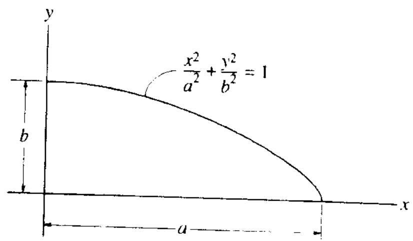
**9-17.** Locate the centroid of the quarter elliptical area.




$dA = y\,dx$

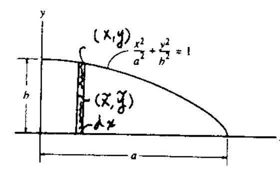
$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$

$$\int_A dA = \int_0^a \sqrt{b^2 - \frac{b^2}{a^2}x^2}\,dx = \frac{b}{2a}\left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\frac{x}{a}\right]_0^a = \frac{\pi}{4}ab$$

$$\int_A \tilde{y}\,dA = \frac{1}{2}\int_0^a \left(b^2 - \frac{b^2}{a^2}x^2\right)dx = \frac{1}{2}\left[b^2 x - \frac{b^2}{3a^2}x^3\right]_0^a = \frac{1}{3}ab^2$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{3}ab^2}{\frac{\pi}{4}ab} = \frac{4b}{3\pi} \qquad \textbf{Ans}$$




$dA = x\,dy$

$\tilde{x} = \frac{x}{2}$

$$\int_A \tilde{x}\,dA = \frac{1}{2}\int_0^b \left(a^2 - \frac{a^2}{b^2}y^2\right)dy = \frac{1}{2}\left[a^2 y - \frac{a^2}{3b^2}y^3\right]_0^b = \frac{1}{3}a^2 b$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\frac{1}{3}a^2 b}{\frac{\pi}{4}ab} = \frac{4a}{3\pi} \qquad \textbf{Ans}$$

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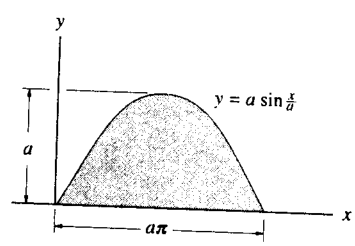
**9-18.** Locate the centroid $(\bar{x}, \bar{y})$ of the shaded area.




***Area and Moment Arm :*** The area of the differential element is $dA = y\,dx = a\sin\frac{x}{a}dx$ and its centroid are $\tilde{x} = x$ and $\tilde{y} = \frac{y}{2} = \frac{a}{2}\sin\frac{x}{a}$.

***Centroid :*** Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{\pi a} x\left(a\sin\frac{x}{a}dx\right)}{\int_0^{\pi a} a\sin\frac{x}{a}dx} = \frac{\left[a^3\sin\frac{x}{a} - x\left(a^2\cos\frac{x}{a}\right)\right]\Big|_0^{\pi a}}{\left(-a^2\cos\frac{x}{a}\right)\Big|_0^{\pi a}} = \frac{\pi}{2}a \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{\pi a} \frac{a}{2}\sin\frac{x}{a}\left(a\sin\frac{x}{a}dx\right)}{\int_0^{\pi a} a\sin\frac{x}{a}dx} = \frac{\left[\frac{1}{4}a^2\left(x - \frac{1}{2}a\sin\frac{2x}{a}\right)\right]\Big|_0^{\pi a}}{\left(-a^2\cos\frac{x}{a}\right)\Big|_0^{\pi a}} = \frac{\pi}{8}a \qquad \textbf{Ans}$$

**9-19.** Locate the centroid of the shaded area.

$$dA = (4 - y)dx = \left(4 - \frac{1}{16}x^2\right)dx$$

$$\tilde{x} = x$$

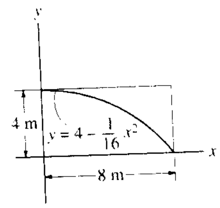
$$\tilde{y} = \frac{4 + y}{2}$$

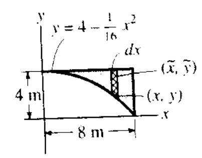
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^8 \left(16 - \left(\frac{1}{16}x^2\right)^2\right)dx}{\int_0^8 \left(4 - \frac{1}{16}x^2\right)dx}$$

$\bar{y} = 2.80$ m **Ans**

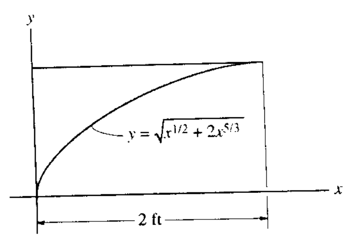
$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^8 x\left(4 - \frac{1}{16}x^2\right)dx}{\int_0^8 \left(4 - \frac{1}{16}x^2\right)dx}$$

$\bar{x} = 3.00$ m **Ans**




***■9-20.** Locate the centroid $\bar{x}$ of the shaded area. Solve the problem by evaluating the integrals using Simpson's rule.

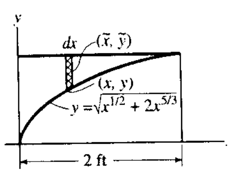



$$\int_A dA = \int_0^2 (2.786 - y)\,dx = \int_0^2 (2.786 - \sqrt{x^{1/2} + 2x^{5/3}})\,dx$$

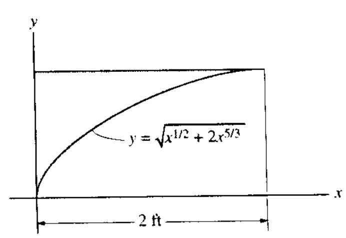
$$= 2.177 \text{ ft}^2$$

$$\int_A \tilde{x}\,dA = \int_0^2 x(2.786 - \sqrt{x^{1/2} + 2x^{5/3}})\,dx = 1.412 \text{ ft}^3$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{1.412}{2.177} = 0.649 \text{ ft}$$ **Ans**




**■9-21.** Locate the centroid $\bar{y}$ of the shaded area. Solve the problem by evaluating the integrals using Simpson's rule.



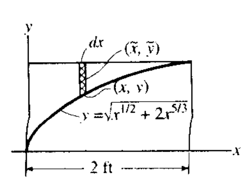

$$\int_A dA = \int_0^2 (2.786 - y)\,dx = \int_0^2 (2.786 - \sqrt{x^{1/2} + 2x^{5/3}})\,dx$$

$$= 2.177 \text{ ft}^2$$

$$\int_A \tilde{y}\,dA = \int_0^2 \left(\frac{2.786 + y}{2}\right)(2.786 - y)\,dx$$

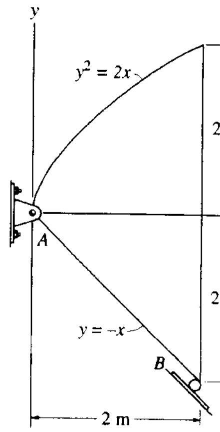
$$= \int_0^2 \frac{1}{2}\{(2.786)^2 - y^2\}dx$$

$$= \frac{1}{2}\int_0^2 [7.764 - (x^{1/2} + 2x^{5/3})]\,dx = 4.440 \text{ ft}^3$$




$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{4.440}{2.177} = 2.04 \text{ ft}$$ **Ans**

**9-22.** The steel plate is 0.3 m thick and has a density of 7850 kg/m³. Determine the location of its center of mass. Also compute the reactions at the pin and roller support.



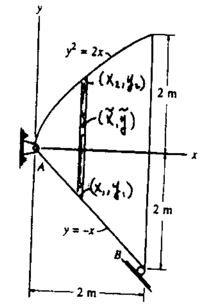

$$y_1 = -x_1$$

$$y_2^2 = 2x_2$$

$$dA = (y_2 - y_1)\,dx = (\sqrt{2x} + x)\,dx$$

$$\tilde{x} = x$$

$$\tilde{y} = \frac{y_2 + y_1}{2} = \frac{\sqrt{2x} - x}{2}$$




$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^2 x(\sqrt{2x} + x)\,dx}{\int_0^2 (\sqrt{2x} + x)\,dx} = \frac{\left[\frac{2\sqrt{2}}{5}x^{5/2} + \frac{1}{3}x^3\right]_0^2}{\left[\frac{2\sqrt{2}}{3}x^{3/2} + \frac{1}{2}x^2\right]_0^2} = 1.2571 = 1.26 \text{ m}$$ **Ans**

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^2 \frac{\sqrt{2x} - x}{2}(\sqrt{2x} + x)\,dx}{\int_0^2 (\sqrt{2x} + x)\,dx} = \frac{\left[\frac{x^2}{2} - \frac{1}{6}x^3\right]_0^2}{\left[\frac{2\sqrt{2}}{3}x^{3/2} + \frac{1}{2}x^2\right]_0^2} = 0.143 \text{ m}$$ **Ans**

$$A = 4.667 \text{ m}^2$$

$$W = 7850(9.81)(4.667)(0.3) = 107.81 \text{ kN}$$

$\circlearrowleft + \Sigma M_A = 0;\quad -1.2571(107.81) + N_B(2\sqrt{2}) = 0$

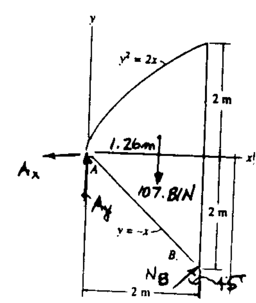
$N_B = 47.92 = 47.9 \text{ kN}$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -A_x + 47.92\sin 45° = 0$

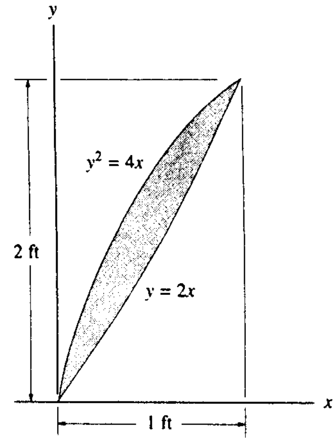
$A_x = 33.9 \text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y + 47.92\cos 45° - 107.81 = 0$

$A_y = 73.9 \text{ kN}$ **Ans**



**9-23.** Locate the centroid $\bar{x}$ of the shaded area.

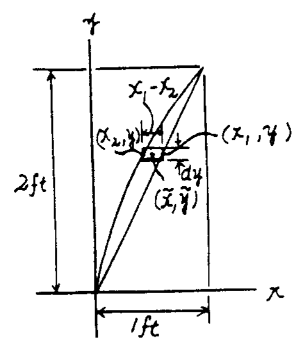



***Area and Moment Arm :*** Here, $x_1 = \frac{y}{2}$ and $x_2 = \frac{y^2}{4}$. The area of the differential element is $dA = (x_1 - x_2)\,dy = \left(\frac{y}{2} - \frac{y^2}{4}\right)dy$ and its centroid is $\tilde{x} = x_2 + \frac{x_1 - x_2}{2} = \frac{1}{2}(x_1 + x_2) = \frac{1}{2}\left(\frac{y}{2} + \frac{y^2}{4}\right)$.

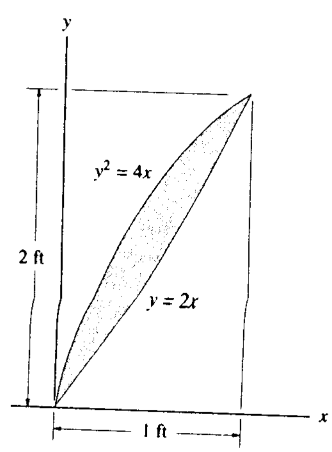
***Centroid :*** Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{2\text{ft}} \frac{1}{2}\left(\frac{y}{2} + \frac{y^2}{4}\right)\left[\left(\frac{y}{2} - \frac{y^2}{4}\right)dy\right]}{\int_0^{2\text{ft}} \left(\frac{y}{2} - \frac{y^2}{4}\right)dy}$$

$$= \frac{\left[\frac{1}{2}\left(\frac{1}{12}y^3 - \frac{1}{80}y^5\right)\right]\Big|_0^{2\text{ft}}}{\left(\frac{1}{4}y^2 - \frac{1}{12}y^3\right)\Big|_0^{2\text{ft}}} = \frac{2}{5}\text{ ft} = 0.4\text{ ft} \qquad \textbf{Ans}$$




***9-24.** Locate the centroid $\bar{y}$ of the shaded area.



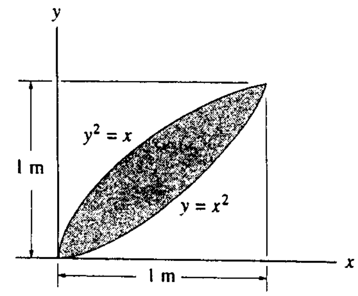

***Area and Moment Arm :*** Here, $x_1 = \frac{y}{2}$ and $x_2 = \frac{y^2}{4}$. The area of the differential element is $dA = (x_1 - x_2)\,dy = \left(\frac{y}{2} - \frac{y^2}{4}\right)dy$ and its centroid is $\tilde{y} = y$.

***Centroid :*** Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{2\text{ft}} y\left[\left(\frac{y}{2} - \frac{y^2}{4}\right)dy\right]}{\int_0^{2\text{ft}} \left(\frac{y}{2} - \frac{y^2}{4}\right)dy}$$

$$= \frac{\left(\frac{1}{6}y^3 - \frac{1}{16}y^4\right)\Big|_0^{2\text{ft}}}{\left(\frac{1}{4}y^2 - \frac{1}{12}y^3\right)\Big|_0^{2\text{ft}}} = 1\text{ ft} \qquad \textbf{Ans}$$

**9-25.** Locate the centroid $\bar{x}$ of the shaded area.

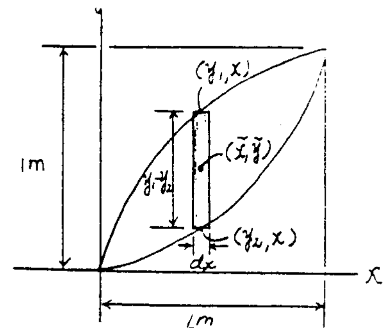



***Area and Moment Arm*** *:* Here, $y_1 = x^{\frac{1}{2}}$ and $y_2 = x^2$. The area of the differential element is $dA = (y_1 - y_2)\,dx = \left(x^{\frac{1}{2}} - x^2\right)dx$ and its centroid is $\tilde{x} = x$.

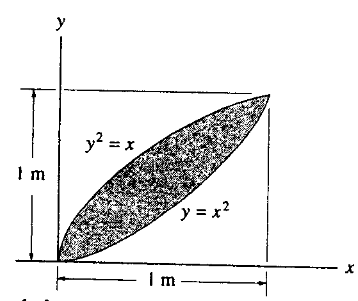
***Centroid*** *:* Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{1\,\text{m}} x\left[\left(x^{\frac{1}{2}} - x^2\right)dx\right]}{\int_0^{1\,\text{m}} \left(x^{\frac{1}{2}} - x^2\right)dx}$$

$$= \frac{\left(\frac{2}{5}x^{\frac{5}{2}} - \frac{1}{4}x^4\right)\Big|_0^{1\,\text{m}}}{\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Big|_0^{1\,\text{m}}} = \frac{9}{20}\ \text{m} = 0.45\ \text{m} \qquad \textbf{Ans}$$




**9-26.** Locate the centroid $\bar{y}$ of the shaded area.



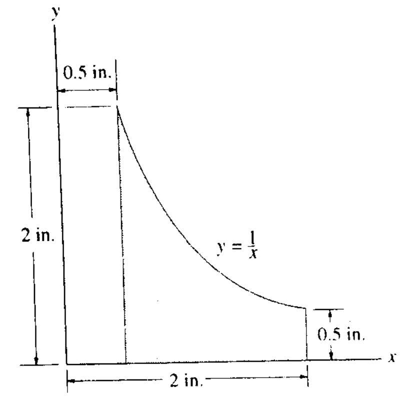

***Area and Moment Arm*** *:* Here, $y_1 = x^{\frac{1}{2}}$ and $y_2 = x^2$. The area of the differential element is $dA = (y_1 - y_2)\,dx = \left(x^{\frac{1}{2}} - x^2\right)dx$ and its centroid is $\tilde{y} = y_2 + \frac{y_1 - y_2}{2} = \frac{1}{2}(y_1 + y_2) = \frac{1}{2}\left(x^{\frac{1}{2}} + x^2\right)$.

***Centroid*** *:* Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\,\text{m}} \frac{1}{2}\left(x^{\frac{1}{2}} + x^2\right)\left[\left(x^{\frac{1}{2}} - x^2\right)dx\right]}{\int_0^{1\,\text{m}} \left(x^{\frac{1}{2}} - x^2\right)dx}$$

$$= \frac{\frac{1}{2}\left(\frac{1}{2}x^2 - \frac{1}{5}x^5\right)\Big|_0^{1\,\text{m}}}{\left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Big|_0^{1\,\text{m}}} = \frac{9}{20}\ \text{m} = 0.45\ \text{m} \qquad \textbf{Ans}$$

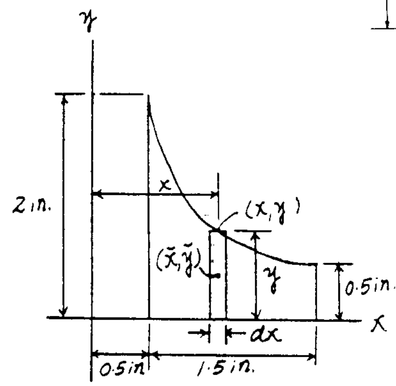
**9-27.** Locate the centroid $x$ of the shaded area.



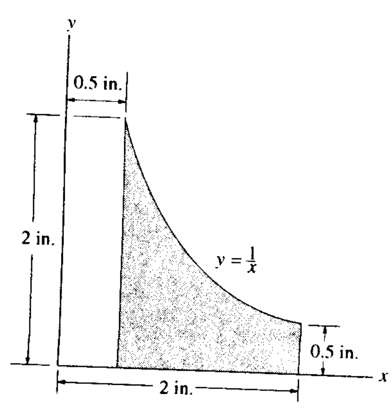

***Area and Moment Arm*** : The area of the differential element is $dA = y\,dx = \frac{1}{x}dx$ and its centroid is $\tilde{x} = x$.

***Centroid*** : Applying Eq. 9 − 6 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_{0.5\,\text{in}}^{2\,\text{in}} x\left(\frac{1}{x}dx\right)}{\int_{0.5\,\text{in}}^{2\,\text{in}} \frac{1}{x}dx} = \frac{x\Big|_{0.5\,\text{in}}^{2\,\text{in}}}{\ln x\Big|_{0.5\,\text{in}}^{2\,\text{in}}} = 1.08 \text{ in.} \qquad \textbf{Ans}$$




***9-28.** Locate the centroid $\bar{y}$ of the shaded area.

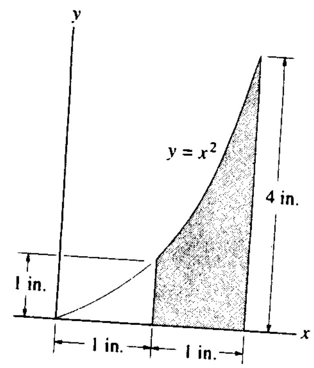



***Area and Moment Arm*** : The area of the differential element is $dA = y\,dx = \frac{1}{x}dx$ and its centroid is $\tilde{y} = \frac{y}{2} = \frac{1}{2x}$.

***Centroid*** : Applying Eq. 9 − 6 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_{0.5\,\text{in}}^{2\,\text{in}} \frac{1}{2x}\left(\frac{1}{x}dx\right)}{\int_{0.5\,\text{in}}^{2\,\text{in}} \frac{1}{x}dx} = \frac{-\frac{1}{2x}\Big|_{0.5\,\text{in}}^{2\,\text{in}}}{\ln x\Big|_{0.5\,\text{in}}^{2\,\text{in}}} = 0.541 \text{ in} \qquad \textbf{Ans}$$

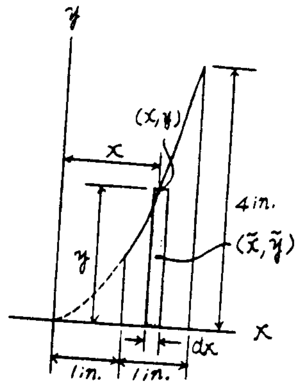
**9-29.** Locate the centroid $\bar{x}$ of the shaded area.




***Area and Moment Arm :*** The area of the differential element is $dA = ydx = x^2dx$ and its centroid is $\tilde{x} = x$.

***Centroid :*** Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{x} = \frac{\int_A \tilde{x}dA}{\int_A dA} = \frac{\int_{1\text{in}}^{2\text{in}} x(x^2dx)}{\int_{1\text{in}}^{2\text{in}} x^2dx} = \frac{\left.\frac{x^4}{4}\right|_{1\text{in}}^{2\text{in}}}{\left.\frac{x^3}{3}\right|_{1\text{in}}^{2\text{in}}} = 1.61 \text{ in} \qquad \textbf{Ans}$$

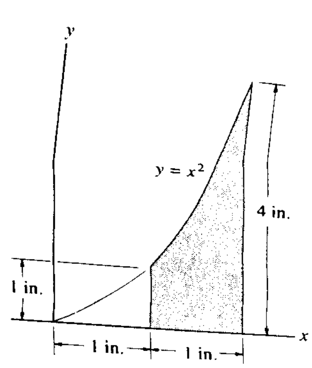



**9-30.** Locate the centroid $\bar{y}$ of the shaded area.

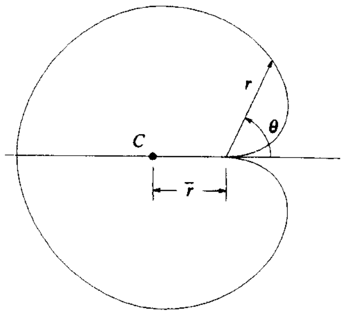
***Area and Moment Arm :*** The area of the differential element is $dA = ydx = x^2dx$ and its centroid is $\tilde{y} = \frac{y}{2} = \frac{1}{2}x^2$.

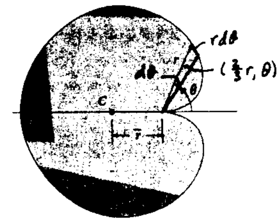
***Centroid :*** Applying Eq. 9 – 6 and performing the integration, we have

$$\bar{y} = \frac{\int_A \tilde{y}dA}{\int_A dA} = \frac{\int_{1\text{in.}}^{2\text{in.}} \frac{1}{2}x^2(x^2dx)}{\int_{1\text{in.}}^{2\text{in.}} x^2dx} = \frac{\left.\frac{x^5}{10}\right|_{1\text{in.}}^{2\text{in.}}}{\left.\frac{x^3}{3}\right|_{1\text{in.}}^{2\text{in.}}} = 1.33 \text{ in.} \qquad \textbf{Ans}$$



**9-31.** Determine the location $\bar{r}$ of the centroid $C$ of the cardioid, $r = a(1 - \cos\theta)$.




$$dA = \frac{1}{2}r^2\,d\theta$$

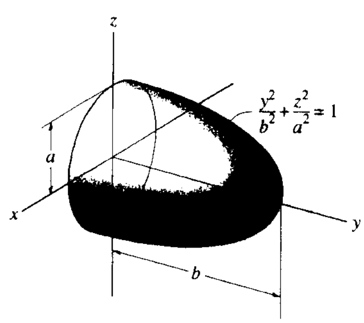
$$A = 2\int_0^{\pi}\frac{1}{2}(a^2)(1-\cos\theta)^2 d\theta = \frac{3}{2}\pi a^2$$

$$\int_A \tilde{r}\,dA = 2\int_0^{\pi}\left(\frac{2}{3}r\cos\theta\right)\left(\frac{1}{2}\right)(a^2)(1-\cos\theta)^2\,d\theta$$

$$= \frac{2}{3}a^3\int_0^{\pi}(1-\cos\theta)^3\cos\theta\,d\theta = 3.927\,a^3$$

$$\bar{r} = \frac{\int_A \tilde{r}\,dA}{\int_A dA} = \frac{3.927\,a^3}{\frac{3}{2}\pi a^2} = 0.833\,a \qquad \textbf{Ans}$$

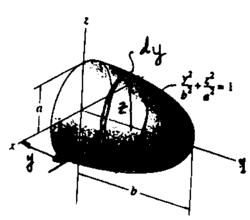
***9-32.** Locate the centroid of the ellipsoid of revolution.




$$dV = \pi z^2\,dy$$

$$\int dV = \int_0^b \pi a^2\left(1-\frac{y^2}{b^2}\right)dy = \pi a^2\left[y - \frac{y^3}{3b^2}\right]_0^b = \frac{2\pi a^2 b}{3}$$

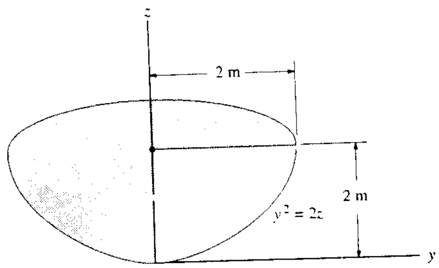
$$\int \tilde{y}\,dV = \int_0^b \pi a^2 y\left(1-\frac{y^2}{b^2}\right)dy = \pi a^2\left[\frac{y^2}{2} - \frac{y^4}{4b^2}\right]_0^b = \frac{\pi a^2 b^2}{4}$$




$$\bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} = \frac{\frac{\pi a^2 b^2}{4}}{\frac{2\pi a^2 b}{3}} = \frac{3}{8}b \qquad \textbf{Ans}$$

$\bar{x} = \bar{z} = 0$ **Ans** **(By symmetry)**

**9-33.** Locate the center of gravity of the volume. The material is homogeneous.




***Volume and Moment Arm:*** The volume of the thin disk differential element is $dV = \pi y^2 dz = \pi(2z)dz = 2\pi z dz$ and its centroid $\tilde{z} = z$.

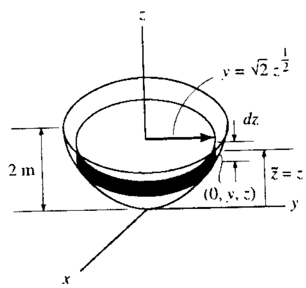
***Centroid:*** Due to symmetry about $z$ axis

$\bar{x} = \bar{y} = 0$ **Ans**

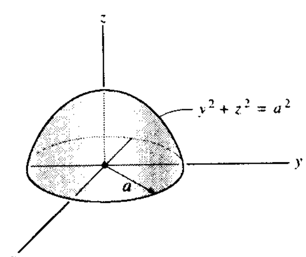
Applying Eq. 9-5 and performing the integration, we have

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^{2\text{m}} z(2\pi z\,dz)}{\int_0^{2\text{m}} 2\pi z\,dz}$$

$$= \frac{2\pi\left(\frac{z^3}{3}\right)\Big|_0^{2\text{m}}}{2\pi\left(\frac{z^2}{2}\right)\Big|_0^{2\text{m}}} = \frac{4}{3}\text{m} \qquad \textbf{Ans}$$




**9-34.** Locate the centroid $\bar{z}$ of the hemisphere.

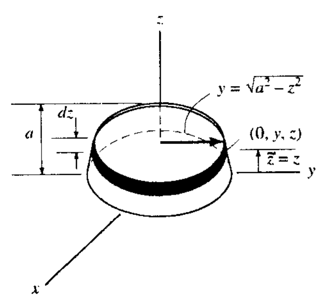



***Volume and Moment Arm:*** The volume of the thin disk differential element is $dV = \pi y^2 dz = \pi(a^2 - z^2)dz$ and its centroid $\tilde{z} = z$.

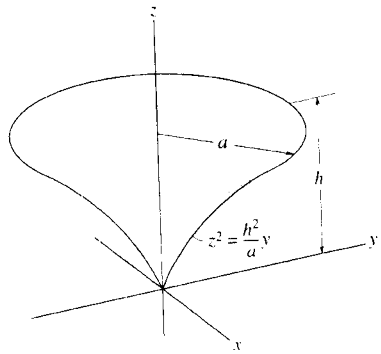
***Centroid:*** Applying Eq. 9-5 and performing the integration, we have

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^a z[\pi(a^2 - z^2)dz]}{\int_0^a \pi(a^2 - z^2)dz}$$

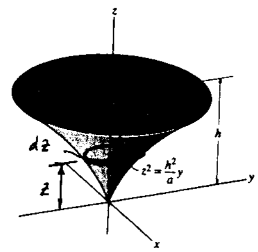
$$= \frac{\pi\left(\frac{a^2z^2}{2} - \frac{z^4}{4}\right)\Big|_0^a}{\pi\left(a^2z - \frac{z^3}{3}\right)\Big|_0^a} = \frac{3}{8}a \qquad \textbf{Ans}$$



**9-35.** Locate the centroid of the solid.




$\bar{x} = \bar{y} = 0$ **Ans** **(By symmetry)**

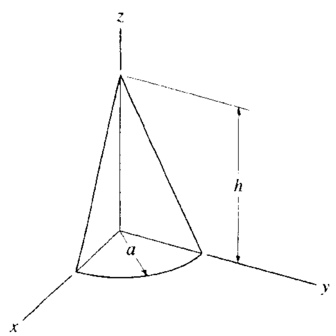


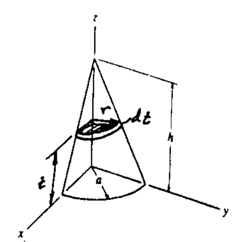

$$\int dV = \int_0^h \pi y^2\, dz = \pi \int_0^h \frac{a^2}{h^4} z^4\, dz = \left[\frac{\pi a^2}{5h^4} z^5\right]_0^h = \frac{\pi a^2 h}{5}$$

$$\int \tilde{z}\, dV = \int_0^h \pi y^2 z\, dz = \frac{\pi a^2}{h^4}\int_0^h z^5\, dz = \left[\frac{\pi a^2}{6h^4} z^6\right]_0^h = \frac{\pi a^2 h^2}{6}$$

$$\bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV} = \frac{\frac{\pi a^2 h^2}{6}}{\frac{\pi a^2 h}{5}} = \frac{5}{6}h$$ **Ans**

***9-36.** Locate the centroid of the quarter-cone.




$$\tilde{z} = z$$

$$r = \frac{a}{h}(h - z)$$

$$dV = \frac{\pi}{4} r^2\, dz = \frac{\pi a^2}{4h^2}(h - z)^2\, dz$$

$$\int dV = \frac{\pi a^2}{4h^2}\int_0^h (h^2 - 2hz + z^2)\, dz = \frac{\pi a^2}{4h^2}\left[h^2 z - h z^2 + \frac{z^3}{3}\right]_0^h$$

$$= \frac{\pi a^2}{4h^2}\left(\frac{h^3}{3}\right) = \frac{\pi a^2 h}{12}$$

$$\int \tilde{z}\, dV = \frac{\pi a^2}{4h^2}\int_0^h (h^2 - 2hz + z^2) z\, dz = \frac{\pi a^2}{4h^2}\left[h^2\frac{z^2}{2} - 2h\frac{z^3}{3} + \frac{z^4}{4}\right]_0^h$$

$$= \frac{\pi a^2}{4h^2}\left(\frac{h^4}{12}\right) = \frac{\pi a^2 h^2}{48}$$

$$\bar{z} = \frac{\int \tilde{z}\, dV}{\int dV} = \frac{\frac{\pi a^2 h^2}{48}}{\frac{\pi a^2 h}{12}} = \frac{h}{4}$$ **Ans**

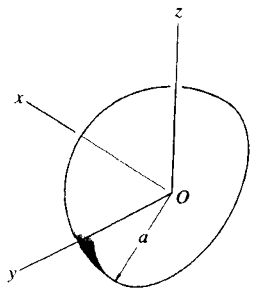
$$\int \tilde{x}\, dV = \frac{\pi a^2}{4h^2}\int_0^h \frac{4r}{3\pi}(h - z)^2\, dz = \frac{\pi a^2}{4h^2}\int_0^h \frac{4a}{3\pi h}(h^3 - 3h^2 z + 3h z^2 - z^3)\, dz$$

$$= \frac{\pi a^2}{4h^2}\left(h^4 - \frac{3h^4}{2} + h^4 - \frac{h^4}{4}\right)$$

$$= \frac{\pi a^2}{4h^2}\left(\frac{a h^3}{3\pi}\right) = \frac{a^3 h}{12}$$

$$\bar{x} = \bar{y} = \frac{\int \tilde{x}\, dV}{\int dV} = \frac{\frac{a^3 h}{12}}{\frac{\pi a^2 h}{12}} = \frac{a}{\pi}$$ **Ans**

**9-37.** Locate the center of MASS $\bar{x}$ of the hemisphere. The density of the material varies linearly from zero at the origin $O$ to $\rho_0$ at the surface. *Suggestion:* Choose a hemispherical shell element for integration.




$$\tilde{x} = \frac{x}{2}$$

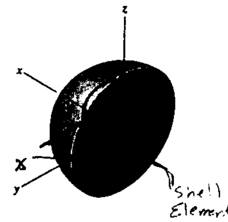
$$\rho = \rho_0\left(\frac{x}{a}\right)$$

$$dV = 2\pi x^2\,dx$$

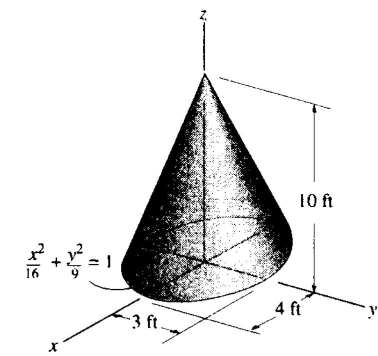
$$dW = \rho\,dV = \left(\frac{2\pi}{a}\right)\rho_0\,x^3\,dx$$

$$\bar{x} = \frac{\int_W \tilde{x}\,dW}{\int_W dW} = \frac{\int_0^a \frac{x}{2}\left(\frac{2\pi}{a}\rho_0\right)x^3\,dx}{\int_0^a \left(\frac{2\pi}{a}\rho_0\right)x^3\,dx}$$

$$= \frac{\frac{1}{2}\left[\frac{x^5}{5}\right]_0^a}{\left[\frac{x^4}{4}\right]_0^a} = 0.4\,a \qquad \textbf{Ans}$$




**9-38.** Locate the centroid $\bar{z}$ of the right-elliptical cone.

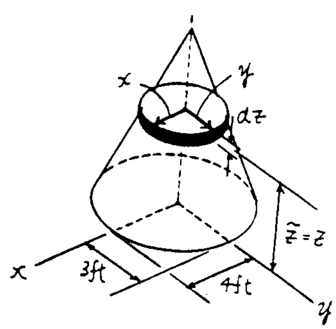


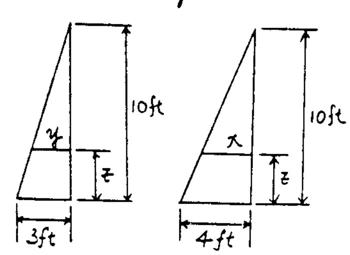

***Volume and Moment Arm*** : From the geometry, $\frac{x}{10-z} = \frac{4}{10}$, $x = 0.4(10-z)$ and $\frac{y}{10-z} = \frac{3}{10}$, $y = 0.3(10-z)$. The volume of the thin disk differential element is $dV = \pi xy\,dz = \pi[0.4(10-z)][0.3(10-z)]\,dz$ $= 0.12\pi\left(z^2 - 20z + 100\right)dz$ and its centroid $\tilde{z} = z$.

