Chapter (2)
Pressure and Head

Pressure

A fluid will exerts a **normal force** on any boundary it is in contact with, and these boundaries may be varies and the force may differ from place to place. So, it is convenient to work in terms of Pressure **(P)** which is **the force per unit area.**

$$\text{Pressure} = \frac{\text{Force}}{\text{Area over which the force is applied}} \rightarrow \text{P} = \frac{\text{F}}{\text{A}}$$

Unit:

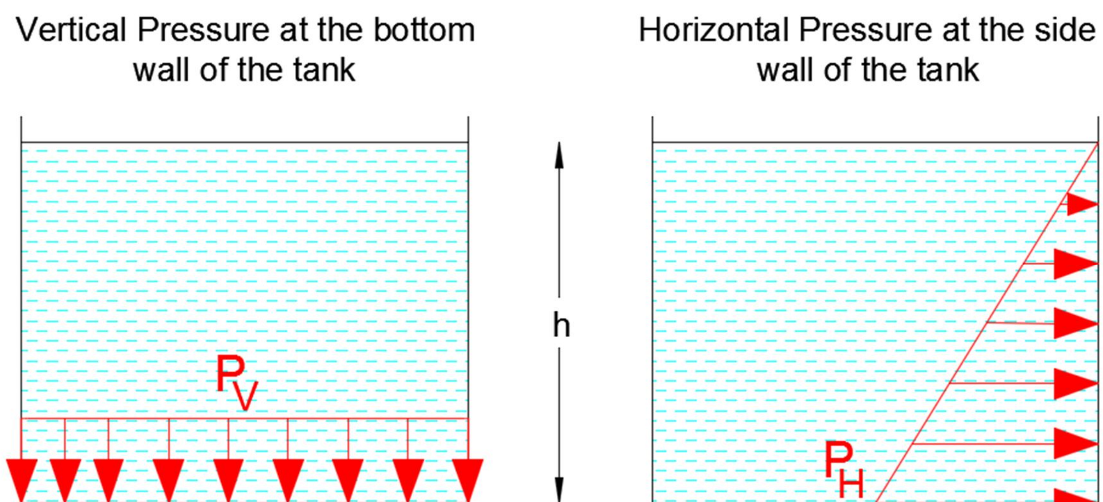
$$\frac{\text{N}}{\text{m}^2} = \text{pascal (Pa)}$$

Also there is a famous unit used for expressing pressure which is **bar** such that $(1\text{bar} = 10^5\text{Pa})$.

Pascal's Law for Pressure at a point

This law applies to **fluid at rest** and it says" **Pressure at any point is the same in all directions"**

The following figure clarifies the concept of **Pascal's law**:

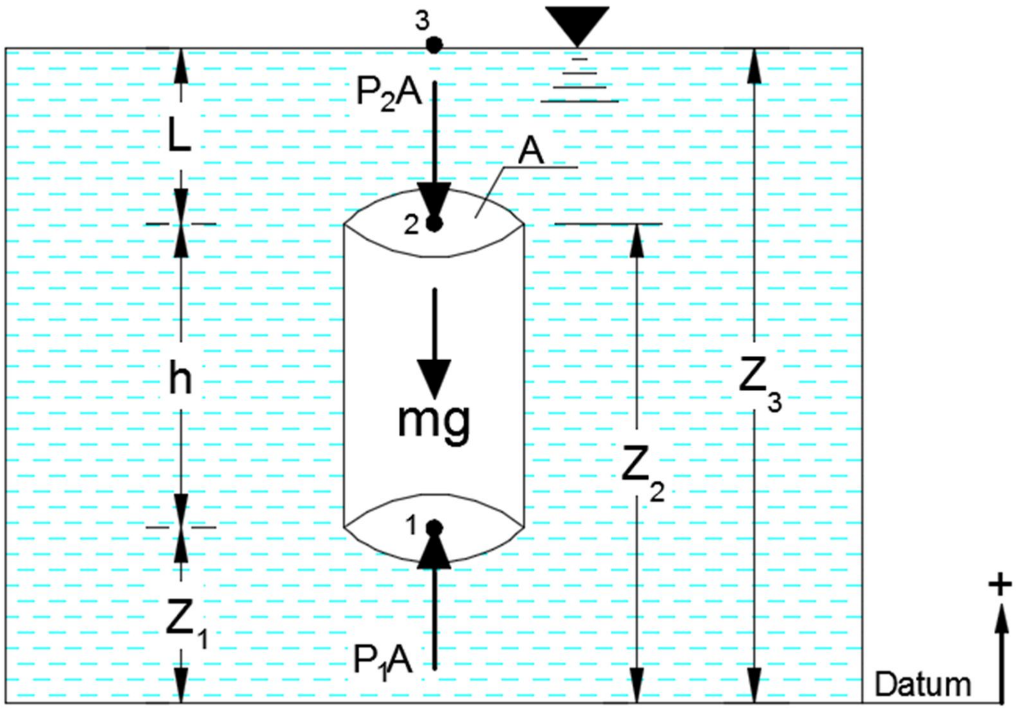


As shown in figure above, a tank contains water and the vertical pressure at depth (h) is P_V and it's applied on the bottom wall of the tank.

Also, the horizontal pressure at the **same depth (**i.e. same point) is P_H and it's applied on the side wall of the tank.

The value of P_V **equal** the value of P_H at the same point (same depth h) and the values of pressure in all directions are the same. (This is Pascal's Law)

Variation of Pressure Vertically in a Fluid under Gravity



Consider the vertical cylinder immersed in fluid of mass density ρ as shown in figure above. The fluid is at rest and in equilibrium so, **all forces in the vertical direction sum to zero.** These forces are**:**

✓ Force due to pressure P_1 at the bottom of the cylinder at level Z_1 on area A and directed **upward** = P_1A.

✓ Force due to pressure P_2 at the top of the cylinder at level Z_2 on area A and directed **downward** = P_2A

✓ Force due to weight of element (cylinder) directed downward $= \mathrm{mg}$

But, $m = \rho V = \rho \times A \times (Z_2 - Z_1) \rightarrow mg = \rho \times g \times A \times (Z_2 - Z_1)$.

