

## CHAPTER 1

1.1 For body weight:

$$4.5 + 4.5 + 12 + 4.5 + 33 + TW = 60$$

$$TW = 1.5\%$$

For total body water:

$$7.5 + 7.5 + 20 + 7.5 + 2.5 + IW = 100$$

$$IW = 55\%$$

1.2

$$Q_{\text{students}} = 30 \text{ ind} \times 80 \frac{\text{J}}{\text{ind s}} \times 15 \text{ min} \times 60 \frac{\text{s}}{\text{min}} \times \frac{\text{kJ}}{1000\text{J}} = 2160 \text{ kJ}$$

$$m = \frac{PVM_{\text{wt}}}{RT} = \frac{(101.325 \text{ kPa})(10 \text{ m} \times 8 \text{ m} \times 3 \text{ m} - 30 \times 0.075 \text{ m}^3)(28.97 \text{ kg/kmol})}{(8.314 \text{ kPa m}^3 / (\text{kmol K}))(20 + 273.15) \text{ K}} = 286.3424 \text{ kg}$$

$$\Delta T = \frac{Q_{\text{students}}}{m C_v} = \frac{2160 \text{ kJ}}{(286.3424 \text{ kg})(0.718 \text{ kJ}/(\text{kg K}))} = 10.50615 \text{ K}$$

Therefore, the final temperature is  $20 + 10.50615 = 30.50615^\circ\text{C}$ .

1.3 This is a transient computation. For the period from ending June 1:

$$\text{Balance} = \text{Previous Balance} + \text{Deposits} - \text{Withdrawals}$$

$$\text{Balance} = 1512.33 + 220.13 - 327.26 = 1405.20$$

The balances for the remainder of the periods can be computed in a similar fashion as tabulated below:

Date	Deposit	Withdrawal	Balance
1-May			\$ 1512.33
	\$ 220.13	\$ 327.26	
1-Jun			\$ 1405.20
	\$ 216.80	\$ 378.61	
1-Jul			\$ 1243.39
	\$ 450.25	\$ 106.80	
1-Aug			\$ 1586.84
	\$ 127.31	\$ 350.61	
1-Sep			\$ 1363.54

$$1.4 \quad Q_{1,\text{in}} = Q_{2,\text{out}} + v_{3,\text{out}}A_3$$

$$A_3 = \frac{Q_{1,\text{in}} - Q_{2,\text{out}}}{v_{3,\text{out}}} = \frac{40 \text{ m}^3/\text{s} - 20 \text{ m}^3/\text{s}}{6 \text{ m/s}} = 3.333 \text{ m}^2$$

$$1.5 \quad \sum M_{\text{in}} - \sum M_{\text{out}} = 0$$

Food + Drink + Air In + Metabolism = Urine + Skin + Feces + Air Out + Sweat

Drink = Urine + Skin + Feces + Air Out + Sweat - Food - Air In - Metabolism

Drink = 1.4 + 0.35 + 0.2 + 0.4 + 0.2 - 1 - 0.05 - 0.3 = 1.2 L

$$1.6 \quad v(t) = \frac{gm}{c}(1 - e^{-(c/m)t})$$

$$\text{jumper \#1: } v(t) = \frac{9.8(70)}{12}(1 - e^{-(12/70)t}) = 46.8714$$

$$\text{jumper \#2: } 46.8714 = \frac{9.8(75)}{15}(1 - e^{-(15/75)t})$$

$$46.8714 = 49 - 49e^{-0.2t}$$

$$0.04344 = e^{-0.2t}$$

$$\ln 0.04344 = -0.2t$$

$$t = \frac{\ln 0.04344}{-0.2} = 15.6818 \text{ s}$$

1.7 You are given the following differential equation with the initial condition,  $v(t=0) = v(0)$ ,

$$\frac{dv}{dt} = g - \frac{c}{m}v$$

The most efficient way to solve this is with Laplace transforms

$$sV(s) - v(0) = \frac{g}{s} - \frac{c}{m}V(s)$$

Solve algebraically for the transformed velocity

$$V(s) = \frac{v(0)}{s + c/m} + \frac{g}{s(s + c/m)} \quad (1)$$

The second term on the right of the equal sign can be expanded with partial fractions