***Centroid*** : Applying Eq. 9 – 5 and performing the integration, we have

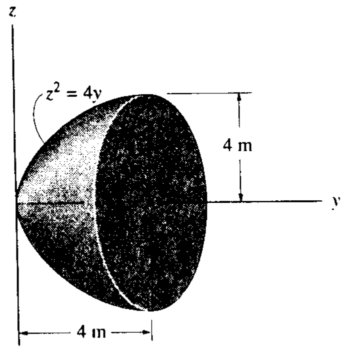
$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^{10\text{ft}} z\left[0.12\pi(z^2 - 20z + 100)\,dz\right]}{\int_0^{10\text{ft}} 0.12\pi(z^2 - 20z + 100)\,dz}$$

$$= \frac{0.12\pi\left(\frac{z^4}{4} - \frac{20z^3}{3} + 50z^2\right)\Big|_0^{10\text{ft}}}{0.12\pi\left(\frac{z^3}{3} - 10z^2 + 100z\right)\Big|_0^{10\text{ft}}} = 2.50\text{ ft} \qquad \textbf{Ans}$$



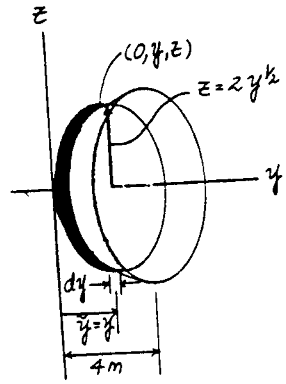



**9-39.** Locate the centroid $\bar{y}$ of the paraboloid.




***Volume and Moment Arm*** **: here, $z = 2y^{\frac{1}{2}}$. The volume of the thin disk differential element is $dV = \pi z^2\,dy = \pi(4y)\,dy$ and its centroid $\tilde{y} = y$.**

***Centroid*** **: Applying Eq. 9 – 5 and performing the integration, we have**

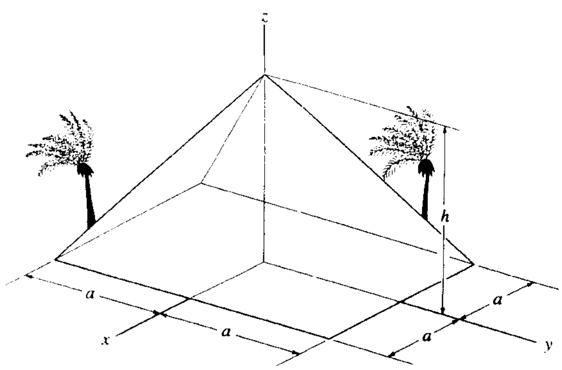


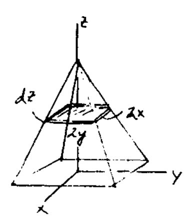

$$\bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} = \frac{\int_0^{4\,\text{m}} y[\pi(4y)\,dy]}{\int_0^{4\,\text{m}} \pi(4y)\,dy}$$

$$= \frac{4\pi\left(\frac{y^3}{3}\right)\Big|_0^{4\,\text{m}}}{4\pi\left(\frac{y^2}{2}\right)\Big|_0^{4\,\text{m}}} = 2.67\ \text{m} \qquad \textbf{Ans}$$

---

***9-40.** The king's chamber of the Great Pyramid of Giza is located at its centroid. Assuming the pyramid to be a solid, prove that this point is at $\bar{z} = \frac{1}{4}h$, *Suggestion:* Use a rectangular differential plate element having a thickness $dz$ and area $(2x)(2y)$.




$$dV = (2x)(2y)\,dz = 4xy\,dz$$

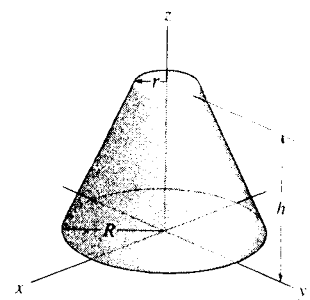
$$x = y = \frac{a}{h}(h - z)$$

$$\int dV = \int_0^h \frac{4a^2}{h^2}(h-z)^2\,dz = \frac{4a^2}{h^2}\left[h^2 z - hz^2 + \frac{z^3}{3}\right]_0^h = \frac{4a^2 h}{3}$$

$$\int \tilde{z}\,dV = \int_0^h \frac{4a^2}{h^2}(h-z)^2 z\,dz = \frac{4a^2}{h^2}\left[h^2\frac{z^2}{2} - 2h\frac{z^3}{3} + \frac{z^4}{4}\right]_0^h = \frac{a^2h^2}{3}$$

$$\bar{z} = \frac{\int \tilde{z}\,dV}{\int dV} = \frac{\frac{a^2h^2}{3}}{\frac{4a^2h}{3}} = \frac{h}{4} \qquad \textbf{(QED)}$$

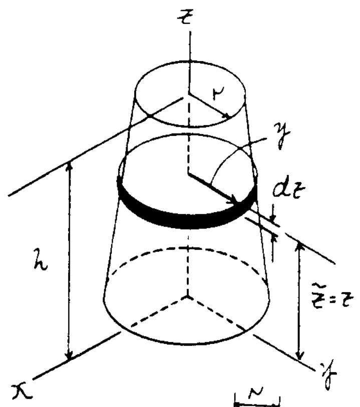
**9-41.** Locate the centroid $\bar{z}$ of the frustum of the right-circular cone.

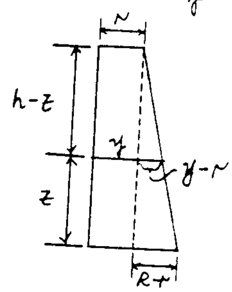


***Volume and Moment Arm :*** From the geometry, $\frac{y-r}{R-r}=\frac{h-z}{h}$, $y=\frac{(r-R)z+Rh}{h}$. The volume of the thin disk differential element is

$$dV=\pi y^2 dz=\pi\left[\left(\frac{(r-R)z+Rh}{h}\right)^2\right]dz$$
$$=\frac{\pi}{h^2}\left[(r-R)^2z^2+2Rh(r-R)z+R^2h^2\right]dz$$

and its centroid $\tilde{z}=z$.

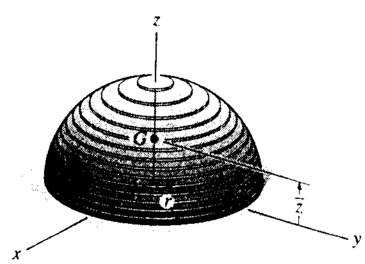




***Centroid :*** Applying Eq. 9 – 5 and performing the integration, we have

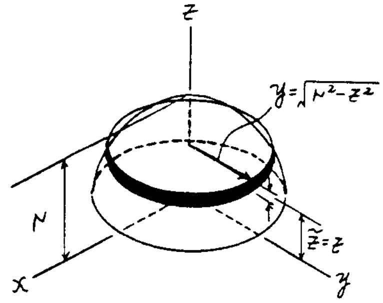
$$\bar{z}=\frac{\int_V \tilde{z}dV}{\int_V dV}=\frac{\int_0^h z\left\{\frac{\pi}{h^2}\left[(r-R)^2z^2+2Rh(r-R)z+R^2h^2\right]dz\right\}}{\int_0^h \frac{\pi}{h^2}\left[(r-R)^2z^2+2Rh(r-R)z+R^2h^2\right]dz}$$
$$=\frac{\frac{\pi}{h^2}\left[(r-R)^2\left(\frac{z^4}{4}\right)+2Rh(r-R)\left(\frac{z^3}{3}\right)+R^2h^2\left(\frac{z^2}{2}\right)\right]\Big|_0^h}{\frac{\pi}{h^2}\left[(r-R)^2\left(\frac{z^3}{3}\right)+2Rh(r-R)\left(\frac{z^2}{2}\right)+R^2h^2(z)\right]\Big|_0^h}$$
$$=\frac{R^2+3r^2+2rR}{4(R^2+r^2+rR)}h \qquad \textbf{Ans}$$

**9-42.** The hemisphere of radius $r$ is made from a stack of very thin plates such that the density varies with height $\rho=kz$, where $k$ is a constant. Determine its mass and the distance $\bar{z}$ to the center of mass $G$.



***Mass and Moment Arm :*** The density of the material is $\rho=kz$. The mass of the thin disk differential element is $dm=\rho dV=\rho\pi y^2dz=kz\left[\pi\left(r^2-z^2\right)dz\right]$ and its centroid $\tilde{z}=z$. Evaluating the integrals, we have

$$m=\int_m dm=\int_0^r kz\left[\pi\left(r^2-z^2\right)dz\right]$$
$$=\pi k\left(\frac{r^2z^2}{2}-\frac{z^4}{4}\right)\Big|_0^r=\frac{\pi k r^4}{4} \qquad \textbf{Ans}$$

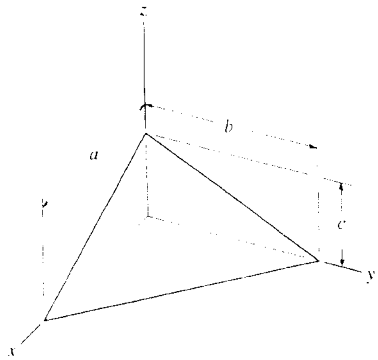


$$\int_m \tilde{z}dm=\int_0^r z\left\{kz\left[\pi\left(r^2-z^2\right)dz\right]\right\}$$
$$=\pi k\left(\frac{r^2z^3}{3}-\frac{z^5}{5}\right)\Big|_0^r=\frac{2\pi k r^5}{15}$$

***Centroid :*** Applying Eq. 9 – 4, we have

$$\bar{z}=\frac{\int_m \tilde{z}dm}{\int_m dm}=\frac{2\pi k r^5/15}{\pi k r^4/4}=\frac{8}{15}r \qquad \textbf{Ans}$$

**9-43.** Determine the location $\bar{z}$ of the centroid for the tetrahedron. *Suggestion:* Use a triangular "plate" element parallel to the $x$-$y$ plane and of thickness $dz$.

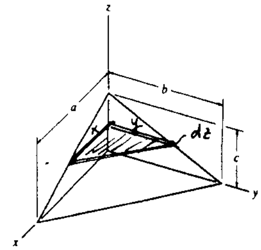



$$z = c(1 - \frac{1}{b}y) = c(1 - \frac{1}{a}x)$$

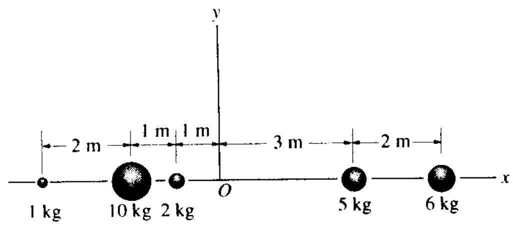
$$\int dV = \int_0^c \frac{1}{2}(x)(y)\,dz = \frac{1}{2}\int_0^c b(1-\frac{z}{c})a(1-\frac{z}{c})\,dz = \frac{abc}{6}$$

$$\int \tilde{z}\,dV = \frac{1}{2}\int_0^c z\,b(1-\frac{z}{c})a(1-\frac{z}{c})\,dz = \frac{abc^2}{24}$$

$$\bar{z} = \frac{\int \tilde{z}\,dV}{\int dV} = \frac{\frac{abc^2}{24}}{\frac{abc}{6}} = \frac{c}{4} \qquad \textbf{Ans}$$




***9-44.** Locate the center of gravity $G$ of the five particles with respect to the origin $O$.

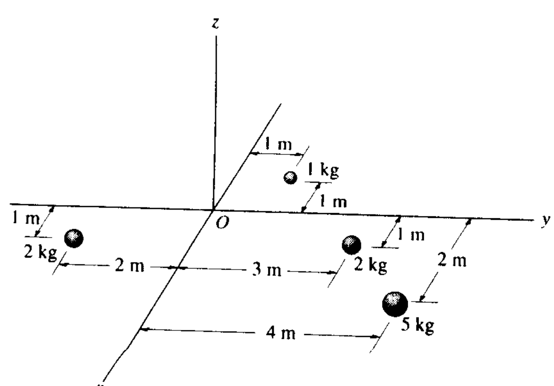



***Center of Gravity :*** The weight of the particles are $W_1 = 5g$, $W_2 = 6g$, $W_3 = 2g$, $W_4 = 10g$ and $W_5 = 1g$ and their respective centers of gravity are $\tilde{x}_1 = 3$ m, $\tilde{x}_2 = 5$ m, $\tilde{x}_3 = -1$ m, $\tilde{x}_4 = -2$ m and $\tilde{x}_5 = -4$ m. Applying Eq. 9 – 8, we have

$$\bar{x} = \frac{\Sigma\tilde{x}W}{\Sigma W} = \frac{3(5g)+5(6g)+(-1)(2g)+(-2)(10g)+(-4)(1g)}{5g+6g+2g+10g+1g}$$

$$= 0.792 \text{ m} \qquad \textbf{Ans}$$

**9-45.** Locate the center of mass $(\bar{x}, \bar{y})$ of the four particles.




***Center of Gravity :*** The weight of the particles are $W_1 = 2$ kg, $W_2 = 5$ kg, $W_3 = 2$ kg and $W_4 = 1$ kg. Their respective centers of mass are $\tilde{x}_1 = 1$ m and $\tilde{y}_1 = 3$ m, $\tilde{x}_2 = 2$ m and $\tilde{y}_2 = 4$ m, $\tilde{x}_3 = 1$ m and $\tilde{y}_3 = -2$ m and $\tilde{x}_4 = -1$ m and $\tilde{y}_4 = 1$ m. Applying Eq. 9 – 8, we have

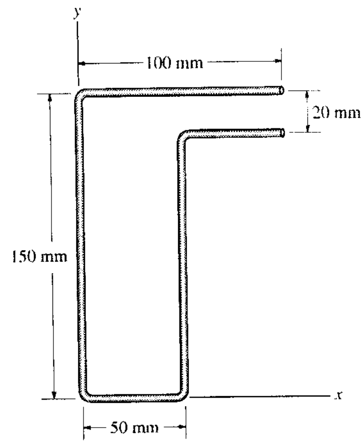
$$\bar{x} = \frac{\Sigma\tilde{x}W}{\Sigma W} = \frac{1(2)+2(5)+1(2)+(-1)(1)}{2+5+2+1}$$

$$= 1.30 \text{ m} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma\tilde{y}W}{\Sigma W} = \frac{3(2)+4(5)+(-2)(2)+1(1)}{2+5+2+1}$$

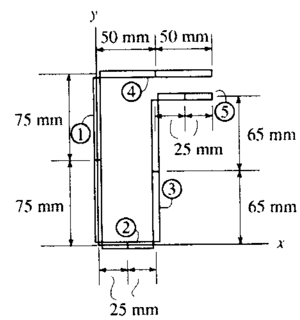
$$= 2.30 \text{ m} \qquad \textbf{Ans}$$

**9-46.** Locate the centroid $(\bar{x}, \bar{y})$ of the uniform wire bent in the shape shown.




***Centroid:*** The length of each segment and its respective centroid are tabulated below.

| Segment | $L$(mm) | $\tilde{x}$(mm) | $\tilde{y}$(mm) | $\tilde{x}L(\text{mm}^2)$ | $\tilde{y}L(\text{mm}^2)$ |
|---|---|---|---|---|---|
| 1 | 150 | 0 | 75 | 0 | 11250 |
| 2 | 50 | 25 | 0 | 1250 | 0 |
| 3 | 130 | 50 | 65 | 6500 | 8450 |
| 4 | 100 | 50 | 150 | 5000 | 15000 |
| 5 | 50 | 75 | 130 | 3750 | 6500 |
| Σ | 480 | | | 16500 | 41200 |

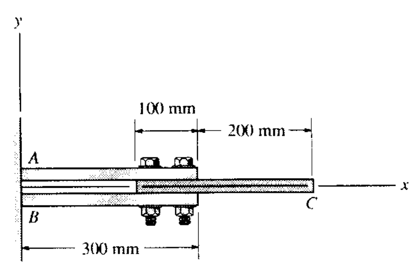



Thus,

$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{16500}{480} = 34.375 \text{ mm} = 34.4 \text{ mm} \quad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{41200}{480} = 85.83 \text{ mm} = 85.8 \text{ mm} \quad \textbf{Ans}$$

**9-47.** The steel and aluminum plate assembly is bolted together and fastened to the wall. Each plate has a constant width in the $z$ direction of 200 mm and thickness of 20 mm. If the density of $A$ and $B$ is $\rho_s = 7.85 \text{ Mg/m}^3$, and for C, $\rho_{al} = 2.71 \text{ Mg/m}^3$, determine the location $\bar{x}$ of the center of mass. Neglect the size of the bolts.




$$\Sigma m = 2[7.85(10)^3(0.3)(0.2)(0.02)] + 2.71(10)^3(0.3)(0.2)(0.02)$$

$$= 22.092 \text{ kg}$$

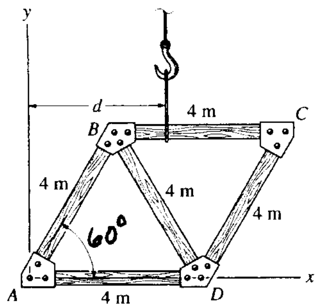
$$\Sigma \tilde{x} m = 150\{2[7.85(10)^3(0.3)(0.2)(0.02)]\}$$

$$+ 350[2.71(10)^3(0.3)(0.2)(0.02)]$$

$$= 3964.2 \text{ kg·mm}$$

$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m} = \frac{3964.2}{22.092} = 179 \text{ mm} \quad \textbf{Ans}$$

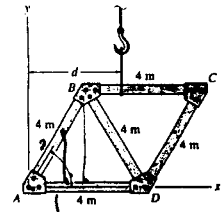
*9-48. The truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. If the mass of the gusset plates at the joints and the thickness of the members can be neglected, determine the distance $d$ to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.



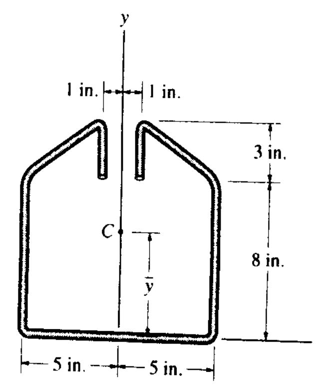

$\Sigma \tilde{x} M = 4(7)(1+4+2+3+5) = 420\ \text{kg}\cdot\text{m}$

$\Sigma M = 4(7)(5) = 140\ \text{kg}$

$d = \bar{x} = \dfrac{\Sigma \tilde{x} M}{\Sigma M} = \dfrac{420}{140} = 3\ \text{m}$ **Ans**



9-49. Locate the centroid for the wire. Neglect the thickness of the material and slight bends at the corners.

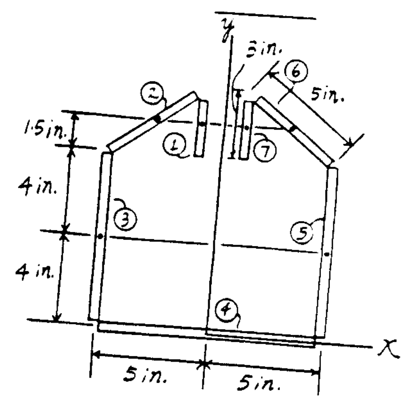



***Centroid*** : The length of each segment and its respective centroid are tabulated below.

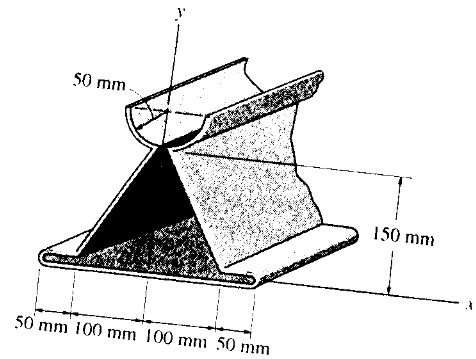
| Segment | $L$(in.) | $\tilde{y}$(in.) | $\tilde{y}L$(in²) |
|---|---|---|---|
| 1 | 3 | 9.5 | 28.5 |
| 2 | 5 | 9.5 | 47.5 |
| 3 | 8 | 4 | 32.0 |
| 4 | 10 | 0 | 0 |
| 5 | 8 | 4 | 32.0 |
| 6 | 5 | 9.5 | 47.5 |
| 7 | 3 | 9.5 | 28.5 |
| $\Sigma$ | 42.0 | | 216.0 |

Due to symmetry about $y$ axis, $\bar{x} = 0$ **Ans**

$\bar{y} = \dfrac{\Sigma \tilde{y} L}{\Sigma L} = \dfrac{216.0}{42.0} = 5.143\ \text{in.} = 5.14\ \text{in.}$ **Ans**

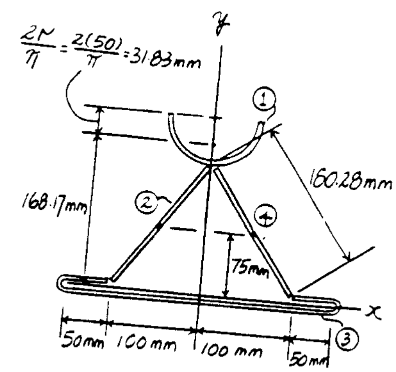


**9-50.** Locate the centroid ($\bar{x}$, $\bar{y}$) of the metal cross section. Neglect the thickness of the material and slight bends at the corners.




***Centroid :*** The length of each segment and its respective centroid are tabulated below.

| Segment | $L$(mm) | $\bar{y}$(mm) | $\bar{y}L$(mm$^2$) |
|---|---|---|---|
| 1 | $50\pi$ | 168.17 | 26415.93 |
| 2 | 180.28 | 75 | 13520.82 |
| 3 | 400 | 0 | 0 |
| 4 | 180.28 | 75 | 13520.82 |
| $\Sigma$ | 917.63 | | 53457.56 |

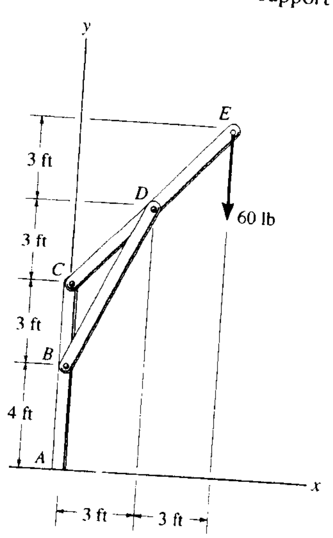



Due to symmetry about $y$ axis, $\bar{x} = 0$ **Ans**

$$\bar{y} = \frac{\Sigma \bar{y}L}{\Sigma L} = \frac{53457.56}{917.63} = 58.26 \text{ mm} = 58.3 \text{ mm}$$ **Ans**

---

**9-51.** The three members of the frame each have a weight per unit length of 4 lb/ft. Locate the position ($\bar{x}$, $\bar{y}$) of the center of gravity. Neglect the size of the pins at the joints and the thickness of the members. Also, calculate the reactions at the fixed support $A$.



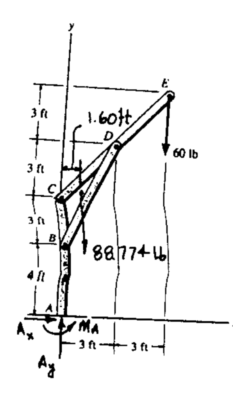

$$\Sigma \bar{x} W = 1.5(4)\sqrt{45} + 3(4)\sqrt{72} = 142.073 \text{ lb}\cdot\text{ft}$$

$$\Sigma W = 4(7) + 4\sqrt{45} + 4\sqrt{72} = 88.774 \text{ lb}$$

$$\bar{x} = \frac{\Sigma \bar{x} W}{\Sigma W} = \frac{142.073}{88.774} = 1.60 \text{ ft}$$ **Ans**

$$\Sigma \bar{y} W = 3.5(4)(7) + 7(4)\sqrt{45} + 10(4)\sqrt{72} = 625.241 \text{ lb.ft}$$

$$\bar{y} = \frac{\Sigma \bar{y} W}{\Sigma W} = \frac{625.241}{88.774} = 7.04 \text{ ft}$$ **Ans**



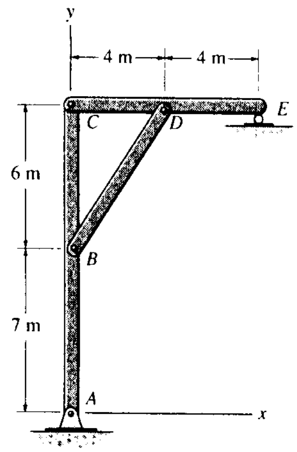

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $A_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;$ $A_y = 88.774 + 60 = 149$ lb **Ans**

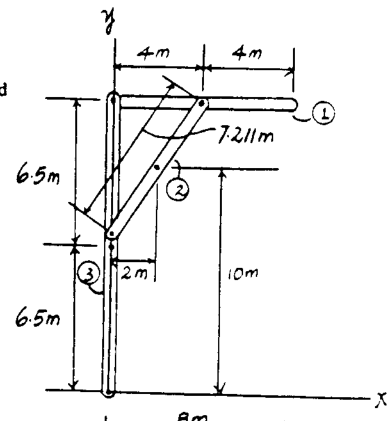
$\curvearrowleft + \Sigma M_A = 0;$ $-60(6) - 88.774(1.60) + M_A = 0$

$M_A = 502$ lb·ft **Ans**

*9-52. Each of the three members of the frame has a mass per unit length of 6 kg/m. Locate the position ($\bar{x}$, $\bar{y}$) of the center of gravity. Neglect the size of the pins at the joints and the thickness of the members. Also, calculate the reactions at the pin $A$ and roller $E$.




**Centroid :** The length of each segment and its respective centroid are tabulated below.



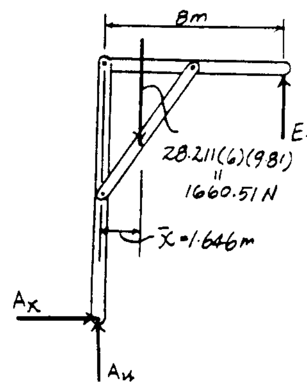

| Segment | $L$(m) | $\bar{x}$(m) | $\bar{y}$(m) | $\bar{x}L$(m²) | $\bar{y}L$(m²) |
|---|---|---|---|---|---|
| 1 | 8 | 4 | 13 | 32.0 | 104.0 |
| 2 | 7.211 | 2 | 10 | 14.42 | 72.11 |
| 3 | 13 | 0 | 6.5 | 0 | 84.5 |
| Σ | 28.211 | | | 46.42 | 260.61 |

Thus,

$$\bar{x} = \frac{\Sigma \bar{x}L}{\Sigma L} = \frac{46.42}{28.211} = 1.646 \text{ m} = 1.65 \text{ m} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \bar{y}L}{\Sigma L} = \frac{260.61}{28.211} = 9.238 \text{ m} = 9.24 \text{ m} \qquad \textbf{Ans}$$

**Equations of Equilibrium :** The total weight of the frame is $W = 28.211(6)(9.81) = 1660.51$ N.

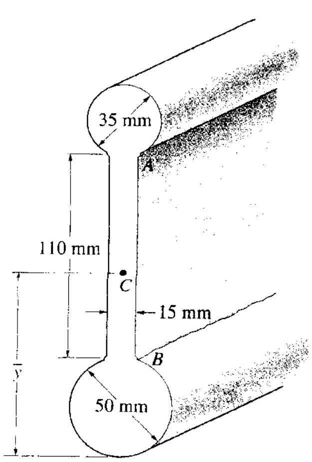



$$+\Sigma M_A = 0; \quad E_y(8) - 1660.51(1.646) = 0$$
$$E_y = 341.55 \text{ N} = 342 \text{ N} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + 341.55 - 1660.51 = 0$$
$$A_y = 1318.95 \text{ N} = 1.32 \text{ kN} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0 \qquad \textbf{Ans}$$

9-53. Determine the location $y$ of the centroid of the beam's cross-sectional area. Neglect the size of the corner elds at $A$ and $B$ for the calculation.

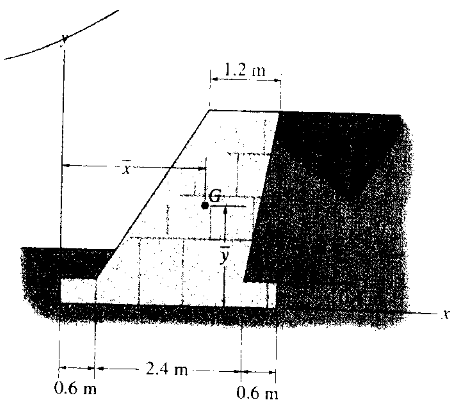



$$\Sigma \bar{y}A = \pi(25)^2(25) + 15(110)(50 + 55) + \pi\left(\frac{35}{2}\right)^2\left(50 + 110 + \frac{35}{2}\right) = 393\,112 \text{ mm}^3$$

$$\Sigma A = \pi(25)^2 + 15(110) + \pi\left(\frac{35}{2}\right)^2 = 4575.6 \text{ mm}^2$$

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{393\,112}{4575.6} = 85.9 \text{ mm} \qquad \textbf{Ans}$$

**9-54.** The gravity wall is made of concrete. Determine the location $(\bar{x}, \bar{y})$ of the center of gravity $G$ for the wall




$$\Sigma \tilde{x}A = 1.8(3.6)(0.4) + 2.1(3)(3) - 3.4\left(\frac{1}{2}\right)(3)(0.6) - 1.2\left(\frac{1}{2}\right)(1.8)(3)$$

$$= 15.192 \text{ m}^3$$

$$\Sigma \tilde{y}A = 0.2(3.6)(0.4) + 1.9(3)(3) - 1.4\left(\frac{1}{2}\right)(3)(0.6) - 2.4\left(\frac{1}{2}\right)(1.8)(3)$$

$$= 9.648 \text{ m}^3$$

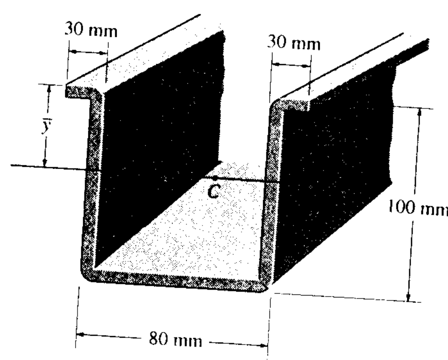
$$\Sigma A = 3.6(0.4) + 3(3) - \frac{1}{2}(3)(0.6) - \frac{1}{2}(1.8)(3)$$

$$= 6.84 \text{ m}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{15.192}{6.84} = 2.22 \text{ m} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{9.648}{6.84} = 1.41 \text{ m} \qquad \textbf{Ans}$$

**9-55.** An aluminum strut has a cross section referred to as a deep hat. Locate the centroid $\bar{y}$ of its area. Each segment has a thickness of 10 mm.

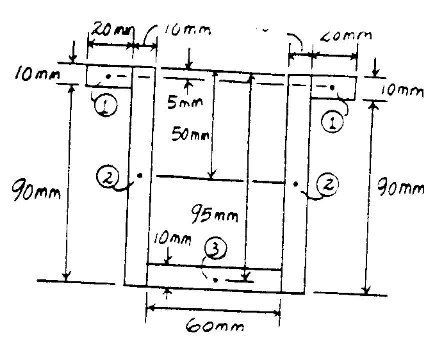



***Centroid :*** The area of each segment and its respective centroid are tabulated below.

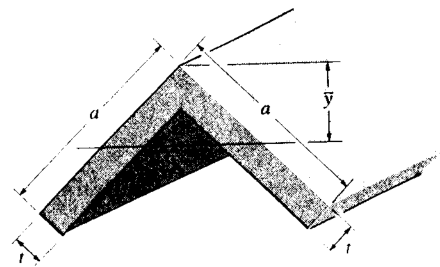
| Segment | $A$ (mm²) | $\tilde{y}$ (mm) | $\tilde{y}A$ (mm³) |
|---|---|---|---|
| 1 | 40(10) | 5 | 2 000 |
| 2 | 100(20) | 50 | 100 000 |
| 3 | 60(10) | 95 | 57 000 |
| $\Sigma$ | 3 000 | | 159 000 |

Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{159\,000}{3\,000} = 53.0 \text{ mm} \qquad \textbf{Ans}$$



***9-56.** Locate the centroid $\bar{y}$ for the cross-sectional area of the angle.



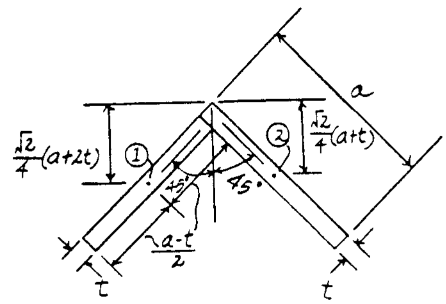

*Centroid :* The area and the centroid for segments 1 and 2 are

$$A_1 = t(a-t)$$

$$\tilde{y}_1 = \left(\frac{a-t}{2} + \frac{t}{2}\right)\cos 45^\circ + \frac{t}{2\cos 45^\circ} = \frac{\sqrt{2}}{4}(a+2t)$$

$$A_2 = at$$

$$\tilde{y}_2 = \left(\frac{a}{2} - \frac{t}{2}\right)\cos 45^\circ + \frac{t}{2\cos 45^\circ} = \frac{\sqrt{2}}{4}(a+t)$$




Listed in a tabular form, we have

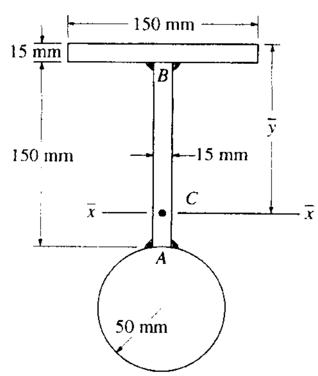
| Segment | $A$ | $\tilde{y}$ | $\tilde{y}A$ |
|---|---|---|---|
| 1 | $t(a-t)$ | $\frac{\sqrt{2}}{4}(a+2t)$ | $\frac{\sqrt{2}t}{4}(a^2+at-2t^2)$ |
| 2 | $at$ | $\frac{\sqrt{2}}{4}(a+t)$ | $\frac{\sqrt{2}t}{4}(a^2+at)$ |
| $\Sigma$ | $t(2a-t)$ | | $\frac{\sqrt{2}t}{2}(a^2+at-t^2)$ |

Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{\frac{\sqrt{2}t}{2}(a^2+at-t^2)}{t(2a-t)}$$

$$= \frac{\sqrt{2}(a^2+at-t^2)}{2(2a-t)} \qquad \textbf{Ans}$$

**9-57.** Determine the location $\bar{y}$ of the centroidal axis $\bar{x}\ \bar{x}$ of the beam's cross-sectional area. Neglect the size of the corner welds at $A$ and $B$ for the calculation.




$$\Sigma\tilde{y}A = 7.5(15)(150) + 90(150)(15) + 215(\pi)(50)^2$$

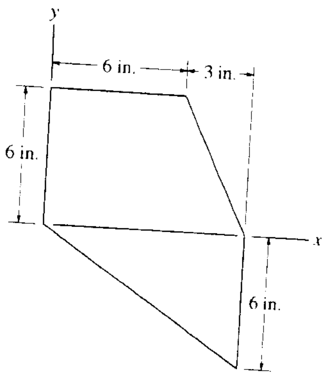
$$= 1\ 907\ 981.05\ \text{mm}^3$$

$$\Sigma A = 15(150) + 150(15) + \pi(50)^2$$

$$= 12\ 353.98\ \text{mm}^2$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{1\ 907\ 981.05}{12\ 353.98} = 154\ \text{mm} \qquad \textbf{Ans}$$

**9-58.** Determine the location $(\bar{x}, \bar{y})$ of the centroid $C$ of the area.




$$\Sigma \tilde{x} A = 3(6)(6) + 7(\frac{1}{2})(6)(3) + 6(\frac{1}{2})(9)(6) = 333 \text{ in}^3$$

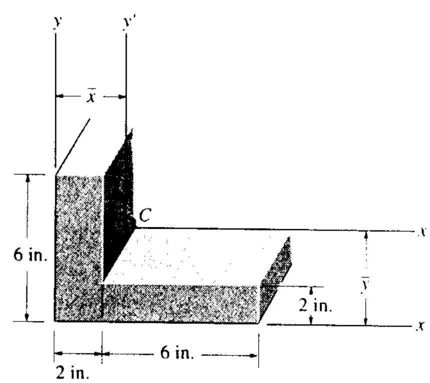
$$\Sigma \tilde{y} A = 3(6)(6) + 2(\frac{1}{2})(6)(3) - 2(\frac{1}{2})(9)(6) = 72 \text{ in}^3$$

$$\Sigma A = 6(6) + \frac{1}{2}(6)(3) + \frac{1}{2}(9)(6) = 72 \text{ in}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{333}{72} = 4.625 = 4.62 \text{ in.}$$ **Ans**

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{72}{72} = 1.00 \text{ in.}$$ **Ans**

**9-59.** Locate the centroid $(\bar{x}, \bar{y})$ for the angle's cross-sectional area.



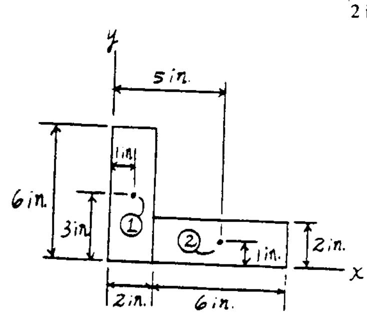

***Centroid :*** The area of each segment and its respective centroid are tabulated below.

| Segment | $A\,(\text{in}^2)$ | $\tilde{x}$ (in.) | $\tilde{y}$ (in.) | $\tilde{x}A\,(\text{in}^3)$ | $\tilde{y}A\,(\text{in}^3)$ |
|---|---|---|---|---|---|
| 1 | 6(2) | 1 | 3 | 12.0 | 36.0 |
| 2 | 6(2) | 5 | 1 | 60.0 | 12.0 |
| Σ | 24.0 | | | 72.0 | 48.0 |

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{72.0}{24.0} = 3.00 \text{ in.}$$ **Ans**

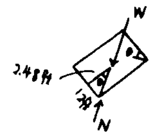
$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{48.0}{24.0} = 2.00 \text{ in.}$$ **Ans**

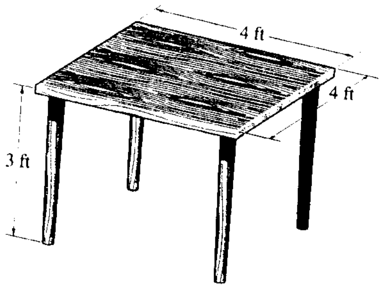


***9-60.** The wooden table is made from a square board having a weight of 15 lb. Each of the legs weighs 2 lb and is 3 ft long. Determine how high its center of gravity is from the floor. Also, what is the angle, measured from the horizontal, through which its top surface can be tilted on two of its legs before it begins to overturn? Neglect the thickness of each leg.