From equilibrium $\rightarrow \sum F_{vertical} = 0.0 \rightarrow P_1 \times A - P_2 \times A - mg = 0.0$

$\rightarrow P_1 \times A - P_2 \times A - \rho \times g \times A \times (Z_2 - Z_1) = 0.0 \rightarrow$ (Divided by A)

$$\rightarrow P_1 - P_2 - \rho \times g \times (Z_2 - Z_1) = 0.0 \rightarrow \Delta P = P_1 - P_2 = +\rho g(Z_2 - Z_1)$$

$$\rightarrow P_2 - P_1 = -\rho g(Z_2 - Z_1)$$

From the above derivation we note the following:

✓ $P_1 > P_2$ because the column height of fluid at 1 is larger than at 2

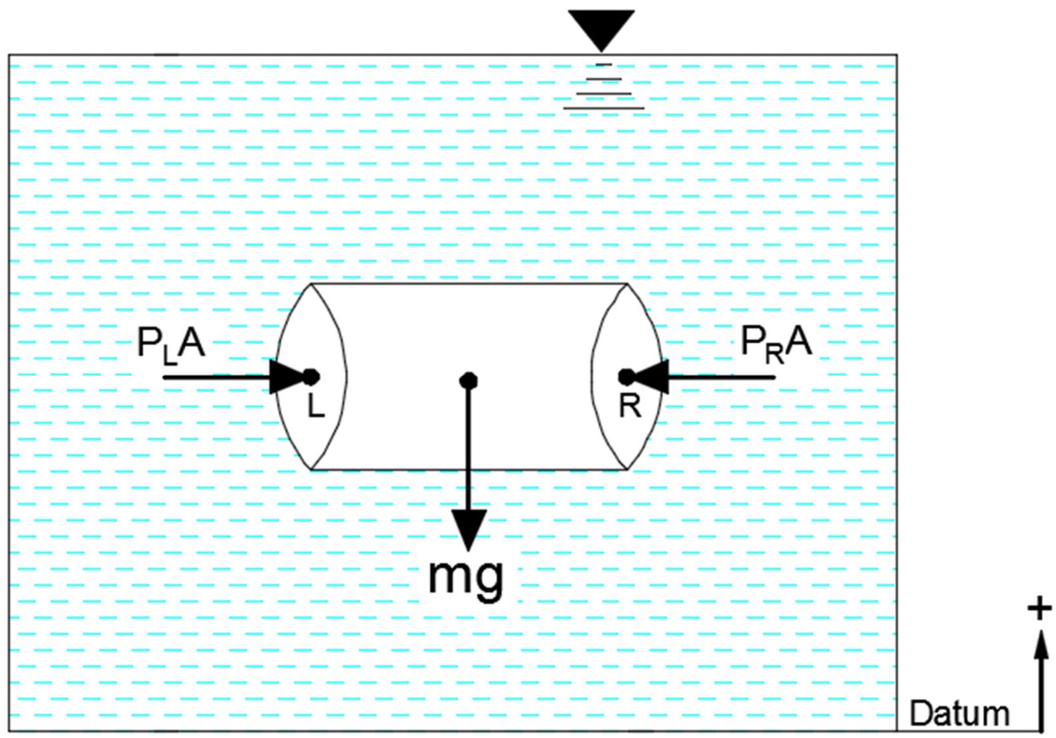
✓ So, as the depth (column height)increase, the pressure will also increase.

✓ Pressure difference (ΔP) always equals (ρgh) such that the distance h called (Pressure Head) and it's the **vertical distance (**difference in elevation with respect to a specific datum) between the two points that's we want to calculate the pressure difference between them.

✓ For example the pressure difference between points 2 and 3 is:

$P_2 - P_3 = \rho gL$ such that $L = Z_3 - Z_2$ and so on.

Equality of Pressure at the Same Level in a Static Fluid



The fluid is at rest and in equilibrium so, **all forces in the horizontal direction sum to zero.**

$\sum F_{horizontal} = 0.0 \rightarrow P_LA - P_RA = 0.0 \rightarrow P_LA = P_RA \rightarrow$ (Divided by A)

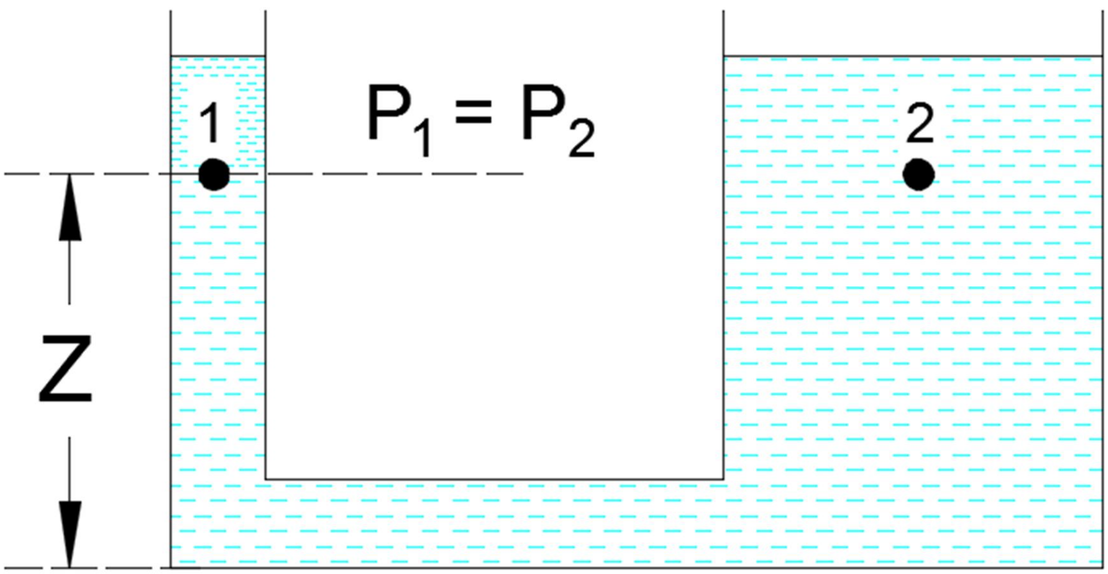
$\rightarrow P_L = P_R$

By another way, $P_L - P_R = \Delta P = \rho gh$ (but $h = 0$) $\rightarrow P_L - P_R = 0 \rightarrow P_L = P_R$

From the above derivation we note that the pressure difference between any points having the same elevation **is the same** but in two conditions:

