CHAPTER 9

9.1

$$\begin{bmatrix} 0 & 2 & 5 \\ 1 & 0 & 1 \\ 8 & 3 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 50 \\ 10 \\ 20 \end{Bmatrix} \qquad [A]^T = \begin{bmatrix} 0 & 1 & 8 \\ 2 & 0 & 3 \\ 5 & 1 & 0 \end{bmatrix}$$

9.2 (a) $[A] = 3\times2$ $[B] = 3\times3$ $[C] = 3\times1$ $[D] = 2\times4$
$[E] = 3\times3$ $[F] = 2\times3$ $[G] = 1\times3$

(b) Square: $[B]$ and $[E]$
Column: $[C]$
Row: $[G]$

(c) $a_{12} = 7$ $b_{23} = 7$ d_{32} = does not exist
$e_{22} = 2$ $f_{12} = 0$ $g_{12} = 6$

(d)

(1) $[E]+[B] = \begin{bmatrix} 5 & 8 & 15 \\ 8 & 4 & 10 \\ 6 & 0 & 10 \end{bmatrix}$ (2) $[A]+[F] = \text{not possible}$

(3) $[B]-[E] = \begin{bmatrix} 3 & -2 & -1 \\ -6 & 0 & 4 \\ -2 & 0 & -2 \end{bmatrix}$ (4) $7[B] = \begin{bmatrix} 28 & 21 & 49 \\ 7 & 14 & 49 \\ 14 & 0 & 28 \end{bmatrix}$

(5) $[E]\times[B] = \begin{bmatrix} 25 & 13 & 74 \\ 36 & 25 & 75 \\ 28 & 12 & 52 \end{bmatrix}$ (6) $\{C\}^T = \lfloor 3 \quad 6 \quad 1 \rfloor$

(7) $[B]\times[A] = \begin{bmatrix} 54 & 76 \\ 41 & 53 \\ 28 & 38 \end{bmatrix}$ (8) $\{D\}^T = \begin{bmatrix} 9 & 2 \\ 4 & -1 \\ 3 & 7 \\ -6 & 5 \end{bmatrix}$

(9) $[A]\times[C] = \text{not possible}$ (10) $[I]\times[B] = [B]$

(11) $[E]^T[E] = \begin{bmatrix} 66 & 19 & 53 \\ 19 & 29 & 46 \\ 53 & 46 & 109 \end{bmatrix}$ (12) $[C]^T[C] = 46$

9.3 (a) Possible multiplications:

$$[A][B] = \begin{bmatrix} 4 & 15 \\ 8 & 29 \\ 9 & 29 \end{bmatrix} \qquad [A][C] = \begin{bmatrix} -16 & 4 \\ -24 & 4 \\ 2 & -10 \end{bmatrix} \qquad [B][C] = \begin{bmatrix} -7 & 1 \\ -5 & 1 \end{bmatrix} \qquad [C][B] = \begin{bmatrix} 1 & 2 \\ -2.5 & -7 \end{bmatrix}$$

Note: Some students might recognize that we can also compute $[B][B]$ and $[C][C]$:

$$[B][B]=\begin{bmatrix}2.5 & 9\\ 1.5 & 5.5\end{bmatrix} \qquad [C][C]=\begin{bmatrix}10 & -6\\ -9 & 7\end{bmatrix}$$

(b) $[B][A]$ and $[C][A]$ are impossible because the inner dimensions do not match:

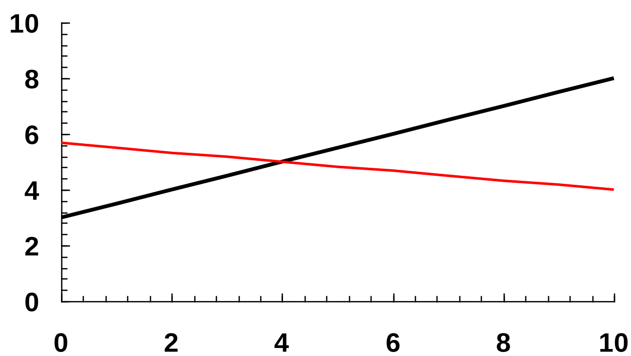
$(2\times2) * (3\times2)$

(c) According to **(a)**, $[B][C] \neq [C][B]$

9.4 The equations can be rearranged into a format for plotting x_2 versus x_1:

$$x_2 = 3 + 0.5x_1$$

$$x_2 = \frac{34}{6} - \frac{1}{6}x_1$$



Therefore, the solution is $x_1 = 4$, $x_2 = 5$. The results can be checked by substituting them back into the original equations:

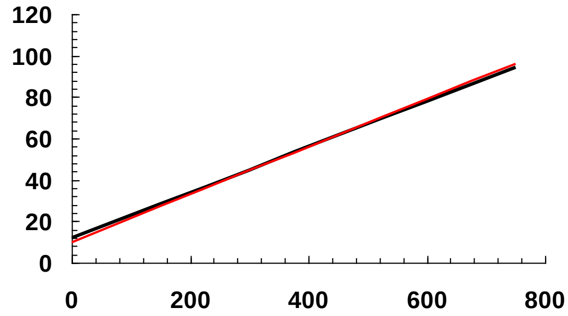
$$4(4) - 8(5) = -24$$

$$4 + 6(5) = 34$$

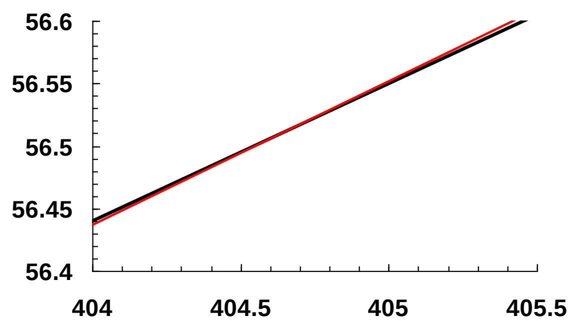
9.5 (a) The equations can be rearranged into a format for plotting x_2 versus x_1:

$$x_2 = 12 + 0.11x_1$$

$$x_2 = 10 + \frac{2}{17.4}x_1$$



If you zoom in, it appears that there is a root at about (404.6, 56.5).



The results can be checked by substituting them back into the original equations:

$$-1.1(404.6) + 10(56.5) = 119.94 \cong 120$$

$$-2(404.6) + 17.4(56.5) = 173.9 \cong 174$$

(b) The plot suggests that the system may be ill-conditioned because the slopes are so similar.

(c) The determinant can be computed as

$$D = -1.1(17.4) - 10(-2) = 0.86$$

which is relatively small. Note that if the system is normalized first by dividing each equation by the largest coefficient,

$$-0.11x_1 + x_2 = 12$$

$$-0.11494x_1 + x_2 = 10$$

the determinant is even smaller

$$D = -0.11(1) - 1(-0.11494) = 0.00494$$

(d) Using Eqs. (9.10) and (9.11) yields

$$x_1 = \frac{17.4(120) - 10(174)}{0.86} = 404.6512$$

$$x_2 = \frac{-1.1(174) - (-2)(120)}{0.86} = 56.51163$$

9.6 (a) The determinant can be computed as:

$$A_1 = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1(0) - 1(1) = -1$$

$$A_2 = \begin{vmatrix} 2 & 1 \\ 3 & 0 \end{vmatrix} = 2(0) - 1(3) = -3$$

$$A_3 = \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = 2(1) - 1(3) = -1$$

$$D = 0(-1) - 2(-3) + 5(-1) = 1$$

(b) Cramer's rule

$$x_1 = \frac{\begin{vmatrix} 9 & 2 & 5 \\ 9 & 1 & 1 \\ 10 & 1 & 0 \end{vmatrix}}{D} = \frac{6}{1} = 6$$

$$x_2 = \frac{\begin{vmatrix} 0 & 9 & 5 \\ 2 & 9 & 1 \\ 3 & 10 & 0 \end{vmatrix}}{D} = \frac{-8}{1} = -8$$

$$x_3 = \frac{\begin{vmatrix} 0 & 2 & 9 \\ 2 & 1 & 9 \\ 3 & 1 & 10 \end{vmatrix}}{D} = \frac{5}{1} = 5$$