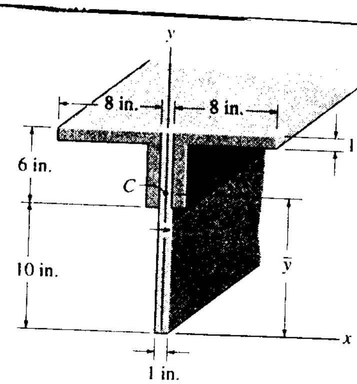
$$\bar{z} = \frac{\Sigma \tilde{z} W}{\Sigma W} = \frac{15(3) + 4(2)(1.5)}{15 + 4(2)} = 2.48 \text{ ft}$$ **Ans**

$$\theta = \tan^{-1}\left(\frac{2}{2.48}\right) = 38.9°$$ **Ans**



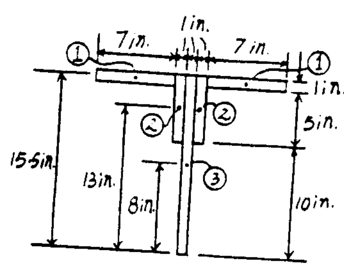




**9-61.** Locate the centroid $\bar{y}$ of the cross-sectional area of the beam.




***Centroid :*** The area of each segment and its respective centroid are tabulated below.

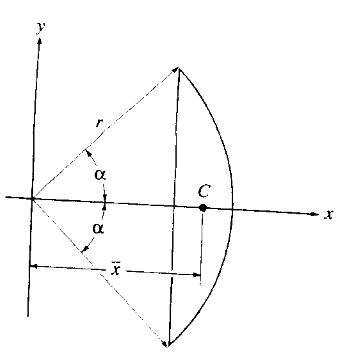
| Segment | $A$ (in$^2$) | $\tilde{y}$ (in.) | $\tilde{y}A$ (in$^3$) |
|---|---|---|---|
| 1 | 14(1) | 15.5 | 217.0 |
| 2 | 6(2) | 13 | 156.0 |
| 3 | 16(1) | 8 | 128.0 |
| $\Sigma$ | 42.0 | | 501.0 |




Thus,

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{501.0}{42.0} = 11.93 \text{ in.} = 11.9 \text{ in.}$$ **Ans**

**9-62.** Determine the location $\bar{x}$ of the centroid $C$ of the shaded area which is part of a circle having a radius $r$.




$$\Sigma \tilde{x} A = \frac{1}{2}r^2\alpha\left(\frac{2r}{3\alpha}\sin\alpha\right) - \frac{1}{2}(r\sin\alpha)(r\cos\alpha)\left(\frac{2}{3}r\cos\alpha\right)$$

$$= \frac{r^3}{3}\sin\alpha - \frac{r^3}{3}\sin\alpha\cos^2\alpha$$

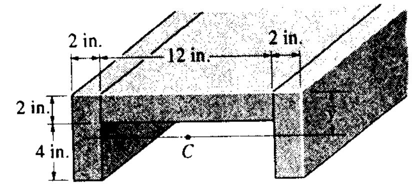
$$= \frac{r^3}{3}\sin^3\alpha$$

$$\Sigma A = \frac{1}{2}r^2\alpha - \frac{1}{2}(r\sin\alpha)(r\cos\alpha)$$

$$= \frac{1}{2}r^2\left(\alpha - \frac{\sin 2\alpha}{2}\right)$$

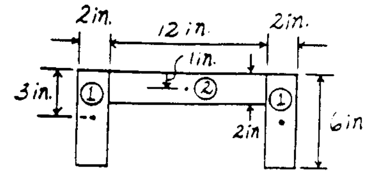
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{\frac{r^3}{3}\sin^3\alpha}{\frac{1}{2}r^2\left(\alpha - \frac{\sin 2\alpha}{2}\right)} = \frac{\frac{2}{3}r\sin^3\alpha}{\alpha - \frac{\sin 2\alpha}{2}}$$ **Ans**

**9-63.** Locate the centroid $\bar{y}$ of the channel's cross-sectional area.




***Centroid*** : The area of each segment and its respective centroid are tabulated below.

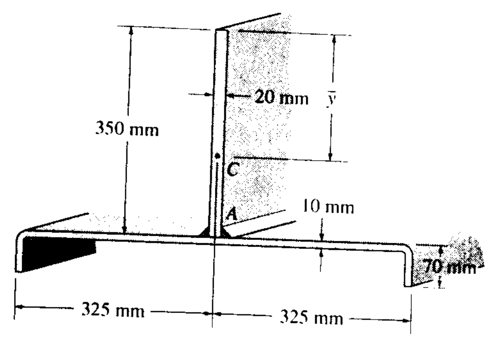
| Segment | $A$ (in²) | $\tilde{y}$ (in.) | $\tilde{y}A$ (in³) |
|---|---|---|---|
| 1 | 6(4) | 3 | 72.0 |
| 2 | 12(2) | 1 | 24.0 |
| Σ | 48.0 | | 96.0 |




Thus,

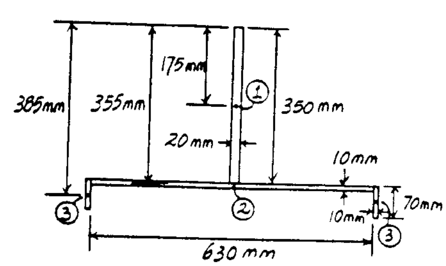
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{96.0}{48.0} = 2.00 \text{ in.}$$ **Ans**

***9-64.** Locate the centroid $\bar{y}$ of the cross-sectional area of the beam constructed from a channel and a plate. Assume all corners are square and neglect the size of the weld at $A$.




***Centroid*** : The area of each segment and its respective centroid are tabulated below.

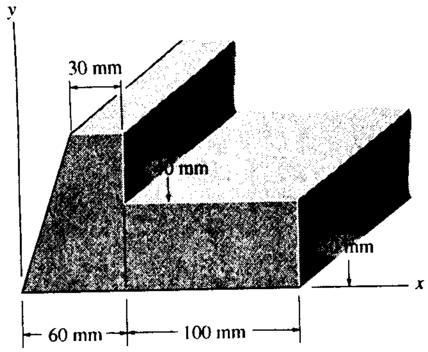
| Segment | $A$ (mm²) | $\tilde{y}$ (mm) | $\tilde{y}A$ (mm³) |
|---|---|---|---|
| 1 | 350(20) | 175 | 1 225 000 |
| 2 | 630(10) | 355 | 2 236 500 |
| 3 | 70(20) | 385 | 539 000 |
| Σ | 14 700 | | 4 000 500 |




Thus,

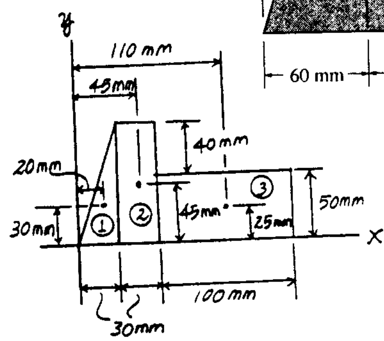
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{4\,000\,500}{14\,700} = 272.14 \text{ mm} = 272 \text{ mm}$$ **Ans**

**9-65.** Locate the centroid $(\bar{x}, \bar{y})$ of the member's cross-sectional area.




***Centroid :*** The area of each segment and its respective centroid are tabulated below.

| Segment | $A\ (\text{mm}^2)$ | $\tilde{x}\ (\text{mm})$ | $\tilde{y}\ (\text{mm})$ | $\tilde{x}A\ (\text{mm}^3)$ | $\tilde{y}A\ (\text{mm}^3)$ |
|---|---|---|---|---|---|
| 1 | $\frac{1}{2}(30)(90)$ | 20 | 30 | 27 000 | 40 500 |
| 2 | 30(90) | 45 | 45 | 121 500 | 121 500 |
| 3 | 100(50) | 110 | 25 | 550 000 | 125 000 |
| Σ | 9 050 | | | 698 500 | 287 000 |

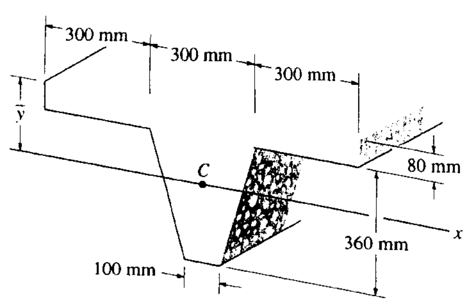



Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{698\,500}{9\,050} = 77.18\text{ mm} = 77.2\text{ mm} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{287\,000}{9\,050} = 31.71\text{ mm} = 31.7\text{ mm} \qquad \textbf{Ans}$$

**9-66.** Locate the centroid $\bar{y}$ of the concrete beam having the tapered cross section shown.

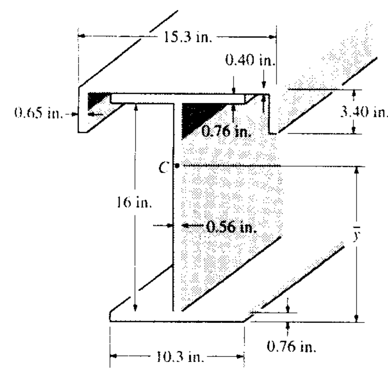



$$\Sigma \tilde{y} A = 900(80)(40) + 100(360)(260) + 2[\tfrac{1}{2}(100)(360)(200)] = 19.44(10^6)\text{ mm}^3$$

$$\Sigma A = 900(80) + 100(360) + 2[\tfrac{1}{2}(100)(360)] = 0.144(10^6)\text{ mm}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{19.44(10^6)}{0.144(10^6)} = 135\text{ mm} \qquad \textbf{Ans}$$

**9-67.** Locate the centroid $\bar{y}$ of the beam's cross-section built up from a channel and a wide-flange beam.

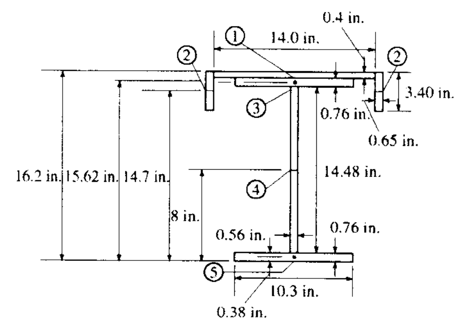



*Centroid:* The area of each segment and its respective centroid are tabulated below.

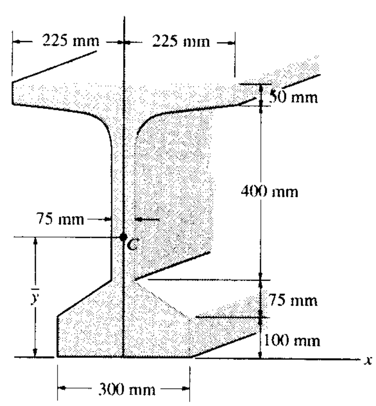
| Segment | $A(\text{in}^2)$ | $\tilde{y}(\text{in.})$ | $\tilde{y}A(\text{in}^3)$ |
|---|---|---|---|
| 1 | 14.0(0.4) | 16.20 | 90.72 |
| 2 | 3.40(1.30) | 14.70 | 64.97 |
| 3 | 10.3(0.76) | 15.62 | 122.27 |
| 4 | 14.48(0.56) | 8.00 | 64.87 |
| 5 | 10.3(0.76) | 0.38 | 2.97 |
| Σ | 33.78 | | 345.81 |

Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{345.81}{33.78} = 10.24 \text{ in.} = 10.2 \text{ in.} \quad \textbf{Ans}$$




***9-68.** Locate the centroid $\bar{y}$ of the bulb-tee cross section.

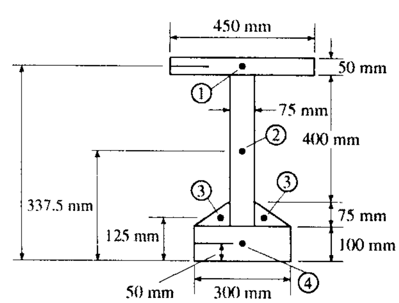



*Centroid:* The area of each segment and its respective centroid are tabulated below.

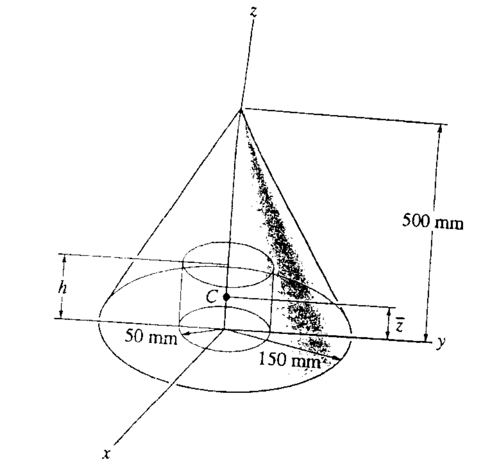
| Segment | $A(\text{mm}^2)$ | $\tilde{y}(\text{mm})$ | $\tilde{y}A(\text{mm}^3)$ |
|---|---|---|---|
| 1 | 450(50) | 600 | 13 500 000 |
| 2 | 475(75) | 337.5 | 12 023 437.5 |
| 3 | $\frac{1}{2}(225)(75)$ | 125 | 1 054 687.5 |
| 4 | 300(100) | 50 | 1 500 000 |
| Σ | 96 562.5 | | 28 078 125 |

Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{28\ 078\ 125}{96\ 562.5} = 290.78 \text{ mm} = 291 \text{ mm} \quad \textbf{Ans}$$



**9-69.** Determine the distance $h$ to which a 100-mm diameter hole must be bored into the base of the cone so that the center of mass of the resulting shape is located at $\bar{z} = 115$ mm. The material has a density of 8 Mg/m$^3$.




$$\frac{\frac{1}{3}\pi(0.15)^2(0.5)(\frac{0.5}{4}) - \pi(0.05)^2(h)(\frac{h}{2})}{\frac{1}{3}\pi(0.15)^2(0.5) - \pi(0.05)^2(h)} = 0.115$$

$$0.4313 - 0.2875\,h = 0.4688 - 1.25\,h^2$$

$$h^2 - 0.230\,h - 0.0300 = 0$$

Choosing the positive root,

$h = 323$ mm **Ans**

**9-70.** Determine the distance $\bar{z}$ to the centroid of the shape which consists of a cone with a hole of height $h = 50$ mm bored into its base.

$$\Sigma\tilde{z}V = \frac{1}{3}\pi(0.15)^2(0.5)(\frac{0.5}{4}) - \pi(0.05)^2(0.05)(\frac{0.05}{2})$$

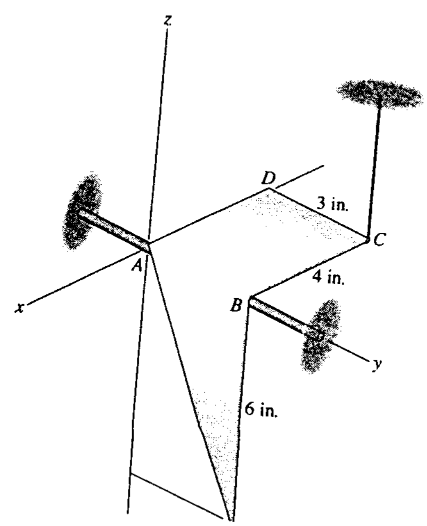
$$= 1.463(10^{-3})\ \text{m}^4$$

$$\Sigma V = \frac{1}{3}\pi(0.15)^2(0.5) - \pi(0.05)^2(0.05)$$

$$= 0.01139\ \text{m}^3$$

$$\bar{z} = \frac{\Sigma\tilde{z}V}{\Sigma V} = \frac{1.463(10^{-3})}{0.01139} = 0.12845\ \text{m} = 128\ \text{mm}$$ **Ans**

**9-71.** The sheet metal part has the dimensions shown. Determine the location $(\bar{x}, \bar{y}, \bar{z})$ of its centroid.




$$\Sigma A = 4(3) + \frac{1}{2}(3)(6) = 21\ \text{in}^2$$

$$\Sigma\tilde{x}A = -2(4)(3) + 0\left(\frac{1}{2}\right)(3)(6) = -24\ \text{in}^3$$

$$\Sigma\tilde{y}A = 1.5(4)(3) + \frac{2}{3}(3)\left(\frac{1}{2}\right)(3)(6) = 36\ \text{in}^3$$

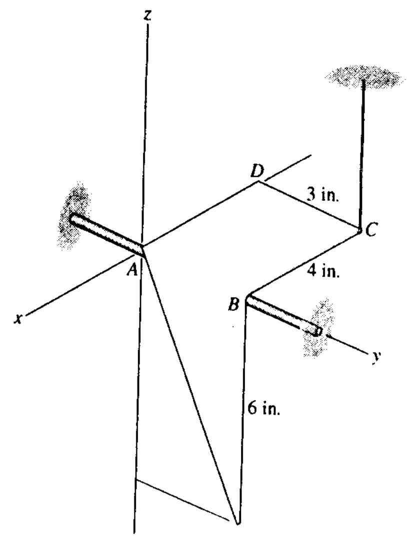
$$\Sigma\tilde{z}A = 0(4)(3) - \frac{1}{3}(6)\left(\frac{1}{2}\right)(3)(6) = -18\ \text{in}^3$$

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-24}{21} = -1.14\ \text{in.}$$ **Ans**

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{36}{21} = 1.71\ \text{in.}$$ **Ans**

$$\bar{z} = \frac{\Sigma\tilde{z}A}{\Sigma A} = \frac{-18}{21} = -0.857\ \text{in.}$$ **Ans**

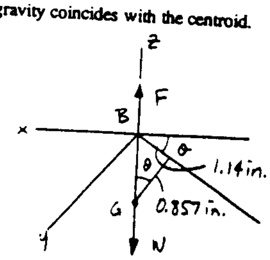
*9-72. The sheet metal part has a weight per unit area of 2 lb/ft$^2$ and is supported by the smooth rod and at $C$. If the cord is cut, the part will rotate about the $y$ axis until it reaches equilibrium. Determine the equilibrium angle of tilt, measured downward from the negative $x$ axis, that $AD$ makes with the $-x$ axis.



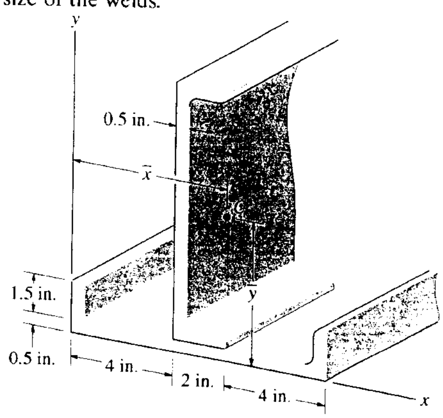

Since the material is homogeneous, the center of gravity coincides with the centroid.

See solution to Prob. 9-71.

$\theta = \tan^{-1}\left(\frac{1.14}{0.857}\right) = 53.1°$ **Ans**




9-73. Determine the location $(\bar{x}, \bar{y})$ of the centroid $C$ of the cross-sectional area for the structural member constructed from two equal-sized channels welded together as shown. Assume all corners are square. Neglect the size of the welds.




$$\Sigma \tilde{x} A = 1.5(0.5)(0.25) + 10(0.5)(5) + 1.5(0.5)(9.75) + 1.5(0.5)(5.25)(2) + 10(0.5)(4.25)$$

$$= 61.625 \text{ in}^3$$

$$\Sigma A = [1.5(0.5) + 10(0.5) + 1.5(0.5)](2) = 13 \text{ in}^2$$

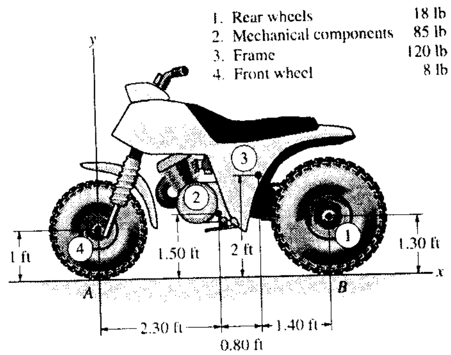
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{61.625}{13} = 4.74 \text{ in.}$$ **Ans**

$$\Sigma \tilde{y} A = 1.5(0.5)(1.25)(2) + 10(0.5)(0.25) + 1.5(0.5)(0.75) + 10(0.5)(5.5) + 1.5(0.5)(10.25)$$

$$= 38.875 \text{ in}^3$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{38.875}{13} = 2.99 \text{ in.}$$ **Ans**

**9-74.** Determine the location $(\bar{x}, \bar{y})$ of the center of gravity of the three-wheeler. The location of the center of gravity of each component and its weight are tabulated in the figure. If the three-wheeler is symmetrical with respect to the $x-y$ plane, determine the normal reactions each of its wheels exerts on the ground.




$$\Sigma \tilde{x} W = 4.5(18) + 2.3(85) + 3.1(120)$$

$$= 648.5 \text{ lb}\cdot\text{ft}$$

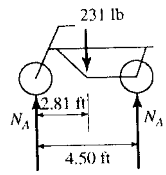
$$\Sigma W = 18 + 85 + 120 + 8 = 231 \text{ lb}$$

$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} = \frac{648.5}{231} = 2.81 \text{ ft} \qquad \textbf{Ans}$$

$$\Sigma \tilde{y} W = 1.30(18) + 1.5(85) + 2(120) + 1(8)$$

$$= 398.9 \text{ lb}\cdot\text{ft}$$

$$\bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} = \frac{398.9}{231} = 1.73 \text{ ft} \qquad \textbf{Ans}$$



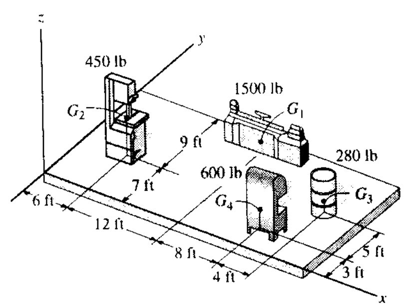

$$\curvearrowleft +\Sigma M_A = 0; \quad 2(N_B)(4.5) - 231(2.81) = 0$$

$$N_B = 72.1 \text{ lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad N_A + 2(72.1) - 231 = 0$$

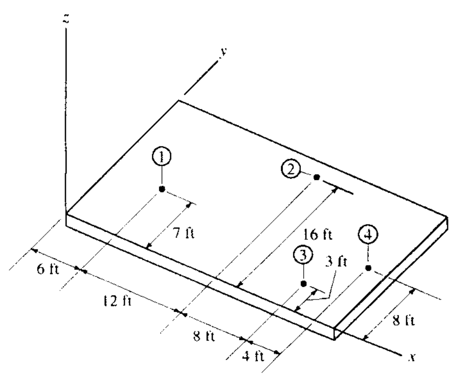
$$N_A = 86.9 \text{ lb} \qquad \textbf{Ans}$$

**9-75.** Major floor loadings in a shop are caused by the weights of the objects shown. Each force acts through its respective center of gravity $G$. Locate the center of gravity $(\bar{x}, \bar{y})$ of all these components.




***Centroid:*** The floor loadings on the floor and its respective centroid are tabulated below.

| Loading | $W$(lb) | $\tilde{x}$(ft) | $\tilde{y}$(ft) | $\tilde{x}W$(lb·ft) | $\tilde{y}W$(lb·ft) |
|---|---|---|---|---|---|
| 1 | 450 | 6 | 7 | 2700 | 3150 |
| 2 | 1500 | 18 | 16 | 27000 | 24000 |
| 3 | 600 | 26 | 3 | 15600 | 1800 |
| 4 | 280 | 30 | 8 | 8400 | 2240 |
| Σ | 2830 | | | 53700 | 31190 |

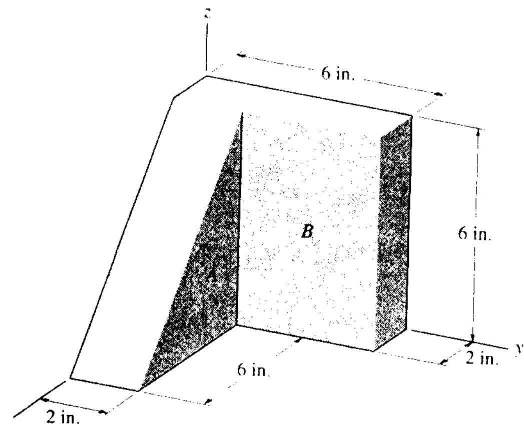



Thus,

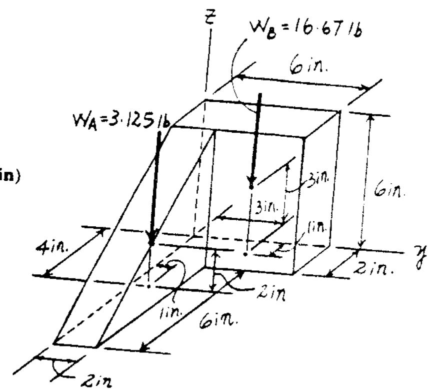
$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} = \frac{53700}{2830} = 18.98 \text{ ft} = 19.0 \text{ ft} \quad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} = \frac{31190}{2830} = 11.02 \text{ ft} = 11.0 \text{ ft} \quad \textbf{Ans}$$

***9-76.** Locate the center of gravity of the two-block assembly. The specific weights of the materials $A$ and $B$ are $\gamma_A = 150\ \text{lb/ft}^3$ and $\gamma_B = 400\ \text{lb/ft}^3$, respectively.




***Centroid*** **:** The weight of block $A$ and $B$ are $W_A = \frac{1}{2}\left(\frac{6}{12}\right)\left(\frac{6}{12}\right)\left(\frac{2}{12}\right)(150) = 3.125\ \text{lb}$ and $W_B = \left(\frac{6}{12}\right)\left(\frac{6}{12}\right)\left(\frac{2}{12}\right)(400) = 16.67\ \text{lb}$. The weight of each block and its respective centroid are tabulated below.




| Block | $W$(lb) | $\bar{x}$(in.) | $\bar{y}$(in.) | $\bar{z}$(in.) | $\bar{x}W$(lb·in) | $\bar{y}W$(lb·in) | $\bar{z}W$(lb·in) |
|---|---|---|---|---|---|---|---|
| A | 3.125 | 4 | 1 | 2 | 12.5 | 3.125 | 6.25 |
| B | 16.667 | 1 | 3 | 3 | 16.667 | 50.0 | 50.0 |
| Σ | 19.792 | | | | 29.167 | 53.125 | 56.25 |

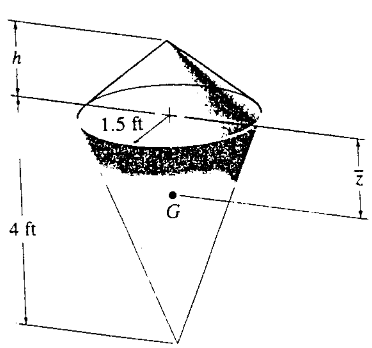
Thus,

$$\bar{x} = \frac{\Sigma \bar{x}W}{\Sigma W} = \frac{29.167}{19.792} = 1.474\ \text{in.} = 1.47\ \text{in.} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \bar{y}W}{\Sigma W} = \frac{53.125}{19.792} = 2.684\ \text{in.} = 2.68\ \text{in.} \qquad \textbf{Ans}$$

$$\bar{z} = \frac{\Sigma \bar{z}W}{\Sigma W} = \frac{56.25}{19.792} = 2.842\ \text{in.} = 2.84\ \text{in.} \qquad \textbf{Ans}$$

**9-77.** The buoy is made from two homogeneous cones each having a radius of 1.5 ft. If $h = 1.2$ ft, find the distance $\bar{z}$ to the buoy's center of gravity $G$.




$$\Sigma \tilde{z} V = \frac{1}{3}\pi(1.5)^2(1.2)\left(-\frac{1.2}{4}\right) + \frac{1}{3}\pi(1.5)^2(4)\left(\frac{4}{4}\right)$$

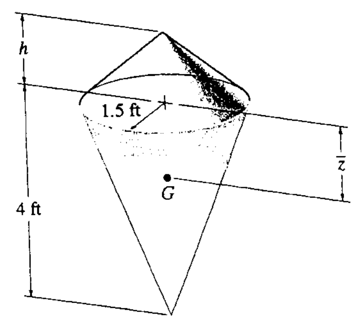
$$= 8.577\ \text{ft}^4$$

$$\Sigma V = \frac{1}{3}\pi(1.5)^2(1.2) + \frac{1}{3}\pi(1.5)^2(4)$$

$$= 12.25\ \text{ft}^3$$

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{8.577}{12.25} = 0.70\ \text{ft} \qquad \textbf{Ans}$$

**9-78.** The buoy is made from two homogeneous cones each having a radius of 1.5 ft. If it is required that the buoy's center of gravity $G$ be located at $\bar{z} = 0.5$ ft, determine the height $h$ of the top cone.




$$\Sigma \tilde{z} V = \frac{1}{3}\pi(1.5)^2(h)\left(-\frac{h}{4}\right) + \frac{1}{3}\pi(1.5)^2(4)\left(\frac{4}{4}\right)$$
$$= -0.5890\,h^2 + 9.4248$$

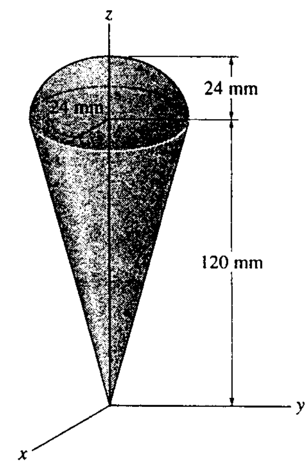
$$\Sigma V = \frac{1}{3}\pi(1.5)^2(h) + \frac{1}{3}\pi(1.5)^2(4)$$
$$= 2.3562\,h + 9.4248$$

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{-0.5890\,h^2 + 9.4248}{2.3562\,h + 9.4248} = 0.5$$

$$-0.5890\,h^2 + 9.4248 = 1.1781\,h + 4.7124$$

$h = 2.00$ ft **Ans**

**9-79.** Locate the centroid $\bar{z}$ of the top made from a hemisphere and a cone.

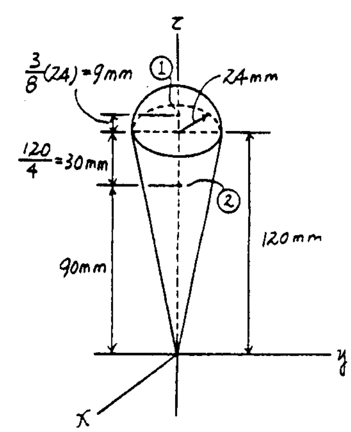



***Centroid :*** The volume of each segment and its respective centroid are tabulated below.

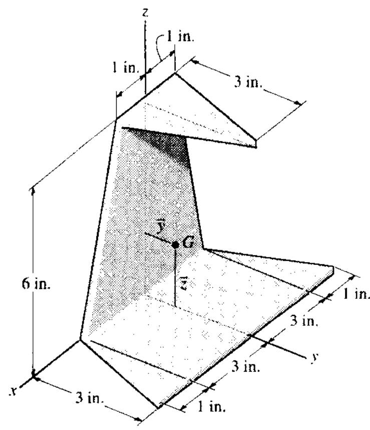
| Segment | $V\,(\text{mm}^3)$ | $\tilde{z}\,(\text{mm})$ | $\tilde{z}V\,(\text{mm}^4)$ |
|---|---|---|---|
| 1 | $\frac{2}{3}\pi(24^3)$ | 129 | $1.188864\pi(10^6)$ |
| 2 | $\frac{1}{3}\pi(24^2)(120)$ | 90 | $2.0736\pi(10^6)$ |
| $\Sigma$ | $32.256\pi(10^3)$ | | $3.262464\pi(10^6)$ |

Thus,

$$\bar{z} = \frac{\Sigma\tilde{z}V}{\Sigma V} = \frac{3.262464\pi(10^6)}{32.256\pi(10^3)} = 101.14 \text{ mm} = 101 \text{ mm} \quad \textbf{Ans}$$



***9-80.** A triangular plate made of homogeneous material has a constant thickness which is very small. If it is folded over as shown, determine the location $\bar{y}$ of the plate's center of gravity $G$.




$$\Sigma A = \frac{1}{2}(8)(12) = 48 \text{ in}^2$$

$$\Sigma \tilde{y} A = 2(1)\left(\frac{1}{2}\right)(1)(3) + 1.5(6)(3) + 2(2)\left(\frac{1}{2}\right)(1)(3)$$

$$= 36 \text{ in}^3$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{36}{48} = 0.75 \text{ in.} \qquad \textbf{Ans}$$

**9-81.** A triangular plate made of homogeneous material has a constant thickness which is very small. If it is folded over as shown, determine the location $\bar{z}$ of the plate's center of gravity $G$.

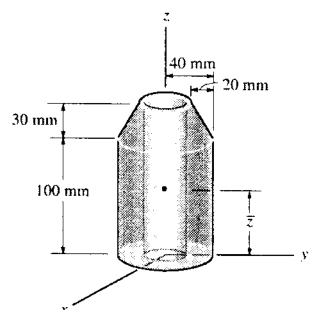
$$\Sigma A = \frac{1}{2}(8)(12) = 48 \text{ in}^2$$

$$\Sigma \tilde{z} A = 2(2)\left(\frac{1}{2}\right)(2)(6) + 3(6)(2) + 6\left(\frac{1}{2}\right)(2)(3)$$

$$= 78 \text{ in}^3$$

$$\bar{z} = \frac{\Sigma \tilde{z} A}{\Sigma A} = \frac{78}{48} = 1.625 \text{ in.} \qquad \textbf{Ans}$$

**9-82.** Locate the center of mass $\bar{z}$ of the assembly. The material has a density of $\rho = 3 \text{ Mg/m}^3$. There is a 30-mm diameter hole bored through the center.