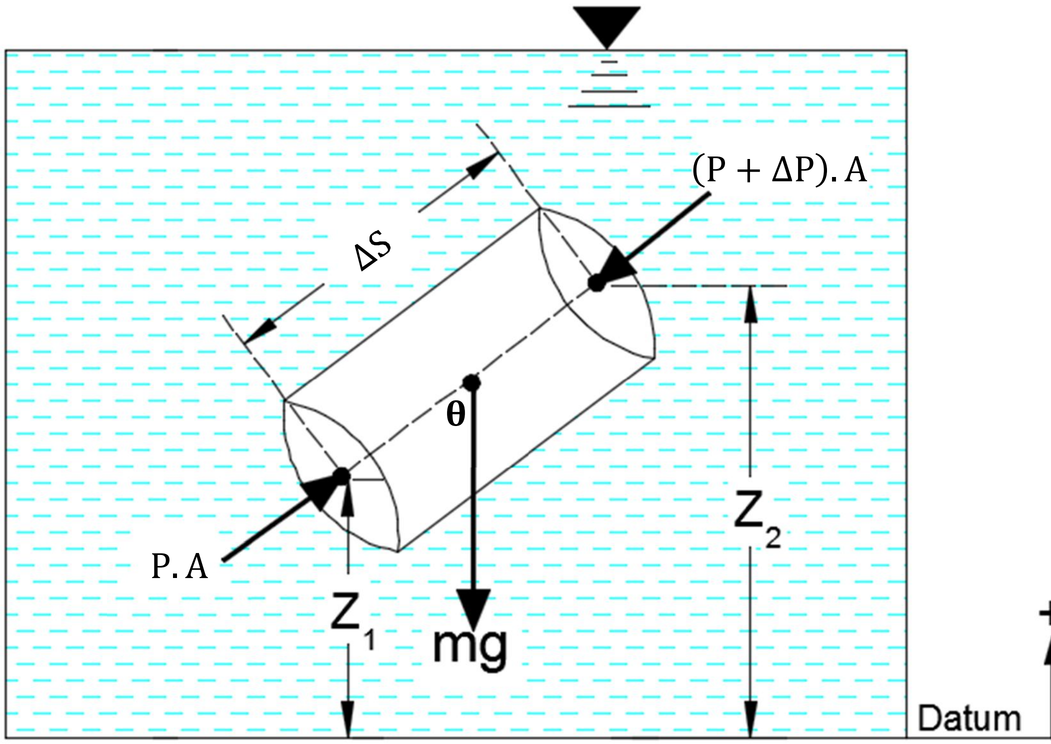
- ✓ The flow is continuous through these points.
- ✓ The fluid is the same fluid at theses points (same mass density, ρ,).

Note: If these two conditions are satisfied the pressure at any points at the same level is equal even if these points don't have the same area because we deal with pressure which not related to area.



Note that the conditions are satisfied (continuous fluid and same fluid), so the pressure at points 1 and 2 is equal. But the **forces** at points 1 and 2 are not equal because the area at point 1 doesn't equal the area at point 2.

General Equation for Variation of Pressure in a Static Fluid



The fluid is at rest and in equilibrium so, **all forces in the all directions sum to zero.**

$\sum F_{\text{oblique plane}} = 0.0 \rightarrow PA - (P + \Delta P)A - mg\cos(\theta) = 0.0$

$mg = \rho Vg = \rho.A.\Delta S.g \rightarrow PA - (P + \Delta P)A - \rho.A.\Delta S.g\cos(\theta) = 0.0$

$\rightarrow$ (Divided by A) $\rightarrow P - (P + \Delta P) - \rho.g.\Delta S.\cos(\theta) = 0.0$

But, $\Delta S.\cos(\theta) = (Z_2 - Z_1)$ = vertical distance (difference in elevation)

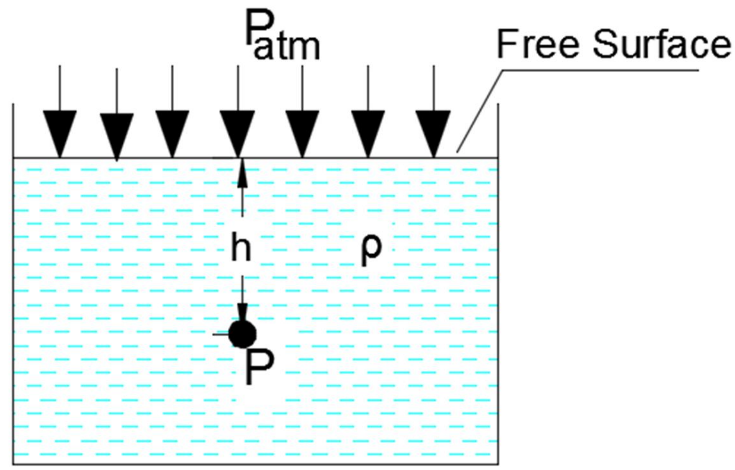
$\rightarrow P - P - \Delta P = \rho.g.(Z_2 - Z_1) \rightarrow \Delta P = -\rho g(Z_2 - Z_1)$

$-$ve Sign is because the pressure at elevation Z_1 is larger than the pressure at elevation Z_2.

Important Note: For all types of surfaces (vertical, horizontal, and oblique) always the difference in pressure between any two points is measured by the relation ($\Delta P = \rho gh$) such that the distance (h) is the vertical distance between these two points.

Pressure and Head

For a static fluid of constant density (ρ) the relation of vertical pressure can be expressed as: $\frac{dP}{dz} = -\rho g \rightarrow dP = -\rho g.dz \rightarrow$ by integrating both sides $\rightarrow \int dP = \int -\rho g \, dz \rightarrow P = -\rho g z + \text{constant}$



As shown in figure above, distance z is measured from the free surface $\rightarrow z = -h \rightarrow P = \rho g h + \text{constant}$.

The constant in the above equation is the pressure at the free surface of fluid which normally is atmospheric pressure $P_{atmospheric} \rightarrow\rightarrow$

$P = \rho g h + P_{atm}$

Atmospheric Pressure: Is the pressure which we live constantly under it, and everything else exists under this pressure. So, **it is convenient to take atmospheric pressure as the datum. Thus,** we measure pressure above or below atmospheric pressure (i.e. atmospheric pressure is a datum).

This measured pressure is known as **gauge pressure**→ for example →

For the above figure, the atmospheric pressure is at the free surface of fluid and the pressure measured from this surface (datum) is $P = \rho g h$ which is the **gauge pressure**(P_{gauge}).

Gauge pressure can be positive (above atmospheric) or negative (lower the atmospheric).

Also negative gauge pressure is known as vacuum pressure.

Atmospheric pressure at any free surface exposed to atmosphere is **zero gauge pressure** (because the atmospheric pressure is a datum for gauge pressure).

Absolute Pressure: is the sum of gauge and atmospheric pressures and it is always positive.

$P_{absolute} = P_{gauge} + P_{atmospheric}$

Notes:

- ✓ In most cases we deal with gauge pressure (i.e. taking atmospheric pressure as a datum having **zero** gauge pressure).
- ✓ By taking $P_{atm} = 0.0 \rightarrow P = \rho g h \rightarrow$ if the density of fluid is known, we can measure pressure by head such that:

  Pressure head (h) $= \frac{\rho g h}{h}$ (m).

The Hydrostatic Paradox

The pressure exerted by the fluid depends only on:

- ✓ Vertical head of fluid (h).
- ✓ Type of fluid (mass density , ρ,)
- ✓ **Not** on the weight of the fluid present (because the weight is already considered in $h, \rho,$ and g).

Whatever the shape or size of the containers that contains the same fluid with same head (h), **the pressure at the bottom of all containers is the same (**this called **Pascal's Paradox).** But the force at the bottom of each container depends on the base area of the container.

