Chapter (6)

Energy Equation and Its Applications

Bernoulli Equation

Bernoulli equation is one of the most useful equations in fluid mechanics and hydraulics. And it's a statement of the principle of conservation of energy along a stream line.

Bernoulli Equation can be written as following:

$$\frac{P}{\rho g}+\frac{v^2}{2g}+z=H_T=\text{constant}$$

All these terms have a unit of length (m)

- $\frac{P}{\rho g}$ = pressure energy per unit weight = pressure head

We know that $P=\rho g h_{pressure}\rightarrow\rightarrow h_{pressure}=\frac{P}{\rho g}$ (m).

- $\frac{v^2}{2g}$ = kinetic energy per unit weight = velocity head

We know that $K.E=\frac{1}{2}mv^2\rightarrow$ divided by weight $\rightarrow\frac{\frac{1}{2}mv^2}{mg}=\frac{v^2}{2g}$ (m).

- z = potential energy per unit weight = (potential elevation head)

We know that $P.E=mgz\rightarrow$ divided by weight $\rightarrow\frac{mgz}{mg}=z$ (m).

- H_T = total energy per unit weight = total head (m).

By using principle of conservation of energy, we can apply Bernoulli equation between two points (1 and 2) on the streamline:

$$\textbf{Total head at (1) = Total head at (2)}$$

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

But!!, this equation no energy losses (e.g. from friction) or energy gains (e.g. from a pump) along a stream line, so the final form for Bernoulli equation is:

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}+z_1+h_P=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}+z_2+h_L+h_T$$

$h_P = q =$ Energy supplied by **pump** per unit weight (m)
$h_T = w =$ work done by **turbine** per unit weight (m)
$h_L =$ Total friction losses per unit weight (m)

Representation of Energy Changes in a Fluid System (HGL and EGL)

It is often convenient to plot mechanical energy **graphically** using heights.

Hydraulic Grade Line or (HGL):

$$HGL = \frac{P}{\rho g} + z$$

It is the line that joins all the points to which water would rise if piezometric tubes were inserted .

Energy Grade Line or (Total Energy Line) EGL:

$$EGL = \text{total head} = \frac{P}{\rho g} + \frac{v^2}{2g} + z$$

It is the line that joins all the points that represent the **total** head (i.e. the EGL is always above HGL by a velocity head $\left(\frac{v^2}{2g}\right)$).

Important Notes (Guidelines) for drawing HGL and EGL:

✓ EGL and HGL are falls in the direction of flow due to friction losses (h_L).
✓ EGL and HGL are vertically downward due to minor losses such as:
[entrance, exit, elbow, valve, increasing or decreasing diameter] if exist.
✓ EGL and HGL are vertically upward due to the head supplied by pump (if exist) and tend vertically downward due to head consumed by turbine (if exist).
✓ If there exist a reservoir, the EGL and HGL are coinciding with the surface of fluid in the reservoir because the velocity is zero and the pressure is atmospheric pressure (zero gauge pressure).
✓ But if the water exits from a nozzle with a specific velocity the HGL will coincide with the water because the pressure is zero, but the EGL will be above HGL by a velocity head.
✓ Another notes will be described in the problems that we will solve.

Problems

1.

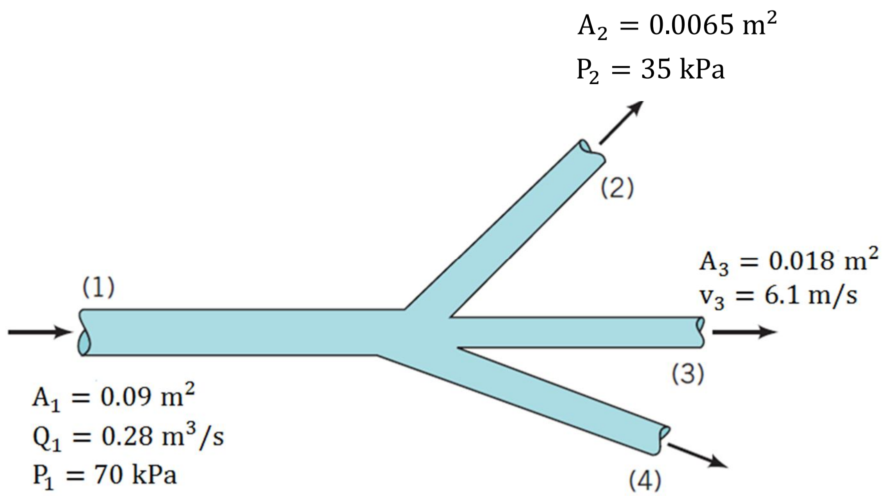
Water flows through the horizontal branching pipe shown below. Calculate:

a) The water speed at section (2).

b) The pressure at section (3).

c) Flow rate at section (4).