(c) The results can be checked by substituting them back into the original equations:

$$2(-8) + 5(5) = 9$$

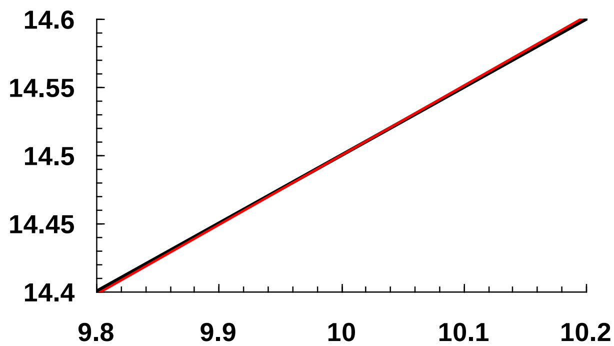
$$2(6) + (-8) + 5 = 9$$

$$3(6) + (-8) = 10$$

9.7 (a) The equations can be rearranged into a format for plotting x_2 versus x_1:

$x_2 = 9.5 + 0.5x_1$

$x_2 = 9.4 + 0.51x_1$



The solution is x_1 = 10, x_2 = 14.5. Notice that the lines have very similar slopes.

(b) The determinant can be computed as

$$D = 0.5(-2) - (-1)1.02 = 0.02$$

(c) The plot and the low value of the determinant both suggest that the system is ill-conditioned.

(d) Using Eqs. (9.10) and (9.11) yields

$$x_1 = \frac{-9.5(-2) - (-1)(-18.8)}{0.02} = 10$$

$$x_2 = \frac{0.5(-18.8) - (-9.5)1.02}{0.02} = 14.5$$

(e) Using Eqs. (9.10) and (9.11) yields

$$x_1 = \frac{-9.5(-2) - (-1)(-18.8)}{-0.02} = -10$$

$$x_2 = \frac{0.52(-18.8) - (-9.5)1.02}{-0.02} = 4.3$$

The ill-conditioned nature of the system is illustrated by the fact that a small change in one of the coefficients results in a huge change in the results.

9.8 (a) The system is first expressed as an augmented matrix:

$$\begin{bmatrix} 10 & 2 & -1 & 27 \\ -3 & -6 & 2 & -61.5 \\ 1 & 1 & 5 & -21.5 \end{bmatrix}$$

Forward elimination:

a_{21} is eliminated by multiplying row 1 by –3/10 and subtracting the result from row 2. a_{31} is eliminated by multiplying row 1 by 1/10 and subtracting the result from row 3.

$$\begin{bmatrix} 10 & 2 & -1 & 27 \\ 0 & -5.4 & 1.7 & -53.4 \\ 0 & 0.8 & 5.1 & -24.2 \end{bmatrix}$$

a_{32} is eliminated by multiplying row 2 by 0.8/(–5.4) and subtracting the result from row 3.

$$\begin{bmatrix} 10 & 2 & -1 & 27 \\ 0 & -5.4 & 1.7 & -53.4 \\ 0 & 0 & 5.351852 & -32.1111 \end{bmatrix}$$

Back substitution:

$$x_3 = \frac{-32.1111}{5.351852} = -6$$

$$x_2 = \frac{-53.4 - 1.7(-6)}{-5.4} = 8$$

$$x_1 = \frac{27 - (-1)(-6) - 2(8)}{10} = 0.5$$

(b) Check:

$$10(0.5) + 2(8) - (-6) = 27$$

$$-3(0.5) - 6(8) + 2(-6) = -61.5$$

$$0.5 + 8 + 5(-6) = -21.5$$

9.9 (a) The system is first expressed as an augmented matrix:

$$\begin{bmatrix} 8 & 2 & -2 & -2 \\ 10 & 2 & 4 & 4 \\ 12 & 2 & 2 & 6 \end{bmatrix}$$

