Notes to instructors

Engineering Fluid Mechanics

Introduction

The following ideas and information are provided to assist the instructor in the design and implementation of the course. Traditionally this course is taught at Washington State University and the University of Idaho as a three-credit semester course which means 3 hours of lecture per week for 15 weeks. Basically the first 11 chapters and Chapter 13 (Flow Measurements) are covered in Mechanical Engineering. Chapters 12 (Compressible Flow) and Chapter 14 (Turbomachinery) may be covered depending on the time available and exposure to compressible flow in other courses (Thermodynamics). Open channel flow (Chapter 15) is generally not covered in Mechanical Engineering. When the text is used in Civil Engineering, Chapters 1-11 and 13 are nominally covered and Chapters 14 and 15 may be included if time permits and exposure to open channel flow may not be available in other courses. The book can be used for 10-week quarter courses by selecting the chapters, or parts of the chapters, most appropriate for the course.

Author Contact

Every effort has been made to insure that the solution manual is error free. If errors are found (and they will be!) please contact Professors Crowe or Elger.

Donald Elger
Mechanical Engineering Dept
University of Idaho
Moscow, ID 83844-0902
Phone (208) 885-7889
Fax (208) 885-9031
e-mail: delger@uidaho.edu

Clayton Crowe
School of Mechanical Eng. & Matl. Science
Washington State University
Pullman, WA 99164-2920
Phone (509) 335-3214
Fax (509) 335-4662
e-mail: crowe@mme.wsu.edu

Design and Computer Problems

This edition, like the 6th edition, includes open-ended design problems. These problems provide a platform for student discussion and group activity. One approach is to divide the class into small groups of three or four and have these groups work on the design problems together. Each group can then report on their design to the rest of the class. This stimulates interest and class discussion. Solutions to the design problems are included in the solution manual.

The seventh edition also includes computer-oriented problems. These problems may best be solved using software such as spreadsheets or MathCad. The choice is left to the student. The answer book also includes the results for the computer-oriented problems.

Instructional Videos

Videos are available from various sources. One source is Ensight Media (2162 Broadway, New York, NY, www.insight-media.com). The videos cover a wide range of subjects in fluid mechanics, turbulence and compressible flow. Video titles which are particularly useful are:

Flow visualization
Kinematics of fluid motion
Pressure fields and acceleration
Fundamentals of boundary layers
The fluid dynamics of drag
Turbulence

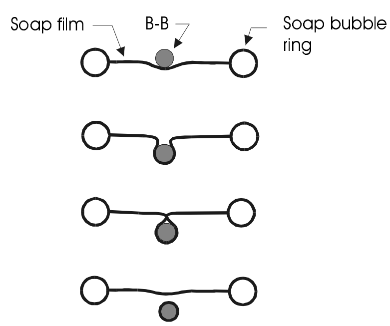
Another source of complementary material is the CD-ROM marketed by Cambridge University Press (40 West 20th Street, New York, NY 1011-421, www.cup.org). This CD-ROM includes video animations and interactive experiments in kinematics, dynamics and boundary layers. It also includes biographical sketches of many early contributors to fluid mechanics.

In Class Experments

In-class experiments are highly effective ways to stimulate student interest. These experiments are simple and require little set-up time Short descriptions of six in-class experiments are provided below.

1. Surface tension:

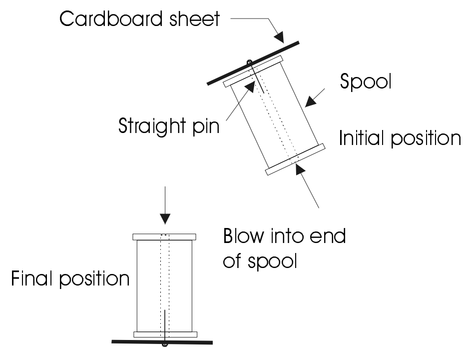
There are several demonstrations that an instructor can do to illustrate surface tension effects. One demonstration is to use the solution for making bubbles available in most toy stores. There is usually a ring included with the package. Generate a soap film supported by the ring and drop a B-B through the ring. The soap bubble will remain intact. The reason is shown in the figure. As the B-B passes through the film, the film is deformed as shown. The surface tension is responsible for the film to neck down behind the B-B and reseal before the B-B penetrates the film. It is usually best to involve some volunteers from the class to do the demonstration.



Surface tension equipment

2. Bernoulli equation:

Take a spool of thread and open the holes at each end. Then take a piece of light cardboard like an index card and cut a 3 in. by 3 in. square. Take a straight pin and put it through the middle of the square. Mount cardboard on the top of the spool with the pin in the hole. Blow into the hole and then gradually turn the spool until the cardboard is on the bottom. The cardboard remains suspended by the pressure forces until you stop blowing. In this test you can then explain that because of the change in area going from the center to the edge of the spool, the velocity at the center must be higher than at the edge (continuity equation). The larger velocity at the center and the atmospheric pressure at the edge of the spool leads to subatmospheric pressure between the cardboard and the spool (the Bernoulli equation). This supports the weight of the cardboard.

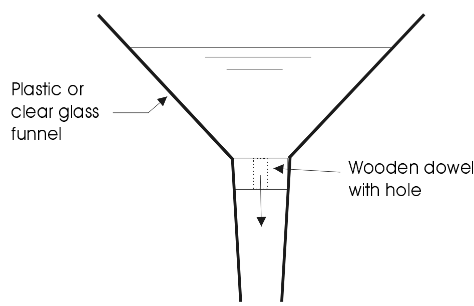


Continuity-Bernoulli experiment

3. Continuity equation:

This experiment involves calculating the emptying time for a funnel. Locate a plastic funnel which is 4 to 6 inches in diameter at the top. Such a funnel is usually available at an auto-parts store. In order to increase the emptying time install in the bottom of the funnel a wooden dowel with a hole as shown. A 1/4 inch hole will work well. Then fill the funnel with water to a certain depth while sealing the hole with your finger. Allow the funnel to drain and measure the time to reach a final depth. Sometimes a vortex is generated in the hole, significantly decreasing the flow rate.

In this demonstration, it is usually best to give the students the dimensions and have them calculate the emptying time before the demonstration. These calculations can be done by the various teams. Each team records their prediction before the experiment. After the demonstration, the team with the prediction closest to the measured time is recognized.

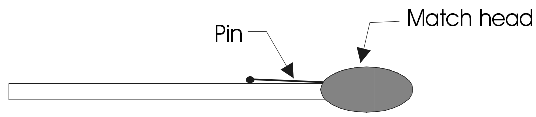


Continuity-Bernoulli experiment

4. Match rocket:

This experiment demonstrates the momentum principle. The materials needed are a wooden match, tin foil and a pin. Position the pin along the shaft of the match with one end at the match head as shown in the figure.

Wrap the tin foil tightly around the head of the match and then remove the pin. The resulting hole is the 'nozzle' of the rocket. Mount the match at an angle. This can be done by deforming a paper clip to serve as the rocket launcher. Heat the tin-foil with another match and when the wrapped match head ignites, the gases discharge through the pin hole giving rise to a thrust on the match.

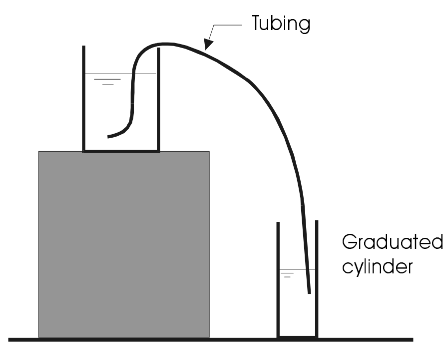


Match rocket

5. Siphon:

Siphon experiments, which are very easy to set up in class, demonstrate the principles in Chapter 7, the energy equation, and Chapter 10, flow in conduits. Basically all that is needed is a jar, a piece of tubing (such as plastic tubing) and a graduated cylinder for measuring the flow rate. Given the length of the tubing and internal diameter, as well as the elevation distance between the water level in the jar and the end of the tube, the flow rate can be predicted. One question to pose is if the flow rate will differ if the end of the tubing becomes submerged in the graduated cylinder as it fills.

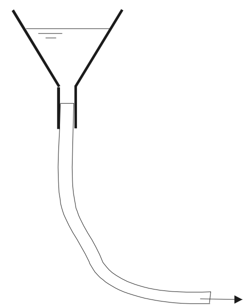
The students, or groups of students, can predict the flow rate before the demonstration and those with the closest prediction can be recognized. There are many variations that can be done with this demonstration such the use of different liquids, tube size and constrictions or fittings in the line.



Siphon experiment

6. Viscosity of common liquids:

Funnel experiments can be used to measure the viscosity of common viscous liquids like syrups. Small glass funnels with spouts the order of 4-5 mm internal diameter can be found in laboratory glassware catalogs. The spouts may be the order of 5 cm long. Calculations should be done beforehand with estimates of the viscosity of the liquid to be sure that the demonstration is feasible. If the hole is too large, plastic tubing can be inserted and glued into the spout as shown in the figure. This is a good design project for the students to make sure the conditions will ensure a laminar flow and that the head loss due to friction is much larger than entrance loss.



Viscosity

There are numerous other in-class experiments which can be desgined. These experiments could include the terminal velocity of a sphere in a liquid flow through a wier and stability of a floating body.

Instructions for Using this Solution Manual

Solution Manual: Rev. 7-1 Instructions for Adobe.

Engineering Fluid Mechanics
Seventh Edition
Clayton T. Crowe, Donald F. Elger and John A. Roberson

The pdf file on this CD ROM contains the solution manual to accompany Engineering Fluid Mechanics.
The pdf file is best viewed in Adobe Acrobat 4.0.
This software may be obtained from the Adobe website at http://www.adobe.com.
The pdf viewing software is free to download and use.
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Chapter Two

2.1 **Information and assumptions**

from Table A.2:
provided in problem statement

$$\begin{aligned} R_{\text{air}} &= 287 \text{ J/kgK} \\ R_{\text{H}_2} &= 4,127 \\ R_{\text{CO}_2} &= 189 \end{aligned}$$

Find

density at 200 kPa and 37.8°C.

Ideal gas law

$$\rho = \frac{P}{RT} = \frac{200,000}{R(37.8 + 273.15)} = \frac{643.2}{R} \text{ kg/m}^3$$

Then

$$\begin{aligned} \rho_{\text{air}} &= \frac{643.2}{287} = \underline{\underline{2.24 \text{ kg/m}^3}} \\ \rho_{\text{H}_2} &= \frac{643.2}{4127} = \underline{\underline{0.156 \text{ kg/m}^3}} \\ \rho_{\text{CO}_2} &= \frac{643.2}{189} = \underline{\underline{3.40 \text{ kg/m}^3}} \end{aligned}$$

2.2 **Information and assumptions**

from Table A.2, $R_{\mathrm{CO_2}} = 189$ J/kgK
provided in problem statement

Find

density and specific weight at 300 kPa and 60°C

Ideal gas law

$$\rho_{\mathrm{CO_2}} = \frac{P}{RT} = \frac{300,000}{189(60+273)} = 4.767 \text{ kg/m}^3$$

from the relation $\gamma = \rho g$

$$\gamma_{\mathrm{CO_2}} = \rho_{\mathrm{CO_2}} \times g = 4.767 \times 9.81 = \underline{\underline{46.764 \text{ N/m}^3}}$$

2.3 **Information and assumptions**

from Table A.2, $R_{\text{He}} = 2077$ J/kgK
provided in problem statement

Find

density and specific weight at 500 kPa and 60°C

Ideal gas law

$$\rho_{\text{He}} = \frac{P}{RT} = \frac{500,000}{2,077(60+273)} = 0.723 \text{ kg/m}^3$$

from the relation $\gamma = \rho g$

$$\gamma_{\text{He}} = \rho_{\text{He}} \times g = 0.723 \times 9.81 = \underline{\underline{7.09 \text{ N/m}^3}}$$

2.4 **Information and assumptions**

provided in problem statement

Find

ratio of final to initial mass in the tank

Solution

definition of density and ideal gas law

$$M = \rho \forall = (p/RT)\forall$$

where p is the absolute pressure. Volume and gas temperature constant so

$$M_2/M_1 = (p_2/p_1)$$

and

$$M_2/M_1 = 300 \text{ kPa}/200 \text{ kPa} = \underline{\underline{1.5}}$$

2.5 **Information and assumptions**

from Table A.2, $R_{\text{air}} = 287$ J/kgK
T=60°C and p=2 atm
provided in problem statement

Find

ratio of density of water to density of air

Ideal gas law

$$\begin{aligned}
\rho_{\text{air}} &= \frac{P}{RT} = \frac{202,600}{287(60+273)} = 2.120 \text{ kg/m}^3 \\
\rho_{\text{water}} &= 983 \text{ kg/m}^3 \quad \text{Then } \frac{\rho_{\text{water}}}{\rho_{\text{air}}} = \frac{983}{2.120} = \underline{\underline{464}}
\end{aligned}$$

2.6 **Information and assumptions**

from Table A.2, $R_{\text{air}} = 1555$ ft·lbf/slug/°R
$\cancel{V}$=4 ft^3, p=200 psia, $T = 50$°F and tank weight=100 lbf
provided in problem statement

Find

weight of tank with oxygen

Ideal gas law

$$\begin{aligned}
\rho &= p/RT \\
p_{\text{abs.}} &= 200 \text{ psia} \times 144 \text{ psf/psi} = 28,800 \text{ psf} \\
R &= 1,555 \text{ ft} \cdot \text{lbf}/(\text{slug} \cdot^{\circ} R) \\
T &= 460 + 50 = 510^{\circ} R \\
\rho &= 28,800/(1,555 \times 510) = 0.0363 \text{ slugs/ft}^3 \\
\text{or } \gamma &= \rho g = 0.0363 \times 32.2 = 1.69 \text{ lb/ft}^3 \\
\text{then } W_{\text{air}} &= 1.169 \text{ lb/ft}^3 \times 4 \text{ ft}^3 = 4.68 \text{ lbf} \\
W_{\text{total}} &= \underline{\underline{104.68 \text{ lbf}}}
\end{aligned}$$

2.7 **Information and assumptions**

from Table A.2, $R = 287$ J/kgK
$p = 445$ kPa, $T = 38^{\rm o}$C
provided in problem statement

Find

specific weight and density

Ideal gas law

$$\begin{aligned}\rho_{\rm air} &= \frac{P}{RT} = \frac{445,000}{287(38+273)} = \underline{\underline{4,986 \text{ kg/m}^3}} \\ \gamma_{\rm air} &= \rho_{\rm air} \times g = 3,865 \times 9.81 = \underline{\underline{48.91 \text{ N/m}^3}}\end{aligned}$$

2.8 **Information and assumptions**

assume the density of air is the value at sea level for standard conditions.
from Table A.2 $\rho_{\text{air}} = 0.00237$ slugs/ft^3.
provided in problem statement

Find

mass in 1 mi^3 of air

Relation between mass, density and volume

$$M = \rho \forall = 0.00237 \times (5,280)^3 = \underline{\underline{3.49 \times 10^8 \text{ slugs}}}$$
$$3.49 \times 10^8 \times 32.2 = \underline{\underline{1.12 \times 10^{10} \text{ lbm}}}$$
$$1.12 \times 10^{10} \times 0.4536 = \underline{\underline{5.08 \times 10^9 \text{ kg}}}$$

The mass will probably be somewhat less than this because the density decreases with altitude.

2.9 **Information and assumptions**

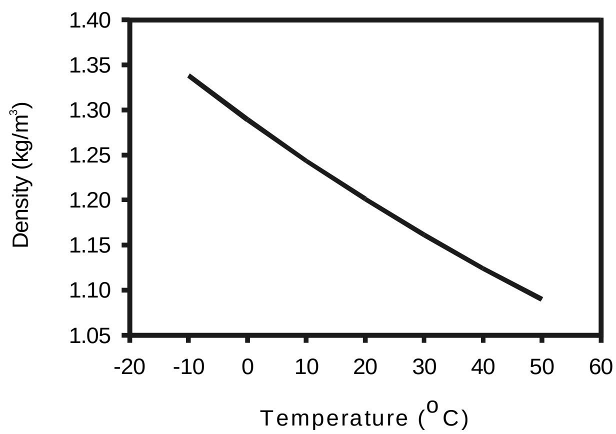
from Table A.2, $R_{\text{air}} = 287$ J/kg/K
$p_{\text{tire}} = 450$ kPa, abs and $T = 20^{o}$C
provided in problem statement

Find

variation in air density from -10^{o}C to 50^{o}C
tire pressure change with temperature if volume constant

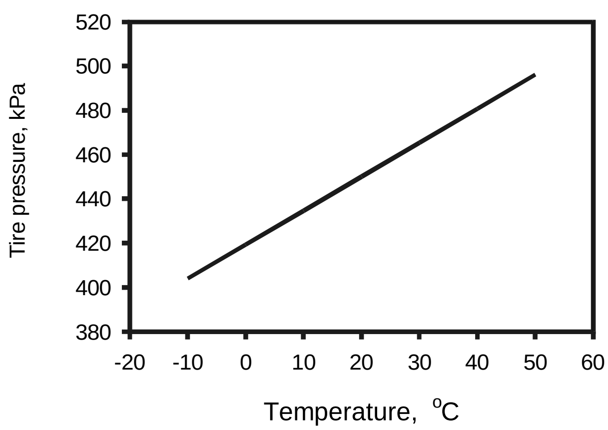
Ideal gas law

$$\rho = \frac{p}{RT} = \frac{101000}{287 \times (273 + T)}$$



with density constant

$$p = p_o \frac{T}{T_o}$$



Observation: Tire pressure decreases with decreasing temperature.

2.10 **Information and assumptions**

from Table A.2, $R_{CO_2} = 189$ J/kgK
from Fig. A.2 $\mu = 1.7 \times 10^{-5}$ N·s/m^2
$p = 250$ kPa, $T = 0°$C
provided in problem statement

Find

density, specific weight, dynamic viscosity and kinematic viscosity

Ideal gas law

$$\rho = p/RT$$
$$\rho = 250 \times 10^3/(189 \times 273) = \underline{\underline{4.84}} \text{ kg/m}^3$$
$$\gamma = \rho g = 4.84 kg/m^3 \times 9.81 m/s^2 = \underline{\underline{47.53 \text{ N/m}^3}}$$
$$\nu = \mu/\rho = 1.7 \times 10^{-5}/4.84 = \underline{\underline{3.51 \times 10^{-6} \text{ m}^2\text{/s}}}$$

2.11 Mass and weight are extensive properties; the remaining properties are intensive.

2.12 **D**erive expression for c_v and c_p in terms of k and R.

$$c_p/c_v = k \qquad c_p - c_v = R$$
$$c_p/c_v - \dot{c}_v/c_v = R/c_v$$
$$k - 1 = R/c_v; \quad c_v = \underline{\underline{R/(k-1)}}$$
$$c_p = R + c_v = R + R/(k-1) = \underline{\underline{kR/(k-1)}}$$

2.13 **Information and assumptions**

from Table A.5 for water:
provided in problem statement

$$\mu_{70} = 4.04 \times 10^{-4} \text{N·s/m}^2$$
$$\mu_{10} = 1.31 \times 10^{-3} \text{N·s/m}^2$$
$$\rho_{70} = 978 \text{ kg/m}^3$$
$$\rho_{10} = 1,000 \text{ kg/m}^3$$

from Table A.3 for air

$$\mu_{70} = 2.04 \times 10^{-5} \text{ N} \cdot \text{s/m}^2$$
$$\mu_{10} = 1.76 \times 10^{-5} \text{ N} \cdot \text{s/m}^2$$
$$\rho_{70} = 1.03 \text{ kg/m}^3$$
$$\rho_{10} = 1.25 \text{ kg/m}^3$$

Find

change in viscosity and density for air and water

Solution

for water

$$\begin{aligned} \Delta\mu &= \mu_{70} - \mu_{10} = \underline{\underline{-9.06 \times 10^{-4} \text{ N} \cdot \text{s/m}^2}} \\ \Delta\rho &= \rho_{70} - \rho_{10} = \underline{\underline{-22 \text{ kg/m}^3}} \end{aligned}$$

for air

$$\begin{aligned} \Delta\mu &= \underline{\underline{+0.28 \times 10^{-5} \text{ N} \cdot \text{s/m}^2}} \\ \Delta\rho &= \underline{\underline{-0.22 \text{ kg/m}^3}} \end{aligned}$$

2.14 **Information and assumptions**

from table A.3, $\nu_{60} = 1.89 \times 10^{-5}\ \mathrm{m^2/s}$, $\nu_{10} = 1.41 \times 10^{-5}\ \mathrm{m^2/s}$
provided in problem statement

Find

change in kinematic viscosity from 10°C to 60°C

$$\Delta v_{\mathrm{air},10\to 60} = (1.89 - 1.41) \times 10^{-5} = \underline{\underline{4.8 \times 10^{-6}\ \mathrm{m^2/s}}}$$

2.15 **Information and assumptions**

from tables A.4, Fig. A.2 and Table A.5.
provided in problem statement

Find

Solution

	Oil (SAE 10W)	kerosene	water
$\mu(\mathrm{N \cdot s/m^2})$	$\underline{\underline{3.6 \times 10^{-2}}}$	$\underline{\underline{1.4 \times 10^{-3}}}$ (Fig. A-2)	$\underline{\underline{6.8 \times 10^{-4}}}$
$\rho(\mathrm{kg/m^2})$	870		993
$\upsilon(\mathrm{m^2/s})$	$\underline{\underline{4.1 \times 10^{-5}}}$	$\underline{\underline{1.7 \times 10^{-6}}}$ (Fig. A-2)	$\underline{\underline{6.8 \times 10^{-7}}}$

2.16 **Information and assumptions**

from Table A.3, $\mu_{\text{air},20^\circ C} = 1.81 \times 10^{-5}$ N·s/m^2; $\nu = 1.51 \times 10^{-5}$ m^2/s
from Table A.5, $\mu_{\text{water},20^\circ C} = 1.00 \times 10^{-3}$ N·s/m^2; $\nu = 1.00 \times 10^{-6}$ m^2/s
provided in problem statement

Find

ratio of dynamic and kinematic viscosities of water and air

Solution

$$\mu_{\text{air}}/\mu_{\text{water}} = \underline{\underline{1.81 \times 10^{-2}}}; \quad \nu_{\text{air}}/\nu_{\text{water}} = \underline{\underline{15.1}}$$

2.17 Computer Problem - no solution.

2.18 The ratio of kinematic viscosities is

$$\frac{\nu}{\nu_o} = \frac{\mu}{\mu_o}\frac{\rho_o}{\rho} = \left(\frac{T}{T_o}\right)^{3/2}\frac{T_o + S}{T + S}\frac{p_o}{p}\frac{T}{T_o}$$

$$\frac{\nu}{\nu_o} = \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o + S}{T + S}$$

2.19 **Information and assumptions**

from Table A.2, S=111 K, $\mu_{\text{air}}(15^{\circ}\text{C})$=$1.78\times10^{-5}$ N·s/m^2.
provided in problem statement

Find

using Sutherland's equation, find μ at 100°C

Sutherland's equation

$$\frac{\mu}{\mu_o} = \left(\frac{T}{T_o}\right)^{3/2} \frac{T_o+S}{T+S} = \left(\frac{373}{288}\right)^{3/2} \frac{288+111}{373+111} = 1.215$$
$$\mu = 1.215 \times 1.78 \times 10^{-5} = \underline{\underline{2.16 \times 10^{-5}}}\ \mathrm{N \cdot s/m^2}.$$

2.20 **Information and assumptions**

from Table A.2, $S = 198$ K
provided in problem statement

Find

by definition

$$\nu = \frac{\mu}{\rho}$$

Ideal-gas law

$$\frac{\nu}{\nu_o} = \frac{\mu}{\mu_o}\frac{\rho_o}{\rho} = \frac{\mu}{\mu_o}\frac{p_o}{p}\frac{T}{T_o}$$

Sutherland's equation

$$\frac{\nu}{\nu_o} = \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o + S}{T + S}$$

so

$$\frac{\nu}{\nu_o} = \frac{1}{2}\left(\frac{473}{288}\right)^{5/2}\frac{473 + 198}{288 + 198} = 1.252$$

and

$$\nu = 1.252 \times 1.08 \times 10^{-5}\ \mathrm{m^2/s} = \underline{\underline{1.35 \times 10^{-5}}}\ \mathrm{m^2/s}$$

2.21 **Information and assumptions**

from Table A.2, S =192^{o}R.
provided in problem statement

Find

μ at 200^{o}F

Sutherland's equation

$$\begin{aligned}
\frac{\mu}{\mu_o} &= \left(\frac{T}{T_o}\right)^{3/2}\frac{T_o+S}{T+S} = \left(\frac{660}{519}\right)^{3/2}\frac{519+192}{660+192} = 1.197 \\
\mu &= 1.197 \times 3.59 \times 10^{-7} = \underline{\underline{4.30 \times 10^{-7}}} \text{ lbf-s/ft}^2
\end{aligned}$$

2.22 **Information and assumptions**

from Table A.2, S =143^{o}R.
provided in problem statement

Find

ν at 30^{o}F

Sutherland's equation

$$\begin{aligned}\frac{\nu}{\nu_o} &= \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o+S}{T+S} = \frac{\nu}{\nu_o} = \frac{1.5}{1}\left(\frac{490}{519}\right)^{5/2}\frac{519+143}{490+143} = 1.36 \\ \nu &= 1.36\times 1.22\times 10^{-3} = \underline{\underline{1.66\times 10^{-3}}}\ \text{ft}^2/\text{s}.\end{aligned}$$

2.23 **Information and assumptions**

provided in problem statement

Find

Sutherland's constant

Solution

Solving **Sutherland's equation** for S/T_o gives

$$\frac{S}{T_o} = \frac{\frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{1/2} - 1}{1 - \frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{3/2}}$$

Substituting $\mu/\mu_o = 1.72$ and $T_o/T = 373/673$ gives

$$\begin{aligned} \frac{S}{T_o} &= 0.964 \\ S &= \underline{\underline{360 \text{ K.}}} \end{aligned}$$

2.24 **Information and assumptions**

provided in problem statement

Find

Sutherland's constant

Solution

Solving for Sutherland's constant

$$\frac{S}{T_o} = \frac{\frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{1/2} - 1}{1 - \frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{3/2}}$$

Substituting values

$$\frac{\mu}{\mu_o} = \frac{3.46 \times 10^{-7}}{2.07 \times 10^{-7}} = 1.671$$

and

$$\frac{T_o}{T} = \frac{528}{852} = 0.6197$$

into equation for S/T_o

$$\begin{aligned} \frac{S}{T_o} &= 1.71 \\ S &= \underline{\underline{903^{\circ}R}} \end{aligned}$$

2.25 **Information and assumptions**

provided in problem statement

Find

$\mu(60^o\mathrm{C})$

Solution

Viscosity variation of a liquid can be expressed as

$$\frac{\mu}{\mu_o} = \exp\left[b(\frac{1}{T} - \frac{1}{T_o})\right]$$

Taking the logarithm and solving for b gives

$$b = \frac{\ln(\mu/\mu_o)}{(\frac{1}{T} - \frac{1}{T_o})}$$

Substituting in the values of $\mu/\mu_o = 0.011/0.067 = 0.164$ and $T = 372$ and $T_o = 311$ gives

$$b = 3430 \text{ (K)}$$

The viscosity ratio at 60°C is

$$\begin{aligned} \frac{\mu}{\mu_o} &= \exp[3430(\frac{1}{333} - \frac{1}{311}) = 0.4833 \\ \mu &= 0.4833 \times 0.067 = \underline{\underline{0.032 \text{ N} \cdot \text{s/m}^2}} \end{aligned}$$

2.26 **Information and assumptions**

provided in problem statement

Find

$\mu(150^o\text{F})$

Solution

Equation for viscosity variation of a liquid can be expressed as

$$\frac{\mu}{\mu_o} = \exp\left[b(\frac{1}{T} - \frac{1}{T_o})\right]$$

Taking the logarithm and solving for b gives

$$b = \frac{\ln(\mu/\mu_o)}{(\frac{1}{T} - \frac{1}{T_o})}$$

Substituting in the values of $\mu/\mu_o = 0.39 \times 10^{-3}/4.43 \times 10^{-3} = 0.0.0880$ and $T = 670$ and $T_o = 560$ gives

$$b = 8293\ (^o\text{R})$$

The viscosity ratio at 150^{o}F is

$$\begin{aligned} \frac{\mu}{\mu_o} &= \exp[8293(\frac{1}{610} - \frac{1}{560}) = 0.299 \\ \mu &= 0.299 \times 4.43 \times 10^{-3} = 1.32 \times 10^{-3}\ \text{lbf-s/ft}^2 \end{aligned}$$

2.27 A computer program is developed for Sutherland's constant from the equation

$$\frac{S}{273} = \frac{\frac{\mu}{\mu_o}\left(\frac{273}{T}\right)^{1/2} - 1}{1 - \frac{\mu}{\mu_o}\left(\frac{273}{T}\right)^{3/2}}$$

Carrying out the calculation for each data point and taking the average for Sutherland's constant gives

$$S = 127 \text{ K}$$

The error is defined as

$$err = 100 \times \left| \frac{\frac{\mu}{\mu_o} - \frac{\mu}{\mu_o}|_{calc}}{\frac{\mu}{\mu_o}} \right|$$

The results are

T(K)	260	270	280	290	300	350	500	1000	1500
$\frac{\mu}{\mu_o}\|_{calc}$	.9606	.991	1.021	1.050	1.079	1.217	1.582	2.489	3.168
error(%)	.013	.039	.084	.118	.108	.366	.486	1.17	3.56

The error is less 0.5% for temperatures up to 500 K but has an error of over 3.5% for a temperature of 1500K.

2.28 **Information and assumptions**

from Table Figure A.2, $\mu = 5.2 \times 10^{-4}$ lb·s/ft^2
provided in problem statement

Find

shear stress

Solution

Rate of strain

$$du/dy = 10/((1/4)/12)s^{-1}$$

Relation between stress and rate of strain

$$\tau = \mu du/dy = 5.2 \times 10^{-4} \times 10 \times 48 = \underline{\underline{0.250 \text{ lb/ft}^2}}$$

2.29 **Information and assumptions**

from Table A.3, $\mu_{\text{air}} = 1.91 \times 10^{-5}$ N·s/m^2
from Table A.5, $\mu_{\text{water}} = 6.53 \times 10^{-4}$ N·s/m^2,$\rho_{\text{water}} = 992$ kg/m^3.
provided in problem statement

Find

kinematic and dynamic viscosities

Ideal gas law

$$\rho_{\text{air}} = p/RT = 170,000(287 \times 313.2) = 1.89 \text{ kg/m}^3$$

For air

$$\begin{aligned}
\mu &= \underline{\underline{1.91 \times 10^{-5} \text{ N} \cdot \text{s/m}^2}} \\
\nu &= \mu/\rho = (1.91 \times 10^{-5})/1.89 = \underline{\underline{10.1 \times 10^{-6} \text{ m}^2/\text{s}}}
\end{aligned}$$

For water

$$\begin{aligned}
\mu &= 6.53 \times 10^{-5} \text{ N} \cdot \text{s/m}^2 \\
\nu &= (6.53 \times 10^{-4}/992) = \underline{\underline{6.58 \times 10^{-7} \text{ m}^2/\text{s}}}
\end{aligned}$$

2.30 **Information and assumptions**

from Table A.5, $\mu = 10^{-3}$ N·s/m^2
provided in problem statement

Find

shear stress at $y = 2$ mm

Stress-strain relationship

$$\begin{aligned}
\tau &= \mu du/dy \\
du/dy &= (1/6)(10)(y)^{-5/6} \text{ s}^{-1} \\
&= (1/6)(10)(.002)^{-5/6} \text{ s}^{-1} \\
&= (10/6)(177.5) \text{ s}^{-1} \\
\tau &= (10^{-3} \text{ N} \cdot \text{s/m}^2)(10/6)(177.5) \text{ s}^{-1} = \underline{\underline{0.296 \text{ N/m}^2}}
\end{aligned}$$

2.31 **Information and assumptions**

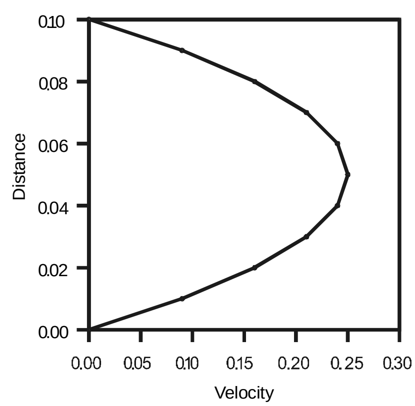
provided in problem statement

Find

shear stress at walls

Solution

$$\begin{aligned}
u &= 100y(0.1-y) = 10y - 100y^2 \\
du/dy &= 10 - 200y \\
(du/dy)_{y=0} &= 10\ \mathrm{s}^{-2} \quad (du/dy)_{y=0.1} = -10\ \mathrm{s}^{-1} \\
\tau_0 &= \mu du/dy = (8\times10^{-5})\times 10 = \underline{\underline{8\times10^{-4}\ \mathrm{lbf/ft}^2}} \\
\tau_{0.1} &= \underline{\underline{-8\times10^{-4}\ \mathrm{lbf/ft}^2}}
\end{aligned}$$



2.32 **Information and assumptions**

provided in problem statement

Find

maximum and minimum shear stress

maximum shear stress at wall

Solution

$$\begin{aligned}
\tau &= \mu dV/dy \\
\tau_{\max} &\approx \mu(\Delta V/\Delta y) \text{ next to wall} \\
\tau_{\max} &= (10^{-3}\text{N}\cdot\text{s/m}^2)((1\text{ m/s})/0.001\text{ m}) = \underline{\underline{1.0\text{ N/m}^2}}
\end{aligned}$$

The minimum shear stress will be zero, midway between the two walls, where the velocity gradient is zero.

2.33 **Information and assumptions**

from Table A.4, $\mu = 6.2 \times 10^{-1}$ N·s/m^2
provided in problem statement

Find

velocity and shear stress at 12 mm from wall, shear stress and velocity at wall

Solution

Velocity

$$\begin{aligned} u &= -(1/2\mu)(dp/dx)(By - y^2); \quad B = 0.05 \text{ m} \\ dp/dx &= -1,600 \text{ N/m}^3 \\ u_{12mm} &= (1,600/(2 \times 0.62))(0.05 \times 0.012 - (0.012)^2) \\ u_{12mm} &= \underline{\underline{0.588 \text{ m/s}}} \end{aligned}$$

Shear stress

$$\begin{aligned} \tau &= \mu du/dy = -(1/2)(dp/dx)(B - 2y) \\ \tau_0 &= (1,600/2)(0.05) = \underline{\underline{40 \text{ N/m}^2}} \\ \tau_{12} &= (1,600/2)(0.05 - 2 \times 0.012) = \underline{\underline{20.8 \text{ N/m}^2}} \end{aligned}$$

2.34 Develop an equation for the shear stress

$$
\begin{aligned}
\tau &= \mu du/dy \\
\mu du/dy &= -\mu(1/2\mu)(dp/ds)(H-2y) + u_t\mu/H \\
\text{Evaluate } \tau \text{ at } y &= H: \\
\tau_H &= -(1/2)(dp/ds)(H-2H) + u_t\mu/H \\
&= (1/2)(dp/ds)H + u_t\mu/H \\
\text{Evaluate } \tau \text{ at } y &= 0 \\
\tau_0 &= -(1/2)(dp/ds)H + u_t\mu/H
\end{aligned}
$$

Observation of the velocity gradient lets one conclude that the pressure gradient dp/ds is negative. Also u_t is negative. Therefore $|\tau_h| > |\tau_0|$. The maximum shear stress occurs at $\underline{\underline{y = H}}$.

2.35 From solution to 2.34

$$\begin{aligned}
\tau &= -(1/2)(dp/ds)(H-2y)+u_t\mu/H \\
\text{Set } \tau &= 0 \text{ and solve for } y \\
0 &= -(1/2)(dp/ds)(H-2y)+u_t\mu/H \\
y &= \underline{\underline{(H/2)-(\mu u_t/(H dp/ds))}}
\end{aligned}$$

2.36 Derive an expression for plate speed

$$\begin{aligned}
\tau &= \mu du/dy = 0 \text{ at } y = 0 \\
du/dy &= -(1/2\mu)(dp/ds)(H - 2y) + u_t/H \\
\text{Then, at } y &= 0 : du/dy = 0 = -(1/2\mu)(dp/ds)H + u_t/H \\
\text{Solve for } u_t &: \underline{u_t = (1/2\mu)(dp/ds)H^2} \\
\textit{Note} &: \text{because } dp/ds < 0, u_t < 0.
\end{aligned}$$

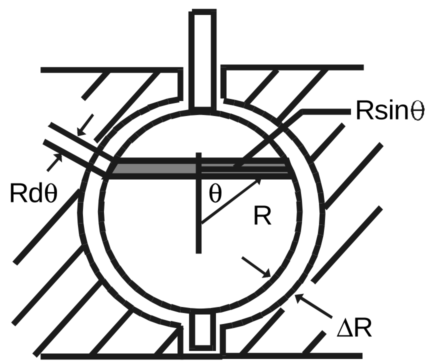
2.37 **Information and assumptions**

from Table A.4, $\mu(38^{o}\text{C})=3.6\times10^{-2}$ N·s/m^2
provided in problem statement

Find

find torque on shaft

Solution



$$
\begin{aligned}
dT &= rdF \\
dT &= r\tau dA \\
\text{where } \tau &= \mu(dV/dy) = \mu(\Delta V/\Delta R) \\
&= \mu(\omega R\sin\theta/\Delta R) \\
&= 3.6\times10^{-2}\ \text{N}\cdot\text{s/m}^2)(10\times2\pi/60)\ \text{rad/s}(0.05\ \text{m}\sin\theta/10^{-3}\ \text{m}) \\
&= 1.885\sin\theta\ \text{N/m}^2 \\
dA &= 2\pi R\sin\theta Rd\theta \\
&= 2\pi R^2\sin\theta Rd\theta \\
&= 2\pi R^2\sin\theta d\theta \\
r &= R\sin\theta \\
\text{Then } dT &= R\sin\theta(1.885\sin\theta)(2\pi R^2\sin\theta d\theta) \\
dT &= 11.84R^3\sin^3\theta d\theta \\
T &= 11.84R^3\int_0^\pi \sin^3\theta d\theta \\
&= 11.84(0.05)^3[-(1/3)\cos\theta(\sin^2\theta+2)]_0^\pi \\
&= 11.84(0.05)^3[-(1/3)(-1)(2)-(-1/3)(1)(2)] \\
\text{Torque} &= \underline{\underline{1.97\times10^{-3}\ \text{Nm}}}
\end{aligned}
$$

2.38 Because the viscosity of gases increases with temperature $\mu_{100}/\mu_{50} > 1$. Correct choice is (c).

2.39 **Information and assumptions**

from Figure A.2, $\mu(10^{\circ}\mathrm{C})$=0.35 N·s/m^2
provided in problem statement

Find

Cylinder fall rate

Solution

$$\begin{aligned}
\tau &= \mu dV/dy \\
W/(\pi d\ell) &= \mu V_{\text{fall}}/[(D-d)/2] \\
V_{\text{fall}} &= W(D-d)/(2\pi d\ell\mu) \\
V_{\text{fall}} &= 20(0.5\times10^{-3})/(2\pi\times0.1\times0.2\times3.5\times10^{-1}) = \underline{\underline{0.23\text{ m/s}}}
\end{aligned}$$

2.40 **Information and assumptions**

from Figure A.2, $\mu(10^{\mathrm{o}}\mathrm{C})$=0.35 N·s/m^2
provided in problem statement

Find

weight of cylinder

Solution

Sum of forces equal zero (no acceleration)

$$\begin{aligned}
&\sum F_z = 0 \\
&-W + F\tau = ma \\
&-W + \pi d\ell\mu V/[(D-d)/2] = W/ga \\
&-W + (\pi \times 0.1 \times 0.2 \times 3.5 \times 10^{-1} V)/(0.5 \times 10^{-3}/2) = Wa/9.81
\end{aligned}$$

Substituting $V = 0.5$ m/s and $a = 14$ m/s^2 and solving yields $\underline{\underline{W = 18.1 \text{ N}}}$

2.41 **Information and assumptions**

assume linear velocity distribution: $dV/dy = V/y = \omega r/y$
provided in problem statement

Find

ratio of shear stress at $r = 2$ cm to shear stress at $r = 3$ cm, speed of oil at contact with cylinder, shear stress at cylinder

Solution

$$\begin{aligned}
\tau &= \mu dV/dy = \mu\omega r/y \\
\tau_2/\tau_3 &= (\mu \times 1 \times 2/y)/(\mu \times 1 \times 3/y) = 2/3 = \underline{\underline{0.667}} \\
V &= \omega r = 2 \times 0.03 = \underline{\underline{0.06 \text{ m/s}}} \\
\tau &= \mu dV/dy = 0.01 \times 0.06/0.002 = \underline{\underline{0.30 \text{ N/m}^2}}
\end{aligned}$$

2.42 **Information and assumptions**

assume a linear velocity distribution
provided in problem statement

Find

torque to rotate disk

Solution

$$
\begin{aligned}
\tau &= \mu dV/dy \\
\tau &= \mu\omega r/y \\
&= 0.01 \times 5 \times r/0.002 = 25r \text{ N/m}^2 \\
d \text{ Torque} &= r\tau dA \\
&= r(10r)2\pi r dr = 50\pi r^3 dr \\
\text{Torque} &= \int_0^{0.05} 50\pi r^3 dr = 50\pi r^4/4 \,\big|_0^{0.5} \\
\text{Torque} &= \underline{\underline{2.45 \times 10^{-4}}} \text{ N} \cdot \text{m}
\end{aligned}
$$

2.43 **Information and assumptions**

assume velocity profiles are linear
provided in problem statement

Find

derive an equation for damping torque

Solution

Relation between shear stress and velocity gradient

$$\tau = \mu dV/dy = \mu r\omega/s$$

On an elemental strip of area of radius r the differential shear force will be τdA or $\tau(2\pi r dr)$. The differential torque will be the product of the differential shear force and the radius r or $dT_{\text{one side}}$

$$\begin{aligned} &= r[\tau(2\pi r dr)] \\ &= r[(\mu r\omega/s)(2\pi r dr)] \\ &= (2\pi\mu\omega/s)r^3 dr \\ dT_{\text{both sides}} &= (r\pi\mu\omega/s)r^3 dr \\ T &= \int_0^{D/2} (4\pi\mu\omega/s)r^3 dr = \underline{\underline{(1/16)\pi\mu\omega D^4/s}} \end{aligned}$$

2.44 One possible design solution is given below.

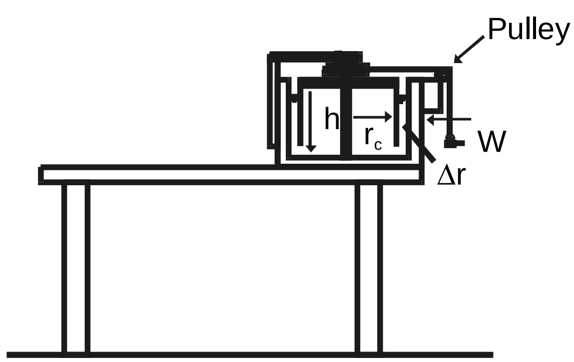
Assumptions:

1. Motor oil is SAE 10W-30; there μ will vary from about 2×10^{-4}lbf-s/ft$^2 to 8 \times 10^{-3}$lbf-s/ft^2 (Fig. A-2)
2. Assume the only significant shear stress develops between the rotating cylinder and the fixed cylinder.
3. Assume we want the maximum, rate of rotation ω, to be 3 rad/s.

Arbitrary decision:

1. Let $h = 4.0$ in. $= 0.333$ ft
2. Let I.D. of fixed cylinder $= 9.00$ in. $= 0.7500$ ft.
3. Let O.D. of rotating cylinder $= 8.900$ in. $= .7417$ ft.

Let the applied torque which drives the rotating cylinder be produced by a force from a thread or small diameter monofilament line acting at a radial distance r_s. Here r_s is the radius of a spool on which the thread of line is wound. The applied force is produced by a weight and pulley system shown in the sketch below.



The relationship between μ, r_s, ω, h, and W is now developed.

$$T = r_c F_s \tag{1}$$

where $T =$ applied torque
$r_c =$ outer radius of rotating cylinder
$F_s =$ shearing force developed at the outer radius of the rotating cylinder but $F_s = \tau A_s$ where $A_s =$ area in shear $= 2\pi r_c h$

$$\tau = \mu dV/dy \approx \mu \Delta V/\Delta r \text{ where } \Delta V = r_c\omega \text{ and } \Delta r = \text{ spacing}$$

Then $T = r_c(\mu \Delta V/\Delta r)(2\pi r_c h)$

$$= r_c \mu (r_c \omega/\Delta r)(2\pi r_c h) \tag{2}$$

But the applied torque $T = Wr_s$ so Eq. (2) become $\quad Wr_s = r_c^3 \mu\omega(2\pi)h/\Delta r$

$$\text{or } \mu = (Wr_s\Delta r)/(2\pi\omega h r_c^3) \tag{3}$$

The weight W will be arbitrarily chosen (say 2 or 3 oz.) and ω will be determined by measuring the time it takes the weight to travel a given distance. So $r_s\omega = V_{\text{fall}}$ or $\omega = V_{\text{fall}}/r_s$. Equation (3) then becomes

$$\mu = (W/V_f)(r_s^2/r_c^3)(\Delta r/(2\pi h))$$

In our design let $r_s = 2$ in. $= 0.1667$ ft. Then

$$\begin{aligned} \mu &= (W/F_f)(0.1667^2/.3708^3)(0.004167/(2\pi \times .3333) \\ \mu &= (W/V_f)(.02779/.05098) \\ \mu &= (W/V_f)(1.085 \times 10^{-3}) \text{ lbf} \cdot \text{s/ft}^2 \end{aligned}$$

Example: If $W = 2\text{oz.} = 0.125\text{lb.}$ and V_f is measured to be 0.24 ft/s than $\mu = (0.125/0.24)(1.085 \times 10^{-3}), = 0.564 \times 10^{-4}$ lbf·s/ft^2

Other things that could be noted or considered in the design:

1. Specify dimensions of all parts of the instrument.
2. Neglect friction in bearings of pulley and on shaft of cylinder.
3. Neglect weight of thread or monofilament line.
4. Consider degree of accuracy.
5. Estimate cost of the instrument.

2.45 **Information and assumptions**

from Table A.5, $E = 2.2 \times 10^9$ Pa
provided in problem statement

Find

volume after pressure applied

Solution

Relation between modulus of elasticity, applied pressure and volume change

$$E = -\Delta p/(\Delta \cancel{V}/\cancel{V})$$

Final volume

$$\begin{aligned}
2.2 \times 10^9 &= -2 \times 10^6/(\Delta \cancel{V}/1,000) \\
\Delta \cancel{V} &= -(2 \times 10^6 \times 1,000)/(2.2 \times 10^9) = -0.909\text{cc} \\
\cancel{V}_{final} &= \cancel{V} - \Delta \cancel{V} = \underline{\underline{999.09\text{cc}}}
\end{aligned}$$

2.46 **Information and assumptions**

from Table A.5, $E = 2.2 \times 10^9$ Pa
provided in problem statement

Find

pressure increase to reduce volume by 1%

Solution

Relation between modulus of elasticity, applied pressure and volume change

$$E = -\Delta p/(\Delta\forall/\forall)$$

For 1% volume change

$$\begin{aligned} 2.2 \times 10^9 &= -\Delta p/(-1/100) \\ \Delta p &= 2.2 \times 10^7 \text{ N/m}^2 = \underline{\underline{22 \text{ MN/m}^3 \text{ (increase)}}} \end{aligned}$$

2.47 Refer to Fig. 2-6(a). The surface tension force, $2\pi r\sigma$, will be resisted by the pressure force acting on the cut section of the spherical droplet or

$$\begin{aligned} p(\pi r^2) &= 2\pi r\sigma \\ p &= 2\sigma/r = \underline{\underline{\underline{4\sigma/d}}} \end{aligned}$$

2.48 **Information and assumptions**

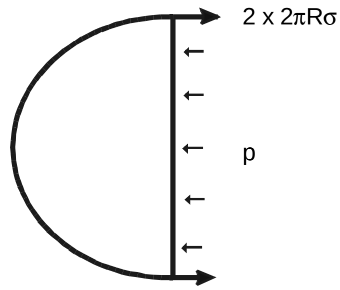
assume effect of thickness is negligible
provided in problem statement

Find

find pressure difference across bubble

Solution

Force balance



$$
\begin{aligned}
\sum F &= 0 \\
\Delta p \pi R^2 - 2(2\pi R\sigma) &= 0 \\
\Delta p &= \underline{\underline{4\sigma/R}} \\
\Delta p_{\text{4mm rad.}} &= (4 \times 7.3 \times 10^{-2}\ \text{N/m})/0.004\ \text{m} = \underline{\underline{73.0\ \text{N/m}^2}}
\end{aligned}
$$

2.49 **Information and assumptions**

from Table A.5, $\sigma = 0.073$ N/m
provided in problem statement

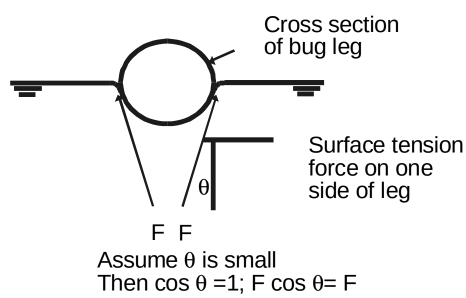
Find

maximum mass of bug

Force equilibrium

$$\sum F_2 = 0$$

surface tension force supporting bug - weight of bug = 0
Consider cross section of bug leg:



$$\begin{aligned}
F_T &= (2/\text{leg})(6 \text{ legs})\sigma\ell \\
&= 12\sigma\ell \\
&= 12(0.073 \text{ N/m})(0.005 \text{ m}) \\
&= 0.00438N \\
&\therefore\ F_T - mg = 0 \\
m &= (0.00438/9.81) \times 1,000 = \underline{\underline{0.446 \text{ gm}}}
\end{aligned}$$

2.50 **Information and assumptions**

from Table A.5 surface tension of water is 0.005 lbf/ft
provided in problem statement

Find

amount of water column due to surface tension effects

Solution

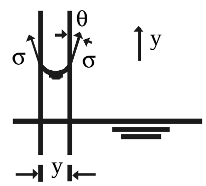
$$\begin{aligned}
\Delta h &= 4\sigma/(\gamma d) = 4 \times 0.005/(62.4 \times d) = 3.21 \times 10^{-4}/d \text{ ft.} \\
d &= 1/4 \text{ in.} = 1/48 \text{ ft.}; \ \Delta h = 3.21 \times 10^{-4}/(1/48) = 0.0154 \text{ ft.} = \underline{\underline{0.185 \text{ in.}}} \\
d &= 1/8 \text{ in.} = 1/96 \text{ ft.}; \ \Delta h = 3.21 \times 10^{-4}/(1/96) = 0.0308 \text{ ft.} = \underline{\underline{0.369 \text{ in.}}} \\
d &= 1/32 \text{ in.} = 1/384 \text{ ft.}; \ \ \Delta h = 3.21 \times 10^{-4}/(1/384) = 0.123 \text{ ft.} = \underline{\underline{1.48 \text{ in.}}}
\end{aligned}$$

2.51 **Information and assumptions**

from Table A.5, surface tension of water is 7.3×10^{-2} N/m
provided in problem statement

Find

capillary rise between two vertical plates



$$\begin{aligned}
\sum F_y &= 0 \\
2\sigma\ell - h\ell t\gamma &= 0 \\
h &= 2\sigma/\gamma t \\
h &= 2 \times 7.3 \times 10^{-2}/(0.0010 \times 9,810) = 0.0149 \text{ m} = \underline{\underline{14.9 \text{ mm}}}
\end{aligned}$$

2.52 **Information and assumptions**

from Table A.5, surface tension of water is 7.3×10^{-2} N/m
provided in problem statement

Find

pressure within 1-mm spherical droplet

Solution

Solution is similar to that for Problem 2-48 except that only one surface exists here.

$$\begin{aligned}
\Delta p \pi R^2 - 2\pi R\sigma &= 0 \\
\Delta p &= 2\sigma/R \\
\Delta p &= 2 \times 7.3 \times 10^{-2}/(0.5 \times 10^{-3}) = \underline{\underline{292 \text{ N/m}^2}}
\end{aligned}$$

2.53 The elevation in a column due to surface tension is

$$\Delta h = \frac{4\sigma}{\gamma d}$$

where γ is the specific weight and d is the tube diameter. For the change in surface tension due to temperature, the change in column elevation would be

$$\Delta h = \frac{4\Delta\sigma}{\gamma d} = \frac{4 \times 0.0167}{9810 \times d} = \frac{6.8 \times 10^{-6}}{d}$$

The change in column elevation for a 1-mm diameter tube would be 6.8 mm. Special equipment, such the optical system from a microscope, would have to be used to measure such a small change in deflection It is unlikely that smaller tubes made of transparent material can be purchased to provide larger deflections.

2.54 **Information and assumptions**

provided in problem statement

Find

depression of mercury surface

Solution

Equating the surface tension forced with the force due to hydrostatic pressure gives

$$\cos\theta\pi d\sigma = \Delta h\gamma\frac{\pi d^2}{4}$$

Solving for Δh results in

$$\Delta h = \frac{4\cos\theta\sigma}{\gamma d}$$

Substituting in the values gives the depression of the mercury surface.

$$\Delta h = \frac{4 \times \cos 40 \times 0.514}{13.6 \times 9810 \times 0.001} = 0.0125 \text{ m} = \underline{\underline{12.5}} \text{ mm}$$

2.55 The bubble will have the greatest pressure because there are two surfaces (two surface tension forces) creating the pressure within the bubble. $\underline{\underline{\text{a)}}}$

2.56 **Information and assumptions**

provided in problem statement

Find

boiling at altitude of 3000 m.

Solution

$$100 - (101 - 69)/3.1 = \underline{\underline{89.7^\circ \text{ C}}}$$

Chapter Three

3.1 **Information and assumptions**

provided in problem statement

Find

% error in gage reading

Solution

$$\begin{aligned} p &= 20/(\pi/4) \times (1)^2 = 25.46 \text{ psi } = 175.4 \text{ kPa} \\ \% \text{ gage error} &= (26 - 25.46) \times 100/25.46 = \underline{\underline{2.10\%}} \end{aligned}$$

3.2 **Information and assumptions**

provided in problem statement

Find

force required to separate shells

Solution

The radius of the seal is 0.1525 m.

$$\begin{aligned} F &= \Delta p A \\ &= (100 \text{ kPa } - 10 \text{ kPa})\pi r^2 \\ &= (90,000 \text{ N/m}^2)\pi(0.1525 \text{ m})^2 \\ &= \underline{\underline{6.58 \text{ kN}}} \end{aligned}$$

3.3 This is an open-ended problem. The area of contact between each tire and pavement is a function of the weight of the car and the air pressure of the tires. The solution to the second part is a function of the air pressure and the diameter of the pump. For example, if the pump diameter is one inch, the force required will be about 24 lbf.

3.4 **Information and assumptions**

assume same force per bolt at B-B
provided in problem statement

Find

number of bolts required at section B-B

Solution

$$\begin{aligned}
F \text{ per bolt at } A-A &= p(\pi/4)D^2/20 \\
p(\pi/4)D^2/20 &= p(\pi/4)d^2/n \\
n &= 20 \times (d/D)^2 = 20 \times (1/2)^2 = \underline{\underline{5}}
\end{aligned}$$

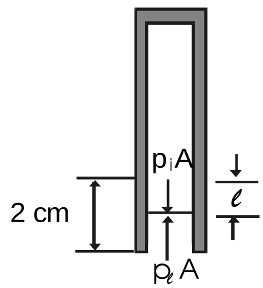
3.5 **Information and assumptions**

assume water wets the glass
provided in problem statement

Find

location of water line in tube.

Solution



Equate forces acting at the liquid surface inside the glass tube.

$$\begin{aligned} \sum F_z &= 0 \\ -p_i A + p_\ell A + \sigma \pi d &= 0 \end{aligned}$$

where p_i is the pressure inside the tube and p_ℓ is the pressure in water at depth ℓ. Also

$$\begin{aligned} p_i \forall_i &= p_{\text{atm}} \forall_{\text{tube}} \\ p_i &= p_{\text{atm}} (\forall_{\text{tube}} / \forall_i) \\ &= p_{\text{atm}} (0.10 A_{\text{tube}} / ((.08 + \ell)(A_{\text{tube}})) \qquad (1) \end{aligned}$$

$$p_i = p_{\text{atm}} (0.10/(.08 + \ell)) \qquad (2)$$

$$p_\ell = p_{\text{atm}} + \gamma \ell \qquad (3)$$

Solving for ℓ with Eqs. (1), (3) and (4) yields

$$\ell = 0.0192m = \underline{\underline{1.92 \text{ cm}}}$$

3.6 Correct graph is $\underline{\underline{\text{(b)}}}$

3.7 (a) The water surface level in the left tube will be higher because of greater surface tension effects for that tube.

3.8 **Information and assumptions**

provided in problem statement

Find

force resisted by piston

Solution

$$\begin{aligned}
p_1 - \gamma\Delta z &= p_2 \\
\text{or } p_2 &= 200 \text{ N}/A \ - 0.85 \times 9,810 \text{ N/m}^3 \times 2 \text{ m} \\
&= [200 \text{ N}/((\pi/4) \times (0.04)^2)] - 16,677 \text{ N/m}^2 \\
&= 142.5 \text{ kPa} \\
F_2 &= p_2 A_2 = 142.5 \times 10^3 (\pi/4) \times (0.10)^2 \\
&= \underline{\underline{1,119 \text{ N}}}
\end{aligned}$$

3.9 **Information and assumptions**

provided in problem statement

Find

ratio of pressure to normal atmospheric pressure

Solution

$$\begin{aligned} p &= \gamma \Delta z = 9,790 \times 50 = 489,500 \text{ N/m}^2 = \underline{\underline{489.5 \text{ kPa}}}, \text{ gage} \\ p_{50}/p_{\text{atm}} &= (489.5 + 101.3)/101.3 = \underline{\underline{5.83}} \end{aligned}$$

3.10 **Information and assumptions**

Assume $\gamma = 9,810 \text{ N/m}^3$
provided in problem statement

Find

The gage pressure at the 10 m depth $= 10\gamma$

Solution

$$= 10 \text{ m} \times 9,810 \text{ N/m}^3 = 98.1 \text{ kPa}$$

The absolute pressure at the 10 m depth

$$= 98.1 \text{ kPa} + 98 \text{ kPa} = 2 \times 98 \text{ kPa}$$

The absolute pressure at the 10 m depth is $\underline{\underline{\text{2 times}}}$ that at the surface.

3.11 **Information and assumptions**

provided in problem statement

Find

gage pressure at bottom of tank

$$\begin{aligned} p &= (\gamma h)_{\text{water}} + (\gamma h)_{\text{kerosene}} \\ &= 9,790 \times 1.6 + 8,010 \times 1.2 = 25,276 \text{ N/m}^2 \\ &= \underline{\underline{25.28}} \text{ kPa} \end{aligned}$$

3.12 **Information and assumptions**

provided in problem statement

Find

pressure at depth of 15 m.

Solution

$$p = 15 \times 8,630 = 129,450 \text{ kPa} = \underline{\underline{129.5}} \text{ kPa}$$

3.13 **Information and assumptions**

provided in problem statement

Find

thickness of atmospheric air layer

Solution

$$p = \gamma t$$

where t is the thickness of the atmosphere.

$$\begin{aligned}
(14.7 \text{ lbf/in}^2)(144 \text{ in}^2/\text{ft}^2) &= \gamma t \\
14.7 \times 144 \text{ lbf/ft}^2 &= (0.00247 \text{ lbf} \cdot \text{s}^2/\text{ft}^4)(32.2 \text{ ft/s}^2)t \\
t &= \underline{\underline{26,615 \text{ ft}}}
\end{aligned}$$

3.14 **Information and assumptions**

provided in problem statement

Find

specific weight and specific gravity

Solution

$$
\begin{aligned}
\gamma &= p/h = 47.9 \times 10^3/6 = \underline{\underline{6.98\ }}\text{kN/m}^3 \\
S &= 6.98/9.81 = \underline{\underline{0.71}}
\end{aligned}
$$

3.15 **Information and assumptions**

provided in problem statement

Find

gage pressure at 10 m depth

Solution

$$
\begin{aligned}
\rho &= \rho_{\text{water}}(1+0.01d) \\
\text{or } \gamma &= \gamma_{\text{water}}(1+0.01d) \\
dp/dz &= -dp/dd = -\gamma \\
dp/dd &= \gamma_{\text{water}}(1+0.01d)
\end{aligned}
$$

Integrating

$$p = \gamma_{\text{water}}(d + 0.01d^2/2) + C$$

For boundary condition $p_{\text{gage}} = 0$ when $d = 0$ gives $C = 0$.

$$
\begin{aligned}
p_{\text{d=10 m}} &= \gamma_{\text{water}}(10 + 0.01 \times 10^2/2) \\
&= \underline{\underline{103.0 \text{ kPa}}}
\end{aligned}
$$

for $\gamma_{\text{water}} = 9,810 \text{ N/m}^3$

3.16 **Information and assumptions**

provided in problem statement

Find

depth where pressure is 60 kPa.

Solution

From solution to Prob 3.15

$$\begin{aligned} p &= \gamma_{\text{water}}(d + 0.01\ d^2/2) \\ 60{,}000\ \text{N/m}^2 &= (9{,}810\ \text{N/m}^3)(d + .005\ d^2) \end{aligned}$$

Solving the above equation for d yields $d =$ $\underline{\underline{5.94\ \text{m}}}$

3.17 **Information and assumptions**

provided in problem statement

Find

pressure at depth of 20 ft.

Solution

$$\begin{aligned}
dp/dz &= -\gamma \\
&= -(50 - 0.1\ z) \\
p &= \int_0^{-20} (50 - 0.1\ z)\ dz \\
&= -50\ z\ + 0.1\ z^2/2\ |_0^{-20} \\
&= 1000 + 0.1 \times 400/2 \\
&= \underline{\underline{1020\ \text{psfg}}}
\end{aligned}$$

3.18 **Information and assumptions**

from Table A.5, $\gamma_{\rm w}$=9790 N/m^3
assume an ideal gas
provided in problem statement

Find

increase of water elevation in manometer.