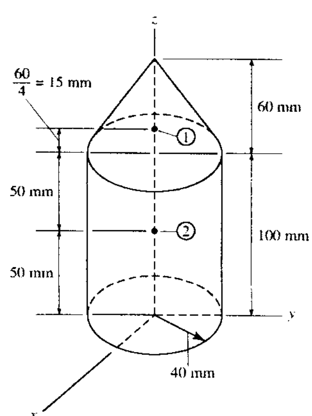


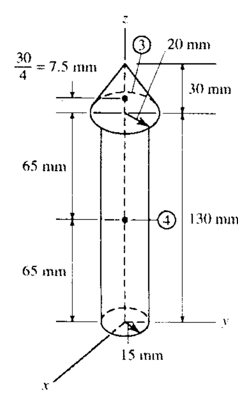

***Centroid:*** Since the density is the same for the whole material, the centroid of the volume coincide with centroid of the mass. The volume of each segment and its respective centroid are tabulated below.

| Segment | $V(\text{mm}^3)$ | $\tilde{z}(\text{mm})$ | $\tilde{z}V(\text{mm}^4)$ |
|---|---|---|---|
| 1 | $\frac{1}{3}\pi(40^2)(60)$ | 115 | $3.68\pi(10^6)$ |
| 2 | $\pi(40^2)(100)$ | 50 | $8.00\pi(10^6)$ |
| 3 | $-\frac{1}{3}\pi(20^2)(30)$ | 137.5 | $-0.550\pi(10^6)$ |
| 4 | $-\pi(15^2)(130)$ | 65 | $-1.901\,25\pi(10^6)$ |
| $\Sigma$ | $158.75\pi(10^3)$ | | $9.228\,75\pi(10^6)$ |

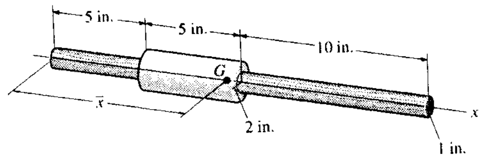
Thus,

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{9.228\,75\pi(10^6)}{158.75\pi(10^3)} = 58.13 \text{ mm} = 58.1 \text{ mm} \qquad \textbf{Ans}$$



**9-83.** The assembly consists of a 20-in. wooden dowel rod and a tight-fitting steel collar. Determine the distance $\bar{x}$ to its center of gravity if the specific weights of the materials are $\gamma_w = 150\ \text{lb/ft}^3$ and $\gamma_{st} = 490\ \text{lb/ft}^3$. The radii of the dowel and collar are shown.




$$\Sigma\tilde{x}W = \{10\pi(1)^2(20)(150) + 7.5\pi(5)(2^2 - 1^2)(490)\}\frac{1}{(12)^3}$$

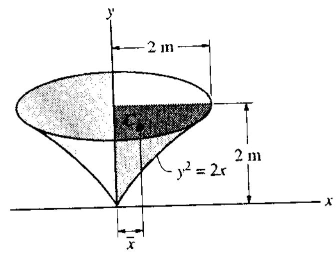
$$= 154.8\ \text{lb}\cdot\text{in.}$$

$$\Sigma W = \{\pi(1)^2(20)(150) + \pi(5)(2^2 - 1^2)(490)\}\frac{1}{(12)^3}$$

$$= 18.82\ \text{lb}$$

$$\bar{x} = \frac{\Sigma\tilde{x}W}{\Sigma W} = \frac{154.8}{18.82} = 8.22\ \text{in.}$$ **Ans**

**9-84.** Using integration, determine both the area and the centroidal distance $\bar{x}$ of the shaded area. Then, using the second theorem of Pappus–Guldinus, determine the volume of the solid generated by revolving the area about the $y$ axis.

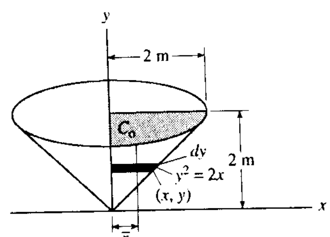



$$\tilde{x} = \frac{x}{2}$$

$$\tilde{y} = y$$

$$dA = x\ dy$$

$$A = \int dA = \int_0^2 \frac{y^2}{2}dy = \left[\frac{y^3}{6}\right]_0^2 = 1.333 = 1.33\ \text{m}^2$$ **Ans**



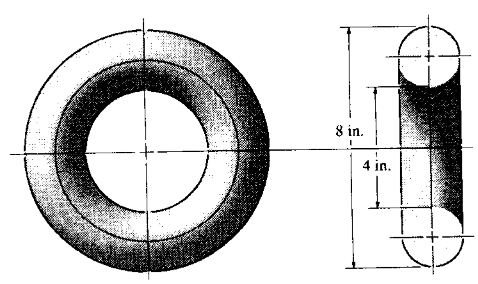

$$\int \tilde{x}dA = \int_0^2 \frac{y^4}{8}dy = \left[\frac{y^5}{40}\right]_0^2 = 0.8\ \text{m}^3$$

$$\bar{x} = \frac{\int \tilde{x}\ dA}{\int dA} = \frac{0.8}{1.333} = 0.6\ \text{m}$$ **Ans**

Thus,

$$V = \theta\bar{r}A = 2\pi(0.6)(1.333) = 5.03\ \text{m}^3$$ **Ans**

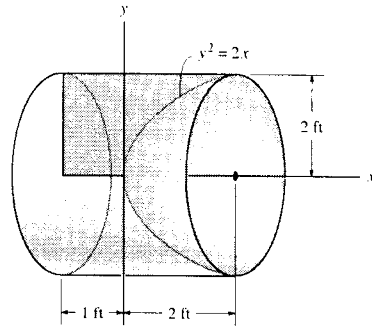
**9-85.** The anchor ring is made of steel having a specific weight of $\gamma_{st} = 490\ \text{lb/ft}^3$. Determine the surface area of the ring. The cross section is circular as shown.




$$A = \theta\bar{r}L = 2\pi(3)2\pi(1)$$

$$= 118\ \text{in}^2$$ **Ans**

**9-86.** Using integration, determine both the area and the distance $\bar{y}$ to the centroid of the shaded area. Then using the second theorem of Pappus–Guldinus, determine the volume of the solid generated by revolving the shaded area about the $x$ axis.

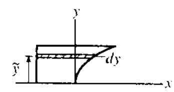



Area of the differential element $dA = \left(1 + \frac{y^2}{2}\right)dy$ and $\tilde{y} = y$

$$A = \int_A dA = \int_0^2 \left(1 + \frac{y^2}{2}\right)dy = 3.333 \text{ ft}^2 = 3.33 \text{ ft}^2 \quad \textbf{Ans}$$

$$\int_A \tilde{y}dA = \int_0^2 y\left(1 + \frac{y^2}{2}\right)dy = 4 \text{ ft}^3$$

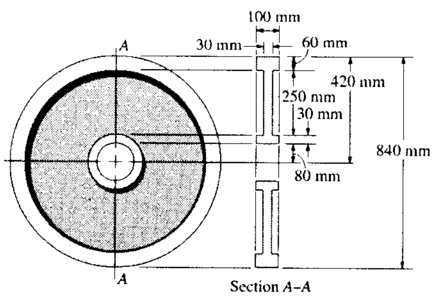
$$\bar{y} = \frac{\int_A \tilde{y}dA}{\int_A dA} = \frac{4}{3.333} = 1.2 \text{ ft} \quad \textbf{Ans}$$



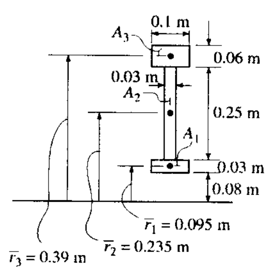

Volume:

$V = \theta \bar{r} A = 2\pi(1.2)(3.333) = 25.1 \text{ ft}^3$ **Ans**

**9-87.** A steel wheel has a diameter of 840 mm and a cross section as shown in the figure. Determine the total mass of the wheel if $\rho = 5 \text{ Mg/m}^3$




***Volume:*** Applying the theorem of Pappus and Guldinus, Eq. 9-12, with $\theta = 2\pi$, $\bar{r}_1 = 0.095$ m, $\bar{r}_2 = 0.235$ m, $\bar{r}_3 = 0.39$ m, $A_1 = 0.1(0.03) = 0.003 \text{ m}^2$, $A_2 = 0.25(0.03) = 0.0075 \text{ m}^2$ and $A_3 = (0.1)(0.06) = 0.006 \text{ m}^2$, we have




$$V = \theta \Sigma \bar{r} A = 2\pi[0.095(0.003) + 0.235(0.0075) + 0.39(0.006)]$$

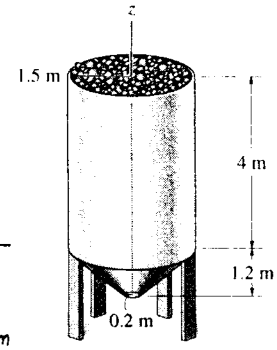
$$= 8.775\pi(10^{-3})\text{m}^3$$

The mass of the wheel is

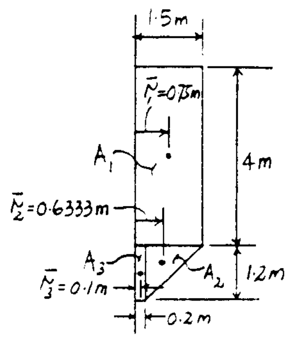
$$m = \rho V = 5(10^3)[8.775(10^{-3})\pi]$$

$$= 138 \text{ kg} \quad \textbf{Ans}$$

*9-88. The hopper is filled to its top with coal. Determine the volume of coal if the voids (air space) are 35 percent of the volume of the hopper.




*Volume* : The volume of the hopper can be obtained by applying the theorem of Pappus and Guldinus, Eq. 9–12, with $\theta = 2\pi$, $\bar{r}_1 = 0.75$ m, $\bar{r}_2 = 0.6333$ m, $\bar{r}_3 = 0.1$ m, $A_1 = 1.5(4) = 6.00\ \text{m}^2$, $A_2 = \frac{1}{2}(1.3)(1.2) = 0.780\ \text{m}^2$ and $A_3 = (0.2)(1.2) = 0.240\ \text{m}^2$.

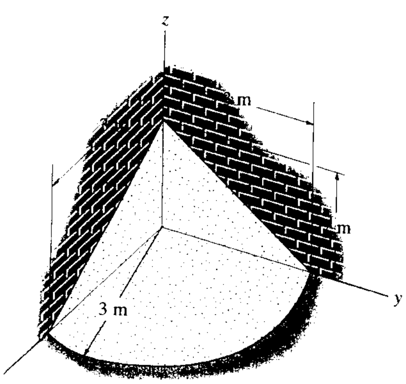



$$V_h = \theta \Sigma \bar{r} A = 2\pi[0.75(6.00) + 0.6333(0.780) + 0.1(0.240)]$$
$$= 10.036\pi\ \text{m}^3$$

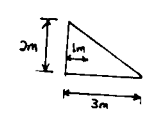
The volume of the coal is

$$V_c = 0.65 V_h = 0.65(10.036\pi) = 20.5\ \text{m}^3 \qquad \textbf{Ans}$$

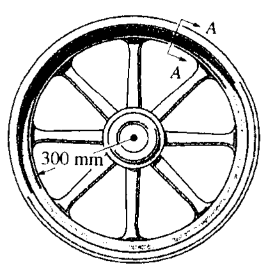
9-89. Sand is piled between two walls as shown. Assume the pile to be a quarter section of a cone and that 26 percent of this volume is voids (air space). Use the second theorem of Pappus–Guldinus to determine the volume of sand.

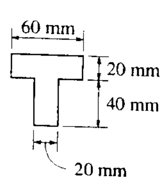



$$V = \theta \bar{r} A = \left[(\frac{\pi}{2})(1)(\frac{1}{2})(3)(2)\right](0.74) = 3.49\ \text{m}^3 \qquad \textbf{Ans}$$




9-90. The *rim* of a flywheel has the cross section *A-A* shown. Determine the volume of material needed for its construction.

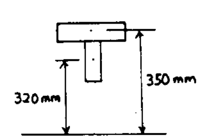


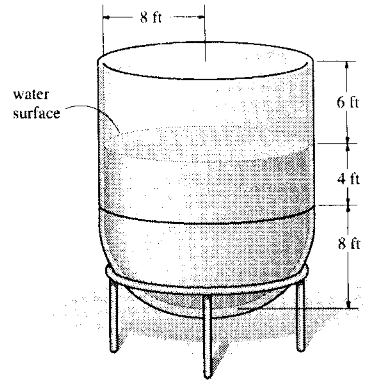

Section *A-A*

$$V = \Sigma \theta \bar{r} A = 2\pi(350)(60)(20) + 2\pi(320)(40)(20)$$

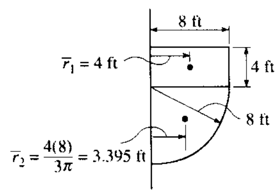
$$V = 4.25(10^6)\ \text{mm}^3 \qquad \textbf{Ans}$$



**9-91.** The open tank is fabricated from a hemisphere and cylindrical shell. Determine the vertical reactions that each of the four symmetrically placed legs exerts on the floor if the tank contains water which is 12 ft deep in the tank. The specific gravity of water is 62.4 lb/ft$^3$. Neglect the weight of the tank.




***Volume:*** The volume of the water can be obtained by applying the theorem of Pappus and Guldinus, Eq. 9-12, with $\theta = 2\pi, \bar{r}_1 = 4$ ft, $\bar{r}_2 = 3.395$ ft, $A_1 = 8(4) = 32.0$ ft$^2$ and $A_2 = \frac{1}{4}\pi(8^2) = 50.27$ ft$^2$.




$$V = \theta\Sigma\bar{r}A = 2\pi[4(32.0) + 3.395(50.27)] = 1876.58 \text{ ft}^3$$

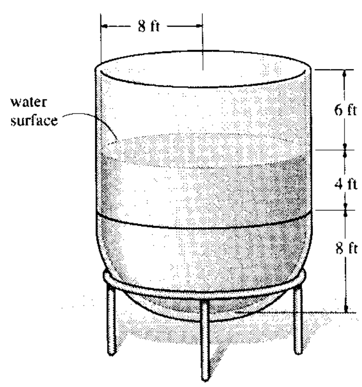
The weight of the water is

$$W = \gamma_w V = 62.4(1876.58) = 117098.47 \text{ lb}$$

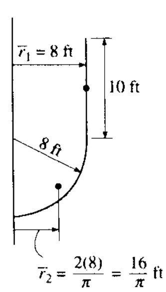
Thus, the reaction of each leg on the floor is

$$R = \frac{W}{4} = \frac{117098.47}{4} = 29274.62 \text{ lb} = 29.3 \text{ kip} \quad \textbf{Ans}$$

***9-92.** Determine the approximate amount of paint needed to cover the outside surface of the open tank. Assume that a gallon of paint covers 400 ft$^2$.




***Surface Area:*** Applying the theorem of Pappus and Guldinus, Eq. 9-11, with $\theta = 2\pi$, $L_1 = 10$ ft, $L_2 = \frac{\pi(8)}{2} = 4\pi$ ft, $\bar{r}_1 = 8$ ft and $\bar{r}_2 = \frac{16}{\pi}$ ft, we have

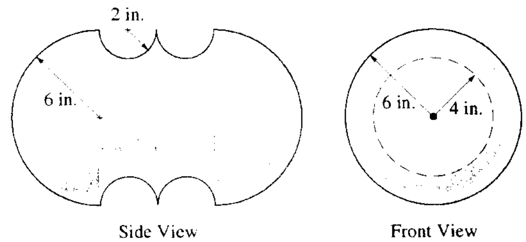



$$A = \theta\Sigma\bar{r}L = 2\pi\left[8(10) + \frac{16}{\pi}(4\pi)\right] = 288\pi \text{ ft}^2$$

Thus,

$$\text{The required amount paint} = \frac{288\pi}{400} = 2.26 \text{ gallon} \quad \textbf{Ans}$$

**9-93.** Determine the volume of material needed to make the casting.



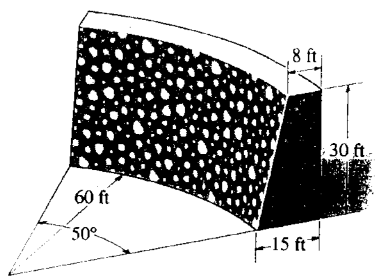

$V = \Sigma \theta A \bar{y}$

$= 2\pi[2(\frac{1}{4}\pi)(6)^2(\frac{4(6)}{3\pi}) + 2(6)(4)(3) - 2(\frac{1}{2}\pi)(2)^2(6 - \frac{4(2)}{3\pi})]$

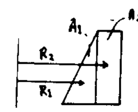
$= 1402.8 \text{ in}^3$

$V = 1.40(10^3) \text{ in}^3$ **Ans**

**9-94.** A circular sea wall is made of concrete. Determine the total weight of the wall if the concrete has a specific weight of $\gamma_c = 150 \text{ lb/ft}^3$.



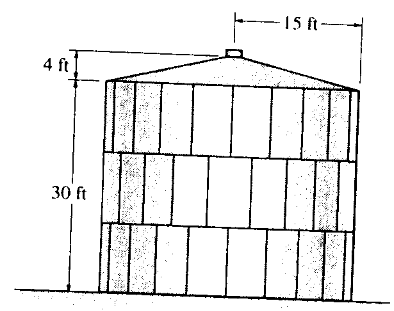

$V = \Sigma \theta \bar{r} A = (\frac{50°}{180°})\pi[(60 + \frac{2}{3}(7))(\frac{1}{2})(30)(7) + 71(30)(8)]$

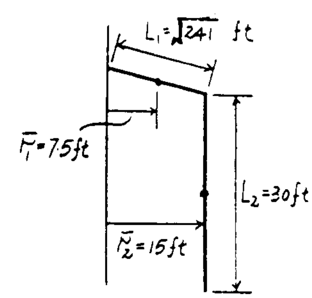



$= 20\,795.6 \text{ ft}^3$

$W = \gamma V = 150(20\,795.6) = 3.12(10^6) \text{ lb}$ **Ans**

**9-95.** Determine the outside surface area of the storage tank.

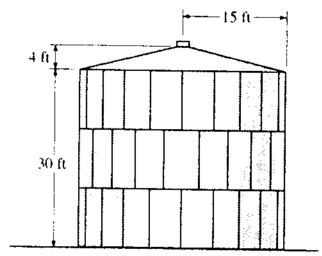



***Surface Area :*** Applying the theorem of Pappus and Guldinus, Eq. 9 – 11, with $\theta = 2\pi$, $L_1 = \sqrt{15^2 + 4^2} = \sqrt{241}$ ft, $L_2 = 30$ ft, $\bar{r}_1 = 7.5$ ft and $\bar{r}_2 = 15$ ft, we have

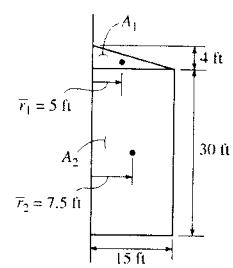
$$A = \theta \Sigma \bar{r} L = 2\pi[7.5(\sqrt{241}) + 15(30)] = 3.56(10^3) \text{ ft}^2 \quad \textbf{Ans}$$

***9-96.** Determine the volume of the storage tank.

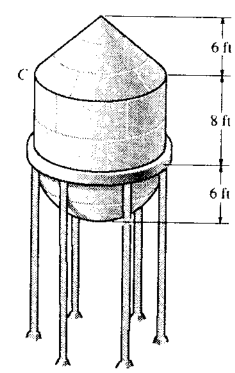



***Volume:*** Applying the theorem of Pappus and Guldinus, Eq. 9-12, with $\theta = 2\pi, \bar{r}_1 = 5$ ft, $\bar{r}_2 = 7.5$ ft, $A_1 = \frac{1}{2}(15)(4) = 30.0\ \text{ft}^2$ and $A_2 = 30(15) = 450\ \text{ft}^2$, we have

$V = \theta\Sigma\bar{r}A = 2\pi[5(30.0) + 7.5(450)] = 22.1(10^3)\ \text{ft}^3$ **Ans**



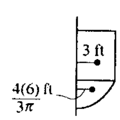

**9-97.** The water-supply tank has a hemispherical bottom and cylindrical sides. Determine the weight of water in the tank when it is filled to the top at C. Take $\gamma_w = 62.4\ \text{lb/ft}^3$.




$$V = \Sigma\theta\bar{r}A = 2\pi\left\{3(8)(6) + \frac{4(6)}{3\pi}\left(\frac{1}{4}\right)(\pi)(6)^2\right\}$$

$V = 1357.17\ \text{ft}^3$

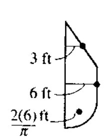
$W = \gamma V = 62.4(1357.17) = 84.7$ kip **Ans**




**9-98.** Determine the number of gallons of paint needed to paint the outside surface of the water-supply tank, which consists of a hemispherical bottom, cylindrical sides, and conical top. Each gallon of paint can cover 250 ft$^2$.

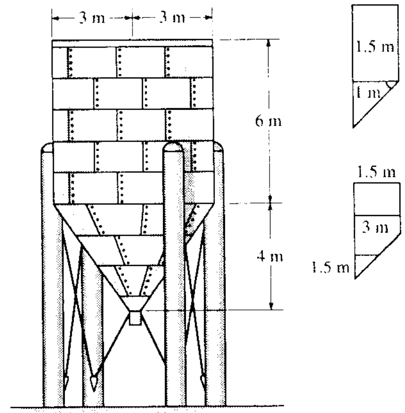
$$A = \Sigma\theta\bar{r}L = 2\pi\left\{3(6\sqrt{2}) + 6(8) + \frac{2(6)}{\pi}\left(\frac{2(6)\pi}{4}\right)\right\}$$

$= 687.73\ \text{ft}^2$




$$\text{Number of gal.} = \frac{687.73\ \text{ft}^2}{250\ \text{ft}^2/\text{gal.}} = 2.75\ \text{gal.}$$ **Ans**

**9-99.** The process tank is used to store liquids during manufacturing. Estimate both the volume of the tank and its surface area. The tank has a flat top and the plates from which the tank is made have negligible thickness.



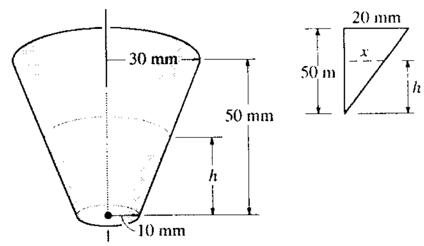

$$V = \Sigma\theta\bar{r}A = 2\pi\left[1\left(\frac{1}{2}\right)(3)(4) + 1.5(3)(6)\right]$$

$V = 207.3\ \text{m}^3 = 207\ \text{m}^3$ **Ans**

$A = \Sigma\theta\bar{r}L = 2\pi[1.5(5) + 3(6) + 1.5(3)]$

$= 188\ \text{m}^2$ **Ans**

***9-100.** Determine the height $h$ to which liquid should be poured into the cup so that it contacts half the surface area on the inside of the cup. Neglect the cup's thickness for the calculation.




$A = \theta\bar{r}L = 2\pi\{20\sqrt{(20)^2 + (50)^2} + 5(10)\}$

$= 2\pi(1127.03)\ \text{mm}^2$

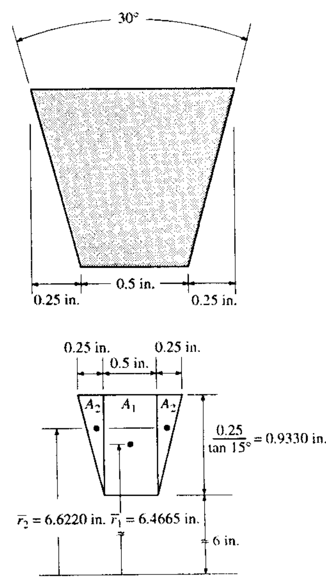
$$x = \frac{20h}{50} = \frac{2h}{5}$$

$$2\pi\left\{5(10) + \left(10 + \frac{h}{5}\right)\sqrt{\left(\frac{2h}{5}\right)^2 + h^2} = \frac{1}{2}(2\pi)(1127.03)\right.$$

$10.77h + 0.2154h^2 = 513.5$

$h = 29.9\ \text{mm}$ **Ans**

**9-101.** A V-belt has as inner radius of 6 in., and a cross-sectional area as shown. Determine the volume of material used in making the V-belt.

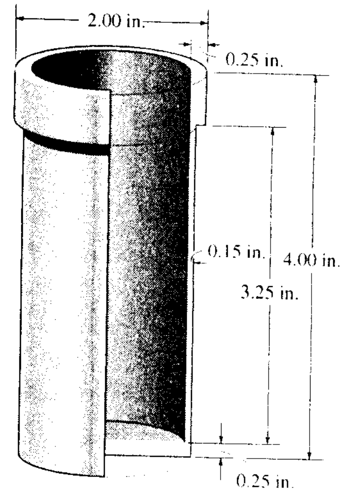



***Volume:*** Applying the theorem of Pappus and Guldinus, Eq. 9-12, with $\theta = 2\pi$, $\bar{r}_1 = 6.4665$ in., $\bar{r}_2 = 6.6220$ in., $A_1 = 0.5(0.9330) = 0.4665\ \text{in}^2$ and $A_2 = \frac{1}{2}(0.5)(0.9330) = 0.2333\ \text{in}^2$, we have

$V = \theta\Sigma\bar{r}A = 2\pi[6.4665(0.4665) + 6.6220(0.2333)]$

$= 28.7\ \text{in}^3$ **Ans**

**9-102.** The full circular aluminum housing is used in an automotive brake system. The cross section is shown in the figure. Determine its weight if aluminum has a specific weight of 169 lb/ft$^3$.

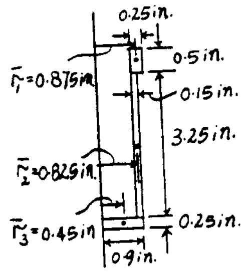



***Volume*** **:** Applying the theorem of Pappus and Guldinus, Eq. 9 – 12, with $\theta = 2\pi$, $\bar{r}_1 = 0.875$ in., $\bar{r}_2 = 0.825$ in., $\bar{r}_3 = 0.45$ in., $A_1 = 0.25(0.5) = 0.125\ \text{in}^2$, $A_2 = 0.15(3.25) = 0.4875\ \text{in}^2$ and $A_3 = 0.25(0.9) = 0.225\ \text{in}^2$, we have

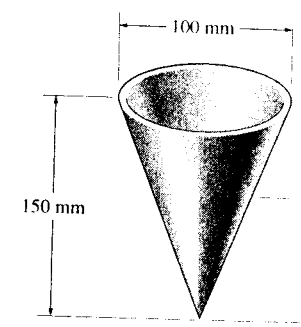
$$V = \theta\Sigma\bar{r}A = 2\pi[0.875(0.125) + 0.825(0.4875) + 0.45(0.225)]$$
$$= 3.850\ \text{in}^3$$

The weight of the housing is

$$W = \gamma V = 169\left(\frac{3.850}{12^3}\right) = 0.377\ \text{lb} \qquad \textbf{Ans}$$




**9-103.** Determine the height $h$ to which liquid should be poured into the conical cup so that it contacts half the surface area on the inside of the cup.

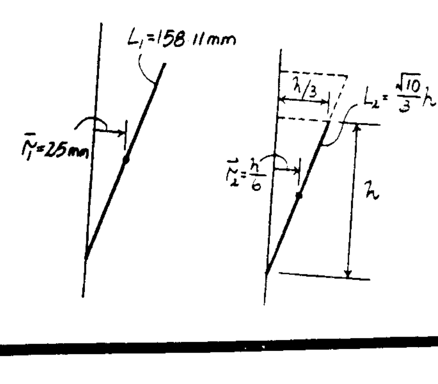



***Surface Area*** **:** This problem requires that $\frac{1}{2}A_1 = A_2$. Applying the theorem of Pappus and Guldinus, Eq. 9 – 9, with $\theta = 2\pi$, $L_1 = \sqrt{50^2 + 150^2} = 158.11$ mm, $L$ $L_2 = \sqrt{h^2 + \left(\frac{h}{3}\right)^2} = \frac{\sqrt{10}}{3}h$, $\bar{r}_1 = 25$ mm and $\bar{r}_2 = \frac{h}{6}$, we have

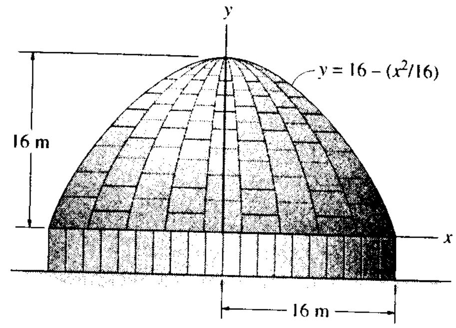
$$\frac{1}{2}(\theta\bar{r}_1 L_1) = \theta\bar{r}_2 L_2$$

$$\frac{1}{2}[2\pi(25)(158.11)] = 2\pi\left(\frac{h}{6}\right)\left(\frac{\sqrt{10}}{3}h\right)$$

$$h = 106\ \text{mm} \qquad \textbf{Ans}$$



***9-104.** Determine the surface area of the roof of the structure if it is formed by rotating the parabola about the $y$ axis.



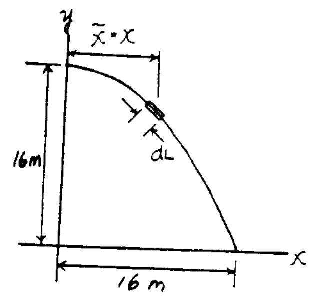

***Centroid*** **:** The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $\tilde{x} = x$. Here, $\frac{dy}{dx} = -\frac{x}{8}$. Evaluating the integrals, we have

$$L = \int dL = \int_0^{16\text{m}} \left(\sqrt{1 + \frac{x^2}{64}}\right)dx = 23.663 \text{ m}$$

$$\int_L \tilde{x}\,dL = \int_0^{16\text{m}} x\left(\sqrt{1 + \frac{x^2}{64}}\right)dx = 217.181 \text{ m}^2$$

Applying Eq. 9 − 7, we have

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{217.181}{23.663} = 9.178 \text{ m}$$




***Surface Area*** **:** Applying the theorem of Pappus and Guldinus, Eq. 9 − 9, with $\theta = 2\pi$, $L = 23.663$ m, $\bar{r} = \bar{x} = 9.178$, we have

$$A = \theta\bar{r}L = 2\pi(9.178)(23.663) = 1365 \text{ m}^2 \qquad \textbf{Ans}$$

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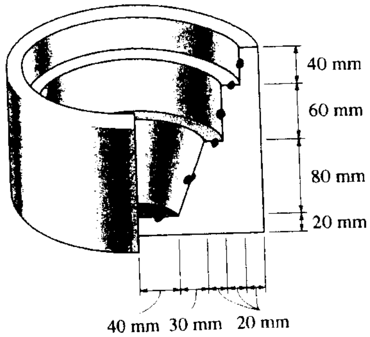
**9-105.** Determine the interior surface area of the brake piston. It consists of a full circular part. Its cross section is shown in the figure.




$$A = \Sigma\theta\bar{r}L = 2\pi[20(40) + 55\sqrt{(30)^2 + (80)^2} + 80(20) + 90(60) + 100(20) + 110(40)]$$

$$A = 119(10^3) \text{ mm}^2 \qquad \textbf{Ans}$$

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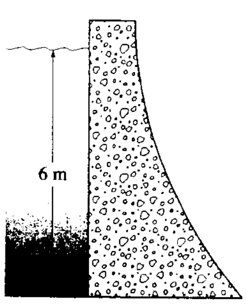
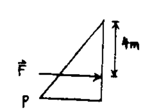
**9-106.** Determine the magnitude of the resultant hydrostatic force acting on the dam and its location, measured from the top surface of the water. The width of the dam is 8 m; $\rho_w = 1.0 \text{ Mg/m}^3$.



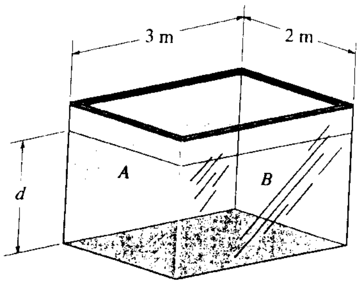

$$p = 6(1)(10^3)(9.81) = 58\,860 \text{ N/m}^2$$

$$F = \frac{1}{2}(6)(8)(58\,860) = 1.41(10^6) \text{ N} = 1.41 \text{ MN} \qquad \textbf{Ans}$$

$$h = \frac{2}{3}(6) = 4 \text{ m} \qquad \textbf{Ans}$$



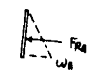
**9-107.** The tank is filled with water to a depth of $d = 4$ m. Determine the resultant force the water exerts on side $A$ and side $B$ of the tank. If oil instead of water is placed in the tank, to what depth $d$ should it reach so that it creates the same resultant forces? $\rho_o = 900\ \text{kg/m}^3$ and $\rho_w = 1000\ \text{kg/m}^3$.




For water

At side $A$ :

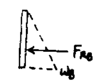
$$w_A = b\,\rho_w\,g\,d$$
$$= 2(1000)(9.81)(4)$$
$$= 78\,480\ \text{N/m}$$




$$F_{R_A} = \frac{1}{2}(78\,480)(4) = 156\,960\ \text{N} = 157\ \text{kN} \qquad \textbf{Ans}$$

At side $B$ :

$$w_B = b\,\rho_w\,g\,d$$
$$= 3(1000)(9.81)(4)$$
$$= 117\,720\ \text{N/m}$$

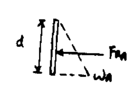



$$F_{R_A} = \frac{1}{2}(117\,720)(4) = 235\,440\ \text{N} = 235\ \text{kN} \qquad \textbf{Ans}$$

For oil

At side $A$ :

$$w_A = b\,\rho_O\,g\,d$$
$$= 2(900)(9.81)d$$
$$= 17\,658\,d$$

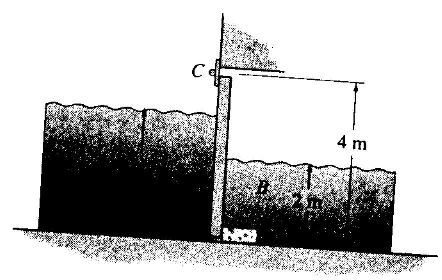



$$F_{R_A} = \frac{1}{2}(17\,658\,d)(d) = 156\,960\ \text{N}$$

$$d = 4.22\ \text{m} \qquad \textbf{Ans}$$

---

***9-108.** When the tide water $A$ subsides, the tide gate automatically swings open to drain the marsh $B$. For the condition of high tide shown, determine the horizontal reactions developed at the hinge $C$ and stop block $D$. The length of the gate is 6 m and its height is 4 m. $\rho_w = 1.0\ \text{Mg/m}^3$.




***Fluid Pressure :*** The fluid pressure at points $D$ and $E$ can be determined using Eq. 9 – 15, $p = \rho g z$.

$$p_D = 1.0(10^3)(9.81)(2) = 19\,620\ \text{N/m}^2 = 19.62\ \text{kN/m}^2$$
$$p_E = 1.0(10^3)(9.81)(3) = 29\,430\ \text{N/m}^2 = 29.43\ \text{kN/m}^2$$

Thus,

$$w_D = 19.62(6) = 117.72\ \text{kN/m}$$
$$w_E = 29.43(6) = 176.58\ \text{kN/m}$$

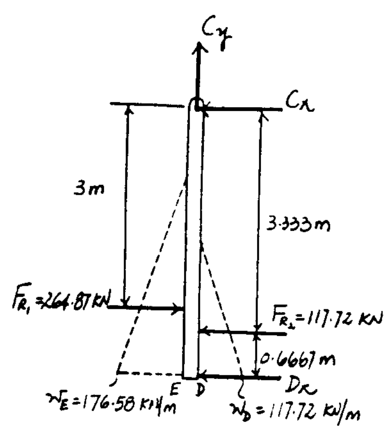
***Resultant Forces :***

$$F_{R_1} = \frac{1}{2}(176.58)(3) = 264.87\ \text{kN}$$
$$F_{R_2} = \frac{1}{2}(117.72)(2) = 117.72\ \text{kN}$$

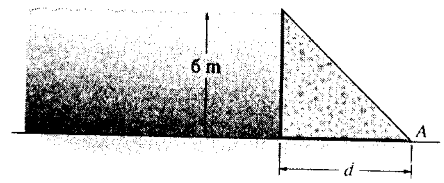
***Equations of Equilibrium :***

$$+\Sigma M_C = 0; \quad 264.87(3) - 117.72(3.333) - D_x(4) = 0$$
$$D_x = 100.55\ \text{kN} = 101\ \text{kN} \qquad \textbf{Ans}$$

$$\xrightarrow{+}\Sigma F_x = 0; \quad 264.87 - 117.72 - 100.55 - C_x = 0$$
$$C_x = 46.6\ \text{kN} \qquad \textbf{Ans}$$



**9-109.** The concrete "gravity" dam is held in place by its own weight. If the density of concrete is $\rho_c = 2.5\ \mathrm{Mg/m^3}$, and water has a density of $\rho_w = 1.0\ \mathrm{Mg/m^3}$, determine the smallest dimension $d$ that will prevent the dam from overturning about its end $A$.

