Chapter (1)

Fluids and their Properties

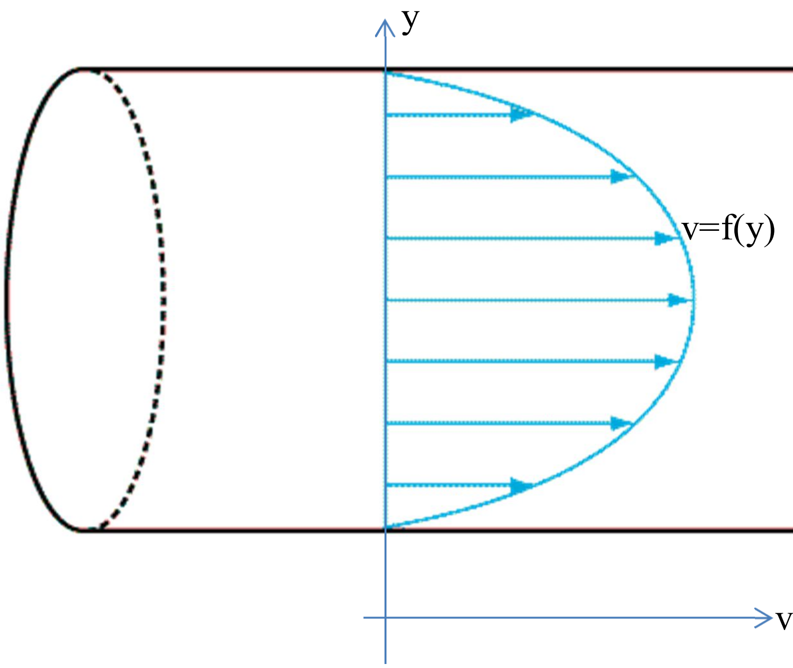
Fluids

- ✓ **(Liquids** or **gases)** which a substance deforms continuously, or flows, when subjected to **shearing forces**.
- ✓ If a fluid is at **rest**, there are **no shearing forces acting**.
- ✓ In fluids we usually deal with **continuous streams** of fluid without beginning or end.

Shear Stress in Moving Particles

If fluid is in motion, shear stress are developed → this occur **if the fluid particles move relative to each other** with different velocities.
However, if the fluid velocity is the same at every point (fluid particles are at rest relative to each other), no shear stress will be produced.

The following figure exhibit the **velocity profile** in a circular pipe:



Note that fluid next to the pipe wall has **zero** velocity (fluid **sticks** to wall), But if the fluid moved away from the wall, velocity increases to **maximum**. Change in velocity (v) with distance(y) is **(velocity gradient):**

$$\text{Velocity gradient} = \frac{dv}{dy}$$

This also called **(rate of shear strain)**

Newton's Law of Viscosity:

$$\tau = \mu \frac{dv}{dy} \quad (Pa = N/m^2)$$

τ = shear stress.

μ = dynamic viscosity (will be discussed later).

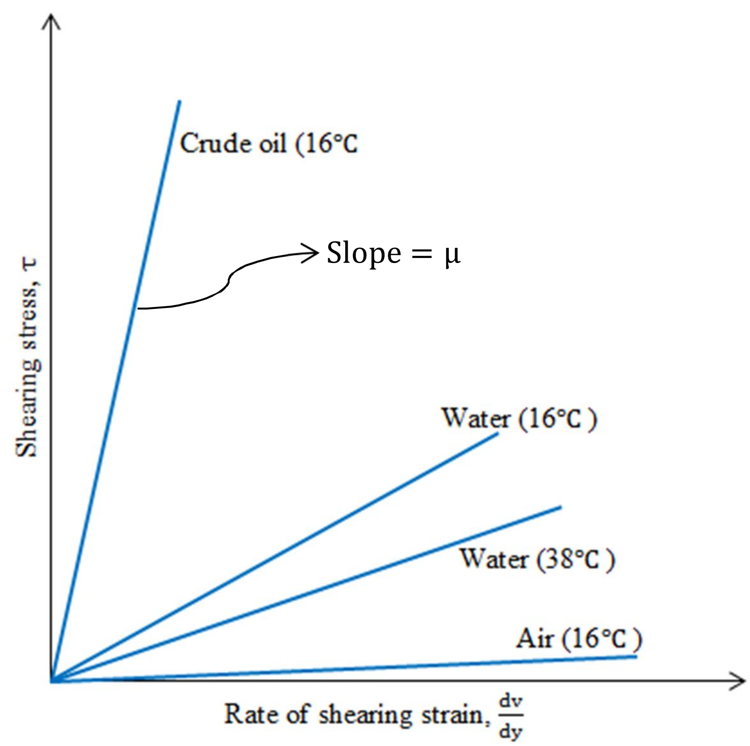
$$\frac{dv}{dy} = \text{velocity gradient}$$

Newtonian & Non-Newtonian Fluid

Newtonian Fluids:

- ✓ Fluids obey (تتبع) Newton's law of viscosity are Newtonian fluids.
- ✓ For this type of fluids, there is a **linear relationship** between shear stress and the velocity gradient.
- ✓ Dynamic viscosity (μ) is the slope of the line.
- ✓ Dynamic viscosity (μ) is constant for a fluid at the same temperature.
- ✓ As temperature increase → (μ) decreases → slope decreases.
- ✓ Most common fluids are Newtonian, for example: Air, Water, Oil, etc…

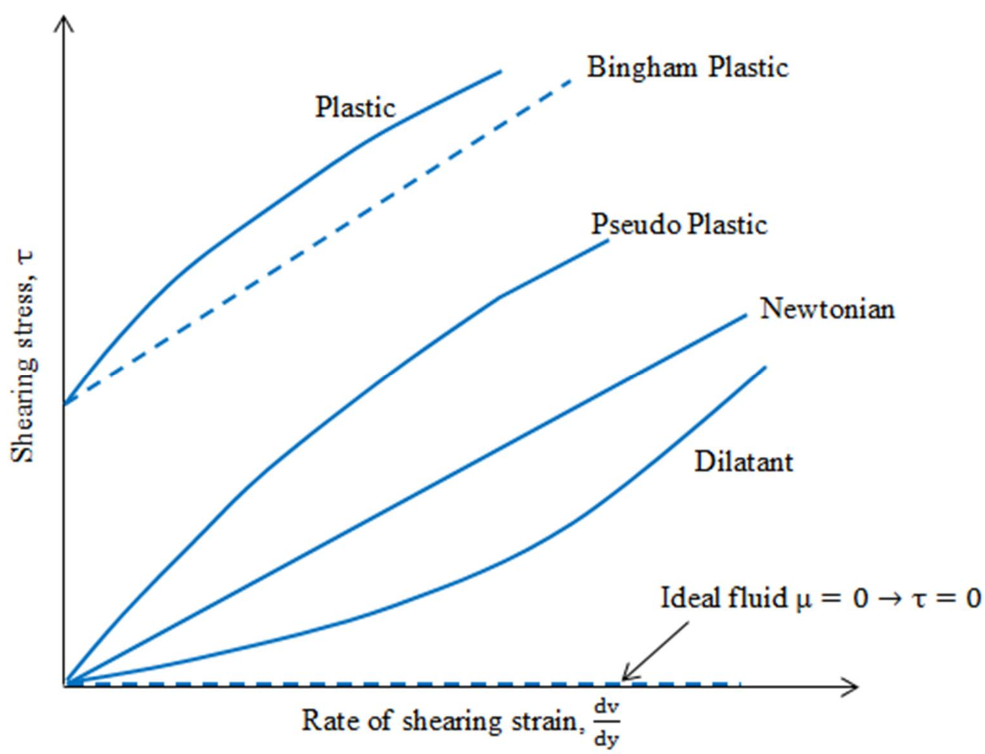
The following graph explains the linear variation of shear stress with rate of shear strain (velocity gradient) for common fluids:



Non-Newtonian Fluids:

- ✓ Fluids don't obey Newton's law of viscosity are Non-Newtonian fluids.
- ✓ For this type of fluids, there is **no linear relationship** between shear stress and the velocity gradient and the slope of the curves is varies.
- ✓ There are several types of Non-Newtonian fluids based on the relationship between shear stress and the velocity gradient.
- ✓ The general relationship is $\tau = A + B\left(\frac{dv}{dy}\right)^n$, the values of A, B, and n depends on the type of Non-Newtonians fluid.
- ✓ For Newtonian fluids $\rightarrow A = 0.0\ ,\ \ B = \mu\ ,\ \ n = 1 \rightarrow$ Newtonian fluids is a special case from the above equation.

The following graph explains the variation of shear stress with rate of shear strain (velocity gradient) for Different types of Non-Newtonian Fluids:



Properties of Fluids

Density: There are many ways of expressing density:

1. **Mass Density (ρ):** Mass per unit volume

$$\rho = \frac{M}{V} \quad (Kg/m^3)$$

Where, M = mass of fluid (Kg), V = volume of fluid (m^3)

Typical values:

Water= $1000\ Kg/m^3$, Mercury = $13546\ Kg/m^3$, Air = $1.23 Kg/m^3$

2. **Specific Weight (γ):** also called (unit weight): is the weight per unit volume.

$$\gamma = \rho\, g = \frac{W}{V} \quad (N/m^3)$$

Where, W = weight of fluid(N) = $M \times g$, V = volume of fluid (m^3)

Typical values:

Water= $9814\ N/m^3$, Mercury = $132943\ N/m^3$, Air = $12.07 Kg/m^3$

3. **Relative Density (σ):** also called (specific gravity "SG"):

$$\sigma = SG = \frac{\rho_{substance}}{\rho_{water\ at\ 4°C}} \quad (Unitless)$$

Typical values:

Water = 1 , Mercury = 1.3

4. **Specific Volume (ν):** is the reciprocal of mass density.

$$\nu = \frac{1}{\rho} (m^3/Kg)$$

Viscosity: it's the ability of fluid to resist shear deformation due to cohesion and interaction between molecules of fluid.
There are two types of viscosity:

1. **Coefficient of Dynamic Viscosity (μ):**

$$\tau = \mu \frac{dv}{dy} \rightarrow \mu = \frac{\tau}{\frac{dv}{dy}} = \frac{\frac{\text{Force}}{\text{Area}}}{\frac{\text{Velocity}}{\text{Distance}}} = \frac{\frac{\text{Force}}{\text{Area}}}{\frac{\text{Distance}}{\text{Distance} \times \text{Time}}} = \frac{\text{Force} \times \text{Time}}{\text{Area}}$$

$$\text{But Force} = \frac{\text{Mass}\times\text{Lenght}}{\text{Time}^2} \rightarrow \mu = \frac{\text{Mass}\times\text{length}\times\text{Time}}{\text{Length}^2\times\text{Time}^2} = \frac{\text{Mass}}{\text{Length}\times\text{Time}}$$

Units: $\frac{Kg}{m\times s} = kg\ m^{-1}\ s^{-1}$

Another Unit: $N = \frac{kg\times m}{S^2} \rightarrow Kg = \frac{N\times S^2}{m} \rightarrow \frac{Kg}{m\times S} = \frac{N\times S^2}{m\times m\times S} = \frac{N}{m^2}.S = Pa.S$

Note that (μ) is often expressed in **Poise (P)** where $10P = 1Pa.S$

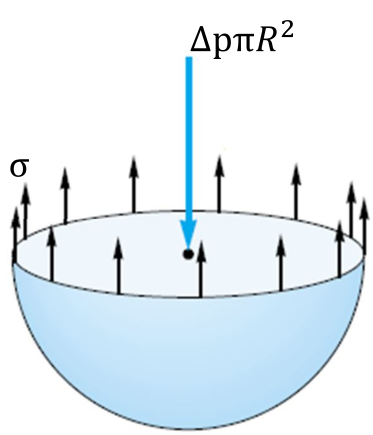
2. **Coefficient of Kinematic Viscosity (ν):**

$$\nu = \frac{\mu}{\rho}$$

Units: $\frac{\frac{Kg}{m\times S}}{\frac{Kg}{m^3}} = \frac{Kg}{m\times S} \times \frac{m^3}{Kg} = \frac{m^2}{S}$

Note that (ν) is often expressed in **Stokes (St)** where $10^4 St = 1\frac{m^2}{S}$

Surface Tension(σ): is a force per unit length (N/m)



R is the radius of the droplet, **σ** is the surface tension, **Δp** is the pressure difference between the inside and outside pressure.
The force developed around the edge due to surface tension along the line:

$$\mathbf{F_{\sigma} = \sigma \times Perimeter = \sigma \times 2\pi R}$$

The force produced from the pressure difference Δp:

$$\mathbf{F_{\Delta p} = \Delta p \times Area = \Delta p \times \pi R^2}$$

(Inside direction, because inside pressure largere thanoutside pressure)