Solution

$$p/\rho = RT$$

but

$$\rho = M/\forall$$

so

$$p\ (M/\forall) = RT;\ p\forall = MRT$$

Because M, R and T are constants for air in the tube

$$\begin{gathered} p_1\forall_1 = p_2\forall_2 \\ p_1 = 100,000\ \text{N/m}^2\ \text{abs} \\ \forall_1 = 1\ \text{m} \times A_{\text{tube}} \\ p_2 = 100,000\ \text{N/m}^2 + \gamma_{\rm w}(1\ \text{m} - \Delta\ell) \\ \text{Also } p_2 = p_1\forall_1/\forall_2 \\ = (100,000\ \text{N/m}^2)(1\ \text{m} \times A_{\text{tube}})/[(1\ \text{m} - \Delta\ell)A_{\text{tube}}] \end{gathered}$$

Equating the equations for p_2

$$\begin{aligned} 100,000 + \gamma_{\rm w}(1 - \Delta\ell) &= (100,000)[1/(1 - \Delta\ell)] \\ \Delta\ell^2 - 12.214\Delta\ell + 1 &= 0 \end{aligned}$$

Solving for $\Delta\ell$

$$\Delta\ell = \underline{\underline{0.082}}\ \text{m}$$

3.19 **Information and assumptions**

provided in problem statement

Find

location of liquid surface in manometer

Solution

The distance Δh is the height of the liquid surface in the manometer above the interface between the water and the heavier liquid. Thus

$$\begin{aligned}\gamma_{\mathrm{H_2O}} \times 0.1 \text{ m} &= 3\gamma_{\mathrm{H_2O}} \times \Delta h \\ \Delta h &= 0.1 \text{ m}/3 \\ &= 0.0333 \text{ m} \\ &= \underline{\underline{3.33}} \text{ cm}\end{aligned}$$

3.20 **Information and assumptions**

provided in problem statement

Find

maximum gage pressure, where will maximum pressure occur, hydrostatic force on side C-D

Solution

$$
\begin{aligned}
0 + 4 \times \gamma_{\mathrm{H_2O}} + 3 \times 3\gamma_{\mathrm{H_2O}} &= p_{\max} \\
p_{\max} &= 13 \times 9,810 = 127,530 \ \mathrm{N/m^2} = \underline{\underline{127.5 \ \mathrm{kPa}}}
\end{aligned}
$$

Maximum pressure will be at the bottom of the liquid with a S of 3.

$$F_{CD} = pA = (127,530 - 1 \times 3 \times 9,810) \times 1 \ \mathrm{m^2} = \underline{\underline{98.1 \ \mathrm{kN}}}$$

3.21 **Information and assumptions**

provided in problem statement

Find

% difference in sea water density at 6 km

Solution

$$
\begin{aligned}
\Delta p - \gamma h &= 10,070 \times 6 \times 10^3 \\
E_V &= \Delta p/(d\rho/\rho) \\
(d\rho/\rho) &= \Delta p/E_v = (10,070 \times 6 \times 10^3)/(2.2 \times 10^9) = 27.46 \times 10^{-3} = \underline{\underline{2.75\%}}
\end{aligned}
$$

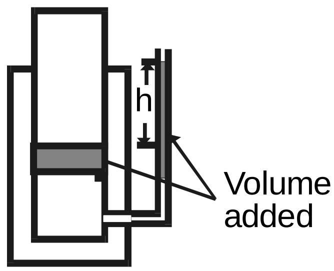
3.22 **Information and assumptions**

provided in problem statement

Find

volume of oil to be added to raise piston by 1 in.

Solution



Volume added is shown in the figure. First get pressure at bottom of piston

$$\begin{aligned} p_p A_p &= 10 \text{ lbf} \\ p_p &= 10/A_p \\ &= 10/((\pi/4) \times 4^2) \\ &= 0.796 \text{ psig } = 114.6 \text{ psfg} \end{aligned}$$

Then

$$\begin{aligned} \gamma_{\text{oil}} h &= 114.6 \text{ psfg} \\ h &= 114.6/(62.4 \times 0.85) = 2.161 \text{ ft} = 25.9 \text{ in} \end{aligned}$$

Finally

$$\begin{aligned} \forall_{\text{added}} &= (\pi/4)(4^2 \times 1 + 1^2 \times 26.9) \\ &= \underline{\underline{33.7 \text{ in.}^3}} \end{aligned}$$

3.23 **Information and assumptions**

air is ideal gas
temperature is constant
provided in problem statement

Find

ratio of density of air in bubble at 20 ft to density at 10 ft.

Solution

$$\begin{aligned}\rho &= p/RT\\ \rho_{10} &= p_{10}/RT;\ \rho_{20} = p_{20}/RT\\ \frac{\rho_{20}}{\rho_{10}} &= \frac{p_{20}}{p_{10}}\end{aligned}$$

where p is absolute pressure.

$$p_{10} = 14.7 \text{ lbf/in}^2 \times 144 \text{ in}^2/\text{ft}^2 + 10 \text{ ft } \times 62.4 \text{ lbf/ft}^3 = 2741 \text{ lbf/ft}^2$$
$$p_{20} = 14.7 \text{ lbf/in}^2 \times 144 \text{ in}^2/\text{ft}^2 + 20 \text{ ft } \times 62.4 \text{ lbf/ft}^2 = 3365 \text{ lbf/ft}^2$$
$$\rho_{20}/\rho_{10} = p_{20}/p_{10} = 3365/2741 = \underline{\underline{1.23}}$$

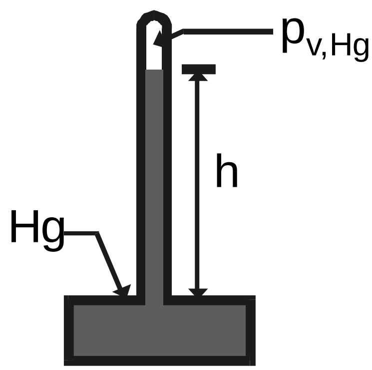
3.24 **Information and assumptions**

assume vapor pressure of mercury is zero
provided in problem statement

Find

reading on mercury barometer

Solution



$$\begin{aligned} p_{v,\mathrm{Hg}} + \gamma_{Hg} h &= p_{\mathrm{atm}} \\ h &= (p_{\mathrm{atm}} - p_{v,\mathrm{Hg}})/\gamma_{Hg} \\ h &= (98,000 - 0)/133,000 = 0.737 \text{ m } = \underline{\underline{737 \text{ mm}}} \end{aligned}$$

3.25 **Information and assumptions**

provided in problem statement

Find

gage pressure at pipe center

Manometer equation

from the pipe to the open end of the manometer

$$p_{\text{pipe}} + (0.5 \text{ ft})(62.4 \text{ lbf/ft}^3) + (1 \text{ ft})(2 \times 62.4 \text{ lbf/ft}^3)$$
$$-(2.5 \text{ ft})(62.4 \text{ lbf/ft}^3) = 0$$
$$p_{\text{pipe}} = (2.5 - 2 - 0.5) \text{ ft } (62.4 \text{ lbf/ft}^3) = \underline{\underline{0}}$$

3.26 **Information and assumptions**

provided in problem statement

Find

gage pressure at pipe center.

Manometer equation

$$p_A + (2.0 \text{ ft})(62.3 \text{ lbf/ft}^3) - (2/12 \text{ ft})(847 \text{ lbf/ft}^3) = 0$$

$$\begin{aligned} p_A &= -124.6 \text{ lbf/ft}^2 + 141.2 \text{ lbf/ft}^2 = +16.6 \text{ lbf/ft}^2 \\ &= \underline{\underline{+0.12 \text{ psi}}} \end{aligned}$$

3.27 **Information and assumptions**

provided in problem statement

Find

gage pressure in pipe A

Manometer equation

$$p_A = 9,810(1 \times 13.55 - 1.5 + 1.3 \times 0.9) = 129,700 \text{ N/m}^2 = \underline{\underline{129.7 \text{ kPa}}}$$

3.28 **Information and assumptions**

$S_{Hg} = 13.55$
$\gamma_{\mathrm{H_2O}} = 62.4 \text{ lbf/ft}^3$
provided in problem statement

Find

gage pressure in pipe A

Manometer equation

from pipe A to the top of the mercury column.

$$\begin{aligned}
p_A - 1.3\gamma_{\text{oil}} + 1.5\gamma_{\mathrm{H_2O}} - 1\gamma_{Hg} &= 0 \\
p_A - 1.3(0.9)\gamma_{\mathrm{H_2O}} + 1.5\gamma_{\mathrm{H_2O}} - 13.55\gamma_{\mathrm{H_2O}} &= 0 \\
p_A &= \gamma_{\mathrm{H_2O}}(13.55 + 1.17 - 1.5) \\
&= (62.4 \text{ lbf/ft}^3)(13.22 \text{ ft}) \\
&= \underline{\underline{825 \text{ psfg}}}
\end{aligned}$$

3.29 **Information and assumptions**

provided in problem statement

Find

estimate gage pressure in pipe A

Solution

$$\begin{aligned}
\Delta h_{\text{surface tension}} &= 4\sigma/(\gamma d) = (4 \times 7.3 \times 10^{-2})/(9,810 \times 1 \times 10^{-3}) \\
&= 0.0298 \text{ m} = 2.98 \text{ cm} \\
p_A &= \gamma h = 9,810(10 - 2.98) \times 10^{-2} = \underline{\underline{689 \text{ Pa}}}
\end{aligned}$$

3.30 **Information and assumptions**

provided in problem statement

Find

pressure at the center of pipe B

Manometer equation

$$
\begin{aligned}
p_B &= 50 \times (3/5 \times 10^{-2} \times 20 \times 10^3 \\
&\quad -10 \times 10^{-2} \times 20 \times 10^3 - 50 \times 10^{-2} \times 10 \times 10^3 \\
p_B &= -1,000 \text{ Pa } = \underline{\underline{-1.00 \text{ kPa}}}
\end{aligned}
$$

3.31 **Information and assumptions**

provided in problem statement

Find

cistern pressure

Solution

$$
\begin{aligned}
(\pi/4)D^2_{\text{tube}} \times \ell &= (\pi/4)D^2_{\text{cistern}} \times (\Delta h)_{\text{cistern}} \\
(\Delta h)_{\text{cistern}} &= (1/8)^2 \times 40 = 0.625 \text{ cm} \\
p_{\text{cistern}} &= (\ell \text{ sin } 10^\circ + \Delta h)\rho g \\
&= (40 \text{ sin } 10^\circ + 0.625) \times 10^{-2} \times 800 \times 9.81 = \underline{\underline{594 \text{ Pa}}}
\end{aligned}
$$

3.32 **Information and assumptions**

provided in problem statement

Find

cistern pressure

Solution

$$\begin{aligned} \Delta h &= (1/10)^2 \times 3 = 0.03 \text{ ft} \\ p_{\text{cistern}} &= (2 \sin 10^\circ + 0.03) \times 50 = \underline{\underline{18.86 \text{ psfg}}} \end{aligned}$$

3.33 First of all neglect the weight of the piston and find the piston area which will give reasonable manometer deflections. Equating the force on the piston, the piston area and the deflection of the manometer gives

$$W = \Delta h \gamma A$$

where γ is the specific weight of the water. Thus, solving for the area one has

$$A = \frac{W}{\gamma \Delta h}$$

For a four foot person weighing 60 lbf, the area for a 4 foot deflection (manometer near eye level of person) would be

$$A = \frac{60}{62.4 \times 4} = 0.24 \text{ ft}^2$$

while for a 250 lbf person 6 feet tall would be

$$A = \frac{250}{62.4 \times 6} = 0.66 \text{ ft}^2$$

It will not be possible to maintain the manometer at the eye level for each person so take a piston area of 0.5 ft^2. This would give a deflection of 1.92 ft for the 4-foot, 60 lbf person and 8 ft for the 6-foot, 250 lbf person. This is a good compromise.

The size of the standpipe does not affect the pressure. The pipe should be big enough so the person can easily see the water level and be able to read the calibration on the scale. A 1/2 inch diameter tube would probably suffice. Thus the ratio of the standpipe area to the piston area would be

$$\frac{A_{pipe}}{A_{piston}} = \frac{0.785 \times 0.5^2}{0.5 \times 144} = 0.0027$$

This means that when the water level rises to 8 ft, the piston will only have moved by $0.0027 \times 8 = 0.0216$ ft or 0.26 inches.

The weight of the piston will cause an initial deflection of the manometer. If the piston weight is 5 lbf or less, the initial deflection of the manometer would be

$$\Delta h_o = \frac{W_{\text{piston}}}{\gamma A_{\text{piston}}} = 0.16 \text{ ft or } 1.92 \text{ inches}$$

This will not significantly affect the range of the manometer (between 2 and 8 feet).

The system would be calibrated by putting knows weights on the scale and marking the position on the standpipe. The scale would be linear.

3.34 **Information and assumptions**

provided in problem statement

Find

gage pressure at center of pipe A

Manometer equation

$$
\begin{aligned}
p_A &= 1.31 \times 847 - 4.59 \times 62.4 = 823.2 \text{ psf} = \underline{\underline{5.72 \text{ psig}}} \\
p_A &= 0.4 \times 1.33 \times 10^5 - 1.4 \times 9,810 = \underline{\underline{39.5 \text{ kPa gage}}}
\end{aligned}
$$

3.35 **Information and assumptions**

provided in problem statement

Find

specific weight of unknown fluid

Solution

Volume of unknown liquid $=\forall= (\pi/4)d^2\ell = 2\text{ cm}^3$

$$\begin{aligned}\forall &= (\pi/4)(0.5)^2\ell = 2 \\ \ell &= 10.186\text{ cm}\end{aligned}$$

Manometer equation from water surface in left leg to liquid surface in right leg:

$$\begin{gathered}0 + (10.186\text{ cm - }5\text{ cm})(10^{-2}\text{ m/cm})(9{,}810\text{ N/m}^3) \\ -(10.186\text{ cm})(10^{-2}\text{ m/cm})\gamma_{\text{liq.}} = 0\end{gathered}$$

$$\begin{aligned}508.7\text{ Pa } - 0.10186\gamma_{\text{liq.}} &= 0 \\ \gamma_{\text{liq.}} &= \underline{\underline{4,995}}\text{ N/m}^3\end{aligned}$$

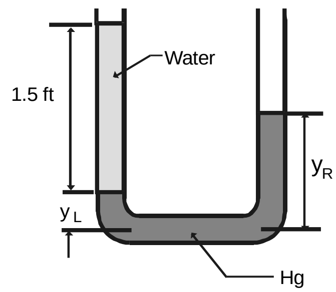
3.36 **Information and assumptions**

provided in problem statement

Find

locate water and mercury surfaces and maximum pressure in tube

Solution



$$
\begin{aligned}
y_L + y_R &= 1 - 2/3 = 1/3 \text{ ft} \\
0 + (1.5 \times 62.4) + (y_L \times 847) - (y_R \times 847) &= 0 \\
y_L - y_R &= -0.1105 \text{ ft} \\
2y_L &= 0.333 - 0.1105 \\
y_L &= \underline{\underline{0.111 \text{ ft}}} \\
y_R &= 0.333 - y_L = \underline{\underline{0.222 \text{ ft}}} \\
p_{\max} &= 0.222 \times 847 = \underline{\underline{188 \text{ psfg}}}
\end{aligned}
$$

3.37 **Information and assumptions**

provided in problem statement

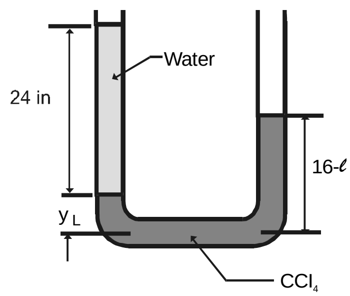
Find

locate water and carbon tetrachloride surfaces

Solution

First assume that the water will all stay in the left leg (it will not occupy any of the horizontal part of the tube). Then the resulting configuration of water and carbon tetrachloride will appear as shown below:

Write the manometer equation from the water surface to the carbon tetrachloride surface



$$\begin{aligned} 0+2\gamma_{\mathrm{H_2O}}+\ell\gamma_{CCL_4}-(1.33-\ell)\gamma_{CCL_4} &= 0 \\ 2\gamma_{\mathrm{H_2O}}-4/3\times1.59\gamma_{\mathrm{H_2O}}+2\ell\times1.59\gamma_{\mathrm{H_2O}} &= 0 \\ -0.12+3.18\ell &= 0 \\ 3.18\ell &= 0.12 \\ \ell &= 0.0377\text{ ft}=0.453\text{ in} \end{aligned}$$

Therefore, the water surface will be $\underline{\underline{24.453}}$ inches above the bottom of the tube and the carbon tetrachloride surface will be $\underline{\underline{15.547}}$ inches above the bottom.

3.38 **Information and assumptions**

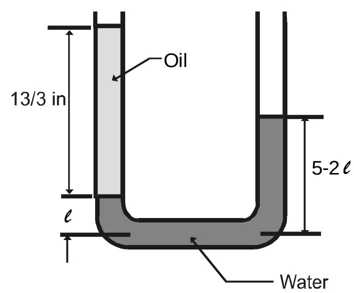
neglect volume of tube associated with bends
provided in problem statement

Find

locate water and oil surfaces

Solution

Assume the water and oil take a configuration in the tube shown below.



The pressure at the bottom of the column of oil is the same as the pressure at a depth of 5" $-\ 2\ell$ in the column of water. Equate these pressures and solve for ℓ.

$$
\begin{aligned}
((5-2\ell)/12)\gamma_w &= ((13/3)/12)\gamma_{\text{oil}} \\
(5-2\ell)\gamma_w &= (13/3)(0.87\gamma_w) \\
\ell &= 0.615 \text{ in.}
\end{aligned}
$$

Thus the interface between the oil and water is 0.615 in. above the centerline of the horizontal part of tube. The surface of the oil is 4.948 in. above the centerline of the horizontal part of the tube and the surface of the water is 4.385 in. above the centerline of the horizontal part of the tube.

3.39 **Information and assumptions**

neglect volume of tube associated with bends
provided in problem statement

Find

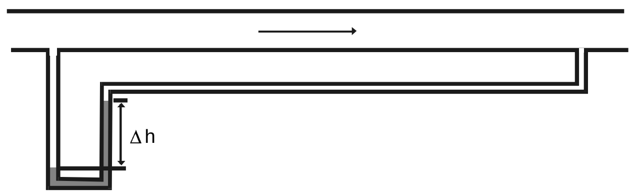
specific gravity of liquid in left leg

Solution

Equating ;pressures at bottom of liquid in left leg

$$(34 - 10) \times 10^{-2} \times 9,810 = 30 \times 10^{-2} \times 9,180 \times S$$
$$S = \underline{\underline{0.80}}$$

3.40 Consider the manometer shown in the figure.



Use a manometer fluid heavier than water. The specific weight of the manometer fluid is identified as γ_m.

Then $\Delta h_{\max} = \Delta p_{\max}/(\gamma_m - \gamma_{\mathrm{H_2O}})$.

If the manometer fluid is carbon-tetrachloride ($\gamma_m = 15,600$), $\Delta h_{\max} = 60 \times 10^3/(15,600 - 9,180) = 13.36$ m —(too large).

If the manometer fluid is mercury ($\gamma_m = 133,000$), $\Delta h_{\max} = 60 \times 10^3/(1333,000 - 9,810) = 0.487$ m—(O.K.). Assume the manometer can be read to ± 2 mm. Then % error $= \pm 2/487 = \pm 0.004 = \pm 0.4\%$. The probable accuracy for full deflection (0.5m) is about 99.6%. For smaller pressure differences the possible degree of error would vary inversely with the manometer deflection. For example, if the deflection were 10 cm = 0.1 m, then the possible degree of error would be $\pm 2\%$ and the expected degree of accuracy would be about 98%.

Note: Error analysis is much more sophisticated than presented above; however, this simple treatment should be enough to let the student have an appreciation for the subject.

3.41 **Information and assumptions**

provided in problem statement

Find

$p_A - p_B$

Manometer equation

$$\begin{aligned} p_B &= p_A + (4+2)62.4 \times 0.8 \\ &\quad + 3 \times 62.4 - (3+2)62.4 \times 0.8 \\ p_A - p_B &= -237 \text{ psf } = \underline{\underline{-1.65 \text{ psi}}} \end{aligned}$$

3.42 **Information and assumptions**

provided in problem statement

Find

$p_A - p_B$

Manometer equation

$$
\begin{aligned}
p_A + (3+1)9,180 \times 0.8 + 2 \times 9,790 - (2+1)9,810 \times 0.8 &= p_B \\
p_A - p_B &= \underline{\underline{-27.43 \text{ kPa}}}
\end{aligned}
$$

3.43 **Information and assumptions**

provided in problem statement

Find

distance z

Manometer equation

$$
\begin{aligned}
(1+3)51 + z \times 180 - (z+3)62.37 &= 2 \times 144 \\
z &= \underline{\underline{2.31 \text{ ft}}}
\end{aligned}
$$

3.44 **Information and assumptions**

provided in problem statement

Find

distance z

Manometer equation

$$\begin{aligned}(0+3)51 + z \times 847 - (z+3)62.4 &= 3 \times 144 \\ z &= \underline{\underline{0.594 \text{ ft}}}\end{aligned}$$

3.45 One possible apparatus might be a simple glass U-tube. Have each leg of the U-tube equipped with a scale so that liquid levels in the tube could be read. The procedure might be as described in steps below:

1. Pour water into the tube so that each leg is filled up to a given level (for example to 15 in. level).
2. Pour liquid with unknown specific weight into the right leg until the water in the left leg rises to a given level (for example to 27 in. level).
3. Measure the elevation of the liquid surface and interface between the two liquids in the right tube. Let the distance between the surface and interface be ℓ ft.
4. The hydrostatic relationship will be $\gamma_{\mathrm{H_2O}}(2') = \gamma_\ell \ell$ or $\gamma_\ell = 2Y_{\mathrm{H_2O}}/\ell$.
5. To accommodate the range of γ specified the tube would have to be about 3 or 4 ft. high.

   The errors that might result could be due to:

1. error in reading liquid level
2. error due to different surface tension
   (a) different surface tension because of different liquids in each leg
   (b) one leg may have slightly different diameter than the other one; therefore, creating different surface tension effect.

Sophisticated error analysis is not expected from the student. However, the student should sense that an error in reading a surface level in the manometer will produce an error in calculation of specific weight. For example, assume that in one test the true value of ℓ were 0.28 ft. but it was actually read as 0.29 ft. Then just by plugging in the formula one would find the true value of γ would be 7.14 $\gamma_{\mathrm{H_2O}}$ but the value obtained by using the erroneous reading would be found to be 6.90 $\gamma_{\mathrm{H_2O}}$. Thus the manometer reading produced a -3.4% error in calculated value of γ. In this particular example the focus of attention was on the measurement of ℓ. However, the setting of the water surface in the left leg of the manometer would also involve a possible reading error, etc.

Other things that could be considered in the design are:

1. Diameter of tubing
2. Means of support
3. Cost
4. How to empty and clean tube after test is made.

3.46 Pressure change

$$p_f = (0.02)(62.4)(50/(1/12))(V^2/64.4)$$
$$= 11.63\, V^2 \;\; \text{lbf/ft}^2$$

Greatest change:

$$p_f = 11.63 \times 15^2 = 2617 \text{ lbf/ft}^2$$
$$= 41.9 \text{ ft of head of water}$$

Smallest change:

$$p_f = 11.63 \times 3^2 = 104.6 \text{ lbf/ft}^2$$
$$= 1.68 \text{ ft of head of water}$$

Since the pressure drop is quite large, we should use a manometer with a fluid that has a large specific weight. Use a mercury-water manometer. Then for a p_f of 41.9 ft. of water the deflection can be calculated as given below:

$$\Delta \times 12.6 = 41.9 \text{ ft.}$$
$$\Delta = 3.325 \text{ ft.}$$

If one could read the deflection to within $\pm 1/8$ in. then the degree of possible error in percent would be

$$(\pm 1/8/(3.325 \times 12)) \times 100 = 0.31\%$$

If one were to use the same manometer to measure the smallest pressure change, then the possible degree of error would be

$$(\pm 1/8/(0.133 \times 12)) \times 100 = 7.81\% \text{ (pretty high)}$$

Other things that could be considered in the design:

1. Cost
2. Kind of tubing and connections between pipe and manometer
3. How to be sure there are no air bubbles in the tubing
4. How to support the manometer.

3.47 **Information and assumptions**

provided in problem statement

Find

pressure at center of pipe A

Manometer equation

$$
\begin{aligned}
p_A &= (0.9 + 0.6 \times 13.6 - 1.8 \times 0.8 + 1.5)9,810 = 89,467 \text{ Pa} \\
p_A &= \underline{\underline{89.47 \text{ kPa}}}
\end{aligned}
$$

3.48 **Information and assumptions**

provided in problem statement

Find

pressure at the center of pipe A

Manometer equation

$$
\begin{aligned}
p_A &= (90 + 60 \times 13.6 - 180 \times 0.8 + 150) \times (1/12) \times 62.4 \\
&= 4,742 \text{ psfg} \\
p_A &= \underline{\underline{32.9 \text{ psig}}}
\end{aligned}
$$

3.49 **Information and assumptions**

provided in problem statement

Find

difference in pressure and difference in piezometric pressure between points A and B

Manometer equation

$$
\begin{aligned}
p_A - 1 \times 0.85 \times 9,810 + 0.5 \times 0.85 \times 9,810 &= p_B \\
p_A - p_B &= 4,169 \text{ Pa } = \underline{\underline{4.169 \text{ kPa}}} \\
(p_A/\gamma + z_A) - (p_B/\gamma + z_B) &= (4,169/0.85 \times 9,810) - 1 \\
&= \underline{\underline{-0.50 \text{ m}}}
\end{aligned}
$$

3.50 **Information and assumptions**

provided in problem statement

Find

find manometer deflection when pressure in tank is doubled

Solution

The deflection on the manometer is given by

$$p - p_{atm} = \gamma h$$

For 150 kPa absolute pressure and an atmospheric pressure of 100 kPa,

$$\gamma h = 150 - 100 = 50 \text{ kPa}$$

For an absolute pressure of 300 kPa

$$\gamma h_{new} = 300 - 100 = 200 \text{ kPa}$$

Dividing the equations to eliminate the specific weight gives

$$\frac{h_{new}}{h} = \frac{200}{50} = 4.0$$

so

$$h_{new} = \underline{\underline{4.0h}}$$

3.51 **Information and assumptions**

from Table A.4, $\gamma_{\mathrm{H}g} = 847\ \mathrm{lbf/ft}^3$
provided in problem statement

Find

difference in pressure and piezometric pressure between points A and B

$$\gamma_{\mathrm{oil}} = (0.85)(62.4\ \mathrm{lbf/ft}^3) = 53.04\ \mathrm{lbf/ft}^3$$

Manometer equation

$$p_A + (18/12)\ \mathrm{ft}\ (\gamma_{\mathrm{oil}}) + (2/12)\ \mathrm{ft.}\ \gamma_{\mathrm{oil}} + (3/12)\ \mathrm{ft}\ \gamma_{\mathrm{oil}} \\ -(3/12)\ \mathrm{ft}\ \gamma_{\mathrm{Hg}} - (2/12)\ \mathrm{ft}\ \gamma_{\mathrm{oil}} = p_B$$

$$\begin{aligned}
p_A - p_B &= (-1.75\ \mathrm{ft.})(53.04\ \mathrm{lbf/ft}^3) + (0.25\ \mathrm{ft.})(847\ \mathrm{lbf/ft}^3) \\
&= \underline{\underline{118.9\ \mathrm{lbf/ft}^2}} \\
h &= (p/\gamma) + z \\
h_A - h_B &= (p_A - p_B)/\gamma_{\mathrm{oil}} + z_A - z_B \\
h_A - h_B &= (118.9\ \mathrm{lbf/ft})/(53.04\ \mathrm{lbf/ft}^3) + (1.5 - 0) \\
&= \underline{\underline{3.74\ \mathrm{ft.}}}
\end{aligned}$$

3.52 **Information and assumptions**

provided in problem statement

Find

difference in deflection between manometers

Solution

The pressure in the tank using manometer b is

$$p_t = p_{atm} - \gamma_w \Delta h_b$$

and using manometer a is

$$p_t = 0.9 p_{atm} - \gamma_w \Delta h_a$$

Equating both equations

$$p_{atm} - \gamma_w \Delta h_b = 0.9 p_{atm} - \gamma_w \Delta h_a$$

or

$$0.1 p_{atm} = \gamma_w (\Delta h_a - \Delta h_b)$$

Solving for the difference in deflection

$$\Delta h_a - \Delta h_b = \frac{0.1 \times 10^5}{9.81 \times 10^3} = \underline{\underline{1.02}} \text{ m}$$

3.53 **Information and assumptions**

provided in problem statement

Find

pressure difference and piezometric pressure difference between points A and B.

Manometer equation

$$p_B = p_A + 0.03\gamma_f - 0.03\gamma_m - 0.1\gamma_f$$

or

$$p_B - p_A = 0.03(\gamma_f - \gamma_m) - 0.1\gamma_f$$

Substituting in the values

$$\begin{aligned} p_B - p_A &= 0.03(9810 - 3 \times 9810) - 0.1 \times 9810 \\ &= -1570 \text{ Pa or -1.57 kPa} \end{aligned}$$

The change in piezometric pressure is

$$\begin{aligned} p_{zB} - p_{zA} &= p_B + \gamma_f z_B - (p_A + \gamma_f z_A) \\ &= p_B - p_A + \gamma_f(z_B - z_A) \end{aligned}$$

But $z_B - z_A$ is equal to 0.1 m so from equation above

$$\begin{aligned} p_{zB} - p_{zA} &= p_B - p_A + 0.1\gamma_f \\ &= 0.03(9810 - 3 \times 9810) \\ &= -588.6 \text{ Pa or } \underline{\underline{-0.5886}} \text{ kPa} \end{aligned}$$

3.54 Using the manometer equation starting at point A gives

$$p_A = p_B + \gamma_f \Delta l + \gamma_m \Delta h - \gamma_f [\Delta h + \Delta l + (z_A - z_B)]$$

The $\gamma_f \Delta l$ terms cancel out and the equation can be rewritten as

$$p_A + \gamma_f z_A - (p_B + \gamma_f z_B) = \Delta h(\gamma_m - \gamma_f)$$

or

$$p_{zA} - p_{zB} = \Delta h(\gamma_m - \gamma_f)$$

3.55 **Information and assumptions**

provided in problem statement

Find

specific gravity of oil and pressure at C

Solution

$$
\begin{aligned}
50,000 \text{ N/m}^2 + \gamma_{\text{oil}} \times 1 \text{ m} &= 58{,}530 \text{ N/m}^2 \\
\gamma_{\text{oil}} &= 8,530 \text{ N/m}^2 \\
S &= (8,530 \text{ N/m}^2)/(9,810 \text{ N/m}^2) \\
&= \underline{\underline{0.87}} \\
p_c &= 58,530 + \gamma_{\text{oil}} \times 0.5 + \gamma_{\text{water}} \times 1 \\
&= 58,530 + 8,530 \times 0.5 + 9,810 \\
&= 72,605 \text{ N/m}^2 = \underline{\underline{72.6 \text{ kPa}}}
\end{aligned}
$$

3.56 **Information and assumptions**

neglect the change of pressure due to the column of air in the tube.
provided in problem statement

Find

depth of liquid in tank

Solution

$$\begin{aligned} p_{\text{gage}} - (d-1)\gamma_{\text{liquid}} &= 0 \\ 20,000 - ((d-1) \times 0.85 \times 9,810) &= 0 \\ d &= (20,000/(0.85 \times 9,810)) + 1 \\ d &= \underline{\underline{3.40 \text{ m}}} \end{aligned}$$

3.57

$$dp/dz = \gamma$$

Because γ becomes smaller with an increase in elevation the ratio of (dp/dz)'s will have a value $\underline{\underline{\text{greater than 1}}}$.

3.58 Let the horizontal gate dimension be given as b and the vertical dimension, h.

$$T_A = F(y_{cp} - \bar{y})$$

where F = the hydrostatic force acting on the gate and $(y_{cp} - \bar{y})$ is the distance between the center of pressure and the centroid of the gate. Thus

$$\begin{aligned} T_A &= \gamma(H - (h/2))(bh)(I/\bar{y}A) \\ &= \gamma(H - (h/2))(bh)(bh^3/12)/(H - (h/2))(bh)) \\ T_A &= \gamma bh^3/12 \end{aligned}$$

Therefore, T_A does not change with H.

$$\begin{aligned} T_B &= F((h/2) + y_{cp} - \bar{y}) \\ &= \gamma(H - (h/2))(bh)((h/2) + y_{cp} - \bar{y}) \\ &= \gamma(H - (h/2))(bh)((h/2) + I(\bar{y}A)) \\ &= \gamma(H - (h/2))(bh)[(h/2) + (bh^3/12)/((H - (h/2))bh)] \\ &= \gamma(H - (h/2))bh^2/2 + \gamma bh^3/12 \end{aligned}$$

Thus, T_A is constant but T_B increases with H. Case (c) is the correct choice.

3.59 The correct answers obtained by looking at the solution to problem 3.58 are that a, b, and e are valid statements.

3.60 **Information and assumptions**

$T_{\text{sea level}} = 296$ K= 23°C
provided in problem statement

Find

boiling point of water at 1500 and 3000 m for standard atmospheric conditions

Solution

For standard atmosphere

$$p = p_0[(T_0 - \alpha(z - z_0))/T_0]^{g/\alpha R} = 101.3[296 - 5.87(z - z_0))/296]^{g/\alpha R}$$

where $g/\alpha R = 9.81/(5.87 \times 10^{-3} \times 287) = 5.823$

$$p_{1,500} = 101.3[(296 - 5.87(1.5))/296]^{5.823} = 84.9 \text{ kPa}$$
$$p_{3,000} = 101.3[(296 - 5.87(3.0))/296]^{5.823} = 70.9 \text{ kPa}$$

From table A-5,

$$T_{\text{boiling, 1,500 m}} \approx \underline{\underline{95°C}} \text{ (interpolated);}$$
$$T_{\text{boiling, 3,000 m}} \approx \underline{\underline{90°C}} \text{ (interpolated)}$$

3.61 **Information and assumptions**

Assume atmospheric pressure is 101 kPa
provided in problem statement

Find

plot pressure variation from 10 m depth to 4000 m in standard atmosphere

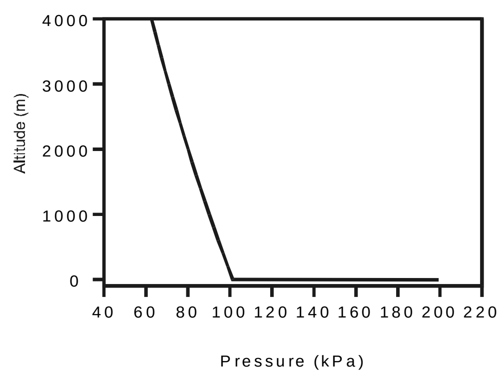
Solution

Pressure in atmosphere

$$p_A = 101.3 \left(1 - \frac{5.87 \times 10^{-3} \times z}{296}\right)^{5.823}$$

Pressure in water

$$p_w = 101.3 + 9.810 \times z$$



3.62 **Information and assumptions**

provided in problem statement

Find

pressure at 6096 m in standard atmosphere

Solution

$$\begin{aligned}
p &= p_0[(T_0 - \alpha(z - z_0))/T_0]^{g/\alpha R} \\
&= 14.7[(520 - 3.221 \times 10^{-3}(20,000 - 0))/520]^{32.17/(3.221\times 10^{-3}\times 1,715)} \\
&= 6.80 \text{ psia} \\
p_a &= 101[(288 - 5.87 \times 10^{-3}(6,096 - 0))/288]^{9.81/(5.87\times 10^{-3}\times 287)} \\
p_a &= \underline{\underline{46.6 \text{ kPa}}} \text{ abs.}
\end{aligned}$$

3.63 **Information and assumptions**

assume volume drawn in per breath is the same
air is ideal gas
provided in problem statement

Find

breathing rate at 18,000 ft.

Solution

Let b$\forall\rho$ = constant where b = breath rate, $\forall$ = volume per breath, and ρ = mass density of air. Assume 1 is sea level and point 2 is 18,000 ft. elevation. Then

$$
\begin{aligned}
b_1\forall_1\rho_1 &= b_2\forall_2\rho_2 \\
b_2 &= b_1(\forall_1/\forall_2)(\rho_1/\rho_2) \\
\text{then } b_2 &= b_1(\rho_1/\rho_2) \text{ but } \rho = (p/RT) \\
\text{Thus, } b_2 &= b_1(p_1/p_2)(T_2/T_1) \\
p_2 &= p_1(T_2/T_1)^{g/\alpha R}; p_1/p_2 = (T_2/T_1)^{-g/\alpha R} \\
\text{Then } b_2 &= b_1(T_2/T_1)^{1-g/\alpha R}
\end{aligned}
$$

Since the volume drawn in per breath is the same

$$b_2 = b_1(\rho_1/\rho_2)$$

Using ideal gas law

$$
\begin{gathered}
b_2 = b_1(p_1/p_2)(T_2/T_1) \\
p_1/p_2 = (T_2/T_1)^{-g/\alpha R} \\
b_2 = b_1(T_2/T_1)^{1-g/\alpha R}
\end{gathered}
$$

where $b_1 = 16$ breaths per minute and $T_1 = 59°F = 519°R$

$$
\begin{aligned}
T_2 &= T_1 - \alpha(z_2 - z_1) = 519 - 3.221 \times 10^{-3}(18,000 - 0) = 461.0 \text{ °R} \\
b_2 &= 16(461.0/519)^{1-32.2/(3.221\times10^{-3}\times1,715)} = \underline{\underline{28.4 \text{ breaths per minute}}}
\end{aligned}
$$

3.64 **Information and assumptions**

provided in problem statement

Find

elevation and temperature when pressure is 75 kPa.

Solution

$$\begin{aligned}
p &= p_0[(T_0 - \alpha(z - z_0))/T_0]^{g/\alpha R} \\
75 &= 95[(283 - 5.87(z - 1))/283]^{9.81/(5.87\times 10^{-3}\times 287)} \\
z &= \underline{\underline{2.91}} \text{ km} \\
T &= T_0 - \alpha(z - z_0) = 10 - 5.87(2.91 - 1) = \underline{\underline{-1.21}}^{\circ}\text{C}
\end{aligned}$$

3.65 **Information and assumptions**

provided in problem statement

Find

elevation when pressure is 10 psia

Solution

$$
\begin{aligned}
p &= p_0[(T_0 - \alpha(z - z_0))/T_0]^{g/\alpha R} \\
10 &= 13.6[((70 + 460) - 3.221 \times 10^{-3}(z - 2,000))/(70 + 460)]^{32.2/(3.221\times 10^{-3}\times 1,715)} \\
z &= \underline{\underline{10,452 \text{ ft}}}
\end{aligned}
$$

3.66 **Information and assumptions**

provided in problem statement

Find

pressure, temperature and density in Denver, CO (mile-high city)

Solution

$$\begin{aligned}
T &= T_0 - \alpha(z - z_0) = 533 - 3.221 \times 10^{-3}(5,280 - 0) = \underline{\underline{516^\circ\text{R}}} \\
&= 296 - 5.87 \times 10^{-3}(1,609 - 0) = \underline{\underline{287\text{ K}}} \\
p &= p_0(T/T_0)^{g/\alpha R} = 14.7(516/533)^{5.823} = \underline{\underline{12.2\text{ psia}}} \\
p_a &= 101.3(287/296)^{9.81/(5.87\times10^{-3}\times287)} = \underline{\underline{86.0\text{ kPa}}} \\
\rho &= p/RT = (12.2 \times 144)/1,715 \times 516) = \underline{\underline{0.00199\text{ slugs/ft}^3}} \\
\rho &= 86,000/(287 \times 287) = \underline{\underline{1.04\text{ kg/m}^3}}
\end{aligned}$$

3.67 **Information and assumptions**

provided in problem statement

Find

outward force on airplane window

Solution

The altitude is less than 13.72 km so the airplane is flying the troposphere where the pressure is given by

$$\frac{p}{p_o} = \left(\frac{T_o - \alpha z}{T_o}\right)^{g/\alpha R}$$

At 10 km altitude, the pressure ratio is

$$\begin{aligned}\frac{p}{p_o} &= \left(\frac{296 - 5.87 \times 10}{296}\right)^{9.81/(5.87 \times 0.287)} \\ &= 0.8017^{5.823} = 0.276\end{aligned}$$

Therefore the pressure outside the airplane is $0.276 \times 101.3 = 28.0$ kPa. The pressure on the window is

$$F = A\Delta p = 0.25 \times 0.3 \times (100 - 28) = \underline{\underline{5.4}} \text{ kN}$$

3.68 **Information and assumptions**

provided in problem statement

Find

pressure at 30 km altitude

Solution

The equation for pressure variation where the temperature increases with altitude is

$$\frac{dp}{dz} = -\gamma = \frac{pg}{R[T_o + \alpha(z - z_o)]}$$

where the subscript o refers to the conditions at 16.8 km and α is the lapse rate above 16.8 km. Integrating this equation gives

$$\frac{p}{p_o} = \left[\frac{T_o + \alpha(z - z_o)}{T_o}\right]^{-g/\alpha R}$$

Substituting in the values gives

$$\begin{aligned}\frac{p}{p_o} &= \left[\frac{215.5 + 1.38 \times (30 - 16.8)}{215.5}\right]^{-9.81/(1.38 \times 0.287)} \\ &= 1.084^{-24.8} = 0.134\end{aligned}$$

Thus the pressure is $0.134 \times 9.85 = \underline{\underline{1.32}}$ kPa.

The density is

$$\rho = \frac{p}{RT} = \frac{1.32}{0.287 \times 234} = \underline{\underline{0.0197}}\ \mathrm{kg/m^3}$$

3.69 The following are sample values obtained using computer calculations.

altitude (km)	temperature (^{o}C)	pressure (kPa)	density (kg/m^3)
10	-35.7	27.9	0.409
15	-57.5	12.8	0.208
25	-46.1	2.75	0.042

3.70 **Information and assumptions**

provided in problem statement

Find

reading on mercury barometer on top of Mount Everest (29,038 ft)

Solution

An altitude of 29,028 ft corresponds to 8.85 km. The temperature at this altitude would be

$$T = 23 - 5.87 \times 8.85 = -28.9^{\circ}\text{C} = 244.1 \text{ K}$$

The pressure ratio is

$$\frac{p}{p_o} = \left(\frac{244.1}{296}\right)^{5.823} = 0.325$$

Thus the pressure is $0.325 \times 101 = 32.8$ kPa. The height of a mercury barometer would be

$$h = \frac{P}{\gamma_m} = \frac{32.8}{13.55 \times 9.81} = 0.247 \text{ m or } \underline{\underline{247 \text{ mm}}}$$

3.71 **Information and assumptions**

provided in problem statement

Find

change in barometer reading from ground to top of Sears tower in Chicago (443 m)

Solution

The temperature at the top of the Sear's tower would be

$$T = 293 - 5.87 \times 0.443 = 290.4 \text{ K}$$

The pressure ratio is given by

$$\frac{p}{p_o} = \left(\frac{290.4}{293}\right)^{5.823} = 0.949$$

The pressure at the top of the tower is

$$p = 0.949 \times 740 = 702$$

The change in barometer reading is

$$\Delta h = 740 - 702 = \underline{\underline{38}} \text{ mm}$$

3.72 **Information and assumptions**

provided in problem statement

Find

moment required to keep gate closed

Solution

Force of slurry on gate $= \bar{p}_s A$ and it acts to the right. Force of water on gate $= \bar{p}_w A$ and it acts to the left

$$\begin{aligned} F_{\text{net}} &= (\bar{p}_s - \bar{p}_w)A \\ &= (8\gamma_s - 8\gamma_w)A \\ &= (8\text{ ft})(16\text{ ft}^2)(150\text{ lbf/ft}^3 - 60\text{ lbf/ft}^3) = \underline{\underline{11{,}520\text{ lbf}}} \end{aligned}$$

Because the pressure is uniform along any horizontal line the moment on the gate is zero; therefore, the moment required to keep the gate closed is zero.

3.73 **Information and assumptions**

provided in problem statement

Find

force of gate on block

Solution

$$
\begin{aligned}
F &= \bar{p}A = 10 \times 9,810 \times 4 \times 4 = 1,569,600 \text{ N} \\
F &= 32.8 \times 62.4 \times 13.1 \times 13.1 = 351,238 \text{ lbf} \\
y_{cp} - \bar{y} &= I/\bar{y}A = (4 \times 4^3/12)/(10 \times 4 \times 4) = 0.133 \text{ m} \\
&= (13.1 \times 13.1^3)/12)/32.8 \times 13.1 \times 13.1) = 0.436 \text{ ft} \\
F_{\text{block}} &= 1,569,600 \times 0.133/2 = \underline{\underline{\underline{104{,}378 \text{ N}}}} \\
&= 351,238 \times 0.435/6.55 = \underline{\underline{\underline{23{,}380 \text{ lbf}}}}
\end{aligned}
$$

3.74 **Information and assumptions**

provided in problem statement

Find

force of the gate on the block

Solution

$$
\begin{aligned}
F &= \bar{p}A \\
&= \gamma_{H_20}\bar{y}A \\
&= (62.4 \text{ lbf/ft}^3)(10 \text{ ft})(16 \text{ ft}^2) \\
&= 9,984 \text{ lbf} \\
y_{cp} - \bar{y} &= I/(\bar{y}A) \\
&= (4 \times 4^3/12)/(10 \times 4^2) \\
&= 0.1333 \text{ ft.}
\end{aligned}
$$

Sum moments about pivot:

$$
\begin{aligned}
\sum M_{\text{pivot}} &= 0 \\
9,984 \times 0.1333 - 2F_{\text{block}} &= 0 \\
F_{\text{block}} &= 666 \text{ lbf}
\end{aligned}
$$

Force of gate on block is $\underline{\underline{\text{666 lbf acting to right}}}$

3.75 **Information and assumptions**

provided in problem statement

Find

force per foot on form and force exerted on bottom tie

Solution

$$
\begin{aligned}
F &= pa = 4.5 \times 150 \times (9 \times 1) = \underline{\underline{6,075}} \text{ lbf} \\
y_{cp} &= y + I/yA = 4.5 + (1 \times 9^3)/(12 \times 4.5 \times 9) = 6.00 \text{ ft} \\
F_{\text{tie}} &= 2 \times F \times y_{cp}/h = 2 \times 6,075 \times 6.00/9 = \underline{\underline{8,100 \text{ lbf}}}
\end{aligned}
$$

3.76 **Information and assumptions**

provided in problem statement

Find

force to keep gate closed

Solution

Force acting on gate:

$$\begin{aligned} F_G &= \bar{p}A \\ p &= \gamma_{\mathrm{H_2O}} \times 2 \\ A &= 40 \text{ ft}^2 \end{aligned}$$

Then $F_G = 62.4 \times 2 \times 40 = 4,992$ lbf. F_G acts at 2/3 depth or 8/3 ft. below water surface. Take moments about hinge to solve for F.

$$\begin{aligned} \sum M &= 0 \\ (F_G \times 8/3) - 4F &= 0 \\ &\underline{\underline{F = 3{,}328 \text{ lbf to the left}}} \end{aligned}$$

3.77 **Information and assumptions**

provided in problem statement

Find

force acting on hinge

Solution

$$
\begin{aligned}
F &= pA = (3+4.5)9,810 \times 9 \times 9 = 5,960,000 \text{ N} \\
y_{cp} &= y + I/\bar{y}A = 7.5 + 9 \times 9^3/(12 \times 7.5 \times 9 \times 9) = 8.40 \text{ m} \\
F_{\text{hinge}} &= F(d - y_{cp})/h = 5,960,000(12 - 8.40)/9 = 2,384,000 \text{ N} = \underline{\underline{2{,}384 \text{ kN}}}
\end{aligned}
$$

3.78 **Information and assumptions**

provided in problem statement

Find

reaction at point A

Solution

$$
\begin{aligned}
F &= pA \\
&= (3\text{ m } + 3\text{ m} \times \cos 30^{\circ})(9,810\text{ N/m}^3) \times 24\text{ m}^2 \\
F &= 1,318,000\text{ N} \\
\bar{y} &= 3 + 3/\cos 30^{\mathrm{o}} = 6.464\text{ m} \\
y_{cp} - y &= I/\bar{y}A \\
&= (4 \times 6^3/12)\text{m}^4/(6.464\text{ m} \times 24\text{ m}^2) \\
&= 0.4641\text{ m}
\end{aligned}
$$

Take moments about the stop

$$
\begin{aligned}
\sum M_{\text{stop}} &= 0 \\
6R_A - (3 - 0.464) \times 1,318,000 &= 0
\end{aligned}
$$

Reaction at point A=$\underline{\underline{557\text{ kN}}}$ to the left inclined downward 30 degrees.

3.79 **Information and assumptions**

provided in problem statement

Find

reaction at point A

Solution

Hydrostatic force on gate is: $F_G = \bar{p}A$ where

$$\begin{aligned}
\bar{p} &= (3 \text{ ft } + 3 \text{ ft } \cos 30^\circ)\gamma_w \\
&= 5.60 \text{ ft } \times 62.4 \text{ lbf/ft}^3 \\
&= 349 \text{ lbf/ft}^2 \\
A &= 6 \times 5 = 30 \text{ ft}^2 \\
\text{Then } F_G &= 349 \times 30 = 10,470 \text{ lbf} \\
y_{cp} - \bar{y} &= I/\bar{y}A \\
\text{where } I &= bh^3/12 = (5 \times 6^3/12) \text{ ft}^4 \\
\bar{y} &= 3 \text{ ft } + 3 \text{ ft}/\cos 30^\circ = 6.46 \text{ ft} \\
A &= 30 \text{ ft}^2 \\
\text{then } y_{cp} - \bar{y} &= (5 \times 6^3/12)/[(6.46)(30)] \\
y_{cp} - \bar{y} &= 0.464 \text{ ft}
\end{aligned}$$

Thus, F_G acts 3.464 ft from hinge or 2.536 ft from stop. Get reaction at A by summing moments about stop.

$$\begin{gathered}
\sum M_{\text{stop}} = 0 \\
F_G \times 2.536 - 6R_A = 0 \\
10,470 \times 2.535 - 6R_A = 0 \\
\underline{\underline{R_A = 4,425 \text{ lbf}}}
\end{gathered}$$

3.80 **Information and assumptions**

provided in problem statement

Find

reaction at point A

Solution

$$
\begin{aligned}
F &= (9+2)62.4 \times 8 \times 6 = 32,947 \text{ lbf} \\
y_{cp} - \bar{y} &= (6 \times 8^3)/(12 \times 22 \times 8 \times 6) = 0.242 \text{ ft} \\
F_A &= (32,947 \times 4.242)/(8 \cos 30^\circ) = \underline{\underline{20,172 \text{ lbf}}}
\end{aligned}
$$

3.81 **Information and assumptions**

provided in problem statement

Find

resisting moment at bottom edge

Solution

$$\begin{aligned} F &= \bar{p}A = (0.4+0.4)9,810 \times 0.8 \times 0.8 \times 1.2 = 6,027 \text{ N} \\ y_{cp} - \bar{y} &= I/\bar{y}A = 1.2 \times 0.8^3/(12 \times 0.8 \times 0.8 \times 1.2) = 0.067 \text{ m} \\ M &= 6,027 \times (0.4 - 0.067) = \underline{\underline{2,007 \text{ N} \cdot \text{m}}} \end{aligned}$$

3.82 **Information and assumptions**

provided in problem statement

Find

torque required to open gate

Solution

$$
\begin{aligned}
F &= \bar{p}A = (5+2.5)9,810 \times 3 \times 5/\sin 60^\circ = 1,274.4 \text{ kN} \\
y_{cp} - \bar{y} &= 3 \times (5/\sin 60^\circ)^3/(12 \times (7.5/\sin 60^\circ)(3 \times 5/\sin 60^\circ)) = 0.321 \text{ m} \\
T &= 0.321 \text{ m } \times 1,274.4 \text{ kN } = \underline{\underline{409 \text{ kN} \cdot \text{m}}}
\end{aligned}
$$

3.83 **Information and assumptions**

provided in problem statement

Find

torque required to open gate

Solution

$$\begin{aligned}
F &= \bar{p}A = (12+6)62.4 \times 612/\sin 60^\circ = 93,381 \text{ lbf} \\
y_{cp} - \bar{y} &= I/\bar{y}A = 6 \times (12/\sin 60^\circ)^3/(12 \times (18/\sin 60^\circ)(6 \times 12/\sin 60^\circ)) \\
&= 0.770 \text{ ft} \\
T &= 0.770 \times 93,381 = \underline{\underline{71{,}903 \text{ ft-lbf}}}
\end{aligned}$$

3.84 **Information and assumptions**

provided in problem statement

Find

force P required to begin to open gate

Solution

The length of gate is $\sqrt{4^2+3^2}=5$ m

$$
\begin{aligned}
\bar{y} &= 2.5 \text{ m} + 5/4 \text{ m} \\
&= 3.75 \text{ m} \\
F &= \bar{p}A \\
&= (4/5)(3.75)(9,810)(2\times 5) \\
&= 294,300 \text{ N} \\
&= 294.3 \text{ kN} \\
y_{cp}-\bar{y} &= I/(\bar{y}A) \\
&= 2\times(5^3/12)/(3.75\times 2\times 5) \\
&= 0.555 \text{ m}
\end{aligned}
$$

Sum moments about hinge:

$$
\begin{gathered}
\sum M_{\text{hinge}} = 0 \\
F\times(2.5+.555)+3P=0 \\
294.3(3.055)+3P=0 \\
\underline{\underline{P=300 \text{ kN}}}
\end{gathered}
$$

3.85 Either a vertical plane gate or a tainter gate could be used. In any case, the horizontal component of hydrostatic force acting on the gate would be at least this much:

$$\begin{aligned} F_{\text{horiz.}} &= \bar{p}A \\ = 10 \times 62.4 \times 20 \times 30 &= \underline{\underline{374,400 \text{ lbf}}} \end{aligned}$$

Many design details such as location, lift mechanism, etc., depends on instructor's requirements.

3.86 **Information and assumptions**

provided in problem statement

Find

h in terms of ℓ to open gate

Solution

$$\begin{aligned}
y_{cp} - \bar{y} &= 0.60\ell - 0.5\ell = 0.10\ell \\
0.10\ell &= I/\bar{y}A = \ell \times \ell^3/(12 \times (h + \ell/2)\ell^2) \\
h &= \underline{\underline{0.333\ell}}
\end{aligned}$$

3.87 **Information and assumptions**

provided in problem statement

Find

length of tank to open gate

Solution

Hydrostatic force on gate: $F_G = \bar{p}A$ where

$$\begin{aligned}
\bar{p} &= 12 \text{ m} \times \gamma_{\text{water}} \\
&= 12 \times 9,810 \\
&= 117,720 \text{ Pa} \\
A &= 12 \times 12 = 144 \text{ m}^2 \\
\text{Then } F_G &= (117,720 \text{ N/m}^2) \times (144 \text{ m}^2) = 16.952 \text{ MN} \\
y_{cp} - \bar{y} &= I/\bar{y}A \\
&= (12 \times 12^3/12)/(12 \times 12^2) = 1 \text{ m}
\end{aligned}$$

Therefore F_G acts 6 m+1 m below hinge

$$F_{\text{tank}} = W = 6 \times 12 \times L \times \gamma_{\text{water}}$$

Force of water in tank

$$= 7.063 \times 10^5 \times L$$

Moment art of $F_{\text{tank}} = L/2$
Sum moments about hinge:

$$\begin{aligned}
\sum M_{\text{hinge}} &= 0 \\
(7.063 \times 10^5 L) \cdot (L/2) &= (16.952 \times 10^6) \times (7) = 0 \\
L^2 &= 336 \text{ m}^2 \\
L &= \underline{\underline{18.3}} \text{ m}
\end{aligned}$$

3.88 **Information and assumptions**

provided in problem statement

Find

torque required to hold valve in position

Solution

$$\begin{aligned}
F &= \bar{p}A \\
&= (30 \text{ ft} \times 62.4 \text{ lb/ft}^3)(\pi \times D^2/4) \text{ ft}^2) \\
&= (30 \times 62.4 \times \pi \times 10^2/4) \text{ lb } = 147,027 \text{ lb} \\
y_{cp} - \bar{y} &= I/\bar{y}A = (\pi r^4/4)/(\bar{y}\pi r^2) = (5^2/4)/(30/.866) \\
&= 0.1804 \text{ ft} \\
\text{Torque} &= 0.1804 \times 147,027 = \underline{\underline{26,520}} \text{ ft-lbf}
\end{aligned}$$

3.89 **Information and assumptions**

provided in problem statement

Find

will gate fall or stay in position

Solution

$$\begin{aligned}
F &= \bar{p}A = (1+1.5)9,810 \times 1 \times 3 \times \sqrt{2} = 104,050 \\
y_{cp} - \bar{y} &= I/\bar{y}A = 1 \times (\sqrt[3]{2})^3/(12 \times (2.5 \times \sqrt{2})(1 \times 3\sqrt{2})) = 0.424 \text{ m} \\
\text{Overturning moment } M_1 &= 90,000 \times 1.5 = 135,000 \text{ N} \cdot \text{m} \\
\text{Restoring moment } M_2 &= 104,050 \times (\sqrt[3]{2}/2 - 0.424) = 176,606 \text{ N} \cdot \text{m} > \text{M}_1
\end{aligned}$$

So the gate will stay.

3.90 **Information and assumptions**

provided in problem statement

Find

will gate fall or stay in position

Solution

$$
\begin{aligned}
F &= (4+3.535)62.4\times(3\times 7.07\sqrt{2}) = 14,103 \text{ lbf} \\
y_{cp}-\bar{y} &= 3\times(7.07\sqrt{2})^3/(12\times 7.535\sqrt{2}\times 3\times 7.07\sqrt{2}) \\
&= 0.782 \text{ ft} \\
\text{Overturning moment } M_1 &= 18,000\times 7.07/2 = 63,630 \text{ N}\cdot\text{m} \\
\text{Restoring moment } M_2 &= 14,103(7.07\sqrt{2}/2-0.782) \\
&= 59,476 \text{ N}\cdot\text{m} < M_1
\end{aligned}
$$

So the gate will fall.

3.91 **Information and assumptions**

provided in problem statement

Find

hydrostatic force on gate

Solution

$$
\begin{aligned}
F &= \bar{p}A = (h + 2h/3)\gamma(Wh/\sin 60^\circ)/2 = \underline{\underline{5\gamma W h^2/3\sqrt{3}}} \\
y_{cp} - \bar{y} &= I/\bar{y}A = W(h/\sin 60^\circ)^3/(36 \times (5h/(3\sin 60^\circ)) \times (Wh/2\sin 60^\circ)) \\
&= h/(15\sqrt{3}) \\
\Sigma M &= 0 \\
R_T h/\sin 60^\circ &= F[(h/(3\sin 60^\circ)) - (h/15\sqrt{3})] \\
R_T/F &= \underline{\underline{3/10}}
\end{aligned}
$$

3.92 **Information and assumptions**

provided in problem statement

Find

hydrostatic force on gate and horizontal force required to keep it closed

Solution

$$\begin{aligned}
F &= \bar{p}A = (1+6)9,810 \times 0.5 \times 4 \times 9 = \underline{\underline{1.236\ }}\text{MN} \\
y_{cp} - \bar{y} &= I/\bar{y}A = (4 \times 9^3)/(36 \times 7 \times 0.5 \times 4 \times 9) = 0.643 \text{ m} \\
p &= 1,236,060 \times (3 - 0.643)/9 = \underline{\underline{323.7 \text{ kN}}}
\end{aligned}$$

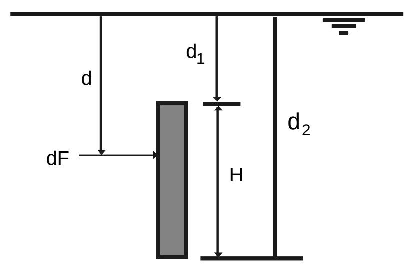
3.93 **Information and assumptions**

provided in problem statement

Find

derive formula for hydrostatic force on one side of plate, center of pressure below or above that for similarly located plate.

Solution



$$
\begin{aligned}
dF &= pdA = \gamma_0(1\_kd/d_0)dd(d)W \\
F &= \gamma_0 W \int_{d_1}^{d_2} d(1+kd/d_0)d(d) \\
F &= \gamma_0 W[1/2(H^2+2d_1H)+(k/3d_0)(H^3+3d_1d_2H)] \\
\text{or } F &= \underline{\underline{\gamma_0 W[1/2(d_2^2-d_1^2)+(k/3d_0)(d_2^3-d_1^3]}}
\end{aligned}
$$

When $d_1 = 0$

$$F = \gamma_0 W(H^2/2 + kH^3/3d_0)$$

Since the specific weight increases with the increase in depth, the location of the center of pressure will be located below that for constant density liquid.

3.94 **Information and assumptions**

provided in problem statement

Find

magnitude of reaction at A and comparison to that for a plane gate.

Solution

a)

$$F_{Hydr} = \bar{p}A = (0.25\ell + 0.5\ell \times 0.707) \times \xi W\ell = 0.6036\gamma W\ell^2$$
$$y_{cp} - \bar{y} = I/\bar{y}A = (W\ell^3/12)/(((0.25\ell/0.707) + 0.5\ell) \times W\ell)$$
$$y_{cp} - \bar{y} = 0.0976\ell$$
$$\sum M_{\text{hinge}} = 0$$
$$\text{Then } -0.70R_A\ell + (0.5\ell + 0.0976\ell) \times 0.6036\gamma W\ell^2 = 0$$
$$\underline{\underline{R_A = 0.510\gamma W\ell^2}}$$

b) The reaction here will be less because if one thinks of the applied hydrostatic force in terms of vertical and horizontal components, the horizontal component will be the same in both cases, but the vertical component will be less because there is less volume of liquid above the curved gate.