$$\frac{g}{s(s+c/m)} = \frac{A}{s} + \frac{B}{s+c/m}$$

Combining the right-hand side gives

$$\frac{g}{s(s+c/m)} = \frac{A(s+c/m) + Bs}{s(s+c/m)}$$

By equating like terms in the numerator, the following must hold

$$g = A \frac{c}{m}$$

$$0 = As + Bs$$

The first equation can be solved for  $A = mg/c$ . According to the second equation,  $B = -A$ . Therefore, the partial fraction expansion is

$$\frac{g}{s(s+c/m)} = \frac{mg/c}{s} - \frac{mg/c}{s+c/m}$$

This can be substituted into Eq. 1 to give

$$V(s) = \frac{v(0)}{s+c/m} + \frac{mg/c}{s} - \frac{mg/c}{s+c/m}$$

Taking inverse Laplace transforms yields

$$v(t) = v(0)e^{-(c/m)t} + \frac{mg}{c} + \frac{mg}{c} e^{-(c/m)t}$$

or collecting terms

$$v(t) = v(0)e^{-(c/m)t} + \frac{mg}{c} (1 - e^{-(c/m)t})$$

The first part is the general solution and the second part is the particular solution for the constant forcing function due to gravity.

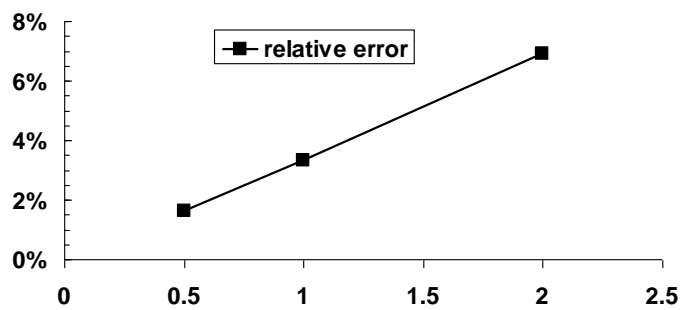
**1.8** At  $t = 10$  s, the analytical solution is 44.87 (Example 1.1). The relative error can be calculated with

$$\text{absolute relative error} = \left| \frac{\text{analytical} - \text{numerical}}{\text{analytical}} \right| \times 100\%$$

The numerical results are:

step	$v(10)$	absolute relative error
2	47.9690	6.90%
1	46.3639	3.32%
0.5	45.6044	1.63%

The error versus step size can then be plotted as



Thus, halving the step size approximately halves the error.

**1.9** (a) You are given the following differential equation with the initial condition,  $v(t = 0) = 0$ ,

$$\frac{dv}{dt} = g - \frac{c'}{m}v^2$$

Multiply both sides by  $m/c'$

$$\frac{m}{c'} \frac{dv}{dt} = \frac{m}{c'} g - v^2$$

Define  $a = \sqrt{mg/c'}$

$$\frac{m}{c'} \frac{dv}{dt} = a^2 - v^2$$

Integrate by separation of variables,

$$\int \frac{dv}{a^2 - v^2} = \int \frac{c'}{m} dt$$

A table of integrals can be consulted to find that

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a} \tanh^{-1} \frac{x}{a}$$

Therefore, the integration yields

$$\frac{1}{a} \tanh^{-1} \frac{v}{a} = \frac{c'}{m} t + C$$

If  $v = 0$  at  $t = 0$ , then because  $\tanh^{-1}(0) = 0$ , the constant of integration  $C = 0$  and the solution is