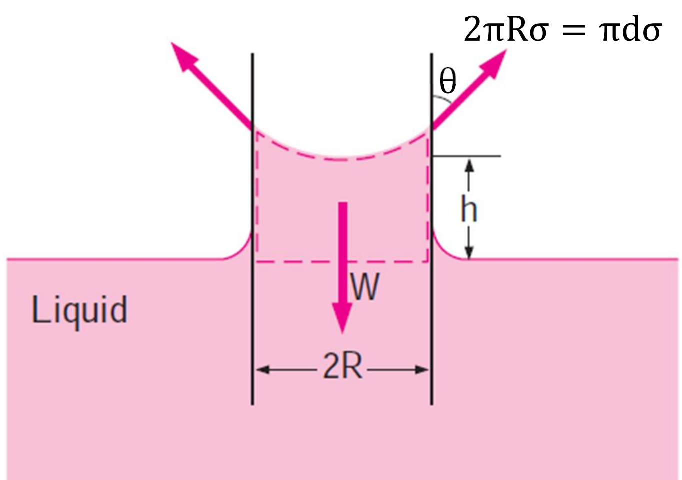
Now, these two forces are in equilibrium so we equating them:

$$\sigma \times 2\pi R = \Delta p \times \pi R^2 \ \rightarrow \Delta p = \frac{2\sigma}{R} = p_i - p_o$$

Capillarity

Capillary effect: is the rise or fall of a liquid in a small-diameter tube caused by surface tension.

h: is the height of water rises.
R: is the radius of the tube (d=2R).
θ: is the **angle of contact** between liquid and solid



The weight of the fluid is balanced with the vertical force caused by surface tension.

$$W = m \times g = \rho \times V \times g$$

$$\rightarrow \pi d\sigma\cos(\theta) = \rho \times \frac{\pi}{4} d^2 \times h \times g$$

$$\rightarrow h = \frac{4\sigma\cos(\theta)}{\rho g d} \text{(only for tube)}$$

if any thing else you should derive the equation with the same concept (see p. 12)

Notes:

- ✓ If $\theta < 90 \rightarrow$ water is rise, If $\theta > 90 \rightarrow$ water is fall, If $\theta = 90 \rightarrow$ no rise or fall ($h = 0.0$).
- ✓ If d is too small → θ is too small and may be neglected.
- ✓ For clean glass in contact with **Water**→ θ = 0.0 (rise).
- ✓ For clean glass in contact with Mercury→ θ = 130 (fall).

Compressibility & Bulk Modulus

All materials are compressible under the application of an external force. The compressibility of a fluid is expressed by its **bulk modulus (K)** such that: $K = \frac{\text{Change in Pressure}}{\text{Volumetric Strain}}$

$\text{Change in Pressure} = \Delta p = P_{final} - P_{initial}$

$$\text{Volumetric Strain} = \frac{V_{final} - V}{V} = \frac{\Delta V}{V}$$

$$K = -\frac{\Delta p}{\Delta V/V}$$

The –ve sign refer to the volume decrease as the pressure increase.
K can be expressed by another form as following:

$$K = \rho\frac{\Delta p}{\Delta \rho}$$

Large values of bulk modulus refer to incompressibility which indicates the large pressures are needed to compress volume slightly.

Famous Unit Conversion

- ✓ 1m=0.3048 ft
- ✓ 1ft=12 inch
- ✓ 1inch=2.54 cm
- ✓ 1Ib = 4.45 N
- ✓ $\gamma_w = 9810\frac{N}{m^3} \times \frac{Ib}{4.45N} \times \frac{(0.3048)^3 m^3}{ft^3} = 62.4\ Ib/ft^3$
- ✓ $g = 9.81\frac{m}{S^2} \times \frac{ft}{0.3048m} = 32.2 ft/S^2$
- ✓ To transform from $\frac{rev}{min}$ to $\frac{rad}{s} \rightarrow$ multiply by $\frac{2\pi}{60}$
- ✓ $v\left(\frac{m}{s}\right) = \omega \times r$ where, ω = velocity in $\frac{rad}{s}$, r = radius of rotation (m)
- ✓ 1slug = 14.59 Kg.
- ✓ Volume of cone $(V_{cone}) = \frac{\pi}{3}R^2 \times h$ (ثلث حجم الاسطوانة)
- ✓ $1m^3 = 10^3$Liter , 1Liter $= 10^3$ ml , $\rightarrow 1m^3 = 10^6$ ml.
- ✓ $1m^3 = 10^6\ cm^3$.

Problems

1.

If 9 m3 of oil weighs 70.5 KN, Calculate its:

A. Specific Weight:

$$\gamma = \frac{\text{Weight}}{\text{Volume}} = \frac{70.5}{9} = 7.833\ \text{KN/m}^3 = 7.833 \times 10^3\ \text{N/m}^3\ ✓.$$

B. Density:

$$\gamma(\text{Newton's}) = \rho \times g \rightarrow \rho = \frac{\gamma}{g} = \frac{7.833 \times 10^3}{9.81} = 798.47\ \text{Kg/m}^3 ✓.$$

C. Specific Volume:

$$\nu = \frac{1}{\rho} = \frac{1}{798.47} = 1.25 \times 10^{-3}\ \text{m}^3/\text{Kg}\ ✓.$$

2.

A cylindrical tube of volume **335 ml** is filled with soda. If the Mass of tube when it's filled by soda is **0.37 Kg** and the weight of tube when it's empty is **0.155 N. Determine** the specific weight, density, and specific gravity for soda.

Solution

$$\gamma_{soda} = \frac{W_{soda}}{V_{soda}}$$

$$V_{soda} = V_{tube} = 335\text{ml} = 335 \times 10^{-6}\text{m}^3$$
$$M_{soda+tube} = 0.37\text{Kg} \rightarrow W_{soda+tube} = M \times g = 0.37 \times 9.81 = 3.63\ \text{N}$$
$$W_{tube} = 0.153\text{N} \rightarrow W_{soda} = W_{soda+tube} - W_{tube} = 3.63 - 0.153 = 3.47\ \text{N}$$
$$\rightarrow \gamma_{soda} = \frac{3.47}{335 \times 10^{-6}} = 10358.2\ \text{N/m}^3 ✓.$$

$$\gamma_{soda} = \rho_{soda} \times g \rightarrow \rho_{soda} = \frac{\gamma_{soda}}{g} = \frac{10358.2}{9.81} = 1055.88\ \text{Kg/m}^3 ✓.$$

$$\text{S.G} = \frac{\rho_{soda}}{\rho_{water@4℃}} = \frac{1055.88}{1000} = 1.0558 ✓.$$

3.

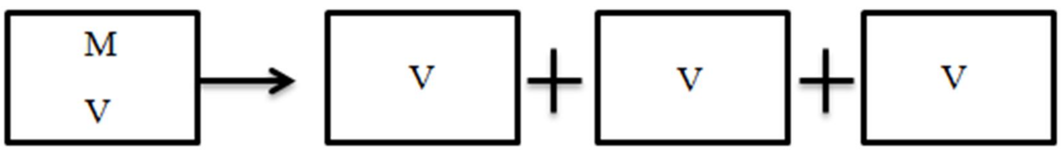
A quantity of soda of mass **M** is filled in a container having volume **V**. The density of soda in this container is 1005 Kg/m^3. If the soda is distributed from this container to **three similar containers** having the same volume **V.** Calculate **specific gravity** and **specific weight** of the soda in each one of three containers after distribution occurs.