3.95 **Information and assumptions**

provided in problem statement

Find

force required to hold gate in place

Solution

Equivalent depth of liquid for 5 psi = $(5 \times 144)/(0.8 \times 62.4) = 14.42$ ft.

$$\begin{aligned} F &= \bar{p}A = (14.42 + 2 + 5)(62.4 \times 0.8)(6 \times 10) = 65.157 \text{ psf} \\ y_{cp} - \bar{y} &= I/\bar{y}A = (6 \times 10^3)/(12 \times 21.42 \times 6 \times 10) = 0.389 \text{ ft} \\ P &= 64,157 \times (5 + 0.389)/10 = \underline{\underline{34,574 \text{ lbf}}} \end{aligned}$$

3.96 **Information and assumptions**

provided in problem statement

Find

force required to hold gate in place

Solution

Equivalent depth of liquid for 40 kPa = 40,000/(0.8×9,810) = 5.10 m.

$$
\begin{aligned}
F &= (5.10 + 1 + 1.5)(0.8 \times 9,810)(3 \times 2) = 357,870 \text{ N} \\
y_{cp} - \bar{y} &= I/\bar{y}A = (2 \times 3^3)/(12 \times 7.60 \times 3 \times 2) = 0.099 \text{ m} \\
P &= 357,870(1.5 + 0.099)/3 = \underline{\underline{190.7 \text{ kN}}}
\end{aligned}
$$

3.97 **Information and assumptions**

provided in problem statement

Find

moment at base of form per meter of length

Solution

$$
\begin{aligned}
F &= \bar{p}A = (1.5/2)24,000 \times (1.5/\sin 60^\circ) = 31,177 \text{ N} \\
y_{cp} - \bar{y} &= I/\bar{y}A \\
&= 1 \times (1.5/\sin 60^\circ)^3/(12 \times (1.5/2\sin 60^\circ)) \times (1.5/\sin 60^\circ)) \\
&= 0.2887 \text{ m} \\
M &= 31,177 \times (1.5/2\sin 60^\circ - 0.2887) = 18,000 \text{ N}\cdot\text{m/m} \\
&= \underline{\underline{18 \text{ kN}\cdot\text{m/m}}}
\end{aligned}
$$

3.98 **Information and assumptions**

provided in problem statement

Find

depth at which gate will automatically open

Solution

A simple check shows that d will have to be less than 4 m. Thus

$$F_{\text{Hydrostatic}} = \bar{p}A = d/2 \times 9,810 \times 2d = 9,810d^2 \text{ N}$$

The hydrostatic force will act 2/3 d below water surface; therefore the moment will be $(4 - (1/3)d)$ below the hinge.

$$\begin{aligned} \sum M_{\text{hinge}} &= 0 \\ 5 \times 60,000 - (4 - (1/3)d)(9,810d^2) &= 0 \end{aligned}$$

Solving for d yields $\underline{\underline{d = 3.23 \text{ m}}}$.

3.99 **Information and assumptions**

provided in problem statement

Find

weight needed for gate to be on verge of opening

Solution

The hydrostatic force acting on the gate will be:

$$
\begin{aligned}
F &= \bar{p}A = (1.5 \times 9,810) \times (2 \times 3) = 88,290 \text{ N} \\
y_{cp} - \bar{y} &= I/\bar{y}A \\
&= (2 \times 3^3/12)/(1.5 \times 2 \times 3) = 0.5 \text{ m} \\
\sum M_{\text{Hinge}} &= 0 \\
W \times 5 - 88,290 \times (1 + 1.5 + 0.5) &= 0 \\
W &= \underline{\underline{52,974 \text{ N}}}
\end{aligned}
$$

3.100 **Information and assumptions**

provided in problem statement

Find

gate is stable or unstable

Solution

$$y_{cp} = (2/3) \times (8/\cos 45^\circ) = 7.54 \text{ m}$$

Point B is (8/cos 45°) m-3.5 m=7.81 m along the gate from the water surface; therefore, the gate is unstable.

3.101 **Information and assumptions**

provided in problem statement

Find

depth h at which gate will automatically open.

Solution

$$
\begin{aligned}
F_{AB,\text{hydrostatic}} &= \bar{p}_{AB}A_{AB} = (h/2)\gamma h = \gamma h^2/2 \\
F_{BC,\text{hydrostatic}} &= \bar{p}_{BC}A_{BC} = \gamma h \times 4 \text{ ft} \\
\sum M_B &= 0 \\
-(\gamma h^2/2)(h/3) + \gamma h \times 4 \text{ ft } \times 2 \text{ ft} &= 0 \\
h &= \underline{\underline{6.93 \text{ ft}}}
\end{aligned}
$$

3.102 **Information and assumptions**

provided in problem statement

Find

minimum volume of concrete to keep gate in closed position

Solution

$$\begin{aligned}
F &= \bar{p}A = 1 \times 9,810 \times 2 \times 1 = 19,620 \text{ N} \\
y_{cp} - \bar{y} &= I/\bar{y}A = (1 \times 2^3)/(12 \times 1 \times 2 \times 1) = 0.33 \text{ m} \\
W &= 19,620 \times (1 - 0.33)/2.5 = 5,258 \text{ N} \\
\forall &= 5,258/(23,600 - 9,810) = \underline{\underline{0.381 \text{ m}^3}}
\end{aligned}$$

3.103 **Information and assumptions**

provided in problem statement

Find

minimum volume of concrete to keep gate in closed position.

Solution

$$\begin{aligned} F &= 2.0 \times 62.4 \times 2 \times 4 = 998.4 \text{ lbf} \\ y_{cp} - \bar{y} &= (2 \times 4^3)/(12 \times 2.0 \times 2 \times 4) = 0.667 \text{ ft} \\ W &= 998.4(2.0 - 0.667)/5 = 266 \text{ lbf} \\ \forall &= 266/(150 - 62.4) = \underline{\underline{3.04 \text{ ft}^3}} \end{aligned}$$

3.104 **Information and assumptions**

provided in problem statement

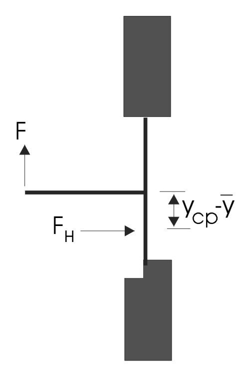
Find

length of chain so that gate just on verge of opening

Solution

Hydrostatic force F_H:

$$
\begin{aligned}
F_H &= \bar{p}A \\
&= 10 \times 9,810 \times \pi D^2/4 \\
&= 98,100 \times \pi(1^2/4) \\
&= 77,048 \text{ N} \\
y_{cp} - \bar{y} &= I/(\bar{y}A) \\
&= (\pi r^4/4)/(10 \times \pi D^2/4) \\
y_{cp} - \bar{y} &= r^2/40 = 0.00625 \text{ m} \\
\sum M_{\text{Hinge}} &= 0 \\
F_H \times (0.00625 \text{ m}) - 1 \times F &= 0 \\
\text{But } F &= F_{\text{buoy}} - W \\
&= A(10 \text{ m} - \ell)\gamma_{\text{H}_2\text{O}} - 200 \\
&= (\pi/4)(.25^2)(10 - \ell)(9,810) - 200 \\
&= 4815.5 \text{ N} - 481.5\ell \text{ N} - 200 \text{ N} \\
&= (4615.5 - 481.5\ell) \text{ N} \\
\text{where } \ell &= \text{length of chain} \\
77,048 \times 0.00625 - 1 \times (4615.5 - 481.5\ell) &= 0 \\
481.55 - 4615.5 + 481.5\ell &= 0 \\
\ell &= \underline{\underline{8.59 \text{ m}}}
\end{aligned}
$$



3.105 The horizontal component of force acting on the walls is the same for each wall. However, walls $A - A'$ and $C - C'$ have vertical components that will require greater resisting moments than the wall $B - B'$. If one thinks of the vertical component as a force resulting from buoyancy, it can be easily shown that there is a greater "buoyant" force acting on wall $A - A'$ than on $C'C'$. Thus, wall $A - A'$ will require the greatest resisting moment.

3.106 **Information and assumptions**

provided in problem statement

Find

specific weight and volume of material

Solution

$$W_{\text{in air}} = 700\text{ N } = \forall\gamma_{\text{block}} \tag{1}$$

$$W_{\text{in water}} = 500\text{ N } = (\forall\gamma_{\text{block}} - \forall\gamma_{\text{water}}) \tag{2}$$

$$\gamma_{\text{water}} = 9,810\text{ N/m}^3 \tag{3}$$

Solving Eqs. (1), (2), and (3) yields:

$$\begin{aligned}\forall &= \underline{\underline{0.0204\text{ m}^3}}\\ \gamma_{\text{block}} &= \underline{\underline{34,313}}\text{ N/m}^3\end{aligned}$$

3.107 Find equation for buoyant force on sphere.

$$dF_z = -\sin\theta \times p \times 2\pi R^2 \cos\theta d\theta$$

$$\text{but } p = \gamma(d - R\sin\theta)$$

$$\therefore dF_z = -\sin\theta(\gamma d - \gamma R\sin\theta)(2\pi R^2\cos\theta d\theta)$$

$$F_z = \int_{-\pi/2}^{\pi/2} -\gamma d 2\pi R^2 \cos\theta\sin\theta d\theta + \int_{-\pi/2}^{\pi/2} \gamma 2\pi R^3 \cos\theta \sin^2\theta d\theta$$

$$F_z = -\gamma d 2\pi R^2 \int_{-\pi/2}^{\pi/2} \cos\theta\sin\theta d\theta + \gamma 2\pi R^3 \int_{-\pi/2}^{\pi/2} \cos\theta\sin^2\theta d\theta$$

$$= -\gamma d 2\pi R \left[(1/2)\sin^2\theta\right]_{-\pi/2}^{\pi/2} + \gamma 2\pi R^3 \left[(1/3)\sin^3\theta\right]_{-\pi/2}^{\pi/2}$$

$$= -\gamma d 2\pi R(0) + \gamma 2\pi R^3 \left[(1/3)(1+1)\right]$$

$$= \gamma 2\pi R^3 (2/3) = \underline{\underline{(4/3\pi R^3 \gamma)}}$$

3.108 **Information and assumptions**

provided in problem statement

Find

weight, volume, specific weight and specific gravity

Solution

$$\begin{aligned}
\forall(\gamma - \gamma_0) &= \forall(\gamma - 9,810 \times 0.8) = 75 \text{ N} \\
\forall(\gamma - \gamma_{Hg}) &= \forall(\gamma - 1333,000) = -100 \text{ N}
\end{aligned}$$

Equating $\forall$'s,

$$\begin{aligned}
\gamma &= \underline{\underline{52,515 \text{ N/m}^3}} \\
\forall &= \underline{\underline{1.68 \times 10^{-3} \text{ m}^3}} \\
W &= \underline{\underline{88.2 \text{ N}}} \\
S &= \underline{\underline{5.35}}
\end{aligned}$$

3.109 **Information and assumptions**

provided in problem statement

Find

maximum altitude of balloon

Solution

$$\begin{aligned}
\forall_0 &= (\pi/6)D_0^3 \\
&= (\pi/6)\text{ m}^3 = 0.524\text{ m}^3 \\
T_0 &= 288\text{ K} \\
p_{0,\text{He}} &= p_{\text{atm.}} + 10,000P_a = 111,300\text{ Pa} \\
\rho_{0,\text{He}} &= (p_{0,\text{He}}/R_{\text{He}}T_0) = 111,300/((2077)(288)) = 0.186\text{ kg/m}^3
\end{aligned}$$

Conservation of mass

$$\begin{gathered}
m_0 = m_{\text{alt.}} \\
\forall_0 \gamma_{0,\text{He}} = \forall_{\text{alt.}}\rho_{\text{He}} \\
\forall_{\text{alt.}} = \forall_0 \rho_{0,\text{He}}
\end{gathered}$$

Newton's second law

$$\begin{aligned}
\sum F_z &= ma = 0 \\
F_{\text{buoy.}} - W &= 0 \\
\forall_{\text{alt.}}\rho_{\text{air}}g - (mg + W_{\text{He}}) &= 0
\end{aligned}$$

Eliminate $\forall_{\text{alt.}}$

$$(\forall_0\rho_0/\rho_{\text{He}})\rho_{\text{air}}g = (mg + \forall_0\rho_{0,\text{He}}g)$$

Eliminate ρ's with equation of state

$$\begin{aligned}
\frac{(V_0\rho_0)(p_{\text{alt.}}/R_{\text{air}})g}{(p_{\text{alt.}} + 10,000)/(R_{\text{He}})} &= (\text{mg } + V_0\rho_0 g) \\
\frac{(0.524)(0.186)(9.81)(2077)p_{\text{alt.}}}{(p_{\text{alt.}} + 10,000)(287)} &= (0.1)(9.81) + (0.524)(0.186)(9.81)
\end{aligned}$$

Solve

$$p_{\text{alt.}} = 3888\text{ Pa}$$

Check to see if $p_{\text{alt.}}$ is in the troposphere or stratosphere. Using Eq. (3.15) solve for pressure at top of troposphere.

$$\begin{aligned}
p &= p_0\left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R} \\
&= 101,300[(296 - 5.87\times 10^{-3})(13,720)/296]^{5.823} \\
&= 15,940\text{ Pa}
\end{aligned}$$

Because $p_{\text{alt.}} < p_{\text{at top of troposphere}}$ we know that $p_{\text{alt.}}$ occurs above the stratosphere. The stratosphere extends to 16.8 km where the temperature is constant at -57.5°C. The pressure at the top of the stratosphere is given by equation 3.16

$$
\begin{gathered}
p = p_0 e^{-(z-z_0)g/RT} \\
= 15.9 \exp(-(16,800 - 13,720) \times 9.81/(287 \times 215.5)) \\
= 9.75 \text{ kPa}
\end{gathered}
$$

Thus the balloon is above the stratosphere where the temperature increases linearly at 1.387°C/km. In this region the pressure varies as

$$
p = p_0 \left[\frac{T_0 + \alpha(z - z_0)}{T_0} \right]^{-g/\alpha R}
$$

Using this equation to solve for the altitude, we have.

$$
\begin{aligned}
\frac{3888}{9750} &= \left[\frac{215.5 + 1.387 \times (z - 16.8)}{215.5} \right]^{-9.81/(0.001387 \times 287)} \\
0.399 &= [1 + 0.00644 \times (z - 16.8)]^{-24.6} \\
z &= 22.8 \text{ km}
\end{aligned}
$$

3.110 **Information and assumptions**

provided in problem statement

Find

volume of rock

Solution

$$\begin{aligned} \forall\gamma &= 918 \text{ N} \\ \forall(\gamma - 9,810) &= 609 \text{ N} \\ \forall &= (918 - 609)/9,810 = \underline{\underline{0.0315 \text{ m}^3}} \end{aligned}$$

3.111

$$
\begin{aligned}
\text{Rod weight } &= (2LA\gamma_W + LA(2\rho_W))g = 4LA\gamma_W g = 4LA\gamma_W \\
\text{Buoyant force } &= \forall\gamma_{\text{Liq}} = 3LA\gamma_{\text{Liq}} \\
\text{Rod weight } &= \text{ Buoyant force} \\
4LA\gamma_W &= 3LA\gamma_{\text{Liq}} \\
\gamma_{\text{Liq}} &= (4/3)\gamma_W.
\end{aligned}
$$

The liquid is more dense than water so is answer $\underline{\underline{\text{c)}}}$

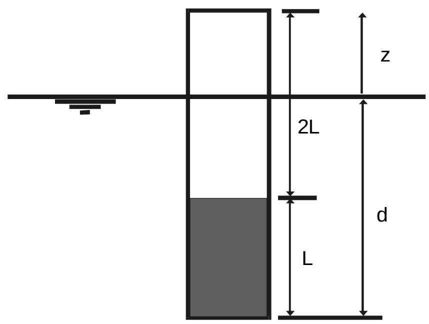
3.112 **Information and assumptions**

provided in problem statement

Find

how deep will rod float.

Solution



$$
\begin{aligned}
\rho &= \rho_{\text{water}}(1+0.3d) \\
\gamma &= \rho g \qquad (1) \\
\sum F_z &= 0 \\
-W_{\text{rod}} + p_{\text{bottom of rod}} A &= 0 \\
-(2LA\rho_{\mathrm{H_2O}} g + LA(2\rho_{\mathrm{H_2O}})) + p_b A &= 0 \\
-(4L\gamma_{\mathrm{H_2O}}) + p_b &= 0 \qquad (2)
\end{aligned}
$$

From given condition of the liquid

$$
\begin{aligned}
\gamma &= \gamma_{\mathrm{H_2O}}(1+0.3d) \\
\gamma &= \gamma_{\mathrm{H_2O}}(1-0.3z) \\
\frac{dp}{dz} &= -\gamma \\
dp &= -\gamma_{\mathrm{H_2O}}(1-0.3z)dz \\
p &= \gamma_{\mathrm{H_2O}} \int (-1+0.3z)dz \\
p_b &= \gamma_{\mathrm{H_2O}}(-z+0.3z^2/2)+C \qquad (3) \\
\text{when } z &= 0 \cdot \ p = 0 \text{ gage so } C = 0
\end{aligned}
$$

Substitute p_b of Eq. (2) into Eq. (1)

$$= 4L_{\mathrm{H_2O}} + \gamma_{\mathrm{H_2O}}(-z + 0.3z^2/2) = 0$$

But $L = 1$ ft so

$$0.15z^2 - z - 4 = 0$$

$$z^2 - 6.667z - 26.67 = 0$$

Solve for z by the quadratic equation

$$z = (+6.667 \pm \sqrt{44.4 + 106.68})/2$$

$z = -2.812$ ft or depth d at which rod will float is $-z$ or $\underline{\underline{2.812 \text{ ft}}}$

3.113 **Information and assumptions**

provided in problem statement

Find

change of water level in pond

Solution

Weight anchor $= 0.50 \text{ ft}^3 \times (2.2 \times 62.4 \text{ lb/ft}^3) = 68.65$ lb.

The water displaced by boat due to weight of anchor

$$= 68.65 \text{ lb}/(62.4 \text{ lb/ft}^3) = 1.100 \text{ ft}^3$$

Therefore, when the anchor is removed from the boat, the boat will rise and the water level in the pond will drop:

$$\Delta h = 1.10 \text{ ft}^3/500 \text{ ft}^2 = 0.0022 \text{ ft}$$

However, when the anchor is dropped into the pond, the poind will rise because of the volume taken up by the anchor. This change in water level in the pond will be:

$$\Delta h = 0.500 \text{ ft}^3/500 \text{ ft}^2 = .001 \text{ ft}$$

Net change =-.0022 ft + .001 ft = -.0012 ft = -.0144 in.

The pond level will drop $\underline{\underline{0.0144}}$ inches.

3.114 **Information and assumptions**

provided in problem statement

Find

change of water level in cone

Solution

$$S = 0.6 \Longrightarrow \gamma_{\text{block}} = 0.6\gamma_{\text{water}}$$

Weight of displaced water = weight of block

$$\begin{aligned}
\forall_W \gamma_W &= \forall_b \gamma_b \\
\forall_W &= (\gamma_b/\gamma_W)\forall_b \\
\forall_W &= 0.6\forall_b = 120 \text{ cm}^3
\end{aligned}$$

Then the total volume below water surface when block is floating in water $= \forall_{W,\text{org.}} + 120 \text{ cm}^3$

$$\begin{aligned}
\forall_{W,\text{orig.}} &= (\pi/3)(10 \text{ cm})^3 \\
&= 1047.2 \text{ cm}^3 \\
\forall_{\text{final}} &= 1047.2 \text{ cm}^3 + 120 \text{ cm}^3 \\
(\pi/3)h_{\text{final}}^3 &= 1167.2 \text{ cm}^3 \\
h_{\text{final}} &= 10.368 \text{ cm} \\
\Delta h &= \underline{\underline{0.368 \text{ cm}}}
\end{aligned}$$

3.115 **Information and assumptions**

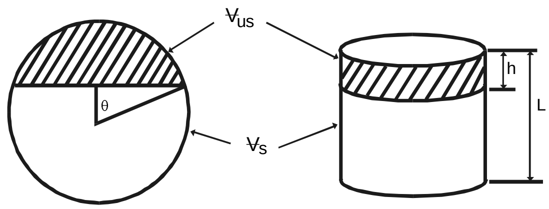
provided in problem statement

Find

height above water when erected

Solution

The same relative volume will be unsubmerged whatever the orientation; therefore,



$$\begin{aligned} \frac{\forall_{\text{u.s.}}}{\forall_{\text{s}}} &= \frac{hA}{LA} = \frac{LA_{\text{u.s.}}}{LA} \\ \text{or } h/L &= A_{\text{u.s.}}/A \end{aligned}$$

Also,

$$\begin{aligned} \cos\theta &= 5'/10' = 0.50 \\ \theta &= 60^\circ \text{ and } 2\theta = 120^\circ \end{aligned}$$

So

$$A_{\text{u.s.}} = (1/3)\pi R^2 - R\cos 60^\circ R \sin 60^\circ$$

Therefore

$$\begin{aligned} h/L &= R^2\left[((1/3)\pi) - \sin 60^\circ \cos 60^\circ)\right]/\pi R^2 = \underline{\underline{0.195}} \\ h &= \underline{\underline{7.80 \text{ m}}} \end{aligned}$$

3.116 **Information and assumptions**

provided in problem statement

Find

change of water level in tank

Solution

The block will displace water equal to the weight of the block:

$$
\begin{aligned}
\Delta\forall_W \gamma_W &= W_{\text{ block}} \\
\Delta\forall_W &= 2 \text{ lbf}/(62.4 \text{ lbf/ft}^3) = 0.03205 \text{ ft}^3 \\
&\therefore \;\; \Delta h A_T = \Delta\forall_W \\
\Delta h &= \Delta\forall_W / A_\tau = 0.03205 \text{ ft}^3/((\pi/4)(1^2) \text{ ft}^2) \\
\Delta h &= 0.0408 \text{ ft}
\end{aligned}
$$

Water in tank will rise 0.0408 ft.

3.117 **Information and assumptions**

provided in problem statement

Find

length of cylinder so that it floats 1 m above water surface

Solution

$$
\begin{aligned}
\sum F_y &= 0 \\
-30,000 - 4 \times 1,000L + 4 \times (\pi/4) \times 1^2 \times 10,000(L-1) &= 0
\end{aligned}
$$

$$L = \underline{\underline{2.24}} \text{ m}$$

3.118 **Information and assumptions**

provided in problem statement

Find

depth block will float

Solution

Assume the block will sink a distance y into the fluid with $S = 1.2$.

$$\begin{aligned} \sum F_y &= 0 \\ -W + pA &= 0 \\ -(6L)^2 \times 3L \times 0.8\gamma_{\text{water}} + (L \times \gamma_{\text{water}} + y \times 1.2\gamma_W)36L^2 &= 0 \end{aligned}$$

$$y = 1.167L$$
$$d = \underline{\underline{2.167L}}$$

3.119 **Information and assumptions**

provided in problem statement

Find

change of water level in container before and after ice melts.

Solution

When the ice is put in the tank the water will rise consistent with the weight of water displaced by the ice:

$$W_{\text{ice}} = \forall_W \gamma_W$$

but

$$\forall_W = 5 \text{ lb}/(62.4 \text{ lb/ft}^3) = 0.0801 \text{ ft}^3$$

The amount of rise of water in tank

$$\begin{aligned} &= 0.0801 \text{ ft}^3/(A_{\text{cyl}}) \\ \Delta h &= 0.0801 \text{ ft}^3/((\pi/4)(2 \text{ ft})^2) \\ &= \underline{\underline{0.0255 \text{ ft}}} \end{aligned}$$

When the ice melts the melted water will simply occupy the same volume of water that the ice originally displaced; therefore, there will be <u>no change in water surface level in the tank when the ice melts</u>.

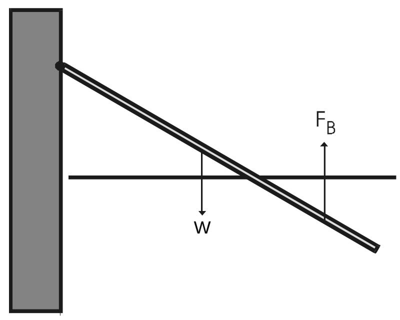
3.120 **Information and assumptions**

provided in problem statement

Find

density of wood

Solution



$$
\begin{aligned}
M_A &= 0 \\
-W_{\text{wood}} \times (0.5L\cos 30^\circ) + F_{B.} \times (5/6)L\cos 30^\circ &= 0 \\
-\gamma_{\text{wood}} \times AL \times (0.5L\cos 30^\circ) + ((1/3)AL\gamma_{\mathrm{H_2O}}) \times (5/6)L\cos 30^\circ &= 0
\end{aligned}
$$

$$
\begin{gathered}
\gamma_{\text{wood}} = (10/18)\gamma_{\mathrm{H_2O}} \\
= \underline{\underline{5,450}}\ \mathrm{N/m^3} \\
\rho_{\text{wood}} = \underline{\underline{556}}\ \mathrm{kg/m^3}
\end{gathered}
$$

3.121 **Information and assumptions**

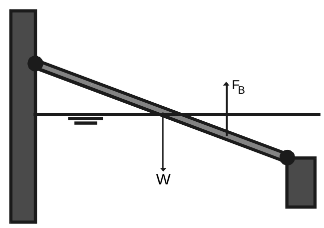
provided in problem statement

Find

if pole will rise or fall

Solution

The summation of moments about A to see if pole will rise or fall. The forces producing moments about A will be the weight of the pole and the buoyant force.



$$\begin{aligned}\sum M_A &= -(1/2)(L\cos\alpha)(L\gamma_p A) + (3/4)(L\cos\alpha)(L/2)\gamma_{\text{liq}}A \\ &= L^2 A\cos\alpha[-(1/2)\gamma_p + (3/8)\gamma_{\text{liq}}] \\ &= K(-80+75)\end{aligned}$$

A negative moment acts on the pole; therefore, it will fall.

3.122 **Information and assumptions**

provided in problem statement

Find

how much the ship will rise or settle

Solution

Draft $= (40,000 \times 2,000)/40,000\gamma = (2,000/\gamma)$ ft.

Since γ of salt water is greater than γ of fresh water, the ship will take a greater draft in fresh water.

$$(2,000/62.4) - (2,000/64.1) = \underline{\underline{0.85 \text{ ft}}}$$

3.123 **Information and assumptions**

provided in problem statement

Find

weight of scrap iron to be sealed in the buoy

Solution

$$
\begin{aligned}
\sum F_V &= 0; F_B - F_s - F_w - F_c = 0 \\
F_s &= F_B - F_w - F_c \\
&= (4/3)\pi(0.6)^3 \times 10,070 - 1,600 - 4,500 \\
&= \underline{\underline{3,011 \text{ N of scrap}}}
\end{aligned}
$$

3.124 **Information and assumptions**

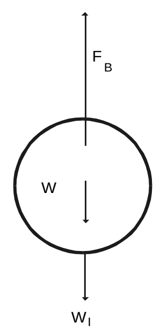
provided in problem statement

Find

diameter of spherical balloon.

Solution

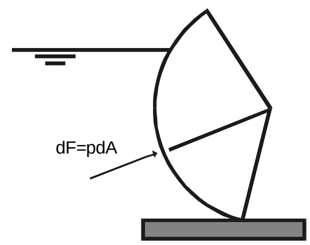
Assuming standard atmospheric temperature condition



$$
\begin{aligned}
T &= 533 - 3.221 \times 10^{-3} \times 15,000 = 485R \\
\rho_{\text{air}} &= (8.3 \times 144)/(1,715 \times 485) \\
&= 0.001437 \text{ slugs/ft}^3 \\
\rho_{\text{He}} &= (8.3 \times 144)/(12,429 \times 485) \\
&= 0.000198 \text{ slugs/ft}^3
\end{aligned}
$$

$$
\begin{aligned}
\sum F &= 0 - F_L - F_b - F_i \\
&= (1/6)\pi D^3 g(\rho_{\text{air}} - \rho_{\text{He}}) - \pi D^2(0.01) - 10 \\
&= D^3 \times 16.88(14.37 - 1.98)10^{-4} - D^2 \times 3.14 \times 10^{-2} - 10 \\
D &= \underline{\underline{8.35 \text{ ft}}}
\end{aligned}
$$

3.125



Consider all the differential pressure forces acting on the radial gate as shown. Because each differential pressure force acts normal to the differential area, then each differential pressure force must act through the center of curvature of the gate. Because all the differential pressure forces will be acting through the center of curvature (the pin), the resultant must also pass through this same point (the pin).

3.126 Volume displaced by the ice is

$$V_{\text{dis}} = W/\gamma_{\text{water}}$$

After the ice melts, the volume of water added to the water in the tank is

$$V_{\text{add}} = M/\rho_{\text{water}} = W/\gamma_{\text{water}}$$

Since the volume displaced is equal to the volume added, there is no net height change in the tank water level.

3.127 **Information and assumptions**

provided in problem statement

Find

location of water level

Solution

The buoyant force is equal to the weight.

$$F_B = W$$

The weight of the buoy is $9.81 \times 460 = 4512$ N.
The volume of the hemisphere at the bottom of the buoy is

$$V = \frac{1}{2}\frac{\pi}{6}D^3 = \frac{\pi}{12}1^3 = \frac{\pi}{12} \text{ m}^3$$

The buoyant force due to the hemisphere is

$$F_B = \frac{\pi}{12}(9.81)(1010) = 2594 \text{ N}$$

Since this is less than the buoy weight, the water line must lie above the hemisphere. Let h is the distance from the top of the buoy. The volume of the cone which lies between the top of the hemisphere and the water line is

$$\begin{aligned} V &= \frac{\pi}{3}r_o^2 h_o - \frac{\pi}{3}r^2 h = \frac{\pi}{3}(0.5^2 \times 0.866 - h^3 \tan^2 30) \\ &= 0.2267 - 0.349h^3 \end{aligned}$$

The additional volume needed to support the weight is

$$V = \frac{4512 - 2594}{9.81 \times 1010} = 0.1936 \text{ m}^3$$

Equating the two volumes and solving for h gives

$$\begin{aligned} h^3 &= \frac{0.0331}{0.349} = 0.0948 \text{ m}^3 \\ h &= \underline{\underline{0.456}} \text{ m} \end{aligned}$$

3.128 **Information and assumptions**

provided in problem statement

Find

vertical and horizontal hydrostatic forces and resultant force

Solution

$$
\begin{aligned}
F_V &= 1 \times 9,810 \times 1 \times +(1/4)\pi \times (1)^2 \times 1 \times 9,810 = \underline{\underline{17,515 \text{ N}}} \\
x &= M_0/F_V \\
&= 1 \times 1 \times 1 \times 9,810 \times 0.5 + 1 \times 9,810 \times \int_0^1 \sqrt{1-x^2}x dx/17,515 \\
&= \underline{\underline{0.467 \text{ m}}} \\
F_H &= \bar{p}A = (1+0.5)9,810 \times 1 \times 1 = \underline{\underline{14,715 \text{ N}}} \\
y_{cp} &= \bar{y} + \bar{I}/\bar{y}A \\
&= 1.5 + (1 \times 1^3)/(12 \times 1.5 \times 1 \times 1) = \underline{\underline{1.555 \text{ m}}} \\
F_R &= \sqrt{(14,715)^2 + (17,515)^2} = \underline{\underline{22,876 \text{ N}}} \\
\tan\theta &= 14,715/17,515; \theta = \underline{\underline{40^\circ 2'}}
\end{aligned}
$$

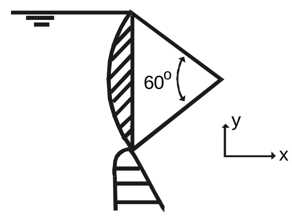
3.129 **Information and assumptions**

provided in problem statement

Find

hydrostatic force acting on gate

Solution



From the reasoning given in the solution to problem 3.119, we know the resultant must pass through the center of curvature of the gate. The horizontal component of hydrostatic force acting on the gate will be the hydrostatic force acting on the vertical projection of the gate or:

$$
\begin{aligned}
F_H &= \bar{p}A \\
&= 25 \text{ ft} \times 62.4 \text{ lb/ft}^3 \times 40 \text{ ft} \times 50 \text{ ft} \\
F_H &= 3,120,000 \text{ lb}
\end{aligned}
$$

The vertical component of hydrostatic force will be the buoyant force acting on the radial gate. It will be equal in magnitude to the weight of the displaced liquid (the weight of water shown by the cross-hatched volume in the above Fig.).

Thus,

$$
\begin{aligned}
F_V &= \gamma\forall \\
\text{where } \forall &= [(60/360)\pi \times 50^2 \text{ ft}^2 - (1/2)50 \times 50 \cos 30^\circ \text{ ft}^2] \times 40 \text{ ft} \\
&= 226.5 \text{ ft}^2 \times 40 \text{ ft} \\
&= 9600 \text{ ft}^3 \\
\text{Then } F_V &= (62.4 \text{ lbf/ft}^3)(9060 \text{ ft}^3) = 565,344 \text{ lbs} \\
F_{\text{result}} &= \underline{\underline{3,120,000\ \mathbf{i}\ + 565,344\ \mathbf{j}\ \text{ lbf}}}
\end{aligned}
$$

acting through the center of curvature of the gate.

3.130 **Information and assumptions**

provided in problem statement

Find

magnitude and direction of horizontal and vertical components

Solution

$$
\begin{aligned}
F_H &= \bar{p}A = -2.5 \times 50 \times (3 \times 1) = -\underline{\underline{375}} \text{ lbf} \\
&\quad \text{(force acts to the right)} \\
F_V &= \forall\gamma = (1 \times 3 + \pi \times 3^2 \times \frac{1}{4})50 = \underline{\underline{503.4}} \text{ lbf (downward)} \\
y_{cp} &= 2.5 + 1 \times 3^3/(12 \times 2.5 \times 1 \times 3) \\
&= \underline{\underline{2.8}} \text{ ft above the water surface}
\end{aligned}
$$

3.131 **Information and assumptions**

provided in problem statement

Find

horizontal and vertical forces on plug

Solution

The horizontal can be calculated by finding the force on a circle with an radius equal to that of the O-ring.

$$F_h = \bar{p}A = \gamma z A = 9810 \times 2 \times \frac{\pi}{4} \times 0.2^2 = \underline{\underline{616}} \text{ N}$$

The vertical force is simply the buoyant force.

$$F_v = \gamma V = 9810 \times \frac{4}{6} \times \pi \times 0.25^3 = \underline{\underline{321}} \text{ N}$$

3.132 **Information and assumptions**

provided in problem statement

Find

force on the hinge of the gate

Solution

The length of one gate is

$$L = 50/\cos 20^o = 53.21 \text{ ft}$$

The force on one gate is

$$F = \bar{p}A$$

where $\bar{p}$ is the pressure at the centroid which is

$$p = \gamma_f \bar{z} = 62.4 \times 50 = 1,560 \text{ psf}$$

The total force on the gate is

$$F = 1,560 \times 53.21 \times 50 = 4.15 \times 10^6 \text{ lbf}$$

Taking the moment about the juncture of the gates, the reactive force on the hinge normal to the gate is

$$F_R = F \times \frac{L/2}{L} = \frac{F}{2} = \underline{\underline{2.07 \times 10^6}} \text{ lbf}$$

3.133 **Information and assumptions**

provided in problem statement

Find

force exerted on chamber by bolts

Solution

Drawing a free body diagram around the steel structure and across the bottom, and equating forces gives

$$F_B + W_w + W_s = pA$$

where F_B is the force on the bolts, W_w is the weight of the water, W_s is the weight of the structures, p is the pressure at the bottom and A is the area at the bottom.

The pressure (gage) is

$$p = \gamma h = 62.4 \times 10 = 624 \text{ psfg}$$

The area is

$$A = \frac{\pi}{4} 2^2 = 3.14 \text{ ft}^2$$

The weight of the water is

$$W_w = \frac{\pi}{4}(2^2 \times 2 + 0.5^2 \times 8) \times 62.4 = 489 \text{ lbf}$$

The force on the bolts is

$$F_B = 624 \times 3.14 - 489 - 600 = \underline{\underline{870}} \text{ lbf}$$

3.134 **Information and assumptions**

provided in problem statement

Find

force exerted by bolts

Solution

$$
\begin{aligned}
W_{\mathrm{H_2O}} &= (2/3)\pi 6^3 \times 62.4 + 12 \times (\pi/4) \times (3/4)^2 \times 62.4 \\
&= 28,559 \text{ lbf} \\
W_{\mathrm{dome}} &= 1,300 \text{ lbf} \\
F_{\mathrm{Pressure}} &= 18 \times 62.4 \times \pi \times (6)^2 = 127,031 \text{ lbf} \\
F_{\mathrm{bolt}} &= F_{\mathrm{pressure}} - W_{\mathrm{H_2O}} - W_{\mathrm{dome}} \\
&= 127,031 - 28,559 - 1,300 = \underline{\underline{97,172 \text{ lbf downward}}}
\end{aligned}
$$

3.135 **Information and assumptions**

provided in problem statement

Find

force exerted by the bolts

Solution

$$\begin{aligned}
\sum F_z &= 0 \\
p_{\text{bottom}}A_{\text{bottom}} + F_{\text{bolts}} - W_{\text{H}_2\text{O}} - W_{\text{dome}} &= 0 \\
\text{where } p_{\text{bottom}}A_{\text{bottom}} &= 4.8 \times 9,810 \times \pi \times 1.6^2 = 378.7 \text{ kN} \\
W_{\text{H}_2\text{O}} &= 9,810(3.2 \times (\pi/4) \times 0.2^2 + (2/3)\pi \times 1.6^3) \\
&= 85.1 \text{ kN} \\
\text{Then } F_{\text{bolts}} &= -378.7 + 85.1 + 6 = \underline{\underline{-287.6 \text{ kN}}}
\end{aligned}$$

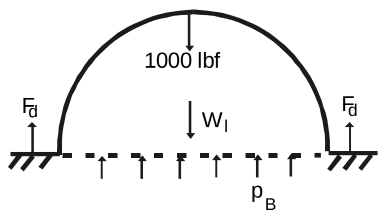
3.136 **Information and assumptions**

provided in problem statement

Find

vertical component of force in metal at base of dome

Solution



$$\begin{aligned}
p_b &= 10 \times 144 - 3 \times 1.5 \times 62.4 \times 1.5 \\
&= \underline{\underline{1{,}019 \text{ psf}}} \\
F_p &= p_b A = 1,019\pi \times 3^2 = 28,811 \text{ lbf} \\
W_\ell &= (2\pi/3)3^3 \times 62.4 \times 1.5 = 5,293 \text{ lbf} \\
\sum F_V &= 0; F_d + F_p - W_\ell - 1,000 = 0 \\
F_d + 28,810 - 5,293 - 1,000 &= 0; F_d = \underline{\underline{-22,518 \text{ lbf}}}
\end{aligned}$$

Metal in tension

3.137 **Information and assumptions**

provided in problem statement

Find

magnitude and direction of force to hold dome in place.

Solution

$$F_H = (1+1)9,810 \times \pi \times (1)^2 = 61,640 \text{ N } = 61.64 \text{ kN}$$

61.64 kN force will act horizontally to the left to hold the dome in place.

$$(y_{cp} - \bar{y}) = I/\bar{y}A = (\pi \times 1^4/4)/(2 \times \pi \times 1^2) = 0.125 \text{ m}$$

The line of action lies 0.125 m below the center of curvature of the dome.

$$F_V = (1/2)(4\pi \times 1^3/3)9,810 = 20,550 \text{ N } = \underline{\underline{20.55 \text{ kN}}}$$

To be applied downward to hold the dome in place.

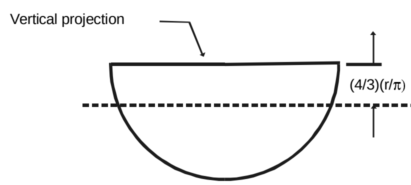
3.138 **Information and assumptions**

provided in problem statement

Find

force on the dome

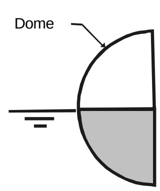
Solution



The horizontal component of the hydrostatic force acting on the dome will be the hydrostatic force acting on the vertical projection of the bottom half of the dome or:

$$\begin{aligned} F_H &= \bar{p}A \\ \bar{p} &= (4/3)(5/\pi) \text{ ft } (62.4 \text{ lbf/ft}^3) \\ &= 132.4 \text{ lbf/ft}^2 \\ F_H &= (132.4 \text{ lbf/ft}^2)(\pi/8)(10^2) \text{ ft}^2 = 5,199 \text{ lbf} \end{aligned}$$

The vertical component of force will be the buoyant force acting on the dome. It will be the weight of water represented by the cross-hatched region shown in the Fig. (below).



Thus,

$$\begin{aligned} F_V &= \gamma\forall \\ &= (62.4 \text{ lbf/ft}^3)((1/6)\pi D^3/4) \text{ ft}^3 \\ F_V &= 8,168 \text{ lbf} \end{aligned}$$

The resultant force is then given by: $\underline{\underline{\mathbf{F}_{\text{result}} = 5,199\mathbf{i} + 8,168\mathbf{j} \text{ lbf and it acts through}}}$ $\underline{\underline{\text{the center of curvature of the dome}}}$.

3.139 **Information and assumptions**

provided in problem statement

Find

weight of hydrometer

Solution

$$
\begin{aligned}
\sum F_2 &= 0 \\
F_{\text{buoy.}} - W &= 0 \\
W &= F_{\text{buoy.}} \\
&= \gamma_W (1\,\text{in}^3 + (2\ \text{in})(0.01\ \text{in}^2))(1/12)^3\ \text{ft}^3/\text{in}^3 \\
&= (62.4\ \text{lbf/ft}^3)(1.02/1728)\ \text{ft}^3 \\
&= \underline{\underline{0.0368\ \text{lbf}}}
\end{aligned}
$$

3.140 **Information and assumptions**

provided in problem statement

Find

weight of hydrometer

Solution

$$\begin{aligned}
F_B &= W \\
0.9\gamma_W \forall &= 0.0368 \text{ lbf} \\
0.9\gamma_W (1.00 + 0.1h)(1/1728) \text{ ft}^3/\text{in}^3 &= 0.0368 \text{ lbf} \\
1.00 + 0.01\ h &= 1.1323 \\
h &= \underline{\underline{13.23 \text{ in.}}}
\end{aligned}$$

3.141 **Information and assumptions**

provided in problem statement

Find

weight of hydrometer

Solution

$$
\begin{aligned}
F_{\text{buoy.}} &= W\ . \\
\forall\gamma_W &= W \\
(1\ \text{cm}^3 + (5.3\ \text{cm})(0.1\ \text{cm}^2))(0.1^3)\ \text{m}^3/\text{cm}^3(\gamma_W) &= W\ . \\
(1.53\ \text{cm}^3)(10^{-6}\ \text{m}^3/\text{cm}^3)(9800\ \text{N/m}^3) &= W. \\
W &= \underline{\underline{1.499\times 10^{-2}\ \text{N}}}
\end{aligned}
$$

3.142 **Information and assumptions**

provided in problem statement

Find

specific gravity of oil

Solution

$$
\begin{aligned}
F_{\text{buoy.}} &= W \\
(1\ \text{cm}^3 + (6.3\ \text{cm})(0.1\ \text{cm}^2))(0.01^3)\ \text{m}^3/\text{cm}^3\gamma_{\text{oil}} &= 0.015\ \text{N} \\
(1 + 0.63) \times 10^{-6}\ \text{m}^3\gamma_{\text{oil}} &= 0.015\ \text{N} \\
\gamma_{\text{oil}} &= 9202\ \text{N/m}^3
\end{aligned}
$$

$$
S = \gamma_{\text{oil}}/\gamma_W = 9202/9810 = \underline{\underline{0.938}}
$$

3.143 **Information and assumptions**

provided in problem statement

Find

specific gravity of the brine

Solution

$$
\begin{aligned}
(1+.33)\times 10^{-6}\ \mathrm{m}^3\gamma_b &= 0.01499\ \mathrm{N} \\
\gamma_b &= 11,271\ \mathrm{N/m}^3 \\
S &= \gamma_b/\gamma_W = 11,271/9,810 = \underline{\underline{1.149}}
\end{aligned}
$$

3.144 **Information and assumptions**

provided in problem statement

Find

weight of each ball

Solution

For a ball to just float, the buoyant force equals the weight

$$F_B = \frac{\pi}{6} D^3 \gamma_{fluid} = W$$

For a one-centimeter ball

$$W = 0.523 \times 10^{-6} \gamma_{\text{fluid}}$$

The following table shows the weights of the balls needed for the required specific gravity intervals.

ball number	1	2	3	4	5	6
sp. gr.	1.01	1.02	1.03	1.04	1.05	1.06
weight (mg)	5.18	5.23	5.28	5.33	5.38	5.44

3.145 **Information and assumptions**

provided in problem statement

Find

range of specific gravities

Solution

When only the bulb is submerged;

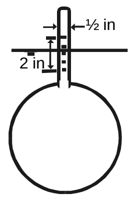
$$\begin{aligned} F_B &= W. \\ (\pi/4)\left[0.02^2 \times 0.08\right] \times 9,810 \times S &= 0.035 \times 9.81 \\ S &= \underline{\underline{1.39}} \end{aligned}$$

When the full stem is submerged;

$$\begin{aligned} & (\pi/4)\left[(0.02)^2 \times (0.08) + (0.01)^2 \times (0.08)\right] 9,810 \times S \\ &= 0.035 \times 9.81 \\ S &= \underline{\underline{1.114}} \end{aligned}$$

Range $\underline{\underline{1.114 \text{ to } 1.39}}$

3.146 Assume that the hydrometer will consist of a stem mounted on a spherical ball as shown in the diagram. Assume also for purposes of design than the diameter of the stem is 0.5 in and the maximum change in depth is 2 in.



Since the weight of the hydrometer is constant, the volumes corresponding to the limiting fluid specific weights can be calculated from

$$W = \gamma_{60}\forall_{60} = \gamma_{70}\forall_{70}$$

or

$$\frac{\forall_{70}}{\forall_{60}} = \frac{60}{70} = 0.857$$

The change in volume can be written as

$$\forall_{60} - \forall_{70} = \forall_{60}(1 - \frac{\forall_{70}}{\forall_{60}}) = 0.143\forall_{60}$$

The change in volume is related to the displacement of the fluid on the stem by

$$\frac{A\Delta h}{\forall_{60}} = 0.143$$

For the parameters given above the volume of the hydrometer when immersed in the 60 lbf/ft^3 liquid is 2.74 in^3. Assume there is one inch of stem between the lower marking and the top of the spherical ball so the volume of the spherical ball would be 2.55 in^3 which corresponds to a ball diameter of 1.7 in. The weight of the hydrometer would have to be

$$W = \gamma_{60}\forall_{60} = 0.0347 \text{ lbf/in}^3 \times 2.74 \text{ in}^3 = 0.095 \text{ lbf}$$

If one could read the displacement on the stem to within 1/10 in, the error would in the reading would be 5%.

Other designs are possible. If one used a longer stem displacement, a larger volume hydrometer would be needed but it would give better accuracy. The design will depend on other constraints like the volume of fluid and space available.

3.147 **Information and assumptions**

provided in problem statement

Find

stability of barge

Solution

$$\begin{aligned}
\text{Draft} &= 400,000/(50 \times 20 \times 62.4) = 6.41 \text{ ft}{<}8 \text{ ft} \\
\text{GM} &= I_{00}/\forall - \text{CG} \\
&= \left[(50 \times 20^3/12)/(6.41 \times 50 \times 20)\right] - (8 - 3.205) \\
&= 0.40 \text{ ft}
\end{aligned}$$

$\underline{\underline{\text{Will float stable}}}$

3.148 **Information and assumptions**

provided in problem statement

Find

location of water line for stability and specific gravity of material

Solution

For neutral stability, the distance to the metacenter is zero. In other words

$$GM = \frac{I_{oo}}{\forall} - GC = 0$$

where GC is the distance from the center of gravity to the center of buoyancy.
The moment of inertia at the waterline is

$$I_{oo} = \frac{w^3 L}{12}$$

where L is the length of the body. The volume of liquid displaced is hwL so

$$GC = \frac{w^3 L}{12hwL} = \frac{w^2}{12h}$$

The value for GC is the distance from the center of buoyancy to the center of gravity, or

$$GC = \frac{w}{2} - \frac{h}{2}$$

So

$$\frac{w}{2} - \frac{h}{2} = \frac{w^2}{12h}$$

or

$$\left(\frac{h}{w}\right)^2 - \frac{h}{w} + \frac{1}{6} = 0$$

Solving for h/w gives 0.789 and 0.211. The first root gives a physically unreasonable solution. Therefore

$$\frac{h}{w} = 0.211$$

The weight of the body is equal to the weight of water displaced.

$$\gamma_b V_b = \gamma_f V$$

Therefore

$$S = \frac{\gamma_b}{\gamma_f} = \frac{whL}{w^2 L} = \frac{h}{w} = \underline{\underline{0.211}}$$

The the specific gravity is smaller than this value, the body will be unstable (floats too high).

3.149 **Information and assumptions**

provided in problem statement

Find

stability

Solution

$$
\begin{aligned}
\text{draft} &= 1 \times 8,000/9,810 = 0.8155 \text{ m} \\
c_{\text{from bottom}} &= 0.8155/2 = 0.4077 \text{ m} \\
G &= 0.500 \text{ m; CG } = 0.500 - 0.4077 = 0.0922 \text{ m} \\
GM &= (I/\forall) - \text{CG} \\
&= ((\pi R^4/4)/(0.8155 \times \pi R^2)) - 0.0922 \\
&= 0.077 \text{ m} - 0.0922 \text{ m (negative)}
\end{aligned}
$$

Thus, block is unstable with axis vertical.

3.150 **Information and assumptions**

provided in problem statement

Find

stability

Solution

$$
\begin{aligned}
\text{draft} &= 5{,}000/9{,}810{=}0.5097 \text{ m} \\
\text{GM} &= I_{00}/\forall - \text{CG} \\
&= \left[(\pi \times 0.5^4/4)/(0.5097 \times \pi \times 0.5^2)\right] - (0.5 - 0.5097/2) \\
&= -0.122 \text{ m, negative}
\end{aligned}
$$

So will not float stable with its ends horizontal.

3.151 **Information and assumptions**

provided in problem statement

Find

stability of floating block

Solution

$$\text{GM} = I_{00}/\forall - \text{CG}$$

Let k = block density/water density (same as S)

$$\text{GM} = (\text{LB}^3/12)/(k\text{B}^2L) - ((\text{B}/2)\text{-}(k\text{B}/2))$$

Condition for impending instability is when GM = 0.
Then solve for k with GM = 0.

$$\begin{aligned}(1/12k) - (1/2)(1-k) &= 0 \\ k^2 - k + 1/6 &= 0\end{aligned}$$

Solve quadratic equation

$$k = 0.211 \text{ and } 0.789$$

Analysis of equation reveals that the block will be stable for $S < 0.211$ and for $S > 0.789$.

3.152 **Information and assumptions**

provided in problem statement

Find

stability

Solution

$$\begin{aligned}\mathrm{GM} &= I_{00}/\forall - \mathrm{CG} \\ &= (3H(2H)^3/(12 \times H \times 2H \times 3H)) - H/2 \\ &= -H/6\end{aligned}$$

Not stable about longitudinal axis

$$\begin{aligned}\mathrm{GM} &= (2H \times (3H)^3/(12 \times H \times 2H \times 3H)) - 3H/4 \\ &= 0\end{aligned}$$

Neutrally stable about transverse axis.

Not stable

3.153 **Information and assumptions**

neglect hydrodynamic drag

Find

oscillation frequency

Solution

With the given assumptions, this is an undamped system where the applied force varies linearly with displacement; therefore, harmonic vibration will occur which has the following solution: $f = \sqrt{k/m/}(2\pi)$ where k is the proportionality constant between force and displacement ($k = F/x$). Here $m = 500$ kg/m$^3 \times \forall = 500$ LA and $F = 9,8100$ Ax; $k = 9,810$ A N/m therefore

$$
\begin{aligned}
f &= \sqrt{(8,810/500 \text{ L})}/2\pi \\
f &= \sqrt{(19.62/L)}/2\pi = \sqrt{19.61/0.02/}2\pi = \underline{\underline{1.58 \text{ Hz}}}
\end{aligned}
$$

Chapter Four

4.1 Non-uniform, unsteady; unsteady, uniform.

4.2 (a) Unsteady, non-uniform.

(b) Local and convective acceleration.

4.3 Non-uniform; steady or unsteady.

4.4 **Information and assumptions**

A fluid flows in a straight conduit. The conduit has a section with constant diameter, followed by a section with changing diameter.

provided in problem statement

Goal

Match the given flow labels with the mathematical descriptions.

Solution

Steady flow corresponds to $\partial V_s/\partial t = 0$

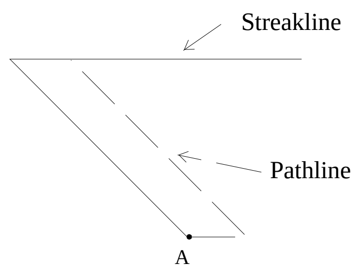
Unsteady flow corresponds to $\partial V_s/\partial t \neq 0$

Uniform flow corresponds to $V_s \partial V_s/\partial_s = 0$

Non-uniform flow corresponds to $V_s \partial V_s/\partial_s \neq 0$

4.5 True statements: (a), (c).

4.6



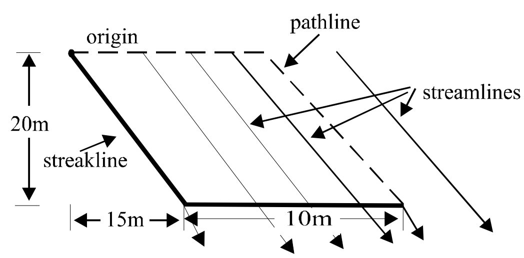
4.7 **Information and assumptions**

For $0 \leq t \leq 5$ seconds, $u = 2$ m/s, $v = 0$
For $0 < t \leq 10$ seconds, $u = 3$ m/s, $v = -4$ m/s
provided in problem statement

Goal

For $t = 10$ s, draw to scale the streakline, pathline of the particle, and streamlines.

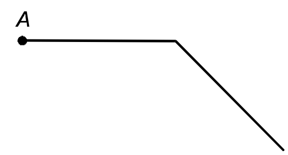
Solution



4.8 **Information and assumptions**

A dye streak is produced in a flow that has a constant speed. The origin of the streak is point A, and the streak was produced during a 10 s interval.

provided in problem statement



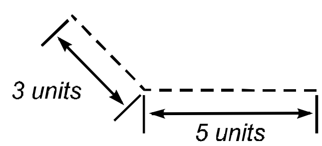
Goals

Sketch a streamline at $t = 8$ s

Sketch a particle pathline at $t = 10$ s, for a particle that was released from point A at $t = 2$ s.

Solution

Streamlines at $t = 8$ s



Particle pathline for a particle released at $t = 2$ s

4.9 **Information and assumptions**

A periodic flow field is described in the textbook.
provided in problem statement

Goal

Sketch a streakline at $t = 2.5$ s.

Solution

From time $t = 0$ to $t = 1$ s dye is emitted from point A and will produce a streak that is 10 meters long (up and to the right of A). See Fig. A below. In the next second the first streak will be transported down and to the right 10 meters and a new streak, 10 ft. long, will be generated down and to the right of point A (see Fig. B below). In the next 0.5 s streaks in Fig. B will move up and to the right a distance of 5 meters and a new streak 5 meters in length will be generated as shown in Fig. C.

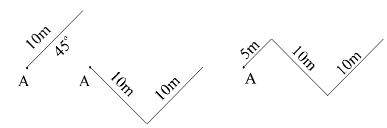
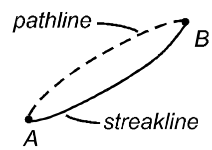


Fig. A t=1s | Fig. B t=25 | Fig. C t=2.5s

4.10 **Information and assumptions**

The figure below shows a pathline and a particle line for a flow. The fluid particle was released from point A at $t = 0$ s. The streakline was produced by releasing dye from point A from $t = 0$ to 5 s.
provided in problem statement



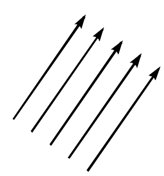
Goal

Sketch a streamline for $t = 0$ s
Is the flow steady or unsteady?

Solution

In the above sketch, the dye released at $t = 0$ s is now at point B. Therefore,a streamline corresponding to $t = 0$ s should be tangent to the streakline at point B. We can reach the same conclusion by using the pathline.

In the above sketch, the path of a fluid particle at $t = 0$ s is shown by the dotted line at point A. There, a streamline corresponding to $t = 0$ s should be tangent to the pathline at point A. Thus, streamlines at $t = 0$ appear as shown below:



The flow is unsteady because the streakline, streamlines and pathlines differ.

4.11 **Information and assumptions**

Consider a velocity field given by $u = 5$ m/s and $v = -2t$ m/s, where t is time.
provided in problem statement

Goals

a.) sketch a streakline for $t = 0$ to 5 s.
b) sketch a pathline for a particle for $t = 0$ to 5 s. The particle is released from the same point as the dye source
c.) sketch streamlines at $t = 5$ s

Particle pathline

Since $u = dx/dt$, we may write $dx = udt$. This can be integrated to give the x-position of a particle at any time t:

$$\begin{aligned} x &= x_o + \int udt = x_o + \int 5dt \\ x &= x_o + 5t \end{aligned}$$

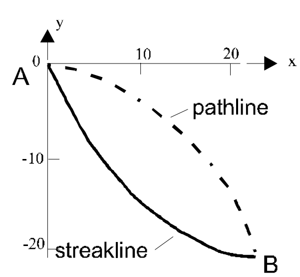
Similarly,

$$\begin{aligned} y &= y_o + \int vdt = 0 + \int -2tdt \\ y &= y_o - t^2 \end{aligned}$$

Letting $x_o = y_o = 0$,we can construct a table of coordinates

t (s)	x (m)	y(m)
0	0	0
1	5	-1
2	10	-4
3	15	-9
4	20	-16
5	25	-25

The (x, y) data from this table are plotted in the figure below



Streakline

To construct the streakline, solve for the displacement of dye particles.

The dye particle released at time $t = 1$ s will reach a position given by

$$\begin{aligned} x &= x_o + \int_1^5 u dt \\ &= 0 + \int_1^5 5 dt = 21 \\ y &= y_o + \int_1^5 v dt \\ &= 0 + \int_1^5 -2t dt = 0 - t^2|_1^5 = -24 \end{aligned}$$

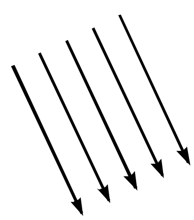
The dye particle released at time $t = 2$ s will reach a position given by

$$\begin{aligned} x &= 0 + \int_2^5 5 dt = 15 \\ y &= 0 + \int_2^5 -2t dt = -21 \end{aligned}$$

Performing similar calculations for each time yields the coordinates of the streakline. These results are plotted in the above figure.

Streamlines (at $t = 5$ s)

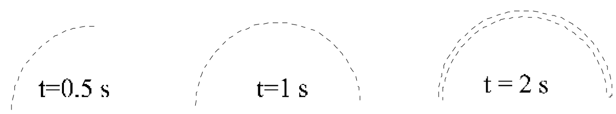
Dye released at $t = 5$ s is at point A in the sketch above. Therefore,a streamline corresponding to $t = 5$ s should be tangent to the streakline at point A. We can reach the same conclusion by using the pathline. The path of a fluid particle at $t = 5$ s is at point B. There, a streamline corresponding to $t = 0$ s should be tangent to the pathline at point B. The streamlines are shown below



4.12 **Pathline**

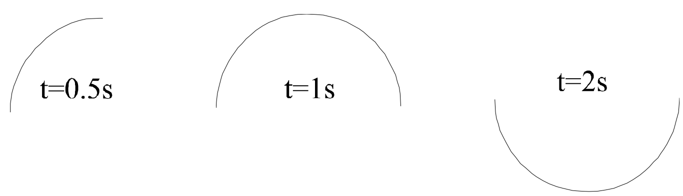
For the first second the particle will follow the circular streamline (clockwise) through an angle of π radians (1/2 circle). Then for the 2nd second the particle reverses its original path and finally ends up at the starting point. Thus, the pathline will be shown:

provided in problem statement

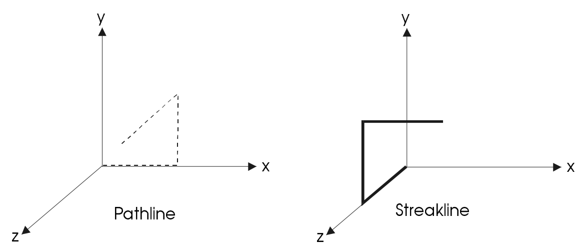


Streakline

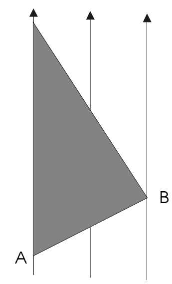
For the first second a stream of dye will be emitted from staring point and the streak from this dye will be generated clockwise along the streamline until the entire top half circle will have a steak of dye at the end of 1 second. When the flow reverses a new dye streak will be generated on the bottom half of the circle and it will be superposed on top of the streak that was generated in the first second. The streakline is shown for t=1/2 sec., 1 sec. & 2 sec.



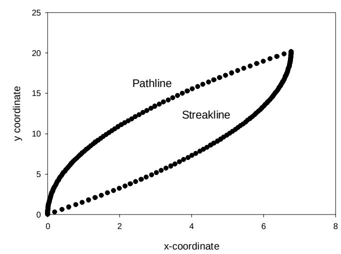
4.13 The final pathline and streakline are shown below.



