F = 1.703m$$
$$\theta = 24.2° \qquad \textbf{Ans}$$

Friction : The maximum friction force that can be developed between the semicylinder and the inclined plane is $(F)_{max} = \mu N = 0.3(9.661m) = 2.898m$. Since $F_{max} > F = 1.703m$, **the semicylinder will not slide down the plane.** **Ans**

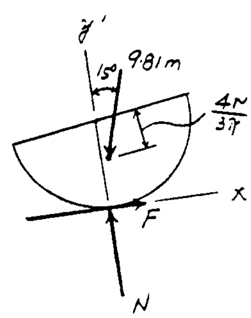
8-33. The semicylinder of mass m and radius r lies on the rough inclined plane. If the inclination $\phi = 15°$, determine the smallest coefficient of static friction which will prevent the semicylinder from slipping.



Equations of Equilibrium :

$\nearrow + \Sigma F_{x'} = 0;\quad F - 9.81m\sin 15° = 0 \quad F = 2.539m$

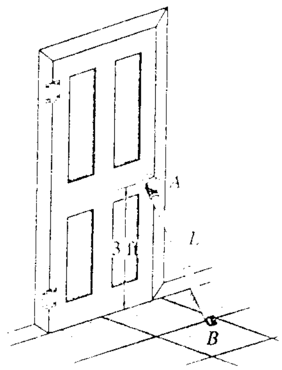
$\nwarrow + \Sigma F_{y'} = 0;\quad N - 9.81m\cos 15° = 0 \quad N = 9.476m$



Friction : If the semicylinder is on the verge of moving, slipping would have to occur. Hence,

$$F = \mu_s N$$
$$2.539m = \mu_s(9.476m)$$
$$\mu_s = 0.268 \qquad \textbf{Ans}$$

8-34. The door brace AB is to be designed to prevent opening the door. If the brace forms a pin connection under the doorknob and the coefficient of static friction with the floor is $\mu_s = 0.5$, determine the largest length L the brace can have to prevent the door from being opened. Neglect the weight of the brace.

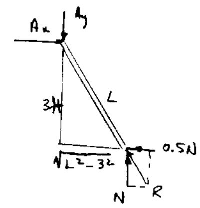


The brace is a two - force menber.

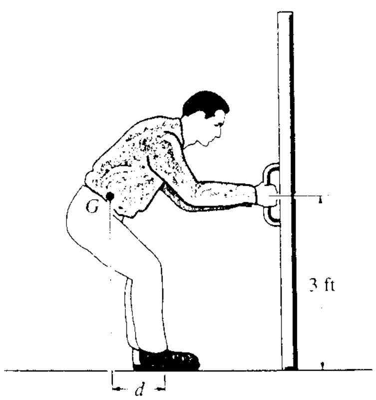
$$\frac{0.5\,N}{N} = \frac{\sqrt{L^2 - (3)^2}}{3}$$

$$1.5 = \sqrt{L^2 - (3)^2}$$

$L = 3.35$ ft **Ans**



8-35. The man has a weight of 200 lb, and the coefficient of static friction between his shoes and the floor is $\mu_s = 0.5$. Determine where he should position his center of gravity G at d in order to exert the maximum horizontal force on the door. What is this force?

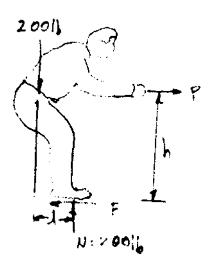


$F_{max} = 0.5\,N = 0.5(200) = 100$ lb

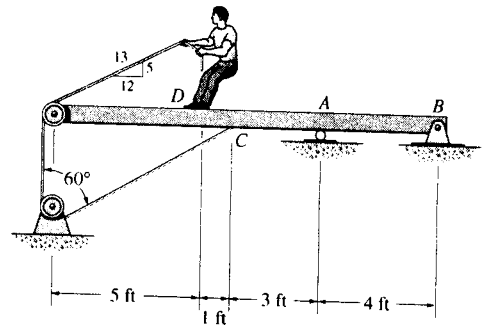
$\xrightarrow{+} \Sigma F_x = 0; \quad P - 100 = 0; \quad P = 100$ lb **Ans**

$\circlearrowleft + \Sigma M_O = 0; \quad 200(d) - 100(3) = 0$

$d = 1.50$ ft **Ans**



***8-36.** The 80-lb boy stands on the beam and pulls on the cord with a force large enough to just cause him to slip. If $(\mu_s)_D = 0.4$ between his shoes and the beam, determine the reactions at A and B. The beam is uniform and has a weight of 100 lb. Neglect the size of the pulleys and the thickness of the beam.



Equations of Equilibrium and Friction : When the boy is on the verge to slipping, then $F_D = (\mu_s)_D N_D = 0.4N_D$. From FBD (a),

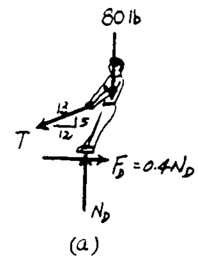
$$+\uparrow \Sigma F_y = 0; \quad N_D - T\left(\frac{5}{13}\right) - 80 = 0 \qquad [1]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.4N_D - T\left(\frac{12}{13}\right) = 0 \qquad [2]$$

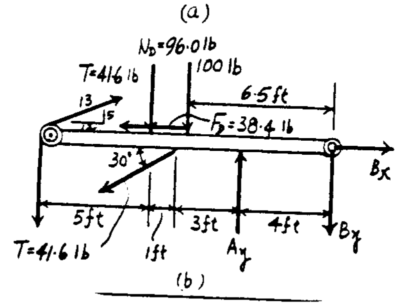
Solving Eqs.[1] and [2] yields

$$T = 41.6 \text{ lb} \qquad N_D = 96.0 \text{ lb}$$

Hence, $F_D = 0.4(96.0) = 38.4$ lb. From FBD (b),



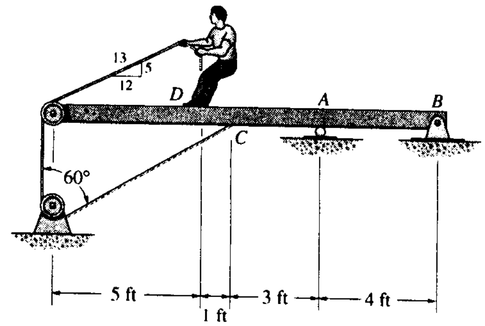
$$\circlearrowleft + \Sigma M_B = 0; \quad 100(6.5) + 96.0(8) - 41.6\left(\frac{5}{13}\right)(13)$$
$$+ 41.6(13) + 41.6\sin 30°(7) - A_y(4) = 0$$
$$A_y = 474.1 \text{ lb} = 474 \text{ lb} \qquad \textbf{Ans}$$



$$\xrightarrow{+} \Sigma F_x = 0; \quad B_x + 41.6\left(\frac{12}{13}\right) - 38.4 - 41.6\cos 30° = 0$$
$$B_x = 36.0 \text{ lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_x = 0; \quad 474.1 + 41.6\left(\frac{5}{13}\right) - 41.6$$
$$- 41.6\sin 30° - 96.0 - 100 - B_y = 0$$
$$B_y = 231.7 \text{ lb} = 232 \text{ lb} \qquad \textbf{Ans}$$

8-37. The 80-lb boy stands on the beam and pulls with a force of 40 lb. If $(\mu_s)_D = 0.4$, determine the frictional force between his shoes and the beam and the reactions at A and B. The beam is uniform and has a weight of 100 lb. Neglect the size of the pulleys and the thickness of the beam.



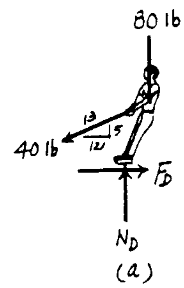
Equations of Equilibrium and Friction : From FBD (a),

$$+\uparrow \Sigma F_y = 0; \quad N_D - 40\left(\frac{5}{13}\right) - 80 = 0 \quad N_D = 95.38 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_D - 40\left(\frac{12}{13}\right) = 0 \quad F_D = 36.92 \text{ lb}$$

Since $(F_D)_{max} = (\mu_s)N_D = 0.4(95.38) = 38.15 \text{ lb} > F_D$, then the boy does not slip. Therefore, the friction force developed is

$$F_D = 36.92 \text{ lb} = 36.9 \text{ lb} \qquad \textbf{Ans}$$



From FBD (b),

$$\circlearrowleft + \Sigma M_B = 0; \quad 100(6.5) + 95.38(8) - 40\left(\frac{5}{13}\right)(13) + 40(13) + 40\sin 30°(7) - A_y(4) = 0$$

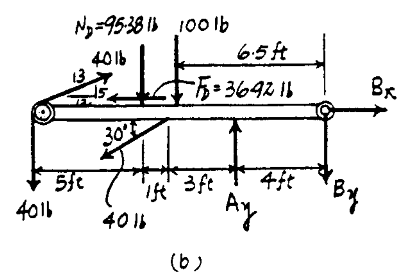
$$A_y = 468.27 \text{ lb} = 468 \text{ lb} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad B_x + 40\left(\frac{12}{13}\right) - 36.92 - 40\cos 30° = 0$$

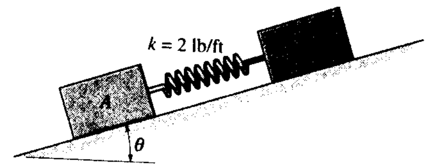
$$B_x = 34.64 \text{ lb} = 34.6 \text{ lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_x = 0; \quad 468.27 + 40\left(\frac{5}{13}\right) - 40 - 40\sin 30° - 95.38 - 100 - B_y = 0$$

$$B_y = 228.27 \text{ lb} = 228 \text{ lb} \qquad \textbf{Ans}$$



8-38. Two blocks A and B have a weight of 10 lb and 6 lb, respectively. They are resting on the incline for which the coefficients of static friction are $\mu_A = 0.15$ and $\mu_B = 0.25$. Determine the incline angle θ for which both blocks begin to slide. Also find the required stretch or compression in the connecting spring for this to occur. The spring has a stiffness of $k = 2$ lb/ft.



Equations of Equilibrium : Using the spring force formula, $F_{sp} = kx = 2x$. From FBD (a),

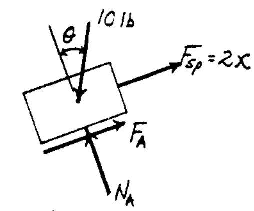
$$+\nearrow \Sigma F_{x'} = 0; \quad 2x + F_A - 10\sin\theta = 0 \qquad [1]$$

$$\nwarrow + \Sigma F_{y'} = 0; \quad N_A - 10\cos\theta = 0 \qquad [2]$$

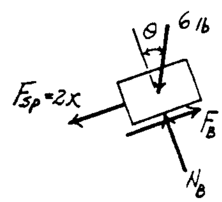
From FBD (b),

$$+\nearrow \Sigma F_{x'} = 0; \quad F_B - 2x - 6\sin\theta = 0 \qquad [3]$$

$$\nwarrow + \Sigma F_{y'} = 0; \quad N_B - 6\cos\theta = 0 \qquad [4]$$



(a)

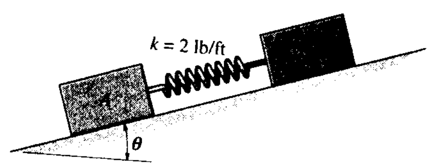


(b)

Friction : If block A and B are on the verge to move, slipping would have to occur at point A and B. Hence, $F_A = \mu_{sA} N_A = 0.15 N_A$ and $F_B = \mu_{sB} N_B = 0.25 N_B$. Substituting these values into Eqs.[1], [2], [3] and [4] and solving, we have

$$\theta = 10.6^\circ \quad x = 0.184 \text{ ft} \qquad \textbf{Ans}$$
$$N_A = 9.829 \text{ lb} \quad N_B = 5.897 \text{ lb}$$

8-39. Two blocks A and B have a weight of 10 lb and 6 lb, respectively. They are resting on the incline for which the coefficients of static friction are $\mu_A = 0.15$ and $\mu_B = 0.25$. Determine the angle θ which will cause motion of one of the blocks. What is the friction force under each of the blocks when this occurs? The spring has a stiffness of $k = 2$ lb/ft and is originally unstretched.



Equations of Equilibrium : Since Block A and B is either not moving or on the verge of moving, the spring force $F_{sp} = 0$. From FBD (a),

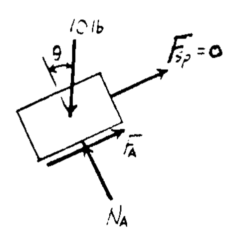
$$+\nearrow \Sigma F_{x'} = 0; \quad F_A - 10\sin\theta = 0 \qquad [1]$$

$$\nwarrow + \Sigma F_{y'} = 0; \quad N_A - 10\cos\theta = 0 \qquad [2]$$

From FBD (b),

$$+\nearrow \Sigma F_{x'} = 0; \quad F_B - 6\sin\theta = 0 \qquad [3]$$

$$\nwarrow + \Sigma F_{y'} = 0; \quad N_B - 6\cos\theta = 0 \qquad [4]$$



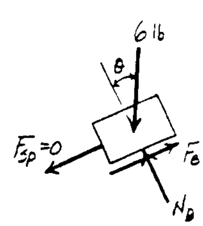
(a)

Friction : Assuming block A is on the verge of slipping, then

$$F_A = \mu_{sA} N_A = 0.15 N_A \qquad [5]$$

Solving Eqs.[1], [2], [3], [4] and [5] yields

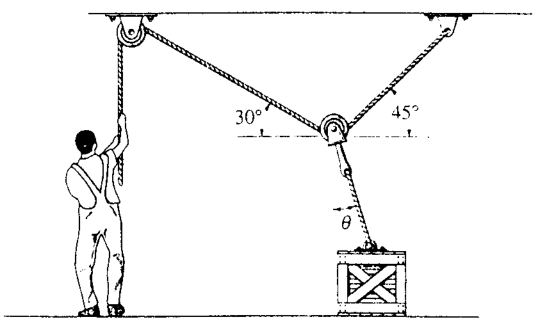
$$\theta = 8.531^\circ \quad N_A = 9.889 \text{ lb} \quad F_A = 1.483 \text{ lb}$$
$$F_B = 0.8900 \text{ lb} \quad N_B = 5.934 \text{ lb}$$



Since $(F_B)_{max} = \mu_{sB} N_B = 0.25(5.934) = 1.483 \text{ lb} > F_B$, block B does not slip. Therefore, the above assumption is correct. Thus

$$\theta = 8.53^\circ \quad F_A = 1.48 \text{ lb} \quad F_B = 0.890 \text{ lb} \qquad \textbf{Ans}$$

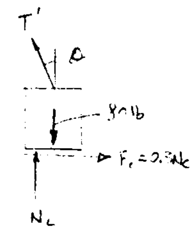
***8-40.** Determine the smallest force the man must exert on the rope in order to move the 80-kg crate. Also, what is the angle θ at this moment? The coefficient of static friction between the crate and the floor is $\mu_s = 0.3$.



Crate:

$\xrightarrow{+} \Sigma F_x = 0; \quad 0.3N_C - T' \sin\theta = 0 \qquad (1)$

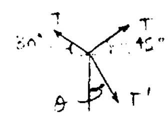
$+\uparrow \Sigma F_y = 0; \quad N_C + T' \cos\theta - 80(9.81) = 0 \qquad (2)$



Pulley:

$\xrightarrow{+} \Sigma F_x = 0; \quad -T\cos 30° + T\cos 45° + T'\sin\theta = 0$

$+\uparrow \Sigma F_y = 0; \quad T\sin 30° + T\sin 45° - T'\cos\theta = 0$



Thus,

$T = 6.29253\, T' \sin\theta$

$T = 0.828427\, T' \cos\theta$

$\theta = \tan^{-1}\left(\frac{0.828427}{6.29253}\right) = 7.50°$ **Ans**

$T = 0.82134\, T' \qquad (3)$

From Eqs. (1) and (2),

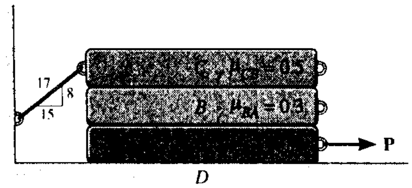
$N_C = 239$ N

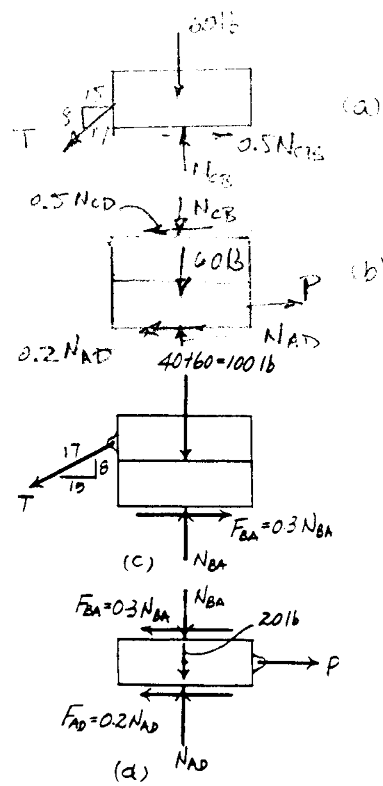
$T' = 550$ N

So that

$T = 452$ N **Ans**

8-41. The three bars have a weight of $W_A = 20$ lb, $W_B = 40$ lb, and $W_C = 60$ lb, respectively. If the coefficients of static friction at the surfaces of contact are as shown, determine the smallest horizontal force P needed to move block A.





Equations of Equilibrium and Friction : If blocks A and B move together, then slipping will have to occur at the contact surfaces CB and AD. Hence, $F_{CB} = \mu_{s\,CB} N_{CB} = 0.5N_{CB}$ and $F_{AD} = \mu_{s\,AD} N_{AD} = 0.2N_{AD}$. From FBD (a)

$$+\uparrow \Sigma F_y = 0; \quad N_{CB} - T\left(\frac{8}{17}\right) - 60 = 0 \qquad [1]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.5N_{CB} - T\left(\frac{15}{17}\right) = 0 \qquad [2]$$

and FBD (b)

$$+\uparrow \Sigma F_y = 0; \quad N_{AD} - N_{CB} - 60 = 0 \qquad [3]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P - 0.5N_{CB} - 0.2N_{AD} = 0 \qquad [4]$$

Solving Eqs.[1], [2], [3] and [4] yields

$T = 46.36$ lb $\quad N_{CB} = 81.82$ lb $\quad N_{AD} = 141.82$ lb
$P = 69.27$ lb

If blocks A move only, then slipping will have to occur at contact surfaces BA and AD. Hence, $F_{BA} = \mu_{s\,BA} N_{BA} = 0.3N_{BA}$ and $F_{AD} = \mu_{s\,AD} N_{AD} = 0.2N_{AD}$. From FBD (c)

$$+\uparrow \Sigma F_y = 0; \quad N_{BA} - T\left(\frac{8}{17}\right) - 100 = 0 \qquad [5]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.3N_{BA} - T\left(\frac{15}{17}\right) = 0 \qquad [6]$$

and FBD (d)

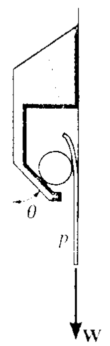
$$+\uparrow \Sigma F_y = 0; \quad N_{AD} - N_{BA} - 20 = 0 \qquad [7]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P - 0.3N_{BA} - 0.2N_{AD} = 0 \qquad [8]$$

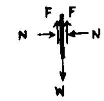
Solving Eqs.[5], [6], [7] and [8] yields

$T = 40.48$ lb $\quad N_{BA} = 119.05$ lb $\quad N_{AD} = 139.05$ lb
$P = 63.52$ lb $= 63.5$ lb (***Control!***) **Ans**

8-42. The friction hook is made from a fixed frame which is shown colored and a cylinder of negligible weight. A piece of paper is placed between the smooth wall and the cylinder. If $\theta = 20°$, determine the smallest coefficient of static friction μ at all points of contact so that any weight W of paper p can be held.



Paper :

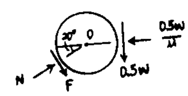


$+\uparrow \Sigma F_y = 0;\quad F = 0.5W$

$F = \mu N;\quad F = \mu N$

$$N = \frac{0.5W}{\mu}$$

Cylinder :



$\curvearrowleft + \Sigma M_O = 0;\quad F = 0.5W$

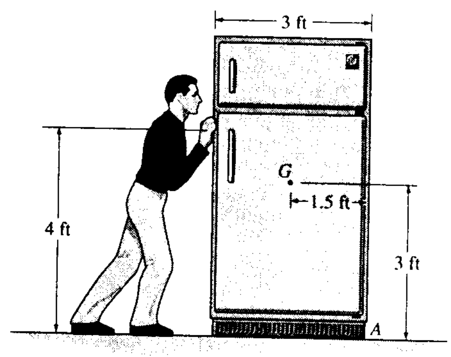
$\xrightarrow{+} \Sigma F_x = 0;\quad N\cos 20° + F\sin 20° - \dfrac{0.5W}{\mu} = 0$

$+\uparrow \Sigma F_y = 0;\quad N\sin 20° - F\cos 20° - 0.5W = 0$

$F = \mu N;\quad \mu^2 \sin 20° + 2\mu \cos 20° - \sin 20° = 0$

$\mu = 0.176$ **Ans**

8-43. The refrigerator has a weight of 180 lb and rests on a tile floor for which $\mu_s = 0.25$. If the man pushes horizontally on the refrigerator in the direction shown, determine the smallest magnitude of force needed to move it. Also, if the man has a weight of 150 lb, determine the smallest coefficient of friction between his shoes and the floor so that he does not slip.



Equations of Equilibrium : From FBD (a),

$+\uparrow \Sigma F_y = 0;\quad N - 180 = 0 \quad N = 180 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad P - F = 0 \qquad [1]$

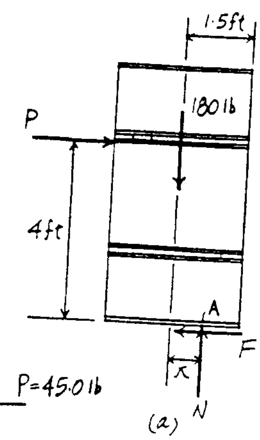
$\curvearrowleft + \Sigma M_A = 0;\quad 180(x) - P(4) = 0 \qquad [2]$

Friction : Assuming the refrigerator is on the verge of slipping, then $F = \mu N = 0.25(180) = 45$ lb. Substituting this value into Eqs. [1], and [2] and solving yields

$$P = 45.0 \text{ lb} \qquad x = 1.00 \text{ ft}$$

Since $x < 1.5$ ft, the refrigerator does not tip. Therefore, the above assumption is correct. Thus

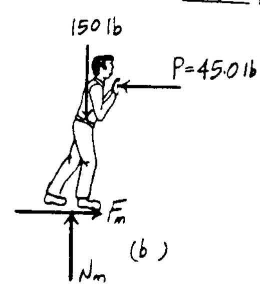
$P = 45.0$ lb **Ans**



From FBD (b),

$+\uparrow \Sigma F_y = 0;\quad N_m - 150 = 0 \quad N_m = 150 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad F_m - 45.0 = 0 \quad F_m = 45.0 \text{ lb}$

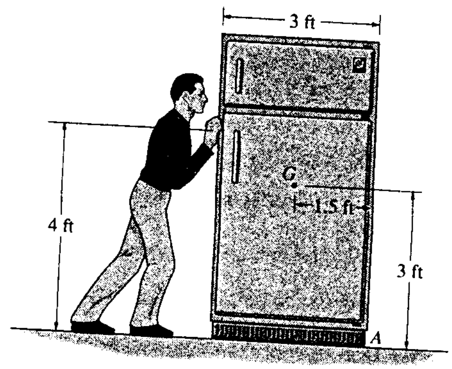


When the man is on the verge of slipping, then

$$F_m = \mu_s' N_m$$
$$45.0 = \mu_s'(150)$$
$$\mu_s' = 0.300$$

Ans

*8-44.** The refrigerator has a weight of 180 lb and rests on a tile floor for which $\mu_s = 0.25$. Also, the man has a weight of 150 lb and the coefficient of static friction between the floor and his shoes is $\mu_s = 0.6$. If he pushes horizontally on the refrigerator, determine if he can move it. If so does the refrigerator slip or tip?



Equations of Equilibrium : From FBD (a),

$$+\uparrow \Sigma F_y = 0; \quad N - 180 = 0 \quad N = 180 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P - F = 0 \qquad [1]$$

$$\circlearrowleft + \Sigma M_A = 0; \quad 180(x) - P(4) = 0 \qquad [2]$$

Friction : Assuming the refrigerator is on the verge of slipping, then $F = \mu N = 0.25(180) = 45$ lb. Substituting this value into Eqs.[1], and [2] and solving yields

$$P = 45.0 \text{ lb} \quad x = 1.00 \text{ ft}$$

Since $x < 1.5$ ft, the refrigerator does not tip. Therefore, the above assumption is correct. Thus, **the refrigerator slips.** **Ans**

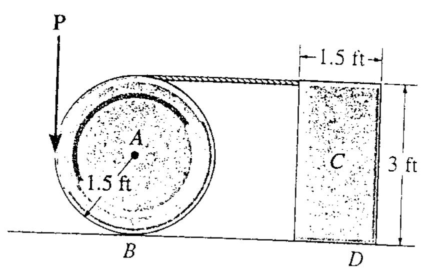
From FBD (b),

$$+\uparrow \Sigma F_y = 0; \quad N_m - 150 = 0 \quad N_m = 150 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_m - 45.0 = 0 \quad F_m = 45.0 \text{ lb}$$

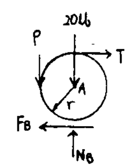
Since $(F_m)_{max} = \mu_s' N_m = 0.6(150) = 90.0 \text{ lb} > F_m$, then the man does not slip. Thus, **The man is capable of moving the refrigerator.** **Ans**

8-45. The wheel weighs 20 lb and rests on a surface for which $\mu_B = 0.2$. A cord wrapped around it is attached to the top of the 30-lb homogeneous block. If the coefficient of static friction at D is $\mu_D = 0.3$, determine the smallest vertical force that can be applied tangentially to the wheel which will cause motion to impend.



Cylinder A :

Assume slipping at B, $\quad F_B = 0.2 N_B$



$$\circlearrowright + \Sigma M_A = 0; \quad F_B + T = P$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_B = T$$

$$+\uparrow \Sigma F_y = 0; \quad N_B = 20 + P$$

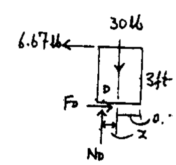
$$N_B = 20 + 2(0.2 N_B)$$

$$N_B = 33.33 \text{ lb}$$

$$F_B = 6.67 \text{ lb}$$

$$T = 6.67 \text{ lb}$$

$$P = 13.3 \text{ lb} \qquad \textbf{Ans}$$



$$\xrightarrow{+} \Sigma F_x = 0; \quad F_D = 6.67 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad N_D = 30 \text{ lb}$$

$$(F_D)_{max} = 0.3(30) = 9 \text{ lb} > 6.67 \text{ lb} \qquad \textbf{(O.K!)}$$

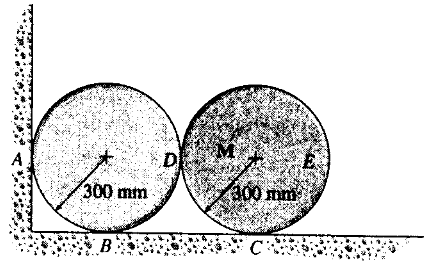
(No slipping occurs)

$$\circlearrowright + \Sigma M_D = 0; \quad -30(x) + 6.67(3) = 0$$

$$x = 0.667 \text{ ft} < \frac{1.5}{2} = 0.75 \text{ ft} \qquad \textbf{(O.K!)}$$

(No tipping occurs)

8-46. Each of the cylinders has a mass of 50 kg. If the coefficients of static friction at the points of contact are $\mu_A = 0.5, \mu_B = 0.5, \mu_C = 0.5$, and $\mu_D = 0.6$, determine the couple moment M needed to rotate cylinder E.