Pressure Measurement by Manometers

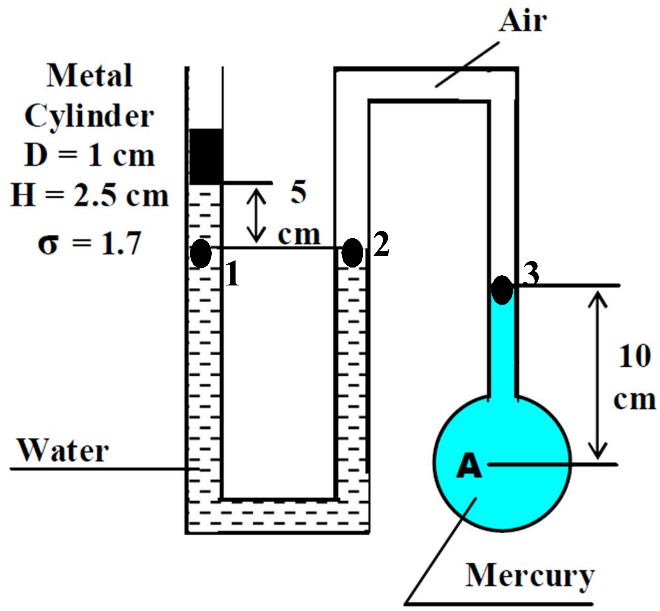
Manometers are devices used to measure the pressure of fluid using the relationship between pressure and head.

Different types of manometers will be discussed in problems of this chapter.

Problems

1.

Calculate gauge pressure at point **A** if ($\rho_{mercury} = 13600\ Kg/m^3$)



Solution

Pressure exerted by metal cylinder:

$P_{metal\ cylinder} = \rho g h$

$= 1.7 \times 1000 \times 9.81 \times 0.025$

$= 416.925$ Pa.

Important Note:

Since the density of air is very small, we can neglect the pressure exerted by air inside the tube. →→

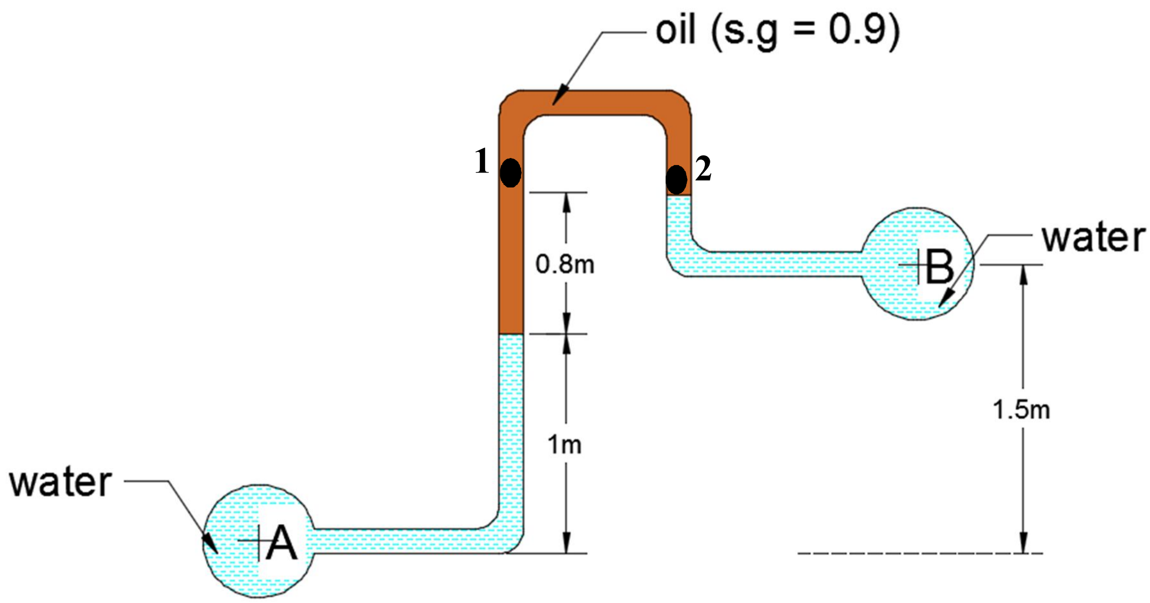
$P_1 = P_2 = P_3$ (solution key)

$$416.935 + 1000 \times 9.81 \times 0.05 + 13600 \times 9.81 \times 0.1 = P_A$$

$P_A = 14249\ Pa = 14.249\ kPa$✓.

2.

Calculate the gauge pressure difference between A and B.



Solution

$P_1 = P_2$ (Solution Key)

Distance from point (2) to gauge (B) = 1.8-1.5= 0.3m

$P_A - 1000 \times 9.81 \times 1 - 0.9 \times 1000 \times 9.81 \times 0.8$

$+1000 \times 9.81 \times 0.3 = P_B$

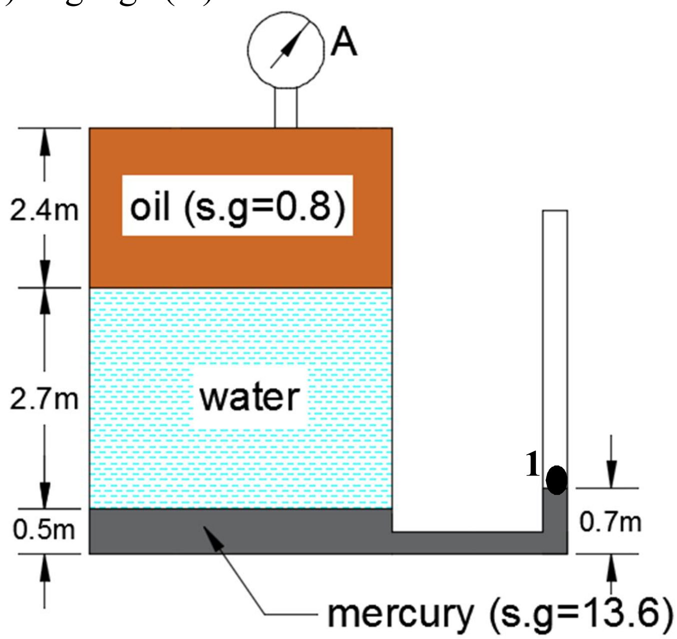
$\rightarrow P_A - P_B = 13{,}8930.2\ \text{Pa} = 13.93\ \text{kPa}$ ✓.

3.

In the figure shown, determine the reading of the gauge A.

Solution

Applying pressure equation from point (1) to gauge (A).