4.14 The vapor will produce a vapor sheet as shown.



4.15 The computed streaklines and pathline are shown below.



4.16

a. Two dimensional
b. One dimensional
c. One dimensional
d. Two dimensional
e. Three dimensional
f. Three dimensional
g. Two dimensional

4.17 **Information and assumptions**

Water flows in a 25 cm diameter pipe. $Q = 0.03 \text{ m}^3/\text{s}$.
provided in problem statement

Find

Mean velocity: V

Solution

$$V = Q/A = 0.03/(\pi/4 \times 0.25^2) = \underline{\underline{0.611 \text{ m/s}}}$$

4.18 **Information and assumptions**

Water flows in a 16 in pipe. $V = 3$ ft/s
provided in problem statement

Find

Discharge in cfs and gpm

Solution

$$\begin{aligned} Q &= VA = (3 \text{ ft/s})(\pi/4 \times 1.333^2) = 4.17 \text{ ft}^3/\text{s} \\ Q &= (4.17 \text{ ft}^3/\text{s})(449 \text{ gpm/ft}^3/\text{s}) = \underline{\underline{1870 \text{ gpm}}} \end{aligned}$$

4.19 **Information and assumptions**

Water flows in a 2 m diameter pipe. $V = 4$ m/s.
provided in problem statement

Find

Discharge in m^3/s and cfs

Solution

$$Q = VA = (4)(\pi/4 \times 2^2) = \underline{\underline{12.6 \text{ m}^3/\text{s}}}$$
$$Q = (12.6 \text{ m}^3/\text{s})(1/0.02832)(\text{ft}^3/\text{s})/(\text{m}^3/\text{s}) = \underline{\underline{444 \text{ cfs}}}$$

4.20 **Information and assumptions**

An 8 in. pipe carries air, $V = 20$ m/s. Air properties: 20 ^{o}C, 200 kPa-absolute.
provided in problem statement

Find

Mass flow rate: $\dot{m}$

Solution

$$\begin{aligned}
\rho &= p/RT = 200,000/(287 \times 293) = \underline{\underline{2.378 \text{ kg/m}^3}} \\
\dot{m} &= \rho V A = 2.378 \times 20 \times (\pi \times 0.08^2/4) \\
&= \underline{\underline{0.239 \text{ kg/s}}}
\end{aligned}$$

4.21 **Information and assumptions**

A 1 m pipe carries natural gas, $V = 15$ m/s. Properties: 15 ^{o}C, 150 kPa-gage.
provided in problem statement

Find

Mass flow rate: $\dot{m}$

Solution

$$\begin{aligned} \rho &= p/RT = (101 + 150)10^3/((518) \times (273 + 15)) = 1.682 \text{ kg/m}^3 \\ \dot{m} &= \rho V A = 1.682 \times 15 \times \pi \times 0.5^2 = \underline{\underline{19.8 \text{ kg/s}}} \end{aligned}$$

4.22 **Information and assumptions**

A pipe for an aircraft engine test has $\dot{m} = 200$ kg/s and $V = 240$ m/s. $p = 50$ kPa-abs, $T = -18$ ^{o}C. provided in problem statement

Find

Pipe diameter: Q

Solution

$$\begin{aligned} \rho &= p/RT = (50 \times 10^3)/((287)(273 - 18)) = 0.683 \text{ kg/m}^3 \\ \dot{m} &= \rho A V \end{aligned}$$

or

$$\begin{aligned} A &= \dot{m}/(\rho V) = (200)/((0.683)(240)) = 1.22 \text{ m}^2 \\ A &= (\pi/4)D^2 = 1.22 \\ D &= ((4)(1.22)/\pi)^{1/2} = \underline{\underline{1.25 \text{ m}}} \end{aligned}$$

4.23 **Information and assumptions**

Air flows in a rectangular air duct with dimensions 1.0×0.2 m. $Q = 1100$ m^3/hr.
provided in problem statement

Find

Velocity: V

Solution

$$V = Q/A = (1,100/(60 \times 60))/(1 \times 0.20) = \underline{\underline{1.53 \text{ m/sec}}}$$

4.24 **Information and assumptions**

A circular duct has a velocity profile of $v(r)/V_o = 1 - r/R$, where V_o is velocity at $r = 0$.
provided in problem statement

Find

Ratio of mean velocity to centerline velocity: $\bar{V}/V_o$

Solution

Discharge

$$Q = \int v dA$$

where $dA = 2\pi r dr$. Then

$$\begin{aligned} Q &= \int_0^R V_0(1 - (r/R))2\pi r dr \\ &= V_0(2\pi)((r^2/2) - (r^3/(3R))) \mid_0^R \\ &= 2\pi V_0((R^2/2) - (R^2/3)) \\ &= (2/6)\pi V_0 R^2 \end{aligned}$$

Average Velocity

$$\begin{aligned} & \bar{V} = Q/A \\ \therefore \quad & \bar{V}/V_0 = (Q/A)/V_0 = [((2/6)\pi V_0 R^2)/(\pi R^2)]/V_0 = \underline{\underline{1/3}} \end{aligned}$$

4.25 **Information and assumptions**

Water flows in a rectangular channel. The velocity profile is $V(x, y) = V_S(1 - 4x^2/W^2)(1 - y^2/D^2)$, where W and D are the channel width and depth, respectively.

provided in problem statement

Find

An expression for the discharge: $Q = Q(V_S,\, D,\, W)$

Solution

$$
\begin{aligned}
Q &= \int \overrightarrow{V} \cdot \overrightarrow{dA} = \int \int V(x, y)dxdy \\
&= \int_{-w/2}^{w/2} \int_{y=0}^{D} V_S(1 - 4x^2/w^2)(1 - y^2/D^2)dxdy \\
&= \underline{\underline{(4/9)V_S W D}}
\end{aligned}
$$

4.26 **Information and assumptions**

Water flows in a 4 ft pipe. The velocity profile is linear. The centerline velocity is $V_{\max} = 15$ ft/s. The velocity at the wall is $V_{\min} = 12$ ft/s.

provided in problem statement

Find

Discharge in cfs and gpm

Solution

$$Q = \int_A V dA = \int_0^{r_0} V 2\pi r dr$$

where $V = V_{\max} - 3r/r_0$.

$$\begin{aligned} Q &= \int_0^{r_0} (V_{\max} - (3r/r_0)) 2\pi r dr = 2\pi r_0^2((V_{\max}/2) - (3/3)) \\ &= 2\pi \times 4.00((15/2) - (3/3)) = \underline{163.4 \text{ cfs}} \\ &= 163.4 \times 449 = \underline{\underline{73,400 \text{ gpm}}} \end{aligned}$$

4.27 **Information and assumptions**

Water flows in a 2 m pipe. The velocity profile is linear. The centerline velocity is $V_{\max} = 8$ m/s. The velocity at the wall is $V_{\min} = 6$ m/s.

provided in problem statement

Find

Discharge: Q

Mean velocity: V

Solution

$$
\begin{aligned}
Q &= 2\pi r_0^2((V_{\max}/2) - (2/3)) \text{ (see problem 4.26 for derivation)} \\
&= 2\pi \times 1((8/2) - (2/3)) = \underline{\underline{20.9 \text{ m}^3/\text{s}}} \\
V &= Q/A = 20.94/(\pi \times 1) = \underline{\underline{6.67 \text{ m/s}}}
\end{aligned}
$$

4.28 **Information and assumptions**

Air flows in a square duct. Details are provided in the textbook provided in problem statement

Find

a.) Volume flow rate: Q
b.) Mean velocity: V
c.) Mass flow rate: $\dot{m}$

Solution

$$
\begin{aligned}
dQ &= VdA \\
dQ &= (20y)dy \\
Q &= 2\int_0^{0.5} VdA \\
&= 2\int_0^{0.5} 20ydy \\
&= 40y^2/2|_0^{0.5} = 20 \times 0.25 = \underline{\underline{5\ \mathrm{m^3/s}}} \\
V &= Q/A = (5\ \mathrm{m^3/s})/(1\ \mathrm{m^2}) = \underline{\underline{5\ \mathrm{m/s}}} \\
\dot{m} &= \rho Q = (1.2\ \mathrm{kg/m^3})(5\ \mathrm{m^3/s}) = \underline{\underline{6.0\ \mathrm{kg/s}}}
\end{aligned}
$$

4.29 **Information and assumptions**

Open channel flow down a 30^o incline. $V = 18$ ft/s. Vertical depth is 4 ft. Width is 25 ft. provided in problem statement

Find

Discharge: Q

Solution

$$Q = V \times A = 18 \times 4 \cos 30^\circ \times 25 = \underline{\underline{1,560 \text{ cfs}}}$$

4.30 **Information and assumptions**

Open channel flow down a 30^o incline. Velocity profile is $u = y^{1/3}$ m/s. Vertical depth is 1 m. Width is 2 m.

provided in problem statement

Find

Discharge: Q

Solution

$$\begin{aligned} Q &= \int_0^{0.866} y^{1/3}(2 \text{ dy}) \\ &= 2\int_0^{0.866} y^{1/3}\text{dy} \\ &= (2/(4/3))y^{4/3}|_0^{0.866 \text{ m}} \\ Q &= \underline{\underline{1.24 \text{ m}^3/\text{s}}} \end{aligned}$$

4.31 **Information and assumptions**

Open channel flow down a 30^o incline. Velocity profile is $u = 10\,(e^y - 1)$ m/s. Vertical depth is 1 m. Width is 2 m.

provided in problem statement

Find

Discharge: Q

Mean velocity: $\bar{V}$

Solution

$$
\begin{aligned}
Q &= \int_0^{0.866} V dy \\
Q &= \int_0^{0.866} (10)(e^y - 1)2 \text{ dy} \\
&= [(2)(10)(e^y - y)]_0^{0.866} \\
&= \underline{\underline{10.23 \text{ m}^3/\text{s}}} \\
\bar{V} &= Q/A = (10.23 \text{ m}^3/\text{s})/(2 \times 0.866 \text{ m}^2) = \underline{\underline{5.91 \text{ m/s}}}
\end{aligned}
$$

4.32 **Information and assumptions**

Water (20^o C, $\gamma = 9790$ N/m^3) enters a weigh tank for 15 min. The weight change is 20 kN.
provided in problem statement

Find

Discharge: Q

Solution

$$
\begin{aligned}
Q &= V/\Delta t \\
&= W/(\gamma \Delta t) \\
&= 20,000/(9790 \times 15 \times 60) \\
&= \underline{\underline{2.27 \times 10^{-3} \text{ m}^3/\text{s}}}
\end{aligned}
$$

4.33 **Information and assumptions**

Water enters a lock for a ship canal through 180 ports. Port area is 2×2 ft. Lock dimensions (plan view) are 105×900 ft. The water in the lock rises at 6 ft/min.

provided in problem statement

Find

Mean velocity in each port: V_{port}

Solution

Using the continuity principle

$$\begin{aligned} \sum V_p A_p &= V_{\text{rise}} \times A_{\text{rise}} \\ 180 \times V_p \times (2 \times 2) &= (6/60) \times (105 \times 900) \\ V_{\text{port}} &= \underline{\underline{13.1 \text{ ft/s}}} \end{aligned}$$

4.34 **Information and assumptions**

Water flows through a rectangular and horizontal open channel. The velocity profile is $u = u_{\max}(y/d)^n$, where y is depth, $u_{\max} = 3$ m/s, $d = 1.2$ m, and $n = 1/6$.

provided in problem statement

Find

Discharge: q(m^3/s per meter of channel width)

Mean velocity: Q

Solution

$$
\begin{aligned}
q &= \int_0^d u_{\max}(y/d)^n dy = u_{\max} d/(n+1) \\
&= 3 \times 1.2/((1/6)+1) = \underline{\underline{3.09 \text{ m}^2/\text{s}}} \\
V &= q/d = 3.09/1.2 = \underline{\underline{2.57 \text{ m/s}}}
\end{aligned}
$$

4.35 **Information and assumptions**

A flow with a linear velocity profile occurs in a triangular-shaped open channel. Details are provided in the text book.

provided in problem statement

Find

Discharge: Q

Solution

$$\begin{aligned} Q &= \int V dA \text{ where } V = 5y \text{ ft/s},\ dA = \text{xdy} = 0.5 \text{ ydy ft}^2 \\ q &= \int_0^1 (5y) \times (0.5 \text{ ydy}) \\ &= (2.5 \text{ y}^3/3)|_0^1 = \underline{\underline{0.833 \text{ cfs}}} \end{aligned}$$

4.36 **Information and assumptions**

Flow in a circular pipe. The velocity profile is $V = V_c(1-(r/r_o))^n$.
provided in problem statement

Find

An expression for mean velocity of the form $V = V(V_c, n)$

Solution

$$\begin{aligned} Q &= \int V dA \\ &= \int V_c(1-(r/r_0^2))^n 2\pi r dr \\ &= -\pi r_0^2 V_c \int_0^{r_o} (1-(r/r_0)^2)^n(-2r/r_p^2)dr \end{aligned}$$

This integral is in the form of

$$\int u^n du = u^{n+1}/(n+1)$$

so the result is

$$\begin{aligned} Q &= -\pi r_0^2 V_c (1-(r/r_0)^2)^{n+1}/(n+1)|_0^{r_0} \\ &= (1/(n+1))V_c \pi r_0^2 \\ V &= Q/A = \underline{\underline{(1/(n+1))V_c}} \end{aligned}$$

4.37 **Information and assumptions**

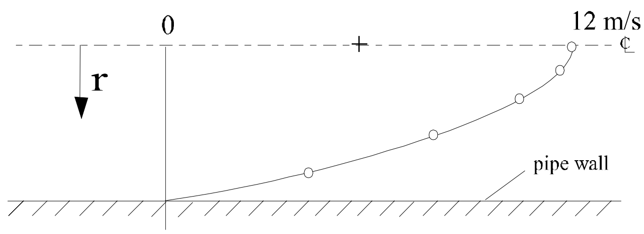
Flow in a pipe with a velocity profile $V = 12(1 - r^2/r_o^2)$ provided in problem statement

Find

Plot the velocity profile
Mean velocity: V
Discharge: Q

Solution

r/r_0	$1-(r/r_0)^2$	V(m/s)
0.0	1.00	12.0
0.2	0.96	11.5
0.4	0.84	10.1
0.6	0.64	7.68
0.8	0.36	4.32
1.0	0.00	0.0



From solution to Prob. 4.36

$$\begin{aligned}
V &= (1/(n+1))V_c \\
V &= (1/2)V_c = \underline{\underline{6 \text{ m/s}}}; \\
Q &= VA \\
&= 6 \times \pi/4 \times 1^2 = \underline{\underline{4.71 \text{ m}^3/\text{s}}}
\end{aligned}$$

4.38 **Information and assumptions**

Water (60 ^{o}F) flows in a 1.5 in. diameter pipe. $\dot{m} = 75$ lbm/min.
provided in problem statement

Find

Mean velocity: V

Solution

$$\begin{aligned} V &= \dot{m}/\rho A \\ V &= (75/60)/\left[(62.37) \times (\pi/4 \times (1.5/12)^2)\right] = \underline{\underline{1.63 \text{ ft/s}}} \end{aligned}$$

4.39 **Information and assumptions**

Water (10 ^{o}C) flows in a 20 cm diameter pipe. $\dot{m} = 1000$ kg/min.
provided in problem statement

Find

Mean velocity: V

Solution

$$
\begin{aligned}
V &= \dot{m}/\rho A \\
&= (1,000/60)/\left[(998) \times (\pi/4 \times 0.20^2)\right] \\
&= \underline{\underline{0.532 \text{ m/s}}}
\end{aligned}
$$

4.40 **Information and assumptions**

Water (60 ^{o}F) enters a weigh tank for 10 min. The weight change is 4765 lbf.
provided in problem statement

Find

Discharge: Q in units of cfs and gpm

Solution

$$\begin{aligned} Q &= V/\Delta t \\ &= \Delta W/(\gamma \Delta t) \\ &= 4765/(62.37 \times 10 \times 60) \\ &= \underline{\underline{0.127 \text{ cfs}}} \\ &= 0.127 \times 449 = \underline{\underline{57.2 \text{ gpm}}} \end{aligned}$$

4.41 **Information and assumptions**

Water (60 ^{o}F) flows in a 4 in. diameter pipe. $V = 8$ ft/s.
provided in problem statement

Find

Discharge: Q in units of cfs and gpm
Mass flow rate: $\dot{m}$

Solution

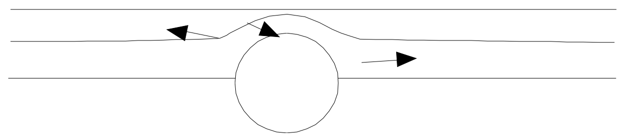
Discharge

$$\begin{aligned} Q &= VA = 8(\pi/4 \times (4/12)^2) = \underline{\underline{0.698 \text{ cfs}}} \\ &= 0.698 \times 449 = \underline{\underline{313 \text{ gpm}}} \end{aligned}$$

Mass flow rate

$$\begin{aligned} \dot{m} &= \rho Q \\ &= 1.94 \times 0.698 = \underline{\underline{1.35 \text{ slugs/s}}} \end{aligned}$$

4.42 (a) Steady. (b) Two-dimensional. (c) No. (d) Yes – see representative vectors below:



(e) $V_A n_A = V_C n_C$ where $n =$ spacing between streamlines.
Then $V_C = V_A(n_A/n_C) = 15(2/1) = \underline{\underline{30 \text{ ft/s}}}$

4.43 **Information and assumptions**

A flow with this velocity field: $u = xt + 2y,\ v = xt^2 - yt,\ w = 0$
Acceleration, **a** at location (1,2) and time3 seconds. Discharge: Q in units of cfs and gpm provided in problem statement

Solution

$$\begin{aligned} a_x &= u\partial u/\partial x + v\partial u/\partial y + w\partial u/\partial z + \partial u/\partial t \\ &= (xt + 2y)(t) + (xt^2 - yt)(2) + 0 + x \end{aligned}$$

Now, let $x = 1$ m, $y = 2$ m and $t = 3$ s

$$\begin{aligned} a_x &= (3 + 4)(3) + (9 - 6)(2) + 1 = 21 + 6 + 1 = 28 \text{ m/s}^2 \\ a_y &= u\partial v/\partial x + v\partial v/\partial y + w\partial v/\partial z + \partial v/\partial t \\ &= (xt + 2y)(t^2) + (xt^2 - yt)(-t) + 0 + (2xt - y) \end{aligned}$$

For $x = 1$ m, $y = 2$ m and $t = 3$ s

$$\begin{aligned} a_y &= (3 + 4)(9) + (9 - 6)(-3) + (6 - 2) = 63 - 9 + 4 = 58 \text{ m/s}^2 \\ \mathbf{a} &= \underline{\underline{28\ \mathbf{i} + 58\,\mathbf{j}\ \text{m/s}^2}} \end{aligned}$$

4.44 **Information and assumptions**

Air is flowing around a sphere. The x-component of velocity along the dividing streamline (from side of sphere) is given by $u = -U_o(1 - r_o^3/x^3)$
provided in problem statement

Find

An expression for the x-component of acceleration of the form $a_x = a_x(x, r_o, U_{o)}$

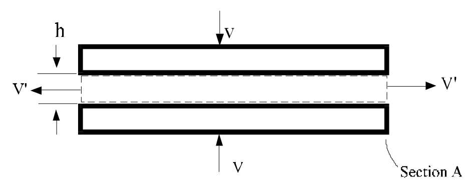
Solution

$$\begin{aligned}
a_x &= u\partial u/\partial x + \partial u/\partial t \\
&= -U_0(1 - r_0^3/x^3)\partial/\partial x(-U_0(1 - r_0^3/x^3)) + \partial/\partial t(-U_0(1 - r_0^3/x^3)) \\
&= U_0^2(1 - r_0^3/x^3)(-3r_0^3/x^3) + 0 \\
&= \underline{\underline{-(3\,U_0^2\,r_0^3\,/\,x^4)(1 - r_0^3\,/\,x^3)}}
\end{aligned}$$

4.45 **Information and assumptions**

As shown in the sketch below, two round plates, each with speed V, move together. At the instant shown, the plate spacing in h. Air flows across section A with a speed V'. Assume V' is constant across section A. Assume the air has constant density.

provided in problem statement



Find

An expression for the radial component of convective acceleration at section A

Solution

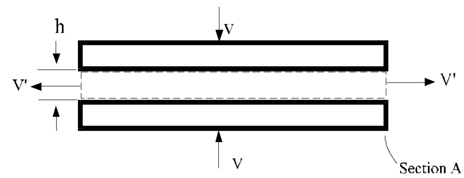
Continuity equation applied to control volume shown above

$$\begin{aligned}\sum \rho \mathbf{V}\cdot\mathbf{A} &= -d/dt\int_{\text{c.v.}}\rho d\cancel{V} \\ -2\rho VA+\rho V'A' &= 0 \\ 2VA &= V'A' \\ V' &= 2VA/A' = 2V(\pi D^2/4)(\pi Dh) = VD/2h = Vr/h\end{aligned}$$

Convective acceleration

$$\begin{aligned}a_c &= V'\partial/\partial r(V') \\ &= Vr/h\,\partial/\partial r(Vr/h) = V^2r/h^2 = \underline{\underline{V^2D/2h^2}}\end{aligned}$$

4.46 As shown in the sketch below, two round plates, each with speed V, move together. At the instant shown, the plate spacing in h. Air flows across section A with a speed V'. Assume V' is constant across section A. Assume the air has constant density.



Find

An expression for the radial component of local acceleration at section A

Solution

Continuity equation applied to control volume shown above

$$\begin{aligned}
\sum \rho \mathbf{V} \cdot \mathbf{A} &= -d/dt \int_{\mathrm{c.v.}} \rho d\cancel{V} \\
-2\rho V A + \rho V' A' &= 0 \\
2VA &= V'A' \\
V' &= 2VA/A' = 2V(\pi D^2/4)(\pi D h) = VD/2h = VR/h
\end{aligned}$$

Introducing time as a parameter

$$h = h_0 - 2Vt$$

so

$$V' = RV/(h_0 - 2Vt)$$

Local acceleration

$$\begin{aligned}
\partial V'/\partial t &= \partial/\partial t[RV(h_0 - 2Vt)^{-1}] = RV(-1)(h_0 - 2Vt)^{-2}(-2V) \\
\partial V'/\partial t &= 2RV^2/(h_0 - 2Vt)^2
\end{aligned}$$

but $h_0 - 2Vt = h$ so

$$\partial V'/\partial t = 2RV^2/h^2 = \underline{\underline{DV^2/h^2}}$$

4.47 **Information and assumptions**

Pipe flows A and B merge into a single pipe with area $A_{\text{exit}} = 0.1\ \text{m}^2$. $Q_A = 0.02t\ \text{m}^3/\text{s}$ and $Q_B = 0.008t^2\ \text{m}^3/\text{s}$. Assume incompressible flow.

provided in problem statement

Find

Velocity at the exit: V_{exit}

Acceleration at the exit: a_{exit}

Solution

Continuity principle

$$\begin{aligned}
Q_{\text{exit}} &= Q_A + Q_B \\
V_{\text{exit}} &= (1/A_{\text{exit}})(Q_A + Q_B) \\
&= (1/0.01\ \text{m}^2)(.02t\ \text{m}^3/\text{s}\ + 0.008t^2\ \text{m}^3/\text{s}) \\
&= 2t\ \text{m/s}\ + 0.8t^2\ \text{m/s}
\end{aligned}$$

Then at $t = 1$ sec,

$$V_{\text{exit}} = \underline{\underline{2.8\ \text{m/s}}}$$

Acceleration

$$a_{\text{exit}} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}$$

Since V varies with time, but not with position, this becomes

$$a_{\text{exit}} = \frac{\partial V}{dt} = 2 + 1.6t\ \text{m/s}$$

Then at $t = 1$ sec

$$\underline{\underline{a_{\text{exit}} = 3.6\ \text{m/s}^2}}$$

4.48 **Information and assumptions**

A velocity field at $r = 10$ m: $V_\theta = 10t$
provided in problem statement

Find

Magnitude of acceleration at $r = 10$ m and $t = 1$ s

Solution

$$\begin{aligned}
V_\theta &= 10t \\
a_{\text{tang.}} &= V_\theta \partial V_\theta / \partial s + \partial V_\theta / \partial t \\
a_{\text{tang.}} &= 0 + 10 \text{ m/s}^2 \\
a_{\text{normal}} &= V_\theta^2 / r \\
&= (10t)^2 / r = 100t^2/10 = 10t^2 \\
\text{at } t &= 1s \; a_{\text{normal}} = 10 \text{ m/s}^2 \\
a_{\text{total}} &= \sqrt{a_{\text{tang.}}^2 + a_{\text{normal}}^2} = \sqrt{200} = \underline{\underline{14.14 \text{ m/s}^2}}
\end{aligned}$$

4.49 **Information and assumptions**

Flow occurs in a tapered passage. The discharge and velocity gradient are given (see solution below). The point of interest is section AA, where the diameter is 50 cm. The time of interest is 0.5 s.

provided in problem statement

Find

Velocity at section AA: V

Local acceleration at section AA: a_ℓ

Convective acceleration at section AA: a_c

Solution

$$\begin{aligned} Q &= Q_0 - Q_1 t/t_0 = 0.985 - 0.5t \quad \text{(given)} \\ \frac{\partial V}{\partial s} &= +2\ \frac{\text{m}}{\text{s}} \text{ per m} \quad \text{(given)} \\ V &= Q/A = (0.985 - 0.5 \times 0.5)/(\pi/4 \times 0.5^2) = \underline{\underline{3.743 \text{ m/s}}} \end{aligned}$$

Acceleration

$$\begin{aligned} a_\ell &= \partial V/\partial t = \partial/\partial t(Q/A) = \partial/\partial t((0.985 - 0.5t)/(\pi/4 \times 0.5^2)) \\ &= -0.5/(\pi/4 \times 0.5^2) = \underline{\underline{-2.55 \text{ m/s}^2}} \\ a_c &= V\partial V/\partial s = 3.743 \times 2 = \underline{\underline{+7.49 \text{ m/s}^2}} \end{aligned}$$

4.50 **Information and assumptions**

One-dimensional flow occurs in a nozzle. Details are provided in the textbook.
provided in problem statement

Find

Convective acceleration: a_c
Local acceleration: a_ℓ

Solution

$$a_c = V dV/ds$$

where $dV/ds = (V_{\text{tip}} - V_{\text{base}})/L$

$$\begin{aligned}
V_{\text{tip}} &= Q/A_{\text{tip}} = 0.40/((\pi/4)(2/12)^2) = 18.34 \text{ ft/s} \\
V_{\text{base}} &= Q/A_{\text{base}} = 4.59 \text{ ft/s} \\
dV/ds &= (18.34 - 4.59)/1.5 = 9.17 \text{ s}^{-1} \\
V_{\text{midway}} &= 4.59 \text{ ft/s} + 9.17\ s^{-1} \times (9/12) \text{ ft} = 11.47 \text{ ft/s}
\end{aligned}$$

Then

$$\begin{aligned}
a_c &= V dV/ds = 11.47 \text{ ft/s} \times 9.17\ s^{-1} = \underline{\underline{105 \text{ ft/s}^2}} \\
a_\ell &= \underline{\underline{0}}
\end{aligned}$$

4.51 **Information and assumptions**

One-dimensional flow occurs in a nozzle. Details are provided in the textbook.
provided in problem statement

Find

Local acceleration: a_ℓ

Solution

$$\begin{aligned} a_\ell &= \partial V/\partial t \\ V &= Q/A \text{ and } Q = 2t \end{aligned}$$

Then

$$\begin{aligned} a_\ell &= \partial/\partial t(2t/A) \\ &= (2/A)\partial/\partial t(t) = 2/A \text{ ft/s}^2 \end{aligned}$$

From solution to Prob. 4.50: $V_{\text{mid}} = 11.47$ ft/s. Therefore

$$A_{\text{mid}} = Q/V = (0.40 \text{ ft}^3/\text{s})/11.47 \text{ ft/s} = 0.0349 \text{ ft}^2$$

Finally

$$a_\ell = 2/A = 2/0.0349 = \underline{\underline{57.3 \text{ ft/s}^2}}$$

4.52 **Information and assumptions**

Flow in a two-dimensional slot, details are provided in the textbook.
provided in problem statement

Find

An expression for local acceleration: a_{local}

Solution

$$\begin{aligned}
V &= q/b = (q_0/t_0)2t/b \text{ but } b = B - (1/2)B(x/4B) \\
V &= (q_0/t_0)(2t)/(B - (1/2)B(x/4B)) \\
a_{\text{local}} &= \partial V/\partial t = (q_0/t_0)(2)/(B - (1/2)B(x/4B))
\end{aligned}$$

Then when $x = 2B$

$$\begin{aligned}
a_{\text{local}} &= 2(q_0/t_0)/(B - (1/2)B(1/2)) = 2(q_0/t_0)/(3/4\ B) \\
a_{\text{local}} &= \underline{\underline{(8/3)(q_0/t_0)/B}}
\end{aligned}$$

4.53 **Information and assumptions**

Flow in a two-dimensional slot, details are provided in the textbook.
provided in problem statement

Find

An expression for convective acceleration: a_{conv}

Solution

$$\begin{aligned} V &= (q_0/t_0)2t/(B-(1/8)x) \\ a_{\text{conv}} &= V\partial V/\partial x = V(q_0/t_0)2t(-1)(-1/8)/(B-(1/8)x)^2 \\ a_{\text{conv}} &= (1/8)(q_0/t_0)^2 4t^2/(B-(1/8)x)^3 \end{aligned}$$

When $x = 2B$

$$a_{\text{conv}} = (1/2)(q_0/t_0)^2 t^2/((3/4)B)^3 = \underline{\underline{32/27(q_0/t_0)^2 t^2/B^3}}$$

4.54 **Information and assumptions**

Flow in a nozzle, details are provided in the textbook.
provided in problem statement

Find

Convective acceleration: a_c
Local acceleration: a_ℓ

Solution

$$\begin{aligned} a_\ell &= \partial V/\partial t = \partial/\partial t[2t/(1-0.5x/L)^2] = 2/(1-0.5x/L)^2 \\ &= 2/(1-0.5\times 0.5L/L)^2 = \underline{\underline{3.56\ \text{ft/s}^2}} \\ a_c &= V(\partial V/\partial x) = [2t/(1-0.5x/L)^2]\partial/\partial x[2t/(1-0.5x/L)^2] \\ &= 4t^2/((1-0.5x/L)^5 l) = 4(3)^2/((1=0.5\times 0.5L/L)^5 4) \\ &= \underline{\underline{37.9\ \text{ft/s}^2}} \end{aligned}$$

4.55 **Information and assumptions**

Air flow downward through a pipe and then outward between to parallel disks. Details are provided in the textbook.

provided in problem statement

Find

Expression for acceleration at point A

Value of acceleration at point A

Velocity in the pipe

Solution

$$\begin{aligned} V_r &= Q/A = Q/(2\pi rh) \\ a_c &= V_r \partial V_r/\partial r \\ &= (Q/(2\pi rh))(-1)(Q)/(2\pi r^2 h) = \underline{\underline{-Q^2/(r(2\pi rh)^2)}} \end{aligned}$$

When $D = 0.1$ m, $p = 0.20$ m, $h = 0.01$ m, and $Q = 0.380$ m^3/s

$$V_{\text{pipe}} = Q/A_{\text{pipe}} = 0.380/((\pi/4) \times 0.1^2) = \underline{\underline{48.4 \text{ m/s}}}$$

Then

$$a_c = -(0.38)^2/((0.2)(2\pi \times 0.2 \times 0.01)^2) = \underline{\underline{-4,570 \text{ m/s}^2}}$$

4.56 **Information and assumptions**

Air flow downward through a pipe and then outward between to parallel disks. Details are provided in the textbook.

provided in problem statement

Find

At $t = 2$ s, acceleration at point A: a_{2s}

At $t = 3$ s, acceleration at point A: a_{3s}

Solution

$$
\begin{aligned}
a_\ell &= \partial V/\partial t = \partial/\partial t(Q/(2\pi rh)) \\
a_\ell &= \partial/\partial t(Q_0(t/t_0)/(2\pi rh)) \\
a_\ell &= (Q_0/t_0)/2\pi rh \\
a_{\ell;2,3} &= (0.1/1)/(2\pi \times 0.20 \times 0.01) = 7.957 \text{ m/s}^2
\end{aligned}
$$

From solution to Problem 4.55

$$a_c = -Q^2/(r(2\pi rh)^2)$$

At $t = 2s, Q = 0.2 \text{ m}^3/\text{s}$

$$
\begin{aligned}
a_{c,2s} &= -1,366 \text{ m/s}^2 \\
a_{2s} &= a_\ell + a_c = 7.957 - 1,266 = \underline{\underline{1,260 \text{ m/s}^2}} \\
a_{c,3s} &= -2,850 \text{ m/s}^2 \\
a_{3s} &= -2,850 + 7.957 = \underline{\underline{-2,840 \text{ m/s}^2}}
\end{aligned}
$$

4.57 **Information and assumptions**

Water flows into a tank through a pipe on the side and then out the bottom of the tank. Details are provided in the textbook.

provided in problem statement

Find

Velocity in the inlet: V_{in}

Solution

Let the control surface surround the liquid in the tank and let it follow the liquid surface at the top.

Continuity Equation (Eq. 4.28):

$$\sum_{\text{cs}} \rho \mathbf{V} \cdot \mathbf{A} = -\frac{d}{dt} \int_{\text{cv}} \rho d\forall$$

$$-\rho V_{\text{in}} A_{\text{in}} + \rho V_{\text{out}} A_{\text{out}} = -\frac{d}{dt}(\rho A_{\text{tank}} h)$$

$$-V_{\text{in}} A_{\text{in}} + V_{\text{out}} A_{\text{out}} = -A_{\text{tank}}(dh/dt)$$

$$-V_{\text{in}}(.0025) + \sqrt{2\text{g}(1)}(.0025) = -0.1(0.1) \times 10^2$$

$$V_{\text{in}} = \sqrt{19.62} + (10^{-4}/(.0025))$$

$$V_{\text{in}} = \underline{\underline{4.47 \text{ m/s}}}$$

4.58 **Information and assumptions**

A bicycle tire ($\forall = 0.04$ ft^3) is inflated with air at an inlet flow rate of $Q_{\text{in}} = 1$ cfm and a density of 0.075 lbm/ft^3. The density of the air in the inflated tire is 0.04 lbm/ft^3.

provided in problem statement

Find

Time needed to inflate the tire: t

Solution

Select a control volume surrounding the air within tire.

Continuity Equation

$$(\rho Q)_{\text{in}} = \frac{d}{dt} M_{\text{cv}}$$

Since the mass of air in the inflated tire (M_{CV}) is constant, this equation may be integrated to give

$$(\rho Q)_{\text{in}}\, t = M_{\text{CV}}$$

or

$$\begin{aligned} t &= \frac{M_{\text{CV}}}{(\rho Q)_{\text{in}}} \\ &= \frac{0.04 \times 0.4}{0.075 \times (1/60)} \\ &= \underline{\underline{12.8 \text{ s}}} \end{aligned}$$

4.59

Case (a)

1) $b = \underline{\underline{1}}$

2) $dB_{\text{syst}}/dt = \underline{\underline{0}}$

3) $\sum b\rho \mathbf{V}\cdot\mathbf{A} = \sum \rho\mathbf{V}\cdot\mathbf{A}$

$= -2\times 10\times 1.5$

$= \underline{\underline{-30 \text{ slugs/s}}}$

4) $d/dt \int_{\text{cv}} b\rho d\forall = \underline{\underline{+30 \text{ slugs/s}}}$

Case (b)

1) $B = \underline{\underline{1}}$

2) $dB_{\text{syst}}/dt = \underline{\underline{0}}$

3) $\sum b\rho \mathbf{V}\cdot\mathbf{A} = \sum \rho\mathbf{V}\cdot\mathbf{A}$

$= 2\times 1\times 2$

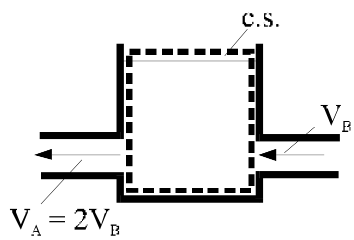
$= 1\times 2\times 2 = \underline{\underline{0}}$

4) $d/dt \int_{\text{cv}} b\rho d\forall = \underline{\underline{0}}$

4.60 Valid statement: (b)

4.61 **Information and assumptions**

The level in the tank (see below) is influenced by the motion of pistons A and B. Each piston moves to the left. $V_A = 2V_B$
provided in problem statement



Find

Determine whether the water level is rising, falling or staying the same

Solution

Select a control volume as shown above. Assume it is coincident with and moves with the water surface.
Continuity principle

$$\begin{aligned}\sum \rho \mathbf{V}\cdot\mathbf{A} &= -d/dt\int_{\text{cv}} \rho d\cancel{V} \\ \rho 2V_B A_A - \rho V_B A_B &= \rho d/dt \int_{\text{cv}} d\cancel{V}\end{aligned}$$

where $A_A = (\pi/4)3^2$; $A_B = (\pi/4)6^2$ and $A_A = (1/4)A_B$. Then

$$\begin{aligned}2V_B(1/4)A_B - V_B A_B &= -d/dt\int_{CV} dV \\ V_B A_B((1/2)-1) &= -d/dt\int_{CV} d\cancel{V} \\ d/dt\int_{CV} d\cancel{V} &= (1/2)V_B A_B \\ d/dt(Ah) &= (/12)V_B A_B \\ Adh/dt &= (1/2)V_B A_B\end{aligned}$$

Because $(1/2)V_B A_B$ is positive dh/dt is positive; therefore, one concludes that the water surface is rising.

4.62 a) True b) True c) True d) True e) True

4.63

1) $b = \underline{\underline{1.0}}$

2) $dB_{\text{syst}}/dt = \underline{\underline{0}}$

3) $\sum b\rho \mathbf{V} \cdot \mathbf{A} = \sum \rho \mathbf{V} \cdot \mathbf{A}$

$$\sum \rho \mathbf{V} \cdot \mathbf{A} = (1.5 \text{ kg/m}^3)(-10 \text{ m/s})(\pi/4) \times (0.04)^2 \text{ m}^2$$
$$+ (1.5 \text{ kg/m}^3)(-6 \text{ m/s})(\pi/4) \times (0.04)^2 \text{ m}^2$$
$$+ (1.2 \text{ kg/m}^3)(6 \text{ m/s})(\pi/4) \times (0.06)^2 \text{ m}^2$$
$$= \underline{\underline{-0.00980 \text{ kg/s}}}$$

4) Because $\sum b\rho \mathbf{V} \cdot \mathbf{A} + d/dt \int b\rho d\cancel{V} = 0$

Then $d/dt \int b\rho d\cancel{V} = -\sum b\rho \mathbf{V} \cdot \mathbf{A}$

or $d/dt \int b\rho d\cancel{V} = \underline{\underline{+0.00980 \text{ kg/s}}}$ (mass is increasing in tank)

4.64 **Information and assumptions**

A plunger moves downward in a concial vessel filled with oil. At a certain instant in time the upward velocity of the oil equals the downward velocity of the plunger.

provided in problem statement

Find

Distance from the bottom of the vessel: y

Solution

Select a control volume as shown above. Rate at which liquid is displaced $= V_{\text{up}}(D^2 - d^2)(\pi/4)$

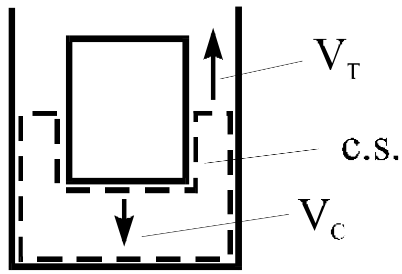
$$\begin{aligned} V_{\text{down}} \times \pi d^2/4 &= V_{\text{up}}(D^2 - d^2)\pi/4 \\ 2d^2 &= D^2 \\ D &= \sqrt{2}d \\ \text{but } y/D &= 24d/2d; \quad D = y/12 \end{aligned}$$

But $y/D = 24d/2d$ so $D = y/12$. Eliminating D in the first equation gives

$$\underline{\underline{y = 12\sqrt{2}d}}$$

4.65 **Information and assumptions**

A 6 in. diameter cylinder falls at a speed $V_C = 3$ ft/s. The container diameter is 8 in.
provided in problem statement



Find

Mean velocity of the liquid in the space between the cylinder and the wall: V_T

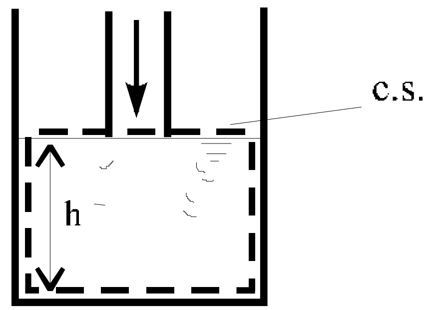
Solution

Apply continuity equation and let the c.s. be fixed except at bottom of cylinder where the c.s. follows the cylinder as it moves down.

$$\begin{aligned}
0 &= d/dt \int \rho d\cancel{V} + \sum \rho \mathbf{V} \cdot \mathbf{A} \\
0 &= d/dt(\cancel{V}) + V_T A_A \\
0 &= V_C A_C + V_T(\pi/4)(8^2 - 6^2) \\
0 &= -3 \times (\pi/4)6^2 + V_T(\pi/4)(8^2 - 6^2) \\
V_T &= 108/(64 - 36) = \underline{\underline{3.86 \text{ ft/s}}} \text{ (upward)}
\end{aligned}$$

4.66 **Information and assumptions**

A round tank ($D = 4$ ft) is being filled with water from a 1 ft diameter pipe. In the pipe, $V = 10$ ft/s. . provided in problem statement



Find

Rate at which the water surface is rising: V_R

Solution

Apply the continuity equation and let the c.s. move up with the water surface in the tank.

$$\begin{aligned} 0 &= d/dt \int_{CV} \rho d\Psi + \sum \rho \mathbf{V} \cdot \mathbf{A} \\ 0 &= d/dt(hA_T) - ((10 + V_R)A_p) \end{aligned}$$

where $A_T =$ tank area, $V_R =$rise velocity and $A_p =$pipe area.

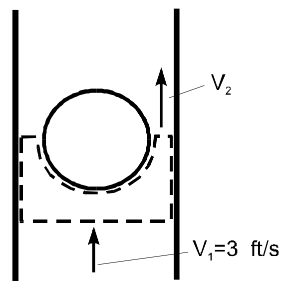
$$0 = A_T dh/dt - 10A_p - V_R A_p$$

but $dh/dt = V_R$ so

$$\begin{aligned} 0 &= A_T V_R - 10A_p - V_R A_p \\ V_R &= (10A_p)/(A_T - A_p) = 10(\pi/4)(1^2)/((\pi/4)4^2 - (\pi/4)1^2) \\ V_R &= \underline{\underline{(2/3) \text{ ft/s}}} \end{aligned}$$

4.67 **Information and assumptions**

An 8 in. sphere is falling at 3 m/s in a 1 ft diameter cylinder filled with water
provided in problem statement



Find

Velocity of water at the midsection of the sphere

Solution

As shown in the above sketch, select a control volume that is attached to the falling sphere. Relative to the sphere, the velocity entering the control volume is V_1 and the velocity exiting is V_2

Continuity equation

$$\begin{aligned}
-\partial/\partial t \int_{CV} \rho dV &= 0 = \sum \rho \mathbf{V} \cdot \mathbf{A} \\
A_1 V_1 &= A_2 V_2 \\
(\pi \times 1.0^2/4) \times 3 &= V_2 \pi (1.0^2 - .67^2)/4;\ V_2 = 5.44 \text{ fps}
\end{aligned}$$

Relative velocity equation: Find the velocity of the water relative to a stationary observer

$$\begin{aligned}
V &= V_2 - V_{sphere} \\
V &= 5.44 - 3.0 \\
&= \underline{\underline{2.44 \text{ ft/s}}}
\end{aligned}$$

4.68 **Information and assumptions**

Air flows in a rectangular duct. $Q = 1.44 \text{ m}^3/\text{s}$.
provided in problem statement

Find

Air speed for a duct of dimensions 20×50 cm: V_1
Air speed for a duct of dimensions 10×40 cm: V_1

Solution

$$\begin{aligned} V_1 &= Q/A_1 \\ &= 1.44/(0.2 \times 0.5) = \underline{\underline{14.4 \text{ m/s}}} \\ V_2 &= 1.44/(0.1 \times 0.4) = \underline{\underline{36.0 \text{ m/s}}} \end{aligned}$$

4.69 **Information and assumptions**

Flow ($Q = 0.3$ m^3/s) entering a pipe wye with inlet diameter 30 cm. Outlet diameters are 20 and 15 cm. Each outlet branch has the same mean velocity.

provided in problem statement

Find

Discharge in each outlet branch: $Q_{20\text{ cm}}$, $Q_{15\text{ cm}}$

Solution

$$\begin{aligned} V &= 0.3/(\pi/4)(0.2^2 + 0.15^2) = 6.11 \text{ m/s} \\ Q_{20\text{ cm}} &= VA_{20} = 6.11 \times (\pi \times 0.1 \times 0.1) = \underline{\underline{0.192\ \ \text{m}^3/\text{s}}} \\ Q_{15\text{ cm}} &= VA_{15} = 6.11 \times (\pi \times 0.075 \times 0.075) = \underline{\underline{0.108\ \text{m}^3/\text{s}}} \end{aligned}$$

4.70 **Information and assumptions**

Flow ($Q = 0.3\ \text{m}^3/\text{s}$) entering a pipe wye with inlet diameter 30 cm. Outlet diameters are 20 and 15 cm. In the larger outlet (20 cm) the flow rate is twice that in the smaller outlet (15 cm).

provided in problem statement

Find

Mean velocity in each outlet branch: V_{15}, V_{20}

Solution

$$Q_{\text{tot.}} = 0.30\ \text{m}^3/\text{s} = Q_{20} + Q_{15}$$

But $Q_{20} = 2Q_{15}$

$$\begin{aligned}
0.30 &= 2Q_{15} + Q_{15} \\
Q_{15} &= 0.10\ \text{m}^3/\text{s}; V_{15} = Q_{15}A_{15} = \underline{\underline{5.66\ \text{m/s}}} \\
Q_{20} &= 0.20\ \text{m}^3/\text{s}; V_{20} = 0.20/A_{20} = \underline{\underline{6.37\ \text{m/s}}}
\end{aligned}$$

4.71 **Information and assumptions**

Water flows through an 8 in. diameter pipe in series with a 6 in pipe. $Q = 898$ gpm.
provided in problem statement

Find

Mean velocity in each pipe: V_6, V_8

Solution

$$\begin{aligned} Q &= 898 \text{ gpm} = 2 \text{ cfs} \\ V_8 &= Q/A_8 = 2/(\pi \times 0.667 \times 0.667/4) = \underline{\underline{5.72 \text{ fps}}} \\ V_6 &= Q/A_6 = 2/(\pi \times 0.5 \times 0.5/4) = \underline{\underline{10.19 \text{ fps}}} \end{aligned}$$

4.72 **Information and assumptions**

Water flows through a tee as shown in the textbook
provided in problem statement

Find

Mean velocity in outlet B: V_B

Solution

Continuity

$$\begin{aligned}
V_B &= (V_A A_A - V_c A_c)/A_B \\
&= [(6 \times \pi/4 \times 4^2) - (4 \times \pi/4 \times 2^2)]/(\pi/4 \times 4^2 \times 4) \\
&= \underline{\underline{5.00 \text{ m/s}}}
\end{aligned}$$

4.73 **Information and assumptions**

Steady flow of gas in a conduit as shown in the textbook.
provided in problem statement

Find

Mean velocity at section 2: V_2

Solution

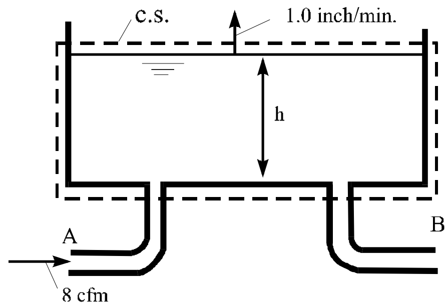
Applying the continuity equation

$$\begin{aligned} V_2 &= (\rho_1 A_1 V_1)/(\rho_2 A_2) = (\rho_1 D_1^2 V_1)/(\rho_2 D_2^2) \\ &= (2.0 \times 1.2^2 \times 15)/(1.5 \times 0.6^2) = \underline{\underline{80.0 \text{ m/s}}} \end{aligned}$$

4.74 **Information and assumptions**

Pipes A and B are connected to an open tank with surface area 80 ft^2. The flowrate in pipe A is $Q_A = 8$ cfm, and the level in the tank is rising at a rate of 1.0 in./min.

provided in problem statement



Find

Discharge in pipe B: Q_B

Is the flow in pipe B entering or leaving the tank?

Solution

Define a control volume as shown in the above sketch. Let the c.s. move upward with the water surface.

Continuity equation:

$$\begin{aligned} 0 &= d/dt \int_{CV} \rho d\cancel{V} + \sum \rho \mathbf{V} \cdot \mathbf{A} \\ 0 &= Adh/dt + Q_B - Q_A \\ Q_B &= Q_A - A\, dh/dt \\ &= 80 - (80)(1.0/12) \\ Q_B &= \underline{\underline{+1.33 \text{ cfm}}} \end{aligned}$$

Because Q_B is positive flow is leaving the tank through pipe B.

4.75 **Information and assumptions**

A tank with one inflow and two outflows is described in the textbook.
provided in problem statement

Find

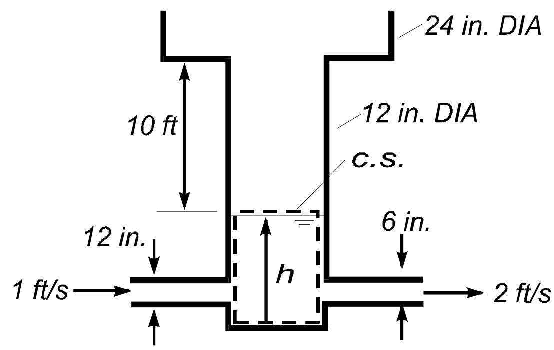
Is the tank filling or emptying?
Rate at which the tank level is changing: $\frac{dh}{dt}$

Solution

$$
\begin{aligned}
\text{Inflow} &= 10 \times \pi \times 2^2/144 = 0.8727 \text{ cfs} \\
\text{Outflow} &= (7 \times \pi \times 3^2/144) + (4 \times \pi \times 1.5^2/144) = 1.571 \text{ cfs} \\
&\ \text{Outflow} > \text{Inflow}, \ \ \text{Thus, } \underline{\underline{\text{tank is emptying}}} \\
\frac{dh}{dt} &= Q/A = (1.571 - 0.8727)/(\pi \times 3^2) = \underline{\underline{0.0247 \text{ ft/s}}}
\end{aligned}
$$

4.76 **Information and assumptions**

The sketch shows a tank filled with water at time $t = 0$ s.
provided in problem statement



Find

At $t = 22$ s, will the water surface be rising or falling?
Rate at which the tank level is changing: $\frac{dh}{dt}$

Solution

Define a control volume. The c.s. is coincident with the water surface and moves with it.
Continuity equation. Evaluate if the water surface is rising or falling:

$$\begin{aligned}
d/dt \int_{cv} \rho d\cancel{V} &= -\sum_{cs} \rho \mathbf{V} \cdot \mathbf{A} \\
d/dt(\rho A h) &= (\rho A V)_{\text{in}} - (\rho A V)_{\text{out}} \\
d/dt(\rho A h) &= \rho(\pi/4 \times 1^2)(1) + \rho(\pi/4 \times 0.5^2)(2) \\
A dh/dt &= (\pi/4) - (\pi/8) \\
A dh/dt &= (\pi/8)
\end{aligned}$$

Since $A dh/dt > 0$, the water level $\underline{\underline{\text{must be rising}}}$. While the water column occupies the 12 in. section, the rate of rise is

$$\begin{aligned}
dh/dt &= (\pi/8)/A \\
&= \pi/(8 \times \pi/4 \times 1^2) \\
&= 1/2 \text{ ft/s}
\end{aligned}$$

Determine the time it takes the water surface to reach the 2 ft. section:

$$\begin{aligned}
10 &= (dh/dt)t; \\
t &= (10)/(1/2) = 20 \text{ secs.}
\end{aligned}$$

Therefore, at the end of 20 sec. the water surface will be in the 2 ft. section. Then the rise velocity will be:

$$
\begin{aligned}
dh/dt &= \pi/(8A) \\
&= \pi/(8 \times \pi/4 \times 2^2) \\
&= \underline{\underline{1/8 \text{ ft/sec}}}
\end{aligned}
$$

4.77 **Information and assumptions**

A lake is fed by one inlet, $Q_{in} = 1000$ cfs. Evaporation is 13 cfs per square mile of lake surface. Lake surface area is $A(h) = 4.5 + 5.5h$, where h is depth in feet.

provided in problem statement

Find

Equilibrium depth of lake

The minimum discharge to prevent the lake from drying up

Solution

Continuity principle: At equilibrium

$$\begin{aligned} Q_{\text{Evap.}} &= Q_{\text{in.}} \\ \left(13 \text{ ft}^3/\text{s/mi}^2\right)(4.5 + 5.5h) \text{ mi}^2 &= 1,000 \text{ ft}^3/\text{s} \\ & \underline{\underline{h = 13.2 \text{ ft.}}} \text{ at equilibrium} \end{aligned}$$

The lake will dry up when $h = 0$ and $Q_{\text{Evap.}} = Q_{\text{in.}}$. For $h = 0$,

$$13(4.5 + 5.5 \times 0) = Q_{\text{in.}}$$

Lake will dry up when $\underline{\underline{Q_{\text{in.}} = 58.5 \text{ ft}^3/\text{s}}}$

4.78 **Information and assumptions**

A nozzle discharges water ($Q_o = 6$ cfs) onto a plate moving towards the nozzle. Plate speed equals half the jet speed.

provided in problem statement

Find

Rate at which the plate deflects water: Q_p

Solution

Select a control volume surrounding the plate and moving with the plate. Continuity is

$$Q_{in} = Q_p$$

Reference velocities to the moving plate. Let V_o be the speed of the water jet relative to the nozzle. From the moving plate, the water has a speed of $V_o + 1/2V_o = 3V_o/2$. Thus

$$\begin{aligned} Q_p &= Q_{in} \\ &= V_{in}A_o \\ &= (3V_o/2)\,(A_o) = (3/2)\,(V_oA_o) \\ &= (3/2)Q_o = \underline{\underline{9.0 \text{ cfs}}} \end{aligned}$$

4.79 **Information and assumptions**

A tank with a depth h has one inflow ($Q = 20$ ft^3/s) and one outflow through a 1 ft diameter pipe. The outflow velocity is $\sqrt{2gh}$

provided in problem statement

Find

Equilibrium depth of liquid

Solution

Continuity principle

$$\begin{aligned}
Q_{\text{in.}} &= Q_{\text{out}} \text{ at equilibrium} \\
20 \text{ ft}^3/\text{s} &= V_{\text{out}} A_{\text{out}} \\
20 &= (\sqrt{2gh})(\pi/4 \times d_{\text{out}}^2) \text{ where } d = 1 \text{ ft.} \\
\text{Solving for } h \text{ yields} &: \quad \underline{\underline{h = 10.1 \text{ ft.}}}
\end{aligned}$$

4.80 **Information and assumptions**

Flows A and B enter a closed tank. Flow C exits the tank. Details are provided in the textbook. Assume steady flow.

provided in problem statement

Find

At section C:

a.) Mass flow rate

b.) Average velocity

c.) Specific gravity of the mixture

Solution

Continuity principle, assuming steady flow,

$$\begin{aligned}
\sum \rho \mathbf{V} \cdot \mathbf{A} &= 0 \\
-\rho_A V_A A_A - \rho_B V_B A_B + \rho_C V_C A_C &= 0 \\
\rho_C V_C A_C &= 0.95 \times 1.94 \times 3 + 0.85 \times 1.94 \times 1 = \underline{\underline{7.18 \text{ slugs/s}}}
\end{aligned}$$

Assuming incompressible flow,

$$\begin{aligned}
V_C A_C &= V_A A_A + V_B A_B = 3 + 1 = 4 \text{ cfs} \quad V_C = Q/A = 4/(\pi/4(1/2)^2 = \underline{\underline{20.4 \text{ ft/s}}} \\
\rho_C &= 7.18/4 = 1.80 \text{ slugs/ft}^3 \qquad S = 1.80/1.94 = \underline{\underline{0.925}}
\end{aligned}$$

4.81 **Information and assumptions**

O_2 and CH_4 enter a mixer, each with a velocity of 5 m/s. Mixer conditions: 200 kPa-abs., 100 ^oC. Outlet density: $\rho = 2.2$ kg/m^3. Flow areas: 1 cm^2 for the CH_4, 3 cm^2 for the O_2, and 3 cm^2 for the exit mixture.
provided in problem statement

Find

Exit velocity of the gas mixture: V_{exit}

Solution

Ideal gas law to find inlet density

$$\begin{aligned} \rho_{0_2} &= p/RT = 200,000/(260 \times 373) = 2.06 \text{ kg/m}^3 \\ \rho_{CH_4} &= 200,000/(518 \times 373) = 1.03 \text{ kg/m}^3 \end{aligned}$$

Continuity principle

$$V_{\text{exit}} = (2.06 \times 5 \times 1.03 \times 5 \times 1)/(2.2 \times 3) = \underline{\underline{5.46 \text{ m/s}}}$$

4.82 **Information and assumptions**

A tank is filled with a compressor. Details are provided in the textbook. provided in problem statement

Find

Time to increased the density of the air in the tank by a factor of 2

Solution

Continuity principle

$$\begin{aligned}
\sum \rho \mathbf{V} \cdot \mathbf{A} &= -\frac{d}{dt}\int_{CV} \rho d\forall \\
-d/dt(\rho\forall) &= -\dot{m} \\
\forall(d\rho/dt) &= 0.5\rho_0/\rho
\end{aligned}$$

Separating variables and integrating

$$\begin{aligned}
\rho d\rho &= 0.5\rho_0 dt/\forall \\
\rho^2/2|_0^f &= 0.5\rho_0 \Delta t/\forall \\
(\rho_f^2 - \rho_0^2)/2 &= 0.5\rho_0 \Delta t/\forall \\
&\therefore \ \Delta t = \forall\rho_0\left((\rho_f^2/\rho_0^2) - 1\right) \\
&= 10(2)(2^2 - 1) \\
&= \underline{\underline{60s}}
\end{aligned}$$

4.83 **Information and assumptions**

A tire (volume 0.5 ft^3) develops a slow leak. In 3 hr, the pressure drops from 30 to 25 psig. The leak rate is $\dot{m} = 0.68A/\sqrt{RT}$, where A is the area of the hole. Tire volume and temperature (60 ^{o}F) remain constant. $p_{atm} = 14$ psia.

provided in problem statement

Find

Area of the leak

Solution

Continuity principle

$$\dot{m}_{out} = -d/dt(\rho \cancel{V})$$

Ideal gas law

$$\rho = p/RT$$

Combining previous 2 equations

$$\dot{m}_{out} = -(\cancel{V}/RT)(dp/dt)$$

Let $\dot{m}_{out} = 0.68A/\sqrt{RT}$ in the above equation

$$0.68A/\sqrt{RT} = -(\cancel{V}/RT)(dp/dt)$$

Separating variables and integrating

$$\begin{aligned}
(1/p)(dp/dt) &= -(0.68A\sqrt{RT})/\cancel{V} \\
\ell n(p_0/p) &= (0.68A\sqrt{RT}t)/\cancel{V}
\end{aligned}$$

Finding area

$$\begin{aligned}
A &= (\cancel{V}/0.68t\sqrt{RT})\ell n(p_0/p) \\
&= (0.5/(0.68 \times 3 \times 3,600))(1,716 \times 520)^{-0.5}\ell n(44/39) \\
&= \underline{\underline{8.69 \times 10^{-9} \text{ ft}^2}} = \underline{\underline{1.25 \times 10^{-6} \text{ in.}^2}}
\end{aligned}$$

4.84 **Information and assumptions**

An O_2 bottle (18 ^{o}C) leaks oxygen through a small orifice ($d = 0.15$ mm). During a time period the pressure drops from 10 to 5 MPa, abs. The leak rate is $\dot{m} = 0.68A/\sqrt{RT}$, where A is the area of the orifice.

provided in problem statement

Find

Time required for the specified pressure change

Solution

Continuity principle

$$\dot{m}_{out} = -d/dt(\rho \cancel{V})$$

Ideal gas law

$$\rho = p/RT$$

Combining previous 2 equations

$$\dot{m}_{out} = -(\cancel{V}/RT)(dp/dt)$$

Let $\dot{m}_{out} = 0.68A/\sqrt{RT}$ in the above equation

$$0.68A/\sqrt{RT} = -(\cancel{V}/RT)(dp/dt)$$

Separating variables and integrating

$$\begin{aligned} (1/p)(dp/dt) &= -(0.68A\sqrt{RT})/\cancel{V} \\ \ell n(p_0/p) &= (0.68A\sqrt{RT}t)/\cancel{V} \end{aligned}$$

Finding time

$$\begin{aligned} t &= (\cancel{V}/0.68A\sqrt{RT})\ell n(p_0/p) \\ &= 0.1\ell n(10/5)/(0.68(\pi/4)(1.5\times10^{-4})\sqrt[2]{260\times291}) = 21,000 \text{ s} \\ &= \underline{\underline{5 \text{ hr. } 50 \text{ min.}}} \end{aligned}$$

4.85 **Information and assumptions**

A tank is draining through an orifice. The water surface drops from 3 to 0.3 m. Other details are provided in the textbook.

provided in problem statement

Find

Time required for the water surface to drop

Solution

From example 4-11:

$$\begin{aligned} t &= (2A_T/\sqrt{2g}A_2)(h_1^{1/2} - h^{1/2}) \\ &= 2 \times (\pi/4 \times 0.6^2)(\sqrt{3} - \sqrt{0.3})/(\sqrt{2 \times 9.81}\pi \times 0.03 \times 0.03/4) \\ &= \underline{\underline{214 \text{ s}}} \end{aligned}$$

4.86 **Information and assumptions**

A cylindrical drum of water is emptying through a pipe on the bottom.
provided in problem statement

$$\begin{aligned} D &= 2 \text{ ft.}, R = 1 \text{ ft.}, V = \sqrt{2gh}; L = 4 \text{ ft.} \\ d &= 2 \text{ in.} = 0.167 \text{ ft.}, h_0 = 1 \text{ ft.} \end{aligned}$$

Find

Time to empty the drum

Solution

Let the control surface surround the water in the tank. Let the c.s.be coincident with the moving water surface. Thus, the control volume will decrease in volume as the tank empties.

Continuity principle:

$$\begin{aligned} \sum_{\text{cs}} \rho \mathbf{V} \cdot \mathbf{A} &= -d/dt \int_{\text{cv}} \rho d\forall \\ +\rho V A &= -d/dt \int_{\text{cv}} \rho d\forall && (1) \\ \rho \sqrt{2gh} A &= -\rho d/dt(\forall) && (2) \\ dt \sqrt{2gh} A &= -d\forall && (3) \end{aligned}$$

Let $d\forall = -L(2x)dy$. Then when this is substituted into Eq. (2) we have

$$dt\sqrt{2gh}A = 2Lxdy \qquad (4)$$

But h can be expressed as a function of y:

$$h = R - y$$

or

$$dt\sqrt{2g(R-y)}A = 2Lxdy$$

Also

$$\begin{aligned} R^2 &= x^2 + y^2 \\ x &= \sqrt{y^2 - R^2} = \sqrt{(y-R)(y+R)} \\ dt\sqrt{2g(R-y)}A &= 2L\sqrt{(y-R)(y+R)}dy \\ dt &= (2L/(\sqrt{2g}A))\sqrt{(y+R)}dy && (5) \end{aligned}$$

Integrate Eq. (4)

$$\begin{aligned} t|_0^t &= (2L/(\sqrt{2g}A)) \int_0^R \sqrt{R+y}dy \\ &= (2L/(\sqrt{2g}A))[(2/3)(R+y)^{3/2}]_0^R \\ t &= (2L/(\sqrt{2g}A))(2/3)((2R)^{3/2} - R^{3/2}) \end{aligned}$$

For $R = 1$

$$t = (2L/(\sqrt{2g}A))(2/3)(2^{2/3} - 1) \quad (6)$$

In Eq. (5) $A = (\pi/4)d^2 = 0.0219$ ft^2. Therefore

$$\begin{aligned} t &= (2 \times 4/\sqrt{64.4} \times 0.0219))(2/3)(1.828) \\ &= \underline{\underline{55.5 \text{ s}}} \end{aligned}$$

Note: The above solution assumes that the velocity of water is uniform across the jet just as it leaves the tank. This is not exactly so, but the solution should yield a reasonable approximation.