$$\frac{1}{a} \tanh^{-1} \frac{v}{a} = \frac{c'}{m} t$$

This result can then be rearranged to yield

$$v = \sqrt{\frac{gm}{c'}} \tanh\left(\sqrt{\frac{gc'}{m}} t\right)$$

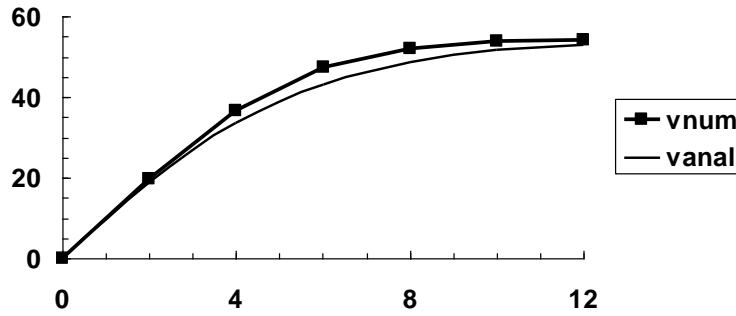
(b) Using Euler's method, the first two steps can be computed as

$$v(2) = 0 + \left[9.8 - \frac{0.225}{68.1}(0)^2\right]2 = 19.6$$

$$v(4) = 19.6 + \left[9.8 - \frac{0.225}{68.1}(19.6)^2\right]2 = 36.6615$$

The computation can be continued and the results summarized and plotted as:

$t$	$v$	$dv/dt$
0	0	9.8
2	19.6	8.53075
4	36.6615	5.35926
6	47.3800	2.38305
8	52.1461	0.81581
10	53.7777	0.24479
12	54.2673	0.07002
$\infty$	54.4622	



Note that the analytical solution is included on the plot for comparison.

**1.10** Before the chute opens ( $t < 10$ ), Euler's method can be implemented as

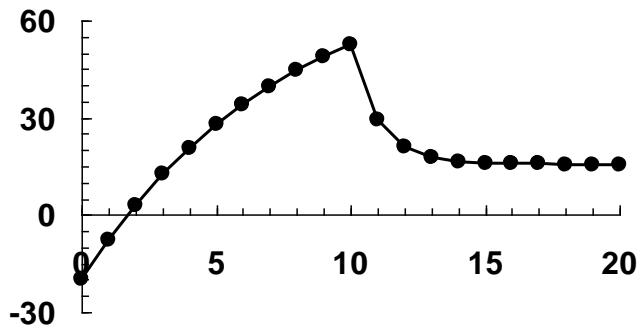
$$v(t + \Delta t) = v(t) + \left[ 9.8 - \frac{10}{80} v(t) \right] \Delta t$$

After the chute opens ( $t \geq 10$ ), the drag coefficient is changed and the implementation becomes

$$v(t + \Delta t) = v(t) + \left[ 9.8 - \frac{50}{80} v(t) \right] \Delta t$$

Here is a summary of the results along with a plot:

Chute closed			Chute opened		
t	v	dv/dt	t	v	dv/dt
0	-20.0000	12.3000	10	52.5134	-23.0209
1	-7.7000	10.7625	11	29.4925	-8.6328
2	3.0625	9.4172	12	20.8597	-3.2373
3	12.4797	8.2400	13	17.6224	-1.2140
4	20.7197	7.2100	14	16.4084	-0.4552
5	27.9298	6.3088	15	15.9531	-0.1707
6	34.2385	5.5202	16	15.7824	-0.0640
7	39.7587	4.8302	17	15.7184	-0.0240
8	44.5889	4.2264	18	15.6944	-0.0090
9	48.8153	3.6981	19	15.6854	-0.0034
			20	15.6820	-0.0013



1.11 (a) The force balance can be written as:

$$m \frac{dv}{dt} = -mg(0) \frac{R^2}{(R+x)^2} + cv$$

Dividing by mass gives

$$\frac{dv}{dt} = -g(0) \frac{R^2}{(R+x)^2} + \frac{c}{m} v$$

(b) Recognizing that  $dx/dt = v$ , the chain rule is

$$\frac{dv}{dt} = v \frac{dv}{dx}$$

Setting drag to zero and substituting this relationship into the force balance gives

$$\frac{dv}{dx} = -\frac{g(0)}{v} \frac{R^2}{(R+x)^2}$$

(c) Using separation of variables

$$v dv = -g(0) \frac{R^2}{(R+x)^2} dx$$

Integrating gives

$$\frac{v^2}{2} = g(0) \frac{R^2}{R+x} + C$$

Applying the initial condition yields

$$\frac{v_0^2}{2} = g(0) \frac{R^2}{R+0} + C$$

which can be solved for  $C = v_0^2/2 - g(0)R$ , which can be substituted back into the solution to give

$$\frac{v^2}{2} = g(0) \frac{R^2}{R+x} + \frac{v_0^2}{2} - g(0)R$$

or

$$v = \pm \sqrt{v_0^2 + 2g(0) \frac{R^2}{R+x} - 2g(0)R}$$

Note that the plus sign holds when the object is moving upwards and the minus sign holds when it is falling.