$0 + (13.6 \times 1000 \times 9.81 \times 0.7)$

$-(13.6 \times 1000 \times 9.81 \times 0.5)$

$-(1000 \times 9.81 \times 2.7)$

$-(0.8 \times 1000 \times 9.81 \times 2.4) = P_A$

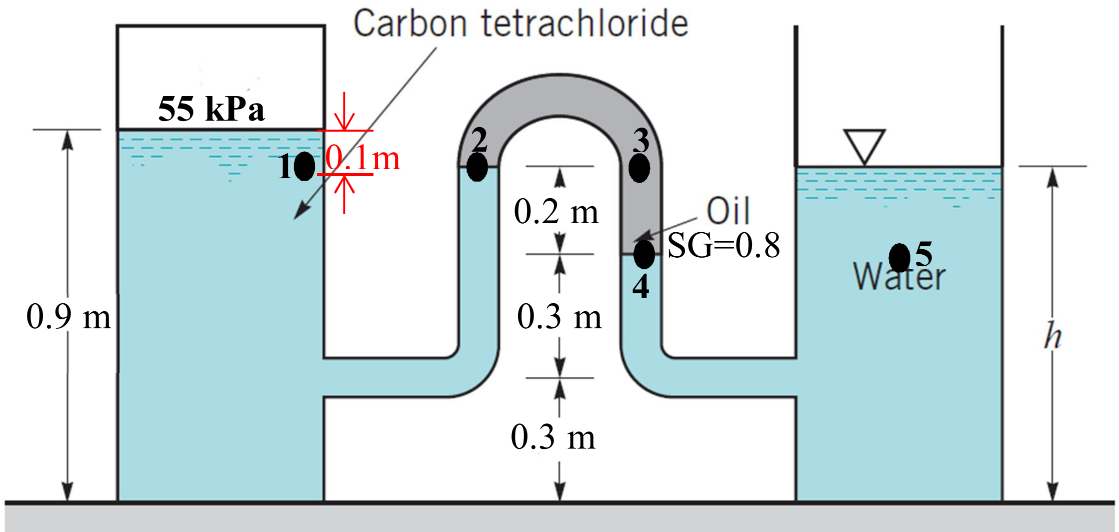
$\rightarrow P_A = -18{,}639 \text{ Pa}$✓.

$= -18.639 \text{ kPa}$✓.

Negative sign means lower than atmospheric or (vacuum pressure)

4.

In the figure shown, the left reservoir contains carbon tetrachloride (SG=1.6) and it is closed and pressurized to 55 kPa. The right reservoir contains water and is open to atmosphere. **Determine the depth of water *h* in the right reservoir.**



Solution

Important Notes:

✓ The best way to solve problems in this chapter is to make one equation starting and ending by points of known pressure.

✓ And you **should always remember** (the pressure for two points at the same level is the same, if the fluid is continuous) because this is the key for solving all problems in this chapter, for example $P_1 = P_2 = P_3$(in figure)

✓ Also there is a general rule in solving all problems which is (if you directed downward you will sum the pressure, however if you directed upward you will subtract the pressure) because pressure increase with depth.

Distance from surface to point (1) $= 0.9 - (0.2 + 0.3 + 0.3) = 0.1\text{m}$

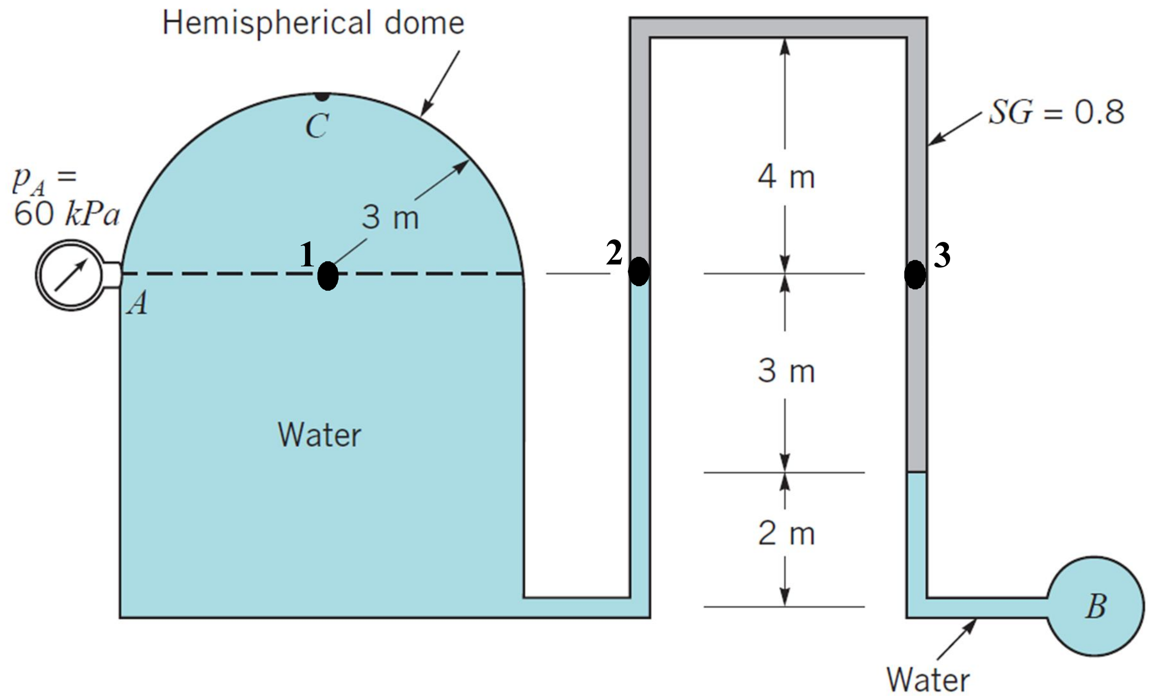
$$55 \times 10^3 + 1.6 \times 1000 \times 9.81 \times (0.1) + 0.8 \times 1000 \times 9.81 \times 0.2$$
$$-1000 \times 9.81 \times \big(h - (0.3 + 0.3)\big) = 0.0 \ (P_{atm} = \text{zero gauge pressure})$$

Solve the equation for h→ $h = 6.52$ m ✓

5.

A closed cylindrical tank filled with water has a hemispherical dome and is connected to an inverted piping system as shown in figure below. The liquid in the top part of the piping system has a specific gravity of 0.8, and the remaining parts of the system are filled with water. If the pressure **gage** reading at A is 60 kPa, **determine:**

1. The pressure at point B.
2. The pressure head at point C in term of mercury ($\rho = 13600\ \mathrm{Kg/m^3}$)



Solution

1. $P_B = ???\ (60 = P_1 = P_2 = P_3) \rightarrow$ solution key

$60 \times 10^3 + 0.8 \times 1000 \times 9.81 \times 3 + 1000 \times 9.81 \times 2 = P_B$

$\rightarrow P_B = 103164\ \mathrm{Pa} = 103.164\ \mathrm{kPa}$ ✓.