4.87 **Information and assumptions**

A pipe with discharge 0.03 ft^3/s fills a funnel. Exit velocity from the funnel is $V_e = \sqrt{2gh}$, and exit diameter is 1 in. Funnel section area is $A_S = 0.1h^2$
provided in problem statement

Find

Level in funnel at steady state: h

Solution

Continuity (steady state)

$$\dot{m}_{in} = \dot{m}_{out}$$

or

$$\rho Q = \rho A_e \sqrt{2gh}$$

Solving for h gives

$$\begin{aligned} h &= \frac{1}{2g}\left(\frac{Q}{A_e}\right)^2 \\ &= \frac{1}{2 \times 32.2}\left(\frac{.03}{\pi/4 \times (1/12)^2}\right)^2 \\ &= \underline{\underline{0.47 \text{ ft}}} \end{aligned}$$

4.88 **Information and assumptions**

Water drains from a pressurized tank. Tank section area: 1 m^2. Exit velocity: $V_e = \sqrt{\frac{2p}{\rho} + 2gh}$. Exit area: 10 cm^3. Supply pressure: $p = 10$ kPa. Initial tank level: $h_o = 2$ m.
provided in problem statement

Find

Time for the tank to empty
a.) with given supply pressure
b.) if supply pressure is zero

Solution

Define a control surface coincident with the tank walls and the top of the fluid in the tank.
Continuity equation

$$\rho \frac{dV}{dt} = -\rho A_e V_e$$

Density is constant. The differential volume is Adh so the above equation becomes

$$-\frac{Adh}{A_e V_e} = -dt$$

or

$$-\frac{Adh}{A_e \sqrt{\frac{2p}{\rho} + 2gh}} = dt$$

Integrating this equation gives

$$-\frac{A}{A_e}\frac{1}{g}\left(\frac{2p}{\rho} + 2gh\right)^{1/2} |_{h_o}^{0} = \Delta t$$

or

$$\Delta t = \frac{A}{A_e}\frac{1}{g}\left[\left(\frac{2p}{\rho} + 2gh_o\right)^{1/2} - \left(\frac{2p}{\rho}\right)^{1/2}\right]$$

and for $A = 1$ m^2, $A_e = 10^{-3}$ m^2, $h_o = 2$ m, $p = 10$ kPa and $\rho = 1000$ kg/m^3 results in

$$\Delta t = 329 \text{ s or } \underline{\underline{5.48 \text{ min}}} \quad \text{(supply pressure of 10 kPa)}$$

For zero pressure in the tank, the time to empty is

$$\Delta t = \frac{A}{A_e}\sqrt{\frac{2h_o}{g}} = 639 \text{ s or } \underline{\underline{10.65 \text{ min}}} \quad \text{(supply pressure of zero)}$$

4.89 **Information and assumptions**

A tapered tank drains through an orifice. Details are provided in the textbook provided in problem statement

Find

a.) A formula for the time to drain

b.) Calculate the time to drain

Solution

$$Q = -A_T(dh/dt); dt = -A_T dh/Q$$

where $Q = \sqrt{2gh}A_j = \sqrt{2gh}(\pi/4)d_j^2$

$$\begin{aligned}
A_T &= (\pi/4)(d + C_1h)^2 = (\pi/4)(d^2 + 2dC_1h + C_1^2h^2) \\
dt &= -(d^2 + 2dC_1h + C_1h^2)dh/(\sqrt{2g}h^{1/2}d_j^2) \\
t &= -\int_{h_0}^{h}(d^2 + 2dC_1h + C_1^2h^2)dh/(\sqrt{2g}h^{1/2}d_j^2) \\
t &= (1/(d_j^2\sqrt{2g}))\int_{h}^{h_0}(d^2h^{-1/2} + 2dC_1h^{1/2} + C_1^2h^{3/2})dh \\
t &= (2/(d_j^2\sqrt{2g}))\left[d^2h^{1/2} + (2/3)dC_1h^{3/2} + (1/5)C_1^2h^{5/2}\right]_h^{h_0} \\
&\underline{\underline{t = (2/(d_j^2\sqrt{2g}))\left[(d^2(h_0^1 - h^{1/2}) + (2/3)dC_1(h_0^{3/2} - h^{3/2}) + (1/5)C_1^2(h_0^{5/2} - h^{5/2})\right]}}
\end{aligned}$$

Then for $h_0 = 1$ m, $h = 0.20$ m, $d = 0.20$ m, $C_1 = 0.4$, and $d_j = 0.05$ m

$$\underline{\underline{t = 18.4 \text{ s}}}$$

4.90 **Information and assumptions**

Water drains out of a trough. Details are provided in the textbook provided in problem statement

Find

a.) A formula for the time to drain to depth h

b.) Calculate the time to drain to 1/2 of the original depth

Solution

Continuity equation:

$$\sum_{c.s.} \rho \mathbf{V} \cdot \mathbf{A} = -d/dt \int_{C.V.} \rho d\cancel{V}$$

$$\rho \sqrt{2gh} A_e = -d/dt \int_{C.V.} \rho d\cancel{V}$$

Mass of water in control volume $= \rho B \times$Face area

$$M = \rho B (W_0 h + h^2 \tan \alpha)$$

Then

$$\begin{aligned} \rho \sqrt{2gh} A_e &= -d/dt\, \rho B (W_0 h + h^2 \tan \alpha) \\ \sqrt{2gh} A_e &= -BW_0 (dh/dt) - 2Bh \tan \alpha (dh/dt) \\ dt &= (1/(\sqrt{2g} A_e))(-BW_0 h^{-1/2} dh - 2B \tan \alpha h^{1/2} dh) \end{aligned}$$

Integrate

$$\begin{aligned} t &= (1/\sqrt{2g} A_e) \int_{h_0}^{h} -BW_0 h^{-1/2} dh - 2B \tan \alpha h^{1/2} dh \\ t &= (1/(\sqrt{2g} A_e))(-2BW_0 h^{1/2} - (4/3) B \tan \alpha h^{3/2})_{h_0}^{h} \\ & \underline{\underline{t = (\sqrt{2} B h_0^{3/2} / (\sqrt{g} A_e))((W_0/h_0)(1 - (h/h_0)^{0.5}) + (2/3) \tan \alpha (1 - (h/h_0)^{1.5}))}} \end{aligned}$$

For $W_0/h_0 = 0.2, \alpha = 30°, A_e g^{0.5}/(h_0^{1.5} B) = 0.01$ sec.$^{-1}$ and h/h$_0$ = 0.5 we get

$$\underline{\underline{t = 43.4 \text{ seconds}}}$$

4.91 **Information and assumptions**

Water drains out of a spherical tank. Tank diameter: 1 m. Hole diameter: 1 cm. Exit velocity: $V_e = \sqrt{2gh}$. At time zero, the tank is half full.

provided in problem statement

Find

Time required to empty the tank

Solution

Select a control volume that is inside of the tank and level with the top of the liquid surface.

Continuity

$$\rho \frac{d\forall}{dt} = -\rho A_e V_e$$

Let

$$\frac{d\forall}{dt} = \frac{d(Ah)}{dt} = A\frac{dh}{dt}$$

Continuity becomes

$$\frac{dh}{dt} = -\frac{A_e}{A}\sqrt{2gh}$$

The cross-sectional area in terms of R and h is

$$A = \pi[R^2 - (R-h)^2] = \pi(2Rh - h^2)$$

Substituting into the differential equation gives

$$\frac{\pi(-2Rh + h^2)}{A_e\sqrt{2gh}} dh = dt$$

or

$$\frac{\pi}{\sqrt{2g}A_e}\left(-2Rh^{1/2} + h^{3/2}\right) dh = dt$$

Integrating this equation results in

$$\frac{\pi}{\sqrt{2g}A_e}\left(-\frac{4}{3}Rh^{3/2} + \frac{2}{5}h^{5/2}\right)|_R^0 = \Delta t$$

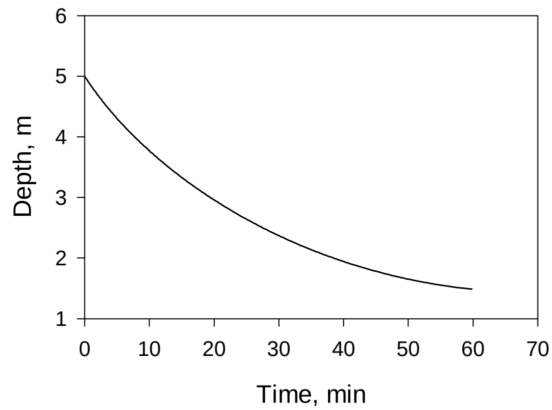
Substituting in the limits yields

$$\frac{\pi}{\sqrt{2g}A_e}\frac{14}{15}R^{5/2} = \Delta t$$

For $R = 0.5$ m and $A_e = 7.85 \times 10^{-5}$ m^2, the time to empty the tank is

$$\Delta t = 1491 \text{ sec or } \underline{\underline{24.8 \text{ min}}}$$

4.92 The numerical solution provides the following results.



4.93 **Information and assumptions**

An end-burning rocket motor is described in the textbook.
provided in problem statement

Find

Gas velocity at nozzle exit plane: V_e

Solution

Ideal gas law

$$\begin{aligned}\rho_e &= p/RT \\ &= 10,000/(415 \times 2273) = 0.0106 \text{ kg/m}^3\end{aligned}$$

Continuity equation

$$\begin{aligned}V_e &= V_M \rho_M A_M / (\rho_e A_e) \\ &= 0.01 \times 1,800 \times (\pi/4 \times 0.1^2) / \left[0.0106 \times \left(\pi/4 \times 0.08^2\right)\right] \\ &= \underline{\underline{2{,}650 \text{ m/s}}}\end{aligned}$$

4.94 **Information and assumptions**

An cylindrical-port rocket motor is described in the textbook.
provided in problem statement

Find

Gas density at the exit: ρ_e

Solution

$$\begin{aligned}
A_g &= \pi DL + 2(\pi/4)(D_0^2 - D^2) \\
&= \pi \times 0.12 \times 0.4 + (\pi/2)(0.2^2 - 0.12^2) = 0.191 \text{ m}^2 \\
\rho_e &= V_g \rho_g A_g/(V_e A_e) = 0.012 \times 2,000 \times 0.191/(2,000 \times (\pi/4) \times (0.20)^2) \\
&= \underline{\underline{0.073 \text{ kg/m}^3}}
\end{aligned}$$

4.95 **Information and assumptions**

A rocket nozzle is described in the textbook.
provided in problem statement

Find

A formula for chamber pressure
Increase in chamber pressure if a crack increases burn area by 20%

Solution

$$\begin{aligned}
\rho_p \dot{r} A_g &= \dot{m} \\
\rho_p a p_c^n A_g &= 0.65 p_c A_t / \sqrt{RT_c} \\
p_c^{1-n} &= (a\rho_0/0.65)(A_g/A_t)(RT_c)^{1/2} \\
& \underline{\underline{p_c = (a\rho_p/0.65)^{1/(1-n)}(A_g/A_t)^{1/(1-n)}(RT_c)^{1/(2(1-n))}}} \\
\Delta p_c &= 3.5(1+0.20)^{1/(1-0.3)} = \underline{\underline{4.54 \text{ MPa}}}
\end{aligned}$$

4.96 **Information and assumptions**

A piston moves in a cylinder and drives exhaust gas out an exhaust port. Assume the gas in the cylinder has a uniform density and pressure. Assume ideal gas.

provided in problem statement

Find

Rate at which the gas density is changing in the cylinder: $d\rho/dt$

Solution

$$
\begin{aligned}
d/dt(\rho V) + 0.65 p_c A_v/\sqrt{RT_c} &= 0 \\
\cancel{V} d\rho/dt + \rho d\cancel{V}/dt + 0.65 p_c A_v/\sqrt{RT_c} &= 0 \\
d\rho/dt &= (\rho/\cancel{V})\ d\cancel{V}/dt - 0.65 p_c A_v/\cancel{V}\sqrt{RT_c} \\
\cancel{V} &= (\pi/4)(0.1)^2(0.1) = 7.854 \times 10^{-4}\ \mathrm{m}^3 \\
(d\cancel{V}/dt) &= -(\pi/4)(0.1)^2(30) = -0.2356\ \mathrm{m}^3/\mathrm{s} \\
\rho &= p/RT = 300,000/(350 \times 873) = 0.982\ \mathrm{kg/m}^3 \\
d\rho/dt &= -(0.982/7.854 \times 10^{-4}) \times (-0.2356) - 0.65 \times 300,000 \\
&\quad \times 1 \times 10^{-4}/(7.854 \times 10^{-4} \times \sqrt{350 \times 873}) \\
&= \underline{\underline{250\ \mathrm{kg/m}^3 \cdot \mathrm{s}}}
\end{aligned}
$$

4.97 **Information and assumptions**

A cyclone has a wind speed of 15 mph at $r = 200$ mi.
provided in problem statement

Find

Wind speed at $r = 50$ and 100 miles: V_{50} & V_{100}

Solution

$$\begin{aligned}
Vr &= \text{Const.} \\
(15 \text{ mph})\ (200 \text{ mi.}) &= \text{Const.} \\
V_{100} &= \text{Const.}/100 \text{ mi.} \\
V_{100} &= (15 \text{ mph})(200 \text{ mi.}/100 \text{ mi.}) = \underline{\underline{30 \text{ mph}}} \\
V_{50} &= (15 \text{ mph})(200/50) = \underline{\underline{60 \text{ mph}}}
\end{aligned}$$

4.98 **Information and assumptions**

A velocity field is given by $u = -\omega y \;\; v = \omega x$
provided in problem statement

Find

a.) Is continuity satisfied?
b.) Vorticity
c.) Rate of rotation

Solution

Check continuity:

$$\partial u/\partial x = 0 \text{ and } \partial v/\partial y = 0$$
$$\underline{\underline{\therefore \partial u/\partial x + \partial v/\partial y = 0 \; continuity \; is \; satisfied}}$$

Rate of rotation

$$\begin{aligned}
\omega_z &= (1/2)(\partial v/\partial x - \partial u/\partial y) \\
&= (1/2)(\omega - (-\omega)) \\
&= (1/2)(2\omega) \\
&= \underline{\underline{\omega}}
\end{aligned}$$

Vorticity is twice the average rate of rotation; therefore, the $\underline{\underline{\text{vorticity} = 2\omega}}$

4.99 **Information and assumptions**

A two-dimensional velocity field is defined in the textbook.
provided in problem statement

Find

a.) Check if continuity is satisfied
b.) Check if flow is irrotational

Solution

$$\begin{aligned}
\partial u/\partial x + \partial v/\partial y &= (= -2Cx/(y^2+x^2)^2) - (2C(y^2-x^2)(2x)/(y^2+x^2)^3) \\
&\quad -(2Cx/(y^2+x^2)^2) + (4Cxy(2y)/(y^2+x^2)^3) \\
&= 0 \quad \underline{\underline{\text{Continuity is satisfied}}} \\
\partial u/\partial y - \partial v/\partial x &= (2Cy/(y^2+x^2)^2) - (2C(y^2-x^2)2y/(y^2+x^2)^3) \\
&\quad +(2Cy/(y^2+x^2)^2) - (4Cxy(2x)/(y^2+x^2)^3) \\
&= 0 \quad \underline{\underline{\text{The flow is irrotational}}}
\end{aligned}$$

4.100 **Information and assumptions**

Use the velocity field of Fig. 4.21
provided in problem statement

Find

a.) Check if continuity is satisfied
b.) Check if flow is rotational or irrotational

Solution

Velocity field

$$u = ky$$

where k is a constant.
Continuity Eq.: $\partial u/\partial x + \partial v/\partial y = 0$

$$0 + 0 = 0 \quad \underline{\underline{\text{continuity is satisfied}}}$$

Irrotational flow: $\partial v/\partial x = \partial u/\partial y$

$$0 \neq k \quad \underline{\underline{\text{flow is rotational}}}$$

4.101 **Information and assumptions**

A velocity field is defined in the textbook.
provided in problem statement

Find

Is continuity satisfied?

Solution

$$\begin{aligned}(\partial u/\partial x)+(\partial v/\partial y)+(\partial w/\partial z) &= U(3x^2+y^2)+U(3y^2+x^2)+0 \\ &\neq 0 \ \ \underline{\underline{\text{Continuity is not satisfied}}}\end{aligned}$$

4.102 **Information and assumptions**

A velocity field is defined by $u = xt + 2y, \; v = xt^2 - yt$.
provided in problem statement

Find

Acceleration at $x = y = 1$ m and $t = 1$ s
Is the flow rotational or irrotational?

Solution

Irrotational flow:

$$\partial u/\partial y = 2; \; \partial v/\partial x = t^2 \quad \partial u/\partial y \neq \partial v/\partial x$$

Therefore, the flow is $\underline{\underline{\text{rotational}}}$.
Determine acceleration:

$$\begin{aligned} a_x &= u\partial u/\partial x + v\partial u/\partial y + \partial u/\partial t \\ a_x &= (xt + 2y)t + 2(xt^2 - yt) + x \\ a_y &= u\partial v/\partial x + v\partial v/\partial y + \partial v/\partial t \\ &= (xt + 2y)t^2 + (xt^2 - yt)(-t) + (2xt - y) \\ \mathbf{a} &= ((xt + 2y)t + 2t(xt - y) + x)\mathbf{i} + (t^2(xt + 2y) - t^2(xt - y) + (2xt - y))\,\mathbf{j} \end{aligned}$$

Then for x =l m, y =l m, and t =l s the acceleration is:

$$\begin{aligned} \mathbf{a} &= ((1 + 2) + 0 + 1)\;\mathbf{i} + ((1 + 2) + 0 + (2 - 1))\;\mathbf{j}\text{ m/s} \\ \mathbf{a} &= \underline{\underline{4\,\mathbf{i} + 4\,\mathbf{j}\text{ m/s}^2}} \end{aligned}$$

4.103 **Information and assumptions**

A velocity field is defined as
provided in problem statement

$$
\begin{aligned}
u &= \frac{y}{(x^2+y^2)^{3/2}} \\
v &= \frac{-x}{(x^2+y^2)^{3/2}}
\end{aligned}
$$

Find

Is continuity satisfied?
Is flow irrotational?

Solution

$$
\begin{aligned}
\partial u/\partial x + \partial v/\partial y &= -3xy/(x^2+y^2)^{5/2} + 3xy/(x^2+y^2)^{5/2} \\
&= 0 \quad \underline{\underline{\text{Continuity is satisfied}}} \\
\partial u/\partial y - \partial v/\partial x &= -3y^2/(x^2+y^2)^{5/2} + 1/(x^2+y^2)^{3/2} \\
&= 3x^2/(x^2+y^2)^{5/2} + 1/(x^2+y^2)^{3/2} \\
&\neq 0 \quad \underline{\underline{\text{Flow is not irrotational}}}
\end{aligned}
$$

4.104 **Information and assumptions**

A u-component of a velocity field is $u = Axy$
provided in problem statement

Find

What is a possible v-component?
What must the v-component be if the flow is irrotational?

Solution

$$
\begin{aligned}
u &= Axy \\
\partial u/\partial x + \partial v/\partial y &= 0 \\
Ay + \partial v/\partial y &= 0 \\
\partial v/\partial y &= -Ay \\
v &= \underline{\underline{(-1/2)Ay^2 + C}} \\
& \text{for irrotationality} \\
\partial u/\partial y - \partial v/\partial x &= 0 \\
Ax - \partial v/\partial x &= 0 \\
\partial v/\partial x &= Ax \\
\text{or } V &= 1/2Ax^2 + C(y)
\end{aligned}
$$

If we let $C(y) = -1/2Ay^2$ then the equation will also satisfy continuity.

$$\underline{\underline{v = 1/2A(x^2 - y^2)}}$$

4.105 **Information and assumptions**

A tornado is modeled as a combined forced and free vortex. The core diameter is 10 miles. At $r = 50$ miles, $V = 20$ mph

provided in problem statement

Find

Wind velocity at edge of core: V_{10}

Centrifugal acceleration at edge of core: a_c

Solution

The velocity variation in a free vortex is

$$Vr = \text{const}$$

Thus

$$V_{50}(50) = V_{10}(10)$$

Therefore

$$V_{10} = V_{50}\frac{50}{10} = 5 \times 20 = 100 \text{ mph}$$

Acceleration (Eulerian formulation)

$$\begin{aligned} V &= 100 \times 5280/3600 = 147 \text{ ft/s} \\ a_c &= V^2/r \\ &= 147^2/(10 \times 5280) = \underline{\underline{0.409 \text{ ft/s}^2}} \end{aligned}$$

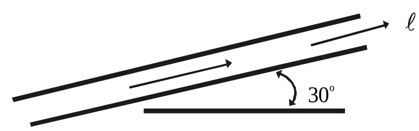
Chapter Five

5.1 **Information and assumptions**

provided in problem statement

Find

pressure gradient in flow direction



Euler's equation

$$\begin{aligned}
\partial/\partial\ell(p+\gamma z) &= -\rho a_\ell \\
\partial p/\partial\ell + \gamma\partial z/\partial\ell &= -\rho a_\ell \\
\partial p/\partial\ell &= -\rho a_\ell - \gamma\partial z/\partial\ell \\
&= -(\gamma/g)\times(-0.30g) - \gamma\sin 30^\circ \\
&= \gamma(0.30-0.50) \\
&= \underline{\underline{-0.20\gamma}}
\end{aligned}$$

5.2 **Information and assumptions**

provided in problem statement

Find

pressure gradient required to accelerate flow

Euler's equation

$$\begin{aligned}
\partial(p+\gamma z)/\partial z &= -\rho a_z = -(\gamma/g)\times 0.20g \\
\partial p/\partial z + \gamma &= -0.20\gamma \\
\partial p/\partial z &= \gamma(-1-0.20) = 0.81\times 62.4(-1.20) = \underline{\underline{-60.7 \text{ lbf/ft}^3}}
\end{aligned}$$

5.3 **Information and assumptions**

provided in problem statement

Find

direction of acceleration

Euler's equation

$$\begin{aligned} \rho a_\ell &= -\partial/\partial\ell(p + \gamma z) \\ a_\ell &= (1/\rho)(-\partial p/\partial\ell - \gamma\partial z/\partial\ell) \end{aligned}$$

Let ℓ be positive upward. Then $\partial z/\partial\ell = +1$ and $\partial p/\partial\ell = (p_A - p_B)/1 = -12,000$ Pa/m. Thus

$$\begin{aligned} a_\ell &= (g/\gamma)(12,000 - \gamma) \\ a_\ell &= g((12,000/\gamma) - 1) \\ a_\ell &= g(1.2 - 1.0) \text{ m/s}^2 \end{aligned}$$

a_ℓ has a positive value; therefore, acceleration is upward. Correct answer is $\underline{\underline{\text{a)}}}$

5.4 **Information and assumptions**

provided in problem statement

Find

pressure at depth of 2 ft. in water column

Euler's equation

$$\rho a_\ell = -\partial/\partial\ell(p + \gamma z)$$

Let ℓ be positive upward.

$$\begin{aligned}
\rho(0.5\text{ g}) &= -\partial p/\partial\ell - \gamma\partial z/\partial\ell \\
(\gamma/\text{g})(0.5\text{g}) &= \partial p/\partial\ell - \gamma(1) \\
\partial p/\partial\ell &= -\gamma(0.5 + 1) = -1.5\gamma
\end{aligned}$$

Thus the pressure decreases upward at a rate of 1.5γ. At a dept of 2 ft.:

$$\begin{aligned}
p_2 &= (1.5\gamma)(2) = 3\gamma \\
p_2 &= 3\text{ ft.} \times 62.4\text{ lbf/ft}^3 = \underline{\underline{187.2\text{ lbf/ft}^2}}
\end{aligned}$$

5.5 **Information and assumptions**

provided in problem statement

Find

acceleration of piston to create a pressure of 9 psi

Euler's equation

$$\partial/\partial s(p+\gamma z)=-\rho a_s$$

$$\begin{aligned}
-\Delta(p+\gamma z) &= 1.94\times 10\times a_s\\
-(p_2-p_1)-\gamma(z_2-z_1) &= 19.4a_s\\
a_s &= (9\times 144-62.4\times 10)/19.4=\underline{\underline{34.6\text{ ft/s}^2}}
\end{aligned}$$

5.6 **Information and assumptions**

provided in problem statement

Find

pressure gradient

Euler's equation with no change in elevation

$$(\partial p/\partial s) = -\rho a_s = -1,000 \times 6 = \underline{\underline{-6,000 \text{ N/m}^3}}$$

5.7 **Information and assumptions**

provided in problem statement

Find

pressure at upstream end

Euler's equation with no change in elevation

$$\begin{aligned}(\partial p/\partial s) &= -\rho a_s = -1,000 \times 6 = -6,000 \text{ N/m}^3 \\ p_{\text{upstream}} &= 90,000 + 6,000 \times 100 = 690,000 \text{ Pa} = \underline{\underline{690 \text{ kPa}}}\end{aligned}$$

5.8 **Information and assumptions**

provided in problem statement

Find

maximum downward acceleration before vaporization

Euler's equation

$$\begin{aligned}
\partial/\partial z(p+\gamma z) &= -\rho a_z \\
\Delta(p+\gamma z) &= -\rho a_z \Delta z \\
(p+\gamma z)_{\text{at water surface}} - (p+\gamma z)_{\text{at piston}} &= -\rho a_z(z_{\text{surface}} - z_{\text{piston}}) \\
p_{\text{atm}} - p_v + \gamma(z_{\text{surface}} - z_{\text{piston}}) &= -12\ \rho a_z \\
14.7 \times 144 - 0 + 62.4(12) &= -12 \times 1.94 a_z \\
a_z &= \underline{\underline{-123.1 \text{ ft/s}^2}}
\end{aligned}$$

5.9 **Information and assumptions**

provided in problem statement

Find

properties of the flow

Euler's equation

$$-\partial/\partial s(p+\gamma z) = \rho a_s$$
$$-\partial p/\partial s - \gamma \partial z/\partial s = \rho a_s$$

where $\partial p/\partial s = (100-170)/2 = -35$ lb/ft^3 and $\partial z/\partial s = \sin 30^\circ = 0.5$. Then

$$\begin{aligned} a_s &= (1/\rho)(35-(100)(0.5)) \\ &= (1/\rho)(-15\ )\ \text{lbf/ft}^3 \end{aligned}$$

Because a_s has a negative value we conclude that the acceleration is in the negative s direction. The flow direction cannot be discerned.

5.10 **Information and assumptions**

provided in problem statement

Find

pressure gradient midway in the nozzle

Euler's equation

$$d/dx(p+\gamma z) = -\rho a_x$$

but z =const.; therefore

$$\begin{aligned} dp/dx &= -\rho a_x \\ a_x &= a_{\text{convective}} = vdv/dx \\ &= (55 \text{ ft/s})(50 \text{ ft/s/ft}) = 2,750 \text{ ft/s}^2 \end{aligned}$$

Finally

$$dp/dx = (-1.94 \text{ slug/ft}^3)(2,750 \text{ ft/s}^2) = \underline{\underline{-5,355 \text{ psf/ft}}}$$

5.11 The valid statement is (b).

5.12 **Information and assumptions**

provided in problem statement

Find

pressure gradient at point A

Solution

Let $y =$ vertical dimension in the duct. Then

$$V_x = q/y$$

where $y = b - 0.1x$ so

$$V_x = 0.2t/(b - 0.1x)$$

with $t_o = 1$ s.

$$a_{\text{local}} = \partial V_x/\partial t = 0.2/(b - 0.1x)$$

At point A, $x = -1$ so $a_{\text{local}} = 0.2/0.3 = 0.667 \text{ m/s}^2$

$$\begin{aligned}
a_{\text{conv}} &= V_x \partial V_x/\partial x \\
\partial V_x/\partial x &= \partial/\partial x[q(b - 0.1x)^{-1}] \\
&= 0.1q/(b - 0.1x)^2 \\
V_x \partial V_x/\partial x &= [0.2t/(b - 0.1x)][0.1 \times 0.2t/(b - 0.1x)^2] \\
a_{\text{conv}} &= 0.004t^2/(b - 0.1x)^3
\end{aligned}$$

For $t = 2s$ and $x = -1$ m, $a_{\text{conv}} = 0.5926 \text{ m/s}^2$

$$\begin{aligned}
a_{\text{tot}} &= 0.5926m/s^2 + 0.667m/s^2 = 1.260m/s^2 \\
\partial p/\partial x &= -\rho a_x = -1,000 \times 1.260 = \underline{\underline{-1,260 \text{ Pa/m}}}
\end{aligned}$$

5.13 **Information and assumptions**

provided in problem statement

Find

pressure gradient

Solution

From solutions to Probs. 4.52 and 4.53.

$$a = (8q_0/3t_0B) + (32t^2q_0^2/(27B^3t_0^2))$$

Then for $q_0 = 0.10$ m^3/s, $t_0 = 0.1$ s, $t = 0.5$ s, and $B = 0.40$ m

$$a = 11.29m/s^2.$$

Euler's equation with no change in elevation

$$\begin{aligned} \partial p/\partial x &= -\rho a_\ell \\ &= -(1,000 \text{ kg/m}^3)(11.29 \text{ m/s}^2) \\ &= \underline{\underline{-11.29 \text{ kPa/m}}} \end{aligned}$$

5.14 **Information and assumptions**

provided in problem statement

Find

pressure gradient in terms of ρ.

Solution

The velocity in the duct is given by

$$V = \frac{Q}{A} = \frac{1}{A}(Q_o - Q_1\frac{t}{t_o})$$

The convective acceleration is

$$\frac{\partial V}{\partial x} = -\frac{1}{A^2}\frac{dA}{dx}(Q_o - Q_1\frac{t}{t_o}) = -\frac{1}{A}\frac{dA}{dx}V$$

For $t = 0.5$

$$\frac{\partial V}{\partial x} = 2 \text{ s}^{-1} \quad \text{and} \quad V = 5.017 - 1.273 = 3.744 \text{ m/s}$$

so

$$-\frac{1}{A}\frac{dA}{dx} = 0.5342 \text{ m}^{-1}$$

At $t = 0.8$ the velocity is

$$V = 5.017 - 2.547 \times 0.8/1.0 = 2.980 \text{ m/s}$$

and the velocity gradient is

$$\frac{\partial V}{\partial x} = -\frac{1}{A}\frac{dA}{dx}V = 0.5342 \times 2.980 = 1.592 \text{ s}^{-1}$$

The convective acceleration is

$$V\frac{\partial V}{\partial x} = 1.592 \times 2.980 = 4.744 \text{ m/s}^2$$

The local acceleration is

$$\frac{\partial V}{\partial t} = -\frac{Q_1}{At_o} = -2.547 \text{ m/s}^2$$

The acceleration in the x-direction is

$$a_x = -2.547 + 4.744 = 2.20 \text{ m/s}^2$$

From **Euler's equation** the pressure gradient is

$$\frac{\partial p}{\partial x} = -\underline{\underline{2.20\rho}} \text{ Pa/m}$$

5.15 **Information and assumptions**

provided in problem statement

Find

convective acceleration at point A

$$V = Q/A = 0.03t/\pi r^2$$

From geometry

$$r = D_0/2 - x/20 = 0.05(1-x)$$

$$\begin{aligned} V &= 0.03t/(\pi \times 0.0025(1-x)^2) = 3.820t(1-x)^{-2} \\ a_{\text{conv}} &= V\partial V/\partial x = (3.820t)^2(1-x)^{-2} \times (-2) \times (1-x)^{-3}(-1) \\ &= 29.18t^2(1-x)^{-5} \end{aligned}$$

At $t = 2$ s and $x = 0.3$ m, $a_{\text{conv}} = \underline{\underline{694.5}}$ m/s^2

$$\begin{aligned} a_{\text{local}} &= \partial V/\partial t = 3.820(1-0.3)^{-2} = \underline{\underline{7.796}}\,\text{m/s}^2 \\ (dp/dx)_{\text{acc}} &= -\rho a_x = -1.6 \times 1000(694.5 + 7.796) \\ &= -1,124\ \text{kN/m}^3 \end{aligned}$$

Euler's equation

$$\begin{aligned} (dp/dx) &= -\rho a_x + \gamma = 1.124 - 1.6 \times 9.810 \\ &= \underline{\underline{1.108}}\ \text{kN/m}^3 \end{aligned}$$

Pressure can be found by integrating the pressure gradient

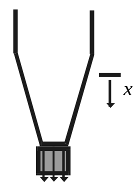
$$\begin{aligned} dp/dx &= -\rho[116.7(1-x)^{-5} + 3.820(1-x)^{-2}] + 1.6 \times 9810 \\ p &= -\rho[29.81(1-x)^{-4} + 3.820(1-x)^{-1}] + 1.6 \times 9810x + C \end{aligned}$$

At $x = 0.6$ m, $p = 0$ so the constant of integration is

$$\begin{aligned} C &= 1000 \times 1.6[29.18(1-0.6)^{-4} + 3.820(1-0.6)^{-1}] \\ -1.6 \times 9810 \times 0.6 &= 1.830\ \text{MPa} \end{aligned}$$

Evaluation of p_A gives

$$\begin{aligned} p_A &= -1.6 \times 1000[29.81(1-0.3)^{-4} + 3.820(1-0.3)^{-1}] + 1.6 \times 9810 \times 0.3 + 1,830,000 \\ &= 1,631,000 \text{ Pa } = \underline{\underline{1.631 \text{ MPa}}} \end{aligned}$$



5.16 Possible method of design:

1. Arbitrarily choose L_1
2. Arbitrarily choose $V_n^2/2g$; solve for V_n
3. Solve for $V_0^2/2g$ with Bernoulli's equation
4. Solve for V_0 and then A_0 and D_0 for given Q
5. Solve for d_n for given Q

5.17 **Information and assumptions**

provided in problem statement

Find

pressure gradient at point A

$$\begin{aligned} a &= -Q^2/(4\pi^2 h^2 r^3) \\ (\partial p/\partial r) &= (\rho Q^2/4\pi^2 h^2) r^{-3} \end{aligned}$$

Euler's equation

$$(\partial p/\partial r) = -\rho a_r = -1.4 \times (-4,572) = 6,401 \text{ N/m}^3$$

Integrating

$$p = -(\rho Q^2/8\pi^2 h^2) r^{-2} + C$$

At $r = r_0$, $p = p_{\text{atm}}$, so

$$C = p_{\text{atm}} + (\rho Q^2/8\pi^2 h^2) r_0^{-2}$$

Therefore

$$\begin{aligned} p &= p_{\text{atm}} + (\rho Q^2/8\pi^2 h^2)(r_0^{-2} - r^{-2}) \\ p &= 100,000 + (1.4 \times 0.380 \times 0.380/(8\pi^2 \times 0.01 \times 0.01)(0.6^{-2} - 0.2^{-2}) \\ &= (100,000 - 569) \text{ Pa} \\ &= 99,431 \text{ Pa} = 99.43 \text{ kPa absolute} \\ \text{or } p_A &= \underline{\underline{-569 \text{ Pa gage}}} \end{aligned}$$

5.18 **Information and assumptions**

provided in problem statement

Find

acceleration of tank

Solution

$$
\begin{aligned}
\tan\alpha &= a_x/g \\
a_x &= g\tan\alpha = 9.81 \times 3/5 = \underline{\underline{5.89\ \mathrm{m/s^2}}}
\end{aligned}
$$

5.19 **Information and assumptions**

provided in problem statement

Find

$p_C - p_A$ and $p_B - p_A$

Euler's equation

$$\begin{aligned}
(dp/dz) &= -\rho(g + a_z) = -1.1 \times 1.94(32.2 - 1.5g) = 34.4 \text{ psf/ft} \\
p_B - p_A &= -34.4 \times 4 = \underline{\underline{-137.6 \text{ psf}}} \\
p_C - p_B &= \rho a_x L = 1.1 \times 1.94 \times 0.9g \times 3 = 185.5 \text{ psf} \\
p_C - p_A &= 185.5 - 137.6 = \underline{\underline{47.9 \text{ lbf/ft}^2}}
\end{aligned}$$

5.20 **Information and assumptions**

provided in problem statement

Find

$p_C - p_A$ and $p_B - p_A$

Euler's equation

$$
\begin{aligned}
(dp/dz) &= -1.3 \times 1,000(9.81 - 6.54) = -4,251 \text{ N/m}^3 \\
p_B - p_A &= 4,251 \times 3 = 12,753 \text{ Pa } = \underline{\underline{12.75 \text{ kPa}}} \\
p_C - p_B &= \rho a_x L = 1.3 \times 1,000 \times 9.81 \times 2 = 22,506 \text{ Pa} \\
p_C - p_A &= 22,506 + 12,753 = 38,259 \text{ Pa} = \underline{\underline{38.26 \text{ kPa}}}
\end{aligned}
$$

5.21 **Information and assumptions**

provided in problem statement

Find

maximum depth before spilling

Solution

$$\begin{aligned}
\tan\alpha &= a_x/g = 8.02/32.2 = 0.2491 \\
\tan\alpha &= h/9 \\
h &= 9\tan\alpha = 9 \times 0.2491 = 2.242 \text{ ft} \\
\text{Maximum depth} &= 7 - 2.242 = \underline{\underline{4.758 \text{ ft}}}
\end{aligned}$$

5.22 **Information and assumptions**

provided in problem statement

Find

maximum speed before water spilled

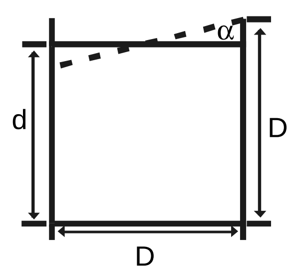
Solution

On straight section

$$\begin{aligned}
\tan\alpha &= a_x/g \\
&= (1/3)g/g \\
&= 1/3 \\
\tan\alpha &= 1/3 = (D-d)/(0.5D) \\
\text{thus } d &= D-(1/6)D = (5/6)D \\
&\quad \text{Tank can be } \underline{\underline{5/6 \text{ full}}} \text{ without spilling}
\end{aligned}$$

On unbanked curve

$$\begin{aligned}
\tan\alpha &= 1/3 \\
\text{Then } 1/3 &= a_n/g \\
a_n &= (1/3)g \\
V^2/r &= (1/3)g \\
\text{or } V &= \sqrt{(1/3)gr} \\
&= \underline{\underline{12.8 \text{ m/s}}}
\end{aligned}$$



5.23 The valid statement is (b).

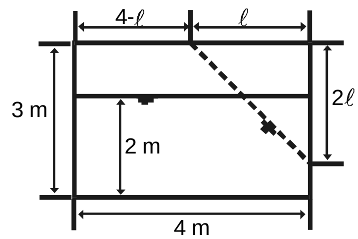
5.24 **Information and assumptions**

provided in problem statement

Find

maximum pressure in tank during acceleration

Solution



$$
\begin{aligned}
\tan\theta &= a_s/g = 2 \\
\text{area of air space} &= \ell^2 = 4 \times 1 \\
\ell^2 &= 4; \ell = 2\text{m} \\
p_{\max}/\gamma &= 4 - 2 + 3 = 5.0 \text{ m} \\
p_{\max} &= \underline{\underline{40.7 \text{ kPa gage}}}
\end{aligned}
$$

5.25 **Information and assumptions**

provided in problem statement

Find

pressure at center at bottom or tank

Rotating flow equation between center bottom of tank and water surface in piezometer.

$$p + \gamma z - \rho r^2 \omega^2/2 = p_p + \gamma z_p - \rho r_p^2 \omega^2/2$$

where $p_p = 0$, $\mathrm{r}_p = 3$ ft and $r = 0$, then

$$\begin{aligned} p &= -(\rho/2)(9 \times 225) + \gamma(z_p - z) \\ &= (1.94/2)(2025) + 62.4 \times 2.5 \\ &= -1808 \text{ psfg} = \underline{\underline{-12.56}} \text{ psig} \end{aligned}$$

5.26 **Information and assumptions**

provided in problem statement

Find

pressure at B

Rotating flow equation from point A to point B

$$\begin{aligned} p_A + \gamma z_A - \rho r_A^2 \omega^2/2 &= p_B + \gamma z_B - \rho r_B^2 \omega^2/2 \\ p_B &= p_A + (\rho/2)(\omega^2)(r_B^2 - r_A^2) + \gamma(z_A - z_B) \end{aligned}$$

where $\omega = V_A/r_A = 20/1.5 = 13.333$ rad/s and $\rho = 0.8 \times 1.94$ slugs/ft^3. Then

$$\begin{aligned} p_B &= 25 + (1.94 \times 0.80/2)(13.33^2)(2.5^2 - 1.5^2) + 62.4 \times 0.8(-1) \\ p_B &= 25 + 551.5 - 49.9 = \underline{\underline{526.6 \text{ psf}}} \end{aligned}$$

5.27 **Information and assumptions**

provided in problem statement

Find

difference in pressure between points A and B, $p_B - p_A$

Let point C be at the center bottom of the tank.

Rotating flow equation between points B & C

$$p_B - \rho r_B^2 \omega^2/2 = p_C - \rho r_C^2 \omega^2/2$$

where $r_B = 0.5$ m, $r_C = 0$ and $\omega = 10$ rad/s. Then

$$\begin{aligned} p_B - p_C &= (\rho/2)(\omega^2)(0.5^2) \\ &= (1200/2)(100)(0.25) \\ &= 15,000 \text{ Pa} \\ p_C - p_A &= 2\gamma + \rho a_z \ell \\ &= 2 \times 11,772 + 1,200 \times 4 \times 2 \\ &= 33,144 \text{ Pa} \end{aligned}$$

Then $p_B - p_A = 48,144$ Pa $= \underline{\underline{48.14}}$ kPa

5.28 **Information and assumptions**

provided in problem statement

Find

maximum rotational speed so that no liquid escapes from the leg

At the condition of imminent spilling, the liquid will be to the top of the outside leg and the leg on the axis of rotation will have the liquid surface at the bottom of its leg.

Rotating flow equation

$$p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2 = p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2$$

Let point 1 be at top of outside leg and point 2 be at surface of liquid of inside leg. Therefore $p_1 = p_2$, $z_1 = .5$ m and $z_2 = 0$

$$\gamma \times 0.5 - (\gamma/g) \times .5^2 \omega^2/2 = 0$$
$$\omega^2 = 4g$$
$$\omega = 2\sqrt{g} = \underline{\underline{6.26}} \text{ rad/s}$$

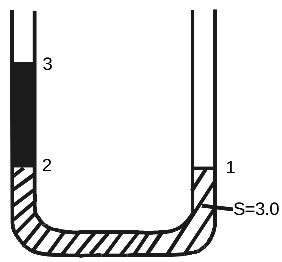
5.29 **Information and assumptions**

provided in problem statement

Find

specific gravity of other fluid

Rotating flow equation between points 1 & 2.



$$p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2 = p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2$$

where $z_2 = z_1$, $r_1 = 0$, $r_2 = 1$ ft. and $\omega = (60/60) \times 2\pi = 2\pi$ rad/s. Then

$$p_2 = (1.94 \times 3)(1^2)(2\pi)^2/2 = 114.9 \text{ psf} \tag{1}$$

Also, by hydrostatics, because there is no acceleration in the vertical

$$p_2 = 0 + \frac{1}{2} \times \gamma_f \tag{2}$$

where γ_f is the specific weight of the other fluid. Solve for γ_f between Eqs. (1) & (2)

$$\begin{aligned} \gamma_f &= 229.8 \text{ lbf/ft}^3 \\ S &= \gamma_f/\gamma_{H_2O} = 229.8/62.4 = \underline{\underline{3.68}} \end{aligned}$$

5.30 **Information and assumptions**

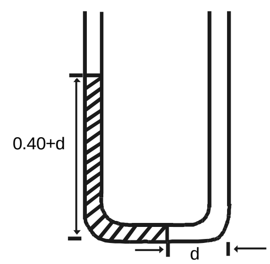
provided in problem statement

Find

new position of water surface in outside leg

Solution

A preliminary check shows that the water will evacuate the axis leg. Thus fluid configuration is shown by the figure.



Rotating flow equation between the water surface in the horizontal part of the tube and the water surface in the vertical part of the tube.

$$p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2 = p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2$$

where $r_1 = d$, $r_2 = 0.30$ m and $(z_2 - z_1) = 0.40 + d$. Then

$$\begin{aligned} (\rho\omega^2/2)(r_2^2 - r_1^2) &= \gamma(0.40 + d) \\ (1,000 \times 32.12^2/2)(0.3^3 - d^2) &= (0.40 + d)9,810 \end{aligned}$$

Solving for d yields $d = 0.278$ m

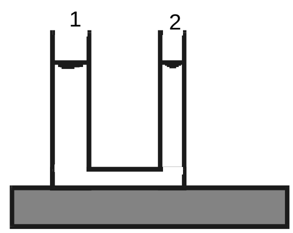
Then $z_2 = 0.40 + 0.278 =$ $\underline{\underline{0.678 \text{ m}}}$

5.31 **Information and assumptions**

provided in problem statement

Find

elevation of liquid in smaller leg of U-tube



Rotating fluid equation between the liquid surface in the large tube and the liquid surface in the small tube for conditions after rotation occurs.

$$\begin{aligned}
\gamma z_1 - (\rho/2)r_1^2\omega^2 &= \gamma z_2 - (\rho/2)r_2^2\omega^2 \\
z_1 - z_2 &= (\rho/2\gamma)(\omega^2)(r_1^2 - r_2^2) \\
&= ((\gamma/g)/(2\gamma))\omega^2(r_1^2 - r_2^2) \\
&= (\omega^2/(2g))(.4^2 - .2^2) \\
&= (4^2/(2g))(0.12) \\
&= 0.0978 \text{ m} = 9.79 \text{ cm}
\end{aligned}$$

Because of the different tube sizes a given increase in elevation in tube (1) will be accompanied by a fourfold decrease in elevation in tube (2). Then $z_1 - z_2 = 5\Delta z$ where Δz = increase in elevation in (1)

$$\Delta z = 9.79 \text{ cm } 5{=}1.96 \text{ cm or } z_1 = \underline{\underline{21.96}} \text{ cm}$$

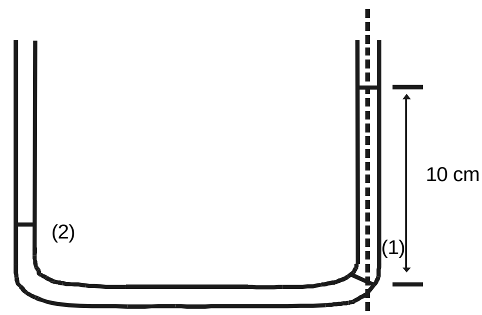
Decrease in elevation of liquid in small tube = $4\Delta z$ = 7.83 cm. Final elevation in small tube = 20 cm −7.83 cm=12.17 cm.

5.32 **Information and assumptions**

provided in problem statement

Find

rotational speed



However $p_1 = (0.10 \text{ m})(\gamma_{H_2O})$ because of hydrostatic pressure distribution in the vertical direction (no acceleration).

Rotating flow equation between pts. (1) & (2);

$$p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2 = p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2$$

where $p_2 = 0$, $z_2 - z_1 = 0.01$ m, $r_1 = 0$ and $r_2 = 1$ m. Then

$$\begin{aligned} 0.1\gamma_{H_2O} + 0 + 0 &= 0 + \gamma_{Hg} \times 0.01 - (\gamma_{Hg}/g) \times 1^2\omega^2/2 \\ \omega^2 &= ((2g)(0.01\gamma_{Hg} - 0.1\gamma_{H_2O}))/\gamma_{Hg} \\ \omega &= (2 \times 9.81)(.01 - (0.1/13.6)) \\ &= \underline{\underline{0.228 \text{ rad/s}}} \end{aligned}$$

5.33 **Information and assumptions**

provided in problem statement

Find

acceleration in g's in leg with greatest amount of oil.

Let leg 1 be the leg on the axis of rotation. Let leg 2 be the other leg of the manometer.

Rotating flow equation between the liquid surfaces of 1 & 2

$$\begin{aligned}
p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2 &= p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2 \\
0 + \gamma z_1 - 0 &= \gamma z_2 - (\gamma/g) r_2^2 \omega^2/2 \\
\omega^2 r_2^2/(2\text{g}) &= z_2 - z_1 \\
a_n &= r\omega^2 \\
&= (z_2 - z_1)(2\text{g})/r \\
&= (0.25)(2\text{g})/r_2 \\
&= (0.25)(2\text{g})/0.1 \\
a_n &= \underline{\underline{5g}}
\end{aligned}$$

5.34 **Information and assumptions**

provided in problem statement

Find

pressure at exit (point A)

Rotating flow equation from liquid surface to point A. Call the liquid surface point 1.

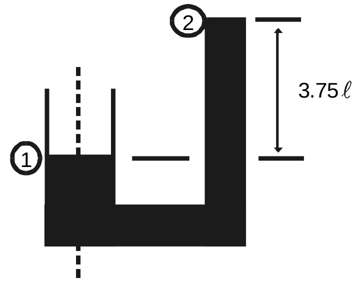
$$p_1 + \gamma z_1 - \rho r_1^2 \omega/2 = p_A + \gamma z_A - \rho r_A^2 \omega^2/2$$
$$p_A = p_1 + (\rho\omega^2/2)(r_A^2 - r_1^2) + \gamma(z_1 - z_A)$$

However $\gamma(z_1 + z_A) = 0$ in zero-g environment. Thus

$$\begin{aligned} p_A &= p_1 + ((800 \text{ kg/m}^3)/2)(6\pi/60 \text{ rad/s})^2(1.5^2 - 1^2) \\ &= 100 \text{ Pa} + 49.3 \text{ Pa} \\ &= \underline{\underline{149.3}} \text{ Pa} \end{aligned}$$

5.35 Start with **rotating flow equation**

$$p_1 + \gamma z_1 - pr_1^2\omega^2/2 = p_2 + \gamma z_2 - \rho r_2^2\omega/2$$



Let point 1 be at the liquid surface in the large tube and point 2 be at the liquid surface in the small tube. Then $p_1 = p_2 = 0$ gage, $r_2 = \ell$, and $z_2 - z_1 = 3.75\ell$.

$$\begin{aligned} \rho r_2^2\omega^2/2 &= \gamma(3.75\ell) \\ (\gamma/(2g))(\ell^2)\omega^2 &= 3.75\gamma\ell \\ \omega^2 &= \frac{7.5g}{\ell} \\ \omega &= \sqrt{7.5g/\ell} \end{aligned}$$

5.36 **Information and assumptions**

provided in problem statement

Find

level of mercury in larger lin larger leg after rotation stops.

Rotating flow equation from the liquid surface in the small tube to the liquid surface in the large tube.

$$p_s + \gamma a_s - \rho r_s^2 \omega^2/2 = p_L + \gamma z_L - \rho r_L^2 \omega^2/2$$

But $p_s = p_L$, $r_s = 0.5\ell$ and $r_L = 1.5\ell$. Then

$$\begin{aligned}
(\rho/2)\omega^2(r_L^2 - r_s^2) &= \gamma(z_L - z_s) \\
(\gamma/2g)\omega^2(1.5^2\ell^2 - 0.5^2\ell^2) &= \gamma(2\ell) \\
\omega^2 &= 2g/\ell \\
\omega &= \sqrt{2g/(1/2)} \\
&= \underline{\underline{11.35 \text{ rad/s}}}
\end{aligned}$$

Change in volume of Hg in small tube is same as in large tube. That is

$$\begin{aligned}
\forall_s &= \forall_L \\
\Delta z_s \pi d^2/4 &= \Delta z_L \pi (2d)^2/4 \\
\Delta z_s &= 4\Delta z_L
\end{aligned}$$

Also

$$\begin{aligned}
\Delta z_s + \Delta z_L &= 2\ell \\
4\Delta z_L + \Delta z_L &= 2 \times 0.5 \text{ ft} \\
\Delta z_L &= 1.0 \text{ ft}/5 = 0.20 \text{ ft}
\end{aligned}$$

Mercury level in large tube will $\underline{\underline{\text{drop 0.2 ft from it original level}}}$.

5.37 **Information and assumptions**

provided in problem statement

Find

force exerted on closed end

Rotating flow equation from the open end of the tube to the closed end.

$$p_1 = \gamma z_1 - \rho r_1^2 \omega^2/2 = p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2$$

where $z_1 = z_2$. Also let point 2 be at the closed end; therefore $r_1 = 0$ and $r_2 = 0.40$ m.

$$\begin{aligned} p_2 &= (\rho/2)(0.40^2)(60.8)^2 \\ &= 500 \times 0.16 \times 3697 \\ &= 295.73 \text{ kPa} \\ \text{Then } F &= p_2 A = 295,730 \times (\pi/4)(.01)^2 \\ &= \underline{\underline{23.2 \text{ N}}} \end{aligned}$$

5.38 **Rotating flow equation** from the mercury surface in the left tube to the mercury surface in the right tube. Then $p_\ell = p_r$ and

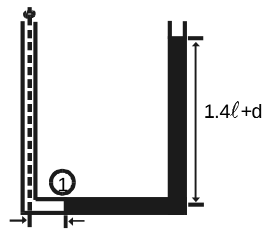
$$\begin{aligned}
\gamma z_\ell - \rho r_\ell^2 \omega^2/2 &= \gamma z_r - \rho r_r^2 \omega^2/2 \\
\omega^2(\gamma/2g)(r_r^2 - r_\ell^2) &= \gamma(z_r - z_\ell) \\
\omega^2 &= 2g(z_r - z_\ell)/(r_r^2 - r_\ell^2) \\
&= 2g(\ell)/(9\ell^2 - \ell^2) \\
\omega &= \sqrt{g/(4\ell)}
\end{aligned}$$

5.39 **Information and assumptions**

provided in problem statement

Find

(a) water level in tube at 5 rad/s, (b) water level for 15 rad/s



Rotating flow equation between the water surface and the left leg and the water surface in the right leg. At these surfaces $p_\ell = p_r = 0$ gage; therefore, for rotation at 5 rad/s

$$\begin{aligned} \gamma z_\ell - \rho r_\ell^2 \omega^2/2 &= \gamma z_r - \rho r_r^2 \omega^2/2 \\ z_\ell - z_r &= -r_r^2 \omega^2/2g = -25\ell^2/2g \end{aligned} \tag{1}$$

Also

$$z_\ell + z_r = 1.4\ell \tag{2}$$

Solving Eqs. (1) and (2) for $\ell = 0.25$ m yields

$$z_\ell = 13.5 \text{ cm and } z_r = 21.5 \text{ cm}$$

For rotation at 15 rad/s

$$\gamma z_1 - \rho r_1^2 \omega^2/2 = \gamma z_r - \rho r_r^2 \omega^2/2$$

where

$$r_1 = d \text{ and } r_r = \ell$$

Then

$$z_1 - z_r = (\omega^2/2g)(d^2 - \ell^2) \tag{3}$$

Also

$$z_r - z_1 = 1.4\ell + d \tag{4}$$

Solving Eqs. (3) and (4) for $\ell = 0.25$ yields $d = 0.140$ m. Thus

$$z_r - z_\ell = 1.4\ell + d = 1.4 \times 0.25 + 0.140 = \underline{\underline{0.49 \text{ m}}}$$

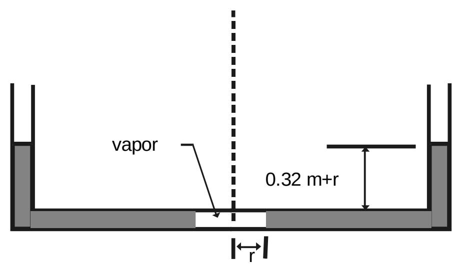
5.40 **Information and assumptions**

assume $p_V = 0$

Find

pressures at points A and B

Rotating flow equation



$$p_A + \gamma z_A - \rho r_A^2 \omega^2/2 = p_R + \gamma z_R - \rho r_R^2 \omega^2/2$$

where $p_R = 0$ gage, $r_A = 0$, $r_R = 0.64$ m and $z_R - z_A = 0.32$ m Then for a rotational speed of 8 rad/s

$$\begin{aligned} p_A &= 0.32\gamma - (\gamma/g) \times 0.64^2 \times 8^2/2 \\ p_A &= \gamma(0.32 - 0.64^2 \times 8^2/(2g)) \\ & \quad -2 \times 9,810(.32 - 1.336) \\ &= -19,936 \text{ Pa} = \underline{\underline{-19.94 \text{ kPa}}} \\ p_B &= 0.32 \times 2 \times 9,810 = \underline{\underline{6.278 \text{ kPa}}} \end{aligned}$$

Now for $\omega = 20$ rad/s solve for p_A as above

$$\begin{aligned} p_A &= \gamma(0.32 - 0.64^2 \times 20^2/(2g)) \\ -2 \times 9,810(-8.031) &= -157,560 \text{ Pa}; \end{aligned}$$

which is not possible because the liquid will vaporize. Therefore, $p_A = p_V = \underline{\underline{-101 \text{ kPa}}}$ abs. To get p_B visualize the liquid as shown in figure. Now

$$p_r - \rho r^2 \omega^2/2 = p_B - \rho \times 0.64^2 \omega^2/2$$

where $p_r = p_V = -101$ kPa, Then

$$\begin{aligned} -101,000 - \rho r^2 \omega^2/2 &= \gamma(0.32 + r) - \rho \times 0.64^2 \omega^2/2 \\ -101,000 - 1,000 r^2 \times 20^2 &= 2 \times 9,810(.32 + 4) - 1,000 \times 0.64^2 \times 20^2 \end{aligned} \tag{1}$$

Solving for r yields $r = 0.352$ m. Therefore, $p_B = (0.32 + 0.352) \times 2 \times 9,81 = 13,184$ Pa=$\underline{\underline{13.18}}$ kPa

5.41 **Information and assumptions**

provided in problem statement

Find

rotational speed when water will begin to spill from open tube.

Solution

When the water is on he verge of spilling from the open tube, the air volume in the closed part of the tube will have doubled. Therefore, we can get the pressure in the air volume with this condition.

$$p_i \forall_i = p_f \forall_f$$

and i and f refer to initial and final conditions

$$\begin{aligned} p_f &= p_i \forall_i / \forall_f = 101 \text{ kPa} \times \frac{1}{2} \\ p_f &= 50.5 \text{ kPa, abs} = -50.5 \text{ kPa, gage} \end{aligned}$$

Rotating fluid equation between water surface in leg A-A to water surface in open leg after rotation.

$$\begin{gathered} p_A + \gamma z_A - \rho r_A^2 \omega^2 / 2 = p_{\text{open}} + \gamma z_{\text{open}} - \rho r_{\text{open}} \omega^2 / 2 \\ -50.5 \times 10^3 + 0 - 0 = 0 + \gamma \times 6\ell - (\gamma/g)(6\ell)^2 \omega^2 / 2 \\ +50.5 \times 10^3 / 9,810 = -6\ell + (36\ell^2 / (2 \times 9.81))\omega^2 \\ w^2 = (+101 + 11.772)/.36 = 313.3 \\ \underline{\underline{\omega = 17.7 \text{ rad/s}}} \end{gathered}$$

5.42 **Information and assumptions**

provided in problem statement

Find

maximum operational height, z

Rotating fluid equation from point 1 in vertical pipe at level of water to point 2 at the outer edge of the rotating disk.

$$\begin{aligned}
p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2 &= p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2 \\
0 + 0 - 0 &= 0 + \gamma z_2 - (\gamma/g)(.05^2)\omega^2/2 \\
0 &= z_2 - 0.05^2 \omega^2/2g
\end{aligned}$$

But

$$\begin{aligned}
\omega &= (2,500 \text{ rev/min})(1 \text{ min}/60 \text{ s})(2\pi \text{ rad/rev}) = 261.8 \text{ rad/s} \\
z_2 &= ((0.05)(261.8))^2/(2 \times 9.81) \\
z_2 &= z = \underline{\underline{8.73}} \text{ m}
\end{aligned}$$

5.43 **Information and assumptions**

provided in problem statement

Find

pressure gradient at $z = \pm 1$

Rotating fluid equation

$$
\begin{aligned}
\partial p/\partial r + \gamma(\partial z/\partial r) &= -\rho r \omega^2 \\
\partial p/\partial z &= -\gamma - \rho r \omega^2
\end{aligned}
$$

when $z = -1$ m

$$
\begin{aligned}
\partial p/\partial z &= -\gamma - \rho\omega^2 = -\gamma - 25\rho = -\gamma(1 + 25/g) \\
&= -9,810(1 + 2.548) = \underline{\underline{-34.8 \text{ kPa/m}}}
\end{aligned}
$$

when $z = +1$ m

$$
\begin{aligned}
\partial p/\partial z &= -\gamma + 25\rho = -9,810(1 - 2.548) = \underline{\underline{15.186 \text{ kPa/m}}} \\
(\partial p/\partial z)_0 &= \underline{\underline{-9.810 \text{ kPa/m}}}
\end{aligned}
$$

5.44 Below the axis both gravity and acceleration cause pressure to increase with decrease in elevation; therefore, the maximum pressure will occur at the bottom of the cylinder. Above the axis the pressure initially decreases with elevation (due to gravity); however, this is counteracted by acceleration due to rotation. Where these two effects completely counter-balance each other is where the minimum pressure will occur ($\partial p/\partial z = 0$). Thus, above the axis:

$$\partial p/\partial z = 0 = -\gamma + r\omega^2\rho \text{ minimum pressure condition}$$

Solving: $r = \gamma/\rho\omega^2$; $p_{\min}$ occurs at $z = +\gamma/\rho\omega^2$
Then

$$p_{\max} - p_{\min} = \Delta p_{\max} = (\rho\omega^2/2)(r_0^2 - r^2) + \gamma(r_0 + r)$$

where r_0 =radius of cylinder and $r = (\gamma/\rho\omega^2)$ is the radius to point of minimum pressure

$$\begin{aligned}
\Delta p_{\max} &= (\rho\omega^2/2)[r_0^2 - (\gamma/\rho\omega^2)^2] + \gamma[r_0 + (\gamma/\rho\omega^2)] \\
\Delta p_{\max} &= \underline{\underline{\rho\omega^2 r_0^2/2 + \gamma r_0 + \gamma^2/(2\rho\omega^2)}}
\end{aligned}$$

5.45 **Information and assumptions**

provided in problem statement

Find

maximum pressure difference in tank and point of minimum pressure

Solution

From solution to Prob. 5.44 $p_{\min}$occurs at $z = \gamma/\rho\omega^2$ where $\omega = (20 \text{ ft/s})/2.0 \text{ ft} = 10 \text{ rad/s}$. Then

$$\begin{aligned}
z_{\min} &= 32.2/10^2 = \underline{\underline{0.322 \text{ ft}}} \text{ above axis} \\
\Delta p_{\max} &= 1.94 \times 10^2 \times 2.0^2/2 + 62.4 \times 2.0 + 62.4^2/(2 \times 1.94 \times 10^2) \\
\Delta p_{\max} &= \underline{\underline{523 \text{ lbf/ft}^2}}
\end{aligned}$$

5.46 **Information and assumptions**

provided in problem statement

Find

pressure at point A

Bernoulli equation

$$
\begin{aligned}
(p_1/\gamma) + (V_1^2/2g) + z_1 &= (p_2/\gamma) + (V_2^2/2g) + z_2 \\
p_1 &= p_2 + \gamma[(V_2^2 - V_1^2)/2g + z_2 - z_1] = 0 \\
p_2 &= ((-256/(2 \times 32.2)) + 15) \times 62.4 = 688 \text{ psf } = \underline{\underline{4.78 \text{ psi}}}
\end{aligned}
$$

5.47 **Information and assumptions**

provided in problem statement

Find

pressure at point A

Bernoulli equation

$$
\begin{aligned}
p_2 &= \gamma(z_1 - V_2^2/2g) = 9,810(15 - 36/(2 \times 9.81)) = 129,150 \text{ Pa} \\
&= \underline{\underline{129.15 \text{ kPa}}}
\end{aligned}
$$

5.48 **Information and assumptions**

provided in problem statement

Find

pressure change between circular and square section

First write the continuity equation between the duct sections:

$$\begin{aligned} V_c A_c &= V_s A_s \\ 100(\pi D^2/4) &= V_s D^2 \\ V_s &= 100(\pi/4) \\ &= 78.54 \text{ ft/s} \end{aligned}$$

Bernoulli equation between the two sections

$$p_c + \rho V_c^2/2 + z_c = p_s + \rho V_s^2/2 + z_s$$

The density is $\rho = \gamma/g = 0.075/32.2 = 0.00233$ slugs/ft^3

Then

$$\begin{aligned} p_c - p_s &= (\rho/2)(V_s^2 - V_c^2) \\ &= (0.00233/2)(78.54^2 - 100^2) \\ p_c - p_s &= \underline{\underline{-4.46}} \text{ lbf/ft}^2 \end{aligned}$$

5.49 **Information and assumptions**

provided in problem statement

Find

deflection of manometer

Solution

Let section 1 be in the large duct where the manometer pipe is connected and section 2 in the smaller duct at the level where the upper manometer pipe is connected. Assume uniform air density.