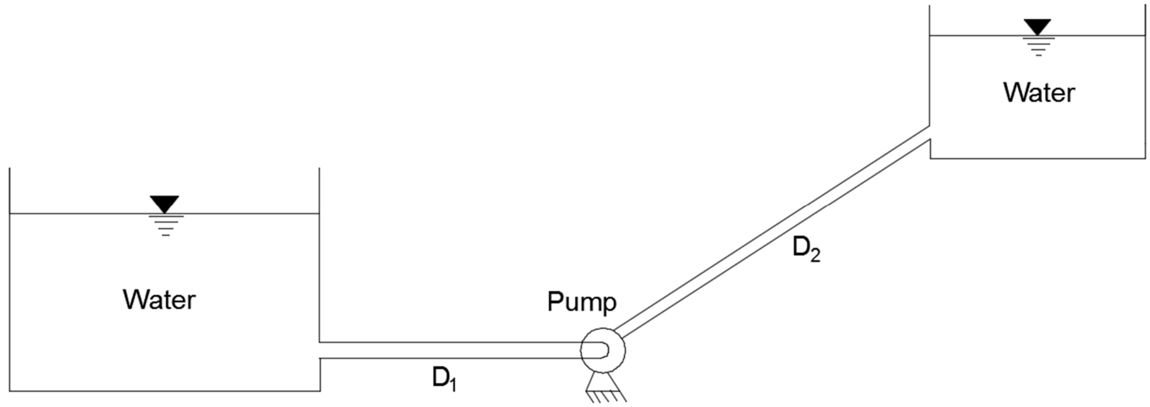
Assume no losses and branch in horizontal plane



Solution

2.

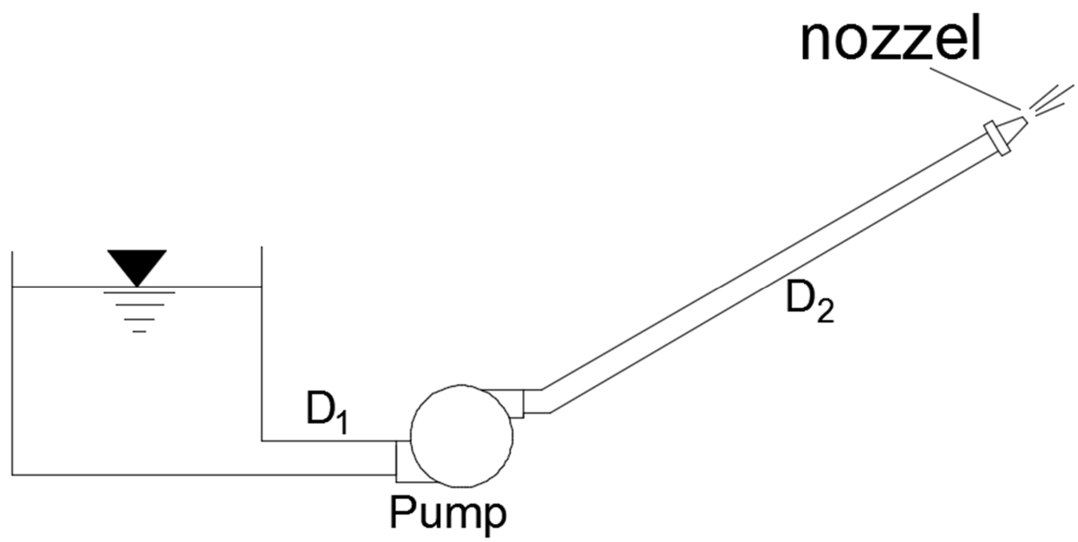
For the shown figure below, if $D_1 > D_2$. Draw the HGL and EGL.



Solution

3.