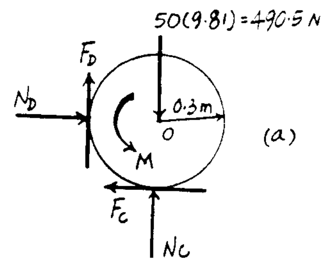


Equations of Equilibrium : From FBD (a),

$\xrightarrow{+} \Sigma F_x = 0; \quad N_D - F_C = 0$ [1]

$+\uparrow \Sigma F_y = 0 \quad N_C + F_D - 490.5 = 0$ [2]

$\curvearrowleft + \Sigma M_O = 0; \quad M - F_C(0.3) - F_D(0.3) = 0$ [3]

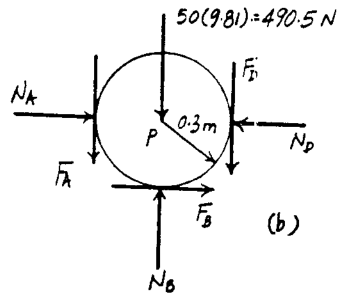


From FBD (b),

$\xrightarrow{+} \Sigma F_x = 0; \quad N_A + F_B - N_D = 0$ [4]

$+\uparrow \Sigma F_y = 0 \quad N_B - F_A - F_D - 490.5 = 0$ [5]

$\curvearrowleft + \Sigma M_P = 0; \quad F_A(0.3) + F_B(0.3) - F_D(0.3) = 0$ [6]



Friction : Assuming cylinder E slips at points C and D and cylinder F does not move, then $F_C = \mu_{s\,C} N_C = 0.5 N_C$ and $F_D = \mu_{s\,D} N_D = 0.6 N_D$. Substituting these values into Eqs. [1], [2] and [3] and solving, we have

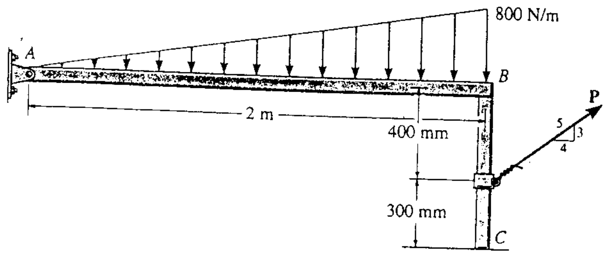
$$N_C = 377.31 \text{ N} \qquad N_D = 188.65 \text{ N}$$
$$M = 90.55 \text{ N}\cdot\text{m} = 90.6 \text{ N}\cdot\text{m}$$ **Ans**

If cylinder F is on the verge of slipping at point A, then $F_A = \mu_{s\,A} N_A = 0.5 N_A$. Substitute this value into Eqs. [4], [5] and [6] and solving, we have

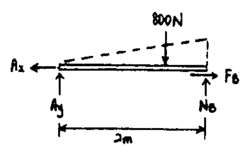
$$N_A = 150.92 \text{ N} \qquad N_B = 679.15 \text{ N} \qquad F_B = 37.73 \text{ N}$$

Since $(F_B)_{max} = \mu_{s\,B} N_B = 0.5(679.15) = 339.58 \text{ N} > F_B$, cylinder F does not move. Therefore the above assumption is correct.

8-47. The beam AB has a negligible mass and thickness and is subjected to a triangular distributed loading. It is supported at one end by a pin and at the other end by a post having a mass of 50 kg and negligible thickness. Determine the minimum force P needed to move the post. The coefficients of static friction at B and C are $\mu_B = 0.4$ and $\mu_C = 0.2$, respectively.



Member AB :



$\circlearrowleft + \Sigma M_A = 0;\qquad -800(\frac{4}{3}) + N_B(2) = 0$

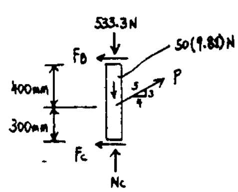
$N_B = 533.3\text{ N}$

Post :

Assume slipping occurs at C; $\qquad F_C = 0.2\,N_C$

$\circlearrowleft + \Sigma M_C = 0;\qquad -\frac{4}{5}P(0.3) + F_B(0.7) = 0$

$\xrightarrow{+} \Sigma F_x = 0;\qquad \frac{4}{5}P - F_B - 0.2N_C = 0$



$+\uparrow \Sigma F_y = 0;\qquad \frac{3}{5}P + N_C - 533.3 - 50(9.81) = 0$

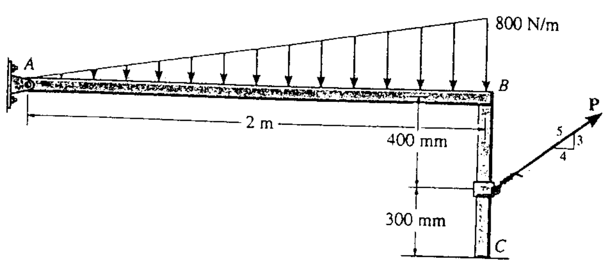
$P = 355\text{ N}$ **Ans**

$N_C = 811.0\text{ N}$

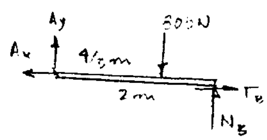
$F_B = 121.6\text{ N}$

$(F_B)_{max} = 0.4(533.3) = 213.3\text{ N} > 121.6\text{ N}$ **(O.K!)**

***8-48.** The beam AB has a negligible mass and thickness and is subjected to a triangular distributed loading. It is supported at one end by a pin and at the other end by a post having a mass of 50 kg and negligible thickness. Determine the two coefficients of static friction at B and at C so that when the magnitude of the applied force is increased to $P = 150$ N, the post slips at both B and C simultaneously.



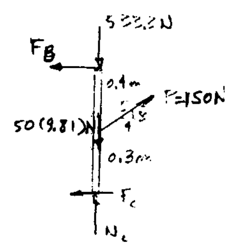
Member AB :



$\circlearrowleft + \Sigma M_A = 0;$ $\quad -800(\frac{4}{3}) + N_B(2) = 0$

$N_B = 533.3$ N

Post :



$+\uparrow \Sigma F_y = 0;$ $\quad N_C - 533.3 + 150(\frac{3}{5}) = 0$

$N_C = 933.84$ N

$\circlearrowleft + \Sigma M_C = 0;$ $\quad -\frac{4}{5}(150)(0.3) + F_B(0.7) = 0$

$F_B = 51.429$ N

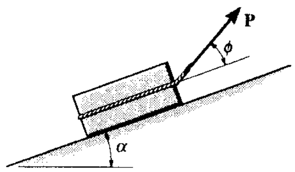
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad \frac{4}{5}(150) - F_C - 51.429 = 0$

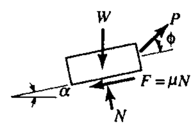
$F_C = 68.571$ N

$\mu_C = \frac{F_C}{N_C} = \frac{68.571}{933.84} = 0.0734$ **Ans**

$\mu_B = \frac{F_B}{N_B} = \frac{51.429}{533.3} = 0.0964$ **Ans**

8-49. The block of weight W is being pulled up the inclined plane of slope α using a force **P**. If **P** acts at the angle ϕ as shown, show that for slipping to occur, $P = W\sin(\alpha+\theta)/\cos(\phi-\theta)$ where θ is the angle of friction; $\theta = \tan^{-1}\mu$.





$\nearrow +\Sigma F_x = 0;\quad P\cos\phi - W\sin\alpha - \mu N = 0$

$+\nwarrow \Sigma F_y = 0;\quad N - W\cos\alpha + P\sin\phi = 0$

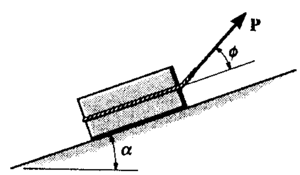
$$P\cos\phi - W\sin\alpha - \mu(W\cos\alpha - P\sin\phi) = 0$$

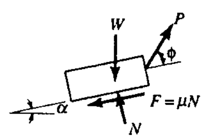
$$P = W\left(\frac{\sin\alpha + \mu\cos\alpha}{\cos\phi + \mu\sin\phi}\right)$$

Let $\mu = \tan\theta$

$$P = W\left(\frac{\sin(\alpha+\theta)}{\cos(\phi-\theta)}\right) \quad \textbf{(QED)}$$

8-50. Determine the angle ϕ at which P should act on the block so that the magnitude of P is as small as possible to begin pushing the block up the incline. What is the corresponding value of P? The block weighs W and the slope α is known.





From Prob. 8-49:

$$P = W\left(\frac{\sin(\alpha+\theta)}{\cos(\phi-\theta)}\right)$$

$$\frac{dP}{d\phi} = W\left(\frac{\sin(\alpha+\theta)\sin(\phi-\theta)}{\cos^2(\phi-\theta)}\right) = 0$$

$$\sin(\alpha+\theta)\sin(\phi-\theta) = 0$$

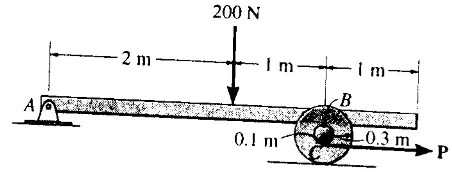
$\sin(\alpha+\theta) = 0$ or $\sin(\phi-\theta) = 0$

$\alpha = -\theta$ $\qquad \phi = \theta$ **Ans**

$P = W\sin(\alpha+\theta)$

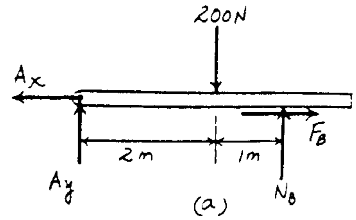
$P = W\sin(\alpha+\phi)$ **Ans**

8-51. The beam AB has a negligible mass and thickness and is subjected to a force of 200 N. It is supported at one end by a pin and at the other end by a spool having a mass of 40 kg. If a cable is wrapped around the inner core of the spool, determine the minimum cable force P needed to move the spool. The coefficients of static friction at B and D are $\mu_B = 0.4$ and $\mu_D = 0.2$, respectively.



Equations of Equilibrium : From FBD (a),

$\circlearrowleft + \Sigma M_A = 0;\quad N_B(3) - 200(2) = 0 \quad N_B = 133.33\text{ N}$



From FBD (b),

$+\uparrow \Sigma F_y = 0 \quad N_D - 133.33 - 392.4 = 0 \quad N_D = 525.73\text{ N}$

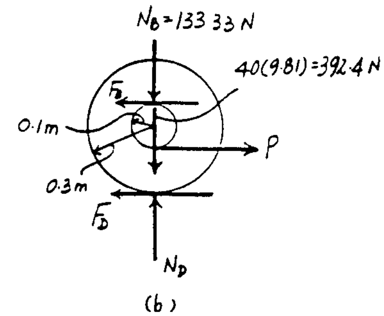
$\xrightarrow{+} \Sigma F_x = 0;\quad P - F_B - F_D = 0 \qquad [1]$

$\circlearrowleft + \Sigma M_D = 0;\quad F_B(0.4) - P(0.2) = 0 \qquad [2]$

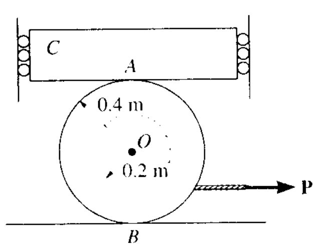
Friction : Assuming the spool slips at point B, then $F_B = \mu_{sB} N_B = 0.4(133.33) = 53.33$ N. Substituting this value into Eqs.[1] and [2] and solving, we have

$F_D = 53.33$ N
$P = 106.67\text{ N} = 107\text{ N}$ **Ans**

Since $(F_D)_{max} = \mu_{sD} N_D = 0.2(525.73) = 105.15\text{ N} > F_B$, the spool does not slip at point D. Therefore the above assumption is correct



***8-52.** Block C has a mass of 50 kg and is confined between two walls by smooth rollers. If the block rests on top of the 40-kg spool, determine the minimum cable force P needed to move the spool. The cable is wrapped around the spool's inner core. The coefficients of static friction at A and B are $\mu_A = 0.3$ and $\mu_B = 0.6$.

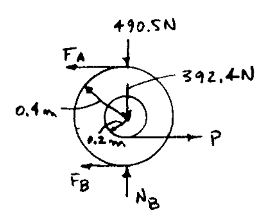


$+\uparrow \Sigma F_y = 0;\quad N_B - 490.5 - 392.4 = 0$

$N_B = 882.9$ N

$\circlearrowleft + \Sigma M_B = 0;\quad F_A(0.4) - F_B(0.4) + P(0.2) = 0$

$\xrightarrow{+} \Sigma F_x = 0;\quad -F_A + P - F_B = 0$



Assume spool slips at A, then

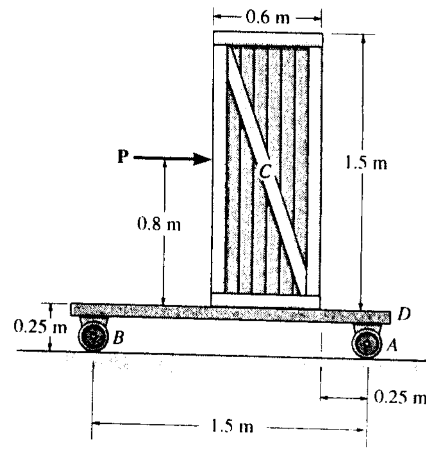
$F_A = 0.3(490.5) = 147.2$ N
Solving,

$F_B = 441.4$ N
$P = 589$ N **Ans**

$N_B = 882.9$ N

Since $(F_B)_{max} = 0.6(882.9) = 529.7\text{ N} > 441.4\text{ N}$ **(O.K!)**

8-53. The uniform 60-kg crate C rests uniformly on a 10-kg dolly D. If the front casters of the dolly at A are locked to prevent rolling while the casters at B are free to roll, determine the maximum force **P** that may be applied without causing motion of the crate. The coefficient of static friction between the casters and the floor is $\mu_f = 0.35$ and between the dolly and the crate, $\mu_d = 0.5$.

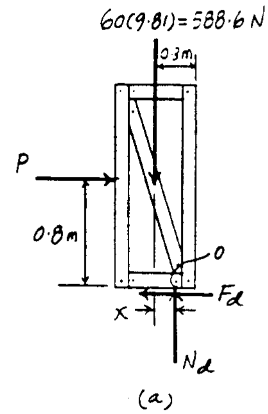


Equations of Equilibrium : From FBD (a),

$$+\uparrow \Sigma F_y = 0; \quad N_d - 588.6 = 0 \quad N_d = 588.6 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P - F_d = 0 \quad [1]$$

$$\circlearrowleft + \Sigma M_A = 0; \quad 588.6(x) - P(0.8) = 0 \quad [2]$$



From FBD (b),

$$+\uparrow \Sigma F_y = 0 \quad N_B + N_A - 588.6 - 98.1 = 0 \quad [3]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P - F_A = 0 \quad [4]$$

$$\circlearrowleft + \Sigma M_B = 0; \quad N_A(1.5) - P(1.05) - 588.6(0.95) - 98.1(0.75) = 0 \quad [5]$$

Friction : Assuming the crate slips on dolly, then $F_d = \mu_{s\,d} N_d = 0.5(588.6) = 294.3$ N. Substituting this value into Eqs. [1] and [2] and solving, we have

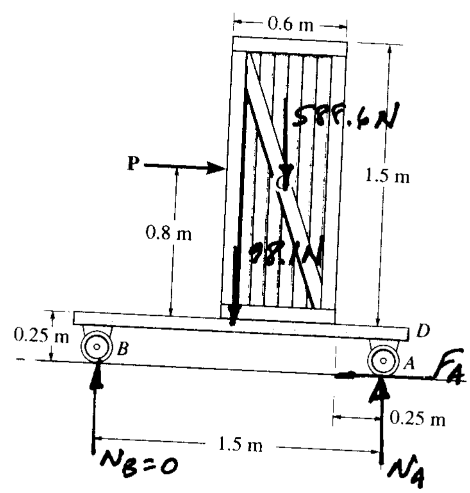
$$P = 294.3 \text{ N} \quad x = 0.400 \text{ m}$$

Since $x > 0.3$ m, the crate tips on the dolly. If this is the case $x = 0.3$ m. Solving Eqs. [1] and [2] with $x = 0.3$ m yields

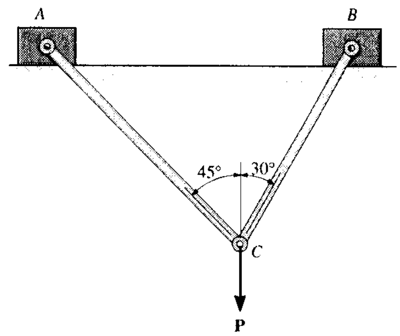
$$P = 220.725 \text{ N} = 221 \text{ N}$$
$$F_d = 220.725 \text{ N}$$

Assuming the dolly slips at A, then $F_A = \mu_{s\,f} N_A = 0.35 N_A$. Substituting this value into Eqs. [3], [4] and [5] and solving, we have

$$N_A = 559 \text{ N} \quad N_B = 128 \text{ N}$$
$$P = 195.6 \text{ N} = 196 \text{ N} \; (\textit{Control!}) \qquad \textbf{Ans}$$



8-54. Two blocks A and B, each having a mass of 6 kg, are connected by the linkage shown. If the coefficients of static friction at the contacting surfaces are $\mu_A = 0.2$ and $\mu_B = 0.8$, determine the largest vertical force P that may be applied to pin C without causing the blocks to slip. Neglect the weight of the links.



Equations of Equilibrium: From FBD (a),

$$+ \quad \Sigma F_{x'} = 0; \quad T_B \cos 15^\circ - P \sin 45^\circ = 0 \quad T_B = 0.7321P$$

$$+ \Sigma F_{y'} = 0; \quad T_A + 0.7321P \sin 15^\circ - P \cos 45^\circ = 0$$

$$T_A = 0.5176P$$

From FBD (b),

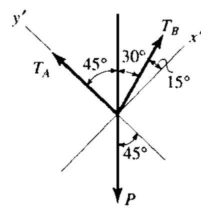
$$+\uparrow \Sigma F_y = 0; \quad N_A - 0.5176P \sin 45^\circ - 58.86 = 0 \qquad [1]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.5176P \cos 45^\circ - F_A = 0 \qquad [2]$$

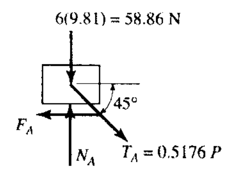
From FBD (c),

$$+\uparrow \Sigma F_y = 0; \quad N_B - 0.7321P \sin 60^\circ - 58.86 = 0 \qquad [3]$$

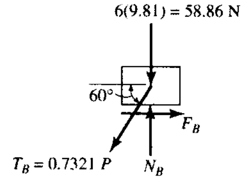
$$\xrightarrow{+} \Sigma F_x = 0; \quad F_B - 0.7321P \cos 60^\circ = 0 \qquad [4]$$



(a)



(b)



(c)

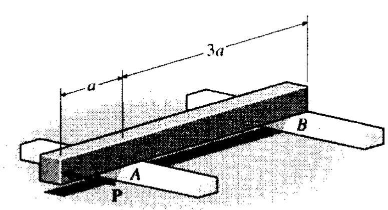
Friction: Assuming block A slips, then $F_A = \mu_{s,A} N_A = 0.2N_A$. Substituting this value into Eqs. [1], [2], [3] and [4] and solving, we have

$$P = 40.20 \text{ N} = 40.2 \text{ N} \qquad \textbf{Ans}$$

$N_A = 73.575$ N $\quad N_B = 84.35$ N $\quad F_B = 14.715$ N

Since $(F_B)_{max} = \mu_{s,B} N_B = 0.8(84.35) = 67.48 \text{ N} > F_B$, block B does not slip. Therefore, the above assumption is correct.

8-55. The uniform beam has a weight W and length $4a$. It rests on the fixed rails at A and B. If the coefficient of static friction at the rails is μ_s determine the horizontal force P, applied perpendicular to the face of the beam, which will cause the beam to move.



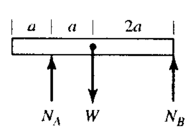
From FBD (a),

$$+\uparrow \Sigma F = 0; \quad N_A + N_B - W = 0$$

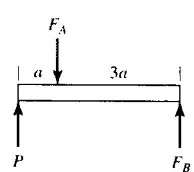
$$\circlearrowleft + \Sigma M_B = 0; \quad -N_A(3a) + W(2a) = 0$$

$$N_A = \frac{2}{3}W \qquad N_B = \frac{1}{3}W$$

Support A can sustain twice as much static frictional force as support B.



(a) Side view



(b) Top view

From FBD (b),

$$+\uparrow \Sigma F = 0; \quad P + F_B - F_A = 0$$

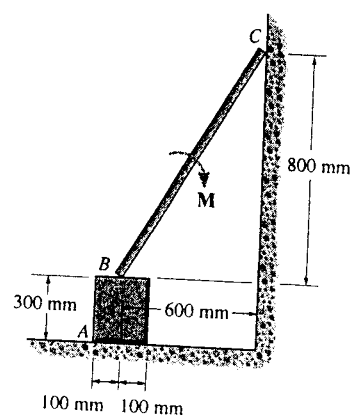
$$\circlearrowleft + \Sigma M_B = 0; \quad -P(4a) + F_A(3a) = 0$$

$$F_A = \frac{4}{3}P \qquad F_B = \frac{1}{3}P$$

The frictional load at A is 4 times as great as at B. The beam will slip at A first.

$$P = \frac{3}{4}(F_A)_{max} = \frac{3}{4}(\mu_s N_A) = \frac{1}{2}\mu_s W \quad \textbf{Ans}$$

***8-56.** The uniform 6-kg slender rod rests on the top center of the 3-kg block. If the coefficients of static friction at the points of contact are $\mu_A = 0.4$, $\mu_B = 0.6$, and $\mu_C = 0.3$, determine the largest couple moment M which can be applied to the rod without causing motion of the rod.



Equations of Equilibrium : From FBD (a),

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_B - N_C = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \quad N_B + F_C - 58.86 = 0 \qquad [2]$$

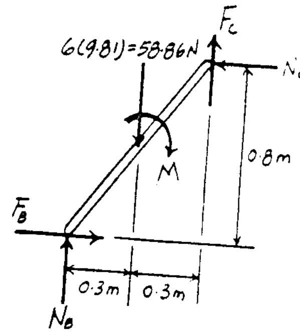
$$+\Sigma M_B = 0; \quad F_C(0.6) + N_C(0.8) - M - 58.86(0.3) = 0 \qquad [3]$$

From FBD (b),

$$+\uparrow \Sigma F_y = 0; \quad N_A - N_B - 29.43 = 0 \qquad [4]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_A - F_B = 0 \qquad [5]$$

$$+\Sigma M_O = 0; \quad F_B(0.3) - N_B(x) - 29.43(x) = 0 \qquad [6]$$



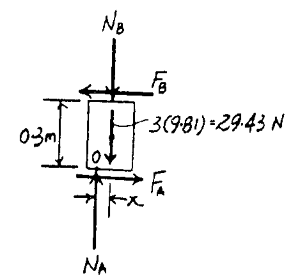
(a)

Friction : Assume slipping occurs at point C and the block tips, then $F_C = \mu_{sC} N_C = 0.3 N_C$ and $x = 0.1$ m. Substituting these values into Eqs.[1], [2], [3], [4], [5] and [6] and solving, we have

$$M = 8.561 \text{ N}\cdot\text{m} = 8.56 \text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

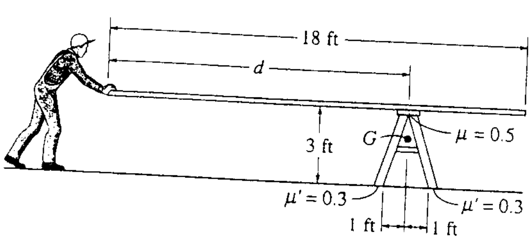
$$N_B = 50.83 \text{ N} \quad N_A = 80.26 \text{ N} \quad F_A = F_B = N_C = 26.75 \text{ N}$$

Since $(F_A)_{max} = \mu_{sA} N_A = 0.4(80.26) = 32.11 \text{ N} > F_A$, the block does not slip. Also, $(F_B)_{max} = \mu_{sB} N_B = 0.6(50.83) = 30.50 \text{ N} > F_B$, then slipping does not occur at point B. Therefore, the above assumption is correct.



(b)

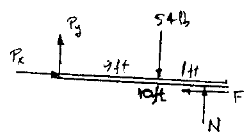
8-57. The carpenter slowly pushes the uniform board horizontally over the top of the saw horse. The board has a uniform weight of 3 lb/ft, and the saw horse has a weight of 15 lb and a center of gravity at G. Determine if the saw horse will stay in position, slip, or tip if the board is pushed forward when d = 10 ft. The coefficients of static friction are shown in the figure.



Board :

$$\circlearrowleft +\Sigma M_P = 0; \quad -54(9) + N(10) = 0$$

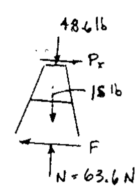
$$N = 48.6 \text{ lb}$$



To cause slipping of board on saw horse :

$$P_x = F_{max} = 0.5\,N = 24.3 \text{ lb}$$

Saw horse :



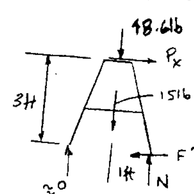
To cause slipping at ground :

$$P_x = F = F_{max} = 0.3(48.6 + 15) = 19.08 \text{ lb}$$

To cause tipping :

$$\circlearrowleft +\Sigma M_B = 0; \quad (48.6 + 15)(1) - P_x(3) = 0$$

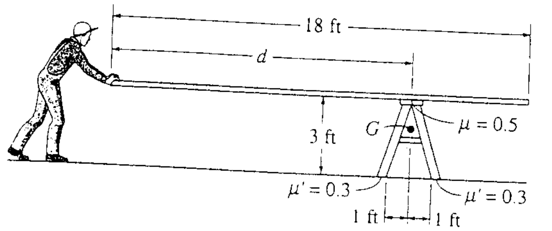
$$P_x = 21.2 \text{ lb}$$



Thus, $\quad P_x = 19.1$ lb $\qquad$ **Ans**

The saw horse will start to slip.

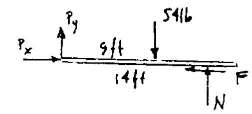
8-58. The carpenter slowly pushes the uniform board horizontally over the top of the saw horse. The board has a uniform weight of 3 lb/ft, and the saw horse has a weight of 15 lb and a center of gravity at G. Determine if the saw horse will stay in position, slip, or tip if the board is pushed forward when d = 14 ft. The coefficients of static friction are shown in the figure.



Board:

$\circlearrowleft + \Sigma M_P = 0; \quad -54(9) + N(14) = 0$

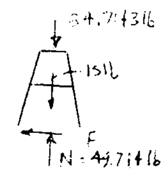
$N = 34.714$ lb



To cause slipping of board on saw horse:

$P_x = F_{max} = 0.5\,N = 17.36$ lb

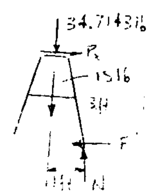
Saw horse:



To cause slipping at ground:

$P_x = F = F_{max} = 0.3(34.714 + 15) = 14.91$ lb

To cause tipping:



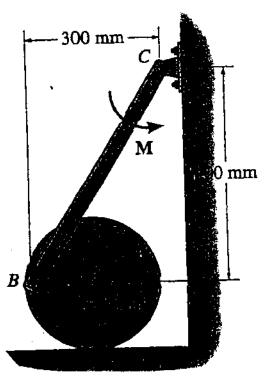
$\circlearrowleft + \Sigma M_B = 0; \quad (34.714 + 15)(1) - P_x(3) = 0$

$P_x = 16.57$ lb

Thus, $\quad P_x = 14.9$ lb $\qquad$ **Ans**

The saw horse will start to slip.

8-59. The 45-kg disk rests on the surface for which the coefficient of static friction is $\mu_A = 0.2$. Determine the largest couple moment M that can be applied to the bar without causing motion.



$\circlearrowleft + \Sigma M_O = 0; \quad F_A = B_y = 0.2\,N_A$

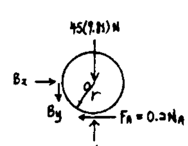
$\xrightarrow{+} \Sigma F_x = 0; \quad B_x - 0.2N_A = 0$

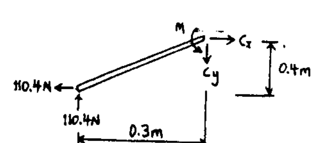
$+\uparrow \Sigma F_y = 0; \quad N_A - B_y - 45(9.81) = 0$

$N_A = 551.8$ N

$B_x = 110.4$ N

$B_y = 110.4$ N

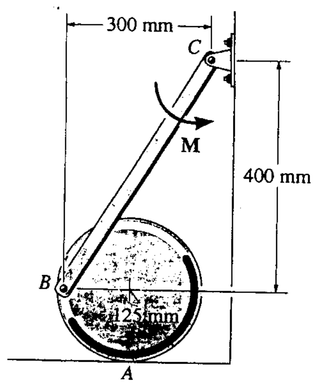




$\circlearrowleft + \Sigma M_C = 0; \quad -110.4(0.3) - 110.4(0.4) + M = 0$

$M = 77.3$ N·m $\qquad$ **Ans**

***8-60.** The 45-kg disk rests on the surface for which the coefficient of static friction is $\mu_A = 0.15$. If $M = 50\ \mathrm{N\cdot m}$, determine the friction force at A.