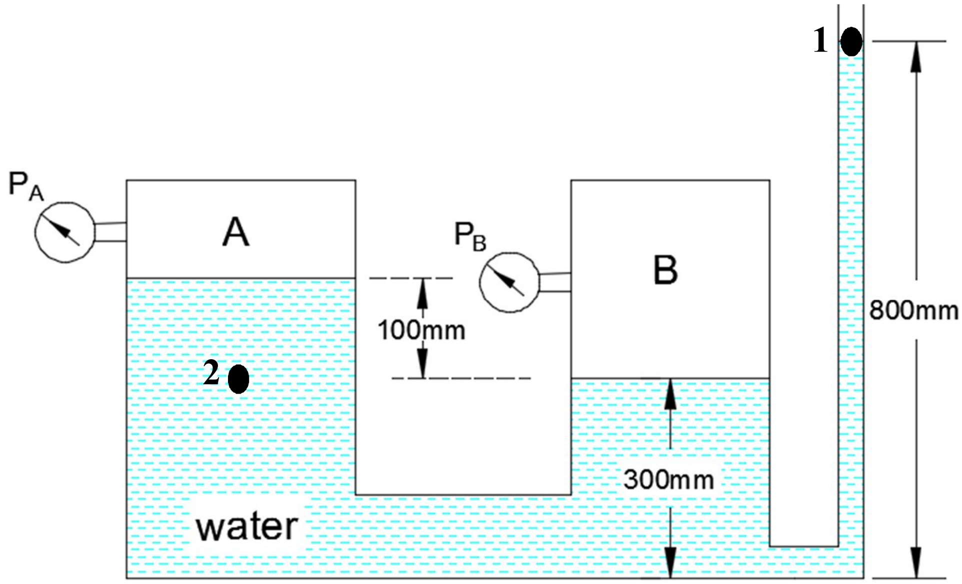
2. h_C (in term of mercury) $= ???$

$60 \times 10^3 - 1000 \times 9.81 \times 3 = P_C \rightarrow P_C = 30570\ \mathrm{Pa} = 30.57\ \mathrm{kPa}.$

$$h_C\ (\text{in terms of mercury}) = \frac{P_C}{\rho_{mercury} \times g} = \frac{30570}{13600 \times 9.81} = 0.229\ \mathrm{m}✓$$

6.

An open tube is attached to a tank shown in figure below, if the water rises to a height of 800 mm in the tube, what are the pressures P_A and P_B of the air above the water?



Solution

To calculate P_B, apply the pressure equation from point (1) to point (B):

$$0 + 1000 \times 9.81 \times 0.8 - 1000 \times 9.81 \times 0.3 = P_B$$
$$\rightarrow P_B = 4905 \text{ Pa} = 4.905 \text{ kPa} \checkmark .$$

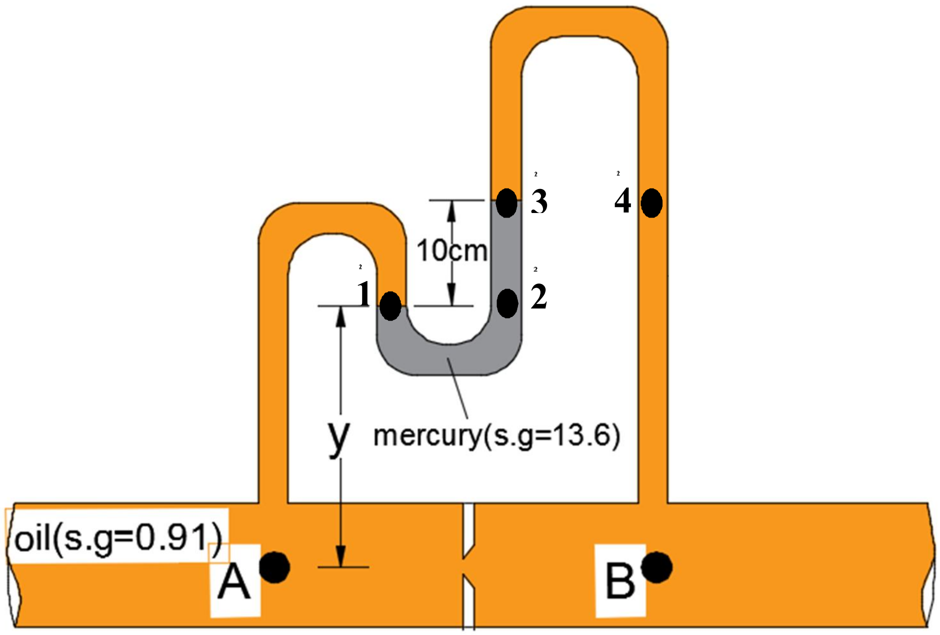
To calculate P_A, apply the pressure equation from point (B) to point (A):

$P_B = P_2 = 4905$ Pa (because same fluid and continuous)

$$4905 - 1000 \times 9.81 \times 0.1 = P_A \rightarrow P_A = 3924 \text{ Pa} = 3.924 \text{ kPa} \checkmark .$$

7.

Calculate the gauge pressure difference between A and B.



Solution

Note that the pressure at A does not equal the pressure at B although the two points have the same elevation, but the fluid does not continuous between them.

$P_1 = P_2$ and $P_3 = P_4$ (Solution Key)

$P_A - 0.91 \times 1000 \times 9.81 \times y - 13.6 \times 1000 \times 9.81 \times 0.1$

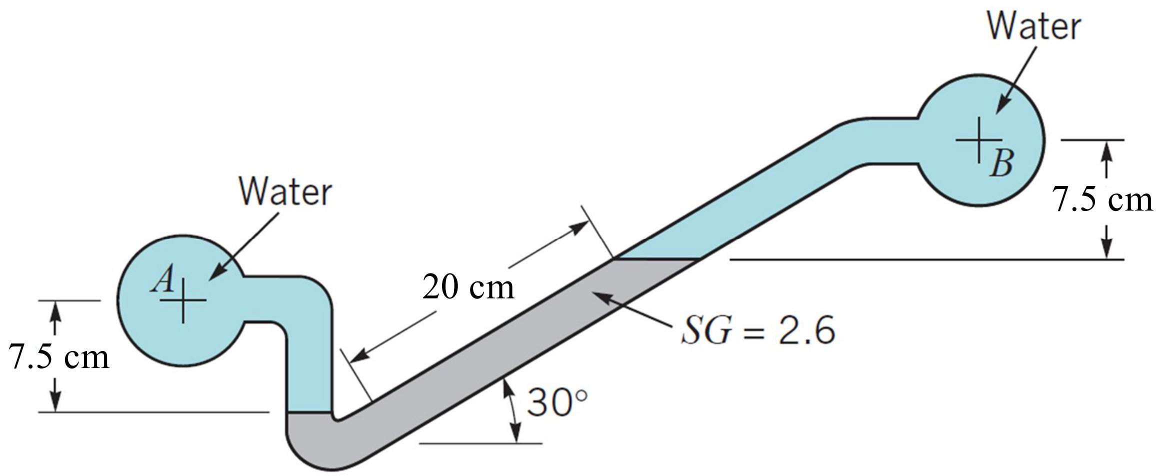
$+0.91 \times 1000 \times 9.81 \times (0.1 + y) = P_B$

$\rightarrow P_A - 8927.1y - 13341.6 + 892.71 + 8927.1 = P_B$

$\rightarrow P_A - P_B = 13341.6 - 892.71 = 12448.89 \text{ Pa} = 12.448 \text{ kPa} \checkmark.$

8.