Forward elimination: First, we pivot by switching rows 1 and 3:

$$\begin{bmatrix} 12 & 2 & 2 & 6 \\ 10 & 2 & 4 & 4 \\ 8 & 2 & -2 & -2 \end{bmatrix}$$

Multiply row 1 by 10/12 = 0.83333 and subtract from row 2 to eliminate a_{21}. Multiply row 1 by 8/12 = 0.66667 and subtract from row 3 to eliminate a_{31}.

$$\begin{bmatrix} 12 & 2 & 2 & 6 \\ 0 & 0.33333 & 2.33333 & -1 \\ 0 & 0.66667 & -3.33333 & -6 \end{bmatrix}$$

Pivot:

$$\begin{bmatrix} 12 & 2 & 2 & 6 \\ 0 & 0.66667 & -3.33333 & -6 \\ 0 & 0.33333 & 2.33333 & -1 \end{bmatrix}$$

Multiply row 2 by 0.33333/0.66667 = 0.5 and subtract from row 3 to eliminate a_{32}.

$$\begin{bmatrix} 12 & 2 & 2 & 6 \\ 0 & 0.66667 & -3.33333 & -6 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

Back substitution:

$$x_3 = \frac{2}{4} = 0.5$$

$$x_2 = \frac{-6-(-3.3333)0.5}{0.66667} = -6.5$$

$$x_1 = \frac{6-2(0.5)-2(-6.5)}{12} = 1.5$$

Check:

$$8(1.5)+2(-6.5)-2(0.5) = -2$$

$$10(1.5)+2(-6.5)+4(0.5) = 4$$

$$12(1.5)+2(-6.5)+2(0.5) = 6$$

9.10 (a) The determinant can be computed as:

$$A_1 = \begin{vmatrix} 2 & -1 \\ -2 & 0 \end{vmatrix} = 2(0)-(-1)(-2) = -2$$

$$A_2 = \begin{vmatrix} 1 & -1 \\ 5 & 0 \end{vmatrix} = 1(0) - (-1)(5) = 5$$

$$A_3 = \begin{vmatrix} 1 & 2 \\ 5 & -2 \end{vmatrix} = 1(-2) - 2(5) = -12$$

$$D = 0(-2) - (-3)5 + 7(-12) = -69$$

(b) Cramer's rule

$$x_1 = \frac{\begin{vmatrix} 2 & -3 & 7 \\ 3 & 2 & -1 \\ 2 & -2 & 0 \end{vmatrix}}{D} = \frac{-68}{-69} = 0.985507$$

$$x_2 = \frac{\begin{vmatrix} 0 & 2 & 7 \\ 1 & 3 & -1 \\ 5 & 2 & 0 \end{vmatrix}}{D} = \frac{-101}{-69} = 1.463768$$

$$x_3 = \frac{\begin{vmatrix} 0 & -3 & 2 \\ 1 & 2 & 3 \\ 5 & -2 & 2 \end{vmatrix}}{D} = \frac{-63}{-69} = 0.913043$$

(c) The system is first expressed as an augmented matrix:

$$\begin{bmatrix} 0 & -3 & 7 & 2 \\ 1 & 2 & -1 & 3 \\ 5 & -2 & 0 & 2 \end{bmatrix}$$

Forward elimination: First, we pivot by switching rows 1 and 3:

$$\begin{bmatrix} 5 & -2 & 0 & 2 \\ 1 & 2 & -1 & 3 \\ 0 & -3 & 7 & 2 \end{bmatrix}$$

Multiply row 1 by 1/5 = 0.2 and subtract from row 2 to eliminate a_{21}. Because a_{31} already equals zero, it does not have to be eliminated.