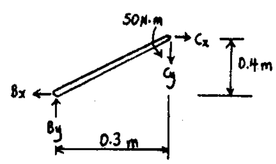


Bar:

$\circlearrowleft + \Sigma M_C = 0;\quad -B_y(0.3) - B_x(0.4) + 50 = 0$

$\xrightarrow{+} \Sigma F_x = 0;\quad B_x = C_x$

$+\uparrow \Sigma F_y = 0;\quad B_y = C_y$

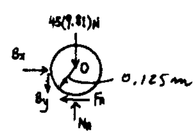


Disk:

$\xrightarrow{+} \Sigma F_x = 0;\quad B_x = F_A$

$+\uparrow \Sigma F_y = 0;\quad N_A - B_y - 45(9.81) = 0$

$\circlearrowleft + \Sigma M_O = 0;\quad B_y(0.125) - F_A(0.125) = 0$



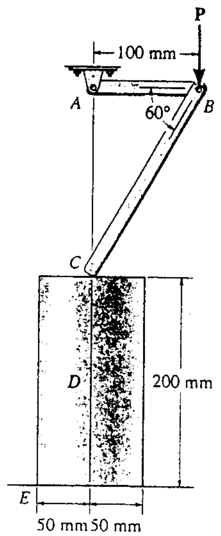
$N_A = 512.9\ \mathrm{N}$

$F_A = 71.4\ \mathrm{N}$ **Ans**

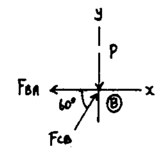
$(F_A)_{max} = 0.15(512.9) = 76.93\ \mathrm{N} > 71.43\ \mathrm{N}$

No motion of disk.

8-61. The end C of the two-bar linkage rests on the top center of the 50-kg cylinder. If the coefficients of static friction at C and E are $\mu_C = 0.6$ and $\mu_E = 0.3$, determine the largest vertical force P which can be applied at B without causing motion. Neglect the mass of the bars.



Joint B :

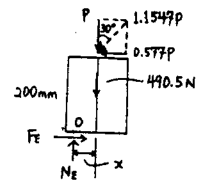


$+\uparrow \Sigma F_y = 0; \quad F_{CB}\sin 60° - P = 0$

$F_{CB} = 1.1547P$

Since $\quad (F_C)_{max} = 0.6P > 1.1547\,P\cos 60° = 0.5774P$

Bar will not slip at C.



$+\uparrow \Sigma F_y = 0; \quad N_E - 1.1547P\cos 30° - 490.5 = 0$

$N_E = 490.5 + P$

$\xrightarrow{+} \Sigma F_x = 0; \quad F_E - 1.1547\sin 30° = 0$

$F_E = 0.5774P$

$\circlearrowleft + \Sigma M_O = 0; \quad -490.5(x) - P(x) + 0.5774P(0.2) = 0$

Assume tipping,

$x = 0.05$ m

$P = 375$ N

$F_E = 216$ N

$N_E = 865$ N

$(F_E)_{max} = 0.3(865) = 259 \text{ N} > 216.5 \text{ N}$ **(O.K!)**

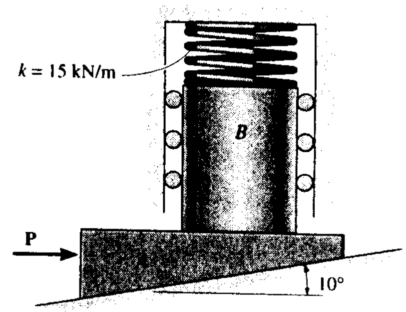
At C,

$(0.6)(375) = 225 > 0.577(375) = 216.4$ **(O.K!)**

Cylinder tips,

$P = 375$ N **Ans**

8-62. Determine the minimum applied force **P** required to move wedge A to the right. The spring is compressed a distance of 175 mm. Neglect the weight of A and B. The coefficient of static friction for all contacting surfaces is $\mu_s = 0.35$. Neglect friction at the rollers.



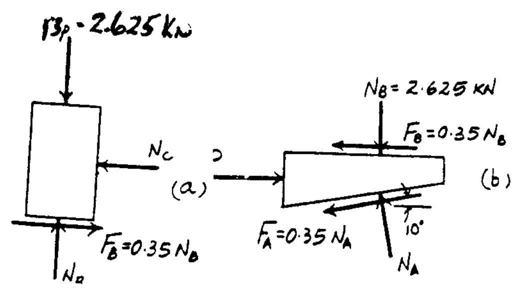
Equations of Equilibrium and Friction : Using the spring formula, $F_{sp} = kx = 15(0.175) = 2.625$ kN. If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces. Thus, $F_A = \mu_s N_A = 0.35N_A$ and $F_B = \mu_s N_B = 0.35N_B$. From FBD (a),

$$+\uparrow \Sigma F_y = 0; \quad N_B - 2.625 = 0 \quad N_B = 2.625 \text{ kN}$$

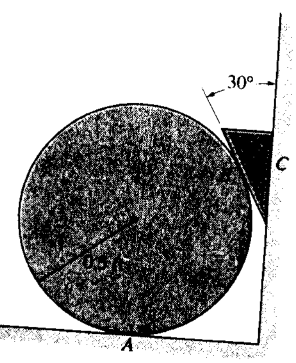
From FBD (b),

$$+\uparrow \Sigma F_y = 0; \quad N_A \cos 10° - 0.35N_A \sin 10° - 2.625 = 0$$
$$N_A = 2.841 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P - 0.35(2.625) - 0.35(2.841)\cos 10° - 2.841 \sin 10° = 0$$
$$P = 2.39 \text{ kN} \qquad \textbf{Ans}$$



8-63. Determine the largest weight of the wedge that can be placed between the 8-lb cylinder and the wall without upsetting equilibrium. The coefficient of static friction at A and C is $\mu_s = 0.5$ and at B, $\mu'_s = 0.6$.



Equations of Equilibrium : From FBD (a),

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_B \cos 30° - F_B \cos 60° - N_C = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \quad N_B \sin 30° + F_B \sin 60° + F_C - W = 0 \qquad [2]$$

From FBD (b),

$$+\uparrow \Sigma F_y = 0; \quad N_A - N_B \sin 30° - F_B \sin 60° - 8 = 0 \qquad [3]$$

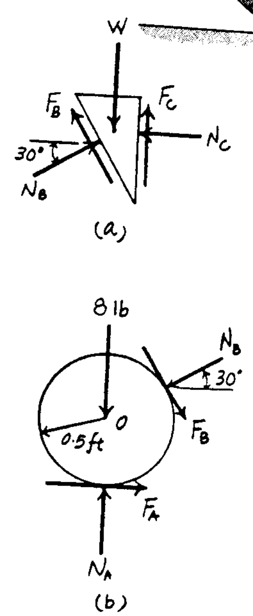
$$\xrightarrow{+} \Sigma F_x = 0; \quad F_A + F_B \cos 60° - N_B \cos 30° = 0 \qquad [4]$$

$$\circlearrowleft + \Sigma M_O = 0; \quad F_A(0.5) - F_B(0.5) = 0 \qquad [5]$$

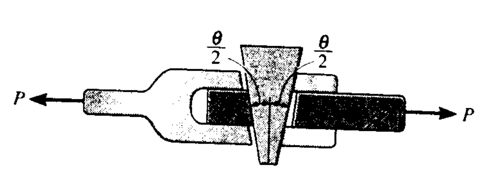
Friction : Assume slipping occurs at points C and A, then $F_C = \mu_s N_C = 0.5N_C$ and $F_A = \mu_s N_A = 0.5N_A$. Substituting these values into Eqs. [1], [2], [3], [4], and [5] and solving, we have

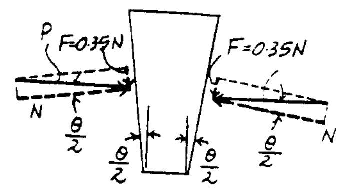
$$W = 66.64 \text{ lb} = 66.6 \text{ lb} \qquad \textbf{Ans}$$
$$N_B = 51.71 \text{ lb} \quad N_A = 59.71 \text{ lb} \quad F_B = N_C = 29.86 \text{ lb}$$

Since $(F_B)_{max} = \mu_s' N_B = 0.6(51.71) = 31.03 \text{ lb} > F_B$, slipping does not occur at point B. Therefore, the above assumption is correct.



***8-64.** The wedge has a negligible weight and a coefficient of static friction $\mu_s = 0.35$ with all contacting surfaces. Determine the angle θ so that it is "self-locking." This requires no slipping for any magnitude of the force **P** applied to the joint.



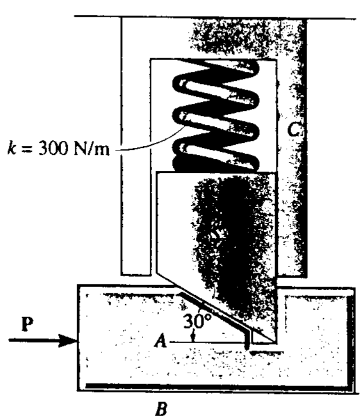


Friction : When the wedge is on the verge of slipping, then $F = \mu N = 0.35N$. From the force diagram (P is the 'locking' force.),

$$\tan\frac{\theta}{2} = \frac{0.35N}{N} = 0.35$$

$$\theta = 38.6^\circ \qquad \textbf{Ans}$$

8-65. If the spring is compressed 60 mm and the coefficient of static friction between the tapered stub S and the slider A is $\mu_{SA} = 0.5$, determine the horizontal force **P** needed to move the slider forward. The stub is free to move without friction within the fixed collar C. The coefficient of static friction between A and surface B is $\mu_{AB} = 0.4$. Neglect the weights of the slider and stub.



Stub :

$+\uparrow\Sigma F_y = 0;\qquad N_A\cos 30^\circ - 0.5N_A\sin 30^\circ - 300(0.06) = 0$

$N_A = 29.22\text{ N}$

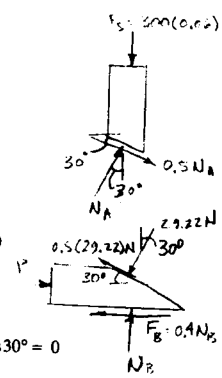
Slider :

$+\uparrow\Sigma F_y = 0;\qquad N_B - 29.22\cos 30^\circ + 0.5(29.22)\sin 30^\circ = 0$

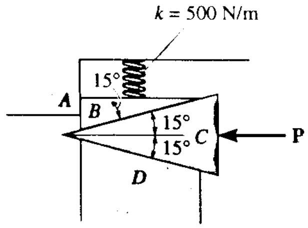
$N_B = 18\text{ N}$

$\xrightarrow{+}\Sigma F_x = 0;\qquad P - 0.4(18) - 29.22\sin 30^\circ - 0.5(29.22)\cos 30^\circ = 0$

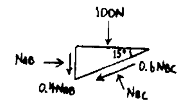
$P = 34.5\text{ N}\qquad$ **Ans**



8-66. The coefficient of static friction between wedges B and C is $\mu_s = 0.6$ and between the surfaces of contact B and A and C and D, $\mu_s' = 0.4$. If the spring is compressed 200 mm when in the position shown, determine the smallest force P needed to move wedge C to the left. Neglect the weight of the wedges.



Wedge B:



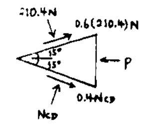
$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad N_{AB} - 0.6N_{BC}\cos 15° - N_{BC}\sin 15° = 0$

$+\uparrow\Sigma F_y = 0;\quad N_{BC}\cos 15° - 0.6N_{BC}\sin 15° - 0.4N_{AB} - 100 = 0$

$N_{BC} = 210.4\text{ N}$

$N_{AB} = 176.4\text{ N}$

Wedge C:



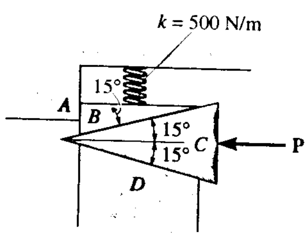
$+\uparrow\Sigma F_y = 0;\quad N_{CD}\cos 15° - 0.4N_{CD}\sin 15° + 0.6(210.4)\sin 15° - 210.4\cos 15° = 0$

$N_{CD} = 197.8\text{ N}$

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 197.8\sin 15° + 0.4(197.8)\cos 15° + 210.4\sin 15° + 0.6(210.4)\cos 15° - P = 0$

$P = 304\text{ N}$ **Ans**

8-67. The coefficient of static friction between the wedges B and C is $\mu_s = 0.6$ and between the surfaces of contact B and A and C and D, $\mu_s' = 0.4$. If $P = 50$ N, determine the largest allowable compression of the spring without causing wedge C to move to the left. Neglect the weight of the wedges.



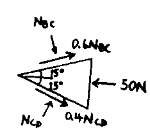
Wedge C:

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad (N_{CD} + N_{BC})\sin 15° + (0.4N_{CD} + 0.6N_{BC})\cos 15° - 50 = 0$

$+\uparrow\Sigma F_y = 0;\quad (N_{CD} - N_{BC})\cos 15° + (-0.4N_{CD} + 0.6N_{BC})\sin 15° = 0$

$N_{BC} = 34.61\text{ N}$

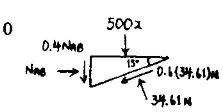
$N_{CD} = 32.53\text{ N}$



Wedge B:

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad N_{AB} - 0.6(34.61)\cos 15° - 34.61\sin 15° = 0$

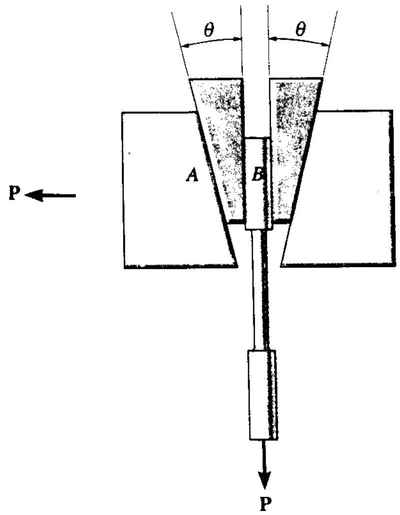
$N_{AB} = 29.01\text{ N}$



$+\uparrow\Sigma F_y = 0;\quad 34.61\cos 15° - 0.6(34.61)\sin 15° - 0.4(29.01) - 500x = 0$

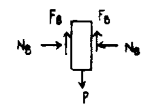
$x = 0.03290\text{ m} = 32.9\text{ mm}$ **Ans**

*8-68. The wedge blocks are used to hold the specimen in a tension testing machine. Determine the design angle θ of the wedges so that the specimen will not slip regardless of the applied load. The coefficients of static friction are $\mu_A = 0.1$ at A and $\mu_B = 0.6$ at B. Neglect the weight of the blocks.



Specimen:

$+\uparrow \Sigma F_y = 0; \quad F_B = \frac{P}{2}$



Wedge:

$\xrightarrow{+} \Sigma F_x = 0; \quad N_A\cos\theta - 0.1N_A\sin\theta - \frac{\frac{P}{2}}{0.6} = 0$

$+\uparrow \Sigma F_y = 0; \quad 0.1N_A\cos\theta + N_A\sin\theta - \frac{P}{2} = 0$

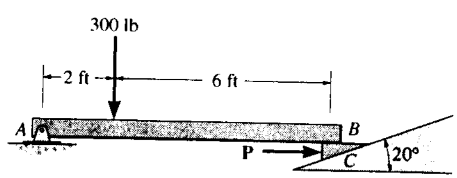
$P = 2N_A(0.1\cos\theta + \sin\theta)$

$0.6N_A(\cos\theta - 0.1\sin\theta) - N_A(0.1\cos\theta + \sin\theta) = 0$

$0.5\cos\theta - 1.06\sin\theta = 0$

$\theta = \tan^{-1}(\frac{0.5}{1.06}) = 25.3°$ **Ans**

8-69. The beam is adjusted to the horizontal position by means of a wedge located at its right support. If the coefficient of static friction between the wedge and the two surfaces of contact is $\mu_s = 0.25$, determine the horizontal force **P** required to push the wedge forward. Neglect the weight and size of the wedge and the thickness of the beam.



Equations of Equilibrium and Friction : If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces. Thus, $F_B = \mu_s N_B = 0.25N_A$ and $F_C = \mu_s N_C = 0.25N_C$. From FBD (a),

$\circlearrowleft + \Sigma M_A = 0; \quad N_B(8) - 300(2) = 0 \quad N_B = 75.0 \text{ lb}$

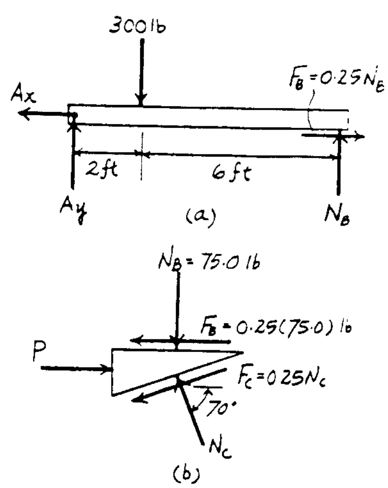
From FBD (b),

$+\uparrow \Sigma F_y = 0; \quad N_C\sin 70° - 0.25N_C\sin 20° - 75.0 = 0$

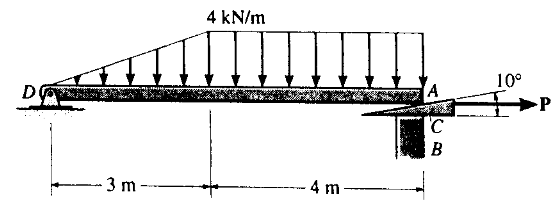
$N_C = 87.80 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad P - 0.25(75.0) - 0.25(87.80)\cos 20° - 87.80\cos 70° = 0$

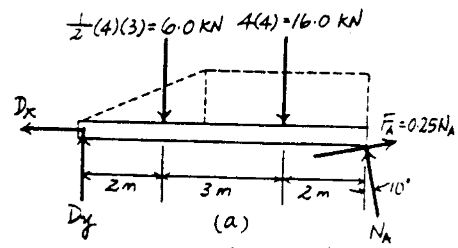
$P = 69.4 \text{ lb}$ **Ans**



8-70. If the beam AD is loaded as shown, determine the horizontal force P which must be applied to the wedge in order to remove it from under the beam. The coefficients of static friction at the wedge's top and bottom surfaces are $\mu_{CA} = 0.25$ and $\mu_{CB} = 0.35$, respectively. If $P = 0$, is the wedge self-locking? Neglect the weight and size of the wedge and the thickness of the beam.



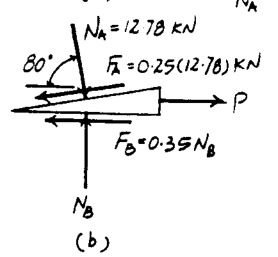
Equations of Equilibrium and Friction : If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces. Thus, $F_A = \mu_{sA} N_A = 0.25 N_A$ and $F_B = \mu_{sB} N_B = 0.35 N_B$. From FBD (a),



$$\circlearrowleft + \Sigma M_D = 0; \quad N_A \cos 10^\circ (7) + 0.25 N_A \sin 10^\circ (7) - 6.00(2) - 16.0(5) = 0$$

$$N_A = 12.78 \text{ kN}$$

From FBD (b),



$$+\uparrow \Sigma F_y = 0; \quad N_B - 12.78 \sin 80^\circ - 0.25(12.78) \sin 10^\circ = 0$$

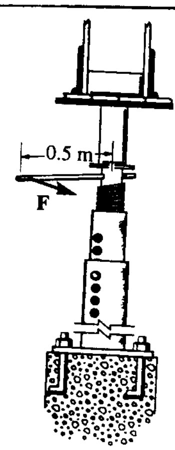
$$N_B = 13.14 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad P + 12.78 \cos 80^\circ - 0.25(12.78) \cos 10^\circ - 0.35(13.14) = 0$$

$$P = 5.53 \text{ kN} \qquad \textbf{Ans}$$

Since a force $P (> 0)$ is required to pull out the wedge, **the wedge will be self-locking when $P = 0$.** **Ans**

8-71. The column is used to support the upper floor. If a force $F = 80$ N is applied perpendicular to the handle to tighten the screw, determine the compressive force in the column. The square-threaded screw on the jack has a coefficient of static friction of $\mu_s = 0.4$, mean diameter of 25 mm, and a lead of 3 mm.



$$M = W(r) \tan(\phi_s + \theta_p)$$

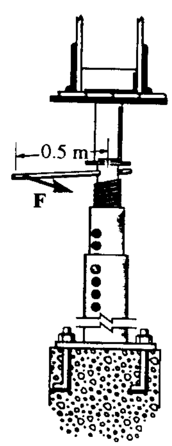
$$\phi_s = \tan^{-1}(0.4) = 21.80^\circ$$

$$\theta_p = \tan^{-1}\left[\frac{3}{2\pi(12.5)}\right] = 2.188^\circ$$

$$80(0.5) = W(0.0125) \tan(21.80^\circ + 2.188^\circ)$$

$$W = 7.19 \text{ kN} \qquad \textbf{Ans}$$

***8-72.** If the force **F** is removed from the handle of the jack in Prob. 8-71, determine if the screw is self-locking.

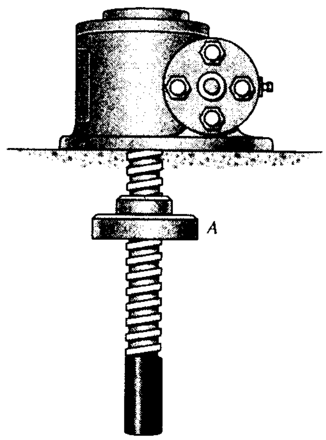


$$\phi_s = \tan^{-1}(0.4) = 21.80^\circ$$

$$\theta_p = \tan^{-1}\left[\frac{3}{2\pi(12.5)}\right] = 2.188^\circ$$

Since $\phi_s > \theta_p$, Screw is self locking. **Ans**

8-73. The square-threaded screw has a mean diameter of 20 mm and a lead of 4 mm. If the weight of the plate A is 5 lb, determine the smallest coefficient of static friction between the screw and the plate so that the plate does not travel down the screw when the plate is suspended as shown.



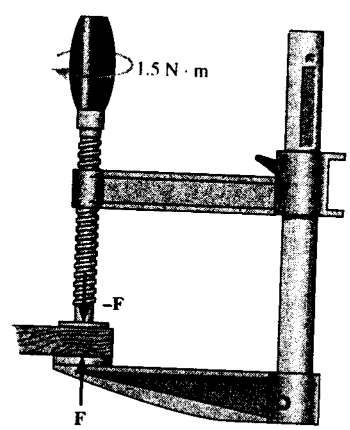
Frictional Forces on Screw **:** This requires a "self-locking" screw where $\phi_s \geq \theta$. Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{4}{2\pi(10)}\right] = 3.643°$.

$\phi_s = \tan^{-1}\mu_s$

$\mu_s = \tan\phi_s$ where $\phi_s = \theta = 3.643°$

$= 0.0637$ **Ans**

8-74. The square threaded screw of the clamp has a mean diameter of 14 mm and a lead of 6 mm. If $\mu_s = 0.2$ for the threads, and the torque applied to the handle is 1.5 N · m, determine the compressive force **F** on the block.



Frictional Forces on Screw **:** Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{6}{2\pi(7)}\right] = 7.768°$, $W = F$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.2) = 11.310°$. Applying Eq. 8 – 3, we have

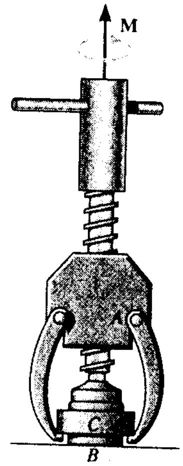
$$M = Wr\tan(\theta + \phi)$$

$$1.5 = F(0.007)\tan(7.768° + 11.310°)$$

$$F = 620 \text{ N}$$ **Ans**

Note : Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if the moment is removed.

8-75. The device is used to pull the battery cable terminal C from the post of a battery. If the required pulling force is 85 lb, determine the torque **M** that must be applied to the handle on the screw to tighten it. The screw has square threads, a mean diameter of 0.2 in., a lead of 0.08 in., and the coefficient of static friction is $\mu_s = 0.5$.

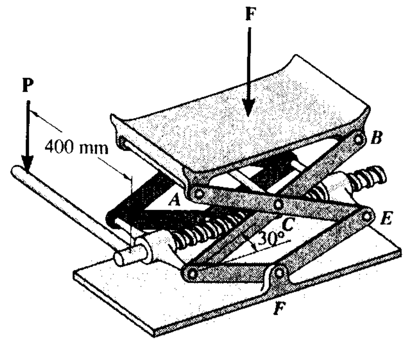


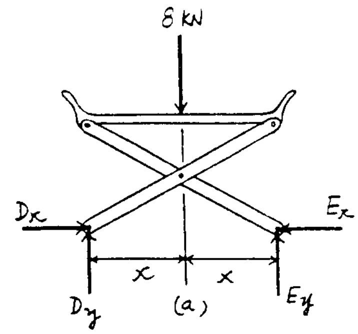
Frictional Forces on Screw : Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{0.08}{2\pi(0.1)}\right] = 7.256°$, $W = 85$ lb and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.5) = 26.565°$. Applying Eq. 8 – 3, we have

$$M = Wr\tan(\theta + \phi)$$
$$= 85(0.1)\tan(7.256° + 26.565°)$$
$$= 5.69 \text{ lb} \cdot \text{in}$$ **Ans**

Note : Since $\phi_s > \theta$, the screw is self - locking. It will not unscrew even if the moment is removed.

8-76. The automobile jack is subjected to a vertical load of $F = 8$ kN. If a square-threaded screw, having a lead of 5 mm and a mean diameter of 10 mm, is used in the jack, determine the force that must be applied perpendicular to the handle to (a) raise the load, and (b) lower the load; $\mu_s = 0.2$. The supporting plate exerts only vertical forces at A and B, and each cross link has a total length of 200 mm.



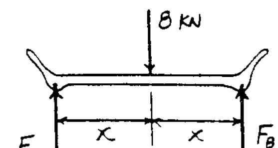


(a)

Equations of Equilibrium : From FBD (a),

$$\circlearrowleft + \Sigma M_E = 0; \quad 8(x) - D_y(2x) = 0 \quad D_y = 4.00 \text{ kN}$$

From FBD (b),

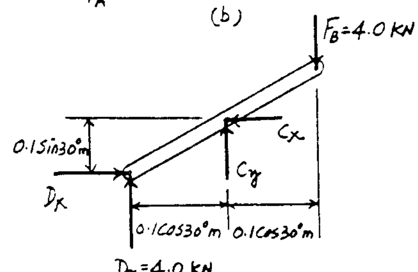


(b)

$$\circlearrowleft + \Sigma M_A = 0; \quad F_B(2x) - 8(x) = 0 \quad F_B = 4.00 \text{ kN}$$

From FBD (c),

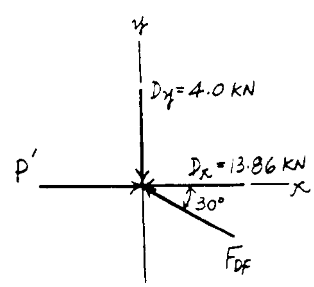
$$\circlearrowleft + \Sigma M_C = 0; \quad D_x(0.1\sin 30°) - 4.00(0.2\cos 30°) = 0$$
$$D_x = 13.86 \text{ kN}$$



(c)

Member DF is a two force member. Analysing the forces that act on pin D[FBD (d)], we have

$$+\uparrow \Sigma F_y = 0; \quad F_{DF}\sin 30° - 4.00 = 0 \quad F_{DF} = 8.00 \text{ kN}$$
$$\xrightarrow{+} \Sigma F_x = 0; \quad P' - 13.86 - 8.00\cos 30° = 0 \quad P' = 20.78 \text{ kN}$$



(d)

Frictional Forces on Screw : Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{5}{2\pi(5)}\right] = 9.043°$, $W = P' = 20.78$ kN, $M = 0.4P$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.2) = 11.310°$. Applying Eq. 8 – 3 if the jack is **raising the load**, we have

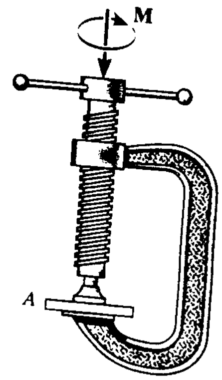
$$M = Wr\tan(\theta + \phi)$$
$$0.4P = 20.78(0.005)\tan(9.043° + 11.310°)$$
$$P = 0.09638 \text{ kN} = 96.4 \text{ N} \qquad \textbf{Ans}$$

Applying Eq. 8 – 5 if the jack is **lowering the load**, we have

$$M'' = Wr\tan(\phi - \theta)$$
$$0.4P = 20.78(0.005)\tan(11.310° - 9.043°)$$
$$P = 0.01028 \text{ kN} = 10.3 \text{ N} \qquad \textbf{Ans}$$

Note : Since $\phi_s > \theta$, the screw is self - locking. It will not unscrew even if force **P** is removed.