For the figure shown below, if the pressure reading at point A is 4.2 kPa. Calculate the pressure at point B.



Solution

As we discussed previously, always we deal with vertical distances between the points that's we want to calculate the pressure between them.

Vertical distance for the fluid of SG=2.6 $= 0.2\sin(30) = 0.1\text{m}$.

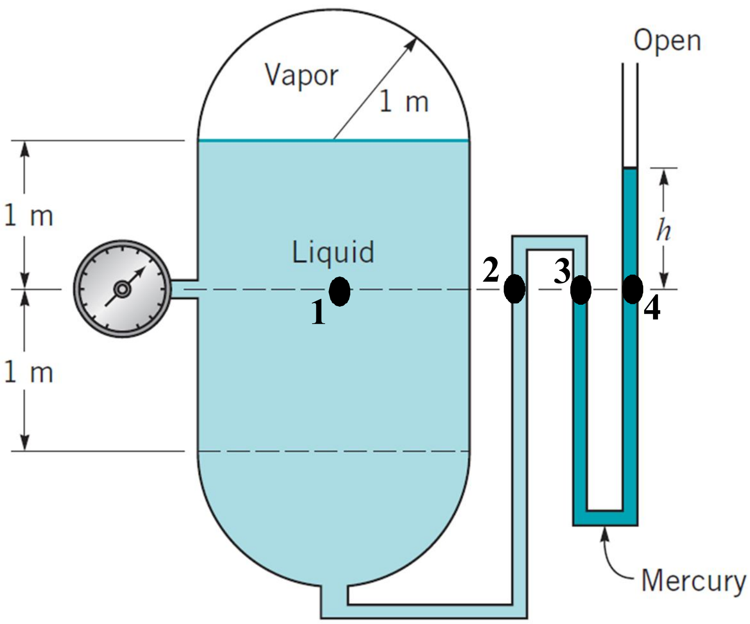
$$\underbrace{4.2\times10^3}_{P_A} + 1000\times9.81\times0.075 - 2.6\times1000\times9.81\times0.1$$

$$-1000\times9.81\times0.075 = P_B \rightarrow P_B = 1649.4\ \text{Pa} = 1.649\ \text{kPa}\checkmark.$$

9.

The cylindrical tank with hemispherical ends shown in figure below contains a volatile liquid and its vapor. The liquid density is 800 Kg/m^3 and its vapor density is negligible. The pressure in the vapor is 120 kPa **(abs),** and the atmospheric pressure is 101 kPa . **Determine:**

(a). the **gage pressure** reading on the pressure gage.
(b). the height**, h,** of the mercury manometer.



Solution

(a).

$$P_{abs} = P_{gauge} + P_{atm}$$

$$\rightarrow P_{gauge} = P_{abs} - P_{atm}$$

For vapor:

$$P_{gauge,vapor} = 120 - 101$$
$$= 19 \text{ kPa.}$$

To find gage reading:

$$19 \times 10^3 + 800 \times 9.81 \times 1 = P_{gage}$$

$$\rightarrow P_{gage} = 26848 \text{ Pa} = 26.848 \text{ kPa} \checkmark.$$

(b).

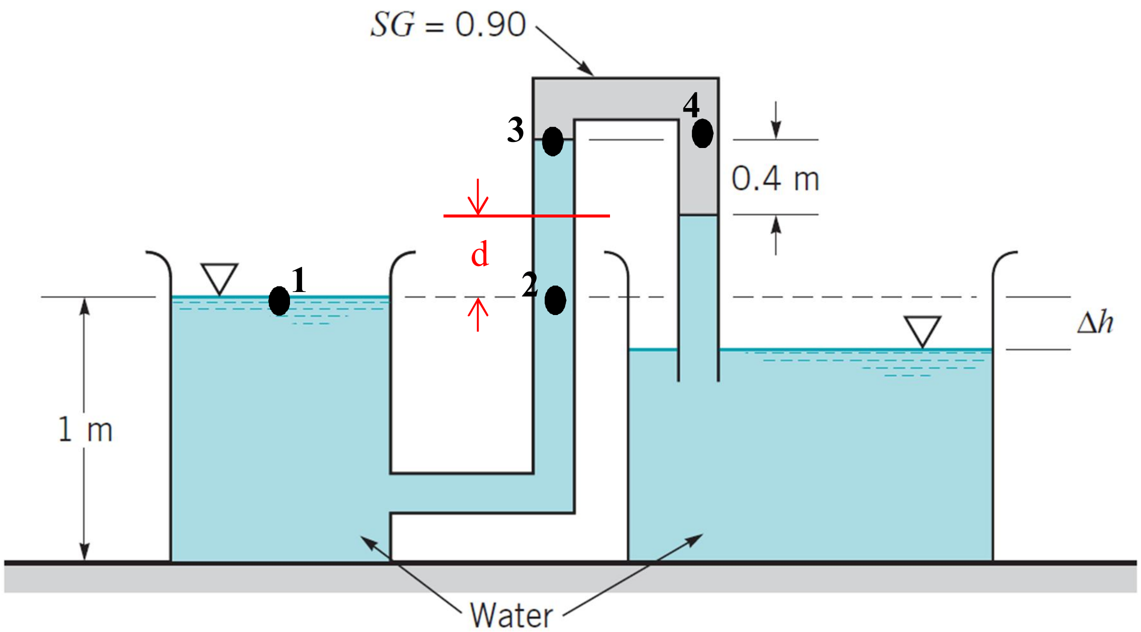
$$P_{gage} = P_1 = P_2 = P_3 = P_4 = 26848 \text{ Pa.}$$

$$26848 - 13600 \times 9.81 \times h = 0.0 \text{ (zero gauge pressure)}$$

$$\rightarrow h = 0.2012 \text{ m} \checkmark.$$

10.

Determine the elevation difference (Δh) between the water levels in the two open tanks shown in figure below.