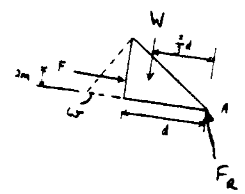



Consider a 1 – m width of dam.

$w = 1000(9.81)(6)(1) = 58\,860\ \mathrm{N/m}$

$F = \frac{1}{2}(58\,860)(6)(1) = 176\,580\ \mathrm{N}$

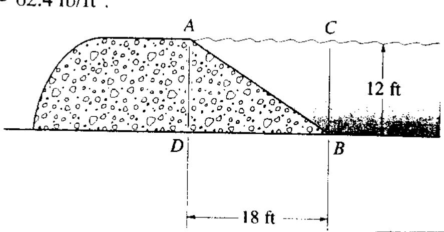
$W = \frac{1}{2}(d)(6)(1)(2500)(9.81) = 73\,575d\ \mathrm{N}$




$\curvearrowleft + \Sigma M_A = 0; \quad -176\,580(2) + 73\,575d\left(\frac{2}{3}d\right) = 0$

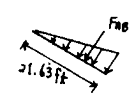
$d = 2.68\ \mathrm{m}$ **Ans**

**9-110.** The concrete dam is designed so that its face $AB$ has a gradual slope into the water as shown. Because of this, the frictional force at the base $BD$ of the dam is increased due to the hydrostatic force of the water acting on the dam. Calculate the hydrostatic force acting on the face $AB$ of the dam. The dam is 60 ft wide. $\gamma_w = 62.4\ \mathrm{lb/ft^3}$.

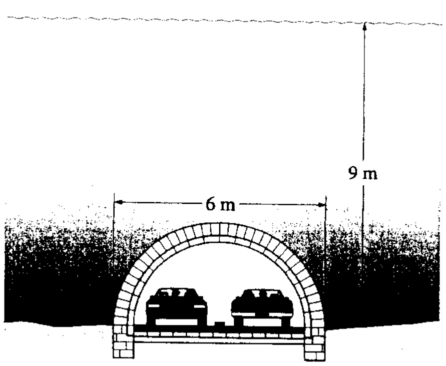



$F_{AB} = \frac{1}{2}[(62.4)(12)(21.63)](60)$

$F_{AB} = 486\ \mathrm{kip}$ **Ans**

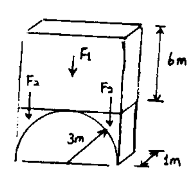



**9-111.** The semicircular tunnel passes under a river which is 9 m deep. Determine the vertical resultant hydrostatic force acting per meter of length along the length of the tunnel. The tunnel is 6 m wide; $\rho_w = 1.0\ \mathrm{Mg/m^3}$.

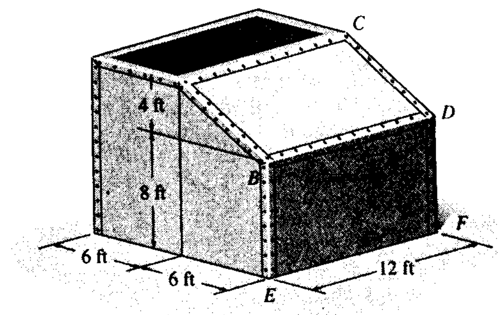



$F = 9.81(1)(6)(6) + 2(1)(9.81)[3(3) - \frac{\pi}{4}(3)^2]$

$F = 391\ \mathrm{kN/m}$ **Ans**



***9-112.** The tank is used to store a liquid having a specific weight of 80 lb/ft$^3$. If it is filled to the top, determine the magnitude of force the liquid exerts on each of its two sides *ABDC* and *BDFE*.




***Fluid Pressure*** : The fluid pressure at points *B* and *E* can be determined using Eq. 9 – 15, $p = \gamma z$.

$$p_B = 80(4) = 320 \text{ lb/ft}^2 \qquad p_E = 80(12) = 960 \text{ lb/ft}^2$$

Thus,

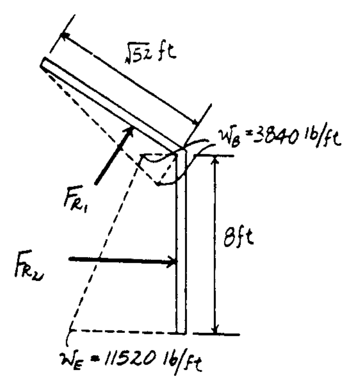
$$w_B = 320(12) = 3840 \text{ lb/ft} \qquad w_E = 960(12) = 11520 \text{ lb/ft}$$

***Resultant Forces*** : The resultant force acts on suface *ABCD* is

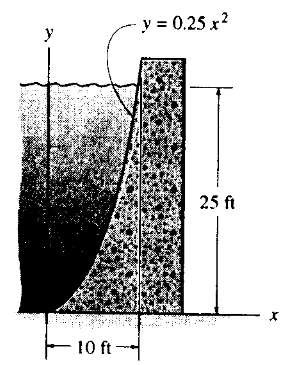
$$F_{R_1} = \frac{1}{2}(3840)\left(\sqrt{52}\right) = 13\,845.31 \text{ lb} = 13.8 \text{ kip} \qquad \textbf{Ans}$$

and acts on surface *BDFE* is

$$F_{R_2} = \frac{1}{2}(3840 + 11520)(8) = 61\,440 \text{ lb} = 61.4 \text{ kip} \qquad \textbf{Ans}$$




**9-113.** Determine the resultant horizontal and vertical force components that the water exerts on the side of the dam. The dam is 25 ft long and $\gamma_w = 62.4$ lb/ft$^3$.




***Fluid Pressure*** : The fluid pressure at the toe of the dam can be determined using Eq. 9 – 15, $p = \gamma z$.

$$p = 62.4(25) = 1560 \text{ lb/ft}^2 = 1.56 \text{ kip/ft}^2$$

Thus,

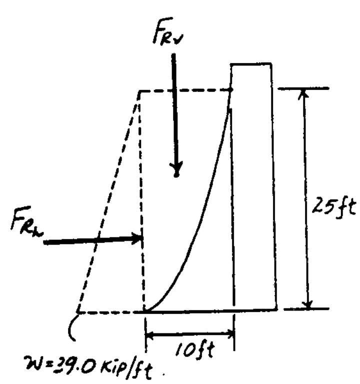
$$w = 1.56(25) = 39.0 \text{ kip/ft}$$

***Resultant Force*** : From the inside back cover of the text, the area of the semiparabolic area is $A = \frac{2}{3}ab = \frac{2}{3}(10)(25) = 166.67 \text{ ft}^2$. Then, the vertical component of the resultant force is

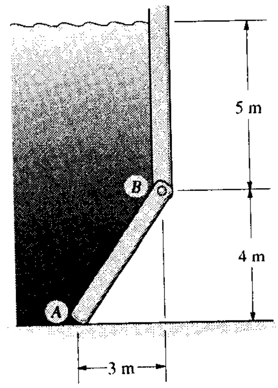
$$F_{R_v} = \gamma V = 62.4[166.67(25)] = 260\,000 \text{ lb} = 260 \text{ kip} \qquad \textbf{Ans}$$

and the horizontal component of the resultant force is

$$F_{R_h} = \frac{1}{2}(39.0)(25) = 487.5 \text{ kip} \qquad \textbf{Ans}$$



**9-114.** The gate $AB$ is 8 m wide. Determine the horizontal and vertical components of force acting on the pin at $B$ and the vertical reaction at the smooth support $A$. $\rho_w = 1.0\ \text{Mg/m}^3$.




***Fluid Pressure:*** The fluid pressure at points $A$ and $B$ can be determined using Eq. 9-15, $p = \rho g z$.

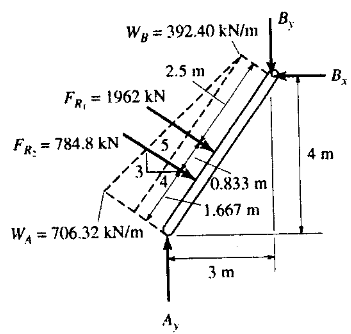
$$P_A = 1.0(10^3)(9.81)(9) = 88\ 290\ \text{N/m}^2 = 88.29\ \text{kN/m}^2$$

$$P_B = 1.0(10^3)(9.81)(5) = 49\ 050\ \text{N/m}^2 = 49.05\ \text{kN/m}^2$$

Thus,

$$w_A = 88.29(8) = 706.32\ \text{kN/m}$$

$$w_B = 49.05(8) = 392.40\ \text{kN/m}$$




***Resultant Forces :***

$$F_{R_1} = 392.4(5) = 1962.0\ \text{kN}$$

$$F_{R_2} = \frac{1}{2}(706.32 - 392.4)(5) = 784.8\ \text{kN}$$

***Equations of Equilibrium:***

$$+\Sigma M_B = 0;\quad 1962.0(2.5) + 784.8(3.333) - A_y(3) = 0$$

$$A_y = 2507\ \text{kN} = 2.51\ \mu\text{N} \qquad \textbf{Ans}$$

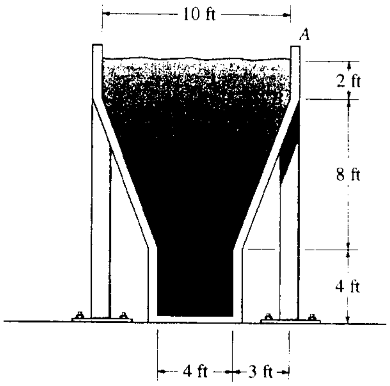
$$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 784.8\left(\frac{4}{5}\right) + 1962\left(\frac{4}{5}\right) - B_x = 0$$

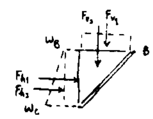
$$B_x = 2197\ \text{kN} = 2.20\ \mu\text{N} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0;\quad 2507 - 784.8\left(\frac{3}{5}\right) - 1962\left(\frac{3}{5}\right) - B_y = 0$$

$$B_y = 859\ \text{kN} \qquad \textbf{Ans}$$

**9-115.** The storage tank contains oil having a specific weight of $\gamma_o = 56\ \text{lb/ft}^3$. If the tank is 6 ft wide, calculate the resultant force acting on the inclined side $BC$ of the tank, caused by the oil, and specify its location along $BC$, measured from $B$. Also compute the total resultant force acting on the bottom of the tank.



$W_B = b\,\rho_O\,h = 6(56)(2) = 672\ \text{lb/ft}$

$W_C = b\,\rho_O\,h = 6(56)(10) = 3360\ \text{lb/ft}$

$F_{h_1} = 8(672) = 5376\ \text{lb}$

$F_{h_2} = \frac{1}{2}(3360 - 672)(8) = 10\,752\ \text{lb}$

$F_{V_1} = 3(2)(6)(56) = 2016\ \text{lb}$

$F_{V_2} = \frac{1}{2}(3)(8)(6)(56) = 4032\ \text{lb}$

$\overset{+}{\rightarrow}\Sigma F_{Rx} = \Sigma F_x;\qquad F_{Rx} = 5376 + 10\,752 = 16\,128\ \text{lb}$

$+\downarrow \Sigma F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 2016 + 4032 = 6048\ \text{lb}$

$F_R = \sqrt{(16\,128)^2 + (6048)^2}$

$= 17\,225\ \text{lb} = 17.2\ \text{kip}$ **Ans**

$\theta = \tan^{-1}\left(\frac{6048}{16\,128}\right) = 20.56°$

$\circlearrowleft + \Sigma M_{R_B} = F_R(d);$

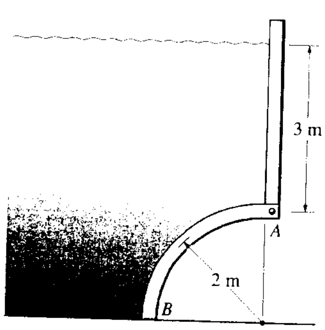
$17\,225\,d = 10\,752\left(\frac{2}{3}\right)(8) + 5376(4) + 2016(1.5) + 4032(2)$

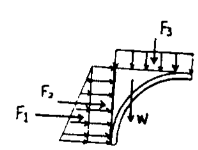
$d = 5.22\ \text{ft}$ **Ans**

At bottom

$F_R = 4(14)(6)(56) = 18\,816\ \text{lb} = 18.8\ \text{kip}$ **Ans**

***9-116.** The arched surface $AB$ is shaped in the form of a quarter circle. If it is 8 m long, determine the horizontal and vertical components of the resultant force caused by the water acting on the surface. $\rho_w = 1.0\ \text{Mg/m}^3$.




$F_3 = 1000(9.81)(3)(2)(8) = 470.88\ \text{kN}$

$F_2 = 1000(9.81)(3)(2)(8) = 470.88\ \text{kN}$

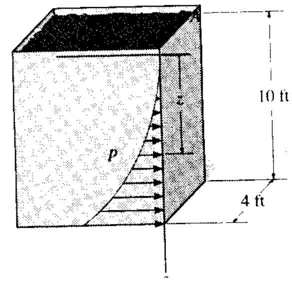
$F_1 = 1000(9.81)(2)\left(\frac{1}{2}\right)(2)(8) = 156.96\ \text{kN}$

$W = \left[(2)^2 - \frac{1}{4}\pi(2)^2\right](8)(1000)(9.81) = 67.37\ \text{kN}$

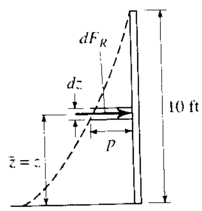
$F_x = 156.96 + 470.88 = 628\ \text{kN}$ **Ans**

$F_y = 470.88 + 67.37 = 538\ \text{kN}$ **Ans**

**9-117.** The rectangular bin is filled with coal, which creates a pressure distribution along wall $A$ that varies as shown, i.e., $p = 4z^3$ lb/ft$^2$, where $z$ is measured in feet. Determine the resultant force created by the coal, and specify its location measured from the top surface of the coal.




***Resultant Force and Its Location:*** The volume of the differential element is $dV = dF_R = 4p\,dz = 4(4z^3)dz = 16z^3 dz$ and its centroid is at $\bar{z} = z$.



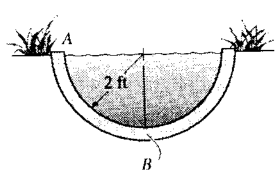

$$F_R = \int_{F_R} dF_R = \int_0^{10\text{ ft}} 16z^3 dz = 4z^4\Big|_0^{10\text{ ft}}$$

$$= 40\,000 \text{ lb} = 40.0 \text{ kip} \qquad \textbf{Ans}$$

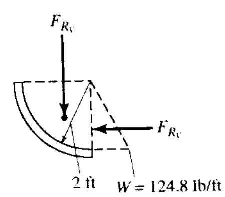
$$\int_{F_R} \bar{z}\,dF_R = \int_0^{10\text{ ft}} z(16z^3 dz) = \frac{16}{5}z^5\Big|_0^{10\text{ ft}} = 320\,000 \text{ lb}\cdot\text{ft}$$

$$\bar{z} = \frac{\int_{F_R} \bar{z}\,dF_R}{\int_{F_R} dF_R} = \frac{320\,000}{40\,000} = 8.00 \text{ ft} \qquad \textbf{Ans}$$

**9-118.** The semicircular drainage pipe is filled with water. Determine the resultant horizontal and vertical force components that the water exerts on the side $AB$ of the pipe per foot of pipe length; $\gamma_w = 62.4$ lb/ft$^3$.




***Fluid Pressure:*** The fluid pressure at the bottom of the drain can be determined using Eq. 9-15, $p = \gamma z$.




$p = 62.4(2) = 124.8$ lb/ft$^2$

Thus,

$w = 124.8(1) = 124.8$ lb/ft

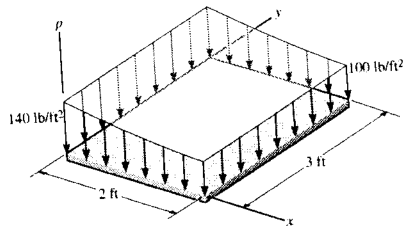
***Resultant Forces:*** The area of the quarter circle is $A = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2^2) = \pi$ ft$^2$. Then, the vertical component of the resultant force is

$F_{R_v} = \gamma V = 62.4[\pi(1)] = 196$ lb **Ans**

and the horizontal component of the resultant force is

$F_{R_h} = \frac{1}{2}(124.8)(2) = 125$ lb **Ans**

**9-119.** The pressure loading on the plate is described by the function $p = 10[6/(x+1)+8]$ lb/ft$^2$. Determine the magnitude of the resultant force and the coordinates $(\bar{x}, \bar{y})$ of the point where the line of action of the force intersects the plate.




$$p = 10\left[\frac{6}{(x+1)} + 8\right]$$

$$F_R = \int_A p\, dA = \int_0^2 10\left[\frac{6}{(x+1)} + 8\right] 3\, dx$$

$F_R = 30[6\ln(x+1) + 8x]|_0^2 = 677.75\text{ lb} = 678\text{ lb}$ **Ans**

$$\int_A \bar{x} p\, dA = \int_0^2 x(10)\left[\frac{6}{(x+1)} + 8\right] 3\, dx$$

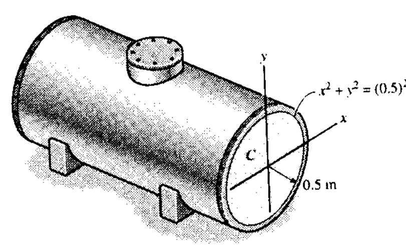
$$= 30[6(x - \ln(1+x)) + 4x^2]_0^2 = 642.250$$

$$\bar{x} = \frac{\int_A \bar{x} p\, dA}{\int_A p\, dA} = \frac{642.250}{677.75} = 0.948\text{ ft}$$ **Ans**

$\bar{y} = 1.50$ ft (by symmetry) **Ans**

---

**9-120.** The tank is filled to the top ($y = 0.5$ m) with water having a density of $\rho_w = 1.0$ Mg/m$^3$ Determine the resultant force of the water pressure acting on the flat end plate $C$ of the tank, and its location, measured from the top of the tank.




$$dF = p\, dA = (1)(9.81)(0.5 - y)\, 2x\, dy$$

$$F = 2(9.81)\int_{-0.5}^{0.5} (0.5 - y)(\sqrt{(0.5)^2 - y^2}\, dy$$

$$= \frac{9.81}{2}\left[y\sqrt{(0.5)^2 - y^2} + 0.5^2 \sin^{-1}\left(\frac{y}{0.5}\right)\right]_{-0.5}^{0.5}$$

$$+ \frac{2(9.81)}{3}[\sqrt{\{(0.5)^2 - y^2\}^3}]_{-0.5}^{0.5}$$

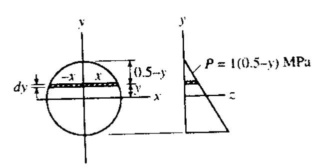
$F = 3.85$ kN **Ans**

$$\int_A y\, dF = 2(9.81)\int_{-0.5}^{0.5}(0.5y - y^2)(\sqrt{(0.5)^2 - y^2}dy = 19.62\left\{\left[-\frac{0.5}{3}\sqrt{\{(0.5)^2 - y^2\}^3}\right]_{-0.5}^{0.5} +\right.$$

$$\left.\frac{y}{4}[\sqrt{\{(0.5)^2 - y^2\}^3}]_{-0.5}^{0.5} - \frac{(0.5)^2}{8}\left[y\sqrt{(0.5)^2 - y^2} + (0.5)^2\sin^{-1}\frac{y}{0.5}\right]_{-0.5}^{0.5}\right\}$$

$$= -0.481\text{ kN m}$$

$$F(-d) = \int y\, dF$$

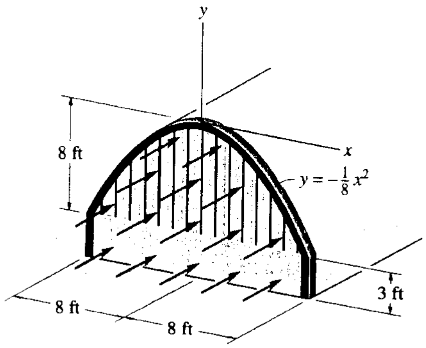



$$d = \frac{-0.481}{3.85} = -0.125\text{ m}$$

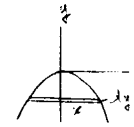
Hence, measured from the top of the tank,

$d' = 0.5 + 0.125 = 0.625$ m **Ans**

**9-121.** The wind blows uniformly on the front surface of the metal building with a pressure of 30 lb/ft$^2$. Determine the resultant force it exerts on the surface and the position of this resultant.




**Parabola:**



$$F_p = \int_0^8 30(2\,x\,dy)$$

$$= 60\int_0^8 \sqrt{8}\,y^{1/2}\,dy$$

$$= 60\sqrt{8}(\frac{2}{3})(8)^{3/2} = 2560 \text{ lb}$$

$$\bar{y} = \frac{\int_0^8 y\,(30)(2\,x\,dy)}{2560} = \frac{60\int_0^8 \sqrt{8}\,y^{3/2}\,dy}{2560}$$

$$\bar{y} = \frac{60\sqrt{8}(\frac{2}{5})(8)^{5/2}}{2560} = 4.80 \text{ ft}$$

Also, from table in back of text:

$$F_p = p(\frac{2}{3}ab) = 60(\frac{2}{3})(8)(8) = 2560$$

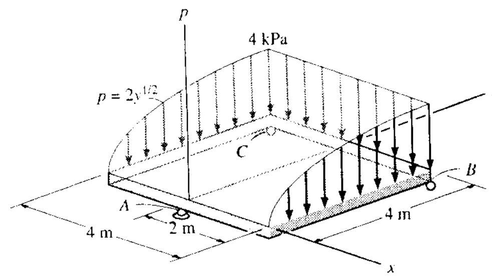
$$\bar{y} = \frac{3}{5}(8) = 4.80 \text{ ft}$$

$F_R = 2560 + 30(3)(16) = 4000 \text{ lb} = 4.00 \text{ kip}$ **Ans**

$4000(\bar{y}) = 4.80(2560) + 9.5(30)(3)(16)$

$\bar{y} = -6.49 \text{ ft}$ **Ans**

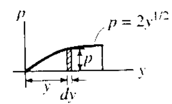
**9-122.** The loading acting on a square plate is represented by a parabolic pressure distribution. Determine the magnitude of the resultant force and the coordinates $(\bar{x}, \bar{y})$ of the point where the line of action of the force intersects the plate. Also, what are the reactions at the rollers $B$ and $C$ and the ball-and-socket joint $A$? Neglect the weight of the plate.




$\tilde{y} = y$

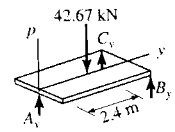
$dA = p\,dy$

$\bar{x} = 0$ **Ans** (Due to symmetry)




$$\int dA = \int_0^4 2y^{1/2}dy = \left[\frac{4}{3}y^{3/2}\right]_0^4 = 10.67 \text{ kN/m}$$

$$\int \tilde{y}\,dA = \int_0^4 2y^{3/2}dy = \left[\frac{4}{5}y^{5/2}\right]_0^4 = 25.6 \text{ kN}$$




$$\bar{y} = \frac{\int \tilde{y}\,dA}{\int dA} = \frac{25.6}{10.67} = 2.40 \text{ m} \qquad \textbf{Ans}$$

$F_R = 10.67(4) = 42.7$ kN **Ans**

$\Sigma M_x = 0;\quad B_y = C_y$

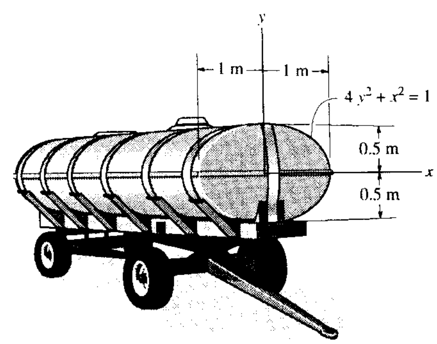
$\Sigma M_x = 0;\quad 42.67(2.40) - 2B_y(4) = 0$

$B_y = C_y = 12.8$ kN **Ans**

$+\uparrow \Sigma F = 0;\quad A_y - 42.67 + 12.8 + 12.8 = 0$

$A_y = 17.1$ kN **Ans**

**9-123.** The tank is filled with a liquid which has a density of 900 kg/m$^3$. Determine the resultant force that it exerts on the elliptical end plate, and the location of the center of pressure, measured from the $x$ axis.

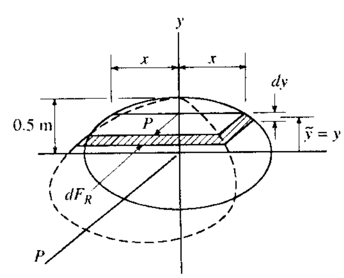



***Fluid Pressure:*** The fluid pressure at an arbitrary point along $y$ axis can be determined using Eq. 9-15, $p = \gamma(0.5 - y) = 900(9.81)(0.5 - y) = 8829(0.5 - y)$.

***Resultant Force and its Location:*** Here, $x = \sqrt{1 - 4y^2}$. The volume of the differential element is $dV = dF_R = p(2x\,dy) = 8829(0.5 - y)[2\sqrt{1 - 4y^2}]dy$. Evaluating the integrals using Simpson's rule, we have

$$F_R = \int_{F_R} dF_R = 17658\int_{-0.5\text{ m}}^{0.5\text{ m}} (0.5 - y)(\sqrt{1 - 4y^2})dy$$

$$= 6934.2 \text{ N} = 6.93 \text{ kN} \qquad \textbf{Ans}$$

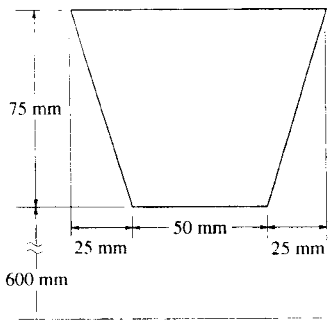



$$\int_{F_R} \tilde{y}\,dF_R = 17658\int_{-0.5\text{ m}}^{0.5\text{ m}} y(0.5 - y)(\sqrt{1 - 4y^2})dy$$

$$= -866.7 \text{ N·m}$$

$$\bar{y} = \frac{\int_{F_R} \tilde{y}\,dF_R}{\int_{F_R} dF_R} = \frac{-866.7}{6934.2} = -0.125 \text{ m} \qquad \textbf{Ans}$$

***9-124.** A circular V-belt has an inner radius of 600 mm and a cross-sectional area as shown. Determine the volume of material required to make the belt.




$$V = \Sigma \theta \bar{r} A$$

$$= 2\pi\left[0.625(2)\left(\tfrac{1}{2}\right)(0.025)(0.075) + 0.6375(0.05)(0.075)\right]$$

$$= 22.4(10)^{-3}\ \text{m}^3 \qquad \textbf{Ans}$$

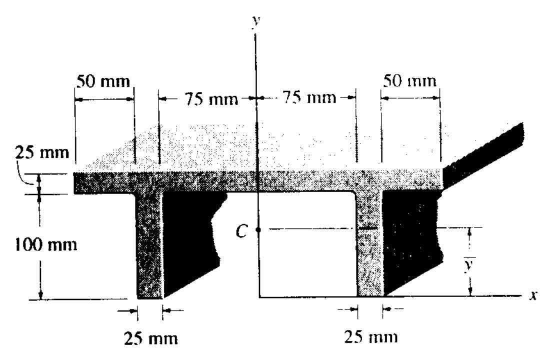
**9-125.** A circular V-belt has an inner radius of 600 mm and a cross-sectional area as shown. Determine the surface area of the belt.

$$A = \Sigma \theta \bar{r} L$$

$$= 2\pi\left[0.6(0.05) + 2(0.6375)\left(\sqrt{(0.025)^2 + (0.075)^2}\right) + 0.675(0.1)\right]$$

$$= 1.25\ \text{m}^2 \qquad \textbf{Ans}$$

**9-126.** Locate the centroid $\bar{y}$ of the beam's cross-sectional area.

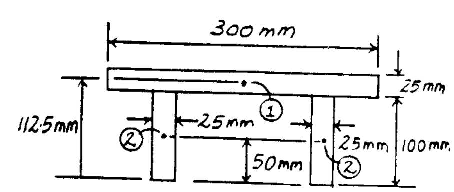



***Centroid*** **: The area of each segment and its respective centroid are tabulated below.**

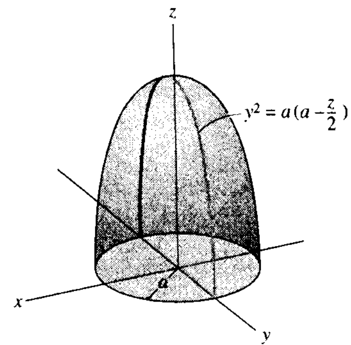
| Segment | $A\ (\text{mm}^2)$ | $\bar{y}\ (\text{mm})$ | $\bar{y}A\ (\text{mm}^3)$ |
|---|---|---|---|
| 1 | 300(25) | 112.5 | 843 750 |
| 2 | 100(50) | 50 | 250 000 |
| $\Sigma$ | 12 500 | | 1 093 750 |

**Thus,**

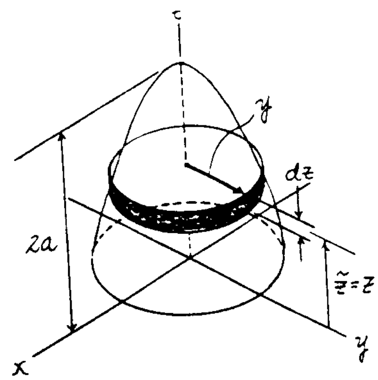
$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{1\,093\,750}{12\,500} = 87.5\ \text{mm} \qquad \textbf{Ans}$$



**9-127.** Locate the centroid of the solid.




***Volume and Moment Arm :*** The volume of the thin disk differential element is $dV = \pi y^2 dz = \pi\left[a\left(a-\frac{z}{2}\right)\right]dz = \pi a\left(a-\frac{z}{2}\right)dz$ and its centroid is at $\tilde{z} = z$.




***Centroid :*** Due to symmetry about the z axis

$$\bar{x} = \bar{y} = 0 \qquad \textbf{Ans}$$

Applying Eq. 9 – 5 and performing the integration, we have

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^{2a} z\left[\pi a\left(a-\frac{z}{2}\right)dz\right]}{\int_0^{2a}\pi a\left(a-\frac{z}{2}\right)dz}$$

$$= \frac{\pi a\left(\frac{az^2}{2}-\frac{z^3}{6}\right)\Big|_0^{2a}}{\pi a\left(az-\frac{z^2}{4}\right)\Big|_0^{2a}} = \frac{2}{3}a \qquad \textbf{Ans}$$

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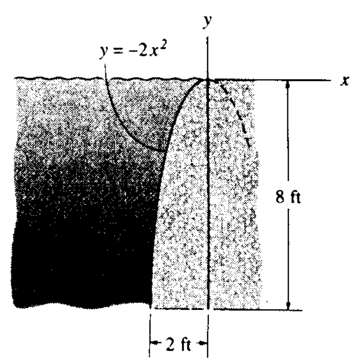
***9-128.** Determine the magnitude of the resultant hydrostatic force acting per foot of length on the sea wall; $\gamma_w = 62.4\ \text{lb/ft}^3$.



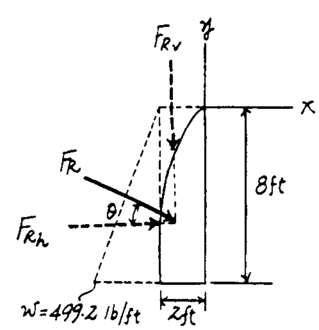

***Fluid Pressure :*** The fluid pressure at the toe of the dam can be determined using Eq. 9 – 15, $p = \gamma z$.

$$p = 62.4(8) = 499.2\ \text{lb/ft}^2$$

Thus,

$$w = 499.2(1) = 499.2\ \text{lb/ft}$$

***Resultant Forces :*** From the inside back cover of the text, the exparabolic area is $A = \frac{1}{3}ab = \frac{1}{3}(8)(2) = 5.333\ \text{ft}^2$. Then, the vertical and horizontal components of the resultant force are




$$F_{R_v} = \gamma V = 62.4[5.333(1)] = 332.8\ \text{lb}$$

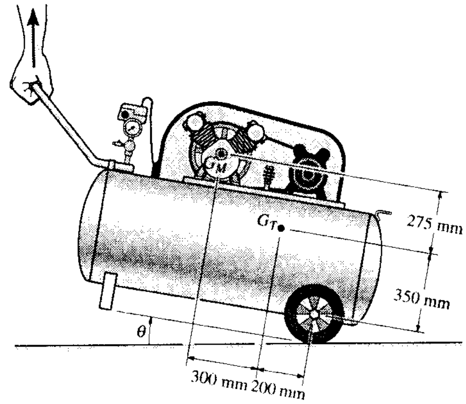
$$F_{R_h} = \frac{1}{2}(499.2)(8) = 1996.8\ \text{lb}$$

The resultant force and is

$$F_R = \sqrt{F_{R_v}^2 + F_{R_h}^2} = \sqrt{332.8^2 + 1996.8^2}$$

$$= 2024.34\ \text{lb} = 2.02\ \text{kip} \qquad \textbf{Ans}$$

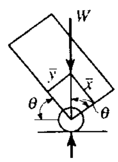
**9-129.** The tank and compressor have a mass of 15 kg and mass center at $G_T$ and the motor has a mass of 70 kg and a mass center at $G_M$. Determine the angle of tilt, $\theta$ of the tank so that the unit will be on the verge of tipping over.



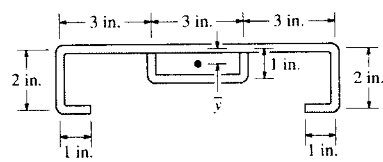

$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} = \frac{0.2(15) + 0.5(70)}{15 + 70} = 0.4471 \text{ m}$$

$$\bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} = \frac{0.35(15) + 0.625(70)}{15 + 70} = 0.57647 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{\bar{x}}{\bar{y}}\right) = \frac{0.4471}{0.57647} = 37.8^\circ \qquad \textbf{Ans}$$



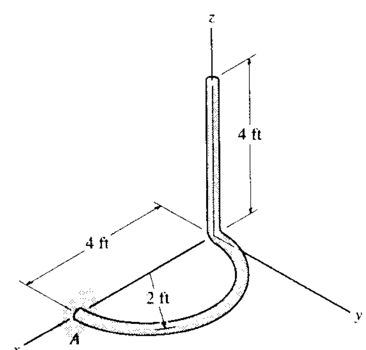

**9-130.** The thin-walled channel and stiffener have the cross section shown. If the material has a constant thickness, determine the location $\bar{y}$ of its centroid. The dimensions are indicated to the center of each segment.