8-77. Determine the clamping force on the board A if the screw of the "C" clamp is tightened with a twist of $M = 8\ \text{N}\cdot\text{m}$. The single square-threaded screw has a mean radius of 10 mm, a lead of 3 mm, and the coefficient of static friction is $\mu_s = 0.35$.



$$\phi_s = \tan^{-1}(0.35) = 19.29°$$

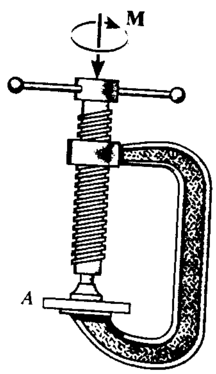
$$\theta_p = \tan^{-1}\left[\frac{3}{2\pi(10)}\right] = 2.734°$$

$$M = W(r)\tan(\phi_s + \theta_p)$$

$$8 = P(0.01)\tan(19.29° + 2.734°)$$

$$P = 1978\ \text{N} = 1.98\ \text{kN} \qquad \textbf{Ans}$$

8-78. If the required clamping force at the board A is to be 50 N, determine the torque M that must be applied to the handle of the "C" clamp to tighten it down. The single square-threaded screw has a mean radius of 10 mm, a lead of 3 mm, and the coefficient of static friction is $\mu_s = 0.35$.



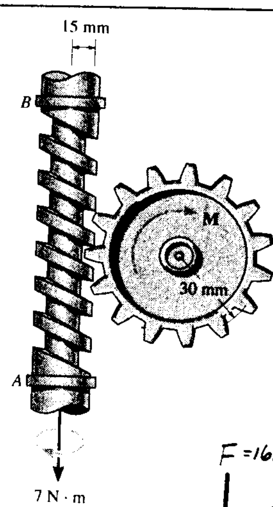
$$\phi_s = \tan^{-1}(0.35) = 19.29°$$

$$\theta_p = \tan^{-1}\left(\frac{P}{2\pi r}\right) = \tan^{-1}\left[\frac{3}{2\pi(10)}\right] = 2.734°$$

$$M = W(r)\tan(\phi_s + \theta_p)$$

$$= 50(0.01)\tan(19.29° + 2.734°) = 0.202\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

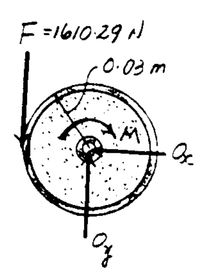
8-79. The shaft has a square-threaded screw with a lead of 8 mm and a mean radius of 15 mm. If it is in contact with a plate gear having a mean radius of 30 mm, determine the resisting torque **M** on the plate gear which can be overcome if a torque of 7 N · m is applied to the shaft. The coefficient of static friction at the screw is $\mu_B = 0.2$. Neglect friction of the bearings located at A and B.



Frictional Forces on Screw : Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{8}{2\pi(15)}\right] = 4.852°$, $W = F$, $M = 7\ \text{N}\cdot\text{m}$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.2) = 11.310°$. Applying Eq. 8 – 3, we have

$$M = Wr\tan(\theta + \phi)$$
$$7 = F(0.015)\tan(4.852° + 11.310°)$$
$$F = 1610.29\ \text{N}$$

Note : Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if force **F** is removed.

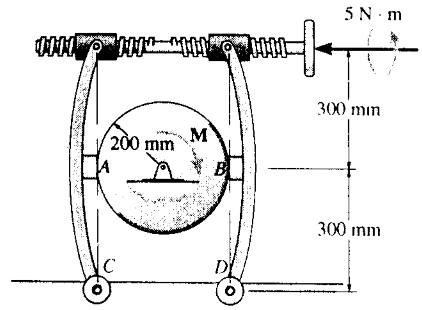


Equations of Equilibrium :

$$+\Sigma M_O = 0; \quad 1610.29(0.03) - M = 0$$

$$M = 48.3\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

***8-80.** The braking mechanism consists of two pinned arms and a square-threaded screw with left and right-hand threads. Thus when turned, the screw draws the two arms together. If the lead of the screw is 4 mm, the mean diameter 12 mm, and the coefficient of static friction is $\mu_s = 0.35$, determine the tension in the screw when a torque of 5 N · m is applied to tighten the screw. If the coefficient of static friction between the brake pads A and B and the circular shaft is $\mu'_s = 0.5$, determine the maximum torque M the brake can resist.



Frictional Forces on Screw : Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{4}{2\pi(6)}\right] = 6.057°$, $M = 5$ N · m and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.35) = 19.290°$. Since friction at two screws must be overcome, then, $W = 2P$. Applying Eq. 8 – 3, we have

$$M = Wr\tan(\theta + \phi)$$
$$5 = 2P(0.006)\tan(6.057° + 19.290°)$$
$$P = 879.61 \text{ N} = 880 \text{ N} \qquad \textbf{Ans}$$

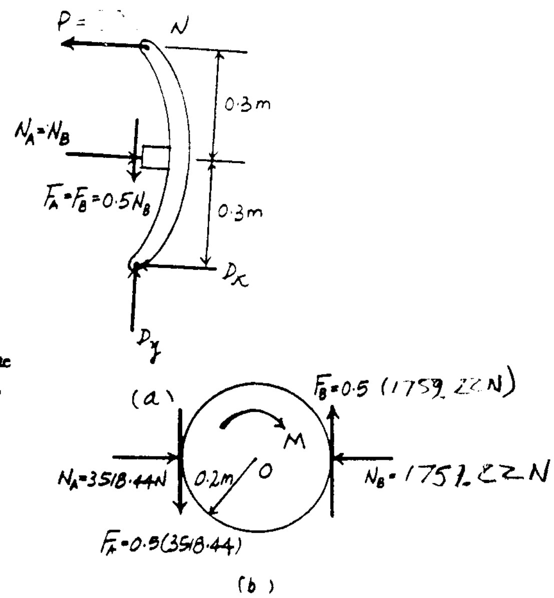
Note : Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if moment M is removed.

Equations of Equilibrium and Friction : Since the shaft is on the verge to rotate about point O, then, $F_A = \mu_s' N_A = 0.5N_A$ and $F_B = \mu_s' N_B = 0.5N_B$. From FBD (a),

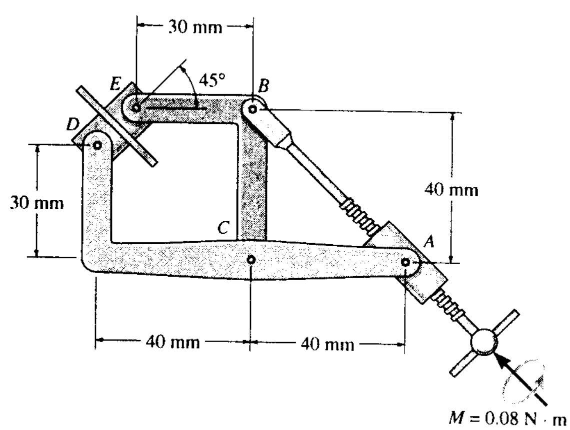
$$\circlearrowleft + \Sigma M_D = 0; \quad 879.61(0.6) - N_B(0.3) = 0 \quad N_B = 1759.22 \text{ N}$$

From FBD (b),

$$\circlearrowleft + \Sigma M_O = 0; \quad 2[0.5(1759.22)](0.2) - M = 0 \quad M = 352 \text{ N} \cdot \text{m} \qquad \textbf{Ans}$$



8-81. The fixture clamp consist of a square-threaded screw having a coefficient of static friction of $\mu_s = 0.3$, mean diameter of 3 mm, and a lead of 1 mm. The five points indicated are pin connections. Determine the clamping force at the smooth blocks D and E when a torque of $M = 0.08$ N · m is applied to the handle of the screw



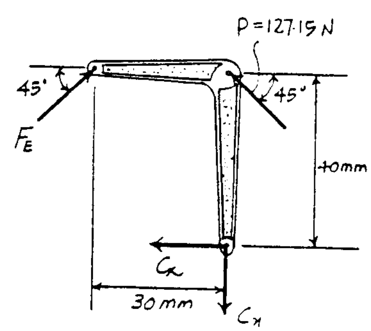
Frictional Forces on Screw : Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{1}{2\pi(1.5)}\right] = 6.057°$, $W = P$, $M = 0.08$ N · m and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$. Applying Eq. 8 – 3, we have

$$M = Wr\tan(\theta + \phi)$$
$$0.08 = P(0.0015)\tan(6.057° + 16.699°)$$
$$P = 127.15 \text{ N}$$

Note : Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if moment M is removed.

Equation of Equilibrium :

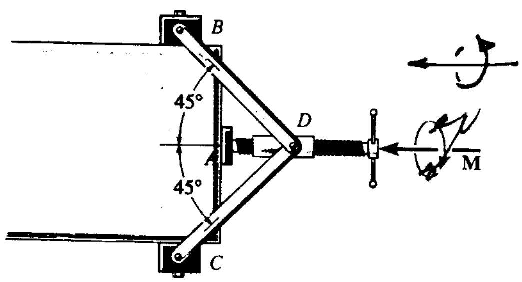
$$\circlearrowleft + \Sigma M_C = 0; \quad 127.15\cos 45°(40) - F_E\cos 45°(40) - F_E\sin 45°(30) = 0$$
$$F_E = 72.65 \text{ N} = 72.7 \text{ N} \qquad \textbf{Ans}$$



The equilibrium of clamped block requires that

$$F_D = F_E = 72.7 \text{ N} \qquad \textbf{Ans}$$

8-82. The clamp provides pressure from several directions on the edges of the board. If the square-threaded screw has a lead of 3 mm, radius of 10 mm, and the coefficient of static friction is $\mu_s = 0.4$, determine the horizontal force developed on the board at A and the vertical forces developed at B and C if a torque of $M = 1.5\ \text{N}\cdot\text{m}$ is applied to the handle to tighten it further. The blocks at B and C are pin-connected to the board.



$$\phi_s = \tan^{-1}(0.4) = 21.801°$$

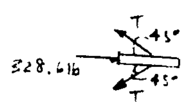
$$\theta_p = \tan^{-1}\left[\frac{3}{2\pi(10)}\right] = 2.734°$$

$$M = W(r)\tan(\phi_s + \theta_p)$$

$$1.5 = A_x(0.01)\tan(21.801° + 2.734°)$$

$$A_x = 328.6\ \text{N} \qquad \textbf{Ans}$$

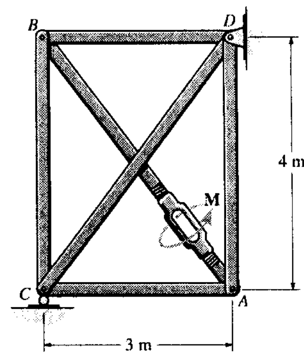
$\xrightarrow{+} \Sigma F_x = 0; \quad 328.6 - 2T\cos 45° = 0$



$$T = 232.36\ \text{N}$$

$$B_y = C_y = 232.36\sin 45° = 164\ \text{N} \qquad \textbf{Ans}$$

8-83. A turnbuckle, similar to that shown in Fig. 8-18, is used to tension member AB of the truss. The coefficient of the static friction between the square threaded screws and the turnbuckle is $\mu_s = 0.5$. The screws have a mean radius of 6 mm and a lead of 3 mm. If a torque of $M = 10\ \text{N}\cdot\text{m}$ is applied to the turnbuckle, to draw the screws closer together, determine the force in each member of the truss. No external forces act on the truss.



Frictional Forces on Screw: Here, $\theta = \tan^{-1}\left(\dfrac{l}{2\pi r}\right) = \tan^{-1}\left[\dfrac{3}{2\pi(6)}\right] = 4.550°$, $M = 5\ \text{N}\cdot\text{m}$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.5) = 26.565°$. Since friction at two screws must be overcome, then, $W = 2F_{AB}$. Applying Eq. 8-3, we have

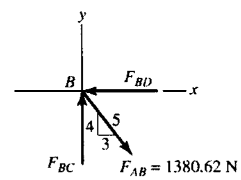
$$M = Wr\tan(\theta + \phi)$$

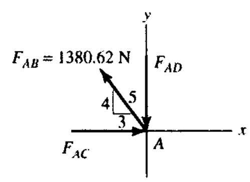
$$10 = 2F_{AB}(0.006)\tan(4.550° + 26.565°)$$

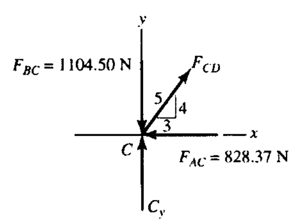
$F_{AB} = 1380.62\ \text{N (T)} = 1.38\ \text{kN(T)}$ **Ans**

Note: Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if moment **M** is removed.

Method of Joints:







Joint B

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 1380.62\left(\dfrac{3}{5}\right) - F_{BD} = 0$

$F_{BD} = 828.37\ \text{N(C)} = 828\ \text{N (C)}$ **Ans**

$+\uparrow\Sigma F_y = 0;\quad F_{BC} - 1380.62\left(\dfrac{4}{5}\right) = 0$

$F_{BC} = 1104.50\ \text{N (C)} = 1.10\ \text{kN (C)}$ **Ans**

Joint A

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{AC} - 1380.62\left(\dfrac{3}{5}\right) = 0$

$F_{AC} = 828.37\ \text{N (C)} = 828\ \text{N (C)}$ **Ans**

$+\uparrow\Sigma F_y = 0;\quad 1380.62\left(\dfrac{4}{5}\right) - F_{AD} = 0$

$F_{AD} = 1104.50\ \text{N (C)} = 1.10\ \text{kN (C)}$ **Ans**

Joint C

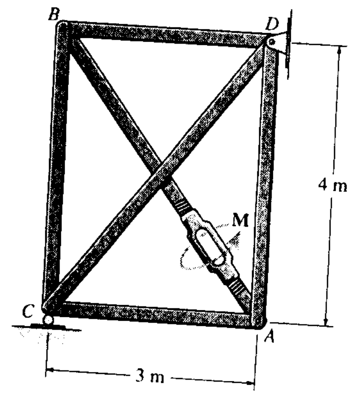
$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad F_{CD}\left(\dfrac{3}{5}\right) - 828.37 = 0$

$F_{CD} = 1380.62\ \text{N (T)} = 1.38\ \text{kN (T)}$ **Ans**

$+\uparrow\Sigma F_y = 0;\quad C_y + 1380.62\left(\dfrac{4}{5}\right) - 1104.50 = 0$

$C_y = 0$ (***No external applied load. check!***)

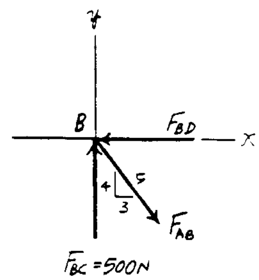
***8-84.** A turnbuckle, similar to that shown in Fig. 8-18, is used to tension member AB of the truss. The coefficient of the static friction between the square-threaded screws and the turnbuckle is $\mu_s = 0.5$. The screws have a mean radius of 6 mm and a lead of 3 mm. Determine the torque M which must be applied to the turnbuckle to draw the screws closer together, so that the compressive force of 500 N is developed in member BC.



Method of Joints :

Joint B

$$+\uparrow \Sigma F_y = 0; \quad 500 - F_{AB}\left(\frac{4}{5}\right) = 0 \quad F_{AB} = 625\text{ N (C)}$$

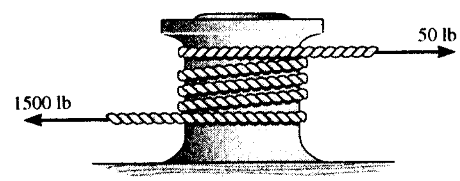


Frictional Forces on Screws : Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{3}{2\pi(6)}\right]$ $= 4.550°$, $M = 5$ N·m and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.5) = 26.565°$. Since friction at two screws must be overcome, then, $W = 2F_{AB} = 2(625) = 1250$ N. Applying Eq. 8 – 3, we have

$$\begin{aligned} M &= Wr\tan(\theta + \phi) \\ &= 1250(0.006)\tan(4.550° + 26.565°) \\ &= 4.53\text{ N}\cdot\text{m} \end{aligned}$$ **Ans**

Note : Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if moment M is removed.

8-85. A "hawser" is wrapped around a fixed "capstan" to secure a ship for docking. If the tension in the rope, caused by the ship, is 1500 lb, determine the least number of complete turns the rope must be rapped around the capstan in order to prevent slipping of the rope. The greatest horizontal force that a longshoreman can exert on the rope is 50 lb. The coefficient of static friction is $\mu_s = 0.3$.



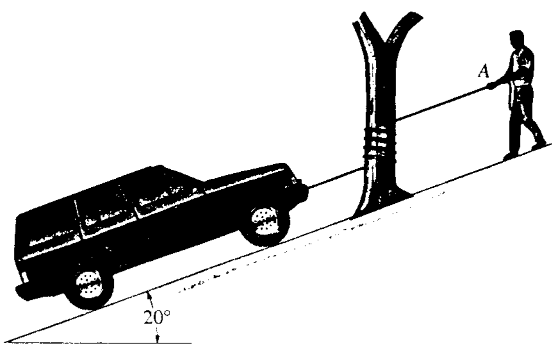
Frictional Force on Flat Belt : Here, $T_1 = 50$ lb and $T_2 = 1500$ lb. Applying Eq. 8 – 6, we have

$$T_2 = T_1 e^{\mu\beta}$$
$$1500 = 50e^{0.3\beta}$$
$$\beta = 11.337\text{ rad}$$

The least number of turns of the rope required is $\frac{11.337}{2\pi} = 1.80$ turns. Thus

Use $n = 2$ turns **Ans**

8-86. The truck, which has a mass of 3.4 Mg, is to be lowered down the slope by a rope that is wrapped around a tree. If the wheels are free to roll and the man at A can resist a pull of 300 N, determine the minimum number of turns the rope should be wrapped around the tree to lower the truck at a constant speed. The coefficient of kinetic friction between the tree and rope is $\mu_k = 0.3$.



$\nearrow^{+}\Sigma F_x = 0;\quad T_2 - 33\,354\sin 20^\circ = 0$

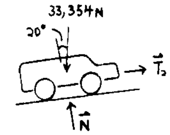
$T_2 = 11\,407.7$

$T_2 = T_1 e^{\mu\beta}$

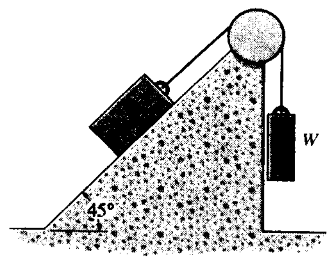
$11\,407.7 = 300\,e^{0.3\beta}$

$\beta = 12.1275$ rad

Approx. 2 turns (695°) **Ans**



8-87. Determine the maximum and the minimum values of weight W which may be applied without causing the 50-lb block to slip. The coefficient of static friction between the block and the plane is $\mu_s = 0.2$, and between the rope and the drum D $\mu'_s = 0.3$.



Equations of Equilibrium and Friction : Since the block is on the verge of sliding up or down the plane, then, $F = \mu_s N = 0.2N$. If the block is on the verge of sliding up the plane [FBD (a)],

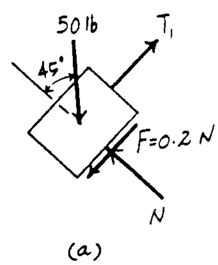
$$\nwarrow^{+}\Sigma F_{y'} = 0;\quad N - 50\cos 45^\circ = 0 \quad N = 35.36 \text{ lb}$$

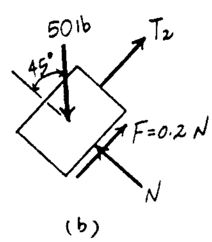
$$\nearrow^{+}\Sigma F_{x'} = 0;\quad T_1 - 0.2(35.36) - 50\sin 45^\circ = 0 \quad T_1 = 42.43 \text{ lb}$$

If the block is on the verge of sliding down the plane[FBD (b)],

$$\nwarrow^{+}\Sigma F_{y'} = 0;\quad N - 50\cos 45^\circ = 0 \quad N = 35.36 \text{ lb}$$

$$\nearrow^{+}\Sigma F_{x'} = 0;\quad T_2 + 0.2(35.36) - 50\sin 45^\circ = 0 \quad T_2 = 28.28 \text{ lb}$$



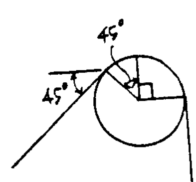


Frictional Force on Flat Belt : Here, $\beta = 45^\circ + 90^\circ = 135^\circ = \frac{3\pi}{4}$ rad.

If the block is on the verge of sliding up the plane, $T_1 = 42.43$ lb and $T_2 = W$.

$$T_2 = T_1 e^{\mu\beta}$$
$$W = 42.43e^{0.3\left(\frac{3\pi}{4}\right)}$$
$$= 86.02 \text{ lb} = 86.0 \text{ lb}$$ **Ans**

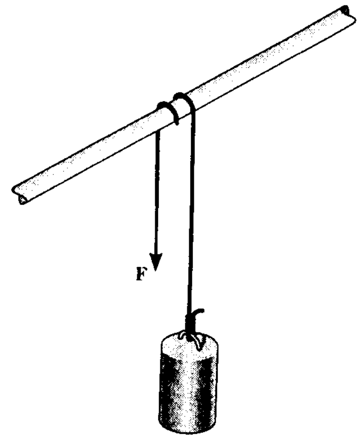
If the block is on the verge of sliding down the plane, $T_1 = W$ and $T_2 = 28.28$ lb.



$$T_2 = T_1 e^{\mu\beta}$$
$$28.28 = We^{0.3\left(\frac{3\pi}{4}\right)}$$

$$W = 13.95 \text{ lb} = 13.9 \text{ lb}$$ **Ans**

***8-88.** A cylinder having a mass of 250 kg is to be supported by the cord which wraps over the pipe. Determine the smallest vertical force **F** needed to support the load if the cord passes (a) once over the pipe, $\beta = 180°$, and (b) two times over the pipe, $\beta = 540°$. Take $\mu_s = 0.2$.



Frictional Force on Flat Belt : Here, $T_1 = F$ and $T_2 = 250(9.81) = 2452.5$ N. Applying Eq. 8 – 6, we have

a) If $\beta = 180° = \pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$
$$2452.5 = Fe^{0.2\pi}$$

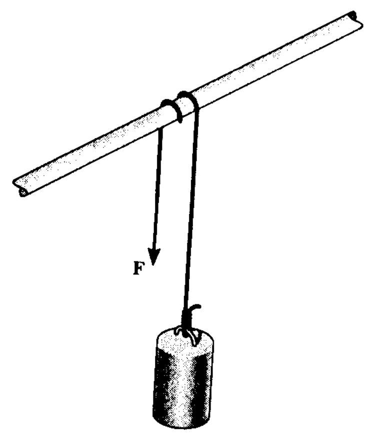
$$F = 1308.38 \text{ N} = 1.31 \text{ kN} \qquad \textbf{Ans}$$

b) If $\beta = 540° = 3\pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$
$$2452.5 = Fe^{0.2(3\pi)}$$

$$F = 372.38 \text{ N} = 372 \text{ N} \qquad \textbf{Ans}$$

8-89. A cylinder having a mass of 250 kg is to be supported by the cord which wraps over the pipe. Determine the largest vertical force **F** that can be applied to the cord without moving the cylinder. The cord passes (a) once over the pipe, $\beta = 180°$, and (b) two times over the pipe, $\beta = 540°$. Take $\mu_s = 0.2$.



Frictional Force on Flat Belt : Here, $T_1 = 250(9.81) = 2452.5$ N and $T_2 = F$. Applying Eq. 8 – 6, we have

a) If $\beta = 180° = \pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$
$$F = 2452.5e^{0.2\pi}$$

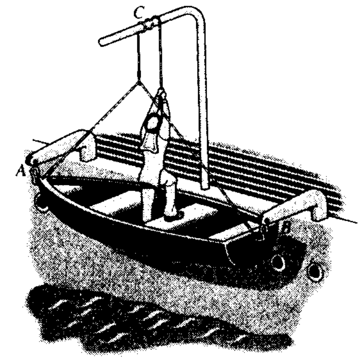
$$F = 4597.10 \text{ N} = 4.60 \text{ kN} \qquad \textbf{Ans}$$

b) If $\beta = 540° = 3\pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$
$$F = 2452.5e^{0.2(3\pi)}$$

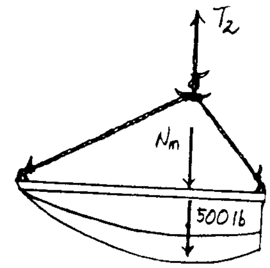
$$F = 15152.32 \text{ N} = 16.2 \text{ kN} \qquad \textbf{Ans}$$

***8-90.** The boat has a weight of 500 lb and is held in position off the side of a ship by the spars at A and B. A man having a weight of 130 lb gets in the boat, wraps a rope around an overhead boom at C, and ties it to the end of the boat as shown. If the boat is disconnected from the spars, determine the *minimum number* of *half turns* the rope must make around the boom so that the boat can be safely lowered into the water at constant velocity. Also, what is the normal force between the boat and the man? The coefficient of kinetic friction between the rope and the boom is $\mu_s = 0.15$. *Hint:* The problem requires that the normal force between the man's feet and the boat be as small as possible.



Frictional Force on Flat Belt : If the normal force between the man and the boat is equal to zero, then, $T_1 = 130$ lb and $T_2 = 500$ lb. Applying Eq. 8 – 6, we have

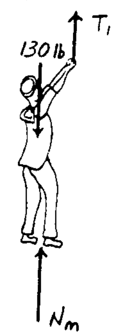
$$T_2 = T_1 e^{\mu\beta}$$
$$500 = 130 e^{0.15\beta}$$
$$\beta = 8.980 \text{ rad}$$



(a)

The least number of half turns of the rope required is $\frac{8.980}{\pi} = 2.86$ turns. Thus

Use $n = 3$ half turns **Ans**



(b)

Equations of Equilibrium : From FBD (a),

$$+\uparrow \Sigma F_y = 0; \quad T_2 - N_m - 500 = 0 \quad T_2 = N_m + 500$$

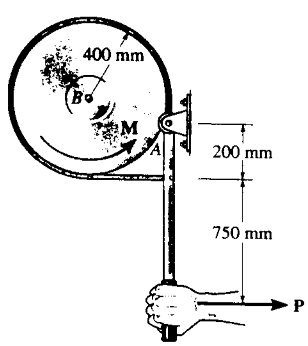
From FBD (b),

$$+\uparrow \Sigma F_y = 0; \quad T_1 + N_m - 130 = 0 \quad T_1 = 130 - N_m$$

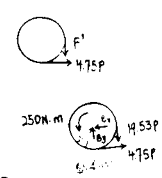
Frictional Force on Flat Belts : Here, $\beta = 3\pi$ rad. Applying Eq. 8 – 6, we have

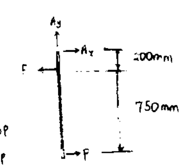
$$T_2 = T_1 e^{\mu\beta}$$
$$N_m + 500 = (130 - N_m) e^{0.15(3\pi)}$$
$$N_m = 6.74 \text{ lb}$$ **Ans**

8-91. Determine the smallest lever force P needed to prevent the wheel from rotating if it is subjected to a torque of $M = 250$ N · m. The coefficient of static friction between the belt and the wheel is $\mu_s = 0.3$. The wheel is pin-connected at its center, B.