(d) Euler's method can be developed as

$$v(x_{i+1}) = v(x_i) + \left[ -\frac{g(0)}{v(x_i)} \frac{R^2}{(R+x_i)^2} \right] (x_{i+1} - x_i)$$

The first step can be computed as

$$v(10,000) = 1,400 + \left[ -\frac{9.8}{1,400} \frac{(6.37 \times 10^6)^2}{(6.37 \times 10^6 + 0)^2} \right] (10,000 - 0) = 1,400 + (-0.007)10,000 = 1,330$$

The remainder of the calculations can be implemented in a similar fashion as in the following table

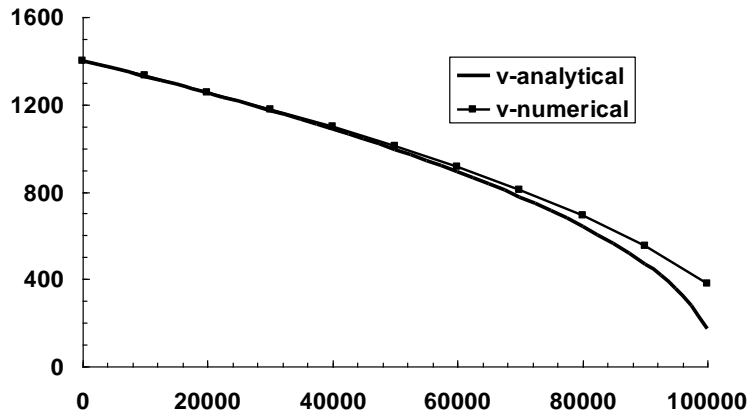
$x$	$v$	$dv/dx$	$v$ -analytical
0	1400.000	-0.00700	1400.000
10000	1330.000	-0.00735	1328.272
20000	1256.547	-0.00775	1252.688
30000	1179.042	-0.00823	1172.500
40000	1096.701	-0.00882	1086.688
50000	1008.454	-0.00957	993.796
60000	912.783	-0.01054	891.612
70000	807.413	-0.01188	776.473
80000	688.661	-0.01388	641.439
90000	549.864	-0.01733	469.650
100000	376.568	-0.02523	174.033

For the analytical solution, the value at 10,000 m can be computed as

$$v = \sqrt{1,400^2 + 2(9.8) \frac{(6.37 \times 10^6)^2}{(6.37 \times 10^6 + 10,000)^2} - 2(9.8)(6.37 \times 10^6)} = 1,328.272$$



The remainder of the analytical values can be implemented in a similar fashion as in the last column of the above table. The numerical and analytical solutions can be displayed graphically.



**1.12 (a)** The first two steps are

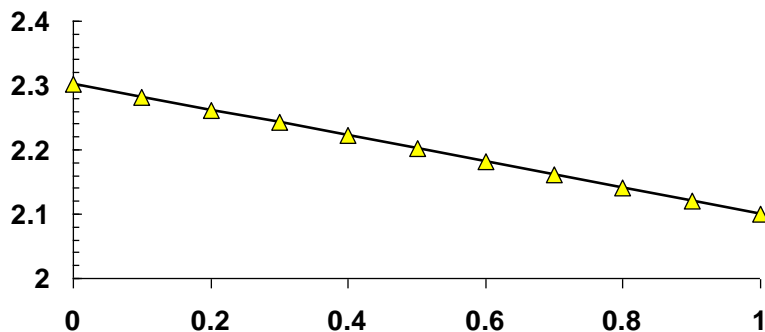
$$c(0.1) = 10 - 0.2(10)0.1 = 9.8 \text{ Bq/L}$$