$$\begin{bmatrix} 5 & -2 & 0 & 2 \\ 0 & 2.4 & -1 & 2.6 \\ 0 & -3 & 7 & 2 \end{bmatrix}$$

Pivot:

$$\begin{bmatrix} 5 & -2 & 0 & 2 \\ 0 & -3 & 7 & 2 \\ 0 & 2.4 & -1 & 2.6 \end{bmatrix}$$

Multiply row 2 by 2.4/(–3) = –0.8 and subtract from row 3 to eliminate a_{32}.

$$\begin{bmatrix} 5 & -2 & 0 & 2 \\ 0 & -3 & 7 & 2 \\ 0 & 0 & 4.6 & 4.2 \end{bmatrix}$$

Back substitution:

$$x_3 = \frac{4.2}{4.6} = 0.913043$$

$$x_2 = \frac{2 - 7(0.913043)}{-3} = 1.463768$$

$$x_1 = \frac{2 - 0(0.913043) - (-2)(1.463768)}{5} = 0.985507$$

(d) Check:

$$-3(1.463768) + 7(0.913043) = 2$$

$$(0.985507) + 2(1.463768) - (0.913043) = 3$$

$$5(0.985507) - 2(1.463768) = 2$$

9.11 (a) The system is first expressed as an augmented matrix:

$$\begin{bmatrix} 2 & -6 & -1 & -38 \\ -3 & -1 & 7 & -34 \\ -8 & 1 & -2 & -20 \end{bmatrix}$$

Forward elimination: First, we pivot by switching rows 1 and 3:

$$\begin{bmatrix} -8 & 1 & -2 & -20 \\ -3 & -1 & 7 & -34 \\ 2 & -6 & -1 & -38 \end{bmatrix}$$

Multiply row 1 by –3/(–8) = 0.375 and subtract from row 2 to eliminate a_{21}. Multiply row 1 by 2/(–8) = –0.25 and subtract from row 3 to eliminate a_{31}.

$$\begin{bmatrix} -8 & 1 & -2 & -20 \\ 0 & -1.375 & 7.75 & -26.5 \\ 0 & -5.75 & -1.5 & -43 \end{bmatrix}$$

Pivot:

$$\begin{bmatrix} -8 & 1 & -2 & -20 \\ 0 & -5.75 & -1.5 & -43 \\ 0 & -1.375 & 7.75 & -26.5 \end{bmatrix}$$

Multiply row 2 by –1.375/–5.75 = 0.23913 and subtract from row 3 to eliminate a_{32}.

$$\begin{bmatrix} -8 & 1 & -2 & -20 \\ 0 & -5.75 & -1.5 & -43 \\ 0 & 0 & 8.108696 & -16.2174 \end{bmatrix}$$

Back substitution:

$$x_3 = \frac{-16.2174}{8.108696} = -2$$

$$x_2 = \frac{-43 - (-1.5)(-2)}{-5.75} = 8$$

$$x_1 = \frac{-20 + 2(-2) - 1(8)}{-8} = 4$$

(b) Check:

$$2(4) - 6(8) - (-2) = -38$$

$$-3(4) - (8) + 7(-2) = -34$$

$$-8(4) + (8) - 2(-2) = -20$$

9.12 The system is first expressed as an augmented matrix:

$$\begin{bmatrix} 2 & 1 & -1 & 1 \\ 5 & 2 & 2 & -4 \\ 3 & 1 & 1 & 5 \end{bmatrix}$$

Normalize the first row and then eliminate a_{21} and a_{31},

$$\begin{bmatrix} 1 & 0.5 & -0.5 & 0.5 \\ 0 & -0.5 & 4.5 & -6.5 \\ 0 & -0.5 & 2.5 & 3.5 \end{bmatrix}$$

Normalize the second row and eliminate a_{12} and a_{32},

$$\begin{bmatrix} 1 & 0 & 4 & -6 \\ 0 & 1 & -9 & 13 \\ 0 & 0 & -2 & 10 \end{bmatrix}$$

Normalize the third row and eliminate a_{13} and a_{23},

$$\begin{bmatrix} 1 & 0 & 0 & 14 \\ 0 & 1 & 0 & -32 \\ 0 & 0 & 1 & -5 \end{bmatrix}$$

Thus, the answers are $x_1 = 14$, $x_2 = -32$, and $x_3 = -5$.