Solution

The mass of soda (M) is **the same** before and after distribution

$M_1 = M$ $\qquad\qquad$ $M_2 = M$

$V_1 = V$ $\qquad\qquad$ $V_2 = 3V$



$M_1 = M_2$(Solution Key)

$\rho = \frac{M}{V} \rightarrow M = \rho V$

$\rightarrow \rho_1 V_1 = \rho_2 V_2 \quad (\rho_1 = 1005\ ,\ V_1 = V\ ,\ \rho_2 = ??\ ,\ V_2 = 3V)$

$\rightarrow 1005V = \rho_2 \times 3V \rightarrow 1005 = 3\rho_2 \rightarrow \rho_2 = 335\ Kg/m^3$

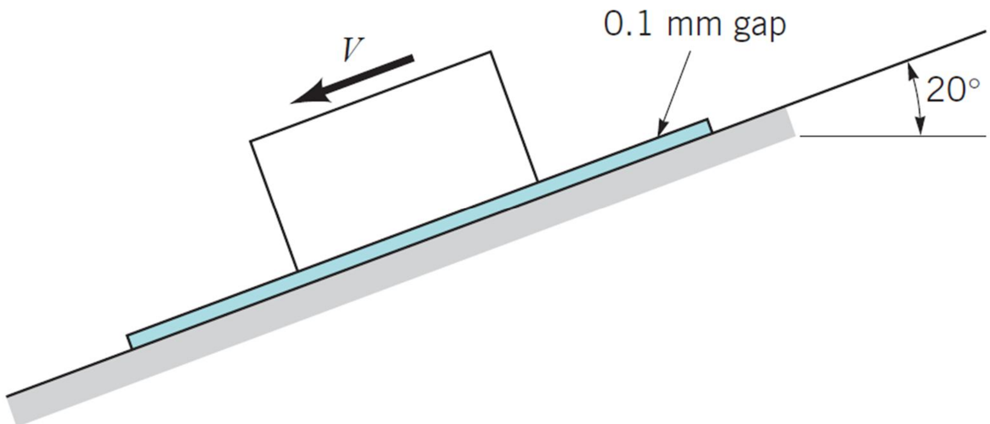
$\rho_2 = 335\ Kg/m^3$ (For soda in each container)

$S.G = \frac{\rho_{soda}}{\rho_{water@4°C}} = \frac{335}{1000} = 0.335$✓ (For soda in each container).

$$\gamma_{soda} = \rho_{soda} \times g = 335 \times 9.81 = 3286.35\ N/m^3 ✓.$$

4.

A 10-kg block slides down a smooth inclined surface as shown in Figure below. Determine the terminal velocity of the block if the 0.1-mm gap between the block and the surface contains oil having dynamic viscosity of 0.38 Pa.s. Assume the velocity distribution in the gap is linear, and the area of the block in contact with the oil is 0.1 m^2.

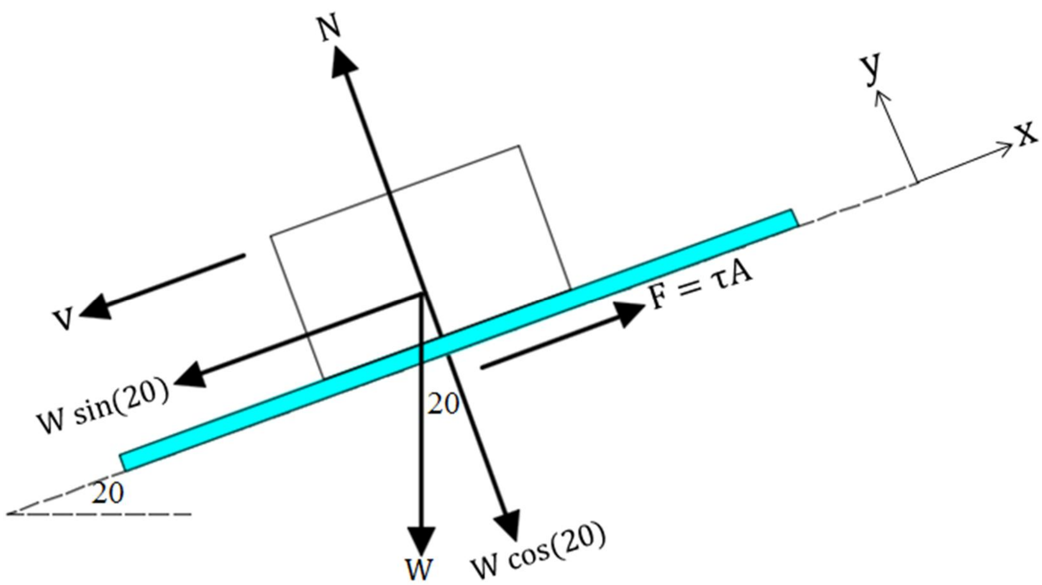


Solution

Important Notes:

- ✓ In all problems, the direction of shear force is in **reverse direction** of movement, because it's the **resistance** of fluid against moving object. So the shear force resists the movement of the object exactly like friction force (if the object moves above rough surface).
- ✓ The summation of **forces** equal zero in all directions, because the velocity is **constant** (we assume it constant) and so the acceleration is zero.

Return to the above problem, the **free body diagram** of the block is:



$$\sum F_x = 0.0 \rightarrow W\sin(20) = \tau A \rightarrow \tau = \frac{W\sin(20)}{A}$$

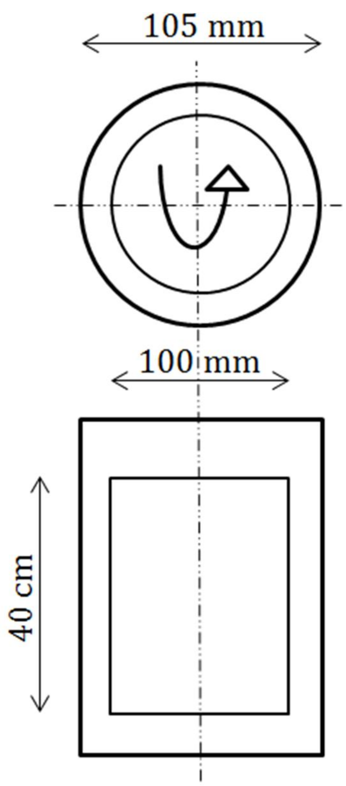
$$W = Mg = 10 \times 9.81 = 98.1\ N \quad , \quad A = 0.1\ m^2\ (\text{given})$$

$$\rightarrow \tau = \frac{98.1 \times \sin(20)}{0.1} = 335.5\ Pa.$$

$$\tau = \mu \frac{v}{y} \rightarrow v = \frac{\tau \times y}{\mu} = \frac{335.5 \times 0.1 \times 10^{-3}}{0.38} = 0.0883\ m/s\checkmark.$$

5.