Continuity equation

$$\begin{aligned} V_1 A_1 &= V_2 A_2 \\ V_2 &= V_1 (A_1/A_2) \\ &= 100(2) \\ &= 200 \text{ ft/s} \end{aligned}$$

Bernoulli equation from 1 to 2

$$\begin{aligned} p_1 + \rho V_1^2/2 &= p_2 + \rho V_2^2/2 \\ p_1 - p_2 &= (1/2)\rho(V_2^2 - V_1^2) \\ &= (1/2)(0.0644/32.2)(40,000 - 10,000) \\ &= 30 \text{ psf} \end{aligned}$$

Manometer deflection

$$\begin{aligned} p_1 - p_2 &= \Delta h(\gamma_{\text{liquid}} - \gamma_{\text{air}}) \\ 30 &= \Delta h(120 - .0644) \\ \Delta h &= \underline{\underline{0.25 \text{ ft.}}} \end{aligned}$$

5.50 $p_A - p_B$

Bernoulli equation

$$p_A - p_B = \gamma[(V_B^2 - V_A^2)/2g - z_A] = 62.4[(400-64)/(2\times 32.2) - 1] = \underline{\underline{\underline{263.2 \text{ psf}}}}$$

5.51 **Information and assumptions**

provided in problem statement

Find

height h jet will rise

Bernoulli equation from the nozzle to the top of the jet. Let point 1 be in the jet at the nozzle and point 2 at the top.

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2$$

where $p_1 = p_2 = 0$ gage

$$\begin{aligned} V_1 &= 20 \text{ ft/s} \\ V_2 &= 0 \\ 0 + (20)^2/2g + z_1 &= 0 + 0 + z_2 \\ z_2 - z_1 &= h = 400/64.4 \\ &= \underline{\underline{6.21 \text{ ft}}} \end{aligned}$$

5.52 **Information and assumptions**

provided in problem statement

Find

force required to drive piston

Continuity equation

$$
\begin{aligned}
V_1 A_1 &= V_2 A_2 \\
V_2 &= V_1 (D/d)^2 = 4 \times (4/2)^2 = 16 \text{ ft/s}
\end{aligned}
$$

Bernoulli equation

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g &= V_2^2/2g \\
p_1 &= \gamma(V_2^2/2g - V_1^2/2g) = 233 \text{ psf}
\end{aligned}
$$

Then $F_{\text{piston}} = p_1 A_1 = 233(\pi/4) \times (4/12)^2 = \underline{\underline{20.3 \text{ lbf}}}$

5.53 **Information and assumptions**

provided in problem statement

Find

gage pressure in pipe

Bernoulli equation

$$p_1/\gamma + V_1^2/2g + z_1 = p_j/\gamma + V_j^2/2g + z_j$$

where 1 and j refer to conditions in pipe and jet, respectively

$$\begin{aligned}
V_1 &= Q/A_1 \\
&= 20/((\pi/4) \times 1.0^2) = 25.5 \text{ ft/s} \\
V_j A_j &= V_1 A_1; V_j = V_1 A_1/A_j \\
V_j &= 25.5 \times 4 = 101.9 \text{ ft/s}
\end{aligned}$$

Also $z_1 = z_j$ and $p_j = 0$. Then

$$\begin{aligned}
p_1/\gamma &= (V_j^2 - V_1^2)2g \\
p_1 &= \gamma(V_j^2 - V_1^2)/2g \\
&= 62.4(101.9^2 - 25.5^2)/64.4 \\
&= \underline{\underline{9,423 \text{ psf}}} \\
&= \underline{\underline{65.4 \text{ psi}}}
\end{aligned}$$

5.54 **Information and assumptions**

provided in problem statement

Find

pressure in the air at $x = r_o$, $1.1r_o$ and $2r_o$

Bernoulli equation

$$p_0 + \rho V_0^2/2 = p_x + \rho V_x^2/2$$

where $p_0 = 0$ gage. Then

$$\begin{aligned}
p_x &= (\rho/2)(V_0^2 - V_x^2) \\
V_x &= u = U_0(1 - r_0^3/x^3) \\
V_{x=r_0} &= U_0(1-1) = 0 \\
V_{x=r_0} &= U_0(1 - 1/1.1^3) = 7.46 \text{ m/s} \\
V_{x=2r_0} &= U_0(1 - 1/2^3) = 26.25 \text{ m/s}
\end{aligned}$$

Finally

$$\begin{aligned}
p_{x=r_0} &= (1.2/2)(30^3 - 0) = \underline{\underline{540}} \text{ Pa, gage} \\
p_{x=1.1r_0} &= (1.2/2)(30^2 - 7.46^2) = \underline{\underline{507}} \text{ Pa, gage} \\
p_{x=2r_0} &= (1.2/2)(30^2 - 26.25^2) = \underline{\underline{127}} \text{ Pa, gage}
\end{aligned}$$

5.55 **Information and assumptions**

provided in problem statement

Find

$$
\begin{aligned}
V &= K/r \\
Q &= \int V dA = \int V L dr = L \int (K/r) dr = KL\ell n(r_2/r_1) \qquad (1) \\
\Delta p &= (1/2)\rho(V_1^2 - V_2^2) \\
\Delta p &= (1/2)\rho((K^2/r_1^2) - (K^2/r_2^2)) = (K^2\rho/2)((r_2^2) - (r_1^2))/(r_1^2 r_2^2) \qquad (2)
\end{aligned}
$$

Solution

Eliminate K between Eqs. (1) and (2) yielding:

$$
\begin{aligned}
(2\Delta p/\rho) &= ((Q^2)/(L^2(\ell n(r_2 r_1))^2))(r_2^2 - r_1^2)/(r_1^2 r_2^2) \\
A_c &= L(r_2 - r_1) \\
&\therefore \; 2\Delta p/\rho = (Q^2/A_c^2)(r_2 - r_1)^2(r_2^2 - r_1^2)/(r_1^2 r_2^2(\ell n(r_2/r_1))^2) \\
Q &= A_c\sqrt{2\Delta p/\rho}(r_1 r_2 \ell n(r_2/r_1))/((r_2 - r_1)(r_2^2 - r_1^2)^{0.5}) \\
Q &= A_c\sqrt{2\Delta p/\rho}\underline{\underline{((r_2/r_1)\ell n(r_2/r_1))/((r_2/r_1 - 1)((r_2^2/r_1^2) - 1)^{0.5})}}
\end{aligned}
$$

For $r_2/r_1 = 1.5$ the $f(r_2/r_1)$ is evaluated

$$
f(r_2/r_1) = 1.5\ell n 1.5/(0.5 \times 1.25^{0.5}) = \underline{\underline{1.088}}
$$

5.56 **Information and assumptions**

provided in problem statement

Find

depth of liquid from inside to outside radius

Bernoulli equation between the outside of the bend at the surface (point 2) and the inside of the bend at the surface (point 1):

$$\begin{aligned}(p_2/\gamma) + V_2^2/2g + z_2 &= (p_1/\gamma) + V_1^2/2g + z_1 \\ 0 + V_2^2/2g + z_2 &= 0 + V_1^2/2g + z_1 \\ z_2 - z_1 &= V_1^2/2g - V_2^2/2g\end{aligned}$$

where $V_2 = (1/3)$ m/s; $V_1 = (1/1)$ m/s. Then

$$z_2 - z_1 = (1/2g)(1^2 - 0.33^2) = \underline{\underline{0.045 \text{ m}}}$$

5.57 **Information and assumptions**

provided in problem statement

Find

pressure at point B

Continuity equation

$$
\begin{aligned}
V_A &= Q/A_A = 70/(\pi/4 \times 6^2) = 2.476 \text{ ft/s} \\
V_B &= Q/A_B = 70/(\pi/4 \times 2^2) = 22.28 \text{ ft/s}
\end{aligned}
$$

Bernoulli equation

$$
\begin{aligned}
p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/2g + z_B \\
p_B/\gamma &= 3500/62.4 - 2.48^2/64.4 - 22.28^2/64.4 - 4 \\
p_B &= 2775 \text{ lbf/ft}^2 = \underline{\underline{19.3}} \text{ lbf/in}^2
\end{aligned}
$$

5.58 **Information and assumptions**

provided in problem statement

Find

pressure at point E

Bernoulli equation

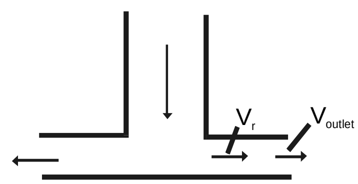
$$
\begin{aligned}
p_c/\gamma + V_c^2/(2g) + z_c &= p_E/\gamma + V_E^2/(2g) + z_E \\
(15 \times 144)/\gamma + 10^2/(2g) + z_c &= p_E/\gamma + 50^2/(2g) + z_E \\
p_E/\gamma &= ((15 \times 144)/\gamma) + (1/2g)(10^2 - 50^2) + z_c - z_E \\
p_E &= 15 \times 144 + ((62.4)/(64.4)(-2,400)) + 62.4(3-1) \\
&= 2,160 \text{ psf } - 2,325 \text{ psf} + 124.8 \text{ psf} \\
p_E &= \underline{\underline{-40.2 \text{ psf }}} = -0.28 \text{ psi}
\end{aligned}
$$

5.59 **Information and assumptions**

provided in problem statement

Find

pressure force on disk



Bernoulli equation from a point at radius r to the outlet:

$$\begin{aligned}
p_r + \rho v_r^2/2 &= 0 + \rho v_{\text{outlet}}^2/2 \\
V &= Q/A \\
&= Q/(2\pi r h) \\
&= Q/(2\pi \times 0.01 r) \\
&= (0.380 \text{ m}^3/\text{s})/(0.02\pi \text{ rm}^2) \\
&= (6.048/r) \text{ m/s}
\end{aligned}$$

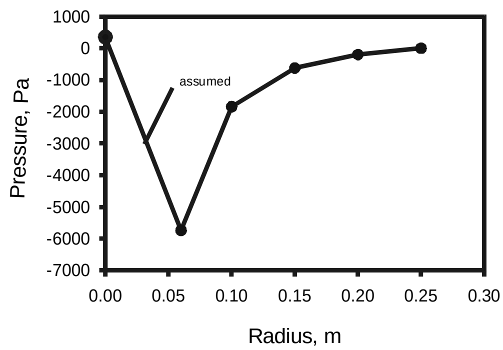
Then

$$\begin{aligned}
v_{\text{outlet}} &= 6.048/0.25 = 24.19 \text{ m/s} \\
p_r &= (\rho/2)(24.19^2 - v_r^2) \\
p_r &= (\rho/2)(24.19^2 - (6.048/r)^2) \\
p_r &= (1.2/2)(585.2 - (36.58)/r^2)
\end{aligned}$$

A table of p_r vs. r is given below:

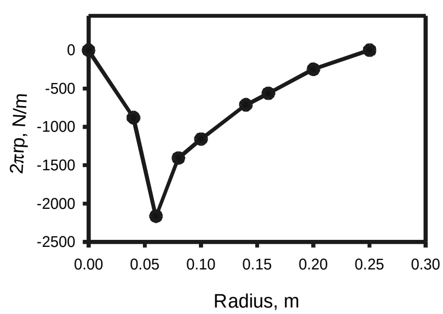
$r(m)$	0.06	0.10	0.15	0.20	0.25
p_r(Pa)	-5,745	-1,844	-624	-198	0

Because the center of the disk is a stagnation point, the pressure there will be $\rho v_{\text{outlet}}^2/2$ or +351 Pa. The pressure variation on the disk is plotted below:



The force on the disk will be $\int pdA = \int p2\rho rdr$. So plot $p \times (2\pi r)$ vs. r. The area under the curve will be desired force. This assumes that zero gage pressure prevails on the other side of the disk.

r, m	0	0.04m	0.06	0.08	0.10	0.14	0.16	0.20	0.25
p, Pa	351	-3,500	-5,745	-2,800	-1,844	-810	-560	-198	0
$2\pi rp$	0	-880	-2,166	-1,407	-1,159	-712	-563	-249	0



Force=175 N acting upward

5.60 **Information and assumptions**

provided in problem statement

Find

deflection of manometer in cm.

Solution

$$\begin{aligned}
C_p &= 1 - 4\sin^2\theta \\
C_{p,50} &= 1 - 4\sin^2 50^\circ \\
&= 1 - 4(0.766)^2 = -1.347 \\
C_{p,10} &= 1 - 4(0.174)^2 = +0.879 \\
C_{p,10} - C_{p,50} &= 0.879 - (-1.347) = 2.226 \\
\Delta p &= \gamma_{\mathrm{H_2O}}\Delta h = \Delta C_p \rho_{\mathrm{air}} V_0^2/2 \\
\Delta h &= 2.226(1.2 \text{ kg/m}^3)(50 \text{ m/s})^2/(2 \times 9,810 \text{ N/m}^3) \\
\Delta h &= 0.340 \text{ m} = \underline{\underline{34.0 \text{ cm}}}
\end{aligned}$$

5.61 **Information and assumptions**

provided in problem statement

Find

pressure difference between highest and lowest pressure.

The maximum pressure will occur at the point of least velocity which will be at the stagnation point where $V = 0$. The point of lowest pressure will be where the velocity is highest ($V = 60$ m/s).

Bernoulli equation between points of highest and lowest pressure and assume hydrostatic effects are negligible.

$$\begin{aligned}
p_h + \rho V_h^2/2 &= p_\ell + \rho V_\ell^2/2 \\
p_h + 0 &= p_\ell + (\rho/2)(80^2) \\
p_h - p_\ell &= (1.2/2)(6,400) \\
&= \underline{\underline{3,840 \text{ Pa}}} \\
&= \underline{\underline{3.84 \text{ kPa}}}
\end{aligned}$$

5.62 The flow is non-uniform. Check to see if it is irrotational by seeing if it satisfies Bernoulli's equation.

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 \\
(10,000/9,810) + (1/(2 \times 9.81)) + 0 &= (7,000/9,810) + 2^2(2 \times 9.81) \\
1.070 &\neq 0.917
\end{aligned}
$$

Flow is rotational. The correct choice is c.

5.63 **Information and assumptions**

provided in problem statement

Find

airspeed

Solution

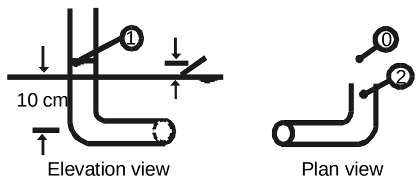
$$\begin{aligned}
\frac{1}{2}\rho V^2 &= \Delta p = \gamma_{\mathrm{H_2O}}(9/12) \\
\rho &= p/(RT) = (10)(144)/((1,716)(483)) = 0.00173 \text{ slugs/ft}^3 \\
V^2 &= 2(62.4 \text{ lbf/ft}^3)(9/12) \text{ ft}/(0.00173 \text{ slugs/ft}^3) \\
&= 54,104 \text{ ft}^2/\text{s}^2 \\
V &= \underline{\underline{233 \text{ ft/sec}}}
\end{aligned}$$

5.64 **Information and assumptions**

provided in problem statement

Find

location of liquid surface in central tube



Rotational flow equation from pt. 1 to pt. 2 where pt. 1 is at liquid surface in vertical part of tube and pt. 2 is just inside the open end of the pitot tube.

$$\begin{aligned} p_1/\gamma - V_1^2/2g + z_1 &= p_2/\gamma - V_2^2/2g + z_2 \\ 0 - 0 + (0.10 + \ell) &= p_2/\gamma - r^2\omega^2/2g - 0 \end{aligned} \tag{1}$$

where $z_1 = z_2$. If we reference the velocity of the liquid to the tip of the pitot tube then we have steady flow and Bernoulli's equation will apply from pt. 0 (point ahead of the pitot tube) to point 2 (point at tip of pitot tube).

$$\begin{aligned} p_0/\gamma - V_0^2/2g + z_0 &= p_2/\gamma - V_2^2/2g + z_2 \\ 0.1\gamma/\gamma + r^2\omega^2/2g &= p_2/\gamma + 0 \end{aligned} \tag{2}$$

Solve Eqs. (1) & (2) for ℓ

$\underline{\underline{\ell = 0}}$, liquid surface in the tube is the same as the elevation as outside liquid surface.

5.65 **Information and assumptions**

provided in problem statement

Find

find velocity

Solution

$$V = (2 \times 32.2 \times 10/12)^{1/2} = \underline{\underline{7.33 \text{ fps}}}$$

5.66 **Information and assumptions**

provided in problem statement

Find

rise in vertical leg relative to water surface

Solution

$$
\begin{aligned}
V &= \sqrt{2gh} \\
h &= V^2/2g = 3^2/(2 \times 9.81) = 0.459 \text{ m} = \underline{\underline{45.9}} \text{ cm}
\end{aligned}
$$

5.67 Because it is a Bourdon tube gage, the difference in pressure that is sensed will be between the stagnation point and the separation zone downstream of the plate.

Therefore

$$
\begin{aligned}
\Delta C_p &= 1-(C_{p,\text{back of plate}}) \\
\Delta C_p &= 1-(\text{neg. number}) \\
\therefore \Delta p/(\rho V_0^2/2) &= 1+\text{postive number} \\
\Delta p &= (\rho V_0^2/2)(1+\text{postive number})
\end{aligned}
$$

case (c) is the correct choice.

5.68 **Information and assumptions**

provided in problem statement

Find

velocity

Ideal gas law

$$\rho = p/RT = 15 \times 144/(1,715)(60+460) = 0.00242 \text{ slugs/ft}$$

Pitot tube equation

$$V = (2\Delta p/\rho)^{1/2} = (2 \times 62.4 \times (2.0/12)/0.00242)^{1/2} = \underline{\underline{92.7 \text{ fps}}}$$

5.69 **Information and assumptions**

provided in problem statement

Find

velocity at station 2

$$p_1/\gamma + V_1^2/2g = p_2/\gamma + V_2^2/2g = p_t/\gamma$$

Manometer equation

$$\begin{aligned} p_1 + 0.1 \times 9810 - \overbrace{0.1 \times 1.2 \times 9.81}^{\text{neglect}} &= p_t \\ p_t - p_1 &= 981 \text{ N/m}^2 = \rho V_1^2/2 \\ V_1 &= 40.4 \text{ m/s} \end{aligned}$$

Continuity equation

$$\begin{aligned} V_2 A_2 &= V_1 A_1 \\ V_2 &= V_1(A_1/A_2) = 2 \times 40.4 = \underline{\underline{80.8}} \text{ m/s} \end{aligned}$$

5.70 **Information and assumptions**

provided in problem statement

Find

free stream velocity, V_o

Let point 1 be the stagnation point and point 2 at 90° around the sphere.

Bernoulli equation between the two points.

$$\begin{aligned}
p_1 + \rho V_1^2/2 &= p_2 + \rho V_2^2/2 \\
p_1 + 0 &= p_2 + \rho(1.5V_0)^2/2;\ p_1 - p_2 = 1.125\rho V_0^2 \\
V_0^2 &= 3,000/(1.125 \times 1,000) = 2.67 \\
\underline{\underline{V_0}} &= \underline{\underline{1.63 \text{ m/s}}}
\end{aligned}$$

5.71 **Information and assumptions**

Assume $z_1 = z_2$:
provided in problem statement

Find

value of θ

Bernoulli equation between the stagnation point and the second pressure tap.

$$\begin{aligned}
p_1 + \rho V_1^2/2 &= p_2 + \rho V_2^2/2 \\
p_1 - p_2 &= \rho V_2^2/2 \\
&= (1.5V_0 \sin\theta)^2(\rho/2) \\
p_1 - p_2 &= \Delta p = 2.25V_0^2 \sin^2\theta(\rho/2) \qquad (1) \\
\text{Given} &: V_0 = \sqrt{\Delta p/\rho}; V_0^2 = \Delta p/\rho \qquad (2)
\end{aligned}$$

Given:$V_0 = \sqrt{\Delta p/\rho}; V_0^2 = \Delta p/\rho$, solve Eqs. (1) & (2) for θ

$$\underline{\underline{\theta = 70.53^\circ}}$$

5.72 **Information and assumptions**

provided in problem statement

Find

a) angle θ for pressure tap, b) equation for free-stream velocity, c) effect of offset angle β

Solution

a) **Bernoulli equation** between the free stream and the location of the pressure tap gives

$$p_o + \frac{1}{2}\rho V_o^2 = p + \frac{1}{2}1.5^2 V_o^2 \sin^2\theta$$

But at the pressure tap location $p = p_o$ so

$$2.25\sin^2\theta = 1$$

Solving for θ gives

$$\theta = \underline{\underline{41.8^o}}$$

b) Writing the Bernoulli equation between the stagnation point, tap A, and pressure tap B gives

$$p_A = p_B + \frac{1}{2}1.5^2\rho V_o^2\sin^2\theta = p_B + \frac{1}{2}1.5^2\rho V_o^2\frac{1}{2.25}$$

or

$$\underline{\underline{V_o = \sqrt{\frac{2(p_A - p_B)}{\rho}}}}$$

c) Let the pressure tap on the axis of the probe be tap A and the other one tap B. The pressure at tap A would be

$$p_A = p_o - \frac{1}{2}\rho V_o^2 1.5^2\sin^2\beta = p_o - 1.125\rho V_o^2\sin^2\beta$$

The pressure at tap B would be

$$p_B = p_o - 1.125\rho V_o^2\sin^2(\beta + 41.8^o)$$

The pressure difference would be

$$p_A - p_B = 1.125\rho V_o^2 \left[\sin^2(\beta + 41.8^o) - \sin^2\beta\right]$$

Solving for the velocity gives

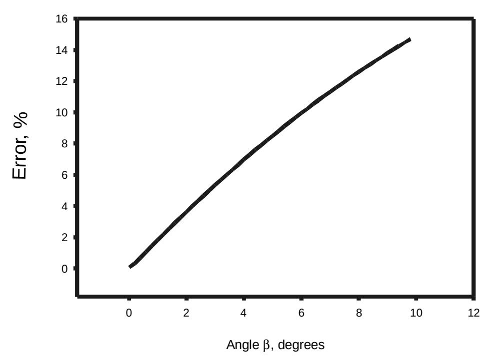
$$V_o = \sqrt{\frac{p_A - p_B}{1.125\rho\left[\sin^2(\beta + 41.8^o) - \sin^2\beta\right]}}$$

which will designated at the "true" velocity, V_T. The "indicated" velocity, V_I, is the one calculated assuming that tap A is at the stagnation point. The ratio of the indicated velocity to the true velocity would be

$$\frac{V_I}{V_T} = \sqrt{2.25\left[\sin^2(\beta + 41.8^o) - \sin^2\beta\right]}$$

The error is

$$\text{error} = \frac{V_T - V_I}{V_T} = 1 - \frac{V_I}{V_T}$$



5.73 Three pressure taps could be located on a sphere at an equal distance from the nominal stagnation point. The taps would be at intervals of 120^o. Then when the probe is mounted in the stream, its orientation could be changed in such a way as to make the pressure the same at the three taps. Then the axis of the probe would be aligned with the freestream velocity.

5.74 **Information and assumptions**

provided in problem statement

Find

possibility of cavitation

Solution

Check minimum pressure value. Minimum pressure will occur where streamlines have smallest spacing (inside of bend). Thus

$$\begin{aligned} p_{\min} + \rho V_m^2/2 &= p_1 + \rho V_1^2/2 \\ V_{\min} \times n_{\min} &= V_2 n_2 \end{aligned}$$

where n is streamline spacing

$$V_{\min} = V_1 \times n_1/n_{\min} = V_1 \times 2.6/1.3$$

scaled from figure, $V_{\min} = 2V_1$

$$p_{\min} = p_1 + (\rho/2)(-3V_1^2) = 110{,}000 + 500(-3 \times 13^2) = \underline{\underline{-143 \text{ kPa, abs}}}$$

$p_{\min}$ is less than p_{vapor}; therefore, <u>cavitation will occur</u>.

5.75 **Information and assumptions**

provided in problem statement

Find

discharge for incipient cavitation

Cavitation will occur when the pressure reaches the vapor pressure of the liquid ($p_V = 1,230$ Pa abs).

Bernoulli equation

$$p_A + \rho V_A^2/2 = p_{\text{throat}} + \rho V_{\text{throat}}^2/2$$

where $V_A = Q/A_A = Q/((\pi/4) \times 0.40^2)$

Continuity equation

$$\begin{gathered} V_{\text{throat}} = Q/A_{\text{throat}} = Q/((\pi/4) \times 0.10^2) \\ \rho/2(V_{\text{throat}}^2 - V_A^2) = p_A - p_{\text{throat}} \\ (\rho Q^2/2)[1/((\pi/4) \times 0.10^2)^2 - 1/[((\pi/4) \times 0.40^2)^2] \\ = 220,000 - 1,230 \\ 500Q^2(16,211 - 63) = 218,770 \\ Q = \underline{\underline{0.165 \text{ m}^3/\text{s}}} \end{gathered}$$

5.76 **Information and assumptions**

provided in problem statement

Find

$$V = \sqrt{2g\Delta h} = \sqrt{2\Delta p/\rho}$$

Solution

The Δp is the same for both; however,

$$\rho_w >> \rho_a$$

Therefore $V_A > V_W$ (case b).

5.77 **Information and assumptions**

provided in problem statement

Find

the velocity

$$\Delta p = \Delta h(\gamma_{\text{HG}} - \gamma_{\text{ker}}) = (6/12)(847 - 51) = 398 \text{ psf}$$

Pitot equation

$$V = (2\Delta p/\rho)^{1/2} = (2 \times 398/1.58)^{1/2} = \underline{\underline{\underline{22.4 \text{ fps}}}}$$

5.78 **Information and assumptions**

provided in problem statement

Find

air velocity

Pitot tube equation

$$V = (2\Delta p/\rho)^{1/2} = (2 \times 3{,}000/1.2)^{1/2} = \underline{\underline{70.7 \text{ m/s}}}$$

5.79 **Information and assumptions**

provided in problem statement

Find

air velocity

Pitot tube equation

$$V = (2 \times 11/0.00237)^{1/2} = \underline{\underline{96.3 \text{ fps}}}$$

5.80 **Information and assumptions**

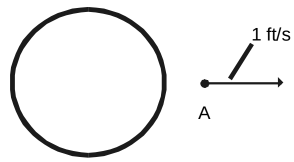
Find

gas velocity in duct

Pitot tube equation

$$
\begin{aligned}
p_{\text{stagn}} - p_{\text{static}} &= \rho V_0^2/2 \\
0.9 \times 144 \text{ psf} &= (0.12/32.2) V_0^2/2 \\
V_0 &= \underline{\underline{264 \text{ ft/sec}}}
\end{aligned}
$$

5.81



By referencing velocities to the spheres a steady flow case will be developed. Thus, for the steady flow case $V_0 = 11$ ft/s and $V_A = 10$ ft/s. Then when Bernoulli's equation is applied between points 0 and A it will be found that $\underline{\underline{p_A/p_0 > 1 \text{ (case c)}}}$.

5.82 **Information and assumptions**

provided in problem statement

Find

pressure difference $p_B - p_C$

Bernoulli equation

$$p_B - p_C = (1,000/2)[(13-3)^2 - (13-5)^2] = 18,000 \text{ Pa} = \underline{\underline{18 \text{ kPa}}} \quad (1)$$

5.83 **Information and assumptions**

provided in problem statement

Find

pressure difference $p_B - p_C$

Bernoulli equation can be applied if all velocities are relative to ship. Thus,

$$
\begin{aligned}
V_{A,\mathrm{rel}} &= \sqrt{0.2^2 + 7^2} = 7.003 \text{ m/s} \\
V_{B,\mathrm{rel}} &= 7.70 \text{ m/s} \\
p_A + \rho V_A^2/2 &= p_B + \rho V_B^2/2 \\
p_A - p_B &= (\rho/2)(V_B^2 - V_A^2) \\
&= (1,050/2)(7.7^2 - 7.003^2) \\
&= 5,380 \text{ Pa} = \underline{\underline{5.380 \text{ kPa}}}
\end{aligned}
$$

5.84 Both gage A and B will read the same, due to hydrostatic pressure distribution in the vertical in both cases.

5.85 **Information and assumptions**

provided in problem statement

Find

air flow rate

The side tube samples the pressure for the undisturbed flow and the central tube senses the stagnation pressure.

Bernoulli equation

$$\begin{aligned} p_0 + \rho V_0^2/2 &= p_{\text{stagn.}} + 0 \\ \text{or } V_0 &= \sqrt{(2/\rho)(p_{\text{stagn.}} - p_0)} \end{aligned}$$

But

$$\begin{aligned} p_{\text{stagn.}} - p_0 &= (0.067 - 0.023)\sin 30^\circ \times 0.7 \times 9,810 = 151.1 \text{ Pa} \\ \rho &= p/RT = 150,000/(287 \times (273 + 20)) = 1.784 \text{ kg/m}^3 \end{aligned}$$

Then

$$\begin{aligned} V_0 &= \sqrt{(2/1.784)(151.3)} = \underline{\underline{13.02 \text{ m/s}}} \\ Q &= VA = 13.02 \times (\pi/4) \times 0.10^2 = \underline{\underline{0.1022 \text{ m}^3\text{/s}}} \end{aligned}$$

5.86 **Information and assumptions**

provided in problem statement

Find

gas velocity

Solution

$$
\begin{aligned}
\Delta C_p &= 1.4 = (p_A - p_B)/(\rho V_0^2/2) \\
V_0^2 &= 2(4,000)/((1.5) \times 1.4); V_0 = \underline{\underline{61.7 \text{ m/s}}}
\end{aligned}
$$

5.87 **Information and assumptions**

provided in problem statement

Find

velocity of stack gases

Solution

$$
\begin{aligned}
\rho &= p/RT = 101,000/(200 \times (250 + 273)) \\
&= 0.966 \text{ kg/m}^2 \qquad (4) \\
\Delta p &= \gamma_{\text{water}} \Delta h = 9,790 \times 0.008 = 78.32 \text{ Pa} \\
(p_A - p_B) &= \Delta p = 78.32 \text{ Pa} \qquad (1) \\
(p_A - p_0)/(\rho V_0^2/2) &= 1.0 \leftarrow c_{p_A} \\
(p_B - p_0)/(\rho V_0^2/2) &= -0.3 \leftarrow c_{p_B} \\
\text{Then } (p_A - p_B)/(\rho V_0^2/2) &= 1.3 \qquad (2)
\end{aligned}
$$

Solving Eq's (1) & (2) yields

$$\rho V_0^2/2 = 78.32/1.3; V_0 = \underline{\underline{11.17 \text{ m/s}}}$$

5.88 **Information and assumptions**

provided in problem statement

Find

free-stream velocity

Let point 1 be at the stagnation point and point 2 be at the 90° position. At the 90° position $U = 1.5U \sin\Theta = 1.5U$.

Bernoulli equation between points 1 and 2.

$$
\begin{aligned}
p_1 + \overbrace{\rho V_1^2/2}^{=0} &= p_2 + \rho V_2^2/2 \\
p_1 - p_2 &= \rho V_2^2/2 \\
(\gamma_{Hg} - \gamma_{H_2O})\Delta h &= (\rho/2)(1.5U)^2 \\
((\gamma_{Hg}/\gamma_{H_2O}) - 1)\Delta h &= (1/2g)(1.5U)^2 \\
(13.6 - 1)\Delta h &= (1/2g)(2.25)U^2 \\
U &= \underline{\underline{2.34}} \text{ m/s}
\end{aligned}
$$

5.89 **Information and assumptions**

provided in problem statement

Find

gage pressure

$$C_p = (p - p_0)/(\rho V^2/2)$$

Bernoulli equation from the free stream to the point of separation:

$$\begin{aligned} p_0 + \rho U^2/2 &= p + \rho u^2/2 \\ p - p_0 &= (\rho/2)(U^2 - u^2) \end{aligned}$$

or

$$(p - p_0)/(\rho U^2/2) = (1 - (u/U)^2)$$

but

$$\begin{aligned} u &= 1.5U \sin\theta \\ u &= 1.5U \sin 120^\circ \\ u &= 1.5U \times 0.866 \end{aligned}$$

At the separation point

$$\begin{aligned} (u/U) &= 1.299 \\ (u/U)^2 &= 1.687 \\ C_p &= 1 - 1.687 \\ &= \underline{\underline{-0.687}} \\ p_{\text{gage}} &= C_p(\rho/2)U^2 \\ &= (-0.687)(1.2/2)(100^2) \\ &= -4,122 \text{ Pa} = \underline{\underline{-4.122 \text{ kPa}}} \end{aligned}$$

5.90 **Information and assumptions**

provided in problem statement

Find

free-stream velocity

$$
\begin{aligned}
u &= 1.5U\sin\theta \\
u_{\theta=90^\circ} &= 1.5U(1) \\
&= 1.5U
\end{aligned}
$$

Bernoulli equation between the stagnation point (forward tap) and the side tap where $u = 1.5U$. Neglect elevation difference.

$$
\begin{aligned}
p_1 + \rho V_1^2/2 &= p_2 + \rho V_2^2/2 \\
p_1 - p_2 &= (\rho/2)(V_2^2 - V_1^2) \\
p_1 - p_2 &= (1.2/2)((1.5U)^2 - 0) \\
120 &= 1.35U^2 \\
U &= \underline{\underline{9.43 \text{ m/s}}}
\end{aligned}
$$

5.91 **Information and assumptions**

provided in problem statement

Find

true airspeed

Pitot tube equation

$$V = K\sqrt{2\Delta p/\rho}$$

then

$$\begin{aligned} V_{\text{calibr.}} &= (K/\sqrt{\rho_{\text{calibr.}}})\sqrt{2\Delta p} \\ V_{\text{true}} &= (K/\sqrt{\rho_{\text{true}}})\sqrt{2\Delta p} \qquad (1) \\ V_{\text{indic.}} &= (K/\sqrt{\rho_{\text{calib.}}})\sqrt{2\Delta p} \qquad (2) \end{aligned}$$

Divide Eq. (1) by Eq. (2):

$$\begin{aligned} V_{\text{true}}/V_{\text{indic.}} &= \sqrt{\rho_{\text{calib.}}/\rho_{\text{true}}} = [(101/70) \times (273 - 6.3)/(273 + 17)]^{1/2} = 1.15 \\ V_{\text{true}} &= 60 \times 1.15 = \underline{\underline{69.3}} \text{ m/s} \end{aligned}$$

5.92 Assume the bottom of the tube through which water will be drawn is 5 in. below the neck of the atomizer. Therefore if the atomizer is to operate at all, the pressure in the necked down portion must be low enough to draw water 5 in. up the tube. In other words p_{neck} must be $-(5/12)\gamma_{\text{water}} = -26$ psfg. Let the outlet diameter of the atomizer be 0.5 in. and the neck diameter be 0.25 in. Assume that the change in area from neck to outlet is gradual enough to prevent separation so that the Bernoulli equation will be valid between these sections. Thus

$$p_n + \rho V_n^2/2 = p_0 + \rho V_0^2/2$$

were n and 0 refer to the neck and outlet sections respectively. But

$$p_n = -26 \text{ psfg and } p_0 = 0$$

or

$$-26 + \rho V_0^2/2 = \rho V_0^2/2 \tag{1}$$

$$\begin{aligned} V_n A_n &= V_0 A_0 \\ V_n &= V_0 A_0/A_n \\ &= V_0(.5/.25)^2 \\ V_n &= 4V_0 \end{aligned} \tag{2}$$

Eliminate V_n between Eqs. (1) and (2)

$$\begin{aligned} -26 + \rho(4V_0)^2/2 &= \rho V_0^2/2 \\ -26 + 16\rho V_0^2/2 &= \rho V_0^2/2 \\ 15\rho V_0^2/2 &= 26 \\ V_0 &= ((52/15)/\rho)^{1/2} \end{aligned}$$

Assume $\rho = 0.0024$ slugs/ft^2

$$\begin{aligned} V_0 &= ((52/15)/0.0024)^{1/2} \\ &= 38 \text{ ft/s} \\ Q &= VA = 38 \times (\pi/4)(.5/12)^2 \\ &= .052 \text{ cfs} \\ &= 3.11 \text{ cfm} \end{aligned}$$

One could use a vacuum cleaner (one that you can hook the hose to the discharge end) to provide the air source for such an atomizer.

5.93 **Information and assumptions**

from Table A.5 $p_v(15°) = 1,700\ Pa$
from Table A.5 $\rho = 999$ kg/m^3

Find

a) velocity of water at exit for maximum lift, b) discharge c) maximum load supportable by suction cup
Venturi exit area, $A_e, = 10^{-3}$ m^2, Venturi throat area, $A_t = (1/4)A_e$, Suction cup area, $A_s = 0.1$ m^2

$$\begin{aligned} p_{\text{atm}} &= 100 \text{ kPa} \\ T_{\text{water}} &= 15° \text{ C} \end{aligned}$$

Bernoulli equation for the Venturi from the throat to exit with the pressure at the throat equal to the vapor pressure of the water. This will establish the maximum lift condition. Cavitation would prevent any lower pressure from developing at the throat.

$$p_v/\gamma + V_t^2/2g + z_t = p_e/\gamma + V_{e\max}^2/2g + z_e \tag{1}$$

Continuity Equation

$$\begin{aligned} V_t A_t &= V_e A_e \\ V_t &= V_e(A_e/A_t) \\ V_t &= 4V_e \end{aligned} \tag{2}$$

Then Eq. (1) can be written as

$$\begin{aligned} 1,700/\gamma + (4V_{e\max})^2/2g &= 100,000/\gamma + V_{e\max}^2/2g \\ V_{e\max} &= ((1/15)(2g/\gamma)(98,300))^{1/2} \\ &= ((1/15)(2/\rho)(98,300))^{1/2} \\ &= \underline{\underline{3.62 \text{ m/s}}} \end{aligned}$$

$$\begin{aligned} Q_{\max} &= V_e A_e \\ &= (3.62 \text{ m/s})(10^{-3}\text{m}^2) \\ &= \underline{\underline{0.00362 \text{ m}^3/\text{s}}} \end{aligned}$$

Find pressure in the suction cup at the level of the suction cup.

$$\begin{aligned} p_t + \gamma\Delta h &= p_{\text{suction}} \\ p_{\text{suction}} &= 1,700 \text{ Pa} + 9{,}800 \times 2 \\ &= 21,300 \text{ Pa} \end{aligned}$$

But the pressure in the water surrounding the suction cup will be $p_{\text{atm}} + \gamma \times 1 = (100 + 9.80)$ kPa, or

$$\begin{aligned} p_{\text{water}} - p_{\text{suction}} &= (109,800 - 21,300) \text{ Pa} \\ &= 88,500 \text{ Pa} \end{aligned}$$

Thus the maximum lift will be:

$$\begin{aligned} \text{Lift}_{\max} &= \Delta p A_s = (p_{\text{water}} - p_{\text{suction}}) A_s \\ &= (88,500 \text{ N/m}^2)(0.1 \text{ m}^2) \\ &= \underline{\underline{8,850 \text{ N}}} \end{aligned}$$

Note: Buoyancy on the object being lifted was neglected.

5.94 **Information and assumptions**

provided in problem statement

Find

maximum and minimum gage pressures

Solution

$$\begin{aligned} p_{\max} &= \rho V_0^2/2 \\ &= 1.1 \times 60^2/2 = \underline{\underline{1{,}980 \text{ Pa, gage}}} \\ p_{\min} &= -0.45\rho V_0^2/2 = -0.45 \times (1,980) = \underline{\underline{-891 \text{ Pa, gage}}} \end{aligned}$$

5.95 **Information and assumptions**

provided in problem statement

Find

pressure difference between bottom and top

Bernoulli equation

$$p_2 - p_1 = (1.2/2)(90^2 - 70^2) = 1,920 \text{ Pa} = \underline{\underline{1.920 \text{ kPa}}}$$

5.96 **Information and assumptions**

provided in problem statement

Find

shape of the water surface

Bernoulli equation can be applied to the free vortex region. The elevation at the juncture of the forced and free vortex and a point far from the vortex center where the velocity is zero is given by

$$z_{10} + \frac{V_{\text{max}}^2}{2g} = z + \frac{V^2}{2g} = 0$$

or

$$z_{10} = -\frac{V_{\text{max}}^2}{2g}$$

In the forced vortex region, the equation relating elevation and speed is

$$z_{10} - \frac{V_{\text{max}}^2}{2g} = z - \frac{V^2}{2g}$$

At the vortex center, $V = 0$, so

$$\begin{aligned} z_0 &= z_{10} - \frac{V_{\text{max}}^2}{2g} = -\frac{V_{\text{max}}^2}{2g} - \frac{V_{\text{max}}^2}{2g} = -\frac{V_{\text{max}}^2}{g} \\ z &= -\frac{10^2}{9.81} = -10.2 \text{ m} \end{aligned}$$

In the forced vortex region

$$V = \frac{r}{10} 10 \text{ m/s} \ = r$$

so the elevation is given by

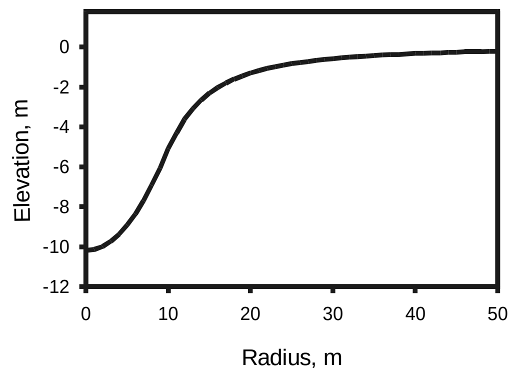
$$z = -10.2 + \frac{r^2}{2g}$$

In the free vortex region

$$V = 10\frac{10}{r}$$

so the elevation is given by

$$z = z_{10} + \frac{V_{\max}^2}{2g} - \frac{100}{2g}\left(\frac{10}{r}\right)^2 = \frac{-510}{r^2}$$



5.97 **Information and assumptions**

provided in problem statement

Find

variation in pressure

Solution

From Eq. 5.28 in the text, the pressure reduction from atmospheric pressure at the vortex center is

$$\Delta p = -\rho V_{\max}^2$$

which gives

$$\Delta p = -1.2 \times (350 \times \frac{1000}{3600})^2 = -11.3 \text{ kPa}$$

or a pressure of $p(0) = 100 - 11.3 = 88.7$ kPa. In the forced vortex region the pressure varies as

$$p(0) = p - \rho \frac{V^2}{2}$$

In this region, the fluid rotates as a solid body so the velocity is

$$V = \frac{r}{50} V_{\max} = 1.94r$$

The equation for pressure becomes

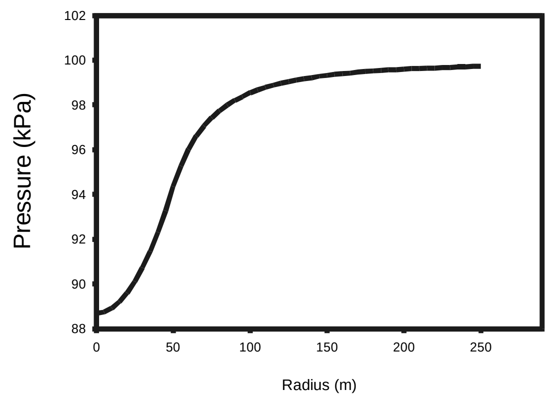
$$p = 88.7 + 2.26r^2/1000 \qquad \text{for } r \leq 50 \text{ m}$$

The factor of 1000 is to change the pressure to kPa. A the point of highest velocity the pressure is 94.3 kPa. In the free vortex region, the **Bernoulli equation** applies

$$p(50) + \frac{1}{2}\rho V_{\max}^2 = p + \frac{1}{2}\rho V^2$$

In the free vortex region so the equation for pressure becomes

$$
\begin{aligned}
p &= p(50) + \frac{1}{2}\rho V_{\max}^2 \left[1 - (\frac{50}{r})^2\right] \qquad \text{for } r \geq 50 \text{ m} \\
p &= 94.3 + 5.65 \times \left[1 - (\frac{50}{r})^2\right]
\end{aligned}
$$



5.98 **Information and assumptions**

provided in problem statement

Find

pressure coefficient versus nondimensional radius

Solution

From Eq. 5.28 in the text, the pressure at the center of a tornado would be $-\rho V_{\max}^2$ so the pressure coefficient at the center would be

$$C_p = \frac{-\rho V_{\max}^2}{\frac{1}{2}\rho V_{\max}^2} = -2$$

For the inner, forced-vortex region the pressure varies as

$$p(0) = p - \frac{1}{2}\rho V^2$$

so the pressure coefficient can be written as

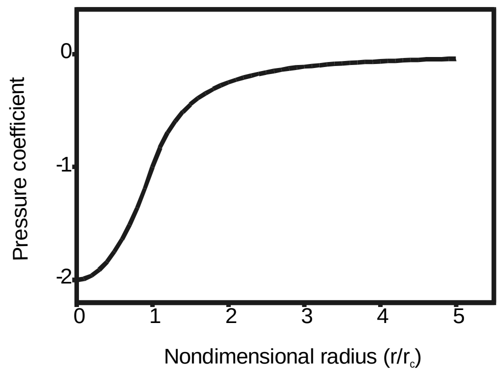
$$\begin{aligned} C_p &= \frac{p - p_o}{\frac{1}{2}\rho V_{\max}^2} = -2 + (\frac{V}{V_{\max}})^2 \qquad \text{for } r \leq r_c \\ C_p &= -2 + (\frac{r}{r_c})^2 \end{aligned}$$

so the pressure coefficient at the edge of the forced vortex is -1. In the free-vortex region, the **Bernoulli equation** applies so

$$p(r_c) + \frac{1}{2}\rho V_{\max}^2 = p + \frac{1}{2}\rho V^2$$

and the pressure coefficient can be written as

$$\begin{aligned}
C_p &= \frac{p - p_o}{\frac{1}{2}\rho V_{\max}^2} = \frac{p(r_c) - p_o}{\frac{1}{2}\rho V_{\max}^2} + [1 - (\frac{r_c}{r})^2] \quad \text{for } r \geq r_c \\
C_p &= -1 + [1 - (\frac{r_c}{r})^2] = -(\frac{r_c}{r})^2
\end{aligned}$$



5.99 The fluid in a tornado moves in a circular path because the pressure gradient provides the force for the centripetal acceleration. For a fluid element of volume $\forall$ the relationship between the centripetal acceleration and the pressure gradient is

$$\rho\frac{V^2}{r} = \forall\frac{dp}{dr}$$

The density of a weather balloon would be less than the local air so the pressure gradient would be higher than the centripetal acceleration so the balloon would move toward the vortex center.

5.100 As the pressure decreases the density becomes less. This means that a smaller pressure gradient is needed to provide the centripetal force to maintain the circular motion. This means that the Bernoulli equation will overpredict the pressure drop.

5.101 **Information and assumptions**

provided in problem statement

Find

speed at which cavitation occurs.

Let p_o be the pressure on the streamline upstream of the sphere. The minimum pressure will occur at the maximum width of the sphere where the velocity is 1.5 times the free stream velocity.

Bernoulli equation between the freestream and the maximum width gives

$$p_o + \frac{1}{2}\rho V_o^2 + \gamma h_o = p + \frac{1}{2}\rho(1.5V_o)^2 + \gamma(h_o + 0.5)$$

Solving for the pressure p gives

$$p = p_o - 0.625\rho V_o^2 - 0.5\gamma$$

The pressure at a depth of 10 ft is 624 lbf/ft^2. The density of water is 1.94 slugs/ft^3 and the specific weight is 62.4 lbf/ft^3. At a temperature of 50^{o}F, the vapor pressure is 0.178 psia or 25.6 psfa. Substituting into the above equation

$$\begin{aligned} 25.6 \text{ psfa} &= 624 \text{ psfa} - 0.625 \times 1.94 \times V_o^2 - 0.5 \times 62.4 \\ 567.2 &= 1.21V_o^2 \end{aligned}$$

Solving for V_o gives

$$V_o = \underline{\underline{21.65 \text{ ft/s}}}$$

5.102 **Information and assumptions**

Assume $p_{\text{atm}} = 101$ kPa abs; $p_{\text{vapor}} = 1,230$ Pa abs.
provided in problem statement

Find

at what speed will cavitation occur

Solution

Considering a point ahead of the foil (at same depth as the foil) and the point of minimum pressure on the foil, and applying the definition of C_p between these two points yields:

$$C_p = (p_{\min} - p_0)/(\rho V_0^2/2)$$

where

$$\begin{aligned} p_0 &= p_{\text{atm}} + 1.8\gamma = 101,000 + 1.8 \times 9,810 = 118,658 \text{ Pa abs.} \\ p_{\min} &= 70,000 \text{ Pa abs; } V_0 = 8 \text{ m/s} \end{aligned}$$

Then

$$C_p = (70,000 - 118,658)/(500 \times 8^2) = -1.521$$

Now use $C_p = -1.521$ (constant) for evaluating V for cavitation where $p_{\min}$ is now p_{vapor}:

$$-1.521 = (1,230 - 118,658)/((1,000/2)V_0^2); V_0 = \underline{\underline{12.4 \text{ m/s}}}$$

5.103 **Information and assumptions**

provided in problem statement

Find

at what speed will cavitation begin

Solution

See solution for Prob. 5.102 for preliminaries. We have the same C_p, but $p_0 = 101,000 + 3\gamma = 130,430$. Then:

$$\begin{aligned} -1.986 &= (1,230 - 130,430)/((1,000/2)V_0^2) \\ V_0 &= \underline{\underline{11.41 \text{ m/s}}} \end{aligned}$$

5.104 **Information and assumptions**

provided in problem statement

Find

at what speed will cavitation begin

Solution

Solution similar to solution for Prob. 5.102.

$$
\begin{aligned}
p_{\min} &= -2.5 \times 144 = -360 \text{ psf gage} \\
p_0 &= 4\gamma = 4 \times 62.4 = 249.6 \text{ psf}
\end{aligned}
$$

Then

$$
\begin{aligned}
C_p &= (p_{\min} - p_0)/(\rho V_0^2/2) = (-360 - 249.6)/((1.94/2) \times 20^2) \\
C_p &= -1.571
\end{aligned}
$$

Now let $p_{\min} = p_{\text{vapor}} = 0.178$ psia $= -14.52$ psia $= -2,091$ psfg
Then

$$
\begin{aligned}
-1.571 &= -(249.6 + 2,091)/((1.94/2)V_0^2) \\
V_0 &= \underline{\underline{39.2 \text{ ft/s}}}
\end{aligned}
$$

5.105 **Information and assumptions**

provided in problem statement

Find

at what speed will cavitation begin

Solution

From solution of Prob. 5.104 we have $C_p = -1.571$ but now $p_0 = 10\gamma = 624$ psf. Then:

$$\begin{aligned} -1.571 &= -(624 + 2,091)/((1.94/2)V_0^2) \\ V_0 &= \underline{\underline{42.2 \text{ ft/s}}} \end{aligned}$$

5.106 **Information and assumptions**

from Table A.5 $p_v(58°) = 0.178$ psia

Find

speed at which cavitation occurs

Bernoulli equation between a point in the free stream to the 90° position where $V = 1.5V_0$. The free stream velocity is the same as the sphere velocity (reference velocities to sphere).

$$\begin{aligned}
\rho V_0^2/2 + p_0 &= p + \rho(1.5V_0)^2/2 \\
\text{where } p_0 &= 18 \text{ psia} \\
\rho V_0^2/2(2.25 - 1) &= (18 - 0.178)(144) \\
V_0^2 &= 2(17.8)(144)/((1.25)(1.94)) \text{ ft}^2/\text{s}^2 \\
V_0 &= \underline{\underline{46.0 \text{ ft/sec}}}
\end{aligned}$$

5.107 **Information and assumptions**

from Table A.2 $p_v(10°C) = 1,230$ Pa

Find

velocity at which cavitation occurs

Solution

$$\begin{aligned} C_p &= (p - p_0)/(\rho V_0^2/2) \\ p_0 &= 100,000 + 1 \times 9,810 \text{ Pa} = 109,810 \text{ Pa} \\ p &= 80,000 \text{ Pa} \\ \text{Thus } C_p &= -2.385 \end{aligned}$$

For cavitation to occur $p = 1,230$ Pa

$$-2.385 = (1,230 - 109,810)/(1,000V_0^2/2); V_0 = \underline{\underline{9.54 \text{ m/s}}}$$

5.108 **Information and assumptions**

provided in problem statement

Find

if cavitation will occur

Solution

$$C_p = (p - p_0)/(\rho V_0^2/2)$$

From Fig. 5-13 $C_{p_{\min}} = -0.45$; therefore,

$$\begin{aligned} p_{\min} &= p_0 + C_{p_{\min}} \rho V_0^2/2 \\ &= 100 - 0.45 \times 1.94 \times 32^2/2 \\ &= \underline{\underline{\text{-347 psfg}}} \end{aligned}$$

Cavitation will not occur because the minimum pressure is much greater than the vapor pressure of the water.

5.109 **Information and assumptions**

provided in problem statement

Find

lift force on Volkswagen

Solution

After separation, the pressure in the wake is the same as the pressure beneath the Volkswagen so, beyond this point there is no contribution to the lift. Using the **Bernoulli equation**, the pressure at angle θ on the surface would be

$$p = p_o + \frac{1}{2}\rho V_o^2(1 - 4\sin^2\theta)$$

Substituting into the equation for lift on the curved section gives

$$F_{\text{lift}} = -\frac{1}{2}WL\int_0^{150^o}[p_o + \frac{1}{2}\rho V_o^2(1 - 4\sin^2\theta)]\sin\theta d\theta$$

The first term can be easily integrated and the equation becomes

$$F_{\text{lift}} = -0.933WLp_o - \frac{1}{4}WL\rho V_o^2\int_0^{150^o}(1 - 4\sin^2\theta)\sin\theta d\theta$$

Finally, evaluating the integral gives

$$F_{\text{lift}} = -0.933WLp_o + 0.849WL\rho V_o^2$$

The lift due to static pressure on the bottom is

$$F_{\text{lift}} = 0.933p_oWL$$

where $0.933L$ is the distance along the bottom of the car to the point below the separation point. Adding the list on the curved surface and beneath the car gives

$$F_{\text{lift,tot}} = 0.849WL\rho V_o^2$$

Substituting in the values gives

$$F_{\text{lift,tot}} = 0.849 \times 2 \times 4 \times 1.2 \times 27.8^2 = \underline{\underline{6299}}\text{ N}$$

5.110 a) Substituting the equation for the streamline into the Euler equation gives

$$\begin{array}{c} u\frac{\partial u}{\partial x}dx + u\frac{\partial u}{\partial y}dy = -g\frac{\partial h}{\partial x}dx \\ v\frac{\partial v}{\partial x}dx + v\frac{\partial v}{\partial y}dy = -g\frac{\partial h}{\partial y}dy \end{array}$$

or

$$\begin{array}{c} \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right)dx + \frac{\partial}{\partial y}(\frac{u^2}{2})dy = -g\frac{\partial h}{\partial x}dx \\ \frac{\partial}{\partial x}\left(\frac{v^2}{2}\right)dx + \frac{\partial}{\partial y}(\frac{v^2}{2})dy = -g\frac{\partial h}{\partial y}dy \end{array}$$

Adding both equations

$$\frac{\partial}{\partial x}\left(\frac{u^2+v^2}{2}\right)dx + \frac{\partial}{\partial y}\left(\frac{u^2+v^2}{2}\right)dy = -g(\frac{\partial h}{\partial x}dx + \frac{\partial h}{\partial y}dy)$$

or

$$d(\frac{u^2+v^2}{2} + gh) = 0$$

b) Substituting the irrotationality condition into Euler's equation gives

$$\begin{array}{c} u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} = -g\frac{\partial h}{\partial x} \\ v\frac{\partial v}{\partial y} + u\frac{\partial u}{\partial y}dy = -g\frac{\partial h}{\partial y} \end{array}$$

or

$$\begin{array}{c} \frac{\partial}{\partial x}(\frac{u^2+v^2}{2} + gh) = 0 \\ \frac{\partial}{\partial y}(\frac{u^2+v^2}{2} + gh) = 0 \end{array}$$

5.111 If one connected a manometer between wall reference points P_l and $P_{1,3}$ then from Fig. 5.8 we see that

$$\begin{aligned} (h-h_0)/(V_0^2/2g &= -3 \\ \text{or } h_0-h &= 3V_0^2/2g \end{aligned} \quad (1)$$

For $V_0 = 5$ m/s; $h_0 - h = 3.823$ meters of water. One could use a mercury-water manometer for sensing the difference in head. Thus, the deflection in such a manometer would be

$$\Delta = (h_0 - h)/12.55 \quad (2)$$

Therefore, for $V_0 = 5$ m/s, Δ would be $3.823/12.55 = 0.305$ m
Solving Eqs. (1) and (2) for V_0 yields

$$V_0 = 9.06\sqrt{\Delta}$$

The discharge would be given as

$$\begin{aligned} Q &= V_0 A \\ &= V_0(4n_0B)\sqrt{\Delta} \\ \text{or } Q &= 9.06\sqrt{\Delta}(4n\, B) \\ &= 36.24\, n_0 B\sqrt{\Delta} \end{aligned}$$

where Δ is the deflection on a mercury-water manometer in meters, n_0 and B are lineal dimensions in meters and Q is the discharge in m^3/s.

The maximum deflection of 0.305 m is not large; therefore, one may want to increase the accuracy of reading by utilizing an inclined tube manometer such as given in Problem 3.32. Also, one could connect the manometer between other points on the wall (such as between points $P_{5.3}$ and P_9) to achieve a greater difference in head for a given discharge.

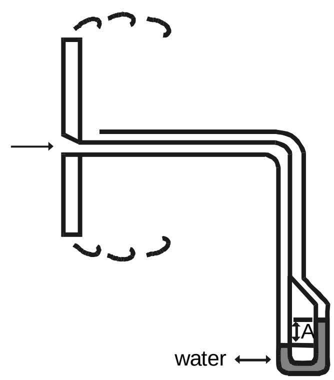
5.112 One possible design is shown. The manometer is an air-water manometer. Then for this setup.

$$\begin{aligned}\Delta p &= 1.45\rho V_0^2/2 \\ \text{or } V_0 &= \sqrt{(2/1.45)\Delta p/\rho_{\text{air}}}\end{aligned}$$

but $\Delta p = \gamma_{\text{H}_2\text{O}}\Delta$ where Δ is the manometer deflection. Then

$$V_0 = \sqrt{(2/1.45)(\gamma_{\text{H}_2\text{O}}/\rho_{\text{air}})\Delta}$$

Once could also use an inclined tube manometer such as given for Problem 3.32 to increase the accuracy of reading.



5.113 The main point to this question is that while inhaling, the air is drawn into your mouth without any separation occurring in the flow that is approaching your mouth. Thus there is no concentrated flow; all air velocities in the vicinity of your face are relatively low. However, when exhaling as the air passes by your lips separation occurs thereby concentrating the flow of air which allows you to easily blow out a candle.

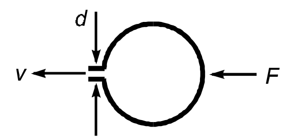
5.114 If a building has a flat roof as air flows over the top of the building separation will occur at the sharp edge between the wall and roof. Therefore, most if not all of the roof will be in the separation zone. Because the zone of separation will have a pressure much lower than the normal atmospheric pressure a net upward force will be exerted on the roof thus tending to lift the roof.