$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{(0)(9) + 2(1)(2) + 2(2)(1) + 2(0.5)(1) + 1(3)}{(9) + 2(2) + 2(1) + 2(1) + 3}$$

$\bar{y} = 0.600$ in. **Ans**

**9-131.** Locate the center of gravity of the homogeneous rod. The rod has a weight of 2lb/ft. Also, compute the $x$, $y$, $z$ components of reaction at the fixed support $A$.




$$\Sigma \tilde{x} L = 0(4) + 2(\pi)(2) = 12.5664 \text{ ft}^2$$

$$\Sigma \tilde{y} L = 0(4) + \frac{2(2)}{\pi}(\pi)(2) = 8 \text{ ft}^2$$

$$\Sigma \tilde{z} L = 2(4) + 0(\pi)(2) = 8 \text{ ft}^2$$

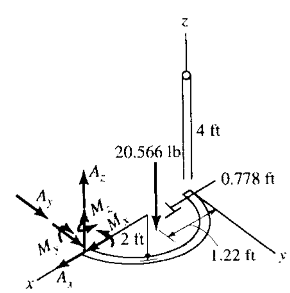
$$\Sigma L = 4 + \pi(2) = 10.2832 \text{ ft}$$

$$\bar{x} = \frac{\Sigma \tilde{x} L}{\Sigma L} = \frac{12.5664}{10.2832} = 1.22 \text{ ft} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{8}{10.2832} = 0.778 \text{ ft} \qquad \textbf{Ans}$$

$$\bar{z} = \frac{\Sigma \tilde{z} L}{\Sigma L} = \frac{8}{10.2832} = 0.778 \text{ ft} \qquad \textbf{Ans}$$

$$W = (2 \text{ lb/ft})(10.2832 \text{ ft}) = 20.566 \text{ lb}$$




$\Sigma M_x = 0;\quad M_{Ax} - 0.778(20.556) = 0$

$M_{Ax} = 16.0$ lb·ft **Ans**

$\Sigma M_y = 0;\quad M_{Ay} - (4 - 1.22)(20.566) = 0$

$M_{Ay} = 57.1$ lb·ft **Ans**

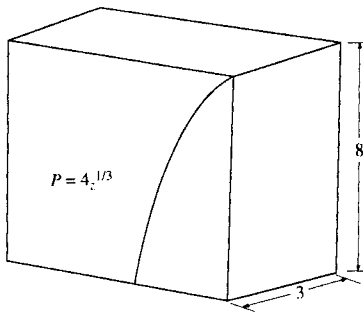
$\Sigma M_z = 0;\quad M_{Az} = 0$ **Ans**

$\Sigma F_x = 0;\quad A_x = 0$ **Ans**

$\Sigma F_y = 0;\quad A_y = 0$ **Ans**

$\Sigma F_z = 0;\quad A_z = 2(10.2832) = 20.6$ lb **Ans**

**9-132.** The rectangular bin is filled with coal, which creates a pressure distribution along wall $A$ that varies as shown, i.e., $P = 4Z^{1/3}$ lb/ft$^2$, where $Z$ is in feet. Compute the resultant force created by the coal, and its location, measured from that top surface of the coal.




$$dF = f\,dA = 4Z^{1/3}(3)dZ$$

$$F = 12\int_0^8 Z^{1/3}\,dZ$$

$$= 12\left[\frac{3}{4}\,Z^{4/3}\right]_0^8$$

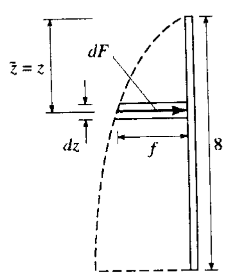
$$= 144 \text{ lb}$$ **Ans**

$$\int_A Z\,dF = 12\int_0^8 Z^{4/3}dZ$$

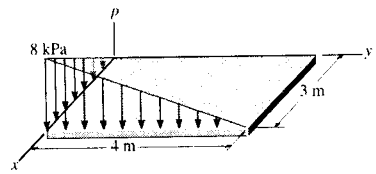
$$= 12\left[\frac{3}{7}\,Z^{7/3}\right]_0^8$$

$$= 658.29 \text{ lb·ft}$$

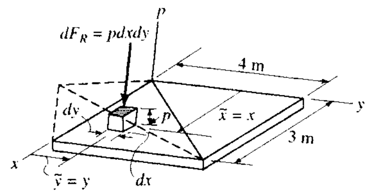
$$\bar{Z} = \frac{658.29}{144} = 4.57 \text{ ft}$$ **Ans**



**9-133.** The load over the plate varies linearly along the sides of the plate such that $p = \frac{2}{3}[x(4-y)]$kPa Determine the resultant force and its position $(\bar{x}, \bar{y})$ on the plate.




***Resultant Force and its Location:*** The volume of the differential element is $dV = dF_R = pdxdy = \frac{2}{3}(xdx)[(4-y)dy]$ and its centroid are $\tilde{x} = x$ and $\tilde{y} = y$.




$$F_R = \int_{F_R} dF_R = \int_0^{3\text{ m}} \frac{2}{3}(xdx)\int_0^{4\text{ m}}(4-y)dy$$

$$= \frac{2}{3}\left[\left(\frac{x^2}{2}\right)\Big|_0^{3\text{ m}}\left(4y - \frac{y^2}{2}\right)\Big|_0^{4\text{ m}}\right] = 24.0\text{ kN} \quad \textbf{Ans}$$

$$\int_{F_R} \tilde{x}dF_R = \int_0^{3\text{ m}} \frac{2}{3}(x^2dx)\int_0^{4\text{ m}}(4-y)dy$$

$$= \frac{2}{3}\left[\left(\frac{x^3}{3}\right)\Big|_0^{3\text{ m}}\left(4y - \frac{y^2}{2}\right)\Big|_0^{4\text{ m}}\right] = 48.0\text{ kN·m}$$

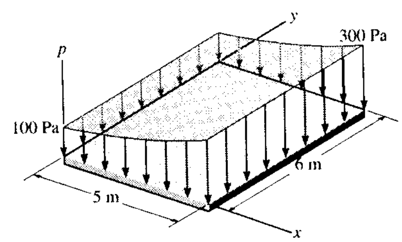
$$\int_{F_R} \tilde{y}dF_R = \int_0^{3\text{ m}} \frac{2}{3}(xdx)\int_0^{4\text{ m}}y(4-y)dy$$

$$= \frac{2}{3}\left[\left(\frac{x^2}{2}\right)\Big|_0^{3\text{ m}}\left(2y^2 - \frac{y^3}{3}\right)\Big|_0^{4\text{ m}}\right] = 32.0\text{ kN·m}$$

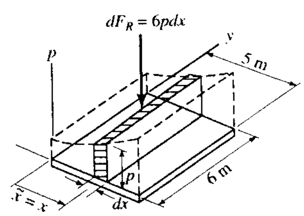
$$\bar{x} = \frac{\int_{F_R}\tilde{x}dF_R}{\int_{F_R}dF_R} = \frac{48.0}{24.0} = 2.00\text{ m} \quad \textbf{Ans}$$

$$\bar{y} = \frac{\int_{F_R}\tilde{y}dF_R}{\int_{F_R}dF_R} = \frac{32.0}{24.0} = 1.33\text{ m} \quad \textbf{Ans}$$

**9-134.** The pressure loading on the plate is described by the function $p = \{-240/(x+1) + 340\}$ Pa. Determine the magnitude of the resultant force and coordinates of the point where the line of action of the force intersects the plate.




***Resultant Force and its Location:*** The volume of the differential element is $dV = dF_R = 6pdx = 6\left(-\frac{240}{x+1} + 340\right)dx$ and its centroid in $\tilde{x} = x$.




$$F_R = \int_{F_R} dF_R = \int_0^{5\text{ m}} 6\left(-\frac{240}{x+1} + 340\right)dx$$

$$= 6[-240\ln(x+1) + 340x^2]|_0^{5\text{ m}}$$

$$= 7619.87\text{ N} = 7.62\text{ kN} \quad \textbf{Ans}$$

$$\int_{F_R} \tilde{x}dF_R = \int_0^{5\text{ m}} 6x\left(-\frac{240}{x+1} + 340\right)$$

$$= [-1440[x - \ln(x+1)] + 1020x^2]|_0^{5\text{ m}}$$

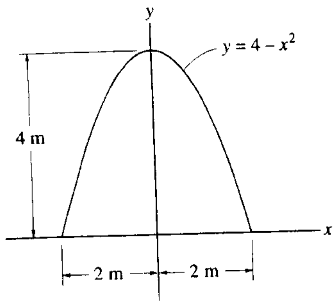
$$= 20880.13\text{ N·m}$$

$$\bar{x} = \frac{\int_{F_R}\tilde{x}dF_R}{\int_{F_R}dF_R} = \frac{20880.13}{7619.87} = 2.74\text{ m} \quad \textbf{Ans}$$

Due to symmetry,

$\bar{y} = 3.00$ m **Ans**

**10-1.** Determine the moment of inertia of the shaded area about the $x$ axis.

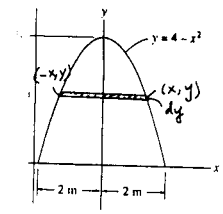



$$I_x = \int_0^4 y^2\,dA = 2\int_0^4 y^2\,(x\,dy)$$

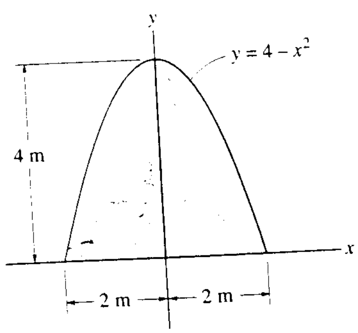
$$= 2\int_0^4 y^2\sqrt{4-y}\,dy$$

$$I_x = 2\left[\frac{2(15\,y^2 + 12(4)(y) + 8(4)^2)(\sqrt{(4-y)^3})}{-105}\right]_0^4$$

$I_x = 39.0\ \text{m}^4$ **Ans**



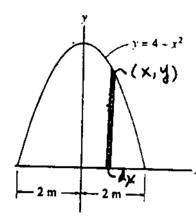

**10-2.** Determine the moment of inertia of the shaded area about the $y$ axis.



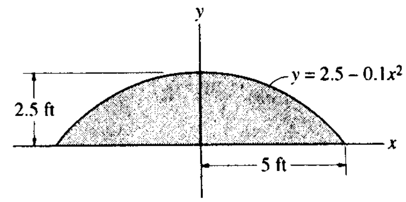

$$I_y = \int_A x^2\,dA = 2\int_0^2 x^2(4-x^2)\,dx$$

$$= 2\left[\frac{4\,x^3}{3} - \frac{x^5}{5}\right]_0^2$$

$I_y = 8.53\ \text{m}^4$ **Ans**

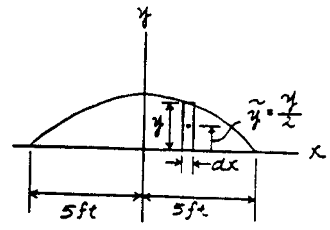


**10-3.** Determine the moment of inertia of the area about the $x$ axis. Solve the problem in two ways, using rectangular differential elements: (a) having a thickness $dx$ and (b) having a thickness of $dy$.




**a)** ***Differential Element :*** The area of the differential element parallel to $y$ axis is $dA = ydx$. The moment of inertia of this element about $x$ axis is

$$\begin{aligned} dI_x &= dI_{x'} + dA\tilde{y}^2 \\ &= \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2 \\ &= \frac{1}{3}\left(2.5 - 0.1x^2\right)^3 dx \\ &= \frac{1}{3}\left(-0.001x^6 + 0.075x^4 - 1.875x^2 + 15.625\right) dx \end{aligned}$$

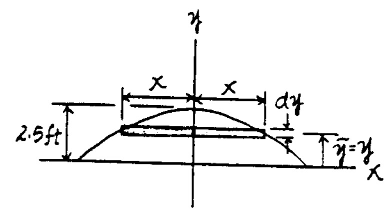


(a)

***Moment of Inertia :*** Performing the integration, we have

$$\begin{aligned} I_x = \int dI_x &= \frac{1}{3}\int_{-5\text{ft}}^{5\text{ft}} \left(-0.001x^6 + 0.075x^4 - 1.875x^2 + 15.625\right) dx \\ &= \frac{1}{3}\left(-\frac{0.001}{7}x^7 + \frac{0.075}{5}x^5 - \frac{1.875}{3}x^3 + 15.625x\right)\Bigg|_{-5\text{ft}}^{5\text{ft}} \\ &= 23.8 \text{ ft}^4 \end{aligned}$$

**Ans**



(b)

**b)** ***Differential Element :*** Here, $x = \sqrt{25 - 10y}$. The area of the differential element parallel to $x$ axisis $dA = 2xdy = 2\sqrt{25 - 10y}dy$.

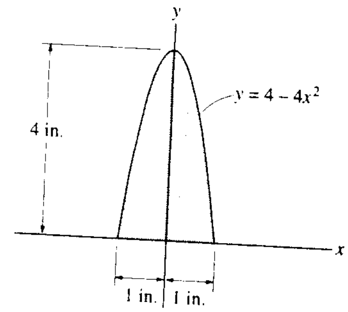
***Moment of Inertia :*** Applying Eq. 10 – 1 and performing the integration, we have

$$\begin{aligned} I_x &= \int_A y^2 dA \\ &= 2\int_0^{2.5\text{ft}} y^2\sqrt{25 - 10y}dy \end{aligned}$$

$$\begin{aligned} &= 2\left[-\frac{y^2}{15}(25 - 10y)^{\frac{3}{2}} - \frac{2y}{375}(25 - 10y)^{\frac{5}{2}} - \frac{2}{13125}(25 - 10y)^{\frac{7}{2}}\right]\Bigg|_0^{2.5\text{ft}} \\ &= 23.8 \text{ ft}^4 \end{aligned}$$

**Ans**

***10-4.** Determine the moment of inertia of the area about the $x$ axis. Solve the problem in two ways, using rectangular differential elements: (a) having a thickness of $dx$, and (b) having a thickness of $dy$.



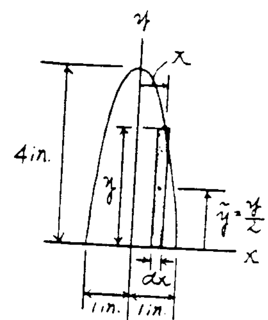

a) ***Differential Element :*** The area of the differential element parallel to $y$ axis is $dA = ydx$. The moment of inertia of this element about $x$ axis is

$$
\begin{aligned}
dI_x &= d\bar{I}_{x'} + dA\,\tilde{y}^2 \\
&= \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2 \\
&= \frac{1}{3}\left(4-4x^2\right)^3 dx \\
&= \frac{1}{3}\left(-64x^6 + 192x^4 - 192x^2 + 64\right) dx
\end{aligned}
$$

***Moment of Inertia :*** Performing the integration, we have

$$
\begin{aligned}
I_x = \int dI_x &= \frac{1}{3}\int_{-1\text{ in.}}^{1\text{ in.}} \frac{1}{3}\left(-64x^6 + 192x^4 - 192x^2 + 64\right) dx \\
&= \frac{1}{3}\left(-\frac{64}{7}x^7 + \frac{192}{5}x^5 - \frac{192}{3}x^3 + 64x\right)\Bigg|_{-1\text{ in.}}^{1\text{ in.}} \\
&= 19.5\text{ in}^4
\end{aligned}
$$

**Ans**



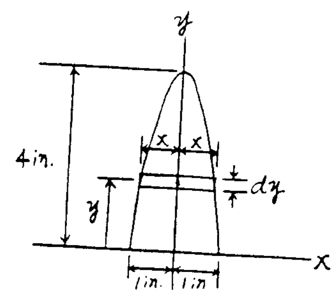
(a)

b) ***Differential Element :*** Here, $x = \frac{1}{2}\sqrt{4-y}$. The area of the differential element parallel to $x$ axis is $dA = 2xdy = \sqrt{4-y}\,dy$.

***Moment of Inertia :*** Applying Eq. 10 − 1 and performing the integration, we have

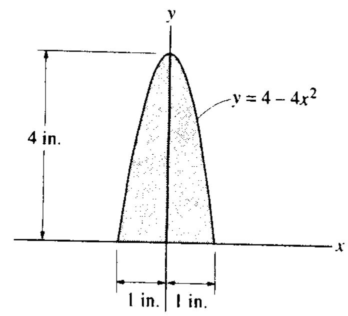
$$
\begin{aligned}
I_x &= \int_A y^2 dA \\
&= \int_0^{4\text{ in.}} y^2\sqrt{4-y}\,dy \\
&= \left[-\frac{2y^2}{3}(4-y)^{\frac{3}{2}} - \frac{8y}{15}(4-y)^{\frac{5}{2}} - \frac{16}{105}(4-y)^{\frac{7}{2}}\right]\Bigg|_0^{4\text{ in.}} \\
&= 19.5\text{ in}^4
\end{aligned}
$$

**Ans**



(b)

**10-5.** Determine the moment of inertia of the area about the $y$ axis. Solve the problem in two ways, using rectangular differential elements : (a) having a thickness of $dx$, and (b) having a thickness of $dy$.




a)*Differential Element :* The area of the differential element parallel to yaxis is $dA = ydx = (4-4x^2)\,dx$ .

*Moment of Inertia :* Applying Eq. 10 − 1 and performing the integration, we have

$$I_y = \int_A x^2 dA = \int_{-1\text{in.}}^{1\text{in.}} x^2(4-4x^2)\,dx$$
$$= \left[\frac{4}{3}x^3 - \frac{4}{5}x^5\right]\Bigg|_{-1\text{in.}}^{1\text{in.}}$$
$$= 1.07\ \text{in}^4 \qquad \textbf{Ans}$$

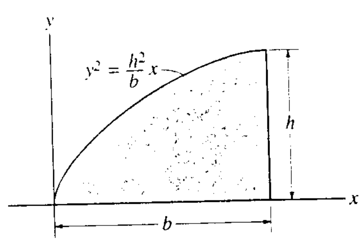
b)*Differential Element :* Here, $x = \frac{1}{2}\sqrt{4-y}$. The moment of inertia of the differential element about y axis is

$$dI_y = \frac{1}{12}(dy)(2x^3) = \frac{2}{3}x^3 dy = \frac{1}{12}(4-y)^{\frac{3}{2}}dy$$

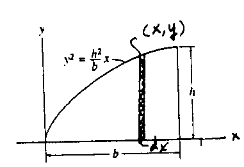
*Moment of Inertia :* Performing the integration, we have

$$I_y = \int dI_y = \frac{1}{12}\int_0^{4\text{in.}}(4-y)^{\frac{3}{2}}dy$$
$$= \frac{1}{12}\left[-\frac{2}{5}(4-y)^{\frac{5}{2}}\right]\Bigg|_0^{4\text{in.}}$$
$$= 1.07\ \text{in}^4 \qquad \textbf{Ans}$$

**10-6.** Determine the moment of inertia of the shaded area about the $x$ axis.

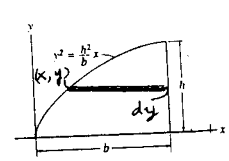



$$dI_x = \frac{1}{3}y^3\,dx$$
$$I_x = \int dI_x$$
$$= \int_0^b \frac{y^3}{3}\,dy = \int_0^b \frac{1}{3}\left(\frac{h^2}{b}\right)^{3/2} x^{3/2}\,dx$$
$$= \frac{1}{3}\left(\frac{h^2}{b}\right)^{3/2}\left(\frac{2}{5}\right)x^{5/2}\Big|_0^b$$
$$= \frac{2}{15}bh^3 \qquad \textbf{Ans}$$

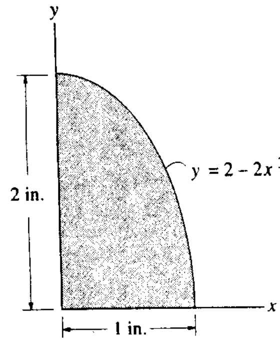



Also,

$$dA = (b-x)\,dy = \left(b - \frac{b}{h^2}y^2\right)dy$$
$$I_x = \int y^2\,dA$$
$$= \int_0^h y^2\left(b - \frac{b}{h^2}y^2\right)dy$$
$$= \left[\frac{b}{3}y^3 - \frac{b}{5h^2}y^5\right]_0^h$$
$$= \frac{2}{15}bh^3 \qquad \textbf{Ans}$$

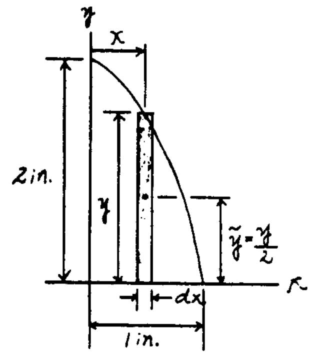


**10-7.** Determine the moment of inertia of the shaded area about the $x$ axis.




***Differential Element :*** The area of the differential element parallel to $y$ axis is $dA = y dx$ . The moment of inertia of this element about $x$ axis is

$$dI_x = d\bar{I}_{x'} + dA\,\tilde{y}^2$$
$$= \frac{1}{12}(dx)y^3 + y dx\left(\frac{y}{2}\right)^2$$
$$= \frac{1}{3}\left(2-2x^3\right)^3 dx$$
$$= \frac{1}{3}\left(-8x^9 + 24x^6 - 24x^3 + 8\right) dx$$




***Moment of Inertia :*** Performing the integration, we have

$$I_x = \int dI_x = \frac{1}{3}\int_0^{1\,\text{in.}} \left(-8x^9 + 24x^6 - 24x^3 + 8\right) dx$$
$$= \frac{1}{3}\left(-\frac{4}{5}x^{10} + \frac{24}{7}x^7 - 6x^4 + 8x\right)\Big|_0^{1\,\text{in.}}$$
$$= 1.54 \text{ in}^4$$

**Ans**

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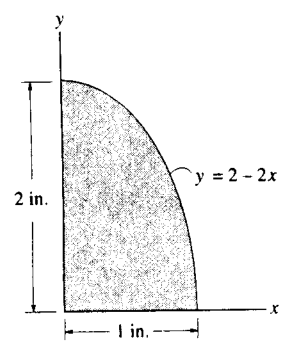
***10-8.** Determine the moment of inertia of the shaded area about the $y$ axis.



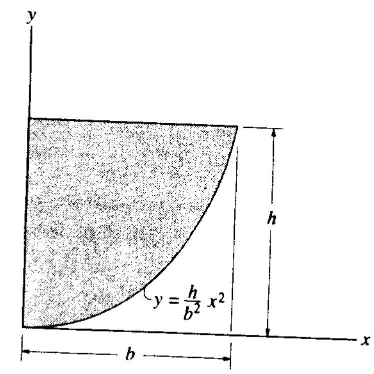

***Differential Element :*** The area of the differential element parallel to yaxis is $dA = y dx = \left(2 - 2x^3\right) dx$ .

***Moment of Inertia :*** Applying Eq. 10 – 1and performing the integration, we have

$$I_y = \int_A x^2 dA = \int_0^{1\,\text{in.}} x^2\left(2-2x^3\right) dx$$
$$= \left[\frac{2}{3}x^3 - \frac{1}{3}x^6\right]\Big|_0^{1\,\text{in.}}$$
$$= 0.333 \text{ in}^4$$

**Ans**

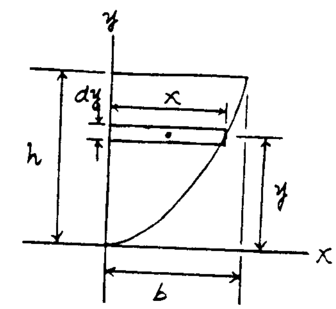
**10-9.** Determine the moment of inertia of the shaded area about the $x$ axis.



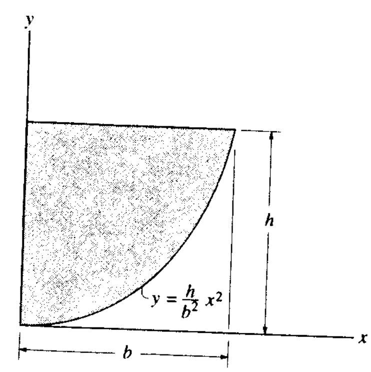

***Differential Element :*** Here, $x = \frac{b}{\sqrt{h}}y^{\frac{1}{2}}$. The area of the differential element parallel to $x$ axis is $dA = x\,dy = \left(\frac{b}{\sqrt{h}}y^{\frac{1}{2}}\right)dy$.

***Moment of Inertia :*** Applying Eq. 10 – 1 and performing the integration, we have

$$I_x = \int_A y^2 dA = \int_0^h y^2\left(\frac{b}{\sqrt{h}}y^{\frac{1}{2}}\right)dy$$
$$= \frac{b}{\sqrt{h}}\left(\frac{2}{7}y^{\frac{7}{2}}\right)\Big|_0^h$$
$$= \frac{2}{7}bh^3 \qquad \textbf{Ans}$$



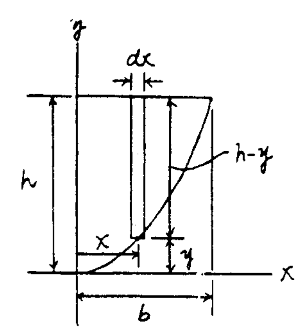

**10-10.** Determine the moment of inertia of the shaded area about the $y$ axis.



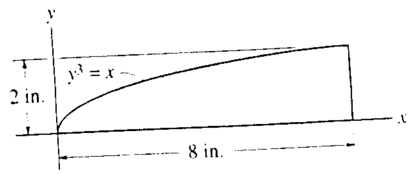

***Differential Element :*** The area of the differential element parallel to $y$ axis is $dA = (h - y)\,dx = \left(h - \frac{h}{b^2}x^2\right)dx$.

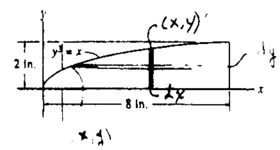
***Moment of Inertia :*** Applying Eq. 10 – 1 and performing the integration, we have

$$I_y = \int_A x^2 dA = \int_0^b x^2\left(h - \frac{h}{b^2}x^2\right)dx$$
$$= \left(\frac{h}{3}x^3 - \frac{h}{5b^2}x^5\right)\Big|_0^b$$
$$= \frac{2}{15}hb^3 \qquad \textbf{Ans}$$



**10-11.** Determine the moment of inertia of the shaded area about the $x$ axis.




$$dI_x = d\bar{I}_x + dA\,\tilde{y}^2$$

$$= \frac{1}{12}\,dx\,y^3 + y\,dx\left(\frac{y}{2}\right)^2$$

$$= \frac{1}{3}y^3\,dx$$

$$I_x = \int dI_x$$

$$= \int_0^8 \frac{1}{3}y^3\,dx$$

$$= \int_0^8 \frac{1}{3}x\,dx$$

$$= \left[\frac{x^2}{6}\right]_0^8 = 10.7\text{ in}^4 \qquad \textbf{Ans}$$

Also,

$$I_x = \int y^2 dA$$

$$= \int_0^2 y^2(8-y^3)\,dy$$

$$= \left[\frac{8y^3}{3} - \frac{y^6}{6}\right]_0^2$$

$$= 10.7\text{ in}^4 \qquad \textbf{Ans}$$

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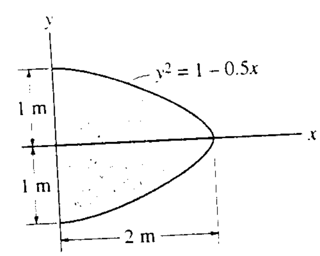
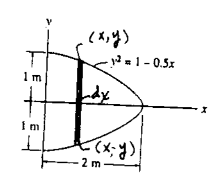
***10-12.** Determine the moment of inertia of the shaded area about the $x$ axis.




$$dI_x = \frac{1}{12}\,dx\,(2y)^3$$

$$I_x = \int dI_x$$

$$= \int_0^2 \frac{2}{3}(1-0.5x)^{3/2}\,dx$$

$$= \frac{2}{3}\left[\frac{2}{5(-0.5)}(1-0.5x)^{5/2}\right]_0^2$$

$$= 0.533\text{ m}^4 \qquad \textbf{Ans}$$

Also,

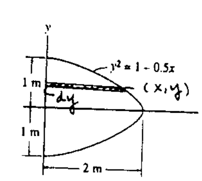
$$dA = x\,dy = 2(1-y^2)\,dy$$

$$I_x = \int y^2\,dA$$

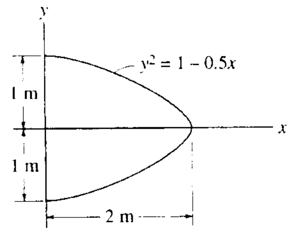
$$= \int_{-1}^{1} 2y^2(1-y^2)\,dy$$

$$= 2\left[\frac{y^3}{3} - \frac{y^5}{5}\right]_{-1}^{1}$$

$$= 0.533\text{ m}^4 \qquad \textbf{Ans}$$



**10-13.** Determine the moment of inertia of the shaded area about the $y$ axis.



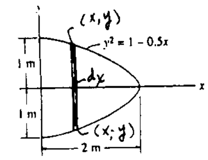

$dA = 2y\,dx$

$I_y = \int x^2\,dA$

$= \int_0^2 2x^2(1-0.5x)^{1/2}\,dx$

$= 2\left[\dfrac{2(8+12(-0.5)x + 15(-0.5)^2x^2)\sqrt{(1-0.5x)^3}}{105(-0.5)^3}\right]_0^2$

$= 2.44\ \text{m}^4$ **Ans**




**Also,**

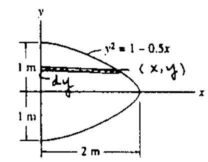
$I_y = \int dI_y$

$= 2\int_0^1 \frac{1}{3}x^3\,dy$

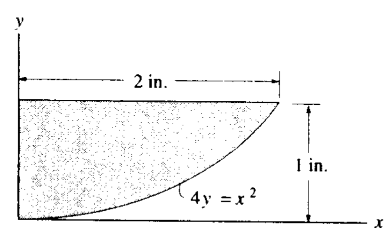
$= 2\int_0^1 \frac{8}{3}(1-y^2)^3\,dy$

$= 2\left(\frac{8}{3}\right)\left[y - y^3 + \frac{3}{5}y^5 - \frac{1}{7}y^7\right]_0^1$

$= 2.44\ \text{m}^4$ **Ans**



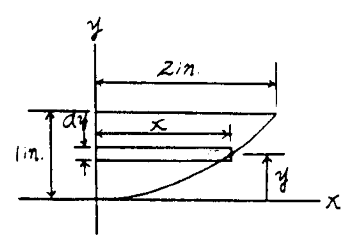

**10-14.** Determine the moment of inertia of the shaded area about the $x$ axis.



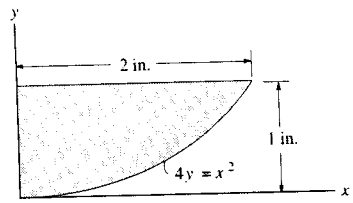

***Differential Element :*** Here, $x = 2y^{\frac{1}{2}}$. The area of the differential element parallel to $x$axis is $dA = xdy = \left(2y^{\frac{1}{2}}\right)dy$.

***Moment of Inertia :*** Applying Eq. 10 – 1 and performing the integration, we have

$$I_x = \int_A y^2 dA = \int_0^{1\,\text{in.}} y^2\left(2y^{\frac{1}{2}}\right)dy$$
$$= \left(\frac{4}{7}y^{\frac{7}{2}}\right)\Big|_0^{1\,\text{in.}}$$
$$= 0.571\ \text{in}^4 \qquad \textbf{Ans}$$



**10-15.** Determine the moment of inertia of the shaded area about the $y$ axis.



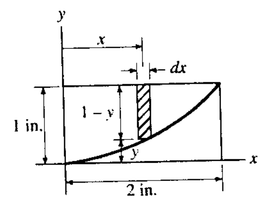

***Differential Element*:** The area of the differential element parallel to $y$ axis is $dA = (1-y)dx = \left(1 - \frac{1}{4}x^2\right)dx$.

***Moment of Inertia*:** Applying Eq. 10-1 and performing the integration, we have

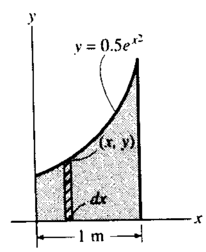
$$I_y = \int_A x^2\,dA = \int_0^{2\text{ in.}} x^2\left(1 - \frac{1}{4}x^2\right)dx$$

$$= \left(\frac{1}{3}x^3 - \frac{1}{20}x^5\right)\Big|_0^{2\text{ in.}}$$

$$= 1.07\ \text{in}^4 \qquad \textbf{Ans}$$



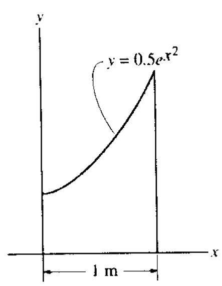

***■10-16.** Determine the moment of inertia of the area about the $y$ axis. Use Simpson's rule to evaluate the integral.




$$I_y = \int x^2\,dA$$

$$= \int_0^1 x^2(0.5e^{x^2}\,dx)$$

$$= 0.314\ \text{m}^4 \qquad \textbf{Ans}$$



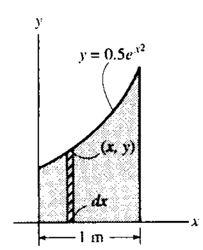

**■10-17.** Determine the moment of inertia of the area about the $x$ axis. Use Simpson's rule to evaluate the integral.

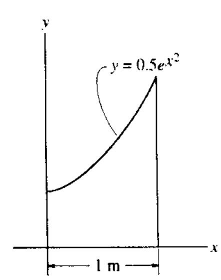
$$dI_x = \frac{1}{3}dx\,y^3$$

$$I_x = \int_0^1 \frac{1}{3}(0.5e^{x^2})^3\,dx$$

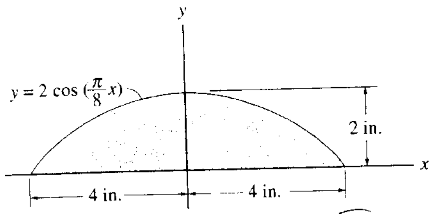
$$= \frac{1}{24}\int_0^1 (e^{x^2})^3\,dx$$

$$= 0.176\ \text{m}^4 \qquad \textbf{Ans}$$



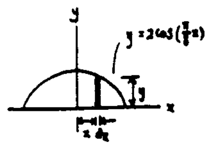



**10-18.** Determine the moment of inertia of the shaded area about the $x$ axis.




$$dI_x = dI_{\bar{x}} + dA\,\tilde{y}^2$$

$$= \frac{1}{12}dx\,y^3 + y\,dx\left(\frac{y}{2}\right)^2 = \frac{1}{3}y^3\,dx$$




$$I_x = \int_A dI_x = \int_{-4}^{4}\frac{8}{3}\cos^3\left(\frac{\pi}{8}x\right)dx$$

$$= \frac{8}{3}\left[\frac{\sin\left(\frac{\pi}{8}x\right)}{\frac{\pi}{8}} - \frac{\sin^3\left(\frac{\pi}{8}x\right)}{\frac{3\pi}{8}}\right]_{-4}^{4} = \frac{256}{9\pi} = 9.05\ \text{in}^4 \quad \textbf{Ans}$$

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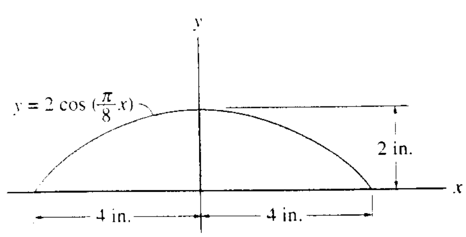
**10-19.** Determine the moment of inertia of the shaded area about the $y$ axis.

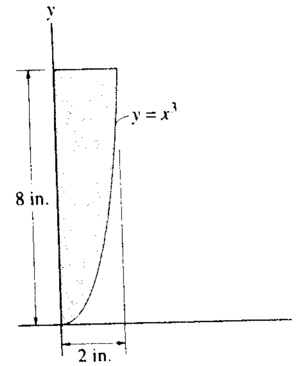



$$I_y = \int_A x^2\,dA = \int_{-4}^{4} x^2\,2\cos\left(\frac{\pi}{8}x\right)dx$$

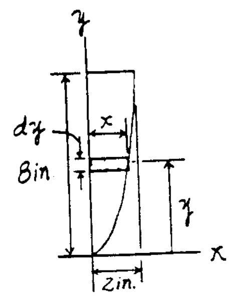
$$= 2\left[\frac{x^2\sin\left(\frac{\pi}{8}x\right)}{\frac{\pi}{8}} + \frac{2x\cos\left(\frac{\pi}{8}x\right)}{\left(\frac{\pi}{8}\right)^2} - \frac{2\sin\left(\frac{\pi}{8}x\right)}{\left(\frac{\pi}{8}\right)^3}\right]_{-4}^{4}$$

$$= 4\left(\frac{128}{\pi} - \frac{1024}{\pi^3}\right) = 30.9\ \text{in}^4 \quad \textbf{Ans}$$

***10-20.** Determine the moment of inertia of the shaded area about the $x$ axis.




***Differential Element :*** Here, $x = y^{\frac{1}{3}}$. The area of the differential element parallel to $x$axis is $dA = xdy = y^{\frac{1}{3}}dy$ .