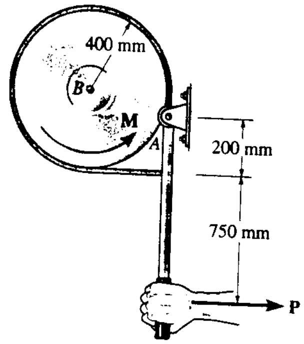
$$\curvearrowleft + \Sigma M_A = 0; \quad -F(200) + P(950) = 0$$
$$F = 4.75\,P$$



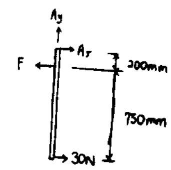


$$T_2 = T_1 e^{\mu\beta}$$
$$F' = 4.75\,P\, e^{0.3(\frac{3\pi}{2})} = 19.53\,P$$
$$\curvearrowleft + \Sigma M_B = 0; \quad -19.53\,P(0.4) + 250 + 4.75P(0.4) = 0$$
$$P = 42.3 \text{ N}$$ **Ans**

*8-92. Determine the torque M that can be resisted by the band brake if a force of $P = 30$ N is applied to the handle of the lever. The coefficient of static friction between the belt and the wheel is $\mu_s = 0.3$. The wheel is pin-connected at its center, B.



$$\circlearrowleft + \Sigma M_A = 0; \qquad -F(200) + 30(950) = 0$$



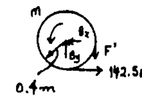
$$F = 142.5 \text{ N}$$

$$T_2 = T_1 e^{\mu\beta}$$

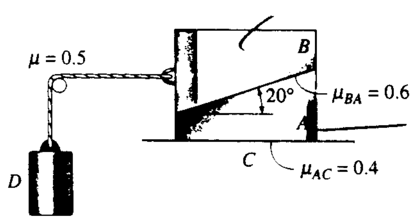
$$F' = 142.5\, e^{0.3(\frac{3\pi}{2})} = 585.8 \text{ N}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad -585.8(0.4) + 142.5(0.4) + M = 0$$

$$M = 177 \text{ N}\cdot\text{m} \qquad \textbf{Ans}$$



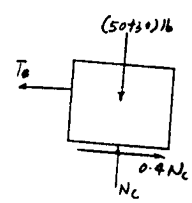
8-93. Blocks A and B weigh 50 lb and 30 lb, respectively. Using the coefficients of static friction indicated, determine the greatest weight of block D without causing motion.



For block A and B : Assuming block B does not slip

$+\uparrow \Sigma F_y = 0; \quad N_C - (50+30) = 0 \qquad N_C = 80$ lb

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 0.4(80) - T_B = 0 \qquad T_B = 32$ lb



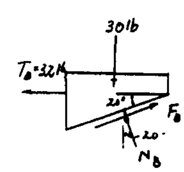
For block B :

$+\uparrow \Sigma F_y = 0; \quad N_B \cos 20° + F_B \sin 20° - 30 = 0 \qquad [1]$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_B \cos 20° - N_B \sin 20° - 32 = 0 \qquad [2]$

Solving Eqs.[1] and [2] yields :

$F_B = 40.32$ lb $\quad N_B = 17.25$ lb

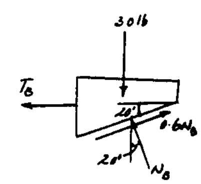


Since $F_B = 40.32$ lb $> \mu N_B = 0.6(17.25) = 10.35$ lb, slipping does occur between A and B. Therefore, the assumption is no good .

Since slipping occurs, $F_B = 0.6\, N_B$.

$+\uparrow \Sigma F_y = 0; \quad N_B \cos 20° + 0.6 N_B \sin 20° - 30 = 0 \qquad N_B = 26.20$ lb

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 0.6(26.20)\cos 20° - 26.20 \sin 20° - T_B = 0 \qquad T_B = 5.812$ lb

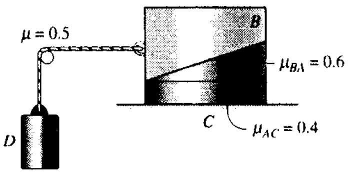


$T_2 = T_1 e^{\mu\beta}$ Where $T_2 = W_D,\ T_1 = T_B = 5.812$ lb, $\beta = 0.5\pi$ rad

$W_D = 5.812 e^{0.5(0.5\pi)}$

$= 12.7$ lb $\qquad$ **Ans**

8-94. Blocks A, B and D weigh 75 lb and 30 lb, respectively. Using the coefficients of static friction indicated, determine the frictional force between blocks A and B and between block A and the floor C.



For the rope, $T_2 = T_1 e^{\mu\beta}$, where $T_2 = 30$ lb, $T_1 = T_B$, and $\beta = 0.5\pi$ rad.

$30 = T_B e^{0.5(0.5\pi)}$

$T_B = 13.678$ lb

$F_C = 13.7$ lb **Ans**

For block B:

$+\uparrow \Sigma F_y = 0;\ N_B \cos 20° + F_B \sin 20° - 75 = 0$ [1]

$\overset{+}{\rightarrow} \Sigma F_x = 0;\ F_B \cos 20° - N_B \sin 20° - 13.678 = 0$ [2]

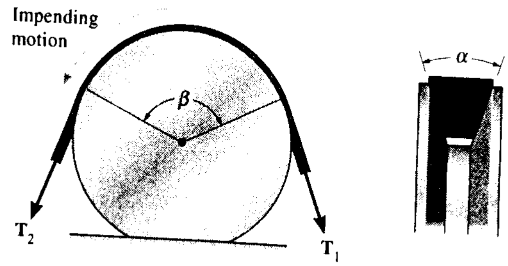
Solving Eqs. [1] and [2] yields:

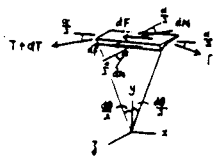
$N_B = 65.8$ lb

$F_B = 38.5$ lb **Ans**

Since $F_B = 38.5$ lb $< \mu N_B = 0.6(65.8) = 39.5$ lb, slipping between A and B does not occur.

8-95. Show that the frictional relationship between the belt tensions, the coefficient of friction μ, and the angular contacts α and β for the V-belt is $T_2 = T_1 e^{\mu\beta/\sin(\alpha/2)}$.





F.B.D of a section of the belt is shown.
Proceeding in the general manner :

$\Sigma F_x = 0;\qquad -(T+dT)\cos\dfrac{d\theta}{2} + T\cos\dfrac{d\theta}{2} + 2\,dF = 0$

$\Sigma F_y = 0;\qquad -(T+dT)\sin\dfrac{d\theta}{2} - T\sin\dfrac{d\theta}{2} + 2\,dN\sin\dfrac{\alpha}{2} = 0$

Replace $\sin\dfrac{d\theta}{2}$ by $\dfrac{d\theta}{2}$,

$\cos\dfrac{d\theta}{2}$ by 1,

$dF = \mu\, dN$

Using this and $(dT)(d\theta) \to 0$, the above relations become

$$dT = 2\mu\, dN$$

$$T\,d\theta = 2\left(dN\sin\frac{\alpha}{2}\right)$$

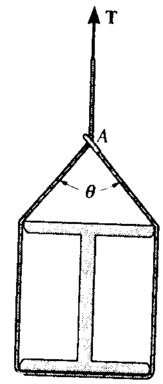
Combine $\qquad \dfrac{dT}{T} = \mu\dfrac{d\theta}{\sin\frac{\alpha}{2}}$

Integrate from $\theta = 0,\ T = T_1$
to $\theta = \beta,\ T = T_2$

we get,

$$T_2 = T_1\, e^{\left(\frac{\mu\beta}{\sin\frac{\alpha}{2}}\right)} \qquad \textbf{Q.E.D}$$

*8-96. The smooth beam is being hoisted using a rope which is wrapped around the beam and passes through a ring at A as shown. If the end of the rope is subjected to a tension $\mathbf{T}$ and the coefficient of static friction between the rope and ring is $\mu_s = 0.3$, determine the angle of θ for equilibrium.

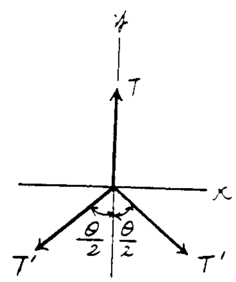


Equation of Equilibrium :

$$+\uparrow \Sigma F_y = 0; \quad T - 2T'\cos\frac{\theta}{2} = 0 \quad T = 2T'\cos\frac{\theta}{2} \qquad [1]$$

Frictional Force on Flat Belt : Here, $\beta = \frac{\theta}{2}$, $T_2 = T$ and $T_1 = T'$. Applying Eq. 8–6 $T_2 = T_1 e^{\mu\beta}$, we have

$$T = T'e^{0.3(\theta/2)} = T'e^{0.15\theta} \qquad [2]$$



Substituting Eqs. [1] into [2] yields

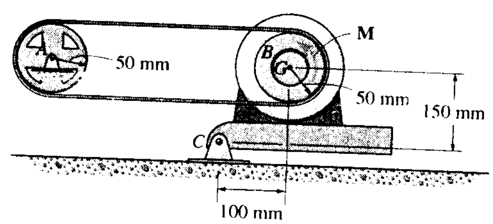
$$2T'\cos\frac{\theta}{2} = T'e^{0.15\theta}$$

$$e^{0.15\theta} = 2\cos\frac{\theta}{2}$$

Solving by trial and error

$$\theta = 1.73104 \text{ rad} = 99.2° \qquad \textbf{Ans}$$

8-97. The 20-kg motor has a center of gravity at G and is pin-connected at C to maintain a tension in the drive belt. Determine the smallest counterclockwise twist or torque $\mathbf{M}$ that must be supplied by the motor to turn the disk B if wheel A locks and causes the belt to slip over the disk. No slipping occurs at A. The coefficient of static friction between the belt and the disk is $\mu_s = 0.3$.



Equations of Equilibrium : From FBD (a),

$$\circlearrowleft + \Sigma M_C = 0; \quad T_2(100) + T_1(200) - 196.2(100) = 0 \qquad [1]$$

From FBD (b),

$$\circlearrowleft + \Sigma M_O = 0; \quad M + T_1(0.05) - T_2(0.05) = 0 \qquad [2]$$

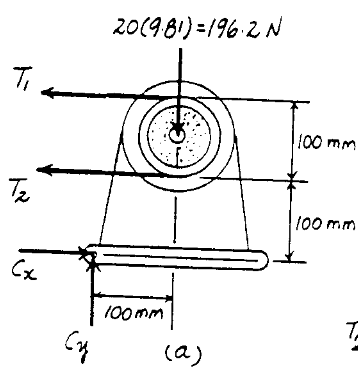
Frictional Force on Flat Belt : Here, $\beta = 180° = \pi$ rad. Applying Eq. 8–6, $T_2 = T_1 e^{\mu\beta}$, we have

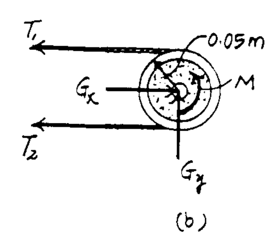
$$T_2 = T_1 e^{0.3\pi} = 2.566T_1 \qquad [3]$$

Solving Eqs. [1], [2] and [3] yields

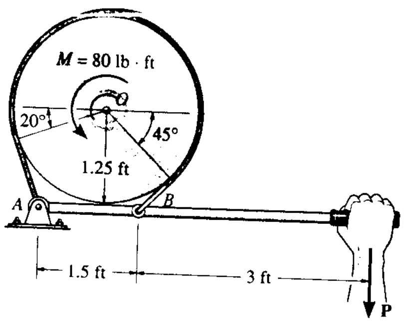
$$M = 3.37 \text{ N}\cdot\text{m} \qquad \textbf{Ans}$$

$$T_1 = 42.97 \text{ N} \quad T_2 = 110.27 \text{ N}$$





8-98. The simple band brake is constructed so that the ends of the friction strap are connected to the pin at A and the lever arm at B. If the wheel is subjected to a torque of $M = 80$ lb·ft, determine the smallest force P applied to the lever that is required to hold the wheel stationary. The coefficient of static friction between the strap and wheel is $\mu_s = 0.5$.



$\beta = 20° + 180° + 45° = 245°$

$\curvearrowleft + \Sigma M_O = 0; \quad T_1(1.25) + 80 - T_2(1.25) = 0$

$T_2 = T_1 e^{\mu\beta}; \quad T_2 = T_1 e^{0.5(245°)(\frac{\pi}{180°})} = 8.4827 T_1$

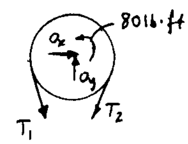
Solving;

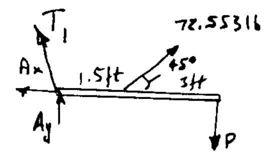
$T_1 = 8.553$ lb

$T_2 = 72.553$ lb

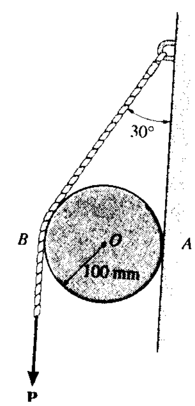
$\curvearrowleft + \Sigma M_A = 0; \quad -72.553(\sin 45°)(1.5) - 4.5P = 0$

$P = 17.1$ lb **Ans**





8-99. The cylinder weighs 10 lb and is held in equilibrium by the belt and wall. If slipping does not occur at the wall, determine the minimum vertical force P which must be applied to the belt for equilibrium. The coefficient of static friction between the belt and the cylinder is $\mu_s = 0.25$.



Equations of Equilibrium :

$\curvearrowleft + \Sigma M_A = 0; \quad P(0.2) + 10(0.1) - T_2 \cos 30°(0.1 + 0.1\cos 30°)$
$- T_2 \sin 30°(0.1 \sin 30°) = 0$ [1]

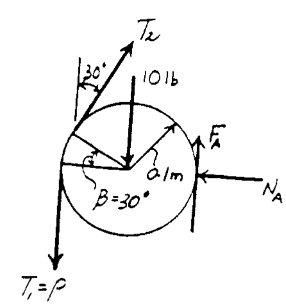
Frictional Force on Flat Belt : Here, $\beta = 30° = \frac{\pi}{6}$ rad and $T_1 = P$. Applying Eq. 8–6, $T_2 = T_1 e^{\mu\beta}$, we have

$$T_2 = Pe^{0.25(\pi/6)} = 1.140P \quad [2]$$

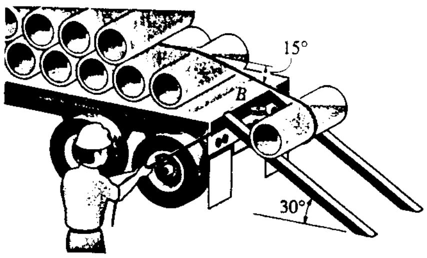
Solving Eqs. [1] and [2] yields

$P = 78.7$ lb **Ans**

$T_2 = 89.76$ lb



***8-100.** The uniform concrete pipe has a weight of 800 lb and is unloaded slowly from the truck bed using the rope and skids shown. If the coefficient of kinetic friction between the rope and pipe is $\mu_k = 0.3$, determine the force the worker must exert on the rope to lower the pipe at constant speed. There is a pulley at B, and the pipe does not slip on the skids. The lower portion of the rope is parallel to the skids.



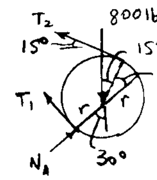
$\curvearrowleft + \Sigma M_A = 0; \quad -800(r \sin 30°) + T_2 \cos 15°(r \cos 15° + r \cos 30°) + T_2 \sin 15°(r \sin 15° + r \sin 15°) =$

$T_2 = 203.466$ lb

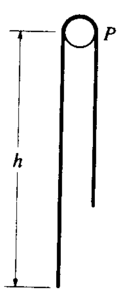
$\beta = 180° + 15° = 195°$

$T_2 = T_1 e^{\mu\beta}, \quad 203.466 = T_1 e^{(0.3)(\frac{195°}{180°})(\pi)}$

$T_1 = 73.3$ lb **Ans**



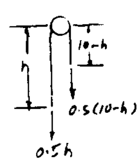
8-101. A cord having a weight of 0.5 lb/ft and a total length of 10 ft is suspended over a peg P as shown. If the coefficient of static friction between the peg and cord is $\mu_s = 0.5$, determine the longest length h which one side of the suspended cord can have without causing motion. Neglect the size of the peg and the length of cord draped over it.



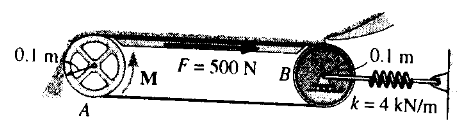
$T_2 = T_1 e^{\mu\beta}$ Where $T_2 = 0.5h$, $T_1 = 0.5(10 - h)$, $\beta = \pi$ rad

$0.5h = 0.5(10 - h)e^{0.5(\pi)}$

$h = 8.28$ ft **Ans**

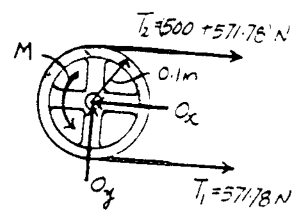


8-102. A conveyer belt is used to transfer granular material and the frictional resistance on the top of the belt is $F = 500$ N. Determine the smallest stretch of the spring attached to the moveable axle of the idle pulley B so that the belt does not slip at the drive pulley A when the torque **M** is applied. What minimum torque **M** is required to keep the belt moving? The coefficient of static friction between the belt and the wheel at A is $\mu_s = 0.2$.



Frictional Force on Flat Belt : Here, $\beta = 180° = \pi$ rad and $T_2 = 500 + T$ and $T_1 = T$. Applying Eq. 8 – 6, , we have

$$T_2 = T_1 e^{\mu\beta}$$
$$500 + T = Te^{0.2\pi}$$
$$T = 571.78 \text{ N}$$



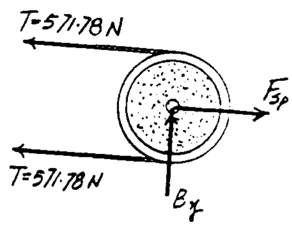
(a)

Equations of Equilibrium : From FBD (a),

$$\circlearrowleft + \Sigma M_O = 0; \quad M + 571.78(0.1) - (500 + 578.1)(0.1) = 0$$
$$M = 50.0 \text{ N} \cdot \text{m} \qquad \textbf{Ans}$$

From FBD (b),

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{sp} - 2(578.71) = 0 \qquad F_{sp} = 1143.57 \text{ N}$$

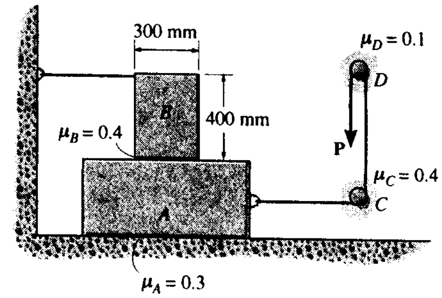


(b)

Thus, the spring stretch is

$$x = \frac{F_{sp}}{k} = \frac{1143.57}{4000} = 0.2859 \text{ m} = 286 \text{ mm} \qquad \textbf{Ans}$$

8-103. Blocks A and B have a mass of 7 kg and 10 kg, respectively. Using the coefficients of static friction indicated, determine the largest vertical force P which can be applied to the cord without causing motion.

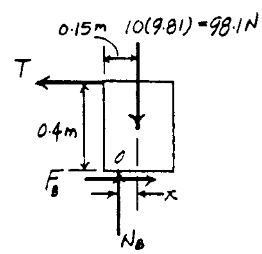


Frictional Forces on Flat Belts : When the cord pass over peg D, $\beta = 180° = \pi$ rad and $T_2 = P$. Applying Eq. 8 – 6, $T_2 = T_1 e^{\mu\beta}$, we have

$$P = T_1 e^{0.1\pi} \qquad T_1 = 0.7304P$$

When the cord pass over peg C, $\beta = 90° = \frac{\pi}{2}$ rad and $T_2' = T_1 = 0.7304P$. Applying Eq. 8 – 6, $T_2' = T_1' e^{\mu\beta}$, we have

$$0.7304P = T_1' e^{0.4(\pi/2)} \qquad T_1' = 0.3897P$$



(a)

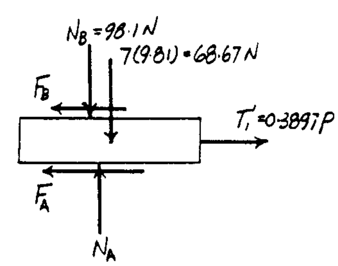
Equations of Equilibrium : From FBD (b),

$$+\uparrow \Sigma F_y = 0; \qquad N_B - 98.1 = 0 \qquad N_B = 98.1 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_B - T = 0 \qquad [1]$$

$$\curvearrowleft + \Sigma M_O = 0; \qquad T(0.4) - 98.1(x) = 0 \qquad [2]$$

From FBD (b),



(b)

$$+\uparrow \Sigma F_y = 0; \qquad N_A - 98.1 - 68.67 = 0 \qquad N_A = 166.77 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3897P - F_B - F_A = 0 \qquad [3]$$

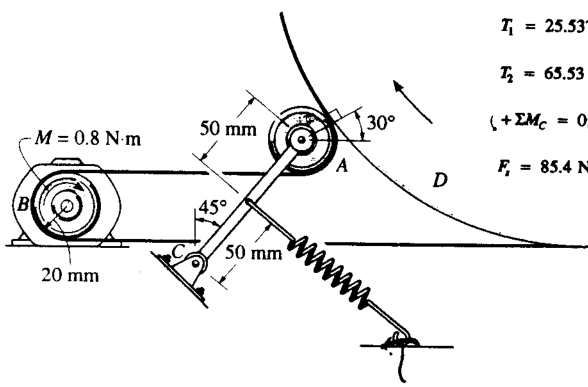
Friction : Assuming the block B is on the verge of tipping, then $x = 0.15$ m. Al for motion to occur, block A will have slip. Hence, $F_A = (\mu_s)_A N_A = 0.3(166.77) = 50.031$ N. Substituting these values into Eqs. [1], [2] and [3] and solving yields

$$P = 222.81 \text{ N} = 223 \text{ N} \qquad \textbf{Ans}$$

$$F_B = T = 36.79 \text{ N}$$

Since $(F_B)_{max} = (\mu_s)_B N_B = 0.4(98.1) = 39.24 \text{ N} > F_B$, block B does not slip but tips. Therefore, the above assumption is correct.

***8-104.** The belt on the portable dryer wraps around the drum D, idler pulley A, and motor pulley B. If the motor can develop a maximum torque of $M = 0.80\ \text{N}\cdot\text{m}$, determine the smallest spring tension required to hold the belt from slipping. The coefficient of static friction between the belt and the drum and motor pulley is $\mu_s = 0.3$.

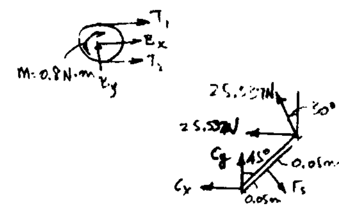


$\circlearrowleft + \Sigma M_B = 0;\quad -T_1(0.02) + T_2(0.02) - 0.8 = 0$

$T_2 = T_1 e^{\mu\beta};\quad T_2 = T_1 e^{(0.3)(\pi)} = 2.5663 T_1$

$T_1 = 25.537\ \text{N}$

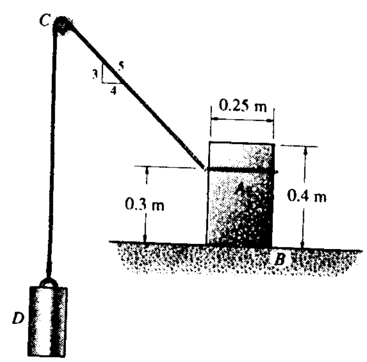
$T_2 = 65.53\ \text{N}$



$\circlearrowleft + \Sigma M_C = 0;\quad -F_s(0.05) + (25.537 + 25.537\sin 30^\circ)(0.1\cos 45^\circ) + 25.537\cos 30^\circ(0.1\sin 45^\circ) = 0$

$F_s = 85.4\ \text{N}$ **Ans**

8-105. Block A has a mass of 50 kg and rests on surface B for which $\mu_s = 0.25$. If the coefficient of static friction between the cord and the fixed peg at C is $\mu'_s = 0.3$, determine the greatest mass of the suspended cylinder D without causing motion.

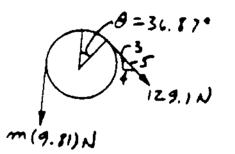


Block A :

Assume block A slips and does not tip.

$+\uparrow \Sigma F_y = 0;\quad N_B + \frac{3}{5}T - 50(9.81) = 0$

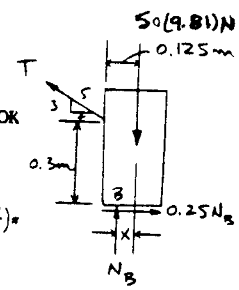
$\xrightarrow{+} \Sigma F_x = 0;\quad 0.25 N_B - \frac{4}{5}T = 0$



$N_B = 413.1\ \text{N}$

$T = 129.1\ \text{N}$

$\circlearrowleft \Sigma M_B = 0;\quad -50(9.81)x + \frac{4}{5}(129.1)(0.3) - \frac{3}{5}(129.1)(0.125 - x) = 0$



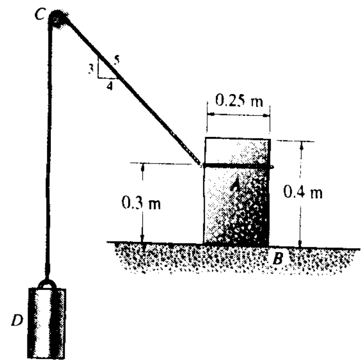
$x = 0.0516\ \text{m} < 0.125\ \text{m}$ OK

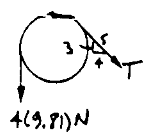
Peg :

$T_2 = T_1 e^{\mu\beta};\quad 9.81\,m = 129.1\,e^{0.3\left(\frac{90^\circ + 36.87^\circ}{180^\circ}\right)\pi}$

$m = 25.6\ \text{kg}$ **Ans**

8-106. Block A rests on the surface for which $\mu_s = 0.25$. If the mass of the suspended cylinder D is 4 kg, determine the smallest mass of block A so that it does not slip or tip. The coefficient of static friction between the cord and the fixed peg at C is $\mu_s' = 0.3$.

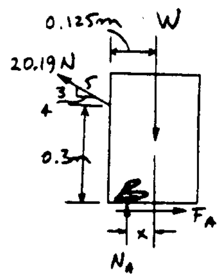




$$T_2 = T_1 e^{\mu\beta}$$

$$4(9.81) = Te^{0.3(\frac{90+36.87}{180})\pi}$$

$$T = 20.19 \text{ N}$$



$\xrightarrow{+}\Sigma F_x = 0;\quad F_A - \frac{4}{5}(20.19) = 0$

$$F_A = 16.152 \text{ N}$$

$+\uparrow\Sigma F_y = 0;\quad N_A + \frac{3}{5}(20.19) - W = 0$

For slipping,

$(F_A)_{max} = 0.25(N_A);\quad 16.152 \text{ N} = 0.25(N_A)$

$N_A = 64.61 \text{ N},\ W = 76.72 \text{ N}$

For tipping, $x = 0.125$ m

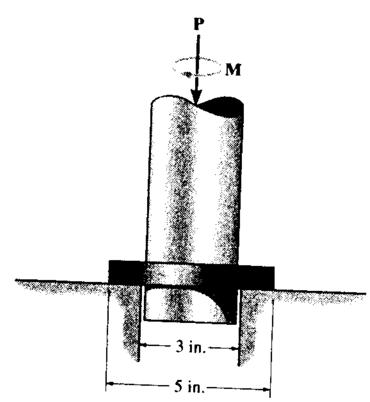
$+\Sigma M_B = 0;\quad -W(0.125 \text{ m}) + \frac{4}{5}(20.19)(0.3) = 0$

$$W = 38.8 \text{ N}$$

Require

$$m = \frac{76.72 \text{ N}}{9.81 \text{ m/s}^2} = 7.82 \text{ kg} \qquad \textbf{Ans}$$

8-107. The collar bearing uniformly supports an axial force of $P = 500$ lb. If the coefficient of static friction is $\mu_s = 0.3$, determine the torque M required to overcome friction.