Check:

$$2(14) + (-32) - (-5) = 1$$

$$5(14) + 2(-32) + 2(-5) = -4$$

$$3(14) + (-32) + (-5) = 5$$

9.13 (a) The system is first expressed as an augmented matrix:

$$\begin{bmatrix} 1 & 1 & -1 & -3 \\ 6 & 2 & 2 & 2 \\ -3 & 4 & 1 & 1 \end{bmatrix}$$

Forward elimination:

a_{21} is eliminated by multiplying row 1 by 6/1 = 6 and subtracting the result from row 2. a_{31} is eliminated by multiplying row 1 by –3/1 = –3 and subtracting the result from row 3.

$$\begin{bmatrix} 1 & 1 & -1 & -3 \\ 0 & -4 & 8 & 20 \\ 0 & 7 & -2 & -8 \end{bmatrix}$$

a_{32} is eliminated by multiplying row 2 by 7/(–4) = –1.75 and subtracting the result from row 3.

$$\begin{bmatrix} 1 & 1 & -1 & -3 \\ 0 & -4 & 8 & 20 \\ 0 & 0 & 12 & 27 \end{bmatrix}$$

Back substitution:

$$x_3 = \frac{27}{12} = 2.25$$

$$x_2 = \frac{20 - 8(2.25)}{-4} = -0.5$$

$$x_1 = \frac{-3-(-1)(2.25)-1(-0.5)}{1} = -0.25$$

(b) The system is first expressed as an augmented matrix:

$$\begin{bmatrix} 1 & 1 & -1 & -3 \\ 6 & 2 & 2 & 2 \\ -3 & 4 & 1 & 1 \end{bmatrix}$$

Forward elimination: First, we pivot by switching rows 1 and 2:

$$\begin{bmatrix} 6 & 2 & 2 & 2 \\ 1 & 1 & -1 & -3 \\ -3 & 4 & 1 & 1 \end{bmatrix}$$

Multiply row 1 by 1/6 = 0.16667 and subtract from row 2 to eliminate a_{21}. Multiply row 1 by –3/6 = –0.5 and subtract from row 3 to eliminate a_{31}.

$$\begin{bmatrix} 6 & 2 & 2 & 2 \\ 0 & 0.66667 & -1.33333 & -3.33333 \\ 0 & 5 & 2 & 2 \end{bmatrix}$$

Pivot:

$$\begin{bmatrix} 6 & 2 & 2 & 2 \\ 0 & 5 & 2 & 2 \\ 0 & 0.66667 & -1.33333 & -3.33333 \end{bmatrix}$$

Multiply row 2 by 0.66667/5 = 0.133333 and subtract from row 3 to eliminate a_{32}.

$$\begin{bmatrix} 6 & 2 & 2 & 2 \\ 0 & 5 & 2 & 2 \\ 0 & 0 & -1.6 & -3.6 \end{bmatrix}$$

Back substitution:

$$x_3 = \frac{-3.6}{1.6} = 2.25$$

$$x_2 = \frac{2-2(2.25)}{5} = -0.5$$

$$x_1 = \frac{2-2(2.25)-2(-0.5)}{6} = -0.25$$

(c) The system is first expressed as an augmented matrix:

$$\begin{bmatrix} 1 & 1 & -1 & -3 \\ 6 & 2 & 2 & 2 \\ -3 & 4 & 1 & 1 \end{bmatrix}$$

Normalize the first row, and then eliminate a_{21} and a_{31},

$$\begin{bmatrix} 1 & 1 & -1 & -3 \\ 0 & -4 & 8 & 20 \\ 0 & 7 & -2 & -8 \end{bmatrix}$$