Even if the building has a peaked roof much of the roof will be in zones of separation. These zones of separation will occur downwind of the peak. Therefore, peaked roof buildings will also tend to have their roofs uplifted in high winds.

Chapter Six

6.1 **Information and assumptions**

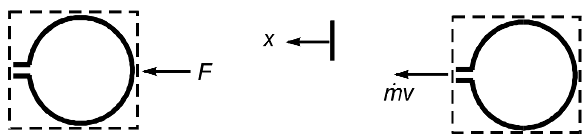
balloon held stationary by a force F
$d = 1$ cm, $v = 40$ m/s, $\rho = 1.2$ kg/m^3
steady flow, constant density



Find

force required to hold balloon stationary: F

Force and momentum diagrams (x-direction terms)

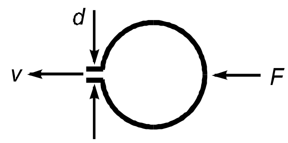


x-momentum

$$
\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
F &= \dot{m}v = \rho A v^2 = (1.2)\left(\pi \times 0.01^2/4\right)\left(40^2\right) \\
F &= \underline{\underline{0.15\text{ N}}}
\end{aligned}
$$

6.2 **Information and assumptions**

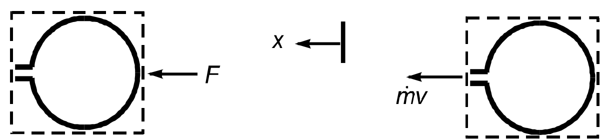
balloon held stationary by a force F
pressure inside balloon: $p = 8$ in.-H_2O $= 1990$ Pa
$d = 1$ cm, $\rho = 1.2$ kg/m^3
steady, irrotational, constant density flow



Find

x-component of force required to hold balloon stationary: F
exit velocity: v

Force and momentum diagrams (x-direction terms)



Bernoulli equation applied from inside the balloon to nozzle exit

$$
\begin{aligned}
p/\rho &= v^2/2 \\
v &= \sqrt{2p/\rho} = \sqrt{2 \times 1990/1.2} \\
v &= \underline{\underline{57.6 \text{ m/s}}}
\end{aligned}
$$

x-momentum

$$
\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
F &= \dot{m}v = \rho A v^2 = (1.2)\left(\pi \times 0.01^2/4\right)\left(57.6^2\right) \\
F &= \underline{\underline{0.31 \text{ N}}}
\end{aligned}
$$

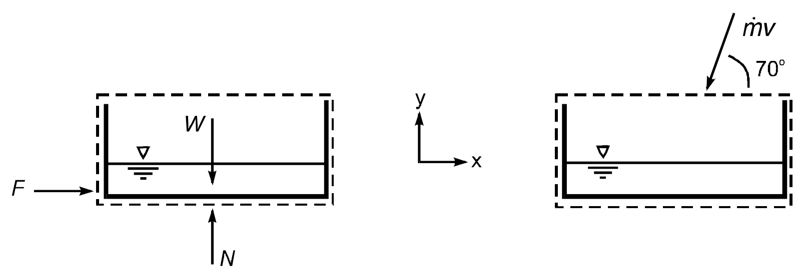
6.3 **Information and assumptions**

water jet filling a tank
jet: $d = 30$ mm, $v = 20$ m/s, $T = 15$ ^{o}C
steady flow
tank mass is 20 kg; tank contains 20 liters of water

Find

force on bottom of tank: N
force acting on stop block: F

Force and momentum diagrams



x-momentum

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
F &= -(-\dot{m}v\cos 70^o) = \rho A v^2 \cos 70^o \\
\rho A v^2 &= (999)\left(\pi \times 0.03^2/4\right)(20^2) = 282.5 \text{ N} \\
F &= \underline{\underline{97 \text{ N}}}
\end{aligned}$$

y-momentum

$$\begin{aligned}
\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\
N - W &= -(-\dot{m}v\sin 70^o) \\
N &= W + \rho A v^2 \sin 70^o \\
W &= (20 + (0.02)(999))9.81 = 392.2 \text{ N} \\
N &= 392.2 + 282.5\sin 70^o \\
N &= \underline{\underline{658 \text{ N}}}
\end{aligned}$$

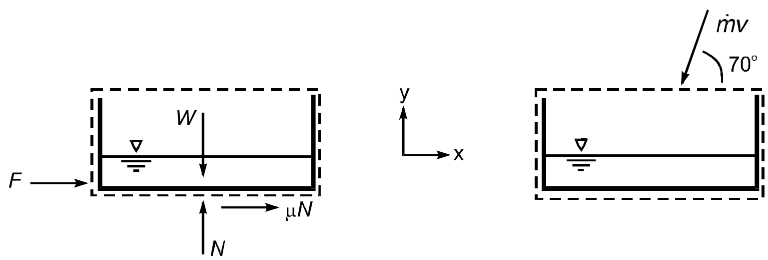
6.4 **Information and assumptions**

water jet filling a tank, friction acts on the bottom of the tank
jet: $d = 2$ in., $v = 50$ ft/s, $T = 70\ ^oF$
steady flow, constant density, steady and irrotational flow
tank mass is 25 lbm; tank contains 5 gallons of water
provided in problem statement

Find

minimum coefficient of friction (μ) so force on stop block is zero

Force and momentum diagrams



y-momentum

$$\begin{aligned} \sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ N - W &= -(-\dot{m}v \sin 70^o) \\ N &= W + \rho A v^2 \sin 70^o \end{aligned}$$

x-momentum

Setting $F = 0$

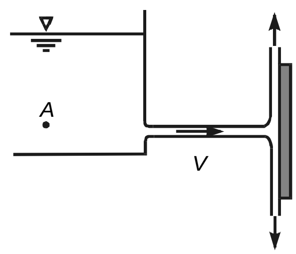
$$\begin{aligned} \mu N &= -(-\dot{m}v \cos 70^o) = \rho A v^2 \cos 70^o \\ \mu &= \left(\rho A v^2 \cos 70^o\right)/N \end{aligned}$$

Calculations

$$\begin{aligned} \rho A v^2 &= (1.94)\left(\pi \times (1/12)^2\right)\left(50^2\right) = 105.8 \text{ lbf} \\ W_{H20} &= \gamma \cancel{V} = (62.37)(5)/(7.481) = 41.75 \text{ lbf} \\ W &= (41.75 + 25) \text{ lbf} = 66.7 \text{ lbf} \\ N &= 66.7 + 105.8 \times \sin 70^o = 166.2 \text{ lbf} \\ \mu &= 105.8 \times \cos 70^o/166.2 \\ \mu &= \underline{\underline{0.22}} \end{aligned}$$

6.5 **Information and assumptions**

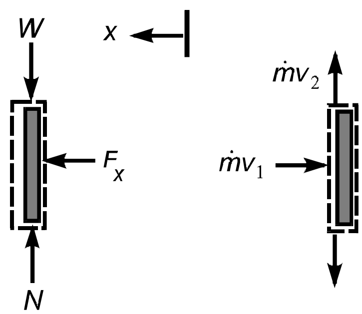
horizontal jet round jet strikes a plate
water at 15^{o}C, $\rho = 999$ kg/m^3, $Q = 0.4$ m^3/s, $p_A = 75$ kPa



Find

Horizontal component of force to hold plate stationary: F_x

Force and momentum diagrams



Bernoulli equation applied from inside of tank to nozzle exit

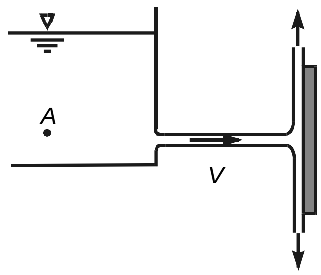
$$\begin{aligned} p_A/\rho &= v_1^2/2 \\ v_1 &= \sqrt{2p/\rho} = \sqrt{2 \times 75000/999} = 12.3 \text{ m/s} \end{aligned}$$

x**-momentum**

$$\begin{aligned} \sum F_x &= -\dot{m}v_{1x} \\ F_x &= -(-\dot{m}v_1) = \rho Q v_1 = (999)\,(0.4)\,(12.3) \\ F_x &= \underline{\underline{4.9 \text{ kN}}} \end{aligned}$$

6.6 **Information and assumptions**

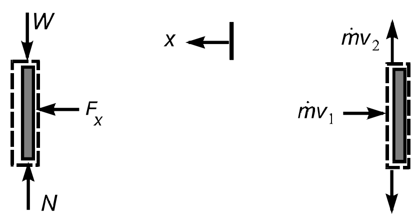
horizontal round jet strikes a plate
water at 70^{o}F, $\rho = 1.94$ slug/ft^3, $Q = 2$ cfs
Horizontal component of force to hold plate stationary: $F_x = 200$ lbf



Find

speed of water jet: v

Force and momentum diagrams

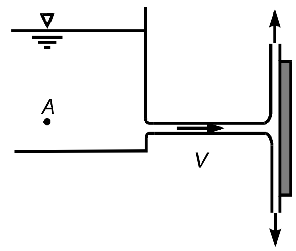


x-momentum

$$
\begin{aligned}
\sum F_x &= -\dot{m}v_{1x} \\
F_x &= -(-\dot{m}v_1) = \rho Q v_1 \\
v_1 &= F_x/\rho Q = (200)/(1.94 \times 2) \\
v_1 &= \underline{\underline{51.5 \text{ ft/s}}}
\end{aligned}
$$

6.7 **Information and assumptions**

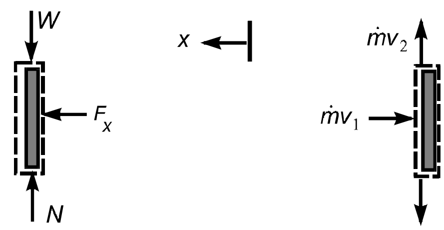
horizontal round jet strikes a plate
water at 70 ^{o}F, $\rho = 1.94$ slug/ft^3
pressure at A: $p_A = 15$ psig
horizontal component of force to hold plate stationary: $F_x = 500$ lbf



Find

diameter of jet: d

Force and momentum diagrams



Bernoulli equation applied from inside of tank to nozzle exit

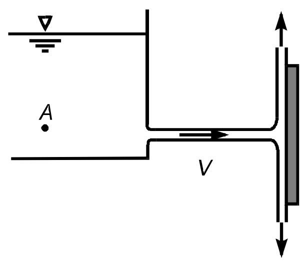
$$
\begin{aligned}
p_A/\rho &= v_1^2/2 \\
v_1 &= \sqrt{2 \times 15 \times 144/1.94} = 47.19 \text{ ft/s}
\end{aligned}
$$

x-momentum

$$
\begin{aligned}
\sum F_x &= -\dot{m}v_{1x} \\
F_x &= -(-\dot{m}v_1) = \rho A v_1^2 \\
A &= F_x/\rho v_1^2 = 500/(1.94 \times 47.19^2) \\
A &= 0.116 \text{ ft}^2 \\
d &= \sqrt{4A/\pi} = \sqrt{4 \times 0.116/\pi} = \underline{\underline{0.38 \text{ ft}}}
\end{aligned}
$$

6.8 **Information and assumptions**

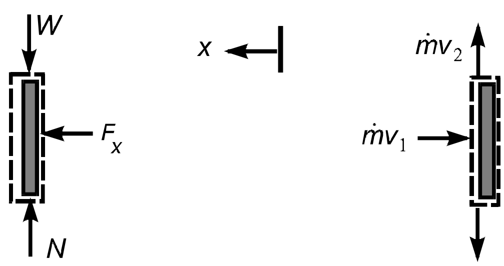
horizontal round jet strikes a plate
water at 15 ^{o}C, $\rho = 999$ kg/m^3
cross sectional area of jet: $A = 0.0015$ m^2
horizontal component of force to hold plate stationary: $F_x = 2.0$ kN



Find

speed of jet: v_1
pressure at location A: p_A

Force and momentum diagrams



x**-momentum**

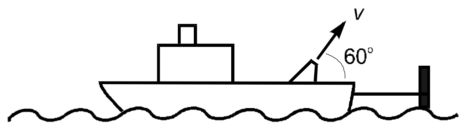
$$\begin{aligned}\sum F_x &= -\dot{m}v_{1x} \\ F_x &= -(-\dot{m}v_1) = \rho A v_1^2 \\ v_1 &= \sqrt{F_x/(\rho A)} = \sqrt{2000/(999 \times 0.015)} = \underline{\underline{11.55 \text{ m/s}}}\end{aligned}$$

Bernoulli equation applied from inside of tank to nozzle exit

$$\begin{aligned}p_A/\rho &= v_1^2/2 \\ p_A &= \rho v_1^2/2 = 999 \times 11.55^2/2 = \underline{\underline{66.7 \text{ kPa-gage}}}\end{aligned}$$

6.9 **Information and assumptions**

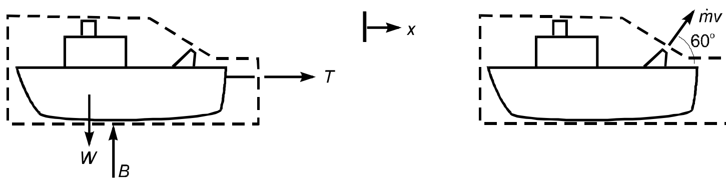
water jet from a fire hose on a boat
diameter of jet: $d = 3$ in., speed of jet: $v = 65$ mph $= 95.33$ ft/s
water at 50 ^{o}F, $\rho = 1.94$ slug/ft^3



Find

tension in cable: T

Force and momentum diagrams

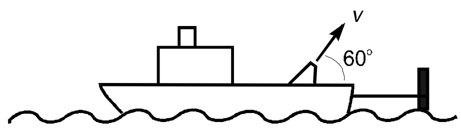


x**-momentum**

$$\begin{aligned}
\sum F &= \dot{m}\left(v_o\right)_x \\
T &= \dot{m}v\cos 60^o \\
\dot{m} &= \rho A v \\
&= (1.94)\left(\pi \times (1.5/12)^2\right)(95.33) = 9.08 \text{ slug/s} \\
T &= (9.08)(95.33)\cos 60^o = \underline{\underline{433 \text{ lbf}}}
\end{aligned}$$

6.10 **Information and assumptions**

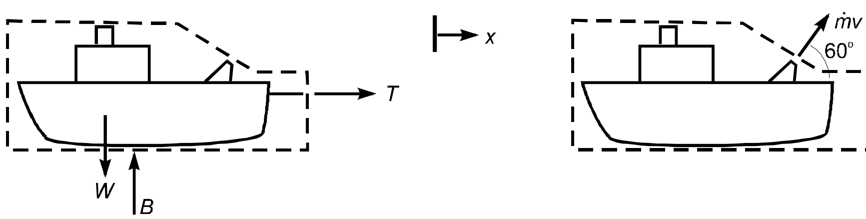
water jet (5 ^{o}C) from a fire hose on a boat
$v = 50$ m/s, $\rho = 1000$ kg/m^3
allowable load on cable: $T = 5.0$ kN



Find

mass flow rate of jet: $\dot{m}$
diameter of jet: d

Force and momentum diagrams



x**-momentum**

$$\begin{aligned}
\sum F &= \dot{m}\,(v_o)_x \\
T &= \dot{m}v\cos 60^o \\
\dot{m} &= T/\,(v\cos 60^o) = 5000/(50\times\cos 60^o) \\
\dot{m} &= \underline{\underline{200 \text{ kg/s}}}
\end{aligned}$$

mass flow rate

$$\begin{aligned}
\dot{m} &= \rho A v = \rho\pi d^2 v/4 \\
d &= \sqrt{4\dot{m}/(\rho\pi v)} = \sqrt{(4\times 200)/(1000\times\pi\times 50)} \\
d &= \underline{\underline{7.13 \text{ cm}}}
\end{aligned}$$

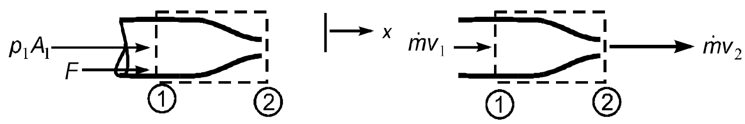
6.11 **Information and assumptions**

water (60 ^{o}F) flows through a nozzle
d_1 =4 in., $d_2 = 1$ in., $p_1 = 3000$ psfg, $p_2 = 0$ psfg
neglect weight, assume steady flow

Find

speed at nozzle exit: v_2
force to hold nozzle stationary: F

Force and momentum diagrams



Continuity

$$\begin{aligned} A_1 v_1 &= A_2 v_2 \\ v_1 &= v_2 \left(d_2/d_1\right)^2 \end{aligned}$$

Bernoulli equation applied from 1 to 2

$$p_1/\rho + v_1^2/2 = v_2^2/2$$

Combining previous two equations

$$\begin{aligned} p_1 &= \rho\left(v_2^2/2\right)\left(1-\left(d_2/d_1\right)^4\right) \\ 3000 &= 1.94 \times \left(v_2^2/2\right) \times \left(1-(1/4)^4\right) \\ v_2 &= 55.72 \text{ ft/s} \\ v_1 &= v_2 \left(d_2/d_1\right)^2 = 55.72 \times (1/4)^2 = 3.483 \text{ ft/s} \end{aligned}$$

Mass flow rate

$$\dot{m}_1 = \dot{m}_2 = \dot{m} = (\rho A v)_2 = 1.94 \times \left(\pi/4 \times (1.0/12)^2\right) \times 55.72 = 0.589 \text{ slug/s}$$

x**-momentum**

$$\begin{aligned} \sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\ F + p_1 A_1 &= \dot{m}\left(v_2 - v_1\right) \\ F &= -p_1 A_1 + \dot{m}\left(v_2 - v_1\right) \\ F &= -(3000) \times \left(\pi/4 \times (4/12)^2\right) + (0.589) \times (55.72 - 3.483) = -231 \text{ lbf} \end{aligned}$$

Force on nozzle = 231 lbf to the left

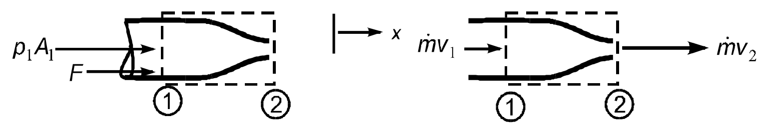
6.12 **Information and assumptions**

water (15 ^{o}C) flows through a nozzle, $\rho = 999$ kg/m^3
$d_1 = 10$ cm., $d_2 = 2$ cm., $v_2 = 25$ m/s
neglect weight, assume steady flow, $p_2 = 0$ kPa-gage

Find

pressure at inlet: p_1
force to hold nozzle stationary: F

Force and momentum diagrams



conservation of mass

$$\begin{aligned}
A_1 v_1 &= A_2 v_2 \\
v_1 &= v_2 \left(d_2/d_1\right)^2 = 25 \times (2/10)^2 = 1.0 \text{ m/s} \\
\dot{m}_1 &= \dot{m}_2 = (\rho A v)_2 = 999 \times \left(\pi/4 \times 0.02^2\right) \times 25 = 7.85 \text{ kg/s}
\end{aligned}$$

Bernoulli equation applied from 1 to 2

$$\begin{aligned}
p_1/\rho + v_1^2/2 &= v_2^2/2 \\
p_1 &= (\rho/2) \times \left(v_2^2 - v_1^2\right) = (999/2) \times (25^2 - 1^2) \\
p_1 &= \underline{\underline{312 \text{ kPa}}}
\end{aligned}$$

x**-momentum**

$$\begin{aligned}
\sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\
F + p_1 A_1 &= \dot{m}\left(v_2 - v_1\right) \\
F &= -p_1 A_1 + \dot{m}\left(v_2 - v_1\right) \\
F &= -(312 \times 10^3) \times (\pi/4 \times 0.1^2) + (7.85) \times (25 - 1) = -2.26 \text{ kN}
\end{aligned}$$

$\underline{\underline{\text{Force on nozzle} = 2.26 \text{ kN to the left}}}$

6.13 The problem involves writing a program for the flow in a nozzle and applying it to problems 6.12 and 6.14

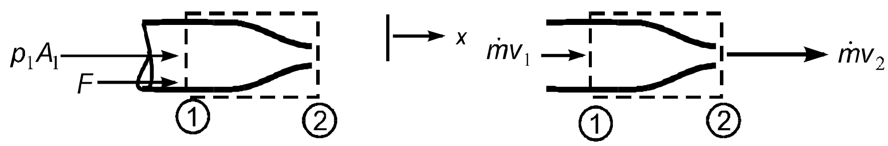
6.14 **Information and assumptions**

water (15 ^{o}C) flows through a nozzle, $\rho = 999$ kg/m^3
$d_1 = 5$ cm., $d_2 = 1$ cm., $v_2 = 10$ m/s
neglect weight, assume steady flow, $p_2 = 0$ kPa-gage

Find

pressure at inlet: p_1
force to hold nozzle stationary: F

Force and momentum diagrams



conservation of mass

$$\begin{aligned} A_1 v_1 &= A_2 v_2 \\ v_1 &= v_2 \left(d_2/d_1\right)^2 = 10 \times (1/5)^2 = 0.4 \text{ m/s} \\ \dot{m}_1 &= \dot{m}_2 = \dot{m} = (\rho A v)_2 = 999 \times \left(\pi/4 \times 0.01^2\right) \times 10 = 0.785 \text{ kg/s} \end{aligned}$$

Bernoulli equation applied from 1 to 2

$$\begin{aligned} p_1/\rho + v_1^2/2 &= v_2^2/2 \\ p_1 &= (\rho/2) \times \left(v_2^2 - v_1^2\right) = (999/2) \times (10^2 - 0.4^2) \\ p_1 &= 49.9 \text{ kPa} \end{aligned}$$

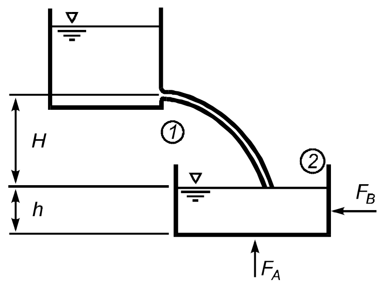
x**-momentum**

$$\begin{aligned} \sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\ F + p_1 A_1 &= \dot{m}\left(v_2 - v_1\right) \\ F &= -p_1 A_1 + \dot{m}\left(v_2 - v_1\right) \\ F &= -(49.9 \times 10^3) \times (\pi/4 \times 0.05^2) + (0.785) \times (10 - 0.4) = -90.4 \text{ N} \end{aligned}$$

Force on nozzle = 90.4 N to the left

6.15 **Information and assumptions**

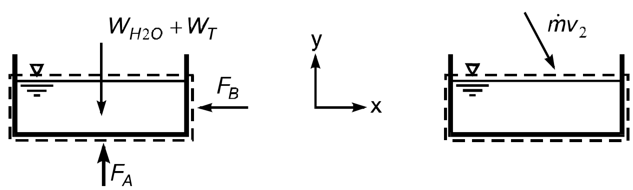
free water jet from upper tank to lower tank, lower tank supported by scales A and B
$Q = 2$ cfs, $d_1 = 4$ in., $h = 1$ ft, $H = 9$ ft
weight of tank: $W_T = 300$ lbf, surface area of lower tank: 4 ft^2
water at 60 ^{o}F: $\rho = 1.94$ slug/ft^3, $\gamma = 62.37$ lbf/ft^3



Find

force on scale A: F_A
force on scale B: F_B

Force and momentum diagrams



Flow rate calculations

$$\begin{aligned} \dot{m} &= \rho Q = 1.94 \times 2.0 = 3.88 \text{ slug/s} \\ v_1 &= Q/A_1 = 2.0/(\pi/4 \times (4/12)^2) = 22.9 \text{ ft/s} \end{aligned}$$

Projectile motion equations

$$\begin{aligned} v_{2x} &= v_1 = 22.9 \text{ ft/s} \\ v_{2y} &= \sqrt{2gH} = \sqrt{2 \times 32.2 \times 9} = 24.1 \text{ ft/s} \end{aligned}$$

x**-momentum**

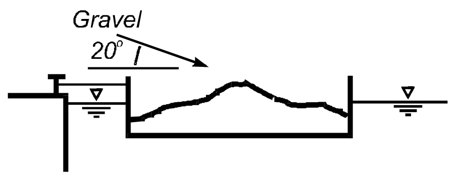
$$
\begin{aligned}
\sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\
-F_B &= -\dot{m}\,(v_{2x}) \\
-F_B &= -3.88 \times 22.9 \\
F_B &= \underline{\underline{88.9 \text{ lbf}}}
\end{aligned}
$$

y-momentum

$$
\begin{aligned}
\sum F_y &= \dot{m}\left[(v_o)_y - (v_i)_y\right] \\
F_A - W_{H2O} - W_T &= -\dot{m}\,(v_{2y}) \\
F_A &= (62.37 \times 4 \times 1) + 300 - (3.88 \times (-24.1)) \\
F_A &= \underline{\underline{643.0 \text{ lbf}}}
\end{aligned}
$$

6.16 **Information and assumptions**

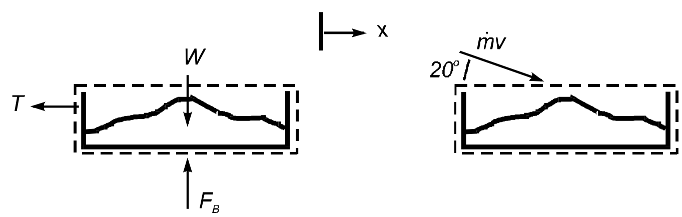
gravel ($\gamma = 120$ lbf/ft^3) flows into a barge that is secured with a hawser
$Q = 50$ yd^3/min $= 22.5$ ft^3/s, $v = 10$ ft/s
assume steady flow



Find

tension in hawser: T

Force and momentum diagrams

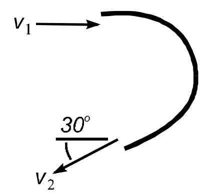


***x*-momentum**

$$
\begin{aligned}
\sum F_x &= \dot{m}\,(v_o)_x - \dot{m}\,(v_i)_x \\
-T &= -\dot{m}(v\cos 20) = -(\gamma/g)Q(v\cos 20) \\
T &= (120/32.2) \times 22.5 \times 10 \times \cos(20) = 788 \text{ lbf} \\
T &= \underline{\underline{788 \text{ lbf}}}
\end{aligned}
$$

6.17 **Information and assumptions**

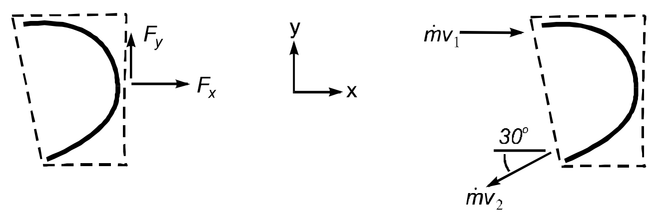
a fixed vane in the horizontal plane; oil ($S = 0.9$)
$v_1 = 28$ m/s, $v_2 = 27$ m/s, $Q = 0.2$ m^3/s



Find

components of force to hold vane stationary: F_x, F_y

Force and momentum diagrams



mass flow rate

$$\dot{m} = \rho Q = 0.9 \times 1000 \times 0.2 = 180 \text{ kg/s}$$

x-momentum

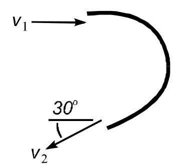
$$\begin{aligned}
\sum F_x &= \dot{m}\left(v_o\right)_x - \dot{m}\left(v_i\right)_x \\
F_x &= \dot{m}(-v_2\cos 30) - \dot{m}v_1 \\
F_x &= -180(27\cos 30 + 28) \\
F_x &= \underline{\underline{-9.25 \text{ kN (acts to the left)}}}
\end{aligned}$$

y-momentum

$$\begin{aligned}
\sum F_y &= \dot{m}\left(v_o\right)_y - \dot{m}\left(v_i\right)_y \\
F_y &= \dot{m}\left(-v_2\sin 30\right) = 180\left(-27\sin 30\right) = -2.43 \text{ kN} \\
F_y &= \underline{\underline{-2.43 \text{ kN (acts downward)}}}
\end{aligned}$$

6.18 **Information and assumptions**

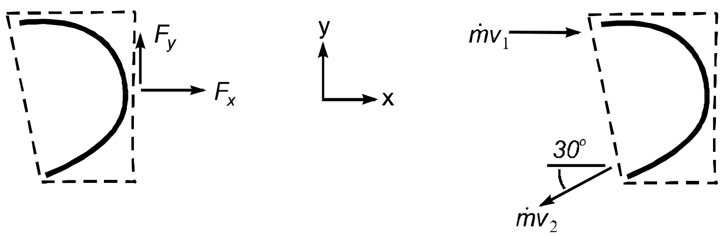
a fixed vane in the horizontal plane; oil ($S = 0.9$)
$v_1 = 90$ ft/s, $v_2 = 85$ ft/s, $Q = 2.0$ cfs



Find

components of force to hold vane stationary: F_x, F_y

Force and momentum diagrams



mass flow rate

$$\dot{m} = \rho Q = 0.9 \times 1.94 \times 2.0 = 3.49 \text{ slug/s}$$

x-momentum

$$\begin{aligned} \sum F_x &= \dot{m}\left(v_o\right)_x - \dot{m}\left(v_i\right)_x \\ F_x &= \dot{m}(-v_2 \cos 30) - \dot{m}v_1 \\ F_x &= -3.49(85 \cos 30 + 90) \\ F_x &= \underline{\underline{-571 \text{ lbf } (F_x \text{ acts to the left})}} \end{aligned}$$

y-momentum

$$\begin{aligned} \sum F_y &= \dot{m}\left(v_o\right)_y - \dot{m}\left(v_i\right)_y \\ F_y &= \dot{m}\left(-v_2 \sin 30\right) = 3.49\left(-85 \sin 30\right) = -148 \text{ lbf} \\ F_y &= \underline{\underline{-148 \text{ lbf (acts downward)}}} \end{aligned}$$

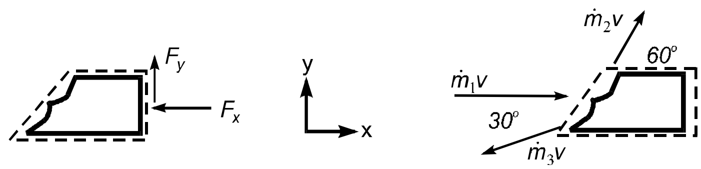
6.19 **Information and assumptions**

a horizontal, two-dimensional water jet deflected by a fixed vane, $\rho = 1.94$ slug/ft^3
$v_1 = 40$ ft/s, width of jets: $w_2 = 0.2$ ft, $w_3 = 0.1$ ft. Neglect gravity.

Find

components of force, per foot of width, to hold vane stationary: F_x, F_y

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v_3 = v = 40 \text{ ft/s}$$

Continuity

$$\begin{aligned} w_1 v_1 &= w_2 v_2 + w_3 v_3 \\ w_1 &= w_2 + w_3 = (0.2 + 0.1) = 0.3 \text{ ft} \end{aligned}$$

x-momentum

$$\begin{aligned} \sum F_x &= \sum \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\ -F_x &= \dot{m}_2 v \cos 60 + \dot{m}_3(-v \cos 30) - \dot{m}_1 v \\ F_x &= \rho v^2(-A_2 \cos 60 + A_3 \cos 30 + A_1) \\ F_x &= 1.94 \times 40^2 \times (-0.2 \cos 60 + 0.1 \cos 30 + 0.3) \\ F_x &= \underline{\underline{889 \text{ lbf/ft (acts to the left)}}} \end{aligned}$$

y-momentum

$$\begin{aligned} \sum F_y &= \sum \dot{m}_o (v_o)_y \\ F_y &= \dot{m}_2 v \sin 60 + \dot{m}_3(-v \sin 30) \\ &= \rho v^2(A_2 \sin 60 - A_3 \sin 30) = 1.94 \times 40^2 \times (-0.2 \sin 60 + 0.1 \sin 30) \\ F_y &= \underline{\underline{-382 \text{ lbf/ft (acts downward)}}} \end{aligned}$$

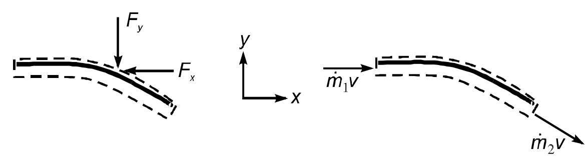
6.20 **Information and assumptions**

a water jet is deflected by a fixed vane, $\dot{m} = 16$ lbm/s $= 0.497$ slug/s
$v_1 = 60$ ft/s

Find

force of the water on the vane: **F**

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v = 60 \text{ ft/s}$$

x**-momentum**

$$\begin{aligned}
\sum F_x &= \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\
-F_x &= \dot{m}v\cos 30 - \dot{m}v \\
F_x &= \dot{m}v(1-\cos 30) = 0.497 \times 60 \times (1-\cos 30) \\
F_x &= 4.00 \text{ lbf to the left}
\end{aligned}$$

y**-momentum**

$$\begin{aligned}
\sum F_y &= \dot{m}_o (v_o)_y \\
-F_y &= \dot{m}(-v\cos 60) = -0.497 \times 60 \times \sin 30 \\
F_y &= 14.9 \text{ lbf downward}
\end{aligned}$$

Equilibrium of forces acting on block

force of water on vane $= \mathbf{F} = -F_x\mathbf{i} - F_y\mathbf{j}$

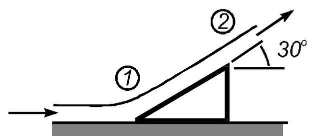
$$\mathbf{F} = \underline{\underline{(4.00 \text{ lbf})\,\mathbf{i} + (14.9 \text{ lbf})\,\mathbf{j}}}$$

6.21 **Information and assumptions**

a water jet strikes a block and the block is held in place by friction–however, we do not know if the frictional force is large enough to prevent the block from sliding

$v_1 = 10$ m/s, $\dot{m} = 1$ kg/s, $\mu = 0.1$, mass of block: $m = 1$ kg

neglect weight of water

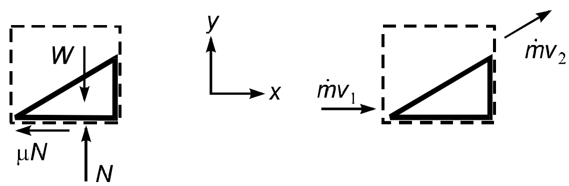


Find

will the block slip?

force of the water jet on the block: **F**

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v = 10 \text{ m/s}$$

x-momentum

$$\begin{aligned}
\sum F_x &= \dot{m}_o\,(v_o)_x - \dot{m}_i\,(v_i)_x \\
-F_f &= \dot{m}v\cos 30 - \dot{m}v \\
F_f &= \dot{m}v(1-\cos 30) = 1.0 \times 10 \times (1-\cos 30) \\
F_f &= 1.34 \text{ N}
\end{aligned}$$

y-momentum

$$\begin{aligned}
\sum F_y &= \dot{m}_o\,(v_o)_y \\
N - W &= \dot{m}(v\sin 30) \\
N &= mg + \dot{m}(v\sin 30) = 1.0 \times 9.81 + 1.0 \times 10 \times \sin 30 = 14.81 \text{ N}
\end{aligned}$$

Friction

F_f (required to prevent block from slipping) $= 1.34$ N

F_f (maximum possible value) $= \mu N = 0.1 \times 14.81 = 1.48$ N

block will not slip

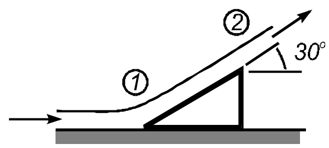
Equilibrium of forces acting on block

force of water on block $= -F_f\,\mathbf{i} + (W - N)\mathbf{j}$

$$\mathbf{F} = \underline{\underline{(1.34\ \text{N})\,\mathbf{i} + (-5.00\ \text{N})\,\mathbf{j}}}$$

6.22 **Information and assumptions**

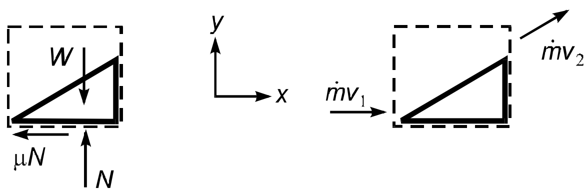
a water jet strikes a block and the block is held in place by friction, $\mu = 0.1$
$\dot{m} = 1$ kg/s, mass of block: $m = 1$ kg
neglect weight of water



Find

maximum velocity such that the block will not slip: v

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v$$

x-momentum

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
-\mu N &= \dot{m}v\cos 30 - \dot{m}v \\
N &= \dot{m}v\left(1 - \cos 30\right)/\mu
\end{aligned}$$

y-momentum

$$\begin{aligned}
\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\
N - W &= \dot{m}(v\sin 30) \\
N &= mg + \dot{m}(v\sin 30)
\end{aligned}$$

Combine previous two equations

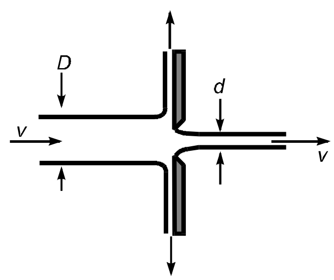
$$\begin{aligned}
\dot{m}v\left(1 - \cos 30\right)/\mu &= mg + \dot{m}(v\sin 30) \\
v &= mg/\left[\dot{m}\left(1/\mu - \cos 30/\mu - \sin 30\right)\right] \\
v &= 1 \times 9.81/\left[1 \times \left(1/0.1 - \cos 30/0.1 - \sin 30\right)\right] \\
v &= \underline{\underline{11.7 \text{ m/s}}}
\end{aligned}$$

6.23 **Information and assumptions**

a water jet strikes plate A and a portion of this jet passes through the sharp-edged orifice at the center of the plate

$v = 30$ m/s, $D = 10$ cm, $d = 5$ cm, $\rho = 999$ kg/m^3

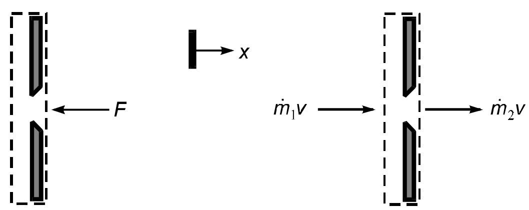
neglect gravity



Find

force required to hold plate stationary: F

Force and momentum diagrams (only x-direction vectors shown)



x-momentum

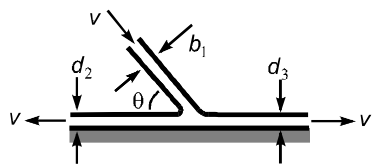
$$\begin{aligned}
\sum \mathbf{F} &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\
-F &= \dot{m}_2 v - \dot{m}_1 v \\
F &= \rho A_1 v^2 - \rho A_2 v^2 = \rho v^2 \pi/4 (D^2 - d^2) = 999 \times 30^2 \times \pi/4 \times (0.1^2 - 0.05^2) \\
F &= \underline{\underline{5.30 \text{ kN}}} \text{ (to the left)}
\end{aligned}$$

6.24 **Information and assumptions**

2D liquid jet strikes a horizontal surface

$v_1 = v_2 = v_3 = v$

assume force associated with shear stress is negligible; let the width of the jet in the z-direction = w



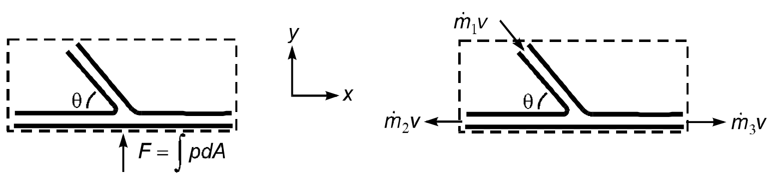
Find

derive formulas for d_2 and d_3 as a function of b_1 and θ

Continuity

$$\begin{aligned}\dot{m}_1 &= \dot{m}_2 + \dot{m}_3 \\ \rho w b_1 v &= \rho w d_2 v + \rho w d_3 v \\ b_1 &= d_2 + d_3\end{aligned}$$

Force and momentum diagrams



x**-momentum**

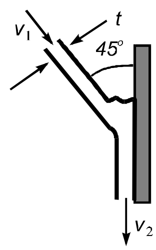
$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\ 0 &= (\dot{m}_3 v + \dot{m}_2(-v)) - \dot{m}_1 v \cos\theta \\ 0 &= \left(\rho w d_3 v^2 - \rho w d_2 v^2\right) - \rho w b_1 v^2 \cos\theta \\ 0 &= d_3 - d_2 - b_1 \cos\theta\end{aligned}$$

Combining x-momentum and continuity equations

$$\begin{aligned}d_3 &= d_2 + b_1 \cos\theta \\ d_3 &= b_1 - d_2 \\ d_2 &= \underline{\underline{b_1(1-\cos\theta)/2}} \\ d_3 &= \underline{\underline{b_1(1+\cos\theta)/2}}\end{aligned}$$

6.25 **Information and assumptions**

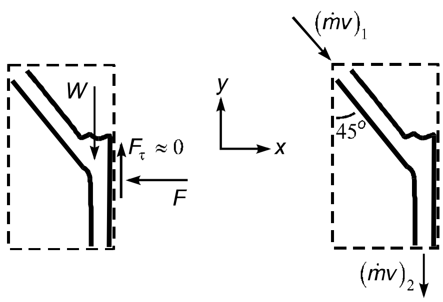
a 2D liquid jet impinges on a vertical wall
$v_1 = v_2 = v$
assume steady flow, assume force associated with shear stress is negligible
let w = the width of the jet in the z-direction



Find

force acting on the wall (per unit width of the jet): F/w
sketch and explain the shape of the liquid surface

Force and momentum diagrams



x-momentum

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ -F &= -\dot{m}v_1 \sin 45^o \\ F &= \rho w t v^2 \sin 45^o\end{aligned}$$

the force on that acts on the wall is in the opposite direction to force pictured on the force diagram, thus

$$F/w = \underline{\underline{\rho t v^2 \sin 45^o}} \text{ (acting to the right)}$$

y-momentum

$$\begin{aligned}
\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\
-W &= \dot{m}\,(-v) - \dot{m}\,(-v)\cos 45^o \\
W &= \dot{m}v(1 - \cos 45^o)
\end{aligned}$$

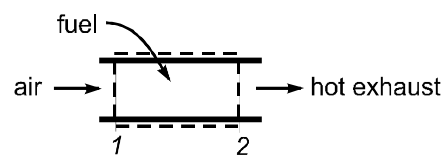
Thus, weight provides the force needed to increase y-momentum flow. This weight is produced by the fluid swirling up to form the shape show in the above sketches.

6.26 **Information and assumptions**

a jet engine (ramjet), the mass flow rate of the fuel is negligible

$$\begin{aligned} \dot{m} &= 50 \text{ kg/s}, v_1 = 200 \text{ m/s} \\ \rho_2 &= 0.25 \text{ kg/m}^3, A_2 = 0.5 \text{ m}^2 \end{aligned}$$

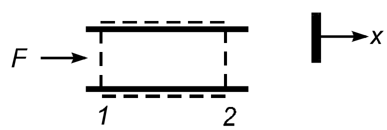
assume steady flow

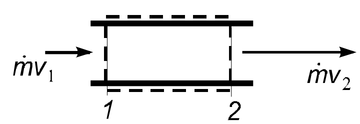


Find

thrust force produced by the ramjet: T

Force and momentum diagrams





let F equal the force required to hold the ramjet stationary

Calculate exit velocity

$$\begin{aligned} \dot{m}_2 &= \rho_2 A_2 v_2 \\ v_2 &= \dot{m}_2/(\rho_2 A_2) = 50/(0.25 \times 0.5) = 400 \text{ m/s} \end{aligned}$$

x-momentum

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ F &= \dot{m}(v_2 - v_1) = 50(400 - 200) = 10.0 \text{ kN} \\ T &= 10.0 \text{ kN (to the left)} \end{aligned}$$

6.27 **Information and assumptions**

A horizontal channel is described in the textbook

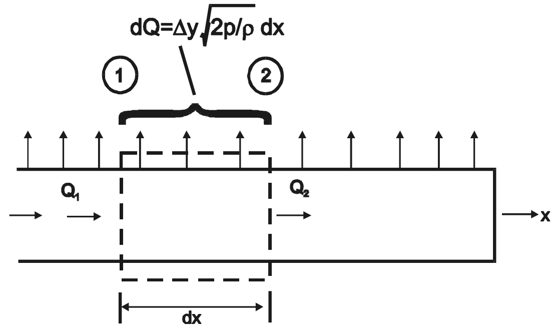
Find

an expression for y_1

x**-momentum** (cs passes through sections 1 and 2)

$$
\begin{aligned}
\sum F_x &= \dot{m}v_2 \\
(\overline{p}A)_1 - (\overline{p}A)_2 &= \rho Q_{v_2} \\
(By_1^2\gamma/2) - (By_2^2\gamma/2) &= \rho Q(Q/y_2B) \\
y_1 &= \underline{\underline{\sqrt{y_2^2 + (2/(gy_2)) \times (Q/B)^2}}}
\end{aligned}
$$

6.28 Obtain the pressure variation along the pipe by applying the momentum equation in steps along the pipe (numerical scheme). The first step would be for the end segment of the pipe. Then move up the pipe solving for the pressure change (Δp) for each segment. Then $p_{\text{end}} + \sum \Delta p$ would give the pressure at a particular section. The momentum equation for a general section is developed below.



x-momentum

$$\begin{aligned}
\Sigma F_x &= \sum_{cs} \dot{m}_o V_{ox} - \sum_{cs} \dot{m}_i V_{ix} \\
p_1 A_1 - p_2 A_2 &= \rho Q_2 (Q_2/A_2) - \rho Q_1 (Q_1/A_1) \\
\text{but } A_1 &= A_2 = A \text{ so we get} \\
p_1 - p_2 &= (\rho/A^2)(Q_2^2 - Q_1^2) \qquad (1)
\end{aligned}$$

As section 1 approaches section 2 in the limit we have the differential form

$$-dp = (\rho/A^2) dQ^2 = 2(\rho/A^2) Q dQ$$

Continuity

$$\begin{aligned}
Q_1 - Q_2 &= \Delta y \sqrt{2p/\rho} \Delta x \\
Q_1 &= Q_2 + \Delta y \sqrt{2p/\rho} dx
\end{aligned}$$

In the limit at $\Delta x \to 0$ we have

$$dQ = -\Delta y \sqrt{2p/\rho} dx$$

The differential equation for pressure becomes

$$dp = 2(\rho/A^2) dQ^2 = 2(\rho/A^2) Q \Delta y \sqrt{2p/\rho} dx$$

Integrating the momentum equation to evaluate Q at location x we have

$$Q = -\Delta y \int^x \sqrt{2p/\rho} d\xi$$

so the equation for pressure distribution is

$$p \,|_0^{\Delta L} = (4/A^2)\Delta y^2 \int_0^{\Delta L} p^{1/2} \left[\int_0^x p^{1/2} d\xi \right] dx$$

where L is some distance along the pipe.

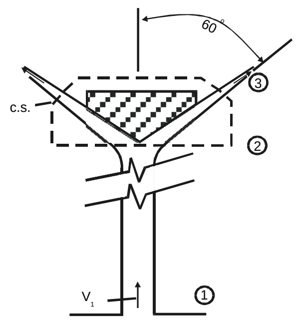
This equation has to be integrated numerically. One can start at the end of the pipe where the pressure is known (atmospheric pressure). The one can assume a linear pressure profile over the interval ΔL. An iterative solution would be needed for each step to select the slope of the pressure curve (pressure gradient). The pressure will decrease in the direction of flow.

6.29 **Information and assumptions**

provided in problem statement

Find

height to which cone will rise



Bernoulli Equation

$$\begin{aligned}
(V_1^2/2g) + 0 &= (V_2^2/2g) + h \\
V_2^2 &= (15)^2 - 2gh \\
V_2^2 &= (15)^2 - 2gh = 225 - 2 \times 9.81h \\
&= 225 - 19.62h
\end{aligned}$$

y-momentum

$$\begin{aligned}
\sum F_y &= \dot{m}_o v_{oy} - \dot{m}_i v_{iy} \\
-w &= \dot{m}(v_{3y} - v_2) \\
-40 &= 1,000 \times 15 \times \pi \times (0.015)^2 (v_2 \sin 30^\circ - v_2) \\
v_2 &= 7.55 \text{ m/s}
\end{aligned}$$

so from the Bernoulli equation

$$\begin{aligned}
(7.55)^2 &= 225 - 19.62h \\
h &= \underline{\underline{8.56 \text{ m}}}
\end{aligned}$$

6.30 **Information and assumptions**

pipe (6 in. diameter) with a 180^o bend
water, Q = 6 cfs, p = 20 psi gage
assume weight acts perpendicular to the flow direction
provided in problem statement

Find

force needed to hold the bend in place in direction parallel to flow

x-momentum

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
2pA - F_x &= -2\dot{m}v
\end{aligned}$$

Calculations

$$\begin{aligned}
pA &= (20 \times 144)\left(\pi/4 \times 0.5^2\right) = 565.5 \text{ lbf} \\
\dot{m}v &= \rho Q^2/A = 1.94 \times 6^2/(\pi/4 \times 0.5^2) = 355.7 \text{ lbf} \\
F_x &= 2(pA + \dot{m}v) = 2 \times (565.5 + 355.7) \text{ lbf} \\
F_x &= \underline{\underline{1840 \text{ lbf}}} \text{ (acting to the left, opposite of inlet flow)}
\end{aligned}$$

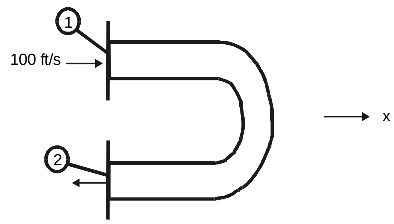
6.31 **Information and assumptions**

A return bend is described in the textbook provided in the problem

Find

force required to hold bend in place

Solution



$$\dot{m} = 1 \text{ lbm/s} = 0.0311 \text{ slugs/s}$$

At section (1):

$$v_1 = +100 \text{ ft/s}, \rho_1 = 0.02 \text{ lbm/ft}^3 = 0.000621 \text{ slugs/ft}^3$$

At section (2):

$$\rho_2 = 0.06 \text{ lbm/ft}^3 = 0.000186 \text{ slugs/ft}^3$$

Continuity

$$\begin{aligned}
\rho_1 v_1 A_1 &= \rho_2 v_2 A_2;\ v_2 = (\rho_1/\rho_2)(A_1/A_2)v_1 \\
v_2 &= (0.02/0.06)(1/1)v_1 = 33.33 \text{ ft/s}
\end{aligned}$$

x-momentum

$$\begin{aligned}
\sum F_x &= \sum_{\subset s} \dot{m}_o v_{o_x} - \sum_{cs} \dot{m}_i v_{i_x} = \dot{m}(v_2 - v_1) \\
F_x &= 0.0311(-33.33 - 100) \\
F_x &= \underline{\underline{-4.147 \text{ lbf}}}
\end{aligned}$$

6.32 Correct choice is (d).

6.33 **Information and assumptions**

provided in problem statement

Find

external force required to hold bend in place

Velocity Calculation

$$v = Q/A = 20/(\pi \times 0.5 \times 0.5) = 25.5 \text{ fps}$$

x-momentum

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ p_1 A_1 + p_2 A_2 + F_x &= \dot{m}(v_2 - v_1) \end{aligned}$$

thus

$$\begin{aligned} F_x &= -2pA - 2\dot{m}v \\ &= -2(15 \times 144(\pi/4 \times 1^2) + 1.94 \times 20 \times 25.5) \\ &= -5,370 \text{ lbf} \end{aligned}$$

y-momentum

$$\begin{aligned} \sum F_y &= 0 \\ -W_{\text{bend}} - W_{H_2 0} + F_y &= 0 \\ F_y &= 200 + 3 \times 62.4 = 387.2 \text{ lbf} \end{aligned}$$

Force required

$$\mathbf{F} = \underline{\underline{(-5,370\mathbf{i} + 387\mathbf{j}) \text{ lbf}}}$$

6.34 **Information and assumptions**

provided in problem statement

Find

force on flanges to hold bend in place

x-momentum

$$v = Q/A = 8.49 \text{ m/s}$$

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
2pA + F_x &= \rho Q(v_2 - v_1)
\end{aligned}$$

$$\begin{aligned}
(2)(100,000)(\pi/4)(0.3^2) + F_x &= (1,000)(0.60)(-8.49 - 8.49) \\
F_x &= -24,325 \text{ N} \\
F_y &= 0 \\
F_z &= 500 + (0.1)(9,810) = 1.481 \text{ kN}
\end{aligned}$$

The force on the flanges is

$$\mathbf{F} = \underline{\underline{(-24.3\mathbf{i} + 0\mathbf{j} + 1.48\mathbf{k}) \text{ kN}}}$$

6.35 **Information and assumptions**

A 90^0 pipe bend is described in the textbook.
provided in problem statement

Find

force on upstream flange to hold bend in place

Velocity calculation

$$v = Q/A = 10/((\pi/4 \times 1.0^2) = 12.73 \text{ ft/s}$$

x-momentum

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
pA + F_x &= \rho Q(0 - v) \\
F_x &= 1.94 \times 10(0 - 12.73) - 4 \times 144 \times \pi/4 \times 1^2 = -699 \text{ lbf}
\end{aligned}$$

y-momentum

$$\begin{aligned}
F_y &= \rho Q(-v - 0) \\
F_y &= -1.94 \times 10 \times 12.73 = -247 \text{ lbf}
\end{aligned}$$

z-momentum

$$-100 - 4 \times 62.4 + F_z = 0$$
$$F_z = +350 \text{ lbf}$$

The force is

$$\mathbf{F} = \underline{\underline{(-699\mathbf{i}\ - 247\mathbf{j}\ + 350\mathbf{k}) \text{ lbf}}}$$

6.36 **Information and assumptions**

A 90^0 pipe bend is described in the textbook.
provided in problem statement

Find

$x-$component of force applied to bend to hold it in place

Velocity calculation

$$v = Q/A = 10/(\pi \times 1^2/4) = 12.73 \text{ m/s}$$

x-momentum

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m} v_{ox} - \sum_{cs} \dot{m} v_{ix} \\
pA + F_x &= \rho Q(0 - v)
\end{aligned}$$

$$\begin{aligned}
300,000 \times \pi \times 0.5^2 + F_x &= 1,000 \times 10 \times (0 - 12.73) \\
F_x &= -362,919 \text{ N } = \underline{\underline{-363 \text{ kN}}}
\end{aligned}$$

6.37 **Information and assumptions**

provided in problem statement

Find

vertical component of force exerted by anchor on bend

Velocity calculation

$$v = Q/A = 31.4/(\pi \times 1 \times 1) - 9.995 \text{ ft/sec}$$

y-momentum

$$\begin{aligned}
\sum F_y &= \rho Q(v_{2y} - v_{1y}) \\
F_a - W_{\text{water}} - W_{\text{bend}} - p_2 A_2 \sin 30^\circ &= \rho Q(v \sin 30^\circ - v \sin 0^\circ) \\
F_a &= \pi \times 1 \times 1 \times 4 \times 62.4 + 300 + 8.5 \times 144 \times \pi \times 1 \times 1 \times 0.5 \\
&\quad +1.94 \times 31.4 \times (9.995 \times 0.5 - 0) \\
F_a &= \underline{\underline{3,310 \text{ lbs}}}
\end{aligned}$$

6.38 **Information and assumptions**

provided in problem statement

Find

magnitude and direction of external force components to hold bend in place.