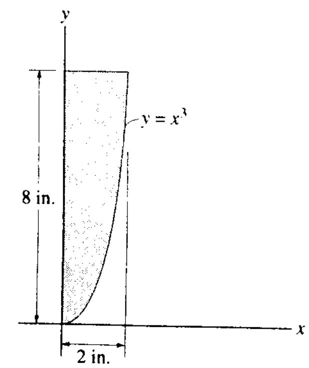



***Moment of Inertia :*** Applying Eq. 10 – 1and performing the integration, we have

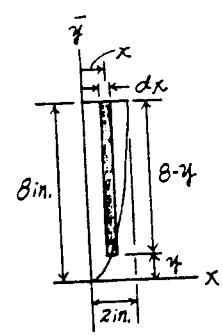
$$I_x = \int_A y^2 dA = \int_0^{8\text{ in.}} y^2\left(y^{\frac{1}{3}}\right)dy$$
$$= \left[\frac{3}{10}y^{\frac{10}{3}}\right]_0^{8\text{ in.}}$$
$$= 307 \text{ in}^4$$

**Ans**

**10-21.** Determine the moment of inertia of the shaded area about the $y$ axis.




***Differential Element :*** The area of the differential element parallel to $y$axis is $dA = (8-y)\,dx = \left(8-x^3\right)dx$ .

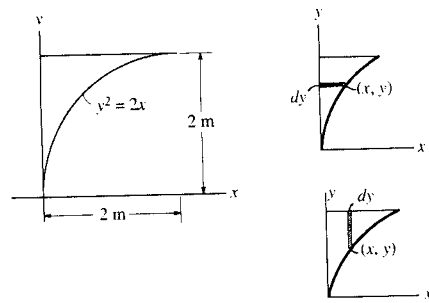



***Moment of Inertia :*** Applying Eq. 10 – 1 and performing the integration, we have

$$I_y = \int_A x^2 dA = \int_0^{2\text{ in.}} x^2\left(8-x^3\right)dx$$
$$= \left(\frac{8}{3}x^3 - \frac{1}{6}x^6\right)\Big|_0^{2\text{ in.}}$$
$$= 10.7 \text{ in}^4$$

**Ans**

**10-22.** Determine the moment of inertia of the shaded area about the $x$ axis.




$$dA = x\,dy = \frac{y^2}{2}\,dy$$

$$I_x = \int y^2\,dA$$

$$= \int_0^2 \frac{y^4}{2}\,dy$$

$$= \left[\frac{y^5}{10}\right]_0^2$$

$$= 3.20\ \text{m}^4 \qquad \textbf{Ans}$$

Also,

$$dA = (2 - \sqrt{2x})dx$$

$$dI_x = dI_{\bar{x}} + dA\,\bar{y}^2$$

$$= \frac{1}{12}dx(2 - \sqrt{2x})^3 + (2 - \sqrt{2x})dx\left(\frac{2 - \sqrt{2x}}{2} + \sqrt{2x}\right)^2$$

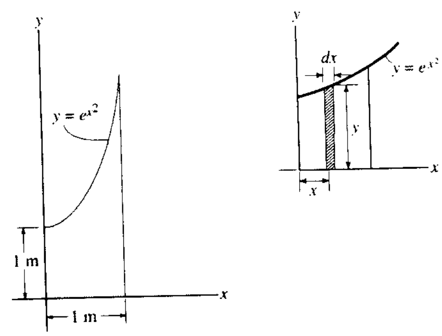
$$= \frac{1}{12}(2 - \sqrt{2x})^3\,dx + \frac{1}{4}(2 - \sqrt{2x})(2 + \sqrt{2x})^2\,dx$$

$$I_x = \int dI_x$$

$$= \int_0^2 \left[\frac{1}{12}(2 - \sqrt{2x})^3 + \frac{1}{4}(2 - \sqrt{2x})(2 + \sqrt{2x})^2\right]dx$$

$$= 3.20\ \text{m}^4 \qquad \textbf{Ans}$$

■**10-23.** Determine the moment of inertia of the shaded area about the $y$ axis. Use Simpson's rule to evaluate the integral.



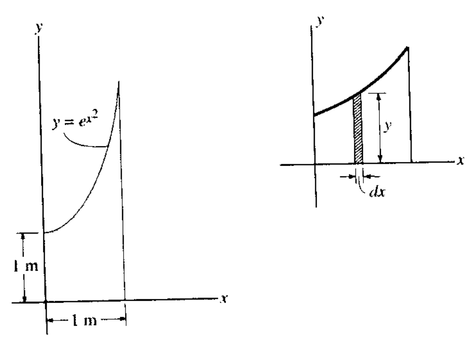

Area of the differential element (shaded) $dA = y dx$ where $y = e^{x^2}$, hence, $dA = y dx = e^{x^2}\,dx$.

$$I_y = \int_A x^2\,dA = \int_0^1 x^2(e^{x^2})dx$$

Use Simpson's rule to evaluate the integral: (to 500 intervals)

$$I_y = 0.628\ \text{m}^4 \qquad \textbf{Ans}$$

■***10-24.** Determine the moment of inertia of the shaded area about the $x$ axis. Use Simpson's rule to evaluate the integral.

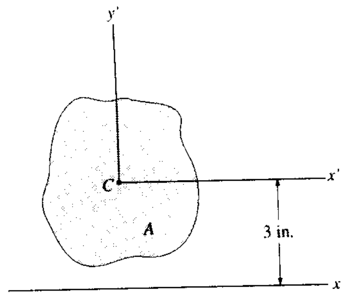



$$dI_x = dI_{\bar{x}} + dA\bar{y}^2$$

$$= \frac{1}{12}dx\,y^3 + y\,dx\left(\frac{y}{2}\right)^2 = \frac{1}{3}y^3\,dx$$

$$I_x = \frac{1}{3}\int_0^1 y^3\,dx = \frac{1}{3}\int_0^1 (e^{x^2})^3\,dx = 1.41\ \text{m}^4 \qquad \textbf{Ans}$$

**10-25.** The polar moment of inertia of the area is $\bar{J}_C = 23\ \text{in}^4$ about the $z$ axis passing through the centroid $C$. If the moment of inertia about the $y'$ axis is $5\ \text{in}^4$, and the moment of inertia about the $x$ axis is $40\ \text{in}^4$, determine the area $A$.



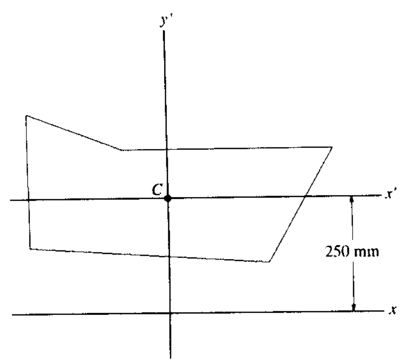

***Moment of Inertia:*** The polar of moment inertia $\bar{J}_C = \bar{I}_{x'} + \bar{I}_{y'}$. Then, $\bar{I}_{x'} = \bar{J}_C - \bar{I}_{y'} = 23 - 5 = 18.0\ \text{in}^4$. Applying the parallel-axis theorem, Eq. 10-3, we have

$$I_x = \bar{I}_{x'} + Ad_y^2$$

$$40 = 18.0 + A(3^2)$$

$$A = 2.44\ \text{in}^2 \qquad \textbf{Ans}$$

**10-26.** The polar moment of inertia of the area is $\bar{J}_C = 548(10^6)\ \text{mm}^4$, about the $z'$ axis passing through the centroid $C$. The moment of inertia about the $y'$ axis is $383(10^6)\ \text{mm}^4$, and the moment of inertia about the $x$ axis is $856(10^6)\ \text{mm}^4$. Determine the area $A$.



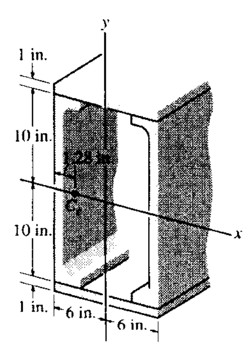

$$I_{x'} = \bar{I}_{x'} + Ad^2 = 856(10^6) - A(250)^2$$

$$\bar{J}_C = \bar{I}_{x'} + \bar{I}_{y'}$$

$$548(10^6) = 856(10^6) - A(250)^2 + 383(10^6)$$

$$A = 11.1(10^3)\text{mm}^2 \qquad \textbf{Ans}$$

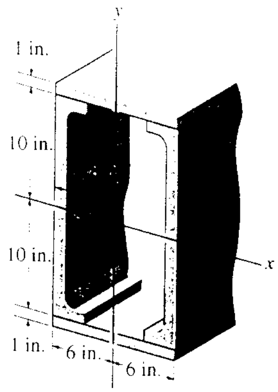
**10-27.** The beam is constructed from the two channels and two cover plates. If each channel has a cross-sectional area of $A_c = 11.8\ \text{in}^2$ and a moment of inertia about a horizontal axis passing through its own centroid, $C_c$, of $(\bar{I}_{\bar{x}})_{C_c} = 349\ \text{in}^4$, determine the moment of inertia of the beam about the $x$ axis.




$$I_x = 2\left[\frac{1}{12}(12)(1)^3 + (1)(12)(10.5)^2\right] + 2(349)$$

$$= 3.35(10^3)\ \text{in}^4 \qquad \textbf{Ans}$$

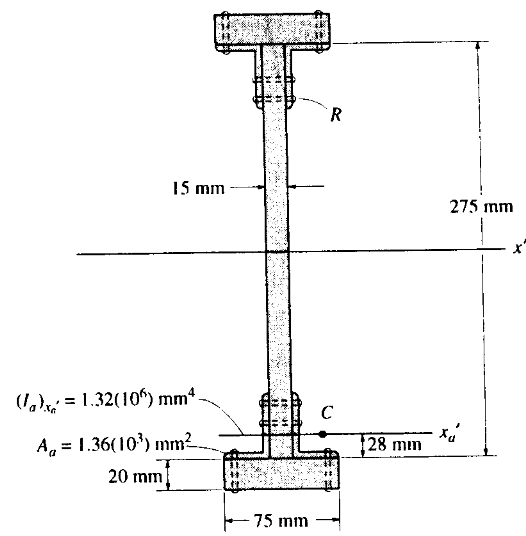
*10-28. The beam is constructed from the two channels and two cover plates. If each channel has a cross-sectional area of $A_c = 11.8\text{ in}^2$ and a moment of inertia about a vertical axis passing through its own centroid, $C_c$, of $(\bar{I}_y)_{C_c} = 9.23\text{ in}^4$, determine the moment of inertia of the beam about the $y$ axis.




$$I_y = 2\left[\frac{1}{12}(1)(12)^3\right] + 2[(9.23) + 11.8(6-1.28)^2]$$

$$= 832\text{ in}^4 \qquad \textbf{Ans}$$

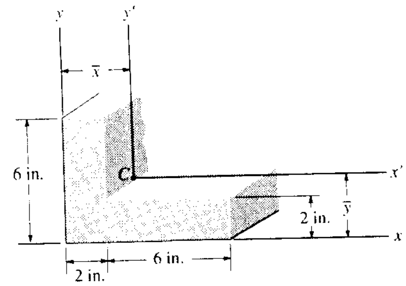
10-29. Determine the moment of inertia of the beam's cross-sectional area with respect to the $x'$ centroidal axis. Neglect the size of all the rivet heads, $R$, for the calculation. Handbook values for the area, moment of inertia, and location of the centroid $C$ of one of the angles are listed in the figure.




$$I_{x'} = \frac{1}{12}(15)(275)^3 + 4\left[1.32(10^6) + 1.36(10^3)\left(\frac{275}{2} - 28\right)^2\right]$$

$$+ 2\left[\frac{1}{12}(75)(20)^3 + (75)(20)\left(\frac{275}{2} + 10\right)^2\right] = 162(10^6)\text{ mm}^4 \qquad \textbf{Ans}$$

**10-30.** Locate the centroid $\bar{y}$ of the cross-sectional area for the angle. Then find the moment of inertia $\bar{I}_{x'}$ about the $x'$ centroidal axis.

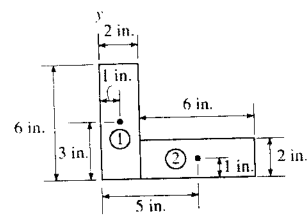



*Centroid:* The area of each segment and its respective centroid are tabulated below.

| Segment | $A(\text{in}^2)$ | $\bar{y}(\text{in.})$ | $\bar{y}A(\text{in}^3)$ |
|---|---|---|---|
| 1 | 6(2) | 3 | 36.0 |
| 2 | 6(2) | 1 | 12.0 |
| Σ | 24.0 | | 48.0 |

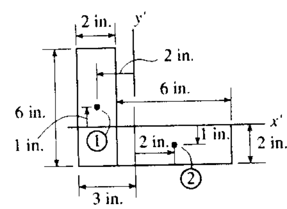
Thus,

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{48.0}{24.0} = 2.00 \text{ in.} \qquad \textbf{Ans}$$




*Moment of Inertia:* The moment of inertia about the $x'$ axis for each segment can be determined using the parallel - axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

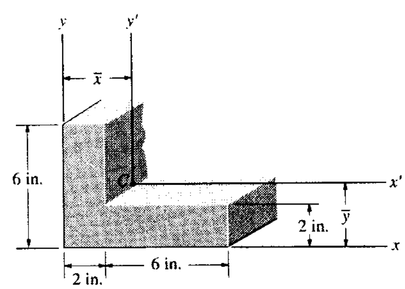
| Segment | $A_i(\text{in}^2)$ | $(d_y)_i(\text{in.})$ | $(\bar{I}_{x'})_i(\text{in}^4)$ | $(Ad_y^2)_i(\text{in}^4)$ | $(I_{x'})_i(\text{in}^4)$ |
|---|---|---|---|---|---|
| 1 | 2(6) | 1 | $\frac{1}{12}(2)(6^3)$ | 12.0 | 48.0 |
| 2 | 6(2) | 1 | $\frac{1}{12}(6)(2^3)$ | 12.0 | 16.0 |




Thus,

$$\bar{I}_{x'} = \Sigma(I_{x'})_i = 64.0 \text{ in}^4 \qquad \textbf{Ans}$$

**10-31.** Locate the centroid $\bar{x}$ of the cross-sectional area for the angle. Then find the moment of inertia $\bar{I}_{y'}$ about the $y'$ centroidal axis.




*Centroid:* The area of each segment and its respective centroid are tabulated below.

| Segment | $A(\text{in}^2)$ | $\bar{x}(\text{in.})$ | $\bar{x}A(\text{in}^3)$ |
|---|---|---|---|
| 1 | 6(2) | 1 | 12.0 |
| 2 | 6(2) | 5 | 60.0 |
| Σ | 24.0 | | 72.0 |

Thus,

$$\bar{x} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{72.0}{24.0} = 3.00 \text{ in.} \qquad \textbf{Ans}$$

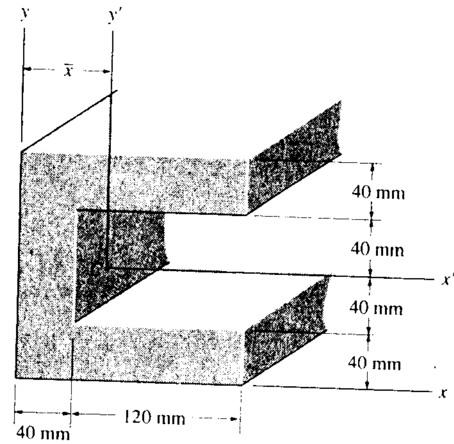
*Moment of Inertia:* The moment of inertia about the $y'$ axis for each segment can be determined using the parallel - axis theorem $I_{y'} = \bar{I}_{y'} + Ad_x^2$.

| Segment | $A_i(\text{in}^2)$ | $(d_x)_i(\text{in.})$ | $(\bar{I}_{y'})_i(\text{in}^4)$ | $(Ad_x^2)_i(\text{in}^4)$ | $(I_{y'})_i(\text{in}^4)$ |
|---|---|---|---|---|---|
| 1 | 6(2) | 2 | $\frac{1}{12}(6)(2^3)$ | 48.0 | 52.0 |
| 2 | 2(6) | 2 | $\frac{1}{12}(2)(6^3)$ | 48.0 | 84.0 |

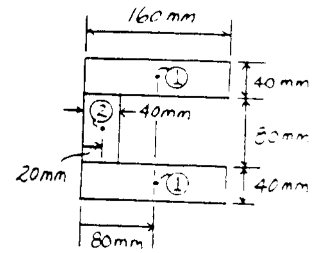
Thus,

$$\bar{I}_{y'} = \Sigma(I_{y'})_i = 136 \text{ in}^4 \qquad \textbf{Ans}$$

***10-32.** Determine the distance $\bar{x}$ to the centroid of the beam's cross-sectional area; then find the moment of inertia about the $y'$ axis.




***Centroid :*** The area of each segment and its respective centroid are tabulated below.

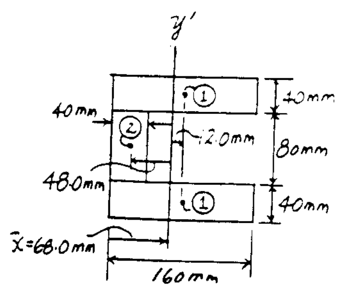



| Segment | $A$ (mm²) | $\tilde{x}$ (mm) | $\tilde{x}A$ (mm³) |
|---|---|---|---|
| 1 | 160(80) | 80 | 1.024(10⁶) |
| 2 | 40(80) | 20 | 64.0(10³) |
| Σ | 16.0(10³) | | 1.088(10⁶) |

Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{1.088(10^6)}{16.0(10^3)} = 68.0\text{ mm} \quad \textbf{Ans}$$

***Moment of Inertia :*** The moment of inertia about the $y'$ axis for each segment can be determined using the parallel − axis theorem $I_{y'} = \bar{I}_{y'} + Ad_x^2$.

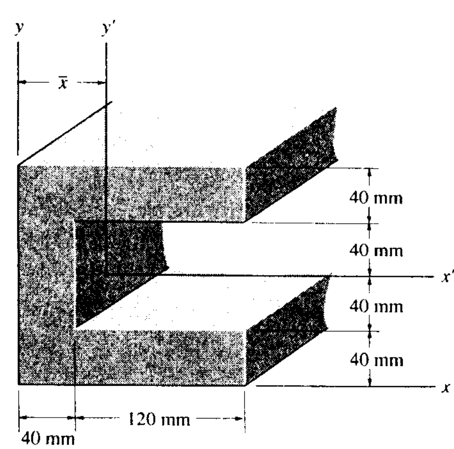



| Segment | $A_i$ (mm²) | $(d_x)_i$ (mm) | $(\bar{I}_{y'})_i$ (mm⁴) | $(Ad_x^2)_i$ (mm⁴) | $(I_{y'})_i$ (mm⁴) |
|---|---|---|---|---|---|
| 1 | 80(160) | 12.0 | $\frac{1}{12}(80)(160^3)$ | 1.8432(10⁶) | 29.150(10⁶) |
| 2 | 80(40) | 48.0 | $\frac{1}{12}(80)(40^3)$ | 7.3728(10⁶) | 7.799(10⁶) |

Thus,

$$I_{y'} = \Sigma(I_{y'})_i = 36.949(10^6)\text{ mm}^4 = 36.9(10^6)\text{ mm}^4 \quad \textbf{Ans}$$

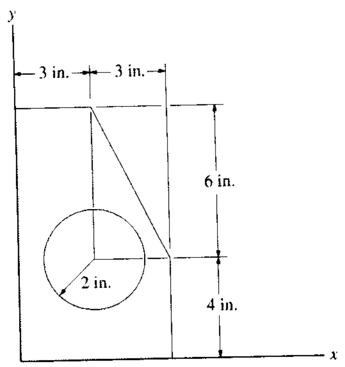
**10-33.** Determine the moment of inertia of the beam's cross-sectional area about the $x'$ axis.




***Moment of Inertia :*** The moment inertia for the rectangle about itscentroidal axis can be determined using the formula, $I_{x'} = \frac{1}{12}bh^3$, given on the inside back cover of the textbook.

$$I_{x'} = \frac{1}{12}(160)(160^3) - \frac{1}{12}(120)(80^3) = 49.5(10^6)\text{ mm}^4 \quad \textbf{Ans}$$

**10-34.** Determine the moments of inertia of the shaded area about the $x$ and $y$ axes.



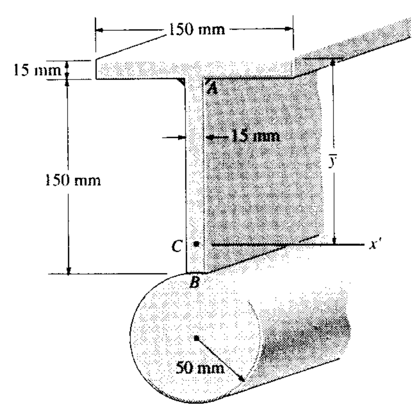

$$I_x = \left[\frac{1}{12}(6)(10)^3 + 6(10)(5)^2\right] - \left[\frac{1}{36}(3)(6)^3 + \left(\frac{1}{2}\right)(3)(6)(8)^2\right]$$

$$- \left[\frac{1}{4}\pi(2)^4 + \pi(2)^2(4)^2\right] = 1.192(10^3) \qquad \textbf{Ans}$$

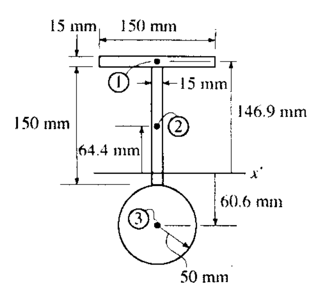
$$I_y = \left[\frac{1}{12}(10)(6)^3 + 6(10)(3)^2\right] - \left[\frac{1}{36}(6)(3)^3 + \left(\frac{1}{2}\right)(6)(3)(5)^2\right]$$

$$- \left[\frac{1}{4}\pi(2)^4 + \pi(2)^2(3)^2\right] = 364.8 \text{ in}^4 \qquad \textbf{Ans}$$

**10-35.** Determine the moment of inertia of the beam's cross-sectional area about the $x'$ axis. Neglect the size of the corner welds at $A$ and $B$ for the calculation, $\bar{y} = 154.4$ mm.




***Moment of Inertia:*** The moment of inertia about the $x'$ axis for each segment can be determined using the parallel - axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

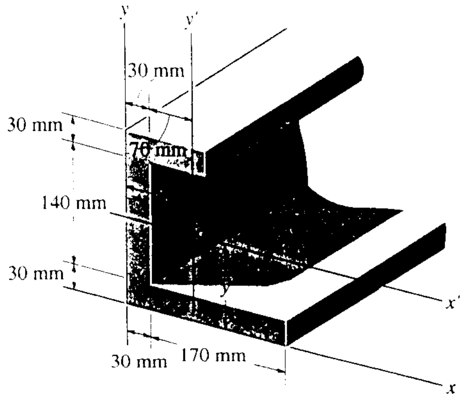



| Segment | $A_i$(mm²) | $(d_y)_i$(mm) | $(\bar{I}_{x'})_i$(mm⁴) | $(Ad_y^2)_i$(mm⁴) | $(I_{x'})_i$(mm⁴) |
|---|---|---|---|---|---|
| 1 | 150(15) | 146.9 | $\frac{1}{12}(150)(15^3)$ | 48.554(10⁶) | 48.596(10⁶) |
| 2 | 15(150) | 64.4 | $\frac{1}{12}(15)(150^3)$ | 9.332(10⁶) | 13.550(10⁶) |
| 3 | $\pi(50^2)$ | 60.6 | $\frac{\pi}{4}(50^4)$ | 28.843(10⁶) | 33.751(10⁶) |

Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 95.898(10^6)\text{ mm}^4 = 95.9(10^6)\text{ mm}^4 \qquad \textbf{Ans}$$

*10-36. Compute the moments of inertia $I_x$ and $I_y$ for the beam's cross-sectional area about the $x$ and $y$ axes.



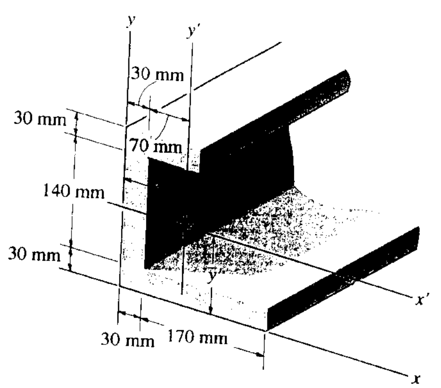

$$I_x = \frac{1}{12}(170)(30)^3 + 170(30)(15)^2 + \frac{1}{12}(30)(170)^3 + 30(170)(85)^2 + \frac{1}{12}(100)(30)^3 + 100(30)(185)^2$$

$I_x = 154(10^6)\ \text{mm}^4$ **Ans**

$$I_y = \frac{1}{12}(30)(170)^3 + 30(170)(115)^2 + \frac{1}{12}(170)(30)^3 + 30(170)(15)^2 + \frac{1}{12}(30)(100)^3 + 30(100)(50)^2$$

$I_y = 91.3(10^6)\ \text{mm}^4$ **Ans**

10-37. Determine the distance $\bar{y}$ to the centroid $C$ of the beam's cross-sectional area and then compute the moment of inertia $\bar{I}_{x'}$ about the $x'$ axis.



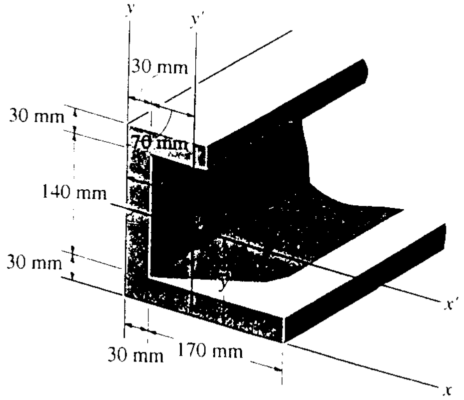

$$\bar{y} = \frac{170(30)(15) + 170(30)(85) + 100(30)(185)}{170(30) + 170(30) + 100(30)}$$

$= 80.68 = 80.7$ mm **Ans**

$$\bar{I}_{x'} = \left[\frac{1}{12}(170)(30)^3 + 170(30)(80.68 - 15)^2\right] + \left[\frac{1}{12}(30)(170)^3 + 30(170)(85 - 80.68)^2\right] + \frac{1}{12}(100)(30)^3 + 100(30)(185 - 80.68)^2$$

$\bar{I}_{x'} = 67.6(10^6)\ \text{mm}^4$ **Ans**

10-38. Determine the distance $\bar{x}$ to the centroid $C$ of the beam's cross-sectional area and then compute the moment of inertia $\bar{I}_{y'}$ about the $y'$ axis.



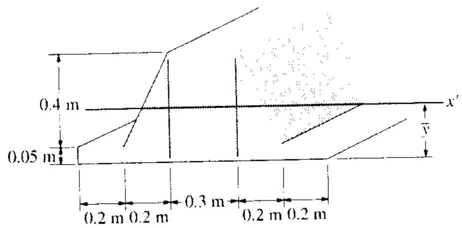

$$\bar{x} = \frac{170(30)(115) + 170(30)(15) + 100(30)(50)}{170(30) + 170(30) + 100(30)}$$

$= 61.59 = 61.6$ mm **Ans**

$$\bar{I}_{y'} = \left[\frac{1}{12}(30)(170)^3 + 170(30)(115 - 61.59)^2\right] + \left[\frac{1}{12}(170)(30)^3 + 30(170)(15 - 61.59)^2\right] + \frac{1}{12}(30)(100)^3 + 100(30)(50 - 61.59)^2$$

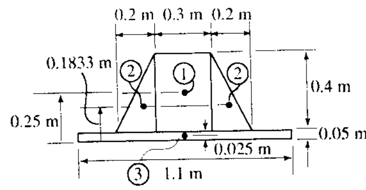
$\bar{I}_{y'} = 41.2(10^6)\ \text{mm}^4$ **Ans**

**10-39.** Locate the centroid $\bar{y}$ of the cross section and determine the moment of inertia of the section about the $x'$ axis.




***Centroid:*** The area of each segment and its respective centroid are tabulated below.

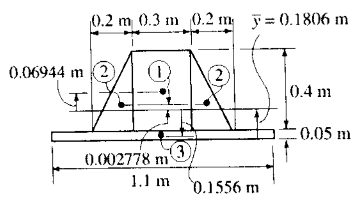
| Segment | $A(\text{m}^2)$ | $\bar{y}(\text{m})$ | $\bar{y}A(\text{m}^3)$ |
|---|---|---|---|
| 1 | 0.3(0.4) | 0.25 | 0.03 |
| 2 | $\frac{1}{2}(0.4)(0.4)$ | 0.1833 | 0.014667 |
| 3 | 1.1(0.05) | 0.025 | 0.001375 |
| $\Sigma$ | 0.255 | | 0.046042 |




Thus,

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.046042}{0.255} = 0.1806 \text{ m} = 0.181 \text{ m} \qquad \textbf{Ans}$$

***Moment of Inertia:*** The moment of inertia about the $x'$ axis for each segment can be determined using the parallel - axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

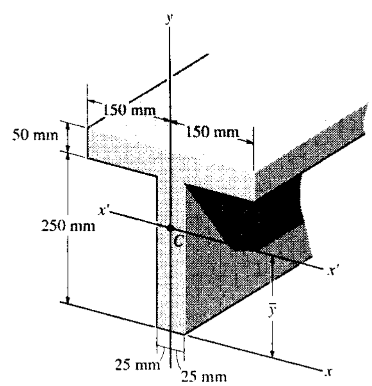



| Segment | $A_i(\text{m}^2)$ | $(d_y)_i(\text{m})$ | $(\bar{I}_{x'})_i(\text{m}^4)$ | $(Ad_y^2)_i(\text{m}^4)$ | $(I_{x'})_i(\text{m}^4)$ |
|---|---|---|---|---|---|
| 1 | 0.3(0.4) | 0.06944 | $\frac{1}{12}(0.3)(0.4^3)$ | $0.5787(10^{-3})$ | $2.1787(10^{-3})$ |
| 2 | $\frac{1}{2}(0.4)(0.4)$ | 0.002778 | $\frac{1}{36}(0.4)(0.4^3)$ | $0.6173(10^{-6})$ | $0.7117(10^{-3})$ |
| 3 | 1.1(0.05) | 0.1556 | $\frac{1}{12}(1.1)(0.05^3)$ | $1.3309(10^{-3})$ | $1.3423(10^{-3})$ |

Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 4.233(10^{-3}) \text{ m}^4 = 4.23(10^{-3}) \text{ m}^4 \qquad \textbf{Ans}$$

***10-40.** Determine $\bar{y}$, which locates the centroidal axis $x'$ for the cross-sectional area of the T-beam, and then find the moments of inertia $\bar{I}_{x'}$ and $\bar{I}_{y'}$.




$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{125(250)(50) + (275)(50)(300)}{250(50) + 50(300)}$$

$$= 206.818 \text{ mm}$$

$$\bar{y} = 207 \text{ mm} \qquad \textbf{Ans}$$

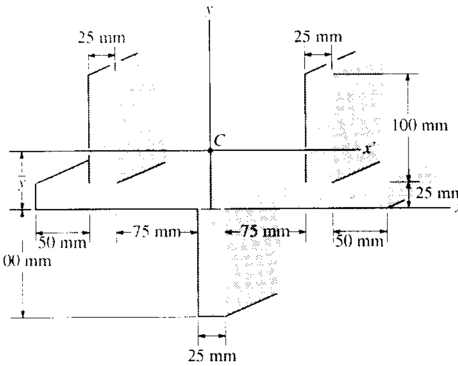
$$\bar{I}_{x'} = \left[\frac{1}{12}(50)(250)^3 + 50(250)(206.818 - 125)^2\right]$$

$$+ \left[\frac{1}{12}(300)(50)^3 + 50(300)(275 - 206.818)^2\right]$$

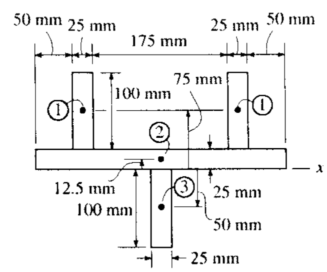
$$\bar{I}_{x'} = 222(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

$$I_y = \frac{1}{12}(250)(50)^3 + \frac{1}{12}(50)(300)^3 = 115(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

**10-41.** Determine the distance $\bar{y}$ to the centroid for the beam's cross-sectional area; then determine the moment of inertia about the $x'$ axis.




*Centroid:* The area of each segment and its respective centroid are tabulated below.




| Segment | $A(mm^2)$ | $\bar{y}$(mm) | $\bar{y}A(mm^3)$ |
|---|---|---|---|
| 1 | 50(100) | 75 | $375(10^3)$ |
| 2 | 325(25) | 12.5 | $101.5625(10^3)$ |
| 3 | 25(100) | −50 | $-125(10^3)$ |
| Σ | $15.625(10^3)$ | | $351.5625(10^3)$ |

Thus,

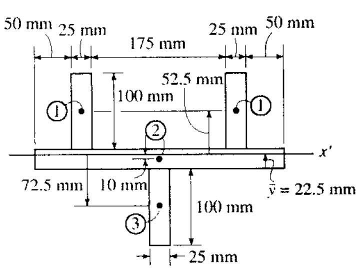
$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{351.5625(10^3)}{15.625(10^3)} = 22.5 \text{ mm} \quad \textbf{Ans}$$

*Moment of Inertia:* The moment of inertia about the $x'$ axis for each segment can be determined using the parallel - axis theorem $I_x = \bar{I}_{x'} + Ad_y^2$.

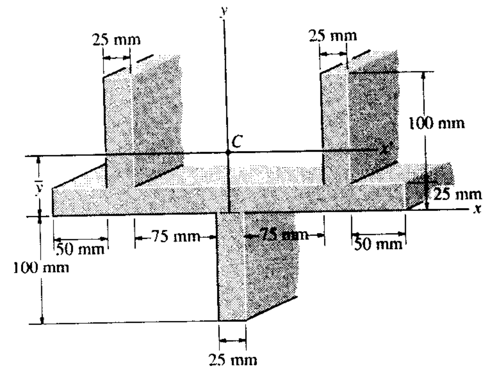
| Segment | $A_i(mm^2)$ | $(d_y)_i$(mm) | $(\bar{I}_{x'})_i(mm^4)$ | $(Ad_y^2)_i(mm^4)$ | $(I_{x'})_i(mm^4)$ |
|---|---|---|---|---|---|
| 1 | 50(100) | 52.5 | $\frac{1}{12}(50)(100^3)$ | $13.781(10^6)$ | $17.948(10^6)$ |
| 2 | 325(25) | 10 | $\frac{1}{12}(325)(25^3)$ | $0.8125(10^6)$ | $1.236(10^6)$ |
| 3 | 25(100) | 72.5 | $\frac{1}{12}(25)(100^3)$ | $13.141(10^6)$ | $15.224(10^6)$ |

Thus,

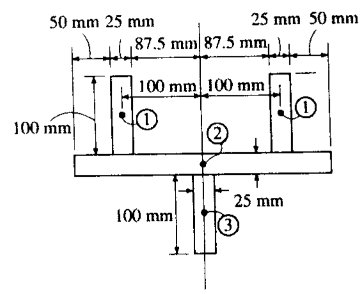
$$I_{x'} = \Sigma(I_{x'})_i = 34.41(10^6) \text{ mm}^4 = 34.4(10^6) \text{ mm}^4 \quad \textbf{Ans}$$



**10-42.** Determine the moment of inertia of the beam's cross-sectional area about the $y$ axis.




*Moment of Inertia:* The moment of inertia about the $y'$ axis for each segment can be determined using the parallel-axis theorem $I_{y'} = \bar{I}_{y'} + Ad_x^2$.