A cylinder of 40 cm length and 10 cm diameter rotates about a vertical axis inside a fixed cylindrical tube of 105 mm diameter, and 0.4 m length. If the space between the tube and the cylinder is filled with liquid of dynamic viscosity of 0.2 N.s/m^2. **Determine** the external torque that led the cylinder to rotate by speed of 700 rev. /min.



Solution

$$\text{Torque} = \text{Shear force} \times \text{radius of rotating cylinder}$$

$$\text{Shear force (F)} = \tau \times \text{Side Area of rotating cylinder } (A_s)$$

$$\tau = \mu \frac{du}{dy}$$

$$\mu = 0.2 N.s/m^2$$

$$du = \text{velocity of rotating cylinder in } (m/s).$$

$$\omega = 700 rev/min = 700 \times \frac{2\pi}{60} = 73.3\ rad/s.$$

$$v = \omega \times \text{radius of rotating cylinder} = 73.3 \times \frac{0.1}{2}$$

$$\rightarrow v = 3.66\ m/s.$$

$$dy = (\text{ always is: distance perpendicular to the direction of shear force})$$

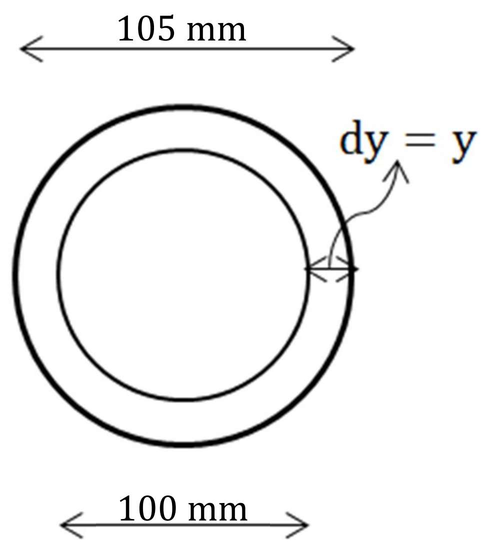
$$dy = \frac{105 - 100}{2} = 2.5mm = 0.0025m$$

$$\rightarrow \tau = 0.2 \times \frac{3.66}{0.0025} = 292.8 \text{ N/m}^2$$

$$A_s = \pi dL = \pi \times 0.1 \times 0.4 = 0.1256 \text{ m}^2$$

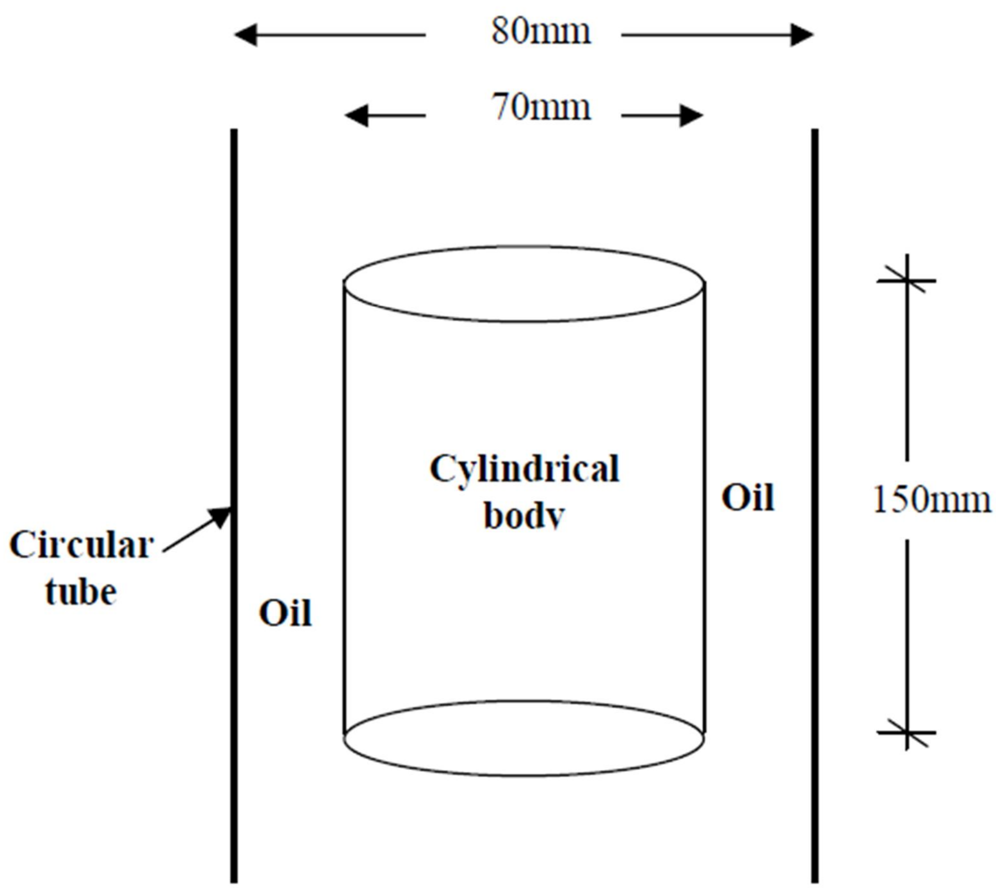
$$\rightarrow \text{Shear force (F)} = 292.8 \times 0.1256 = 36.7 \text{ N.}$$

$$\rightarrow \text{Torque} = 36.7 \times 0.05 = 1.838 \text{ N.m} \checkmark.$$



6.