$$c(0.2) = 9.8 - 0.2(9.8)0.1 = 9.604 \text{ Bq/L}$$

The process can be continued to yield

$t$	$c$	$dc/dt$
0	10.0000	-2.0000
0.1	9.8000	-1.9600
0.2	9.6040	-1.9208
0.3	9.4119	-1.8824
0.4	9.2237	-1.8447
0.5	9.0392	-1.8078
0.6	8.8584	-1.7717
0.7	8.6813	-1.7363
0.8	8.5076	-1.7015
0.9	8.3375	-1.6675
1	8.1707	-1.6341

**(b)** The results when plotted on a semi-log plot yields a straight line



The slope of this line can be estimated as

$$\frac{\ln(8.1707) - \ln(10)}{1} = -0.20203$$

Thus, the slope is approximately equal to the negative of the decay rate.

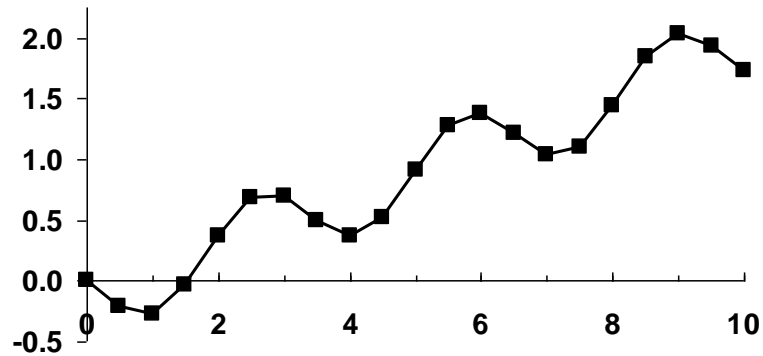
**1.13** The first two steps yield

$$y(0.5) = 0 + \left[ 3 \frac{500}{1200} \sin^2(0) - \frac{500}{1200} \right] 0.5 = 0 + [0 - 0.41667] 0.5 = -0.20833$$

$$y(1) = -0.20833 + [\sin^2(0.5) - 0.41667] 0.5 = -0.27301$$

The process can be continued to give

$t$	$y$	$dy/dt$	$t$	$y$	$dy/dt$
0	0.00000	-0.41667	5.5	1.27629	0.20557
0.5	-0.20833	-0.12936	6	1.37907	-0.31908
1	-0.27301	0.46843	6.5	1.21953	-0.35882
1.5	-0.03880	0.82708	7	1.04012	0.12287
2	0.37474	0.61686	7.5	1.10156	0.68314
2.5	0.68317	0.03104	8	1.44313	0.80687
3	0.69869	-0.39177	8.5	1.84656	0.38031
3.5	0.50281	-0.26286	9	2.03672	-0.20436
4	0.37138	0.29927	9.5	1.93453	-0.40961
4.5	0.52101	0.77779	10	1.72973	-0.04672
5	0.90991	0.73275			