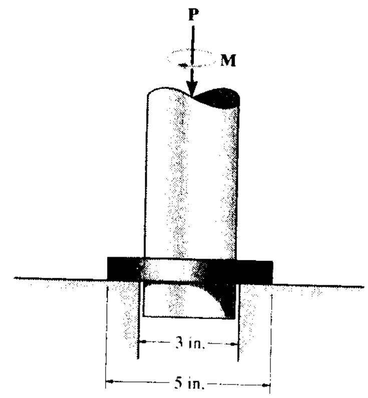
Bearing Friction : **Applying Eq. 8–7 with $R_2 = 1.5$ in., $R_1 = 1$ in., $\mu_s = 0.3$ and $P = 500$ lb, we have**

$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

$$= \frac{2}{3}(0.3)(500)\left(\frac{1.5^3 - 1^3}{1.5^2 - 1^2}\right)$$

$$= 190 \text{ lb}\cdot\text{in} = 15.8 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

*8-108. The collar bearing uniformly supports an axial force of $P = 500$ lb. If a torque of $M = 3$ lb · ft is applied to the shaft and causes it to rotate at constant velocity, determine the coefficient of kinetic friction at the surface of contact.



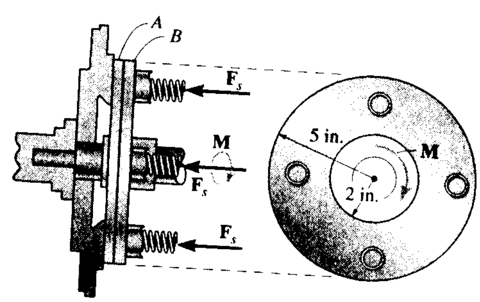
Bearing Friction **: Applying Eq. 8 – 7 with $R_2 = 1.5$ in., $R_1 = 1$ in., $M = 3(12) = 36$ lb · in and $P = 500$ lb, we have**

$$M = \frac{2}{3}\mu_k P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

$$36 = \frac{2}{3}(\mu_k)(500)\left(\frac{1.5^3 - 1^3}{1.5^2 - 1^2}\right)$$

$$\mu_k = 0.0568 \qquad \textbf{Ans}$$

8-109. The *disk clutch* is used in standard transmissions of automobiles. If four springs are used to force the two plates A and B together, determine the force in each spring required to transmit a moment of $M = 600$ lb · ft across the plates. The coefficient of static friction between A and B is $\mu_s = 0.3$.



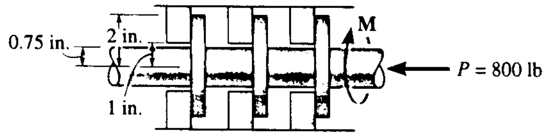
Bearing Friction **: Applying Eq. 8 – 7 with $R_2 = 5$ in., $R_1 = 2$ in., $M = 600(12) = 7200$ lb · in, $\mu_s = 0.3$ and $P = 4F_{sp}$, we have**

$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

$$7200 = \frac{2}{3}(0.3)(4F_{sp})\left(\frac{5^3 - 2^3}{5^2 - 2^2}\right)$$

$$F_{sp} = 1615.38 \text{ lb} = 1.62 \text{ kip} \qquad \textbf{Ans}$$

8-110. The annular ring bearing is subjected to a thrust of 800 lb. If $\mu_s = 0.35$, determine the torque M that must be applied to overcome friction.



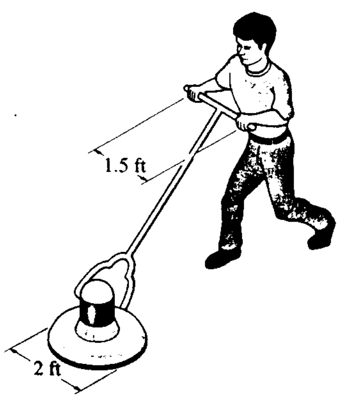
$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

$$= \frac{2}{3}(0.35)(800)\left[\frac{(2)^3 - 1^3}{(2)^2 - 1^2}\right]$$

$$= 435.6 \text{ lb·in.}$$

$M = 36.3$ lb·ft **Ans**

8-111. The floor-polishing machine rotates at a constant angular velocity. If it has a weight of 80 lb. determine the couple forces F the operator must apply to the handles to hold the machine stationary. The coefficient of kinetic friction between the floor and brush is $\mu_k = 0.3$. Assume the brush exerts a uniform pressure on the floor.

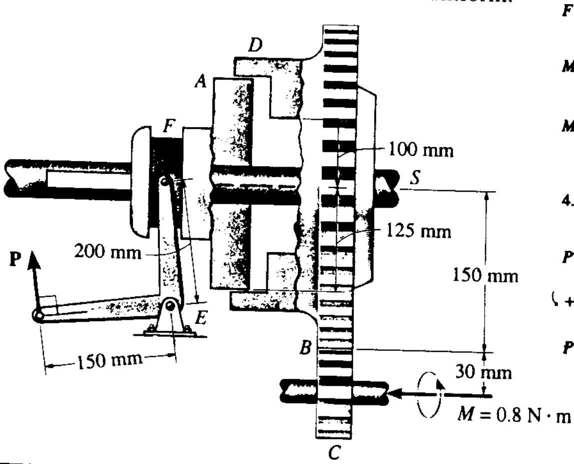


$$M = \frac{2}{3}\mu P R$$

$$F(1.5) = \frac{2}{3}(0.3)(80)(1)$$

F = ~~[illegible]~~ 10.7 lb **Ans**

***8-112.** The plate clutch consists of a flat plate A that slides over the rotating shaft S. The shaft is fixed to the driving plate gear B. If the gear C, which is in mesh with B, is subjected to a torque of $M = 0.8$ N·m, determine the smallest force P, that must be applied via the control arm, to stop the rotation. The coefficient of static friction between the plates A and D is $\mu_s = 0.4$. Assume the bearing pressure between A and D to be uniform.



$$F = \frac{0.8}{0.03} = 26.667 \text{ N}$$

$$M = 26.667(0.150) = 4.00 \text{ N·m}$$

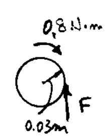
$$M = \frac{2}{3}\mu P'\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

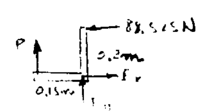
$$4.00 = \frac{2}{3}(0.4)(P')\left(\frac{(0.125)^3 - (0.1)^3}{(0.125)^2 - (0.1)^2}\right)$$

$$P' = 88.525 \text{ N}$$

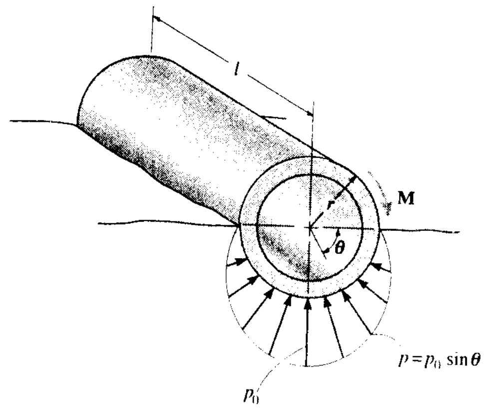
$$\circlearrowleft +\Sigma M_F = 0; \quad 88.525(0.2) - P(0.15) = 0$$

$P = 118$ N **Ans**





8-113. A tube has a total weight of 200 lb, length l = 8 ft, and radius = 0.75 ft. If it rests in sand for which the coefficient of static friction it is $\mu_s = 0.23$, determine the torque **M** needed to turn it. Assume that the pressure distribution along the length of the tube is defined by $p = p_0 \sin\theta$. For the solution it is necessary to determine p_0, the peak pressure, in terms of the weight and tube dimensions.



Equations of Equilibrium and Friction : Here, $dN = plrd\theta = p_0 lr\sin\theta d\theta$. Since the tube is on the verge of slipping, $dF = \mu_s dN = p_0\mu_s lr\sin\theta d\theta$.

$+\uparrow\Sigma F_y = 0;\quad 2\int_0^{\frac{\pi}{2}} dN\sin\theta - W = 0$

$$2\int_0^{\frac{\pi}{2}} p_0 lr\sin^2\theta d\theta = W$$

$$p_0 lr\int_0^{\frac{\pi}{2}} (1-\cos 2\theta)\, d\theta = W$$

$$p_0 lr\left(\frac{\pi}{2}\right) = W$$

$$p_0 = \frac{2W}{\pi lr} \qquad [1]$$

$\circlearrowleft + \Sigma M_O = 0;\quad 2\int_0^{\frac{\pi}{2}} dF(r) - M = 0$

$$M = 2\int_0^{\frac{\pi}{2}} p_0\mu_s lr^2 \sin\theta d\theta = 2p_0\mu_s lr^2 \qquad [2]$$

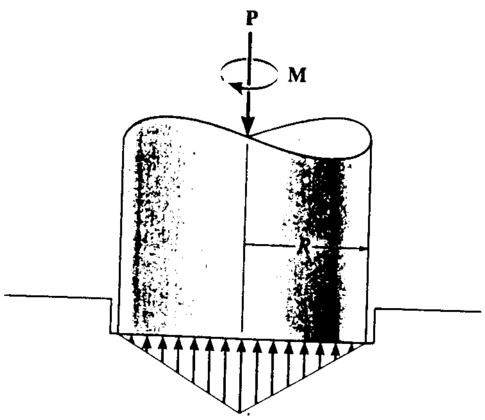
Substituting Eq.[1] into [2] yields

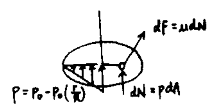
$$M = \frac{4W\mu_s r}{\pi}$$

However, W = 200 lb, μ_s = 0.23 and r = 0.75 ft, then

$$M = \frac{4(200)(0.23)(0.75)}{\pi} = 43.9 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

8-114. Because of wearing at the edges, the pivot bearing is subjected to a conical pressure distribution at its surface of contact. Determine the torque M required to overcome friction and turn the shaft, which supports an axial force $\mathbf{P}$. The coefficient of static friction is μ_s. For the solution, it is necessary to determine the peak pressure p_0 in terms of P and the bearing radius R.





$$dM = r\,dF = r\,\mu\,dN = r\,\mu\,p\,dA = r\,\mu\,p\,(r\,d\theta\,dr)$$

$$M = \int dM = \int_0^R \mu\left(p_0 - \frac{p_0}{R}r\right) r^2\,dr \int_0^{2\pi} d\theta$$

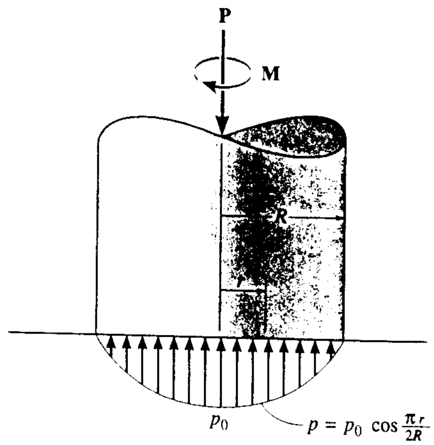
$$= \frac{\pi}{6}\,\mu\,p_0\,R^3$$

$$P = \int p\,dA = \int_0^R \left(p_0 - \frac{p_0}{R}r\right) r\,dr \int_0^{2\pi} d\theta$$

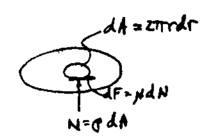
$$= \frac{\pi}{3} p_0 R^2$$

Thus, $\quad M = \dfrac{1}{2}\mu P R \quad$ **Ans**

8-115. The pivot bearing is subjected to a pressure distribution at its surface of contact which varies as shown. If the coefficient of static friction is μ, determine the torque M required to overcome friction if the shaft supports an axial force **P**.



$$dF = \mu\, dN = \mu\, p_0 \cos(\frac{\pi r}{2R})\, dA$$



$$M = \int_A r\,\mu\, p_0 \cos(\frac{\pi r}{2R})\, r\,dr\,d\theta$$

$$= \mu\, p_0 \int_0^R (r^2 \cos(\frac{\pi r}{2R}) dr) \int_0^{2\pi} d\theta$$

$$= \mu\, p_0 \left[\frac{2r}{(\frac{\pi}{2R})^2}\cos(\frac{\pi r}{2R}) + \frac{(\frac{\pi}{2R})^2 r^2 - 2}{(\frac{\pi}{2R})^3}\sin(\frac{\pi r}{2R})\right]_0^R (2\pi)$$

$$= \mu p_0 (\frac{16R^3}{\pi^2})\left[(\frac{\pi}{2})^2 - 2\right]$$

$$= 0.7577\, \mu\, p_0 R^3$$

$$P = \int_A dN = \int_0^R p_0 (\cos(\frac{\pi r}{2R}) r dr \int_0^{2\pi} d\theta$$

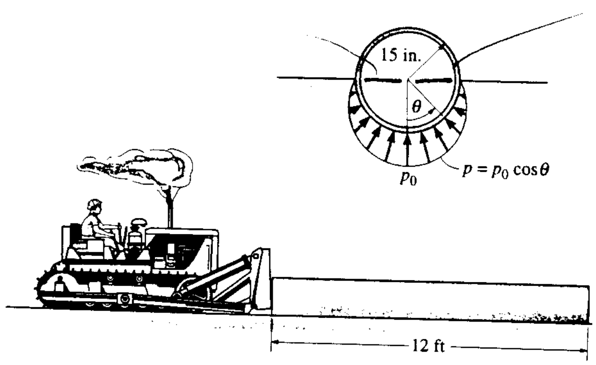
$$= p_0 \left[\frac{1}{(\frac{\pi}{2R})^2}\cos(\frac{\pi r}{2R}) + \frac{r}{(\frac{\pi}{2R})}\sin(\frac{\pi r}{2R})\right]_0^R (2\pi)$$

$$= 4p_0 R^2 (1 - \frac{2}{\pi})$$

$$= 1.454 p_0 R^2$$

Thus, $M = 0.521\, P\mu R$ **Ans**

***8-116.** The tractor is used to push the 1500-lb pipe. To do this it must overcome the frictional forces at the ground, caused by sand. Assuming that the sand exerts a pressure on the bottom of the pipe as shown, and the coefficient of static friction between the pipe and the sand is $\mu_s = 0.3$, determine the force required to push the pipe forward. Also, determine the peak pressure p_0.



$$+\uparrow \Sigma F_y = 0; \quad 2\,l \int_0^{\pi/2} p_0 \cos\theta\,(r\,d\theta)\cos\theta - W = 0$$

$$2p_0\, l\, r \int_0^{\pi/2} \cos^2\theta\, d\theta = W$$

$$2p_0\, r\, l (\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta)\Big|_0^{\frac{\pi}{2}} = W$$

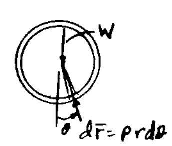
$$2(p_0)\, r\, l (\frac{\pi}{4}) = W$$

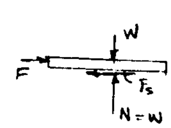
$$2\,p_0(15)(12)(12)(\frac{\pi}{4}) = 1500$$

$p_0 = 0.442$ psi **Ans**

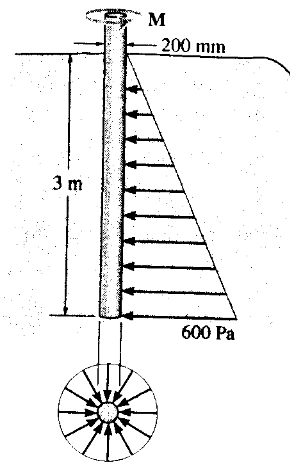
$$F = \int_{-\pi/2}^{\pi/2} (0.3)(0.442\ \text{lb/in}^2)(12\ \text{ft})(12\ \text{in./ft})(15\ \text{in.})\, d\theta$$

$F = 573$ lb **Ans**



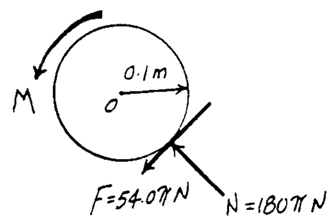


8-117. A 200-mm diameter post is driven 3 m into sand for which $\mu_s = 0.3$. If the normal pressure acting *completely around the post* varies linearly with depth as shown, determine the frictional torque **M** that must be overcome to rotate the post.

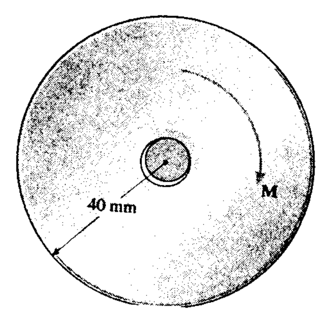


Equations of Equilibrium and Friction : The resultant normal force on the post is $N = \frac{1}{2}(600+0)(3)(\pi)(0.2) = 180\pi$ N. Since the post is on the verge of rotating, $F = \mu_s N = 0.3(180\pi) = 54.0\pi$ N.

$$\circlearrowleft + \Sigma M_O = 0; \qquad M - 54.0\pi(0.1) = 0$$
$$M = 17.0\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

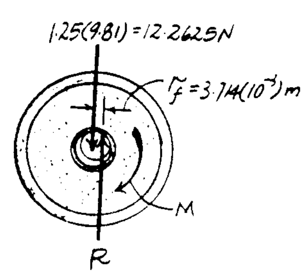


8-118. A pulley having a diameter of 80 mm and mass of 1.25 kg is supported loosely on a shaft having a diameter of 20 mm. Determine the torque M that must be applied to the pulley to cause it to rotate with constant motion. The coefficient of kinetic friction between the shaft and pulley is $\mu_k = 0.4$. Also calculate the angle θ which the normal force at the point of contact makes with the horizontal. The shaft itself cannot rotate.



Frictional Force on Journal Bearing : Here, $\phi_k = \tan^{-1}\mu_k = \tan^{-1}0.4 = 21.80°$. Then the radius of friction circle is $r_f = r\sin\phi_k = 0.01\sin 21.80° = 3.714(10^{-3})$ m. The angle for which the normal force makes with horizontal is

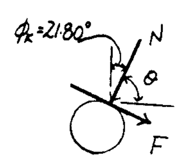
$$\theta = 90° - \phi_k = 68.2° \qquad \textbf{Ans}$$



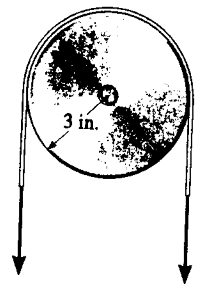
Equations of Equilibrium :

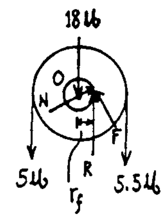
$$+\uparrow \Sigma F_y = 0; \qquad R - 12.2625 = 0 \qquad R = 12.2625\ \text{N}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad 12.2625(3.714)(10^{-3}) - M = 0$$
$$M = 0.0455\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$



8-119. The pulley has a radius of 3 in. and fits loosely on the 0.5-in.-diameter shaft. If the loadings acting on the belt cause the pulley to rotate with constant angular velocity, determine the frictional force between the shaft and the pulley and compute the coefficient of kinetic friction. The pulley weighs 18 lb.





$+\uparrow \Sigma F_y = 0; \quad R - 18 - 10.5 = 0$

$R = 28.5 \text{ lb}$

$\circlearrowleft + \Sigma M_O = 0; \quad -5.5(3) + 5(3) + 28.5\, r_f = 0$

$r_f = 0.05263 \text{ in.}$

$r_f = r \sin \phi_k$

$$0.05263 = \frac{0.5}{2} \sin\phi_k$$

$\phi_k = 12.15°$

$\mu = \tan \phi_k = \tan 12.15° = 0.215$ **Ans**

Note also by approximation,

$r_f = r\mu$

$$0.05263 = \frac{0.5}{2}\mu$$

$\mu = 0.211$ **Ans** (approx.)

Also,

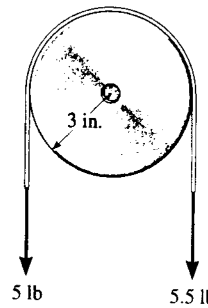
$$\circlearrowleft + \Sigma M_O = 0; \quad -5.5(3) + 5(3) + F\left(\frac{0.5}{2}\right) = 0$$

$F = 6 \text{ lb}$ **Ans**

$N = \sqrt{R^2 - F^2} = \sqrt{(28.5)^2 - 6^2} = 27.86 \text{ lb}$

$$\mu = \frac{F}{R} = \frac{6}{27.86} = 0.215$$ **Ans**

***8-120.** The pulley has a radius of 3 in. and fits loosely on the 0.5-in.-diameter shaft. If the loadings acting on the belt cause the pulley to rotate with constant angular velocity, determine the frictional force between the shaft and the pulley and compute the coefficient of kinetic friction. Neglect the weight of the pulley.

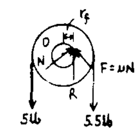


$+\uparrow \Sigma F_y = 0; \quad R - 5 - 5.5 = 0$

$R = 10.5$ lb

$\circlearrowleft + \Sigma M_O = 0; \quad -5.5(3) + 5(3) + F(0.25) = 0$

$F = 6$ lb **Ans**



$N = \sqrt{(10.5)^2 - 6^2} = 8.617$ lb

$\mu_k = \frac{F}{N} = \frac{6}{8.617} = 0.696$ **Ans**

Also,

$\circlearrowleft + \Sigma M_O = 0; \quad -5.5(3) + 5(3) + 10.5(r_f) = 0$

$r_f = 0.1429$ in.

$0.1429 = \frac{0.5}{2} \sin \phi_k$

$\phi_k = 34.85°$

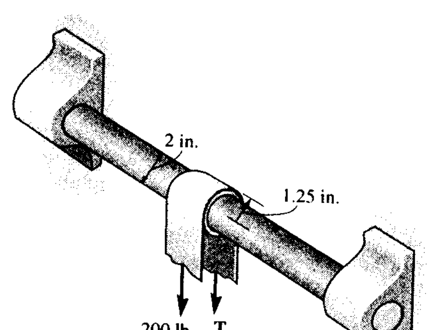
$\mu_k = \tan 34.85° = 0.696$ **Ans**

By approximation,

$r_f = r\mu_k$

$\mu_k = \frac{0.1429}{0.25} = 0.571$ **Ans** (approx.)

8-121. Determine the tension **T** in the belt needed to overcome the tension of 200 lb created on the other side. Also, what are the normal and frictional components of force developed on the collar bushing? The coefficient of static friction is $\mu_s = 0.21$.



Frictional Force on Journal Bearing : Here, $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.21 = 11.86°$. Then the radius of friction circle is

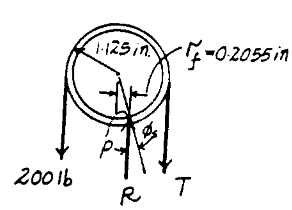
$$r_f = r\sin\phi_k = 1\sin 11.86° = 0.2055 \text{ in.}$$

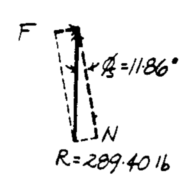
Equations of Equilibrium :

$\circlearrowleft + \Sigma M_P = 0; \quad 200(1.125 + 0.2055) - T(1.125 - 0.2055) = 0$

$T = 289.41 \text{ lb} = 289 \text{ lb}$ **Ans**

$+\uparrow F_y = 0; \quad R - 200 - 289.4 = 0 \quad R = 489.41$ lb



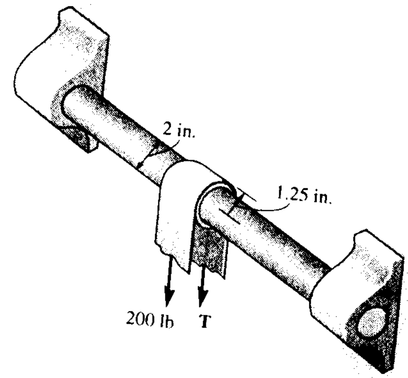


Thus, the normal and friction force are

$N = R\cos\phi_s = 489.41\cos 11.86° = 479$ lb **Ans**

$F = R\sin\phi_s = 489.41\sin 11.86° = 101$ lb **Ans**

8-122. If a tension force $T = 215$ lb is required to pull the 200-lb force around the collar bushing, determine the coefficient of static friction at the contacting surface. The belt does not slip on the collar.



Equation of Equilibrium :

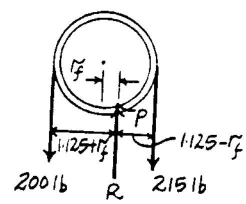
$$\curvearrowleft + \Sigma M_P = 0; \quad 200(1.125 + r_f) - 215(1.125 - r_f) = 0$$
$$r_f = 0.04066 \text{ in.}$$

Frictional Force on Journal Bearing : The radius of friction circle is

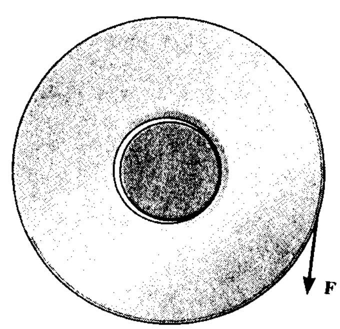
$$r_f = r \sin \phi_k$$
$$0.04066 = 1 \sin \phi_k$$
$$\phi_k = 2.330°$$

and the coefficient of static friction is

$$\mu_s = \tan \phi_s = \tan 2.330° = 0.0407 \qquad \textbf{Ans}$$



8-123. A disk having an outer diameter of 120 mm fits loosely over a fixed shaft having a diameter of 30 mm. If the coefficient of static friction between the disk and the shaft is $\mu_s = 0.15$ and the disk has a mass of 50 kg, determine the smallest vertical force **F** acting on the rim which must be applied to the disk to cause it to slip over the shaft.



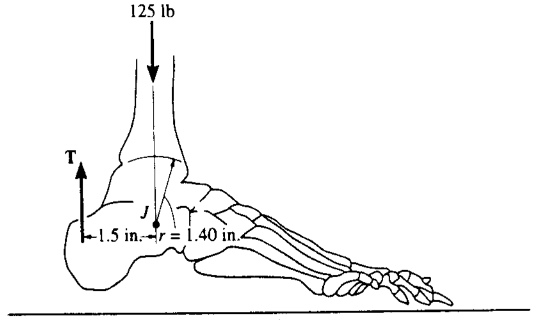
Frictional Force on Journal Bearing : Here, $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.15 = 8.531°$. Then the radius of friction circle is

$$r_f = r \sin \phi_s = 0.015 \sin 8.531° = 2.225(10^{-3}) \text{ m}$$

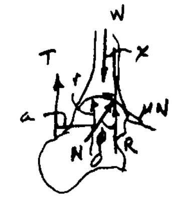
Equation of Equilibrium :

$$\curvearrowleft + \Sigma M_P = 0; \quad 490.5(2.225)(10^{-3}) - F[0.06 - (2.225)(10^{-3})] = 0$$
$$F = 18.9 \text{ N} \qquad \textbf{Ans}$$

***8-124.** The weight of the body on the tibiotalar joint J is 125 lb. If the radius of curvature of the talus surface of the ankle is 1.40 in., and the coefficient of static friction between the bones is $\mu_s = 0.1$, determine the force T developed in the Achilles tendon necessary to rotate the joint.



With a addition of force T, the resultant force $W + T$ acts a distance x horizontally from W.



$F_{R_x} = \Sigma M_O;\quad -(W+T)x = -Ta \quad x = \frac{Ta}{W+T}$

Friction :

$\tan\phi = \frac{\mu N}{N} = \mu$

However, from geometry r is the radius of curvature.

$\sin\phi = \frac{x}{r}$

Since ϕ is small $\quad \sin\phi \approx \tan\phi = \mu = \frac{x}{r}$, subsitute $x = \frac{Ta}{W+T}$ yields

$\mu = \frac{Ta}{r(W+T)}$

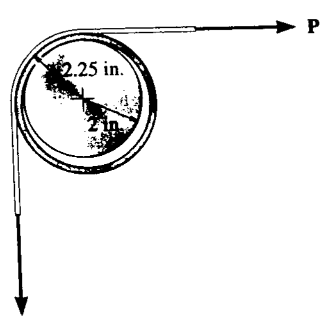
$T = \frac{\mu r W}{a - \mu r}$

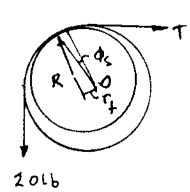
Here $\quad W = 125$ lb, $r = 1.40$ in, $\mu = 0.1, a = 1.50$ in.

$T = \frac{0.1(1.40)(125)}{1.50 - 0.1(1.40)}$

$= 12.9$ lb **Ans**

8-125. The collar fits *loosely* around a fixed shaft that has a radius of 2 in. If the coefficient of kinetic friction between the shaft and the collar is $\mu_k = 0.3$, determine the force P on the horizontal segment of the belt so that the collar rotates counterclockwise with a constant angular velocity. Assume that the belt does not slip on the collar; rather, the collar slips on the shaft. Neglect the weight and thickness of the belt and collar. The radius, measured from the center of the collar to the mean thickness of the belt, is 2.25 in.