A cylindrical body of 70 mm diameter and 150 mm length falls freely in a 80 mm diameter circular tube as shown in the figure below. If the space between the cylindrical body and the tube is filled with oil of viscosity 0.9 poise. **Determine the weight of the body when it falls at a speed of 1.5 m/s.**



Solution

The weight of cylindrical body (downward) equals **the resist shear force** (upward) due to the **equilibrium**.

$$\sum F = 0.0 \rightarrow W = \tau \times A$$

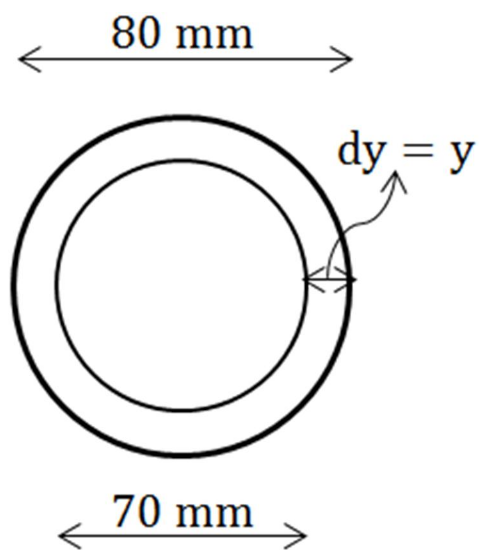
The shear stress caused by cylindrical tube is:

$$\tau = \mu \frac{du}{dy}$$

$\mu = 0.9 \text{ pois, but } 10\text{pois} = 1\text{pa.s} \rightarrow 0.9\text{poise} = \frac{0.9}{10} = 0.09 \text{ pa.s.}$

$du = \text{velocity of freely falling} = 1.5\text{m/s.}$

$$\rightarrow dy = \frac{80 - 70}{2} = 5\text{mm} = 0.005\text{m}$$



$$\rightarrow \tau = 0.09 \times \frac{1.5}{0.005} = 27 \text{ pa.}$$

$$\text{Shear force} = \text{side area of cylindrical body} \times \tau$$

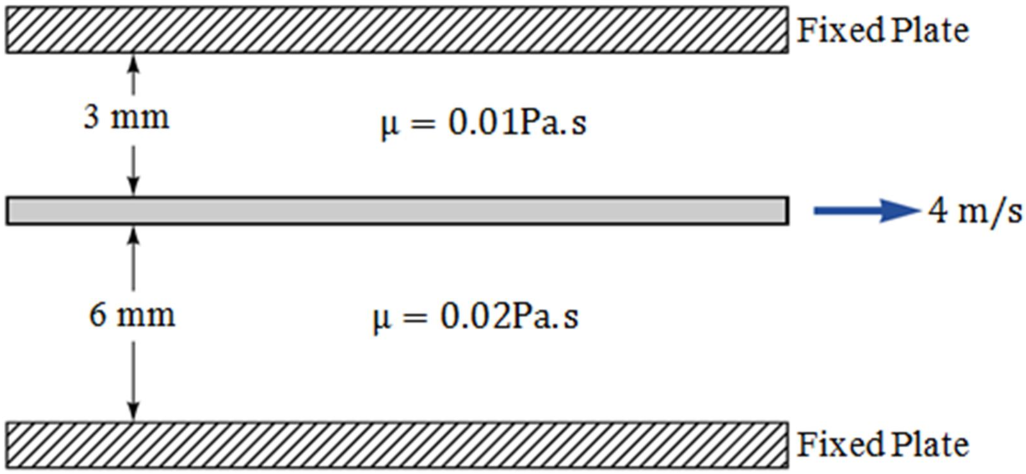
$$F_{shear} = \pi \times d \times L \times \tau = 3.14 \times 0.07 \times 0.15 \times 27$$

$$\rightarrow F_{shear} = 0.89 \text{ N}$$

$$\text{The weight of cylindrical body} = F_{shear} = 0.89 \text{ N}\checkmark.$$

7.

A movable plate of $0.5m^2$ area (for each face) is located between two large fixed plates as shown in the figure below. Two **different** Newtonian fluids having the viscosities indicated are contained between the plates.
Determine the magnitude of the force acting on the movable plate when it moves at a speed of 4.0 m/s.



Solution

The free body diagram of the plate is:

$$\sum F = 0.0 \rightarrow F = \tau_1 \times A + \tau_2 \times A \rightarrow F = A \times (\tau_1 + \tau_2)$$

$u_1 = u_2 = 4\text{m/s}.$

$$\tau_1 = \mu_1 \frac{u_1}{y_1} = 0.01 \times \frac{4}{0.003} = 13.33 \text{ Pa.}$$

$$\tau_2 = \mu_2 \frac{u_2}{y_2} = 0.02 \times \frac{4}{0.006} = 13.33 \text{ Pa.}$$

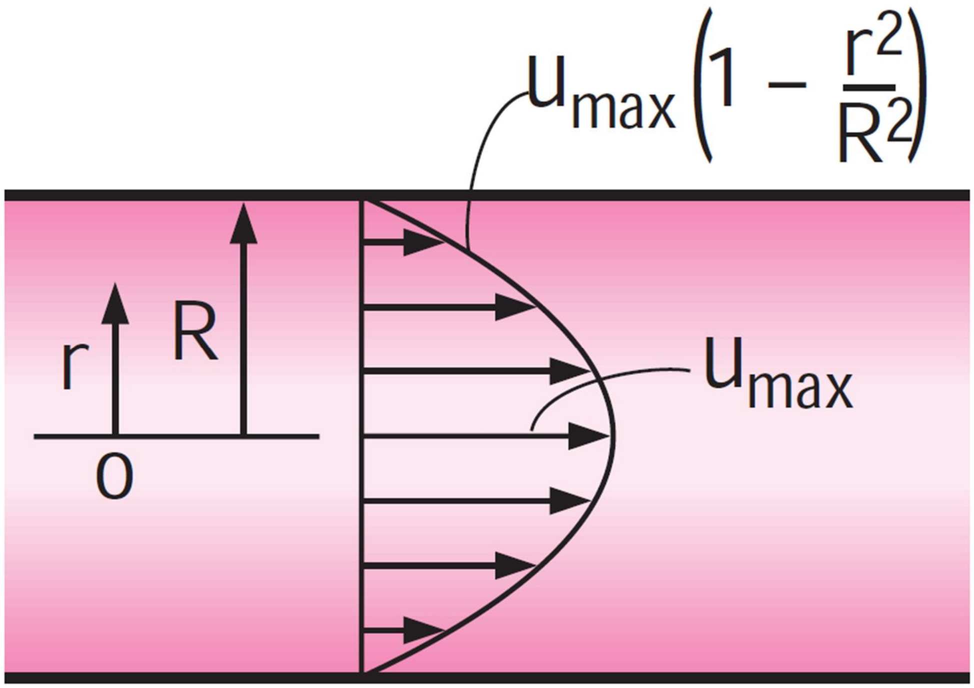
$\rightarrow F = 0.5 \times (13.33 + 13.33) = 13.33 \text{ N} \checkmark.$

8.