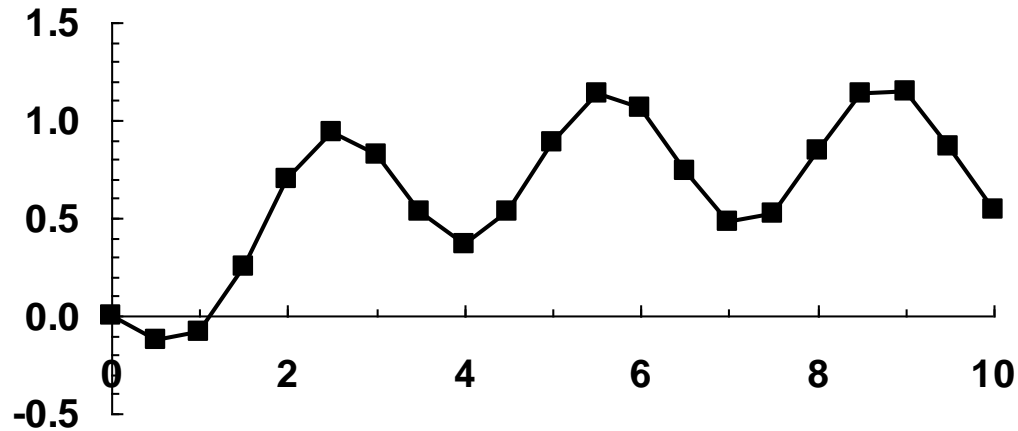
1.14 The first two steps yield

$$y(0.5) = 0 + \left[ 3 \frac{500}{1200} \sin^2(0) - \frac{300(1+0)^{1.5}}{1200} \right] 0.5 = 0 + [0 - 0.25] 0.5 = -0.125$$

$$y(1) = -0.125 + \left[ \sin^2(0.5) - \frac{300(1-0.125)^{1.5}}{1200} \right] 0.5 = -0.08366$$

The process can be continued to give

$t$	$y$	$dy/dt$
0	0.00000	-0.25000
0.5	-0.12500	0.08269
1	-0.08366	0.66580
1.5	0.24924	0.89468
2	0.69658	0.48107
2.5	0.93711	-0.22631
3	0.82396	-0.59094
3.5	0.52849	-0.31862
4	0.36918	0.31541
4.5	0.52689	0.72277
5	0.88827	0.50073
5.5	1.13864	-0.15966
6	1.05881	-0.64093
6.5	0.73834	-0.51514
7	0.48077	0.08906
7.5	0.52530	0.62885
8	0.83973	0.59970
8.5	1.13958	0.01457
9	1.14687	-0.57411
9.5	0.85981	-0.62702
10	0.54630	-0.11076



1.15 The volume of the droplet is related to the radius as

$$V = \frac{4\pi r^3}{3} \quad (1)$$

This equation can be solved for radius as

$$r = \sqrt[3]{\frac{3V}{4\pi}} \quad (2)$$

The surface area is

$$A = 4\pi r^2 \quad (3)$$

Equation (2) can be substituted into Eq. (3) to express area as a function of volume

$$A = 4\pi \left( \frac{3V}{4\pi} \right)^{2/3}$$

This result can then be substituted into the original differential equation,

$$\frac{dV}{dt} = -k4\pi \left( \frac{3V}{4\pi} \right)^{2/3} \quad (4)$$

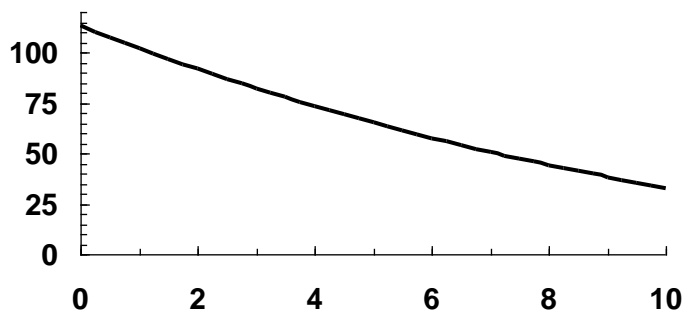
The initial volume can be computed with Eq. (1),

$$V = \frac{4\pi r^3}{3} = \frac{4\pi(3)^3}{3} = 113.0973\text{mm}^3$$

Euler's method can be used to integrate Eq. (4). Here are the beginning and last steps

$t$	$V$	$dV/dt$
0	113.0973	-11.3097
0.25	110.2699	-11.1204
0.5	107.4898	-10.9327
0.75	104.7566	-10.7466
1	102.07	-10.5621
•		
•		
•		
9	38.29357	-5.49416
9.25	36.92003	-5.36198
9.5	35.57954	-5.2314
9.75	34.27169	-5.1024
10	32.99609	-4.97499

A plot of the results is shown below:



Eq. (2) can be used to compute the final radius as

$$r = \sqrt[3]{\frac{3(32.99609)}{4\pi}} = 1.9897$$

Therefore, the average evaporation rate can be computed as

$$k = \frac{(3 - 1.9897) \text{ mm}}{10 \text{ min}} \frac{60 \text{ min}}{\text{hr}} = 0.10103 \frac{\text{mm}}{\text{min}}$$

which is approximately equal to the given evaporation rate.