$\phi_k = \tan^{-1}\mu_k = \tan^{-1}0.3 = 16.699°$

$r_f = 2\sin 16.699° = 0.5747$ in.

Equilibrium :

$+\uparrow \Sigma F_y = 0;\quad R_y - 20 = 0 \quad R_y = 20$ lb

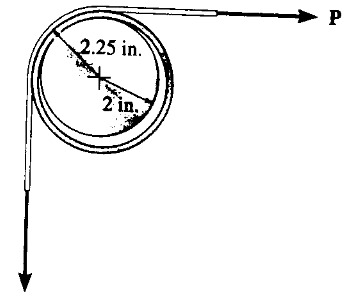
$\xrightarrow{+} \Sigma F_x = 0;\quad T - R_x = 0 \quad R_x = T$

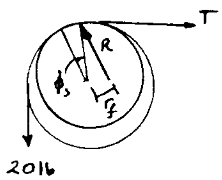
Hence $\quad R = \sqrt{R_x^2 + R_y^2} = \sqrt{T^2 + 20^2}$

$\circlearrowleft + \Sigma M_O = 0;\quad -\left(\sqrt{T^2+20^2}\right)(0.5747) + 20(2.25) - T(2.25) = 0$

Choose the smallest root $\quad T = 13.8$ lb **Ans**

8-126. The collar fits *loosely* around a fixed shaft that has a radius of 2 in. If the coefficient of kinetic friction between the shaft and the collar is $\mu_k = 0.3$, determine the force P on the horizontal segment of the belt so that the collar rotates clockwise with a constant angular velocity. Assume that the belt does not slip on the collar; rather, the collar slips on the shaft. Neglect the weight and thickness of the belt and collar. The radius, measured from the center of the collar to the mean thickness of the belt, is 2.25 in.





$\phi_k = \tan^{-1}\mu_k = \tan^{-1}0.3 = 16.699°$

$r_f = 2\sin 16.699° = 0.5747$ in.

Equilibrium :

$+\uparrow\Sigma F_y = 0;\quad R_y - 20 = 0 \quad R_y = 20$ lb

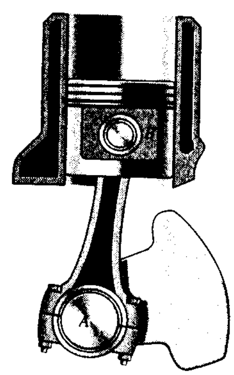
$\xrightarrow{+}\Sigma F_x = 0;\quad T - R_x = 0 \quad R_x = T$

Hence $\quad R = \sqrt{R_x^2 + R_y^2} = \sqrt{T^2 + 20^2}$

$\circlearrowleft+\Sigma M_O = 0;\quad \left(\sqrt{T^2+20^2}\right)(0.5747) + 20(2.25) - T(2.25) = 0$

Choose the largest root $\quad T = 29.0$ lb $\quad$ **Ans**

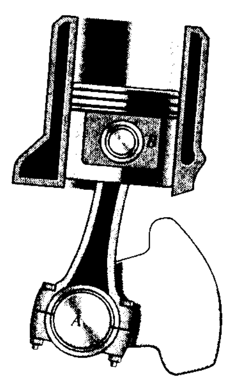
8-127. The connecting rod is attached to the piston by a 0.75-in.-diameter pin at B and to the crank shaft by a 2-in.-diameter bearing A. If the piston is moving downwards, and the coefficient of static friction at these points is $\mu_s = 0.2$, determine the radius of the friction circle at each connection.



$(r_f)_A = r_A\mu_s = 0.2$ in. $\quad$ **Ans**

$(r_f)_B = r_B\mu_s = \dfrac{0.75(0.2)}{2} = 0.075$ in. $\quad$ **Ans**

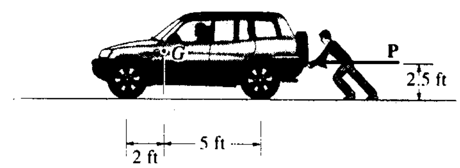
***8-128.** The connecting rod is attached to the piston by a 20-mm-diameter pin at B and to the crank shaft by a 50-mm-diameter bearing A. If the piston is moving upwards, and the coefficient of static friction at these points is $\mu_s = 0.3$, determine the radius of the friction circle at each connection.



$(r_f)_A = r_A\mu_s = 25\,(0.3) = 7.50$ mm $\quad$ **Ans**

$(r_f)_B = r_B\mu_s = 10\,(0.3) = 3$ mm $\quad$ **Ans**

8-129. The vehicle has a weight of 2600 lb and center of gravity at G. Determine the horizontal force **P** that must be applied to overcome the rolling resistance of the wheels. The coefficient of rolling resistance is 0.5 in. The tires have a diameter of 2.75 ft.



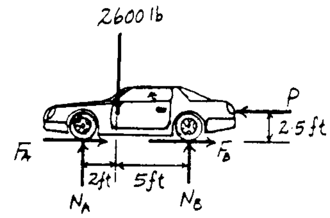
Equations of Equilibrium :

$$\curvearrowleft + \Sigma M_A = 0; \quad N_B(7) + P(2.5) - 2600(2) = 0$$

$$N_B = \frac{5200 - 2.5P}{7}$$

$$\curvearrowleft + \Sigma M_B = 0; \quad P(2.5) + 2600(5) - N_A(7) = 0$$

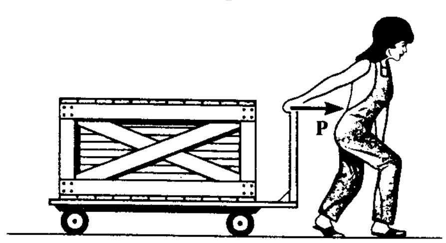
$$N_A = \frac{13000 + 2.5P}{7}$$



Rolling Resistance : Here, $W = N_A + N_B = \frac{5200 - 2.5P}{7} + \frac{13000 + 2.5P}{7}$ $= 2600$ lb, $a = 0.5$ in. and $r = \left(\frac{2.75}{2}\right)(12) = 16.5$ in.. Applying Eq. 8 – 11, we have

$$P = \frac{Wa}{r} = \frac{2600(0.5)}{16.5} = 78.8 \text{ lb} \qquad \textbf{Ans}$$

8-130. The hand cart has wheels with a diameter of 80 mm. If a crate having a mass of 500 kg is placed on the cart so that each wheel carries an equal load, determine the horizontal force P that must be applied to the handle to overcome the rolling resistance. The coefficient of rolling resistance is 2 mm. Neglect the mass of the cart.

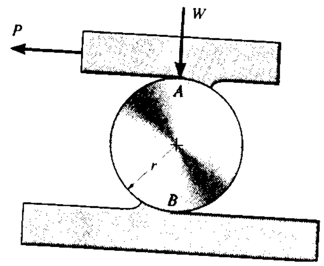


$$P = \frac{Wa}{r}$$

$$= 500(9.81)\left(\frac{2}{40}\right)$$

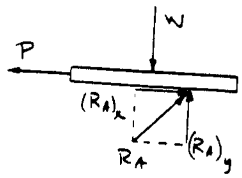
$$P = 245 \text{ N} \qquad \textbf{Ans}$$

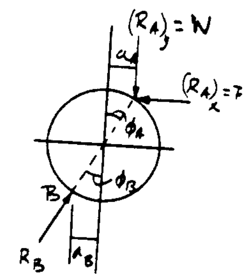
8-131. The cylinder is subjected to a load that has a weight W. If the coefficients of rolling resistance for the cylinder's top and bottom surfaces are a_A and a_B, respectively, show that a force having a magnitude of $P = [W(a_A + a_B)]/2r$ is required to move the load and thereby roll the cylinder forward. Neglect the weight of the cylinder.



$\xrightarrow{+} \Sigma F_x = 0;\quad (R_A)_x - P = 0 \quad (R_A)_x = P$

$+\uparrow \Sigma F_y = 0;\quad (R_A)_y - W = 0 \quad (R_A)_y = W$



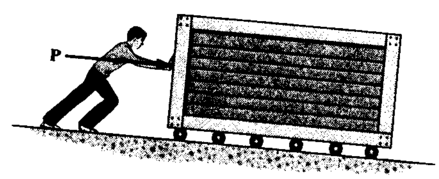


$\curvearrowleft + \Sigma M_B = 0;\quad P(r\cos\phi_A + r\cos\phi_B) - W(a_A + a_B) = 0 \qquad (1)$

Since ϕ_A and ϕ_B are very small, $\cos\phi_A = \cos\phi_B = 1$. Hence, from Eq. (1)

$$P = \frac{W(a_A + a_B)}{2r} \qquad \textbf{(QED)}$$

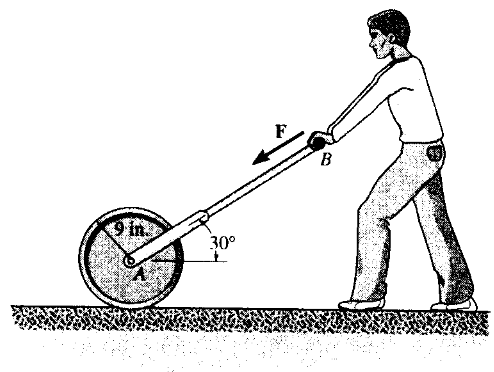
***8-132.** A large crate having a mass of 200 kg is moved along the floor using a series of 150-mm-diameter rollers for which the coefficient of rolling resistance is 3 mm at the ground and 7 mm at the bottom surface of the crate. Determine the horizontal force **P** needed to push the crate forward at a constant speed. *Hint:* Use the result of Prob. 8-131.



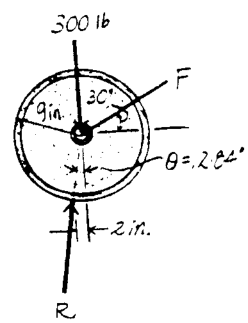
Rolling Resistance : Applying the result obtained in Prob. 8 – 131, $P = \dfrac{W(a_A + a_B)}{2r}$, with $a_A = 7$ mm, $a_B = 3$ mm, $W = 200(9.81) = 1962$ N, and $r = 75$ mm, we have

$$P = \frac{1962(7+3)}{2(75)} = 130.8 \text{ N} = 131 \text{ N} \qquad \textbf{Ans}$$

8-133. The lawn roller weighs 300 lb. If the rod BA is held at an angle of 30° from the horizontal and the coefficient of rolling resistance for the roller is 2 in., determine the force **F** needed to push the roller at constant speed. Neglect friction developed at the axle and assume that the resultant force acting on the handle is applied along BA.



Rolling Resistance : The angle $\theta = \sin^{-1}\frac{2}{9} = 12.84°$. From the eqilibrium of the lawn roller, we have



$$\xrightarrow{+} \Sigma F_x = 0; \quad R\sin 12.84° - F\cos 30° = 0 \qquad [1]$$

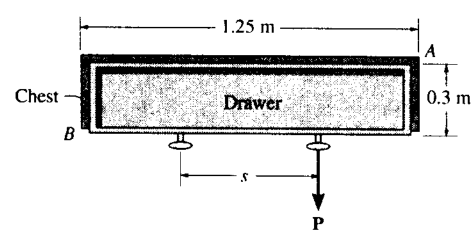
$$+\uparrow \Sigma F_y = 0; \quad R\cos 12.84° - 300 - F\sin 30° = 0 \qquad [2]$$

Solving Eq.[1] and [2]

$F = 90.9$ lb **Ans**

$R = 354.31$ lb

8-134. A single force **P** is applied to the handle of the drawer. If friction is neglected at the bottom side and the coefficient of static friction along the sides is $\mu_s = 0.4$, determine the largest spacing s between the symmetrically placed handles so that the drawer does not bind at the corners A and B when the force **P** is applied to one of the handles.



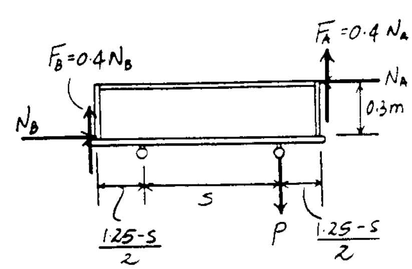
Equations of Equilibrium and Friction : If the drawer does not bind at corners A and B, slipping would have to occur at points A and B. Hence, $F_A = \mu N_A = 0.4N_A$ and $F_B = \mu N_B = 0.4N_B$.

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_B - N_A = 0 \quad N_A = N_B = N$$

$$+\uparrow \Sigma F_y = 0; \quad 0.4N + 0.4N - P = 0 \quad P = 0.8N$$

$$\circlearrowleft + \Sigma M_B = 0; \quad N(0.3) + 0.4N(1.25) - 0.8N\left(\frac{s+1.25}{2}\right) = 0$$

$$N\left[0.3 + 0.5 - 0.8\left(\frac{s+1.25}{2}\right)\right] = 0$$

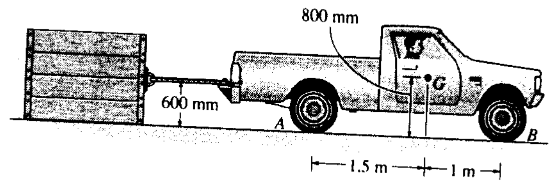


Since $N \neq 0$, then

$$0.3 + 0.5 - 0.8\left(\frac{s+1.25}{2}\right) = 0$$

$s = 0.750$ m **Ans**

8-135. The truck has a mass of 1.25 Mg and a center of mass at G. Determine the greatest load it can pull if (a) the truck has rear-wheel drive while the front wheels are free to roll, and (b) the truck has four-wheel drive. The coefficient of static friction between the wheels and the ground is $\mu_s = 0.5$, and between the crate and the ground, it is $\mu_s' = 0.4$.



a) The truck with rear wheel drive.

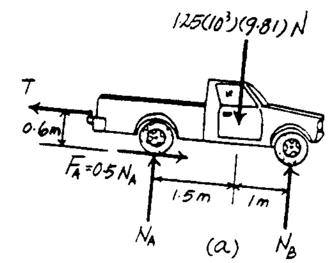
Equations of Equilibrium and Friction : It is required that the rear wheels of the truck slip. Hence $F_A = \mu_s N_A = 0.5N_A$. From FBD (a),

$$\circlearrowleft + \Sigma M_B = 0; \quad 1.25(10^3)(9.81)(1) + T(0.6) - N_A(2.5) = 0 \qquad [1]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.5N_A - T = 0 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$N_A = 5573.86 \text{ N} \qquad T = 2786.93 \text{ N}$$



Since the crate moves, $F_C = \mu_s' N_C = 0.4N_C$. From FBD (c),

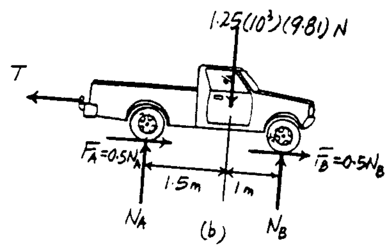
$$+\uparrow \Sigma F_y = 0; \quad N_C - W = 0 \quad N_C = W$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 2786.93 - 0.4W = 0$$

$$W = 6967.33 \text{ N} = 6.97 \text{ kN} \qquad \textbf{Ans}$$

b) The truck with four wheel drive.

Equations of Equilibrium and Friction : It is required that the rear wheel and front wheels of the truck slip. Hence $F_A = \mu_s N_A = 0.5N_A$ and $F_B = \mu_s N_B = 0.5N_B$. From FBD (b),



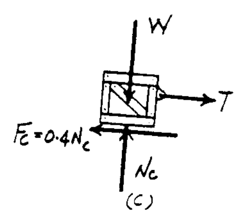
$$\circlearrowleft + \Sigma M_B = 0; \quad 1.25(10^3)(9.81)(1) + T(0.6) - N_A(2.5) = 0 \qquad [3]$$

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(2.5) + T(0.6) - 1.25(10^3)(9.81)(1.5) = 0 \qquad [4]$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 0.5N_A + 0.5N_B - T = 0 \qquad [5]$$

Solving Eqs. [3], [4] and [5] yields

$$N_A = 6376.5 \text{ N} \qquad N_B = 5886.0 \text{ N} \qquad T = 6131.25 \text{ N}$$



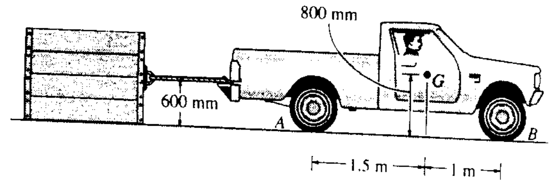
Since the crate moves, $F_C = \mu_s' N_C = 0.4N_C$. From FBD (c),

$$+\uparrow \Sigma F_y = 0; \quad N_C - W = 0 \quad N_C = W$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 6131.25 - 0.4W = 0$$

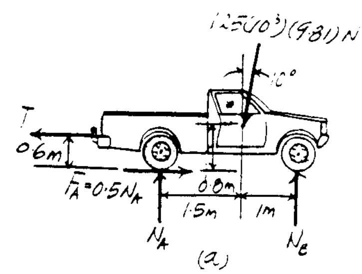
$$W = 15328.125 \text{ N} = 15.3 \text{ kN} \qquad \textbf{Ans}$$

*8-136. Solve Prob. 8-135 if the truck and crate are traveling up a 10° incline.



a) The truck with rear wheel drive.

Equations of Equilibrium and Friction : It is required that the rear wheel of the truck slip hence $F_A = \mu_s N_A = 0.5N_A$. From FBD (a),



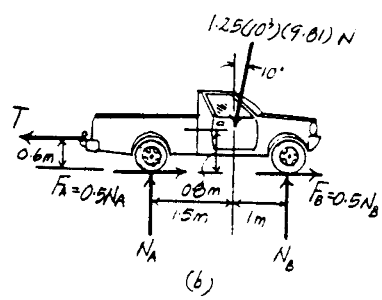
$$\curvearrowleft + \Sigma M_B = 0; \quad 1.25(10^3)(9.81)\cos 10°(1) + 1.25(10^3)(9.81)\sin 10°(0.8) + T(0.6) - N_A(2.5) = 0 \quad [1]$$

$$+ \; \Sigma F_{x'} = 0; \quad 0.5N_A - 1.25(10^3)(9.81)\sin 10° - T = 0 \quad [2]$$

Solving Eqs. [1] and [2] yields

$$N_A = 5682.76 \text{ N} \qquad T = 712.02 \text{ N}$$

Since the crate moves, $F_C = \mu_s' N_C = 0.4N_C$. From FBD (c),



$$+\Sigma F_{y'} = 0; \quad N_C - W\cos 10° = 0 \quad N_C = 0.9848W$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 712.02 - W\sin 10° - 0.4(0.9848W) = 0$$

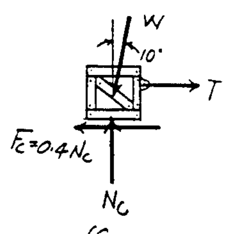
$$W = 1254.50 \text{ N} = 1.25 \text{ kN} \qquad \textbf{Ans}$$

b) The truck with four wheel drive.

Equations of Equilibrium and Friction : It is required that the rear wheels of the truck slip hence $F_A = \mu_s N_A = 0.5N_A$. From FBD (a),

$$\curvearrowleft + \Sigma M_B = 0; \quad 1.25(10^3)(9.81)\cos 10°(1) + 1.25(10^3)(9.81)\sin 10°(0.8) + T(0.6) - N_A(2.5) = 0 \quad [3]$$

$$\curvearrowleft + \Sigma M_A = 0; \quad -1.25(10^3)(9.81)\cos 10°(1.5) + 1.25(10^3)(9.81)\sin 10°(0.8) + T(0.6) + N_B(2.5) = 0 \quad [4]$$



$$\overset{+}{\rightarrow} \Sigma F_{x'} = 0; \quad 0.5N_A + 0.5N_B - 1.25(10^3)(9.81)\sin 10° - T = 0 \quad [5]$$

Solving Eqs. [3], [4] and [5] yields

$$N_A = 6449.98 \text{ N} \qquad N_B = 5626.23 \text{ N} \qquad T = 3908.74 \text{ N}$$

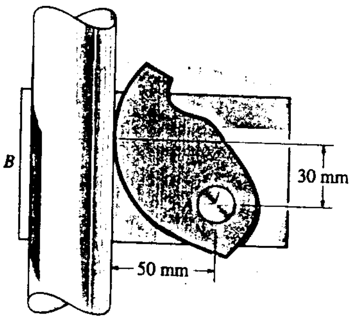
Since the crate moves, $F_C = \mu_s' N_C = 0.4N_C$. From FBD (c),

$$+\Sigma F_{y'} = 0; \quad N_C - W\cos 10° = 0 \quad N_C = 0.9848W$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 3908.74 - W\sin 10° - 0.4(0.9848W) = 0$$

$$W = 6886.79 \text{ N} = 6.89 \text{ kN} \qquad \textbf{Ans}$$

8-137. The cam or short link is pinned at A and is used to hold mops or brooms against a wall. If the coefficient of static friction between the broomstick and the cam is $\mu_s = 0.2$, determine if it is possible to support the broom having a weight W. The surface at B is smooth. Neglect the weight of the cam.



The cam is a two-force member.

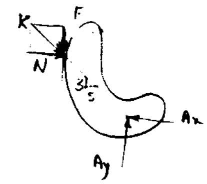
Require

$$\frac{F}{3} = \frac{N}{5}$$

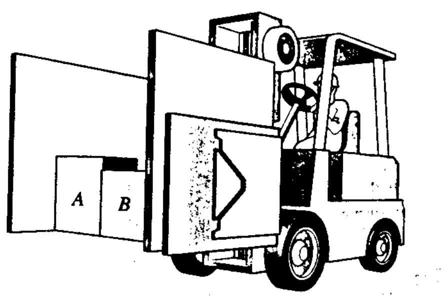
$$F = 0.6\,N$$

However $\quad F_{max} = \mu_s N = 0.2\,N$

Therefore, the cam **cannot** support the broom. **Ans**



8-138. The carton clamp on the forklift has a coefficient of static friction of $\mu_s = 0.5$ with any cardboard carton, whereas a cardboard carton has a coefficient of static friction of $\mu_s' = 0.4$ with any other cardboard carton. Compute the smallest horizontal force P the clamp must exert on the sides of a carton so that two cartons A and B each weighing 30 lb can be lifted. What smallest clamping force P' is required to lift three 30-lb cartons? The third carton C is placed between A and B.

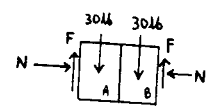


If two cartons against the clamp,

$+\uparrow \Sigma F_y = 0; \quad 2F = 60$

$2(0.5\,N) = 60$

$N = 60$ lb

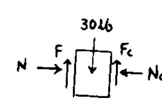


If the cartons slide against each other,

$+\uparrow \Sigma F_y = 0; \quad F + F_C = 30$

$0.5\,N + 0.4\,N = 30$

$N = 33.33$ lb



Thus, $\quad P = 60$ lb for two cartons. **Ans**

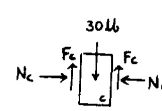
For three cartons:

If two cartons slide against each other,

$+\uparrow \Sigma F_y = 0; \quad 2F_C = 30$

$2(0.4\,N_C) = 30$

$N_C = 37.5$ lb

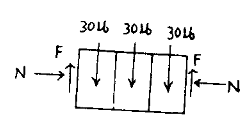


If the cartons slide against the camp,

$+\uparrow \Sigma F_y = 0; \quad 2F = 90$ lb

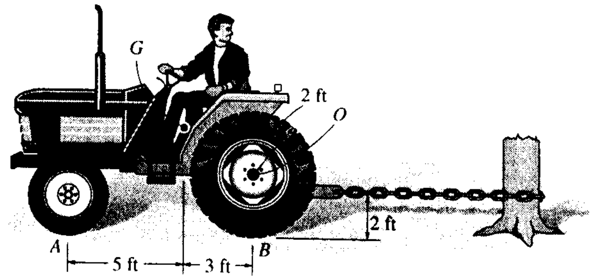
$2(0.5\,N) = 90$

$N = 90$ lb



$P = 90$ lb for three cartons. **Ans**

8-139. The tractor pulls on the fixed tree stump. Determine the torque that must be applied by the engine to the rear wheels to cause them to slip. The front wheels are free to roll. The tractor weighs 3500 lb and has a center of gravity at G. The coefficient of static friction between the rear wheels and the ground is $\mu_s = 0.5$.

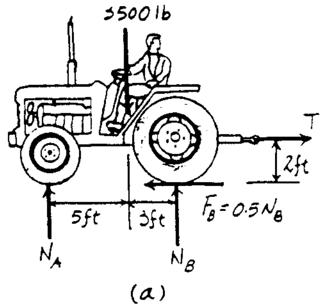


Equations of Equilibrium and Friction : Assume that the rear wheels B slip. Hence $F_B = \mu_s N_B = 0.5N_B$.

$$\circlearrowleft + \Sigma M_A = 0 \qquad N_B(8) - T(2) - 3500(5) = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B + N_A - 3500 = 0 \qquad [2]$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T - 0.5N_B = 0 \qquad [3]$$



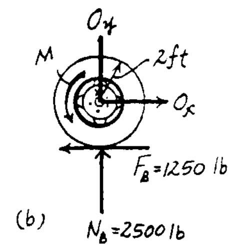
Solving Eqs.[1], [2] and [3] yields

$$N_A = 1000 \text{ lb} \qquad N_B = 2500 \text{ lb} \qquad T = 1250 \text{ lb}$$

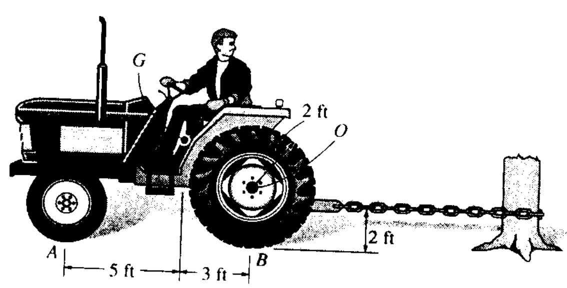
Since $N_A > 0$, the front wheels do not lift up. Therefore the rear wheels slip as assumed. Thus, $F_B = 0.5(2500) = 1250$ lb. From FBD (b),

$$\circlearrowleft + \Sigma M_O = 0, \qquad M - 1250(2) = 0$$

$$M = 2500 \text{ lb} \cdot \text{ft} = 2.50 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$



*8-140. The tractor pulls on the fixed tree stump. If the coefficient of static friction between the rear wheels and the ground is $\mu_s = 0.6$, determine if the rear wheels slip or the front wheels lift off the ground as the engine provides torque to the rear wheels. What is the torque needed to cause the motion? The front wheels are free to roll. The tractor weighs 2500 lb and has a center of gravity at G.

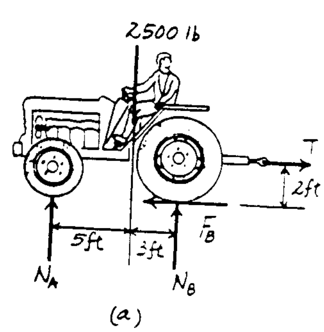


Equations of Equilibrium and Friction : Assume that the rear wheels B slip. Hence $F_B = \mu_s N_B = 0.6N_B$.

$$\circlearrowleft + \Sigma M_A = 0 \qquad N_B(8) - T(2) - 2500(5) = 0 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B + N_A - 2500 = 0 \qquad [2]$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T - 0.6N_B = 0 \qquad [3]$$



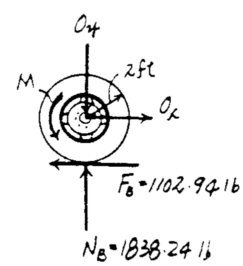
Solving Eqs.[1], [2] and [3] yields

$$N_A = 661.76 \text{ lb} \qquad N_B = 1838.24 \text{ lb} \qquad T = 1102.94 \text{ lb}$$

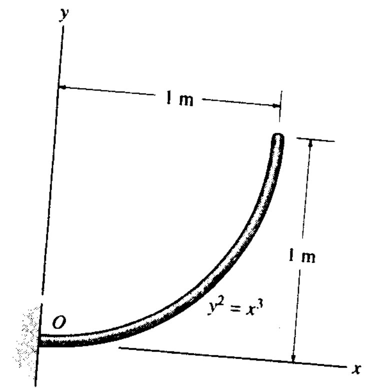
Since $N_A > 0$, **the front wheels do not lift off the ground.** Therefore the rear wheels slip as assumed. Thus, $F_B = 0.6(1838.24) = 1102.94$ lb. From FBD (b),

$$+\Sigma M_O = 0, \qquad M - 1102.94(2) = 0$$

$$M = 2205.88 \text{ lb} \cdot \text{ft} = 2.21 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$



9-1. Determine the distance $\bar{x}$ to the center of mass of the homogeneous rod bent into the shape shown. If the rod has a mass per unit length of 0.5 kg/m, determine the reactions at the fixed support O.