Fluid flow through a circular pipe is one-dimensional, and the velocity profile for laminar flow is given by $u(r) = u_{max}\left(1 - \frac{r^2}{R^2}\right)$ where R is the radius of the pipe, r is the radial distance from the center of the pipe, and u_{max} is the maximum flow velocity, which occurs at the center. **Determine:**

(a). a relation for the drag force applied by the fluid on a **section of the pipe** of length L.

(b). the value of the drag force for water flow at 20°C with $R = 0.08m, L = 15m, u_{max} = 3m/s, \mu = 0.001kg/m.s.$



Solution

a)

The shear stress at the **surface of the pipe (at r = R)** is given by:

$$\tau = -\mu\frac{du}{dr} \rightarrow \text{(Negative sign is because u decresed with r increased)}$$

$$\tau = -\mu \times \frac{d}{dr}\left(u_{max}\left(1 - \frac{r^2}{R^2}\right)\right) \quad (\text{But}, u_{max} \text{ is constant}) \rightarrow$$

$$\tau = -\mu u_{max} \times \frac{d}{dr}\left(1 - \frac{r^2}{R^2}\right) = -\mu u_{max} \times \left(0 - \frac{2r}{R^2}\right) \text{ (because R is constant)}$$

$$\rightarrow \tau = \frac{2r\mu u_{max}}{R^2} \text{ (at the surface of the pipe} \rightarrow r = R) \rightarrow \tau = \frac{2\mu u_{max}}{R}.$$

Note: Always we calculate the shear stress at the **fixed surface** (like pipe surface in the above problem) because it gives maximum shear stress.
The drag force that causes shear stress on the pipe surface is:

$$\sum F = 0.0 \rightarrow F_D = \tau \times A$$

$$(A = \text{side area of tube} = 2\pi RL) \rightarrow F_D = \frac{2\mu u_{max}}{R} \times 2\pi RL$$

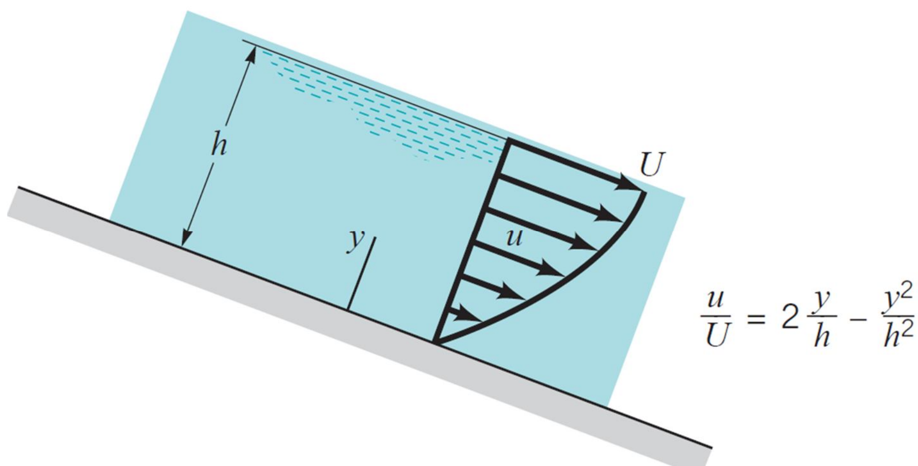
$$\rightarrow F_D = 4\pi L\mu u_{max} \checkmark.$$

b)

$$F_D = 4\pi L\mu u_{max} \rightarrow F_D = 4\pi \times 15 \times 0.001 \times 3 = 0.565\ N \checkmark.$$

9.

A layer of water flows down an inclined fixed surface with the velocity profile shown in Figure below. Determine the **magnitude** of shearing stress that the water exerts on the **fixed surface** for U= 2m/s and h=0.1m $\mu = 1.12 \times 10^{-3}$ pa.s



Solution

The shear stress at the **fixed surface (at y = 0.0)** is given by:

$$\tau = +\mu\frac{du}{dy} \rightarrow (\text{Positive sign is because u incresed with y increased})$$

$$\frac{u}{U} = 2\frac{y}{h} - \frac{y^2}{h^2} \rightarrow u = U\left(2\frac{y}{h} - \frac{y^2}{h^2}\right) \ (\text{But}, U = 2m/s \ \text{and} \ h = 0.1m) \rightarrow$$

$$u = 2\left(\frac{2y}{0.1} - \frac{y^2}{0.1^2}\right) \rightarrow u = 40y - 200y^2$$

$$\rightarrow \frac{du}{dy} = 40 - 400y \ \ (\text{But, at the fixed surface } y = 0.0) \rightarrow \frac{du}{dy} = 40$$

$$\rightarrow \tau = 1.12 \times 10^{-3} \times 40 = 44.8 \times 10^{-3} \text{N/m}^2 \checkmark.$$

10.

When a 2-mm-diameter tube is inserted into a liquid in an open tank, the liquid is observed to rise 10 mm above the free surface of the liquid. The contact angle between the liquid and the tube is **zero**, and the specific weight of the liquid is 1.2 x 10^4 N/m^3. **Determine** the value of the surface tension for this liquid.

Solution

For tube the capillary rise can be expressed by the relation tha we previously derived:

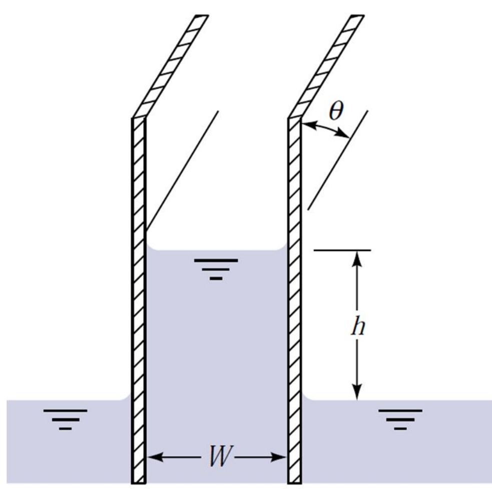
$$h = \frac{4\sigma\cos(\theta)}{\rho g d} \rightarrow \sigma = \frac{h\rho g d}{4\cos(\theta)}$$

$$h = 0.01\text{m}, \rho g = \gamma = 1.2 \times 10^4 \text{N/m}^3, d = 0.002\text{m}, \theta = 0 \rightarrow$$

$$\sigma = \frac{0.01 \times 1.2 \times 10^4 \times 0.002}{4\cos(0)} = 0.06 \text{ N/m} \checkmark.$$

11.

Derive an expression for the capillary height change **h** for a fluid of surface tension σ and contact angle θ between **two** vertical parallel **plates** a distance W apart, as in figure.