Normalize the second row and eliminate a_{12} and a_{32},

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 12 & 27 \end{bmatrix}$$

Normalize the third row and eliminate a_{13} and a_{23},

$$\begin{bmatrix} 1 & 0 & 0 & -0.25 \\ 0 & 1 & 0 & -0.5 \\ 0 & 0 & 1 & 2.25 \end{bmatrix}$$

9.14 In a fashion similar to Example 9.11, vertical force balances can be written to give the following system of equations,

$$\begin{aligned} m_1 g - T_{12} - c_1 v &= m_1 a \\ m_2 g + T_{12} - c_2 v - T_{23} &= m_2 a \\ m_3 g - c_3 v + T_{23} - T_{34} &= m_3 a \\ m_4 g - c_4 v + T_{34} - T_{45} &= m_4 a \\ m_5 g - c_5 v + T_{45} &= m_5 a \end{aligned}$$

After substituting the known values, the equations can be expressed in matrix form (g = 9.8),

$$\begin{bmatrix} 55 & 1 & 0 & 0 & 0 \\ 75 & -1 & 1 & 0 & 0 \\ 60 & 0 & -1 & 1 & 0 \\ 75 & 0 & 0 & -1 & 1 \\ 90 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} a \\ T_{12} \\ T_{23} \\ T_{34} \\ T_{45} \end{Bmatrix} = \begin{Bmatrix} 449 \\ 627 \\ 453 \\ 591 \\ 792 \end{Bmatrix}$$

The system can be solved for

a = 8.202817 T_{12} = –2.15493 T_{23} = 9.633803
T_{34} = –29.5352 T_{12} = –53.7465

9.15 Using the format of Eq. 9.27,

$$[A] = \begin{bmatrix} 3 & 4 \\ 0 & 1 \end{bmatrix} \qquad [B] = \begin{bmatrix} 2 & 0 \\ -1 & 0 \end{bmatrix}$$

$$\{U\} = \begin{Bmatrix} 2 \\ 3 \end{Bmatrix} \qquad \{V\} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

The set of equations to be solved are

$$\begin{aligned} 3x_1 + 4x_2 - 2y_1 \qquad &= 2 \\ x_2 + y_1 \qquad &= 3 \\ 2x_1 \qquad + 3y_1 + 4y_2 &= 1 \\ -x_1 \qquad\qquad + y_2 &= 0 \end{aligned}$$

These can be solved for $x_1 = -0.53333$, $x_2 = 1.6$, $y_1 = 1.4$, and $y_2 = -0.53333$. Therefore, the solution is $z_1 = -0.53333 + 1.4i$ and $z_2 = 1.6 - 0.53333i$.

9.16 Here is a VBA program to implement matrix multiplication and solve Prob. 9.3 for the case of $[A]\times[B]$.

```
Option Explicit

Sub Mult()
Dim i As Integer, j As Integer
Dim l As Integer, m As Integer, n As Integer
Dim a(10, 10) As Double, b(10, 10) As Double
Dim c(10, 10) As Double
l = 2
m = 2
n = 3
a(1, 1) = 1: a(1, 2) = 6
a(2, 1) = 3: a(2, 2) = 10
a(3, 1) = 7: a(3, 2) = 4
b(1, 1) = 1: b(1, 2) = 3
b(2, 1) = 0.5: b(2, 2) = 2
Call Mmult(a, b, c, m, n, l)
For i = 1 To n
  For j = 1 To l
    MsgBox c(i, j)
  Next j
Next i
End Sub

Sub Mmult(a, b, c, m, n, l)
Dim i As Integer, j As Integer, k As Integer
Dim sum As Double
For i = 1 To n
  For j = 1 To l
    sum = 0
    For k = 1 To m
      sum = sum + a(i, k) * b(k, j)
    Next k
    c(i, j) = sum
  Next j
Next i
End Sub
```