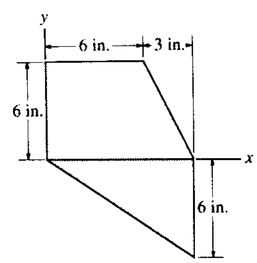



| Segment | $A_i\,(\text{mm}^2)$ | $(d_x)_i\,(\text{mm})$ | $(\bar{I}_{y'})_i\,(\text{mm}^4)$ | $(Ad_x^2)_i\,(\text{mm}^4)$ | $(I_{y'})_i\,(\text{mm}^4)$ |
|---|---|---|---|---|---|
| 1 | $2[100(25)]$ | 100 | $\frac{1}{12}(100)(25^3)$ | $50.0(10^6)$ | $50.130(10^6)$ |
| 2 | $25(325)$ | 0 | $\frac{1}{12}(25)(325^3)$ | 0 | $71.519(10^6)$ |
| 3 | $100(25)$ | 0 | $\frac{1}{12}(100)(25^3)$ | 0 | $0.130(10^6)$ |

Thus,

$$I_{y'} = \Sigma(I_{y'})_i = 121.78(10^6)\text{ mm}^4 = 122(10^6)\text{ mm}^4 \qquad \textbf{Ans}$$

**10-43.** Determine the moment of inertia $I_x$ of the shaded area about the $x$ axis.



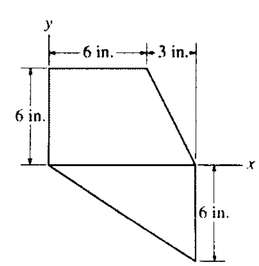

$$I_x = \left[\frac{1}{12}(6)(6)^3 + 6(6)(3)^2\right]$$

$$+\left[\frac{1}{36}(3)(6)^3 + \left(\frac{1}{2}\right)(3)(6)(2)^2\right]$$

$$+\left[\frac{1}{36}(9)(6)^3 + \frac{1}{2}(6)(9)(2)^2\right]$$

$$I_x = 648\text{ in}^4 \qquad \textbf{Ans}$$

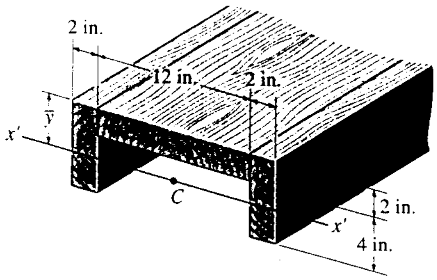
***10-44.** Determine the moment of inertia $I_y$ of the shaded area about the $y$ axis.




$$I_y = \left[\frac{1}{12}(6)(6)^3 + 6(6)(3)^2\right] + \left[\frac{1}{36}(6)(3)^3 + \frac{1}{2}(6)(3)(6+1)^2\right]$$

$$+\left[\frac{1}{36}(6)(9)^3 + \frac{1}{2}(6)(9)(6)^2\right] = 1971\text{ in}^4 \qquad \textbf{Ans}$$

**10-45.** Locate the centroid $\bar{y}$ of the channel's cross-sectional area, and then determine the moment of inertia with respect to the $x'$ axis passing through the centroid.



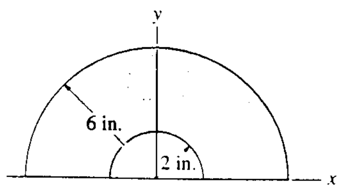

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{(1)(12)(2) + 2[(3)(6)(2)]}{12(2) + 2(6)(2)}$$

$$= 2 \text{ in.} \qquad \textbf{Ans}$$

$$\bar{I}_{x'} = \left[\frac{1}{12}(12)(2)^3 + 12(2)(1)^2\right] + 2\left[\frac{1}{12}(2)(6)^3 + 6(2)(3-2)^2\right]$$

$$\bar{I}_{x'} = 128 \text{ in}^4 \qquad \textbf{Ans}$$

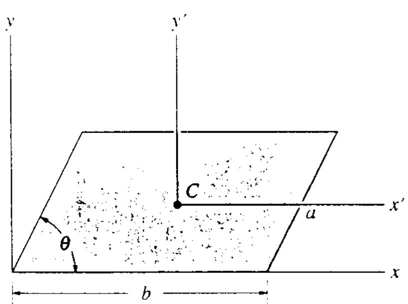
**10-46.** Determine the moments of inertia $I_x$ and $I_y$ of the shaded area.




$$I_x = I_y = \frac{\pi (6)^4}{8} - \frac{\pi (2)^4}{8}$$

$$= 503 \text{ in}^4 \qquad \textbf{Ans}$$

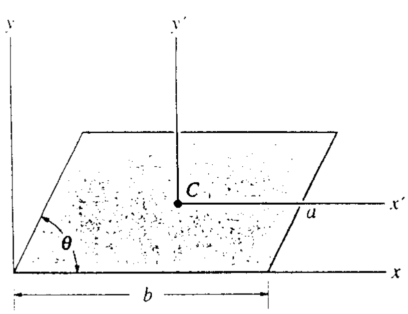
**10-47.** Determine the moment of inertia of the parallelogram about the $x'$ axis, which passes through the centroid $C$ of the area.




$$h = a\sin\theta$$

$$\bar{I}_{x'} = \tfrac{1}{12}bh^3 = \tfrac{1}{12}(b)(a\sin\theta)^3 = \tfrac{1}{12}a^3 b \sin^3\theta \qquad \textbf{Ans}$$

***10-48.** Determine the moment of inertia of the parallelogram about the $y'$ axis, which passes through the centroid $C$ of the area.



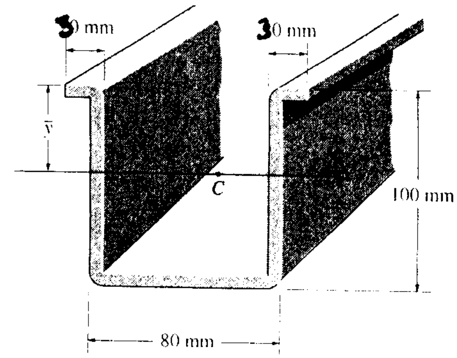

$$\bar{x} = a\cos\theta + \frac{b - a\cos\theta}{2} = \tfrac{1}{2}(a\cos\theta + b)$$

$$\bar{I}_{y'} = 2\left[\tfrac{1}{36}(a\sin\theta)(a\cos\theta)^3 + \tfrac{1}{2}(a\sin\theta)(a\cos\theta)\left(\tfrac{b}{2} + \tfrac{a}{2}\cos\theta - \tfrac{2}{3}a\cos\theta\right)^2\right]$$

$$+\tfrac{1}{12}(a\sin\theta)(b - a\cos\theta)^3$$

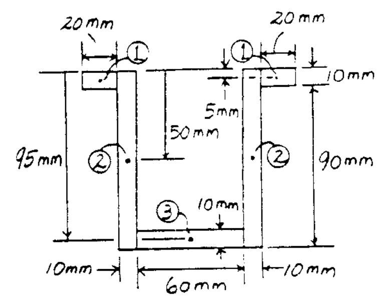
$$= \frac{ab\sin\theta}{12}\left(b^2 + a^2\cos^2\theta\right) \qquad \textbf{Ans}$$

**10-49.** An aluminum strut has a cross section referred to as a deep hat. Determine the location $\bar{y}$ of the centroid of its area and the moment of inertia of the area about the $x'$ axis. Each segment has a thickness of 10 mm.




***Centroid :*** The area of each segment and its respective centroid are tabulated below.

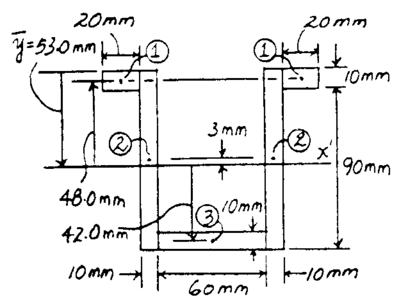
| Segment | $A\,(\mathrm{mm^2})$ | $\tilde{y}\,(\mathrm{mm})$ | $\tilde{y}A\,(\mathrm{mm^3})$ |
|---|---|---|---|
| 1 | 40(10) | 5 | $2.00(10^3)$ |
| 2 | 20(100) | 50 | $100.0(10^3)$ |
| 3 | 60(10) | 95 | $57.0(10^3)$ |
| $\Sigma$ | $3.00(10^3)$ | | $159.0(10^3)$ |




Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{159.0(10^3)}{3.00(10^3)} = 53.0 \text{ mm} \qquad \textbf{Ans}$$

***Moment of Inertia :*** The moment of inertia about the $x'$ axis for each segment can be determined usin the parallel − axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

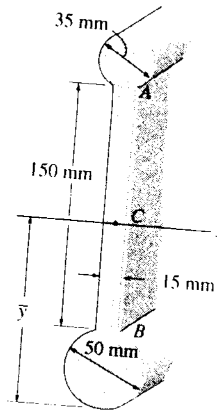



| Segment | $A_i\,(\mathrm{mm^2})$ | $(d_y)_i\,(\mathrm{mm})$ | $(\bar{I}_{x'})_i\,(\mathrm{mm^4})$ | $(Ad_y^2)_i\,(\mathrm{mm^4})$ | $(I_{x'})_i\,(\mathrm{mm^4})$ |
|---|---|---|---|---|---|
| 1 | 40(10) | 48.0 | $\frac{1}{12}(40)(10^3)$ | $0.9216(10^6)$ | $0.9249(10^6)$ |
| 2 | 20(100) | 3.00 | $\frac{1}{12}(20)(100^3)$ | $0.018(10^6)$ | $1.6847(10^6)$ |
| 3 | 60(10) | 42.0 | $\frac{1}{12}(60)(10^3)$ | $1.0584(10^6)$ | $1.0634(10^6)$ |

Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 3.673(10^6)\ \mathrm{mm^4} = 3.67(10^6)\ \mathrm{mm^4} \qquad \textbf{Ans}$$

**10-50.** Determine the moment of inertia of the beam's cross-sectional area with respect to the $x'$ axis passing through the centroid $C$ of the cross section. Neglect the size of the corner welds at $A$ and $B$ for the calculation, $\bar{y} = 104.3$ mm.

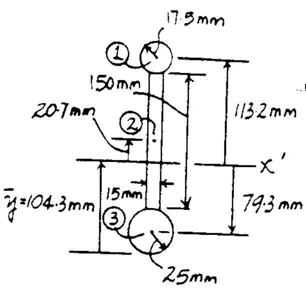



***Moment of Inertia :*** The moment of inertia about the $x'$ axis for each segment can be determined usin the parallel – axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

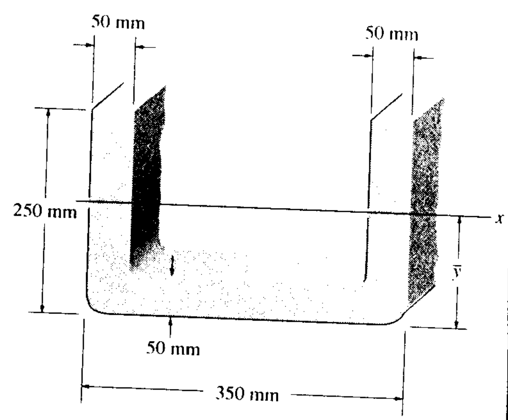
| Segment | $A_i$ (mm²) | $(d_y)_i$ (mm) | $(\bar{I}_{x'})_i$ (mm⁴) | $(Ad_y^2)_i$ (mm⁴) | $(I_{x'})_i$ (mm⁴) |
|---|---|---|---|---|---|
| 1 | $\pi(17.5^2)$ | 113.2 | $\frac{\pi}{4}(17.5^4)$ | $12.329(10^6)$ | $12.402(10^6)$ |
| 2 | $15(150)$ | 20.7 | $\frac{1}{12}(15)(150^3)$ | $0.964(10^6)$ | $5.183(10^6)$ |
| 3 | $\pi(25^2)$ | 79.3 | $\frac{\pi}{4}(25^4)$ | $12.347(10^6)$ | $12.654(10^6)$ |

Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 30.24(10^6)\ \text{mm}^4 = 30.2(10^6)\ \text{mm}^4 \quad \textbf{Ans}$$

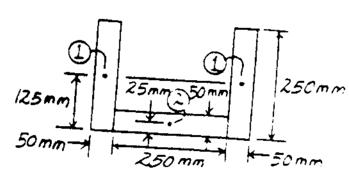



**10-51.** Determine the location $\bar{y}$ of the centroid of the channel's cross-sectional area and then calculate the moment of inertia of the area about this axis.




***Centroid :*** The area of each segment and its respective centroid are tabulated below.

| Segment | $A$ (mm²) | $\tilde{y}$ (mm) | $\tilde{y}A$ (mm³) |
|---|---|---|---|
| 1 | 100(250) | 125 | $3.125(10^6)$ |
| 2 | 250(50) | 25 | $0.3125(10^6)$ |
| $\Sigma$ | $37.5(10^3)$ | | $3.4375(10^6)$ |

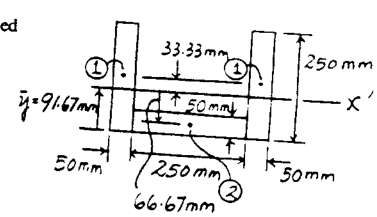



Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{3.4375(10^6)}{37.5(10^3)} = 91.67\ \text{mm} = 91.7\ \text{mm} \quad \textbf{Ans}$$

***Moment of Inertia :*** The moment of inertia about the $x'$ axis for each segment can be determined using the parallel – axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.

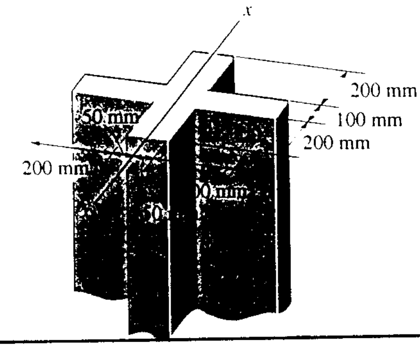
| Segment | $A_i$ (mm²) | $(d_y)_i$ (mm) | $(\bar{I}_{x'})_i$ (mm⁴) | $(Ad_y^2)_i$ (mm⁴) | $(I_{x'})_i$ (mm⁴) |
|---|---|---|---|---|---|
| 1 | 100(250) | 33.33 | $\frac{1}{12}(100)(250^3)$ | $27.778(10^6)$ | $157.99(10^6)$ |
| 2 | 250(50) | 66.67 | $\frac{1}{12}(250)(50^3)$ | $55.556(10^6)$ | $58.16(10^6)$ |




Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 216.15(10^6)\ \text{mm}^4 = 216(10^6)\ \text{mm}^4 \quad \textbf{Ans}$$

*10-52. Determine the radius of gyration $k_x$ for the column's cross-sectional area.

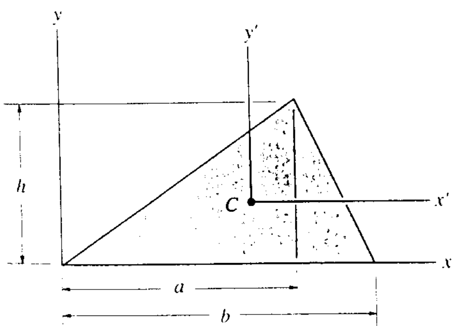



$$I_x = \frac{1}{12}(500)(100)^3 + 2\left[\frac{1}{12}(100)(200)^3 + (100)(200)(150)^2\right]$$

$$= 1.075(10^9)\ \text{mm}^4$$

$$k_x = \sqrt{\frac{1.075(10^9)}{90(10^3)}} = 109\ \text{mm} \qquad \textbf{Ans}$$

10-53. Determine the moments of inertia of the triangular area about the $x'$ and $y'$ axes, which pass through the centroid $C$ of the area.



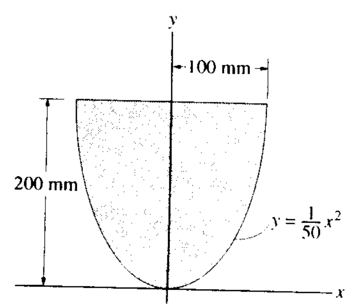

$$\bar{I}_{x'} = \tfrac{1}{36}(a)h^3 + \tfrac{1}{36}(b-a)h^3 = \tfrac{1}{36}bh^3 \qquad \textbf{Ans}$$

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{\frac{2}{3}a\left[\frac{1}{2}(h)(a)\right] + \left[a+\frac{b-a}{3}\right]\left[\frac{1}{2}h(b-a)\right]}{\frac{1}{2}(h)(a) + \frac{1}{2}h(b-a)} = \frac{b+a}{3}$$

$$\bar{I}_{y'} = \tfrac{1}{36}ha^3 + \tfrac{1}{2}ha\left(\tfrac{b+a}{3} - \tfrac{2}{3}a\right)^2 + \tfrac{1}{36}h(b-a)^3 + \tfrac{1}{2}h(b-a)\left(a + \tfrac{b-a}{3} - \tfrac{b+a}{3}\right)^2$$

$$= \tfrac{1}{36}hb\left(b^2 - ab + a^2\right) \qquad \textbf{Ans}$$

*10-54. Determine the product of inertia of the shaded portion of the parabola with respect to the $x$ and $y$ axes.



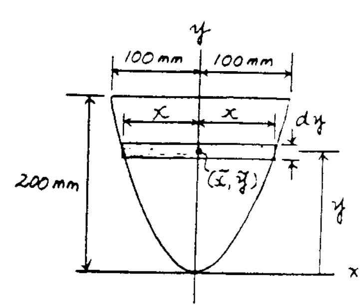

***Differential Element*** : Here, $x = \sqrt{50}y^{\frac{1}{2}}$. The area of the differential element parallel to the $x$ axis is $dA = 2x\,dy = 2\sqrt{50}y^{\frac{1}{2}}dy$. The coordinates of the centroid for this element are $\tilde{x} = 0$, $\tilde{y} = y$. Then the product of inertia for this element is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\tilde{x}\tilde{y}$$
$$= 0 + \left(2\sqrt{50}y^{\frac{1}{2}}dy\right)(0)(y)$$
$$= 0$$

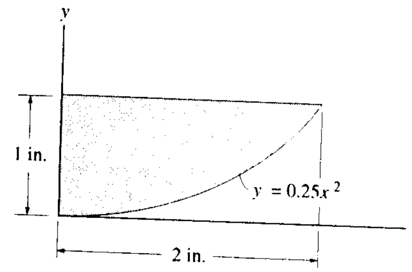
***Product of Inertia*** : Performing the integration, we have

$$I_{xy} = \int dI_{xy} = 0 \qquad \textbf{Ans}$$

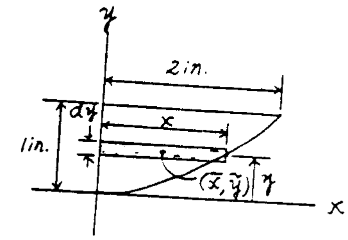
**Note** : By inspection, $I_{xy} = 0$ since the shaded area is symmetrical about the $y$ axis.



**10-55.** Determine the product of inertia of the shaded area with respect to the $x$ and $y$ axes.




***Differential Element*** : Here, $x = 2y^{\frac{1}{2}}$. The area of the differential element parallel to the $x$ axis is $dA = x dy = 2y^{\frac{1}{2}}dy$. The coordinates of the centroid for this element are $\tilde{x} = \frac{x}{2} = y^{\frac{1}{2}}$, $\tilde{y} = y$. Then the product of inertia for this element is

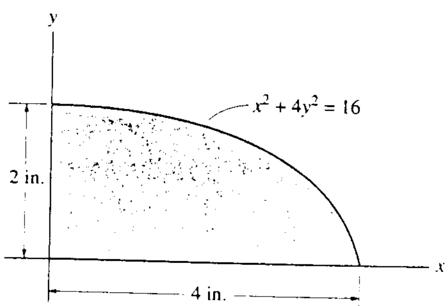



$$dI_{xy} = d\bar{I}_{x'y'} + dA\tilde{x}\tilde{y}$$
$$= 0 + \left(2y^{\frac{1}{2}}dy\right)\left(y^{\frac{1}{2}}\right)(y)$$
$$= 2y^2 dy$$

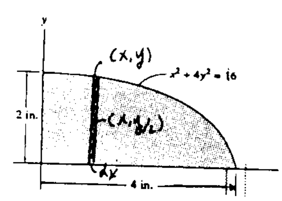
***Product of Inertia*** : Performing the integration, we have

$$I_{xy} = \int dI_{xy} = \int_0^{1\,\text{in.}} 2y^2 dy = \frac{2}{3}y^3\Big|_0^{1\,\text{in.}} = 0.667\ \text{in}^4 \qquad \textbf{Ans}$$

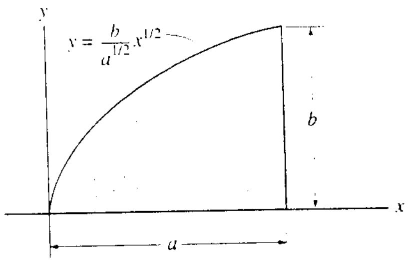
***10-56.** Determine the product of inertia of the shaded area of the ellipse with respect to the $x$ and $y$ axes.




$$I_{xy} = \int_A \tilde{x}\tilde{y}\, dA = \int_0^4 \left(\frac{y}{2}\right)(xy)\, dx$$
$$= \frac{1}{2}\int_0^4 y^2 x\, dx$$
$$= \frac{1}{2}\int_0^4 \frac{1}{4}(16 - x^2)x\, dx$$
$$= \frac{1}{8}\int_0^4 (16x - x^3)\, dx$$
$$= \frac{1}{8}\left[8x^2 - \frac{1}{4}x^4\right]_0^4$$
$$I_{xy} = 8\ \text{in}^4 \qquad \textbf{Ans}$$



**10-57.** Determine the product of inertia of the parabolic area with respect to the $x$ and $y$ axes.

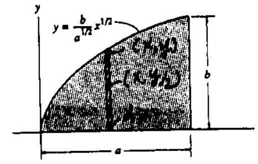



$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$

$dA = y\,dx$

$dI_{xy} = \frac{xy^2}{2}\,dx$



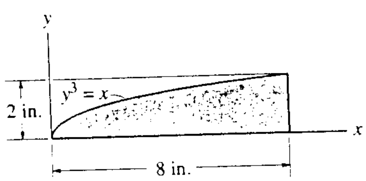
$I_{xy} = \int dI_{xy}$

$= \int_0^a \frac{1}{2}\left(\frac{b^2}{a}\right)x^2\,dx$

$= \frac{1}{6}\left[\left(\frac{b^2}{a}\right)x^3\right]_0^a$

$= \frac{1}{6}a^2 b^2$ **Ans**

**10-58.** Determine the product of inertia of the shaded area with respect to the $x$ and $y$ axes.

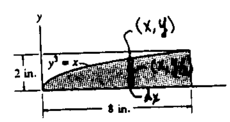



$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$

$dA = y\,dx$

$dI_{xy} = \frac{xy^2}{2}\,dx$



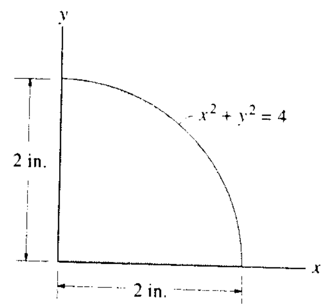
$I_{xy} = \int dI_{xy}$

$= \frac{1}{2}\int_0^8 x^{5/3}\,dx$

$= \frac{1}{2}\left(\frac{3}{8}\right)\left[x^{8/3}\right]_0^8$

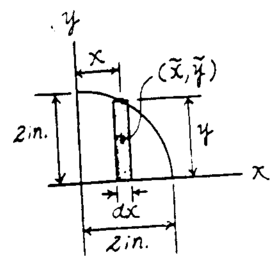
$= 48\text{ in}^4$ **Ans**

**10-59.** Determine the product of inertia of the shaded area with respect to the $x$ and $y$ axes.




***Differential Element :*** Here, $y = \sqrt{4-x^2}$. The area of the differential element parallel to the $y$ axis is $dA = ydx = \sqrt{4-x^2}dx$ . The coordinates of the centroid for this element are $\tilde{x} = x$, $\tilde{y} = \frac{y}{2} = \frac{1}{2}\sqrt{4-x^2}$. Then the product of inertia for this element is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\tilde{x}\tilde{y}$$
$$= 0 + \left(\sqrt{4-x^2}dx\right)(x)\left(\frac{1}{2}\sqrt{4-x^2}\right)$$
$$= \frac{1}{2}\left(4x - x^3\right)dx$$




***Product of Inertia :*** Performing the integration, we have

$$I_{xy} = \int dI_{xy} = \frac{1}{2}\int_0^{2\text{in.}}\left(4x - x^3\right)dx$$
$$= \frac{1}{2}\left(2x^2 - \frac{x^4}{4}\right)\Bigg|_0^{2\text{in.}} = 2.00 \text{ in}^4 \qquad \textbf{Ans}$$

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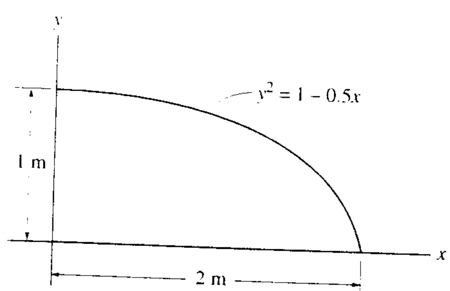
***10-60.** Determine the product of inertia of the shaded area with respect to the $x$ and $y$ axes.

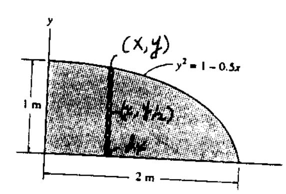



$$\tilde{x} = x$$

$$\tilde{y} = \frac{y}{2}$$

$$dA = y\,dx$$

$$d\,I_{xy} = \frac{xy^2}{2}\,dx$$



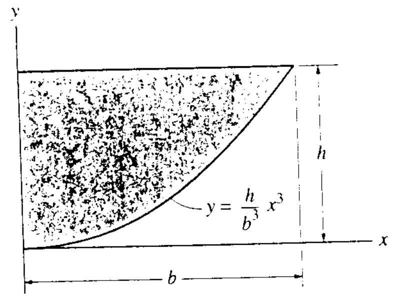

$$I_{xy} = \int d\,I_{xy}$$

$$= \int_0^2 \frac{1}{2}(x - 0.5x^2)\,dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2} - \frac{1}{6}x^3\right]_0^2$$

$$= 0.333 \text{ m}^4 \qquad \textbf{Ans}$$

**10-61.** Determine the product of inertia of the shaded area with respect to the $x$ and $y$ axes.




$\tilde{x} = x$

$\tilde{y} = \frac{y}{2}$

$dA = x\,dy$

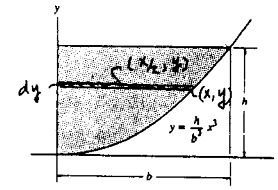
$dI_{xy} = \frac{x^2 y}{2}\,dy$

$$I_{xy} = \int dI_{xy}$$

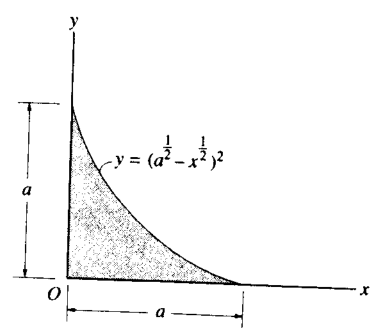
$$= \int_0^h \frac{1}{2}\left(\frac{b}{h^{1/3}}\right)^2 y^{5/3}\,dy$$

$$= \frac{1}{2}\left[\left(\frac{b^2}{h^{2/3}}\right)\left(\frac{3}{8}\right)y^{8/3}\right]_0^h$$

$$= \frac{3}{16}b^2h^2 \qquad \textbf{Ans}$$




***10-62.** Determine the product of inertia of the shaded area with respect to the $x$ and $y$ axes.




***Differential Element :*** The area of the differential element parallel to the $y$ axis is $dA = y\,dx = \left(a^{\frac{1}{2}} - x^{\frac{1}{2}}\right)^2 dx$. The coordinates of the centroid for this element are $\tilde{x} = x$, $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(a^{\frac{1}{2}} - x^{\frac{1}{2}}\right)^2$. Then the product of inertia for this element is

$$dI_{xy} = d\bar{I}_{x'y'} + dA\tilde{x}\tilde{y}$$

$$= 0 + \left[\left(a^{\frac{1}{2}} - x^{\frac{1}{2}}\right)^2 dx\right](x)\left[\frac{1}{2}\left(a^{\frac{1}{2}} - x^{\frac{1}{2}}\right)^2\right]$$

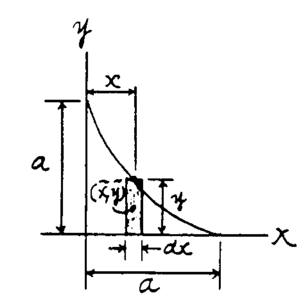
$$= \frac{1}{2}\left(x^3 + a^2x + 6ax^2 - 4a^{\frac{3}{2}}x^{\frac{3}{2}} - 4a^{\frac{1}{2}}x^{\frac{5}{2}}\right)dx$$

***Product of Inertia :*** Performing the integration, we have

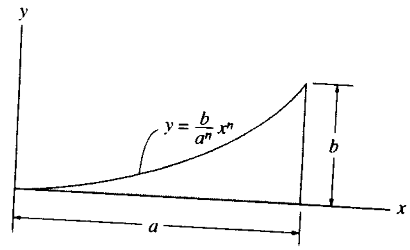
$$I_{xy} = \int dI_{xy} = \frac{1}{2}\int_0^a \left(x^3 + a^2x + 6ax^2 - 4a^{\frac{3}{2}}x^{\frac{3}{2}} - 4a^{\frac{1}{2}}x^{\frac{5}{2}}\right)dx$$

$$= \frac{1}{2}\left(\frac{x^4}{4} + \frac{a^2}{2}x^2 + 2ax^3 - \frac{8}{5}a^{\frac{3}{2}}x^{\frac{5}{2}} - \frac{8}{7}a^{\frac{1}{2}}x^{\frac{7}{2}}\right)\Bigg|_0^a$$

$$= \frac{a^4}{280} \qquad \textbf{Ans}$$



**1`63.** Determine the product of inertia of the shaded are. with respect to the $x$ and $y$ axes.



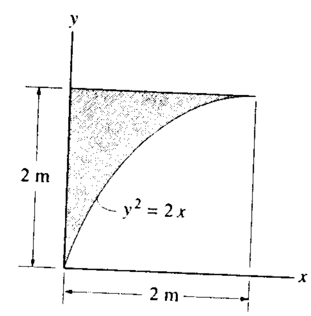

$$dI_{xy} = d\bar{I}_{xy} + \tilde{x}\tilde{y}\,dA$$

$$I_{xy} = 0 + \int_0^a (x)\left(\frac{y}{2}\right)(y\,dx) = \frac{1}{2}\int_0^a \left(\frac{b^2}{a^{2n}}\right)x^{2n+1}dx$$

$$= \left(\frac{b^2}{2a^{2n}}\right)\left(\frac{1}{2n+2}\right)x^{2n+2}\Big|_0^a = \frac{b^2a^{2n+2}}{4(n+1)\,a^{2n}}$$

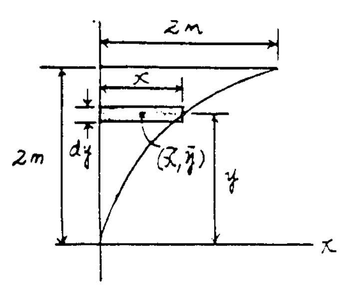
$$= \frac{a^2b^2}{4(n+1)} \quad \textbf{Ans}$$

***10-64.** Determine the product of inertia of the shaded area with respect to the $x$ and $y$ axes.




***Differential Element :*** Here, $x = \frac{y^2}{2}$. The area of the differential element parallel to the $x$ axis is $dA = x\,dy = \frac{y^2}{2}dy$. The coordinates of the centroid for this element are $\tilde{x} = \frac{x}{2} = \frac{y^2}{4}$, $\tilde{y} = y$. Then the product of inertia for this element is

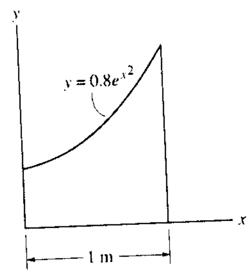
$$dI_{xy} = d\bar{I}_{x'y'} + dA\,\tilde{x}\tilde{y}$$
$$= 0 + \left(\frac{y^2}{2}dy\right)\left(\frac{y^2}{4}\right)(y)$$
$$= \frac{1}{8}y^5\,dy$$

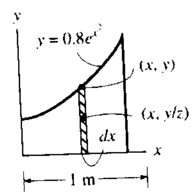



***Product of Inertia :*** Performing the integration, we have

$$I_{xy} = \int dI_{xy} = \frac{1}{8}\int_0^{2\,\text{m}} y^5\,dy = \frac{1}{48}y^6\Big|_0^{2\,\text{m}} = 1.33\ \text{m}^4 \quad \textbf{Ans}$$

■**10-65.** Determine the product of inertia of the shaded area with respect to the $x$ and $y$ axes. Use Simpson's rule to evaluate the integral.




$\bar{x} = x$

$\bar{y} = \dfrac{y}{2}$

$dA = y\,dx$

$dI_{xy} = \dfrac{xy^2}{2}\,dx$

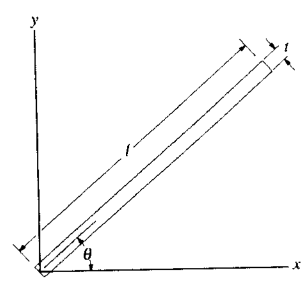
$I_{xy} = \int dI_{xy}$

$= \int_0^1 \frac{1}{2}x(0.8\,e^{x^2})^2\,dx$

$= 0.32\int_0^1 x\,e^{2x^2}\,dx$

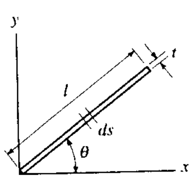
$= 0.511\text{ m}^4$ **Ans**

**10-66.** Determine the product of inertia of the thin strip of area with respect to the $x$ and $y$ axes. The strip is oriented at an angle $\theta$ from the $x$ axis. Assume that $t \ll l$.

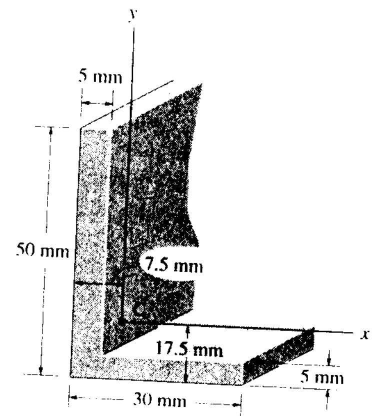



$I_{xy} = \int_A xy\,dA = \int_0^l (s\cos\theta)(s\sin\theta)t\,ds = \sin\theta\cos\theta t\int_0^l s^2\,ds$

$= \frac{1}{6}l^3 t\sin 2\theta$ **Ans**

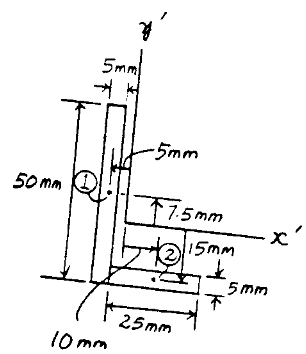


**10-67.** Determine the product of inertia of the beam's cross-sectional area with respect to the $x$ and $y$ axes that have their origin located at the centroid $C$.




***Product of Inertia :*** The area for each segment, its centroid and product of inertia with respect to $x$ and $y$ axes are tabulated below.

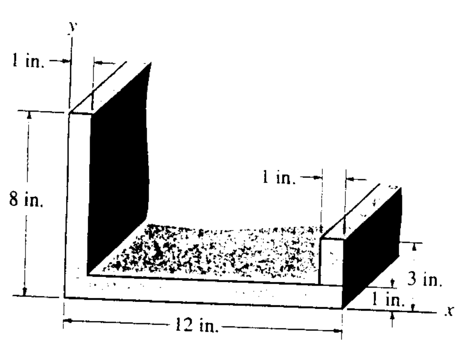
| Segment | $A_i$ (mm²) | $(d_x)_i$ (mm) | $(d_y)_i$ (mm) | $(I_{xy})_i$ (mm⁴) |
|---|---|---|---|---|
| 1 | 50(5) | −5 | 7.5 | −9.375(10³) |
| 2 | 25(5) | 10 | −15 | −18.75(10³) |




Thus,

$$I_{xy} = \Sigma (I_{xy})_i = -28.125(10^3)\ \text{mm}^4 = -28.1(10^3)\ \text{mm}^4 \quad \textbf{Ans}$$

***10-68.** Determine the product of inertia of the beam's cross-sectional area with respect to the $x$ and $y$ axes.




$$I_{xy} = 0.5(4)(8)(1) + 6(0.5)(10)(1) + 11.5(1.5)(3)(1)$$

$$= 97.8\ \text{in}^4 \quad \textbf{Ans}$$