Solution

Assume the side length of each plate is (b).

$$\sum F = 0.0 \rightarrow F_{\sigma} = \text{Weight of water column}$$

$$F_{\sigma} = 2 \times \sigma \times b \times \cos(\theta) \qquad \text{(2 because there are two plates)}$$

$$\text{Weight} = M \times g = \rho \times V \times g \qquad (V = W \times h \times b)$$

$$\rightarrow \text{Weight} = W \times h \times b \times \rho \times g$$

$$\rightarrow 2 \times \sigma \times b \times \cos(\theta) = W \times h \times b \times \rho \times g \rightarrow h = \frac{2\sigma \cos(\theta)}{\rho g W} \checkmark.$$

12.

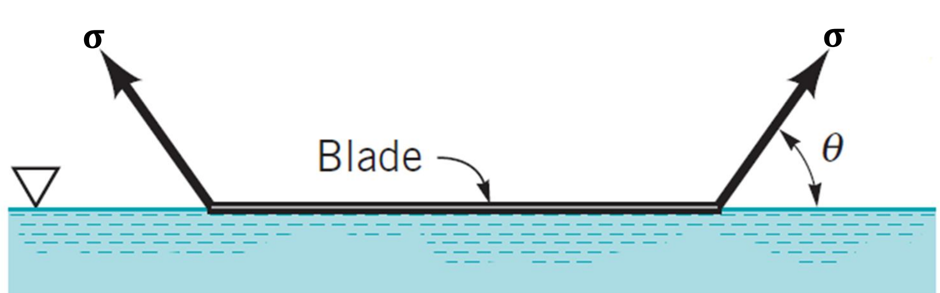
Assume that the surface tension of 7.34×10^{-2} N/m act at an angle θ relative to the water surface as shown in Figure below.

a.

If the mass of the double-edge blade is 0.64×10^{-3} Kg, and the **total** length of its **sides** is 206 mm. Determine the value of θ required to maintain equilibrium between the blade weight and the resultant surface tension force.

b.

If the mass of the single-edge blade is 2.61×10^{-3} Kg, and the total length of its sides is 154 mm. Explain why this blade sinks. Support your answer with the necessary calculations.



Solution

a.

Note: The surface tension σ exists about all sides of the blade so to calculate the surface tension force F_σ we should multiply the value of σ by the total length of blade sides as following:

$F_\sigma = \sigma \times \text{total length} = 7.34 \times 10^{-2} \times 0.206 = 0.0152\ \text{N}$

$\rightarrow F_\sigma = 0.0152\ \text{N}$ with inclination of θ with horizontal →→

The weight of the blade is:

$W = Mg = 0.64 \times 10^{-3} \times 9.81 = 6.278 \times 10^{-3}\text{N}$ (downward)

By Equilibrium→ $\sum F_{\text{vertical}} = 0.0$

$\rightarrow 0.0152 \sin(\theta) = 6.278 \times 10^{-3} \rightarrow \theta = 24.4^{\circ}$ ✓.

b.

$F_\sigma = \sigma \times \text{total length} = 7.34 \times 10^{-2} \times 0.154 = 0.0113\ \text{N}$

$F_{\sigma,\text{vertical}} = 0.0113 \sin(\theta)$

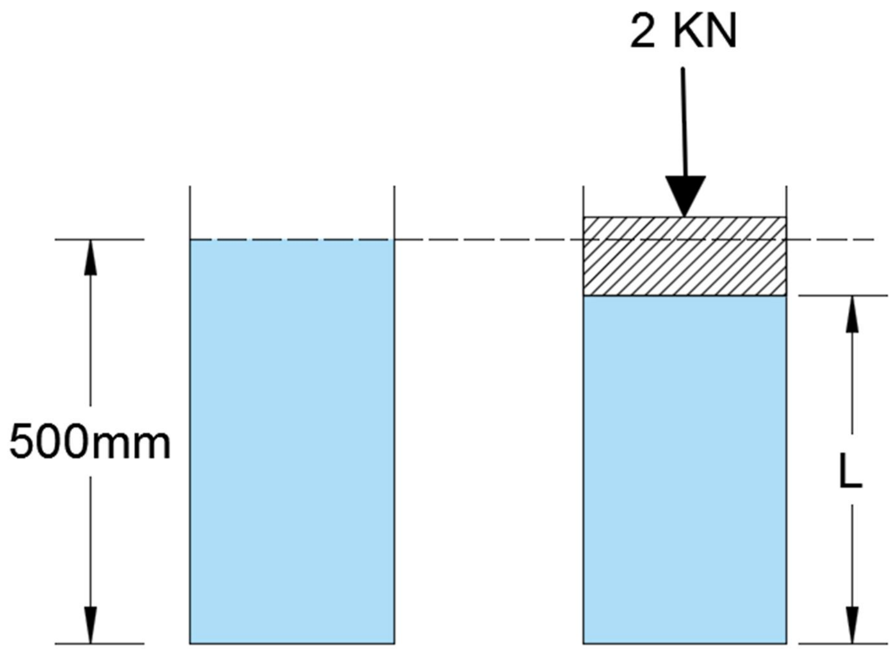
$W = Mg = 2.61 \times 10^{-3} \times 9.81 = 0.0256\text{N}$

Now, the maximum value of $\sin(\theta) = 1$ so, the maximum value of $F_{\sigma,\text{vertical}} = 0.0113$ N.

Note that the value of $W = 0.0256\text{N}$ (downward) is greater than the value of $F_{\sigma,\text{vertical}} = 0.0113$ N so, the blade will sink downward ✓.

13.

A rigid cylinder (15 mm inside diameter) contains column of water 500mm length. If the bulk modulus of water is $K_{water} = 2.05 \times 10^9\ N/m^2$. **What will be the column length if a 2 KN force is applied it's end by frictionless plunger?** Assume no leakage.



Solution

$$K = -\frac{\Delta p}{\Delta V/V}$$

$K = 2.05 \times 10^9$ Pa.

$\Delta p = P_f - P_i$ $\quad$ ($P_i = 0.0$ because no applied forces in first case)

$$P_f = \frac{\text{Force}}{\text{Area}} = \frac{2 \times 10^3}{\frac{\pi}{4} \times 0.015^2} = 11.317 \times 10^6 \text{Pa.}$$

$$\rightarrow \Delta p = 11.317 \times 10^6 - 0 = 11.317 \times 10^6 \text{Pa.}$$

$$\frac{\Delta V}{V} = \frac{V_f - V}{V} = \frac{L \times A - 0.5 \times A}{0.5 \times A} = \frac{L - 0.5}{0.5}$$

$$\rightarrow K = -\frac{\Delta p}{\frac{\Delta V}{V}} \rightarrow 2.05 \times 10^9 = -\frac{11.317 \times 10^6}{\frac{L - 0.5}{0.5}} \rightarrow L = 0.497 \text{ m} \checkmark.$$