Solution

$P_1 = P_2 = 0.0$ and $P_3 = P_4$ (Solution Key)

$0.0 - 1000 \times 9.81 \times (d + 0.4) + 0.9 \times 1000 \times 9.81 \times 0.4$

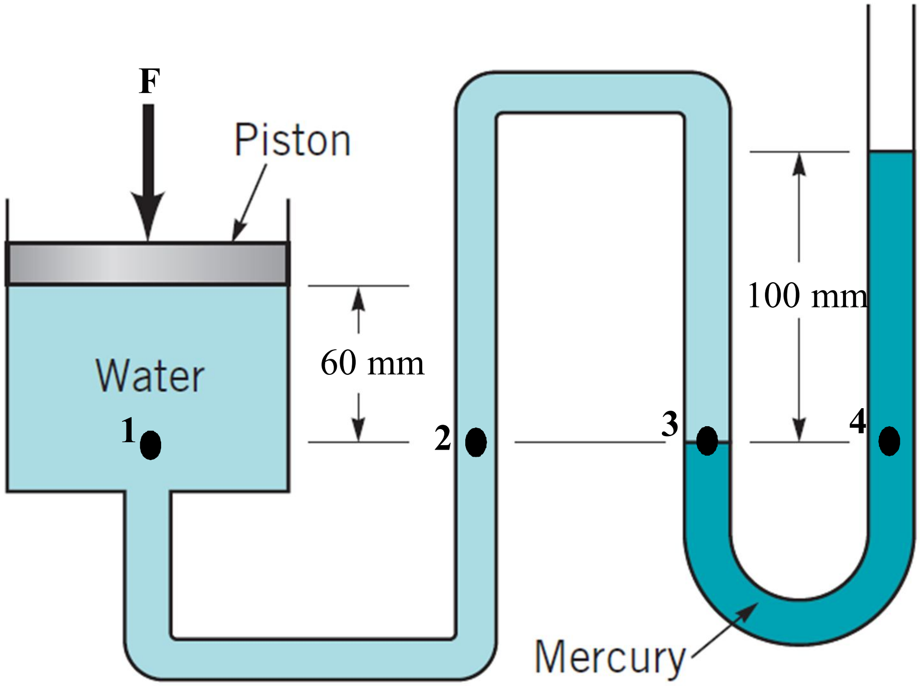
$+1000 \times 9.81 \times d + 1000 \times 9.81 \times \Delta h = 0.0$

$-9810\ d - 3924 + 3531.6 + 9810\ d + 9810\ \Delta h = 0.0$

$\Delta h = 0.04$ m✓.

11.

A piston having a cross-sectional area of $0.07 m^2$ is located in a cylinder containing water as shown in figure below. What is the value of the applied force?, F, acting on the piston? The weight of the piston is negligible.



Solution

The force applied on the piston exerts pressure on the piston and this pressure will transfer to water.

$$P_{piston} = \frac{F}{Area} = \frac{F}{0.07} = 14.28\ F$$

$$P_1 = P_2 = P_3 = P_4$$

$$14.28\ F + 1000 \times 9.81 \times 0.06 - 13600 \times 9.81 \times 0.1 = 0.0$$

$$\rightarrow F = 893.067\ N\ \checkmark.$$

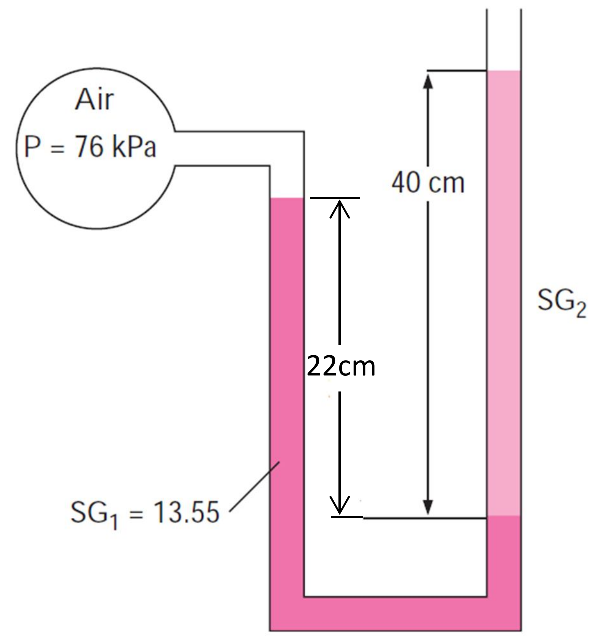
12.

If the absolute pressure of air is 76 kPa and the atmospheric pressure is 100 kPa. Determine the specific gravity (SG_2) for the right fluid.

Solution

$P_{\text{gauge,air}} = 76 - 100$

$= -24 \times 10^3 \text{Pa}$



$(-24 \times 10^3$

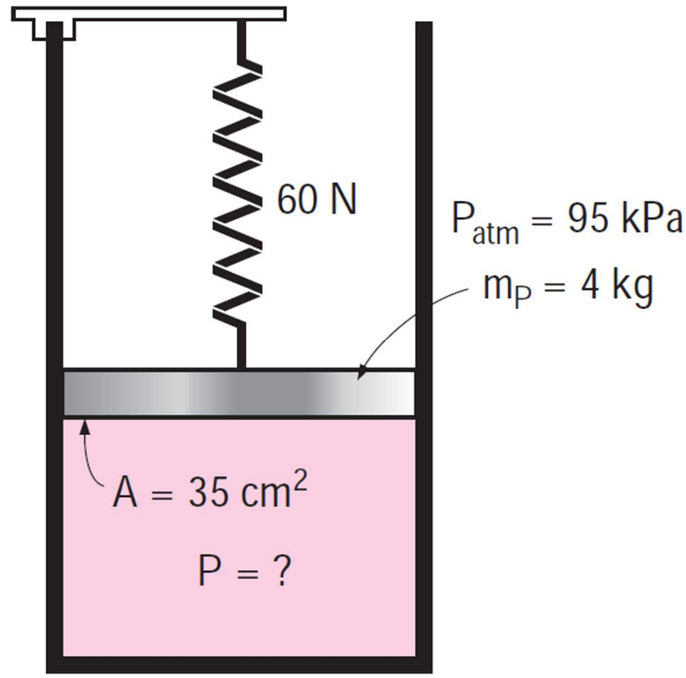
$+13.55 \times 1000 \times 9.81 \times 0.22$

$-SG_2 \times 1000 \times 9.81 \times 0.4 = 0.0)$

$SG_2 = 1.34 \checkmark.$

13.

A gas is contained in a vertical, frictionless piston– cylinder device. The piston has a mass of 4 kg and a cross sectional area of 35 cm2. A compressed spring above the piston exerts a force of 60 N on the piston. If the atmospheric pressure is 95 kPa, determine the pressure inside the cylinder.



Solution

The pressure inside the cylinder equals the total applied pressure by piston weight and spring force in addition to atmospheric pressure to get **absolute pressure**.

$$P_{abs} = \frac{60 + (4 \times 9.81)}{35 \times 10^{-4}} + 95 \times 10^3$$

$$P_{abs} = 123354.28 \text{ Pa}$$
$$= 123.35 \text{ KPa} ✓.$$

If we want to calculate gauge pressure:

$$P_{gauge} = P_{abs} - P_{atm}$$

$$P_{gauge} = 123.35 - 95 = 28.35 \text{ kPa} ✓.$$