9.17 Here is a VBA program to implement the matrix transpose and solve Prob. 9.3 for the case of $[A]^T$.

```
Option Explicit

Sub TransTest()
Dim i As Integer, j As Integer
Dim m As Integer, n As Integer
Dim a(10, 10) As Double, aT(10, 10) As Double
n = 3
m = 2
a(1, 1) = 1: a(1, 2) = 6
a(2, 1) = 3: a(2, 2) = 10
a(3, 1) = 7: a(3, 2) = 4
Call Transpose(a, aT, n, m)
For i = 1 To m
  For j = 1 To n
    MsgBox aT(i, j)
  Next j
Next i
End Sub

Sub Transpose(a, b, n, m)
Dim i As Integer, j As Integer
For i = 1 To m
  For j = 1 To n
    b(i, j) = a(j, i)
  Next j
Next i
End Sub
```

9.18 Here is a VBA program to implement the Gauss elimination algorithm and solve the test case in Prob. 9.16.

```
Option Explicit

Sub GaussElim()
Dim n As Integer, er As Integer, i As Integer
Dim a(10, 10) As Double, b(10) As Double, x(10) As Double
n = 3
a(1, 1) = 1: a(1, 2) = 2: a(1, 3) = -1
a(2, 1) = 5: a(2, 2) = 2: a(2, 3) = 2
a(3, 1) = -3: a(3, 2) = 5: a(3, 3) = -1
b(1) = 2: b(2) = 9: b(3) = 1
Call Gauss(a, b, n, x, er)
If er = 0 Then
  For i = 1 To n
    MsgBox "x(" & i & ") = " & x(i)
  Next i
Else
  MsgBox "ill-conditioned system"
End If
End Sub

Sub Gauss(a, b, n, x, er)
Dim i As Integer, j As Integer
Dim s(10) As Double
Const tol As Double = 0.000001
er = 0
For i = 1 To n
```

```
  s(i) = Abs(a(i, 1))
  For j = 2 To n
    If Abs(a(i, j)) > s(i) Then s(i) = Abs(a(i, j))
  Next j
Next i
Call Eliminate(a, s, n, b, tol, er)
If er <> -1 Then
  Call Substitute(a, n, b, x)
End If
End Sub

Sub Pivot(a, b, s, n, k)
Dim p As Integer, ii As Integer, jj As Integer
Dim factor As Double, big As Double, dummy As Double
p = k
big = Abs(a(k, k) / s(k))
For ii = k + 1 To n
  dummy = Abs(a(ii, k) / s(ii))
  If dummy > big Then
    big = dummy
    p = ii
  End If
Next ii
If p <> k Then
  For jj = k To n
    dummy = a(p, jj)
    a(p, jj) = a(k, jj)
    a(k, jj) = dummy
  Next jj
  dummy = b(p)
  b(p) = b(k)
  b(k) = dummy
  dummy = s(p)
  s(p) = s(k)
  s(k) = dummy
End If
End Sub

Sub Substitute(a, n, b, x)
Dim i As Integer, j As Integer
Dim sum As Double
x(n) = b(n) / a(n, n)
For i = n - 1 To 1 Step -1
  sum = 0
  For j = i + 1 To n
    sum = sum + a(i, j) * x(j)
  Next j
  x(i) = (b(i) - sum) / a(i, i)
Next i
End Sub

Sub Eliminate(a, s, n, b, tol, er)
Dim i As Integer, j As Integer, k As Integer
Dim factor As Double
For k = 1 To n - 1
  Call Pivot(a, b, s, n, k)
  If Abs(a(k, k) / s(k)) < tol Then
    er = -1
    Exit For
  End If
  For i = k + 1 To n
    factor = a(i, k) / a(k, k)
```

```
    For j = k + 1 To n
      a(i, j) = a(i, j) - factor * a(k, j)
    Next j
    b(i) = b(i) - factor * b(k)
  Next i
Next k
If Abs(a(k, k) / s(k)) < tol Then er = -1
End Sub
```

Its application yields a solution of (1, 1, 1).