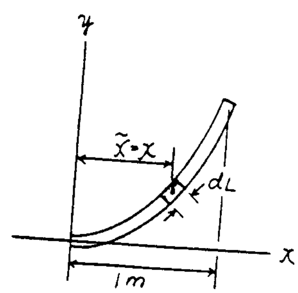
Length and Moment Arm : The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $\tilde{x} = x$. Here, $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$. Performing the integration, we have

$$L = \int dL = \int_0^{1\,\mathrm{m}} \left(\sqrt{1 + \frac{9}{4}x}\right)dx = \frac{8}{27}\left(1 + \frac{9}{4}x\right)^{\frac{3}{2}}\Bigg|_0^{1\,\mathrm{m}} = 1.4397\ \mathrm{m}$$

$$\int_L \tilde{x}\,dL = \int_0^{1\,\mathrm{m}} x\sqrt{1 + \frac{9}{4}x}\,dx$$

$$= \left[\frac{8}{27}x\left(1 + \frac{9}{4}x\right)^{\frac{3}{2}} - \frac{64}{1215}\left(1 + \frac{9}{4}x\right)^{\frac{5}{2}}\right]\Bigg|_0^{1\,\mathrm{m}}$$

$$= 0.7857$$



Centroid : Applying Eq. 9 − 7 , we have

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{0.7857}{1.4397} = 0.5457\ \mathrm{m} = 0.546\ \mathrm{m} \qquad \textbf{Ans}$$

Equations of Equilibrium :

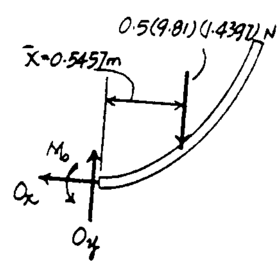
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad O_x = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \qquad O_y - 0.5(9.81)(1.4397) = 0$$

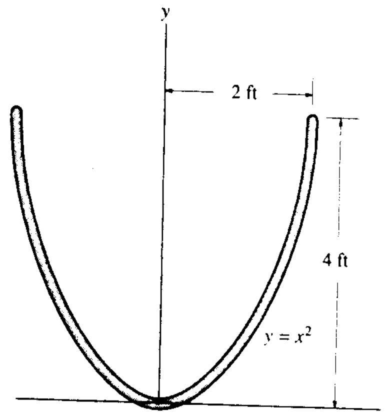
$$O_y = 7.06\ \mathrm{N} \qquad \textbf{Ans}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad M_O - 0.5(9.81)(1.4397)(0.5457) = 0$$

$$M_O = 3.85\ \mathrm{N \cdot m} \qquad \textbf{Ans}$$



9-2. Determine the location $(\bar{x}, \bar{y})$ of the centroid of the wire.



Length and Moment Arm : The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $\tilde{y} = y = x^2$. Here, $\frac{dy}{dx} = 2x$.

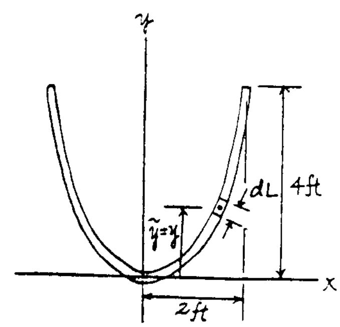
Centroid : Due to symmetry

$$\bar{x} = 0 \qquad \textbf{Ans}$$

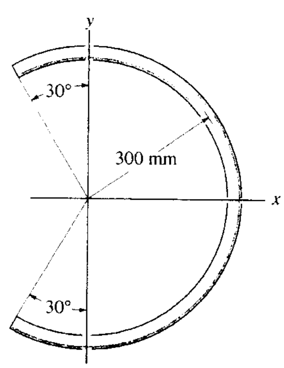
Applying Eq. 9 – 7 and performing the integration, we have

$$\bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_{-2\,\text{ft}}^{2\,\text{ft}} x^2\sqrt{1+4x^2}\,dx}{\int_{-2\,\text{ft}}^{2\,\text{ft}} \sqrt{1+4x^2}\,dx}$$

$$= \frac{16.9423}{9.2936} = 1.82 \text{ ft} \qquad \textbf{Ans}$$



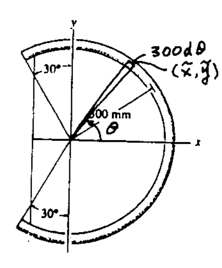
9-3. Locate the center of mass of the homogeneous rod bent into the shape of a circular arc.



$dL = 300\,d\theta$

$\tilde{x} = 300\cos\theta$

$\tilde{y} = 300\sin\theta$



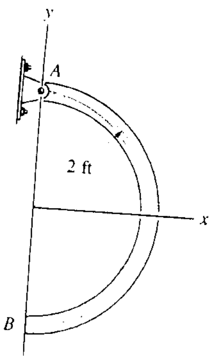
$$\bar{x} = \frac{\int \tilde{x}\,dL}{\int dL} = \frac{\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} 300\cos\theta\,(300\,d\theta)}{\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} 300\,d\theta}$$

$$= \frac{(300)^2[\sin\theta]_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}}}{300(\frac{4}{3}\pi)}$$

$$= 124 \text{ mm} \qquad \textbf{Ans}$$

$$\bar{y} = 0 \qquad \textbf{Ans} \quad \textbf{(By symmetry)}$$

***9-4.** Locate the center of gravity $\bar{x}$ of the homogeneous rod bent in the form of a semicircular arc. The rod has a weight per unit length of 0.5 lb/ft. Also, determine the horizontal reaction at the smooth support B and the x and y components of reaction at the pin A.



$\tilde{x} = 2\cos\theta$

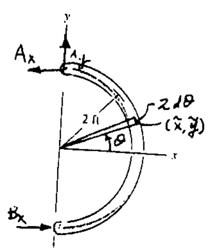
$\tilde{y} = 2\sin\theta$

$dL = 2\,d\theta$

$$\bar{x} = \frac{\int \tilde{x}\,dL}{\int dL} = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos\theta\, 2d\theta}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2d\theta}$$

$$= \frac{4[\sin\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}{[2\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}$$

$$= \frac{4}{\pi} \qquad \textbf{Ans}$$



Arc length $= \pi r = 2\pi$

$W = 2\pi(0.5)$ lb

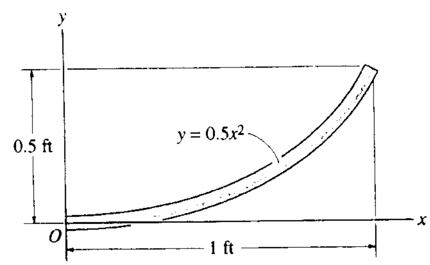
$\circlearrowleft + \Sigma M_A = 0; \qquad -2\pi(0.5)(\frac{4}{\pi}) + B_x(4) = 0$

$B_x = 1$ lb **Ans**

$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 1$ lb **Ans**

$+\uparrow \Sigma F_y = 0; \qquad A_y = 3.14$ lb **Ans**

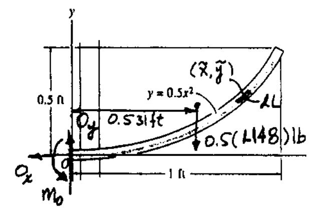
9-5. Determine the distance $\bar{x}$ to the center of gravity of the homogeneous rod bent into the parabolic shape. If the rod has a weight per unit length of 0.5 lb/ft, determine the reactions at the fixed support O.



$$dL = \sqrt{dx^2 + dy^2}$$

$$dy = x\,dx$$

$$\bar{x} = \frac{\int \tilde{x}\,dL}{\int dL} = \frac{\int_0^1 x\sqrt{dx^2 + x^2 dx^2}}{\int_0^1 \sqrt{dx^2 + x^2 dx^2}}$$



Let $x = \tan\theta$

$$dx = \sec^2\theta\,d\theta$$

$$\bar{x} = \frac{\int_0^{\frac{\pi}{4}} \tan\theta\sqrt{1 + \tan^2\theta}\sec^2\theta\,d\theta}{\int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2\theta}\sec^2\theta\,d\theta}$$

$$= \frac{[\frac{\sec^3\theta}{3}]_0^{\frac{\pi}{4}}}{[\frac{\sec\theta\tan\theta}{2} + \frac{1}{2}\{\ln|\sec\theta + \tan\theta|\}]_0^{\frac{\pi}{4}}}$$

$\bar{x} = 0.531$ ft **Ans**

Also,

$$dL = \sqrt{dx^2 + dy^2}$$

$$L = \int\sqrt{1 + (\frac{dy}{dx})^2}\,dx$$

$$= \int_0^1 \sqrt{1 + x^2}\,dx$$

$= 1.148$ ft

$$\int \tilde{x}\,dL = \int_0^1 x\sqrt{1 + x^2}\,dx$$

$= 0.6095$

$\bar{x} = \frac{0.6095}{1.148} = 0.531$ ft **Ans**

$\xrightarrow{+}\Sigma F_x = 0;$ $O_x = 0$ **Ans**

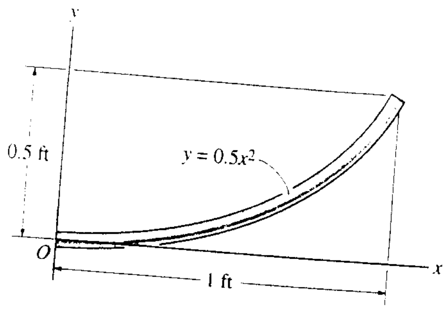
$+\uparrow\Sigma F_y = 0;$ $O_y - 0.5(1.148) = 0$

$O_y = 0.574$ lb **Ans**

$\circlearrowleft +\Sigma M_O = 0;$ $M_O - 0.5(1.148)(0.531) = 0$

$M_O = 0.305$ lb·ft **Ans**

9-6. Determine the distance $\bar{y}$ to the center of gravity of the homogeneous rod bent into the parabolic shape.



$$dL = \sqrt{dx^2 + dy^2}$$

$$L = \int \sqrt{1 + (\frac{dy}{dx})^2}\, dx$$

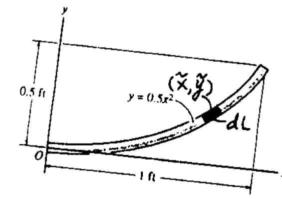
$$= \int_0^1 \sqrt{1 + x^2}\, dx$$

$$= 1.148 \text{ ft}$$

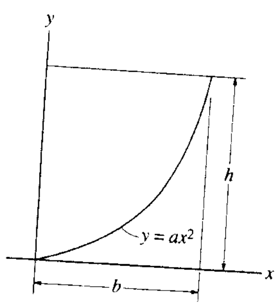
$$\int \tilde{y}\, dL = \int_0^1 0.5x^2\sqrt{1 + x^2}\, dx$$

$$= 0.2101 \text{ ft}$$

$$\bar{y} = \frac{0.2101}{1.148} = 0.183 \text{ ft}$$ **Ans**



9-7. Locate the centroid of the parabolic area.



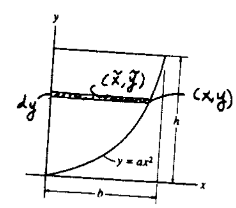
$$dA = x\, dy$$

$$\tilde{x} = \frac{x}{2}$$

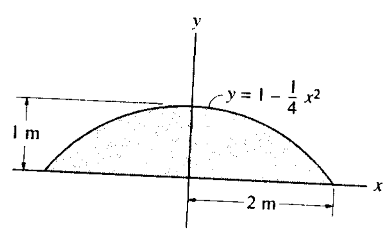
$$\tilde{y} = y$$

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_0^h \frac{y}{2a}\, dy}{\int_0^h \sqrt{\frac{y}{a}}\, dy} = \frac{[\frac{y^2}{4a}]_0^h}{[\frac{2y^{\frac{3}{2}}}{3\sqrt{a}}]_0^h} = \frac{3}{8}\sqrt{\frac{h}{a}} = \frac{3}{8}b$$ **Ans**

$$\bar{y} = \frac{\int_A \tilde{y}\, dA}{\int_A dA} = \frac{\int_0^h \frac{y^{3/2}}{\sqrt{a}} dy}{\int_0^h \sqrt{\frac{y}{a}}\, dy} = \frac{[\frac{2y^{5/2}}{5\sqrt{a}}]_0^h}{[\frac{2y^{\frac{3}{2}}}{3\sqrt{a}}]_0^h} = \frac{3}{5}h$$ **Ans**



***9-8.** Locate the centroid $(\bar{x}, \bar{y})$ of the shaded area.



Area and Moment Arm : The area of the differential element is $dA = ydx = \left(1 - \frac{1}{4}x^2\right)dx$ and its centroid is $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(1 - \frac{1}{4}x^2\right)$.

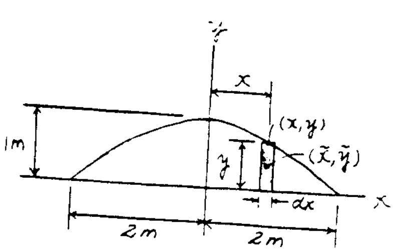
Centroid : Due to symmetry

$$\bar{x} = 0$$ **Ans**

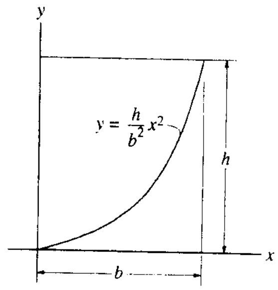
Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y} dA}{\int_A dA} = \frac{\int_{-2\text{m}}^{2\text{m}} \frac{1}{2}\left(1 - \frac{1}{4}x^2\right)\left(1 - \frac{1}{4}x^2\right)dx}{\int_{-2\text{m}}^{2\text{m}} \left(1 - \frac{1}{4}x^2\right)dx}$$

$$= \frac{\left(\frac{x}{2} - \frac{x^3}{12} + \frac{x^5}{160}\right)\Big|_{-2\text{m}}^{2\text{m}}}{\left(x - \frac{x^3}{12}\right)\Big|_{-2\text{m}}^{2\text{m}}} = \frac{2}{5} \text{ m}$$ **Ans**



9-9. Locate the centroid of the shaded area.



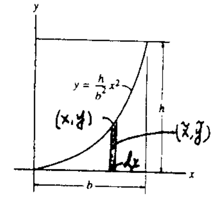
$dA = y\,dx$

$\tilde{x} = x$

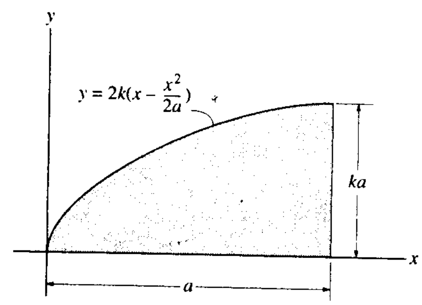
$\tilde{y} = \frac{y}{2}$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^b \frac{h}{b^2}x^3\,dx}{\int_0^b \frac{h}{b^2}x^2\,dx} = \frac{[\frac{h}{4b^2}x^4]_0^b}{[\frac{h}{3b^2}x^3]_0^b} = \frac{3}{4}b \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^b \frac{h^2}{2b^4}x^4\,dx}{\int_0^b \frac{h}{b^2}x^2\,dx} = \frac{[\frac{h^2}{10b^4}x^5]_0^b}{[\frac{h}{3b^2}x^3]_0^b} = \frac{3}{10}h \qquad \textbf{Ans}$$



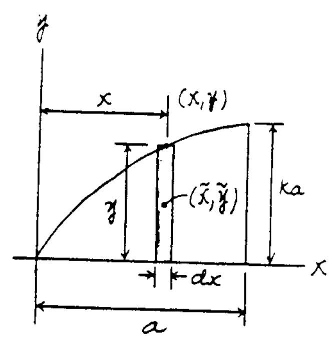
9-10. Locate the centroid $\bar{x}$ of the shaded area.



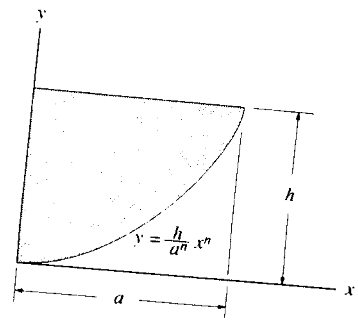
Area and Moment Arm : The area of the differential element is $dA = y\,dx = 2k\left(x - \frac{x^2}{2a}\right)dx$ and its centroid is $\tilde{x} = x$.

Centroid : Applying Eq. 9 – 6 and performing the integration, we have

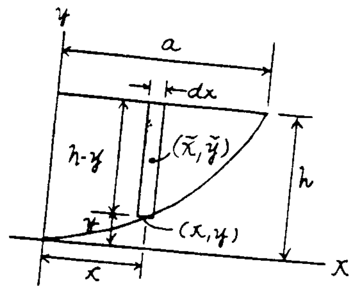
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a x\left[2k\left(x - \frac{x^2}{2a}\right)dx\right]}{\int_0^a 2k\left(x - \frac{x^2}{2a}\right)dx} = \frac{2k\left(\frac{x^3}{3} - \frac{x^4}{8a}\right)\Big|_0^a}{2k\left(\frac{x^2}{2} - \frac{x^3}{6a}\right)\Big|_0^a} = \frac{5a}{8} \qquad \textbf{Ans}$$



9-11. Locate the centroid $\bar{x}$ of the shaded area.



Area and Moment Arm : The area of the differential element is $dA = (h-y)\,dx = h\left(1-\frac{x^n}{a^n}\right)dx$ and its centroid is $\tilde{x} = x$.



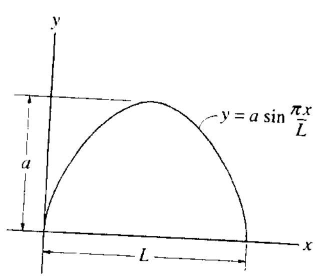
Centroid : Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a x\left[h\left(1-\frac{x^n}{a^n}\right)dx\right]}{\int_0^a h\left(1-\frac{x^n}{a^n}\right)dx}$$

$$= \frac{h\left(\frac{x^2}{2} - \frac{x^{n+2}}{(n+2)a^n}\right)\Big|_0^a}{h\left(x - \frac{x^{n+1}}{(n+1)a^n}\right)\Big|_0^a}$$

$$= \frac{n+1}{2(n+2)}\,a$$ **Ans**

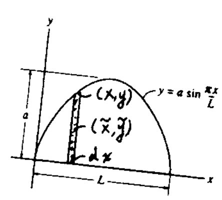
***9-12.** Locate the centroid of the shaded area.



$dA = y\,dx$

$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$



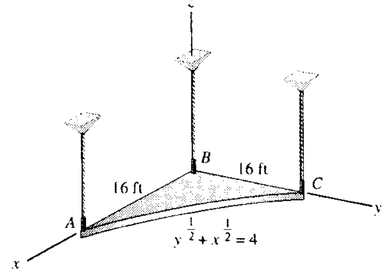
$$\int_A dA = \int_0^L a \sin\frac{\pi x}{L}\,dx = \left[-\frac{a\cos\frac{\pi x}{L}}{\frac{\pi}{L}}\right]_0^L = \frac{2aL}{\pi}$$

$$\int_A \tilde{y}\,dA = \frac{1}{2}\int_0^L a^2 \sin^2\frac{\pi x}{L}\,dx = \frac{a^2}{2}\left[-\frac{\sin\frac{2\pi x}{L}}{\frac{4\pi}{L}} + \frac{x}{2}\right]_0^L = \frac{a^2 L}{4}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{a^2 L}{4}}{\frac{2aL}{\pi}} = \frac{a\pi}{8}$$ **Ans**

$$\bar{x} = \frac{L}{2}$$ **Ans** **(By symmetry)**

9-13. The plate has a thickness of 0.25 ft and a specific weight of $\gamma = 180\ \text{lb/ft}^3$. Determine the location of its center of gravity. Also, find the tension in each of the cords used to support it.



Area and Moment Arm: Here, $y = x - 8x^{\frac{1}{2}} + 16$. The area of the differential element is $dA = ydx = (x - 8x^{\frac{1}{2}} + 16)dx$ and its centroid is $\tilde{x} = x$ and $\tilde{y} = \frac{1}{2}(x - 8x^{\frac{1}{2}} + 16)$. Evaluating the integrals, we have

$$A = \int_A dA = \int_0^{16\ \text{ft}} (x - 8x^{\frac{1}{2}} + 16)dx$$

$$= \left(\frac{1}{2}x^2 - \frac{16}{3}x^{\frac{3}{2}} + 16x\right)\Bigg|_0^{16\ \text{ft}} = 42.67\ \text{ft}^2$$

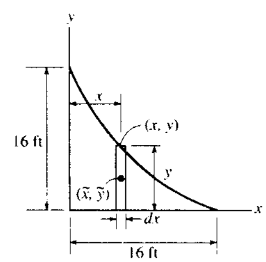
$$\int_A \tilde{x}dA = \int_0^{16\ \text{ft}} x[(x - 8x^{\frac{1}{2}} + 16)dx]$$

$$= \left(\frac{1}{3}x^3 - \frac{16}{5}x^{\frac{5}{2}} + 8x^2\right)\Bigg|_0^{16\ \text{ft}} = 136.53\ \text{ft}^3$$

$$\int_A \tilde{y}dA = \int_0^{16\ \text{ft}} \frac{1}{2}(x - 8x^{\frac{1}{2}} + 16)[(x - 8x^{\frac{1}{2}} + 16)dx]$$

$$= \frac{1}{2}\left(\frac{1}{3}x^3 - \frac{32}{5}x^{\frac{5}{2}} + 48x^2 - \frac{512}{3}x^{\frac{3}{2}} + 256x\right)\Bigg|_0^{16\ \text{ft}}$$

$$= 136.53\ \text{ft}^3$$

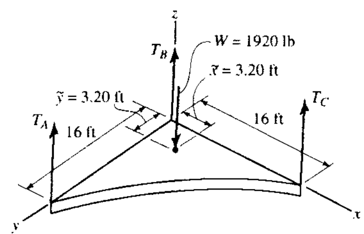


Centroid: Applying Eq. 9-6, we have

$$\bar{x} = \frac{\int_A \tilde{x}dA}{\int_A dA} = \frac{136.53}{42.67} = 3.20\ \text{ft} \quad \textbf{Ans}$$

$$\bar{y} = \frac{\int_A \tilde{y}dA}{\int_A dA} = \frac{136.53}{42.67} = 3.20\ \text{ft} \quad \textbf{Ans}$$

Equations of Equilibrium: The weight of the plate is $W = 42.67(0.25)(180) = 1920$ lb.



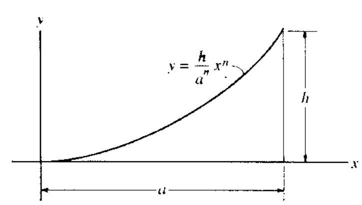
$\Sigma M_x = 0;\quad 1920(3.20) - T_A(16) = 0 \quad T_A = 384\ \text{lb}$ **Ans**

$\Sigma M_y = 0;\quad T_C(16) - 1920(3.20) = 0 \quad T_C = 384\ \text{lb}$ **Ans**

$\Sigma F_z = 0;\quad T_B + 384 + 384 - 1920 = 0$

$T_B = 1152\ \text{lb} = 1.15\ \text{kip}$ **Ans**

9-14. Locate the centroid $\bar{y}$ of the shaded area.



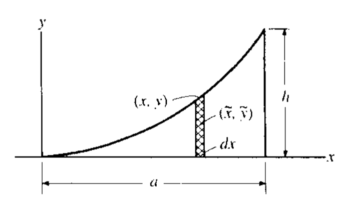
$dA = y\ dx$

$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$

$$\bar{x} = \frac{\int_A \tilde{x}\ dA}{\int_A dA} = \frac{\int_0^a \frac{h}{a^n}x^{n+1}dx}{\int_0^a \frac{h}{a^n}x^n dx} = \frac{\frac{h(a^{n+2})}{a^n(n+2)}}{\frac{h(a^{n+1})}{a^n(n+1)}}$$

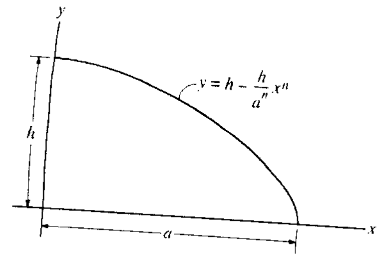
$$= \frac{(n+1)}{2(n+2)}a \quad \textbf{Ans}$$



$$\bar{y} = \frac{\int_A \tilde{y}\ dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a \frac{h^2}{a^{2n}}x^{2n}dx}{\int_0^a \frac{h}{a^n}x^n dx} = \frac{\frac{h^2(a^{2n+1})}{2a^{2n}(2n+1)}}{\frac{h(a^{n+1})}{a^n(n+1)}}$$

$$= \frac{n+1}{2(2n+1)}h \quad \textbf{Ans}$$

9-15. Locate the centroid of the shaded area.



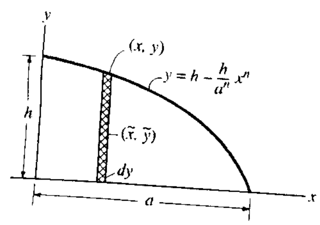
$dA = y\,dx$

$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a \left(hx - \frac{h}{a^n}x^{n+1}\right)dx}{\int_0^a \left(h - \frac{h}{a^n}x^n\right)dx}$$

$$= \frac{\left[\frac{h}{2}x^2 - \frac{h(x^{n+2})}{a^n(n+2)}\right]_0^a}{\left[hx - \frac{h(x^{n+1})}{a^n(n+1)}\right]_0^a}$$



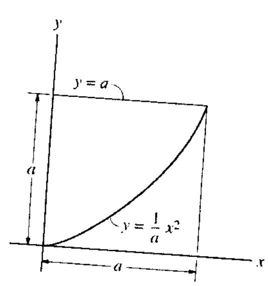
$$\bar{x} = \frac{\left(\frac{h}{2} - \frac{h}{n+2}\right)a^2}{\left(h - \frac{h}{n+1}\right)a} = \frac{n - 1}{2(n+2)}a$$ **Ans**

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a \left(h^2 - 2\frac{h^2}{a^n}x^n - \frac{h^2}{a^{2n}}x^{2n}\right)dx}{\int_0^a \left(h - \frac{h}{a^n}x\right)dx}$$

$$= \frac{\frac{1}{2}\left[h^2x - \frac{2h^2(x^{n+1})}{a^n(n+1)} + \frac{h^2(x^{2n})}{a^{2n}(2n1)}\right]_0^a}{\left[hx - \frac{h(x^{n+1})}{a^n(n+1)}\right]_0^a}$$

$$\bar{y} = \frac{\frac{2n^2}{2(n+1)(2n+1)}h}{\frac{n}{n+1}} = \frac{nh}{2n}$$ **Ans**

***9-16.** Locate the centroid the shaded area bounded by the parabola and the line = a



$dA = x\,dy$

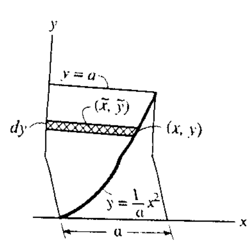
$\tilde{x} = \frac{x}{2}$

$\tilde{y} = y$

$$\int_A dA = \int_0^a x\,dy = \int_0^a \sqrt{ay}\,dy = \sqrt{a}\left(\frac{2}{3}a^{3/2}\right) = \frac{2}{3}a^2$$

$$\int_A \tilde{x}\,dA = \int_0^a \frac{x^2}{2}dy = \quad dy = \frac{1}{4}a^3$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\frac{1}{4}a^3}{\frac{2}{3}a^2}$$ **Ans**



$$\int_A \tilde{y}\,dA = \int_0^a xy\,\sqrt{a}y^{3/2}\,dy = \sqrt{a}\left(\frac{2}{5}a^{5/2}\right) = \frac{2}{5}a^3$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} \quad a$$ **Ans**