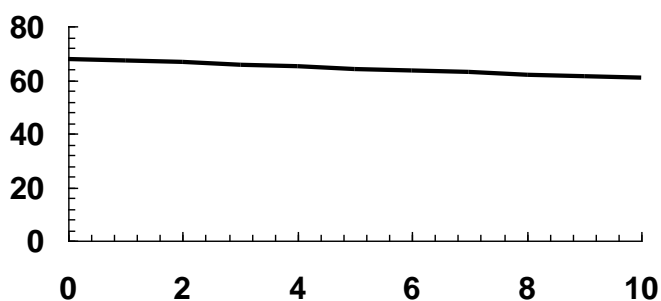
**1.16** The first two steps can be computed as

$$T(1) = 68 + [-0.017(68 - 21)]1 = 68 + (-0.799)1 = 67.201$$

$$T(2) = 67.201 + [-0.017(67.201 - 21)]1 = 68 + (-0.7854)1 = 66.41558$$

The remaining results are displayed below along with a plot

$t$	$T$	$dT/dt$
0	68.00000	-0.79900
1	67.20100	-0.78542
2	66.41558	-0.77206
3	65.64352	-0.75894
4	64.88458	-0.74604
5	64.13854	-0.73336
6	63.40519	-0.72089
7	62.68430	-0.70863
8	61.97566	-0.69659
9	61.27908	-0.68474
10	60.59433	-0.67310



1.17 (a) The solution of the differential equation is

$$N = N_0 e^{\mu t}$$

The doubling time can be computed as the time when  $N = 2N_0$ ,

$$2N_0 = N_0 e^{\mu(20)}$$

$$\mu = \frac{\ln 2}{20 \text{ hrs}} = \frac{0.693}{20 \text{ hrs}} = 0.034657/\text{hr}$$

(b) The volume of an individual spherical cell is

$$\text{cell volume} = \frac{\pi d^3}{6} \quad (1)$$

The total volume is

$$\text{volume} = \frac{\pi d^3}{6} N \quad (2)$$

The rate of change of  $N$  is defined as

$$\frac{dN}{dt} = \mu N \quad (3)$$

If  $N = N_0$  at  $t = 0$ , Eq. 3 can be integrated to give

$$N = N_0 e^{\mu t} \quad (4)$$

Therefore, substituting (4) into (2) gives an equation for volume

$$\text{volume} = \frac{\pi d^3}{6} N_0 e^{\mu t} \quad (5)$$

(c) This equation can be solved for time

$$t = \frac{\ln \frac{6 \times \text{volume}}{\pi d^3 N_0}}{\mu} \quad (6)$$

The volume of a 500  $\mu\text{m}$  diameter tumor can be computed with Eq. 2 as 65,449,847. Substituting this value along with  $d = 20 \mu\text{m}$ ,  $N_0 = 1$  and  $\mu = 0.034657/\text{hr}$  gives

$$t = \frac{\ln \left( \frac{6 \times 65,449,847}{\pi 20^3 (1)} \right)}{0.034657} = 278.63 \text{ hr} = 11.6 \text{ d} \quad (6)$$

**1.18** Continuity at the nodes can be used to determine the flows as follows:

$$Q_1 = Q_2 + Q_3 = 0.6 + 0.4 = 1 \frac{\text{m}^3}{\text{s}}$$

$$Q_{10} = Q_1 = 1 \frac{\text{m}^3}{\text{s}}$$

$$Q_9 = Q_{10} - Q_2 = 1 - 0.6 = 0.4 \frac{\text{m}^3}{\text{s}}$$

$$Q_4 = Q_9 - Q_8 = 0.4 - 0.3 = 0.1 \frac{\text{m}^3}{\text{s}}$$

$$Q_5 = Q_3 - Q_4 = 0.4 - 0.1 = 0.3 \frac{\text{m}^3}{\text{s}}$$

$$Q_6 = Q_5 - Q_7 = 0.3 - 0.2 = 0.1 \frac{\text{m}^3}{\text{s}}$$

Therefore, the final results are