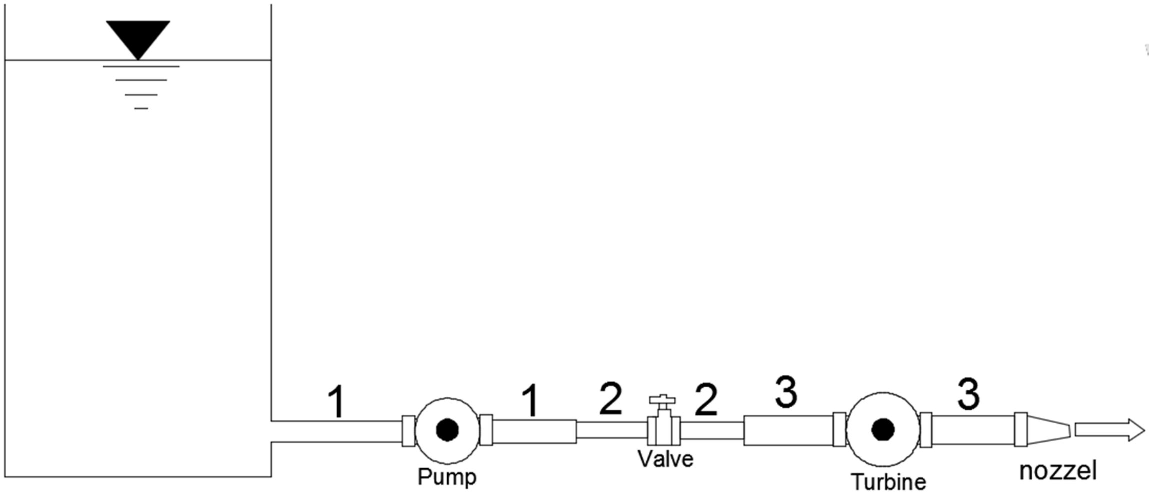
For the shown figure below, if $D_1 > D_2$. Draw the HGL and EGL.



Solution

4.

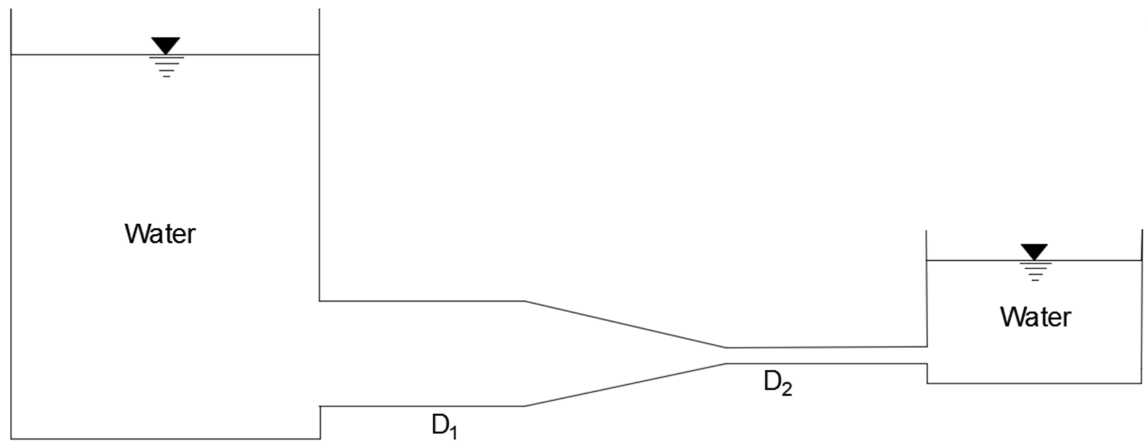
For the shown figure below, if $D_3 > D_1 > D_2$. Draw the HGL and EGL.



Solution

5.

For the shown figure below. Draw the HGL and EGL.



Solution

6.

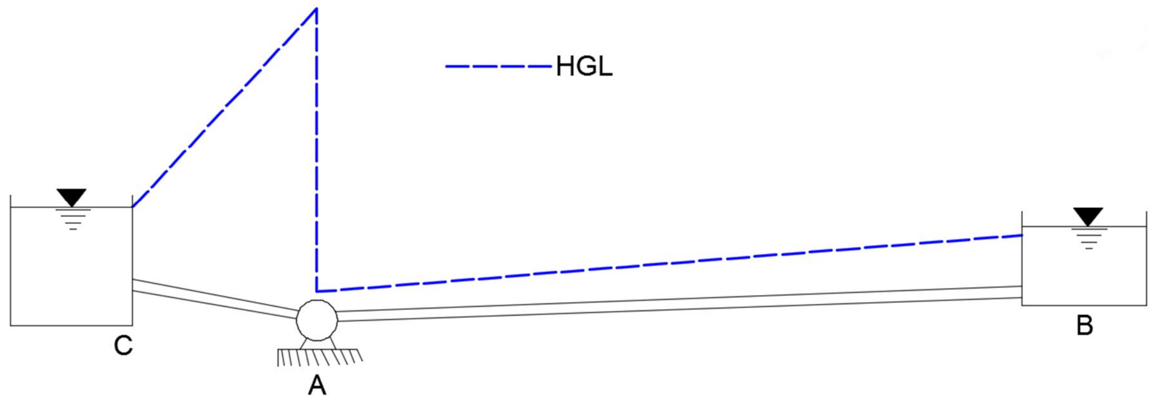
For the system shown below.

a) What is the direction of flow?

b) What kind of machine is at A?

c) Do you think both pipes, AB and CA, are the same diameter?

d) Sketch the EGL for the system.



Solution