Velocity and discharge

$$\begin{aligned} v_1 &= 10/4 = 2.5 \text{ m/s} \\ Q &= A_1 v_1 = \pi \times 0.3 \times 0.3 \times 2.5 = 0.707 \text{ m}^3/\text{s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_1 &= p_2 + (\rho/2)(v_2^2 - v_1^2) \\ &= 0 + (1,000/2)(10 \times 10 - 2.5 \times 2.5) \\ &= 46,875 \text{ Pa} \end{aligned}$$

x**-momentum**

$$\begin{aligned} F_x + p_1 A_1 &= \rho Q(-v_2 \cos\ 60^\circ - v_1) \\ F_x &= -46,875 \times \pi \times 0.3 \times 0.3 + 1,000 \times 0.707 \times (-10\cos 60^\circ - 2.5) \\ &= -18,560 \text{ N} \end{aligned}$$

y**-momentum**

$$\begin{aligned} F_y &= \rho Q(-v_2 \sin 60^\circ - v_1) \\ F_y &= 1,000 \times 0.707 \times (-10 \sin 60^\circ - 0) = -6,123 \text{ N} \end{aligned}$$

z**-momentum**

$$\begin{aligned} F_z - W_{\mathrm{H_20}} - W_{\mathrm{bend}} &= 0 \\ F_z &= (0.25 \times 9,810) + (250 \times 9.81) = 4,905 \text{ N} \end{aligned}$$

Net force

$$\mathbf{F} = \underline{\underline{(-18.6\mathbf{i} - 6.12\mathbf{j} + 4.91\mathbf{k}) \text{ kN}}}$$

6.39 **Information and assumptions**

provided in problem statement

Find

vertical force applied to nozzle at flange

Continuity

$$\begin{aligned} v_1 A_1 &= v_2 A_2 \\ v_1 &= v_2 A_2 / A_1 = 62.5 \text{ ft/s} \\ Q &= v_2 A_2 = (125 \text{ ft/s})(0.5 \text{ ft}^2) \\ &= 62.5 \text{ cfs} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_1/\gamma + v_1^2/2g + z_1 &= p_2/\gamma + v_2^2/2g + z_2 \\ p_1/\gamma &= 0 + (125^2/2g) + 2 - (62.5^2/2g) \\ p_1 &= 62.4(242.6 + 2 - 60.7) = 11,479 \text{ lbf/ft}^2 \end{aligned}$$

y**-momentum**

$$p_1 A_1 - W_{H_2 0} - W_{\text{nozzle}} + F_y = \rho Q (v_2 \sin 30^\circ - v_1)$$

Calculations

$$\rho Q (v_2 \sin 30^\circ - v_1) = (1.94)(62.5)\left[(100 \sin 30^\circ) - 50\right] = 0 \text{ lbf}$$

Thus,

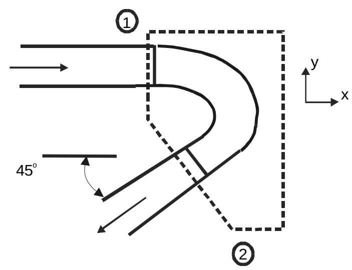
$$\begin{aligned} F_y &= W_{H_2 0} + W_{\text{nozzle}} - p_1 A_1 = (1.8 \times 62.4) + (100) - (11,479 \times 1) = -11,300 \text{ lbf} \\ F_y &= \underline{\underline{11,300 \text{ lbf (acting downward)}}} \end{aligned}$$

6.40 **Information and assumptions**

provided in problem statement

Find

external force required to hold bend



Discharge

$$Q = vA = 15 \times \pi/4 \times 1^2 = 11.78 \text{ cfs}$$

x-momentum

$$\begin{aligned}
\sum F_x &= \rho Q(v_{2x} - v_{1x}) \\
p_1A_1 + p_2A_2\cos 45^\circ + F_x &= \rho Q(-v_2\cos 45^\circ - v_1) \\
F_x &= -pA(1+\cos 45^\circ) - \rho Qv(1+\cos 45^\circ) \\
&= -(1,440) \times (\pi/4 \times 1^2)(1+\cos 45^\circ) - (0.8 \times 1.94)(11.78)(15)(1+\cos 45^\circ) \\
&= -2,400 \text{ lbf}
\end{aligned}$$

y-momentum

$$\begin{aligned}
\sum F_y &= \rho Q(v_{2y} - v_{1y}) \\
p_2A_2\sin 45^\circ + F_y &= \rho Q(-v_2\sin 45^\circ - 0) \\
F_y &= -pA\sin 45^\circ - \rho Qv_2\sin 45^\circ \\
F_y &= -(1,440)(\pi/4 \times 1^2)\sin 45^\circ - (0.8 \times 1.94)(11.78)(15)\sin 45^\circ \\
F_y &= -994 \text{ lbf}
\end{aligned}$$

Net force

$$\mathbf{F} = \underline{\underline{(-2,400\mathbf{i} - 994\mathbf{j}) \text{ lbf}}}$$

6.41 **Information and assumptions**

provided in problem statement

Find

external force required to hold bend

Discharge

$$Q = 8 \times \pi/4 \times 0.15 \times 0.15 = 0.141 \text{ m}^3/\text{s}$$

$x-$**momentum**

$$\begin{aligned}
\sum F_x &= \dot{m}(v_{2x} - v_{1x}) \\
p_1A_1 + p_2A_2\cos 45^\circ + F_x &= \rho Q(-v_2\cos 45^\circ - v_1) \\
F_x &= -pA(1+\cos 45^0) - \rho Q v(1+\cos 45^\circ) \\
&= -(100,000)(\pi/4 \times 0.15^2)(1+\cos 45^\circ) - (1000 \times 0.8)(0.141)(8)(1+\cos 45^\circ) \\
&= -4557 \text{ N}
\end{aligned}$$

$y-$**momentum**

$$\begin{aligned}
\sum F_y &= \rho Q(v_{2y} - v_{1y}) \\
p_2A_2\sin 45^\circ + F_y &= -\rho Q v_2 \sin 45^\circ \\
&= -(100,000)(\pi/4 \times 0.15^2)\sin 45^\circ - (1,000 \times 0.8)(0.141)(8)\sin 45^\circ \\
&= -1,888 \text{ N}
\end{aligned}$$

Net force

$$\mathbf{F} = \underline{\underline{(-4.56\mathbf{i} - 1.89\mathbf{j}) \text{ kN}}}$$

6.42 **Information and assumptions**

provided in problem statement

Find

horizontal force required to hold bend in place

Bernoulli equation

$$\begin{aligned} v_1 &= v_2 A_2/A_1 = 50(1/10) = 5 \text{ m/s} \\ p_1 + \rho v_1^2/2 &= p_2 + \rho v_2^2/2 \end{aligned}$$

Let $p_2 = 0$, then

$$\begin{aligned} p_1 &= -(1,000)/2)(5^2) + (1,000/2)(50^2) \\ p_1 &= 1,237.5 \text{ kPa} \end{aligned}$$

x-momentum

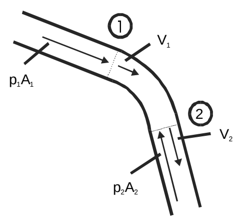
$$\begin{aligned} \sum F_x &= \dot{m}(v_{2x} - v_{1x}) \\ p_1 A_1 + F_x &= \rho A_2 v_2 (v_2 \cos 60^\circ - v_1) \\ F_x &= -1,237,000 \times 0.001 + 1,000 \times 0.0001 \times 50(50 \cos 60^\circ - 5) \\ F_x &= \underline{\underline{1,140 \text{ N}}} \end{aligned}$$

6.43 **Information and assumptions**

provided in problem statement

Find

force thrust block must exert on bend



Velocity vectors

$$\mathbf{v}_1 = (Q/A)[(13/\ell_1)\mathbf{j} - (10/\ell_1)\mathbf{k}]$$

where $\ell_1 = \sqrt{13^2 + 10^2}$. Thus

$$\begin{aligned}
\mathbf{v}_1 &= (Q/A)[0.793\mathbf{j} - 0.6097\mathbf{k}] \\
\mathbf{v}_2 &= (Q/A)[(13/\ell_2)\mathbf{i} + (19/\ell_2)\mathbf{j} - (20/\ell_2)\mathbf{k}]
\end{aligned}$$

where $\ell_2 = \sqrt{13^2 + 19^2 + 20^2}$. Then

$$\mathbf{v}_2 = (Q/A)[0.426\mathbf{i} + 0.623\mathbf{j} - 0.656\mathbf{k}]$$

Pressure forces

$$\begin{aligned}
\mathbf{F}_{p_1} &= p_1 A_1 (0.793\mathbf{j} - 0.6097\mathbf{k}) \\
\mathbf{F}_{p_2} &= p_2 A_2 (-0.426\mathbf{i} - 0.623\mathbf{j} + 0.656\mathbf{k})
\end{aligned}$$

Weight

$$\mathbf{W} = -3 \times 9,810\mathbf{k}$$

x**-momentum**

$$\begin{aligned}
\sum F_x &= \rho Q(v_{2x} - v_{1x}) \\
F_x - 0.426 \times p_2 A_2 &= \rho Q[0.426 Q/A - 0]
\end{aligned}$$

where

$$\begin{aligned}
p_2 A_2 &= 25,000 \times (\pi/4) \times (1.3)^2 = 33,183 \text{ N} \\
\text{and } Q/A &= 15/((\pi/4) \times (1.3)^2) = 11.30 \text{ m/s}
\end{aligned}$$

thus

$$F_x = 1,000 \times 15 \times 0.426 \times 11.3 + 0.426 \times 33,183 = 86,343 \text{ N}$$

y-momentum

$$\begin{aligned} \sum F_y &= \rho Q(v_{2y} - v_{1y}) \\ F_y + 0.793 p_1 A_1 - 0.623 p_2 A_2 &= \rho Q[0.623(Q/A) - 0.793 Q/A] \end{aligned}$$

where

$$\begin{aligned} p_1 A_1 &= 20,000 \times (\pi/4)(1.3)^2 = 26,546 \text{ N} \\ F_y &= 1,000 \times 15(11.3)(-0.170) - 0.793 \times 26,546 \\ &\quad +0.623 \times 33,183 \\ &= -28,815 - 21,051 + 20,673 = \underline{\underline{-29,193 \text{ N}}} \end{aligned}$$

z-momentum

$$\sum F_z = \rho Q(v_{2_z} - v_{1_z})$$
$$F_z - 0.6097 p_1 A_1 + 0.656 p_2 A_2 - W = 1,000 \times 15[-0.656(Q/A) - (-0.6097 Q/A)]$$
$$F_z = -7,848 + 3 \times 9,810 + 10,000 - 0.656 \times 33,183 + 0.6097 \times 26,546 = \underline{\underline{25,999 \text{ N}}}$$

Net force

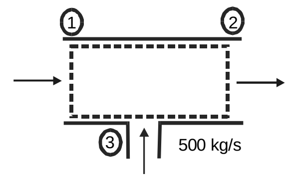
$$\mathbf{F} = \underline{\underline{(86.4\mathbf{i} - 29.2\mathbf{j} + 26.0\mathbf{k}) \text{ kN}}}$$

6.44 **Information and assumptions**

provided in problem statement

Find

pressure difference between 1 and 2



Continuity

$$
\begin{aligned}
\dot{m}_1 + 500 \text{ kg/s} &= \dot{m}_2 \\
\dot{m}_1 &= (10 \text{ m/s})(0.10 \text{ m}^2)(1,000 \text{ kg/m}^3) = 1,000 \text{ kg/s} \\
\dot{m}_2 &= 1,000 + 500 = 1,500 \text{ kg/s} \\
v_2 &= (\dot{m}_2)/(\rho A_2) = (1,500)/((1,000)(0.1)) = 15 \text{ m/s}
\end{aligned}
$$

***x*-momentum**

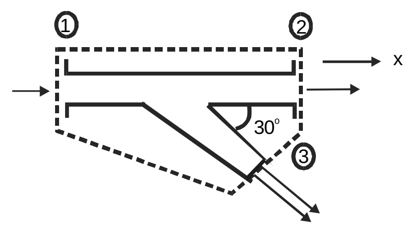
$$
\begin{aligned}
\sum F_x &= \dot{m}_2 v_{2x} - \dot{m}_1 v_{1x} - \dot{m}_3 v_{3x} \\
p_1 A_1 + p_2 A_2 &= \dot{m}_2 v_2 - \dot{m}_1 v_1 - 0 \\
A(p_1 - p_2) &= (-1,000)(10) + (1,500)(15) \\
p_1 - p_2 &= (22,500 - 10,000)/0.10 \\
&= 125,000 \text{ Pa} = \underline{\underline{125 \text{ kPa}}}
\end{aligned}
$$

6.45 **Information and assumptions**

provided in problem statement

Find

$x-$component of force to hold wye in place



Velocity and flow rate calculations

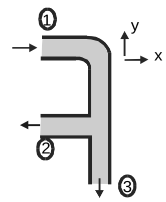
$$\begin{aligned} v_1 &= Q_1/A_1 = 20 \text{ ft/s} \\ v_2 &= Q_2/A_2 = 12 \text{ ft/s} \\ Q_3 &= 20 - 12 = 8 \text{ ft}^3/\text{s} \\ v_3 &= Q_3/A_3 = 32 \text{ ft/s} \end{aligned}$$

x-momentum

$$\begin{aligned} \sum F_x &= \dot{m}_2 v_2 + \dot{m}_3 v_3 \cos 30^\circ - \dot{m}_1 v_1 \\ F_x + p_1 A_1 - p_2 A_2 &= (20\rho)(-20) + (12\rho)(+12) + (32\cos 30^\circ)(\rho)(8) \\ F_x + (1,000)(1) - (900)(1) &= -400\rho + 144\rho + \rho(8)(32)(0.866) \\ F_x &= 100 - 1.94(-34.3) \\ F_x &= \underline{\underline{-166.5 \text{ lbf (acting to the left)}}} \end{aligned}$$

6.46 **Information and assumptions**

Water flow through a horizontal bend and T section provided in problem statement



$$\begin{aligned} \dot{m}_1 &= 10 \text{ lbm/s}, \dot{m}_2 = \dot{m}_3 = 5 \text{ lbm/s} \\ A_1 &= A_2 \ = \ A_3 \ = \ 5 \text{ in}^2, \ p_1 = 5 \text{ psig}, \ p_2 = p_3 = 0 \end{aligned}$$

Find

Horizontal component of force to hold fitting stationary: F_x

Velocity calculations

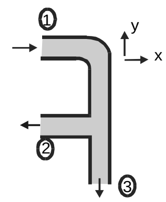
$$\begin{aligned} v_1 &= \dot{m}_1/\rho A_1 = (10/32.2)/\left[(1.94)(5/144)\right] = 4.61 \text{ ft/s} \\ v_2 &= \dot{m}_2/\rho A_2 = (5/32.2)/\left[(1.94)(5/144)\right] = 2.31 \text{ ft/s} \end{aligned}$$

x-momentum

$$\begin{aligned} \sum F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\ p_1 A_1 + F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\ F_x &= -p_1 A_1 - \dot{m}_2 v_2 - \dot{m}_1 v_1 \\ &= -(5 \times 5) - (5/32.2)(2.31) - (10/32.2)(4.61) \\ F_x &= \underline{\underline{-26.8 \text{ lbf}}} \end{aligned}$$

6.47 **Information and assumptions**

Water flow through a horizontal bend and T section provided in problem statement



$$\begin{aligned} v_1 &= 6 \text{ m/s}, \, p_1 = 4.8 \text{ kPa}, \, v_2 = v_3 = 3 \text{ m/s}, \, p_2 = p_3 = 0 \\ A_1 &= A_2 \, = \, A_3 \, = \, 0.20 \text{ m}^2 \end{aligned}$$

Find

Components of force (F_x, F_y) needed to hold bend stationary.

Discharge

$$\begin{aligned} Q_1 &= A_1 v_1 = 0.2 \times 6 = 1.2 \text{ m}^3/\text{s} \\ Q_2 &= Q_3 = A_2 v_2 = 0.2 \times 3 = 0.6 \text{ m}^3/\text{s} \end{aligned}$$

x**-momentum**

$$\begin{aligned} \sum F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\ p_1 A_1 + F_x &= -\rho(Q_2 v_2 + Q_1 v_1) \\ F_x &= -p_1 A_1 - \rho(Q_2 v_2 + Q_1 v_1) \\ &= -4,800 \times 0.2 - 1,000(0.6 \times 3 + 1.2 \times 6) \\ F_x &= \underline{\underline{-9.96 \text{ kN (acts to the left)}}} \end{aligned}$$

y**-momentum**

$$\begin{aligned} \sum F_y &= \dot{m}_3(-v_3) \\ F_y &= -\rho Q_3 V_3 = -1,000 \times 0.6 \times 3 \\ F_y &= \underline{\underline{-1.8 \text{ kN (acts downward)}}} \end{aligned}$$

6.48 **Information and assumptions**

provided in problem statement

Find

$x-$ and $y-$components necessary to retain section

Velocity calculations

$$\begin{aligned} v_1 &= 0.25/(\pi \times 0.075 \times 0.075) = 14.15 \text{ m/s} \\ v_2 &= 0.15/(\pi \times 0.05 \times 0.05) = 19.10 \text{ m/s} \\ v_3 &= (0.25 - 0.15)/(\pi \times 0.075 \times 0.075) = 5.66 \text{ m/s} \end{aligned}$$

x-momentum

$$F_x = -100,000 \times \pi \times 0.075 \times 0.075 + 80,000 \times \pi \times 0.075 \times 0.075$$
$$-1,000 \times 14.15 \times 0.25 + 1,000 + \times 5.66 \times 0.10 = -3,325 \text{ N} = -3.325 \text{ kN}$$

y-momentum

$$\begin{aligned} F_y &= -1,000 \times 19.10 \times 0.15 - 70,000 \times \pi \times 0.05 \times 0.05 \\ &= -3,415 \text{ N } = -3.415 \text{ kN} \end{aligned}$$

Net force

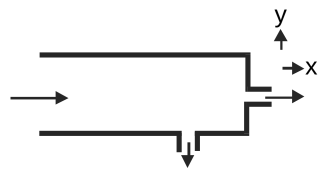
$$\mathbf{F} = \underline{\underline{(-3.325\mathbf{i} - 3.415\mathbf{j}) \text{ kN}}}$$

6.49 **Information and assumptions**

provided in problem statement

Find

force at flange to hold nozzle in place



Continuity

$$
\begin{aligned}
v_p A_p &= \sum v_j A_j \\
v_p &= 2 \times 30 \times 0.01/0.10 = 6.00 \text{ m/s}
\end{aligned}
$$

Bernoulli equation

$$
p_{\text{pipe}}/\gamma + v_p^2/2g = p_{\text{jet}}/\gamma + v_j^2/2g
$$

Then

$$
\begin{aligned}
p_p &= (\gamma/2g)(v_j^2 - v_p^2) \\
&= 500(900 - 36) = 432,000 \text{ Pa}
\end{aligned}
$$

x-momentum

$$
\begin{aligned}
p_p A_p + F_x &= -v_p \rho v_p A_p + v_j \rho v_j A_j \\
F_x &= -1,000 \times 6^2 \times 0.10 + 1,000 \times 30^2 \times 0.01 - 432,000 \times 0.1 \\
F_x &= -37,800 \text{ N}
\end{aligned}
$$

y-momentum

$$
\begin{aligned}
F_y &= \dot{m}(-v_j) = -v_j \rho v_j A \\
&= -30 \times 1,000 \times 30 \times 0.01 = -9,000 \text{ N}
\end{aligned}
$$

z-momentum

$$
\begin{aligned}
\sum F_z &= 0 \\
-200 - \gamma V + F_z &= 0 \\
F_z &= 200 + 9,810 \times 0.1 \times 0.4 = 592 \text{ N}
\end{aligned}
$$

Net force

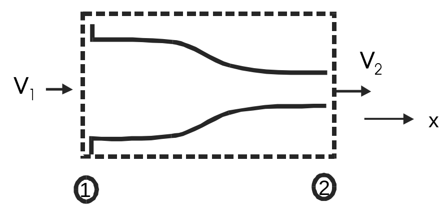
$$
\mathbf{F} = \underline{\underline{(-37.8\mathbf{i} - 9.0\mathbf{j} + 0.59\mathbf{k}) \text{ kN}}}
$$

6.50 **Information and assumptions**

provided in problem statement

Find

force at flange to hold nozzle in place



Velocity calculations

$$
\begin{aligned}
v_1 &= Q/A_1 = 15/((\pi/4) \times 1^2) = 19.098 \text{ ft/s} \\
v_2 &= Q/A_2 = 15/((\pi/4)(8/12)^2) = 42.97 \text{ ft/s}
\end{aligned}
$$

Bernoulli equation

$$
\begin{aligned}
p_1 + \rho v_1^2/2 &= p_2 + v_2^2/2 \\
p_1 &= 0 + (\rho/2)(v_2^2 - v_1^2) \\
&= 1,437 \text{ lbf/ft}^2
\end{aligned}
$$

x**-momentum**

$$
\begin{aligned}
\sum F_x &= \dot{m}v_2 - \dot{m}v_1 \\
p_1 A_1 + F &= 42.97(1.94)(15) - (19.098)(1.94)(15) \\
(1,437)(\pi/4)(1^2) + F &= (1.94)(15)(42.97 - 19.098) \\
F &= \underline{\underline{-434 \text{ lbf}}} \text{ (acts to left)}
\end{aligned}
$$

6.51 **Information and assumptions**

provided in problem statement

Find

force at flange to hold nozzle in place

Velocity calculation

$$\begin{aligned} v_1 &= 0.3/(\pi \times 0.15 \times 0.15) = 4.244 \text{ m/s} \\ v_2 &= 4.244 \times 9 = 38.196 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_1 = 0 + (1,000/2)(38,196^2 - 4.244^2) = 720 \text{ kPa}$$

x-momentum

$$\begin{aligned} F_x &= -720,000 \times \pi \times 0.15^2 + 1,000 \times 0.3(38.196 - 4.244) \\ F_x &= \underline{\underline{-40.7 \text{ kN}}} \text{ (acts to the left)} \end{aligned}$$

6.52 **Information and assumptions**

provided in problem statement

Find

x-component of force through flange bolts to hold nozzle in place

Velocity calculation

$$\begin{aligned} v_A &= v_B = 16 \times 144/[(\pi/4)(4 \times 4 + 4.5 \times 4.5)] = 80.93 \text{ fps} \\ v_1 &= 16/(\pi \times 0.5 \times 0.5) = 20.37 \text{ fps} \end{aligned}$$

Bernoulli equation

$$p_1 = 0 + (1.94/2)(80.93 \times 80.93 - 20.37 \times 20.37) = 5,951 \text{ psf}$$

x-momentum

$$F_x = -5,951 \times \pi \times 0.5 \times 0.5 \times \sin 30^\circ - 80.93 \times 1.94 \times 80.93 \times \pi \times 2$$
$$\times 2/144 - 20.37 \times 1.94 \times 16.0 \sin 30^\circ = \underline{\underline{-3,762 \text{ lbf}}}$$

6.53 **Information and assumptions**

provided in problem statement

Find

x-component of force acting through flange bolts required to keep nozzle in place

Velocity calculation

$$\begin{aligned} v_A &= v_B = 0.5/(\pi \times 0.05 \times 0.05 + \pi \times 0.06 \times 0.06) = 26.1 \text{ m/s} \\ v_1 &= 0.5/(\pi \times 0.15 \times 0.15) = 7.07 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_1 = (1,000/2)(26.1^2 - 7.07^2) = 315,612 \text{ Pa}$$

x-momentum

$$\sum F_x = \dot{m}_o v_{ox} - m_i v_{ix}$$

$$F_x + p_1 A_1 \sin 30 = -\dot{m} v_A - \dot{m} v_i \sin 30$$

$$F_x = -315,612 \times \pi \times 0.15^2 \times \sin 30^\circ - 26.1 \times 1,000 \times 26.1$$

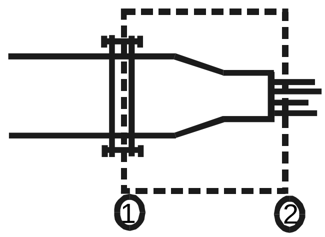
$$\times \pi \times 0.05^2 - 7.07 \times 1,000 \times 0.5 \sin 30^\circ = -18,270 \text{ N} = \underline{\underline{-18.27 \text{ kN}}}$$

6.54 **Information and assumptions**

provided in problem statement

Find

tension load in each bolt



Continuity

$$v_2 = (A_1/A_2)v_1 = 4v_1$$

Bernoulli equation

$$\begin{aligned}(v_1^2/2g) + (p_1/\gamma) &= (v_2^2/2g) + (p_2/\gamma)\\ 15(v_1^2/2g) &= (200,000/9,810)\\ v_1 &= 5.16 \text{ m/s}\\ v_2 &= 20.66 \text{ m/s}\\ Q &= 0.365 \text{ m}^3/\text{s}\end{aligned}$$

x-momentum

$$\begin{aligned}\sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix}\\ &= F_{\text{bolts}} + p_1 A_1 = \rho Q(v_2 - v_1)\end{aligned}$$

Thus

$$\begin{aligned}F_{\text{bolts}} &= -200,000 \times \pi \times 0.15^2 + 1,000 \times 0.365(20.66 - 5.16)\\ &= -7,441 \text{ N}\end{aligned}$$

Force per bolt = $\underline{\underline{1{,}240 \text{ N}}}$

6.55 **Information and assumptions**

provided in problem statement

Find

pressure at the gage

pressure and force per unit foot on end plates

Velocity calculation

$$\begin{aligned} v_b &= 5/(1/4) = 20 \text{ ft/sec} \\ v_B &= 20 \times (3/8) = 7.5 \text{ ft/sec} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_B &= (\rho/2)(v_b^2 - v_B^2) \\ &= (1.94/2)(20^2 - 7.5^2) = 333.4 \text{ psf} \end{aligned}$$

Hydrostatic equation

$$p_{\text{gage}} = 333.4 - 62.4 \times 4/12 = \underline{\underline{312.6 \text{ psf} = 2.171 \text{ psi}}}$$

x-momentum

$$\begin{aligned} \sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} = \rho Q(v_b - v_B) \\ F_x + p_B A_B &= \rho Q(v_b - v_B) \end{aligned}$$

thus,

$$F_x = -333.4 \times 8/12 + 1.94 \times 5 \times (20 - 7.5) = \underline{\underline{-101 \text{ lbf/ft}}}$$

6.56 **Information and assumptions**

provided in problem statement

Find

pressure at the gage

pressure and force per unit meter on end plates.

Velocity calculation

$$\begin{aligned} v_b &= 0.4/0.07 = 5.71 \text{ m/s} \\ v_B &= 0.40/0.20 = 2.00 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_B = (1,000/2)(5.71^2 - 2.00^2) = 14.326 \text{ kPa}$$

Hydrostatic equation

$$p_{\text{gage}} = 14,326 - 9.810 \times 0.1 = \underline{\underline{13.3 \text{ kPa}}}$$

x-momentum

$$\begin{aligned} \sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\ F_x + p_B A_B &= \rho Q(v_b - v_B) \end{aligned}$$

thus

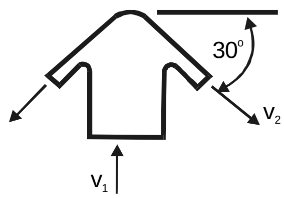
$$F_x = -14,326 \times 0.2 + 1,000 \times 0.4(5.71 - 2.00) = -1,381 \text{ N} = \underline{\underline{-1.38 \text{ kN/m}}}$$

6.57 **Information and assumptions**

provided in problem statement

Find

force acting through the bolts to keep the spray head on.



Velocity calculation

$$v_1 = Q/A_1 = 3/(\pi/4 \times 0.5^2) = 15.28 \text{ ft/s}$$

y-momentum

$$\begin{aligned}
\sum F_y &= \dot{m}_o v_{oy} - \dot{m}_i v_{iy} \\
F_y + p_1 A_1 &= \rho Q(-v_2 \sin 30^\circ - v_1) \\
F_y &= (-3872.8)(\pi/4 \times 0.5^2) + 1.94 \times 3(-65 \sin 30^\circ - 15.28) \\
&= \underline{\underline{-1,040 \text{ lbf}}}
\end{aligned}$$

6.58 **Information and assumptions**

provided in problem statement

Find

force required at flange to hold nozzle in place

Continuity

$$v_1 = Q/A = (2 \times 80.2 \times \pi/4 \times 0.5^2)/(\pi/4 \times 2^2) = 10.025 \text{ fps}$$

x-momentum

$$\begin{aligned}
\sum F_x &= \dot{m}_{ox} - \dot{m}_i v_{ix} \\
p_1 A_1 + F_x &= \dot{m}_2 v_{2x} + \dot{m}_3 v_{3x} - \dot{m}_1 v_{1x} \\
F_x &= -43 \times \pi \times 2^2 + 1.94 \times 80.2^2 \times \pi \times .5^2/144 \\
&\quad -(1.94 \times 80.2 \times \pi \times 0.5^2/144) \times 80.2 \sin 30 \\
&\quad -(1.94 \times 10.025 \times \pi \times 0.1667^2) \times 10.025 \\
&= -523.3 \text{ lbf}
\end{aligned}$$

y-momentum

$$\begin{aligned}
\sum F_y &= \dot{m}_{oy} - \dot{m}_i v_{iy} \\
F_y &= \dot{m}_3 v_{3y} = \rho A v_3(-v_3 \cos 30^\circ) \\
&= -1.94(\pi/4 \times (1/12)^2) 80.2^2 \cos 30^\circ \\
&= -58.94 \text{ lbf}
\end{aligned}$$

Net force

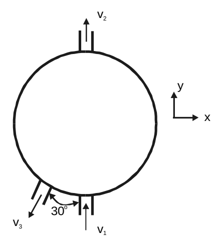
$$\mathbf{F} = \underline{\underline{(-523.3\mathbf{i} - 58.9\mathbf{j}) \text{ lbf}}}$$

6.59 **Information and assumptions**

provided in problem statement

Find

force in the inlet pipe wall required to hold sphere



Continuity

$$v_3 = (40 \times 2^2 - 100 \times 1^2)/1^2 = 60 \text{ ft/s}$$

x-momentum

$$\begin{aligned} F_x &= \dot{m}_3 v_{3x} \\ &= -\rho A_3 v_3^2 \sin 30^\circ \\ &= -(1.94 \times 1.2)(\pi/4 \times 0.0833^2)(60^2) \sin 30^\circ \\ &= -22.86 \text{ lbf} \end{aligned}$$

y-momentum

$$F_y - W + p_1 A_1 = \dot{m}_2 v_{2y} + \dot{m}_3 v_{3y} - \dot{m}_1 v_{1y}$$

thus

$$F_y = W - p_1 A_1 + \dot{m}_2 v_2 - m_3 v_3 \cos 30^\circ - \dot{m}_1 v_1$$

Calculations

$$\begin{aligned} & W_1 - p_1 A_1 - 200 - 60 \times \pi \times 1^2 = 11.50 \text{ lbf} \\ \dot{m}_2 v_2 &= \rho A_2 v_2^2 = (1.2 \times 1.94)(\pi/4 \times 0.0833^2)(100^2) = 126.97 \text{ lbf} \\ \dot{m}_3 v_3 \cos 30^\circ &= \rho A_3 v_3^2 \cos 30^\circ = (1.2 \times 1.94)(\pi/4 \times 0.0833^2)(60^2) \cos 30^\circ = 39.59 \text{ lbf} \\ \dot{m}_1 v_1 &= \rho A_1 v_1^2 = (1.2 \times 1.94)(\pi \times 0.0833^2)(40^2) = 81.26 \text{ lbf} \end{aligned}$$

thus,

$$F_y = (11.50 + 126.97 - 39.59 - 81.26) = 17.62 \text{ lbf}$$

Net Force

$$\mathbf{F} = \underline{\underline{(-22.9\mathbf{i} + 17.6\mathbf{j}) \text{ lbf}}}$$

6.60 **Information and assumptions**

provided in problem statement

Find

force required in pipe wall to hold sphere in place

Continuity

$$v_3 = (10 \times 5^2 - 30 \times 2.5^2)/(2.5^2) = 10 \text{ m/s}$$

x-momentum

$$F_x = -10 \sin 30^\circ \times 1,500 \times 10 \times \pi \times 0.0125^2 = -36.8 \text{ N}$$

y-momentum

$$\begin{aligned} F_y &= -400,000 \times \pi \times 0.025^2 + 600 + (1,500\pi) \\ &\quad \times (-10^2 \times 0.025^2 + 30^2 \times 0.0125^2 \\ &\quad -10^2 \times 0.0125^2 \cos 30^\circ) \\ &= 119 \text{ N} \end{aligned}$$

Net Force

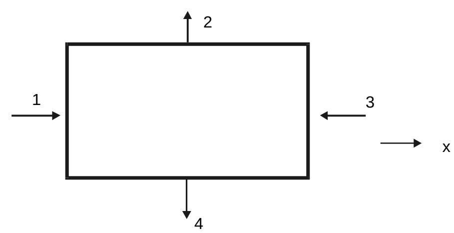
$$\mathbf{F} = \underline{\underline{(-36.8\mathbf{i} + 119\mathbf{j}) \text{ N}}}$$

6.61 **Information and assumptions**

provided in problem statement

Find

force required to hold "black box" in place



Continuity

$$\begin{aligned} Q_4 &= 0.6 - 0.10 \\ &= 0.50 \text{ m}^3/\text{s} \end{aligned}$$

x-momentum

$$F_x = -\dot{m}_1 v_{1_x} - \dot{m}_3 v_{3_x} = -\dot{m}_1 v_1 + \dot{m} v_3 = 0$$

y-momentum

$$\begin{aligned} F_y &= \dot{m}_2 v_{2_y} + \dot{m}_4 v_{4_y} \\ F_y &= \rho Q_2 v_2 - \rho Q_4 v_4 \\ &= (2.0 \times 1,000)(0.1)(20) - (2.0 \times 1,000)(0.5)(15) \\ &= -11.0 \text{ kN} \end{aligned}$$

Net Force

$$\mathbf{F} = \underline{\underline{(0\mathbf{i} - 11.0\mathbf{j}) \text{ kN}}}$$

6.62 To verify Eq. (6.11) the quantities Q, v_1, v_2, b, y_1, y_2 and F_G will have to be measured. Since a laboratory is available for your experiment it is assumed that the laboratory has equipment to obtain Q. The width b can be measured by a suitable scale. The depths y_1 and y_2 can be measured by means of piezometer tubes attached to openings in the bottom of the channel or by means of point gages by which the actual level of the surface of the water can be determined. Then v_1 and v_2 can be calculated from $v = Q/A = Q/(\text{by})$.

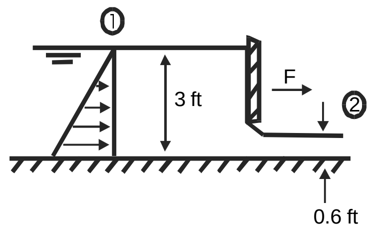
The force on the gate can be indirectly evaluated by measuring the pressure distribution on the face of the gate. This pressure may be sensed by piezometers or pressure transducer attached to small openings (holes) in the gate. The pressure taps on the face of the gate could all be connected to a manifold, and by appropriate valving the pressure at any particular tap could be sensed by a piezometer or pressure transducer. The pressures at all the taps should be measured for a given run. Then by integrating the pressure distribution over the surface of the gate one can obtain F_G. Then compare the measured F_G with the value obtained from the right hand side of Eq. (6.11). The design should be such that air bubbles can be purged from tubes leading to piezometer or transducer so that valid pressure readings are obtained.

6.63 **Information and assumptions**

provided in problem statement

Find

force of water per unit width of sluice gate



Bernoulli equation

$$
\begin{aligned}
v_1^2/2g + z_1 &= v_2^2/2g + z_2 \\
(0.6/3)^2 v_2^2/2g + 3 &= v_2^2/2g + 0.6 \\
v_2 &= 12.69 \text{ fps} \\
v_1 &= 2.54 \\
Q &= 7.614 \text{ cfs/ft}
\end{aligned}
$$

x-momentum

$$
\begin{aligned}
\sum F_x &= \rho Q(v_{2x} - v_{1x}) \\
& F_x + \overline{p}_1 A_1 - \overline{p}_2 A_2 = \rho Q(v_2 - v_1) \\
F_x &= -62.4 \times 3.0 \times 3.0/2 + 62.4 \times 0.6 \times 0.6/2 + 1.94 \times 7.614 \\
\times(12.69 - 2.54) &= \underline{\underline{-120 \text{ lbf/ft}}}
\end{aligned}
$$

6.64 **Information and assumptions**

provided in problem statement

Find

Solution

Derive a formula for the resisting shear force: F_τ

x-momentum

$$\begin{aligned}\sum F_x &= \int_{cs} \rho v(v \cdot dA) \\ p_1A_1 - p_2A_2 - F_\tau &= \int_{A_2} \rho u_2^2 dA - (\rho A u_1)u_1 \\ p_1A - p_2A - F_\tau &= -\rho u_1^2 A + \int_{A_2} \rho u_2^2 dA \qquad (1)\end{aligned}$$

Integration of profile

$$\begin{aligned}u_2 &= u_{\max}(1-(r/r_0)^2)^2 \\ u_2^2 &= u_{\max}^2(1-(r/r_0)^2)^2 \\ \int_{A_2} \rho u_2^2 dA &= \int_0^{r_0} \rho u_{\max}^2(1-(r/r_0)^2)^2 2\pi r dr \\ &= -\rho u_{\max}^2 \pi r_0^2 \int_0^{r_0} (1-(r/r_0)^2)^2(-2r/r_0^2)dr\end{aligned}$$

The above integral is in the form $\int u^n du = u^{n+1}/(n+1)$
Thus

$$\begin{aligned}\int_{A_2} \rho u_2^2 dA &= -\rho u_{\max}^2 \pi r_0^2 (1-(r/r_0)^2)^3/3|_0^{r_0} \\ &= -\rho u_{\max}^2 \pi r^2 (0 - 1/3) \\ &= +\rho u_{\max}^2 \pi r_0^2/3 \qquad (2)\end{aligned}$$

Continuity

$$\begin{aligned}
UA &= \int udA \\
&= \int_0^{r_0} u_{\max}(1-(r/r_0)^2)2\pi \text{ rdr} \\
&= -u_{\max}\pi r_0^2 \int_0^{r_0} (1-(r/r_0)^2)(-2r/r_0^2) \text{ dr} \\
&= -u_{\max}\pi r_0^2(1-(r/r_0)^2)^2/2|_0^{r_0} \\
&= u_{\max}\pi r_0^2/2
\end{aligned}$$

Therefore

$$u_{\max} = 2U$$

Substituting back into Eq. 2 gives

$$\int_{A_2} \rho u_2^2 dA = 4\rho U^2 \pi r_0^2/3$$

Finally substituting back into Eq. 1, and letting $u_1 = U$, the shearing force is given by

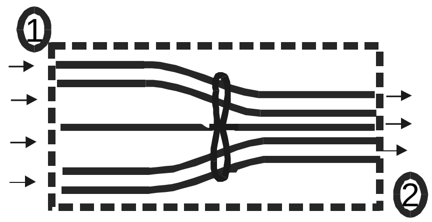
$$\underline{\underline{F_\tau = A[p_1 - p_2 - (1/3)\rho U^2]}}$$

6.65 **Information and assumptions**

provided in problem statement

Find

propulsive force when the boat is not moving and when it is moving at 30 ft/s.



Calculate flow rate

$$\begin{aligned} Q &= 80 \times \pi \times 1.5^2 = 565.5 \text{ cfs} \\ v_2 &= 80 \text{ fps} \end{aligned}$$

x−momentum (boat not moving)

$$\sum F_x = \rho Q(v_{2x} - v_{1x})$$

assume $v_1 \approx 0$

$$\begin{aligned} F_x &= \rho Q v_2 \\ F_x &= 0.00228 \times 565.5(80 - 0) = \underline{\underline{103.1 \text{ lbf}}} \end{aligned}$$

x−momentum (boat moving at 30 ft/s). Take control volume with respect to moving boat. Then $v_1 = 30$ ft/s

$$\begin{aligned} \sum F_x &= \rho Q(v_{2x} - v_{1x}) \\ F_x &= \rho Q(v_2 - v_1) \\ F_x &= 0.00228 \times 565.5(80 - 30) = \underline{\underline{64.5 \text{ lbf}}} \end{aligned}$$

6.66 **Information and assumptions**

provided in problem statement

Find

thrust on windmill

Continuity

$$v_2 = 10 \times (3/4.5)^2 = 4.44 \text{ m/s}$$

x**-momentum**

$$\begin{aligned}
\sum F_x &= \dot{m}(v_2 - v_1) \\
F_x &= \dot{m}(v_2 - v_1) \\
&= (1.2)(\pi/4 \times 3^3)(10)(4.44 - 10) \\
F_x &= -472.0 \text{ N (acting to the left)} \\
T &= \underline{\underline{472 \text{ N (acting to the right)}}}
\end{aligned}$$

6.67 **Information and assumptions**

provided in problem statement

Find

Derive formula for pressure increase across a jet pump and evaluate pressure change for water if $A_j/A_o = 1/3$, $v_j = 15$ m/s and $v_o = 2$ m/s

Continuity

$$\begin{aligned} v_1 &= v_0 D_0^2/(D_0^2 - D_j^2) \qquad (1) \\ v_2 &= (v_0 D_0^2 + v_j D_j^2)/D_0^2 \qquad (2) \end{aligned}$$

x-momentum

$$\sum F_x = \dot{m}(v_2 - v_1)$$

$$(p_1 - p_2)\pi D_0^2/4 = -\rho v_1^2 \pi (D_0^2 - D_j^2)/4 - \rho v_j^2 \pi D_j^2/4 + \rho v_2^2 \pi D_0^2/4$$

thus,

$$\underline{\underline{(p_2 - p_1) = \rho v_1^2 (D_0^2 - D_j^2)/D_0^2 + \rho v_j^2 \times D_j^2/D_0^2 - \rho v_2^2}} \qquad (3)$$

Calculations

from Eq. (1) and (2)

$$\begin{aligned} v_1 &= v_0/(1 - (D_j/D_0)^2) \\ &= 2/(1 - (1/3)) \\ &= 3 \text{ m/s} \\ v_2 &= v_0 + v_j (D_j^2/D_0^2) \\ &= 2 + 15(1/3) \\ &= 7 \text{ m/s} \end{aligned}$$

from Eq. (3)

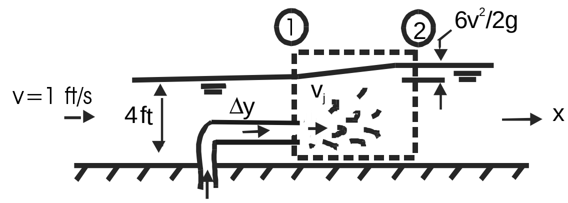
$$\begin{aligned} p_2 - p_1 &= \rho \left[v_1^2 (1 - (D_j/D_0)^2 + v_j^2 (D_j/D_0)^2 + v_2^2 \right] \\ &= 1,000 \left[3^2 (1 - 1/3) + 15^2 (1/3) - 7^2 \right] \\ &= 32 \text{ kPa} \end{aligned}$$

6.68 **Information and assumptions**

provided in problem statement

Find

Carry out a design for a jet pump



***x*-momentum**

Carry out the analysis for a section 1 ft wide (unit width) and neglect bottom friction.

$$\begin{aligned}
\sum F_x &= \dot{m}_2 v_2 - \dot{m}_1 v_1 - \dot{m}_j v_j \\
\gamma y_1^2/2 - \gamma y_2^2/2 &= -1\rho(1 \times (4 - \Delta y)) - v_j \rho(v_j \Delta y) + v_2 \rho(v_2 y_2) \qquad (1) \\
\text{but } y_2 &= 4 \text{ ft} + 6\ v^2/2g \\
&= 4 + 6/2g = 4.0932 \text{ ft}
\end{aligned}$$

Continuity

$$\begin{aligned}
v_2 y_2 &= v_1(4 - \Delta y) + v_j \Delta y \\
v_2 &= v_1(4 - \Delta y)/y_2 + v_j \Delta y / y_2
\end{aligned}$$

Assume

$$\Delta y = 0.10 \text{ ft}$$

Then

$$v_2 = 1(3.9)/(4.093) + v_j \times 0.1/4.0392 = 0.9528 + 0.02476 v_j \qquad (2)$$

Combine Eqs. (1) and (2)

$$\begin{aligned}
v_j^2 - (0.9528 + 0.02476 v_j^2 \times 40.932 &= 5g(y_2^2 - y_1^2) - 39.0 \\
&= 82.44 \text{ ft}^2/\text{s}^2
\end{aligned}$$

Solving:

$$v_j = 12.1 \text{ ft/s} \quad A_j = 0.10 \text{ ft}^2$$

If circular nozzles were used, then $A_j = (\pi/4)d_j^2;\ d_j = 4.28$ in. Therefore, one could use 8 nozzles of about 4.3 in. in diameter discharging water at 12.1 ft/s.

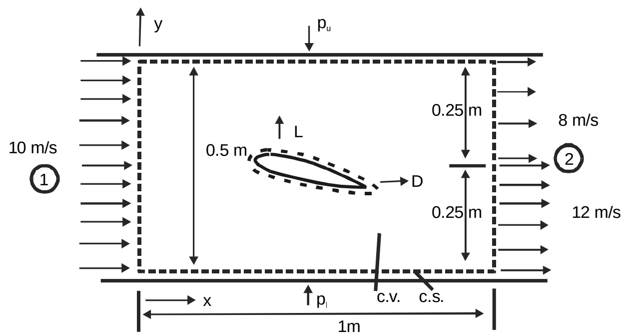
Other combinations of $d_j,\ v_j$ and number of jets are possible to achieve the desired result.

6.69 **Information and assumptions**

provided in the problem statement

Find

lift and drag on airfoil



***x*-momentum**

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}v_0 - \dot{m}_1 v_1 \\
-D + p_1A_1 - p_2A_2 &= v_1(-\rho v_1 A_1) + v_a(\rho v_a A/2) + v_b(\rho v_b A/2) \\
-D/A &= p_2 - p_1 - \rho v_1^2 + \rho v_a^2/2 + \rho v_b^2/2
\end{aligned}$$

where

$$\begin{aligned}
p_1 &= p_u(x=0) = p_\ell(x=0) = 100 \text{ Pa, gage} \\
p_2 &= p_u(x=1) = p_\ell(x=1) = 90 \text{ Pa, gage}
\end{aligned}$$

then

$$\begin{aligned}
-D/A &= 90 - 100 + 1.2(-100 + 32 + 72) \\
-D/A &= -5.2 \\
D &= 5.2 \times 0.5^2 = 1.3 \text{ N}
\end{aligned}$$

***y*-momentum**

$$\sum F_y = 0$$

$$-L + \int_1^2 p_\ell B dx - \int_0^1 p_u B dx = 0 \text{ where } B \text{ is depth of tunnel}$$

$$-L + \int_0^1 (100 - 10x + 20x(1-x))0.5dx - \int_0^1 (100 - 10x - 20x(1-x))0.5dx = 0$$

$$-L + 0.5(100x - 5x^2 + 10x^2 - (20/3)x^3)|_0^1 - 0.5(100x - 5x^2 - 10x^2 + (20/3)x^3|_0^1 = 0$$

thus,

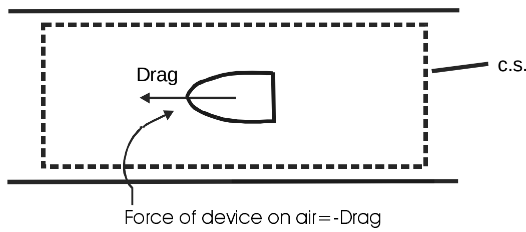
$$\begin{aligned} -L + 49.167 - 45.833 &= 0 \\ L &= \underline{\underline{3.334\ \mathrm{N}}} \end{aligned}$$

6.70 **Information and assumptions**

provided in problem statement

Find

drag on device and support vanes



Solution

Mass flow rate

$$\dot{m} = \rho v A = 0.0026 \times 100 \times (\pi/4)(3.0)^2 = \underline{\underline{1.838 \text{ slugs/s}}}$$

Find exit velocity profile

$$\int_0^0 v dA = Q$$

But v is linearly distributed, so $v = (r/r_0)v_{\max}$

Thus

$$\begin{aligned} \int_0^{r_0} ((r/r_0)v_{\max})2\pi r dr &= \overline{v}A \\ 2v_{\max}r_0^2/3 &= \overline{v}r_0^2 \\ v_{\max} &= 1.5\overline{v} = \underline{\underline{150 \text{ ft/s}}} \end{aligned}$$

***x*-momentum**

$$\sum F_x = -v_1 p A v_1 + \int_0^{r_0} p v_2^2 dA$$

$$p_1 A_1 - p_2 A_2 - D = -100 \times .0026 \times (\pi/4) \times 3^2 \times 100 + \int_0^{r_0} \rho v^2 dA$$

$$144 \times (0.24 \times \pi \times 1.5^2 - 0.10 \times \pi \times 1.5^2) - D = -184 + \int_0^{r_0} \rho(r/r_0 1.5\overline{v})^2 2\pi r dr$$

$$1.42 - D = -184 + \rho(4.5\pi/4)\overline{v}^2 r_0^2$$

Then for $\overline{v} = 100$ ft/s and $r_0 = 1.5$ ft

$$D = 142 + 184 - 207 = \underline{\underline{119 \text{ lbf}}}$$

6.71 **Information and assumptions**

provided in problem statement

Find

acceleration of sled at time t

Solution

This type of problem is directly analogous to the rocket problem except that the weight does not directly enter as a force term and $p_e = p_{\text{atm}}$. Therefore, the appropriate equation is

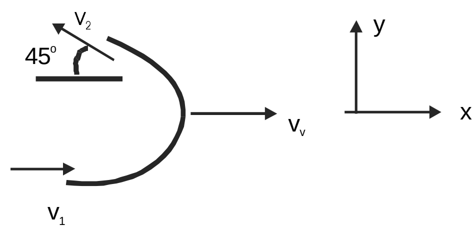
$$\begin{aligned} M\ dv_s/dt &= \rho v_e^2 A_e - F_f \\ a &= (1/M)(\rho v_e^2 (\pi/4) d_e^2 - \mu W) \end{aligned}$$

where μ =coefficient of sliding friction and W is the weight

$$\begin{aligned} W &= 350 + 0.1 \times 1000 \times 981 = 1,331 \text{ N} \\ a &= (g/W)(1,000 \times 25^2 (\pi/4) \times 0.015^2 - (1,331 \times 0.05)) \\ &= (9.81/1,331)(43.90) \text{ m/s}^2 = \underline{\underline{0.324 \text{ m/s}^2}} \end{aligned}$$

6.72 **Information and assumptions**

A fluid jet strikes a wave that is moving at a speed $v_v = 7$ m/s. $D_1 = 6$ cm. Speed of the fluid jet is 20 m/s, relative to a fixed frame.

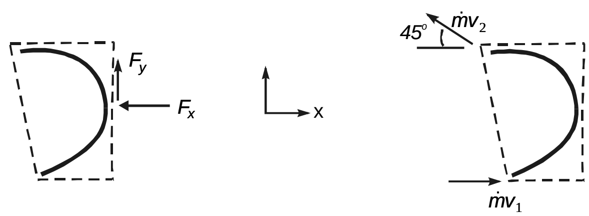


Find

force of water on the vane

Force and momentum diagrams

Select a control volume surrounding the wave and moving with the vane. Select a reference frame fixed to the moving wave.



x-momentum

$$\begin{aligned}\sum F_x &= \dot{m}v_{2x} - \dot{m}v_{1x} \\ -F_x &= -\dot{m}v_2 \cos 45^\circ - \dot{m}v_1\end{aligned}$$

y-momentum

$$\begin{aligned}\sum F_y &= \dot{m}v_{2y} - \dot{m}v_{1y} \\ F_y &= \dot{m}v_2 \sin 45^\circ\end{aligned}$$

Velocity analysis

v_1 is relative to the reference frame = (20-7)=13.

$\dot{m} = \rho A v$ use v which is relative to the control surface. In this case $v = (20 - 7) = 13$ m/s

v_2 is relative to the reference frame $v_2 = v_1 = 13$ m/s

Mass flow rate

$$\dot{m} = \rho A v = (1,000 \text{ kg})(\pi/4 \times 0.06^2)(13) = 36.76 \text{ kg/s}$$

Evaluate forces

$$
\begin{aligned}
F_x &= \dot{m}v_1(1+\cos 45) \\
&= 36.76 \times 13(1+\cos 45) = 815.8 \text{ N}
\end{aligned}
$$

which is in the negative $x-$direction.

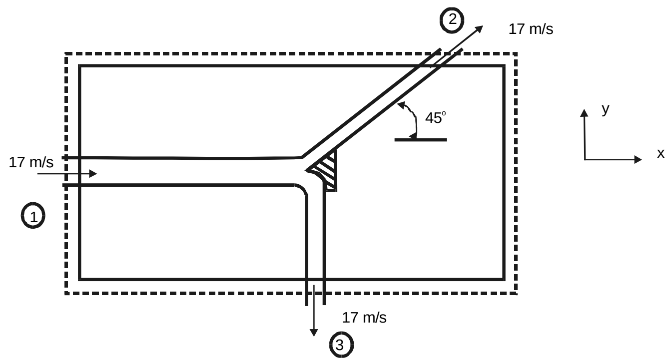
$$
\begin{aligned}
F_y &= \dot{m}v_2 \sin 45 \\
&= 36.76 \times 13 \sin 45 = 338.0 \text{ N}
\end{aligned}
$$

The force of the water on the vane is the negative of the force of the vane on the water. Thus the force of the water on the vane is

$$
\mathbf{F} = \underline{\underline{(815.8\mathbf{i}{-}338\mathbf{j}) \text{ N}}}
$$

6.73 **Information and assumptions**

A cart is moving with steady speed



Find

force exerted by the vane on the jet

Solution

Make the flow steady by referencing all velocities to the moving vane and let the c.v. move with the vane as shown.

x-momentum

$$\begin{aligned}
F_x &= \dot{m}_2 v_{2x} - \dot{m}_1 v_1 \\
F_x &= (17^2 \cos 45^\circ)(1,000)(\pi/4)(0.1^2)/2 - (17)(1,000)(-17)(\pi/4)(0.1^2) \\
&= +802 - 2,270 = -1,467 \text{ N}
\end{aligned}$$

y-momentum

$$\begin{aligned}
F_y &= \dot{m}_2 v_{2y} - \dot{m} v_{3y} \\
&= (17)(1,000)(\sin 45^\circ)(17)(\pi/4)(0.1^2)/2 - (17)^2(1,000)(\pi/4)(0.1^2)/2 \\
&= -333 \text{ N} \\
\mathbf{F} \text{ (water on vane)} &= \underline{\underline{(1,467\mathbf{i} + 330\mathbf{j}) \text{ N}}}
\end{aligned}$$

6.74 **Information and assumptions**

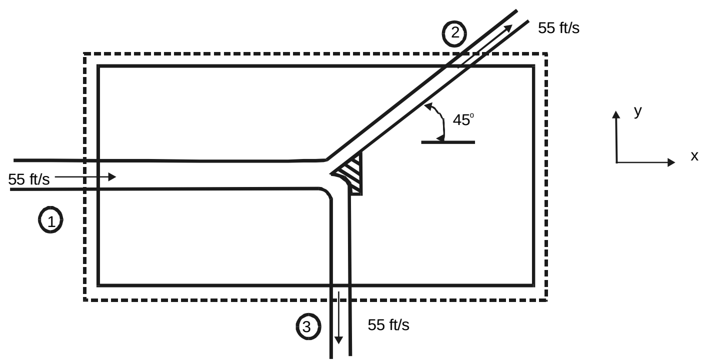
provided in problem statement

Find

rolling resistance of cart

Solution

Let the control surface surround the cart and let it move with the cart at 5 ft/s. Then we have a steady flow situation and the relative jet velocities are shown below.



x-momentum

$$\sum F_x = \dot{m}_2 v_{2_x} - \dot{m}_1 v_1$$

Velocity analysis

$$\begin{aligned} v_2 &= V_2 = 55 \text{ m/s} \\ v_1 &= V_1 = 55 \text{ m/s} \end{aligned}$$

Calculations

$$\begin{aligned} \dot{m}_1 &= \rho A_1 V_1 = (1.94)(\pi/4 \times 0.1^2)55 \\ &= 0.838 \text{ kg/s} \\ \dot{m}_2 &= 0.838/2 = 0.419 \text{ kg/s} \\ F_{\text{rolling}} &= \dot{m}_1 v_1 - \dot{m}_2 v_2 \cos 45^\circ \\ &= 0.838 \times 55 - 0.419 \times 55 \cos 45^\circ \\ F_{\text{rolling}} &= \underline{\underline{29.8 \text{ lbf}}} \text{ (acting to the left)} \end{aligned}$$

6.75 **Information and assumptions**

provided in problem statement

Find

external horizontal force needed to move cone.

Solution

Select a control volume surrounding the moving cone. Select a reference frame fixed to the cone.

Velocity analysis

$$\begin{aligned} v_1 &= V_1 = 43 \text{ m/s} \\ v_2 &= 43 \text{ m/s} \end{aligned}$$

x**-momentum**

$$\begin{aligned} F_x &= \dot{m}(v_{2x} - v_1) \\ F_x &= 1,000 \times \pi \times (0.05)^2 \times 43 \times (27.64 - 43) = -5,187 \text{ N} \\ F_x &= \underline{\underline{5.19 \text{ kN (acting to left)}}} \end{aligned}$$

6.76 **Information and assumptions**

provided in problem statement

Find

power per foot of jet transmitted to vane.

Solution

Select a control volume surrounding the moving cone. Select a reference frame fixed to the cone.

Velocity analysis

$$\begin{aligned} v_1 &= V_1 = 40 \text{ ft/s} \\ v_2 &= 40 \text{ ft/s} \end{aligned}$$

$x-$**momentum**

$$\begin{aligned} \sum F_x &= \dot{m}(v_{2x} - v_1) \\ F_x &= 1.94 \times 40 \times 0.3 \times (40 \cos 50 - 40) \\ &= -332.6 \text{ lbf} \end{aligned}$$

Calculate power

$$\begin{aligned} P &= Fv \\ &= 332.6 \times 60 \\ &= \underline{\underline{19{,}956 \text{ ft-lbf/s}=36.3 \text{ hp}}} \end{aligned}$$

6.77 **Information and assumptions**

A sled of mass $m_s = 1,000$ kg is decelerated by placing a scoop of width $w = 20$ cm into water at a depth $d = 8$ cm

Find

Deceleration of sled: a_s

Analysis

Select a moving control volume surrounding the scoop and sled. Select a stationary reference frame.

x-momentum

$$0 = \frac{d}{dt}(m_s v_s) + \dot{m} v_{2_x} - \dot{m} v_{1_x}$$

Velocity analysis

$$\begin{aligned} v_{1_x} &= 0 \\ V_1 &= 100 \text{ m/s} \\ V_2 &= 100 \text{ m/s} \\ \mathbf{v}_2 &= 100 \text{ m/s}[-\cos 60\mathbf{i} + \sin 60\mathbf{j}] + 100\mathbf{i} \text{ m/s} \\ v_{2x} &= 50 \text{ m/s} \end{aligned}$$

The x-momentum equation simplifies to

$$0 = m_s a_s + \dot{m} v_{2_x}$$

where

$$\begin{aligned} \dot{m} &= = \rho A_1 V_1 = 1,000 \times 0.2 \times 0.08 \times 100 = 1,600 \text{ kg/s} \\ a_s &= -\frac{\dot{m} v_{2x}}{m_s} = \frac{(-1,600)(50)}{1,000} = \underline{\underline{-80 \text{ m/s}^2}} \end{aligned}$$

6.78 **Information and assumptions**

provided in problem statement

Find

power required for snow removal

x-momentum

Select a control volume surrounding the snow-plow blade. Attach a reference frame to the moving blades.

$$\sum F_x = \rho Q(v_{2x} - v_1)$$

Velocity analysis

$$\begin{aligned} V_1 &= v_1 = 40 \text{ ft/s} \\ v_{2_x} &= -40 \cos 60^\circ \cos 30^\circ \\ &= -17.32 \text{ ft/s} \end{aligned}$$

Calculations

$$\begin{aligned} \sum F_x &= 1.94 \times 0.2 \times 40 \times 2 \times (1/4)(-17.32 - 40) \\ &= -444.8 \text{ lbf} \\ P &= FV = 444.8 \times 40 = 17,792 \text{ ft-lbf/s} \\ P &= \underline{\underline{32.3 \text{ hp}}} \end{aligned}$$

6.79 Maximum force occurs at the beginning; hence, the tank will accelerate immediately after opening the cap. However, as water leaves the tank the force will decrease, but acceleration may decrease or increase because mass will also be decreasing. In any event, the tank will go faster and faster until the last drop leaves, assuming no aerodynamic drag.

6.80 **Information and assumptions**

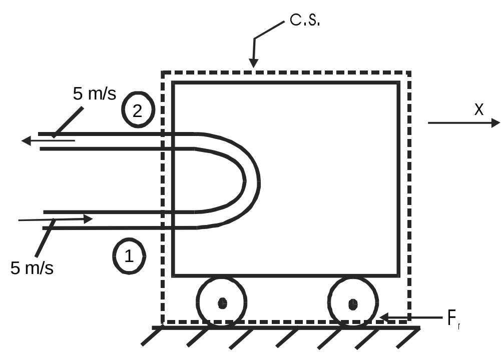
provided in problem statement

Find

resistive force on cart.

Solution

Assume the resistive force (F_r) is caused primarily by rolling resistance (bearing friction, etc.); therefore, the resistive force will act on the wheels at the ground surface. Select a reference frame fixed to the moving cart. The velocities and resistive force are shown below.



Velocity analysis

$$\begin{aligned} V_1 &= v_1 = v_2 = 5 \text{ m/s} \\ \dot{m} &= pA_1V_1 = (1,000)(0.001)(5) = 5 \text{ kg/s} \end{aligned}$$

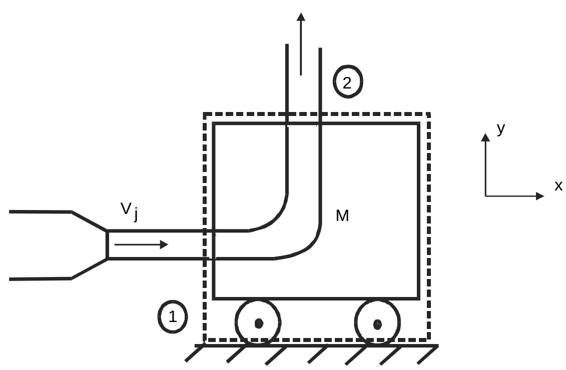
x**-momentum**

$$\begin{aligned} \sum F_x &= \dot{m}(v_2 - v_1) \\ -F_r &= 5(-5-5) = -50 \text{ N} \\ F_r &= \underline{\underline{50 \text{ N (acting to the left)}}} \end{aligned}$$

6.81 **Information and assumptions**

A jet with speed v_j strikes a cart ($M = 10$ kg), causing the cart to accelerate. The deflection of the jet is normal to the cart [when cart is not moving] jet speed: $v_j = 10$ m/s; jet discharge: $Q = 0.1$ m^3/s.

Neglect rolling resistance. Neglect mass of water within the cart.



Find

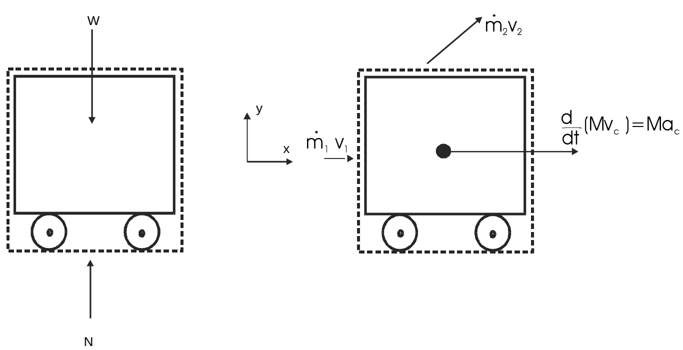
An expression for acceleration of cart

Acceleration when $v_c = 5$ m/s

Analysis

Select a control surface surrounding the moving cart. Select a reference frame fixed to the nozzle. Note that a reference frame fixed to the cart would be non-inertial.

Force and momentum diagrams



x**-momentum**

$$\sum F_x = \frac{d}{dt}(mv_c) + \dot{m}_2 v_{2_x} = -\dot{m}_1 v_1$$

Momentum accumulation

Note that the cart is accelerating. Thus,

$$\begin{aligned}\frac{d}{dt}\int_{cv} v_x \rho dV &= \frac{d}{dt} v_c \int_{cv} \rho dV = \frac{d}{dt}(Mv_c) \\ &= ma_c\end{aligned}$$

Velocity analysis

$$\begin{aligned} V_1 &= v_j - v_c \text{ [relative to control surface]} \\ v_1 &= v_j \text{ [relative to reference frame (nozzle)]} \end{aligned}$$

from conservation of mass

$$\begin{aligned} v_{2y} &= (v_j - v_c) \\ v_{2_x} &= v_c \\ \dot{m}_2 &= \dot{m}_1 \end{aligned}$$

Combining terms

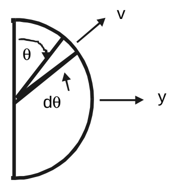
$$\begin{aligned} \sum F_x &= \frac{d}{dt}(Mv_c) + \dot{m}(v_{2x} - v_1) \\ 0 &= Ma_c + \rho A_1(v_j - v_c)(v_c - v_j) \\ a_c &= \underline{\underline{\frac{(\rho Q/v_j)(v_j - v_c)^2}{M}}} \end{aligned}$$

Calculations

$$\begin{aligned} a_c &= \frac{1,000 \times 0.1/10(10-5)^2}{10} \\ a_c &= \underline{\underline{25 \text{ m/s}^2}} \text{ (when } v_c = 5 \text{ m/s)} \end{aligned}$$

6.82 **Information and assumptions**

A hemispherical nozzle sprays a sheet of liquid at a speed v through a 180° arc. Sheet thickness is t.



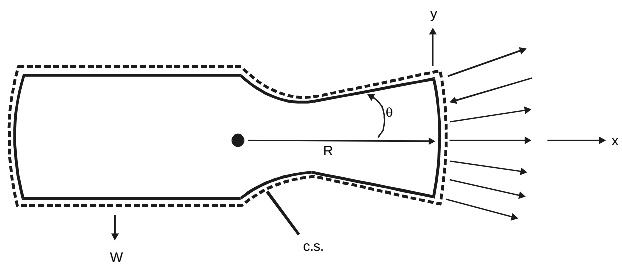
Find

Force in y-direction to hold nozzle: $F_y = F_y(\rho, v, r, t)$

y-momentum

$$\begin{aligned} F_y &= \int_{cs} v_y \rho \mathbf{V} \cdot \mathbf{dA} \\ &= \int_0^\pi (v \sin\theta) \rho v (tr d\theta) \\ &= \rho v^2 tr \int_0^\pi \sin\theta d\theta \\ F_y &= \underline{\underline{2\rho V^2 tr}} \end{aligned}$$

6.83 Define A_e as the projection of the exit area on the y plane. Use the momentum equation to solve this problem and let the control surface surround the nozzle and fuel chamber as shown above. The forces acting on the system are the pressure forces and thrust, T. The pressure forces in the x-direction are from p_0 and p_e. Writing the momentum equation in the x-direction we have:



$$
\begin{aligned}
T + p_0 A_e - p_e A_e &= \int_A v_x \rho \mathbf{V} \cdot \mathbf{dA} \\
T + p_0 A_e - p_e A_e &= \int 2(v\cos\theta)\rho(-vLRd\theta) \\
T + p_0 A_e - p_e A_e &= -2v^2\rho LR \int_0^\theta \cos\theta d\theta \\
T + p_0 A_e - p_e A_e &= -2v^2\rho LR \sin\theta
\end{aligned}
$$

But

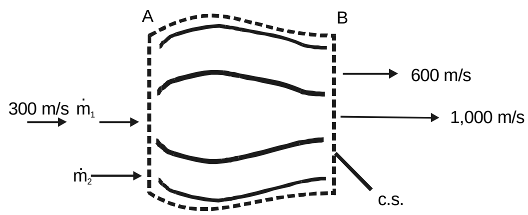
$$
\begin{aligned}
\dot{m} &= 2\int_0^\theta \rho v dA = 2\int_0^\theta \rho vLRd\theta = 2\rho vLR\theta \\
T + p_0 A_e - p_e A_e &= -2\rho v^2 LR\theta(\sin\theta/\theta) \\
T + p_0 A_e - p_e A_e &= -v\dot{m}\sin\theta/\theta \\
T &= \dot{m}v(-\sin\theta/\theta) + p_e A_e - p_0 A_e \\
T &= \dot{m}vf(\theta) + A_e(p_e - p_0)\lambda(\theta)
\end{aligned}
$$

where

$$
\begin{aligned}
f(\theta) &= -\sin\theta/\theta \\
\lambda(\theta) &= 1.
\end{aligned}
$$

6.84 **Information and assumptions**

provided in problem statement



Find

thrust

Calculate mass flow rates

$$\begin{aligned}\dot{m} &= \rho v A = \rho Q \\ \dot{m}_A &= \dot{m}_B = 300 \text{ kg/s} \\ \dot{m}_A &= \dot{m}_1 + \dot{m}_2 \text{ but } \dot{m}_2 = 2\dot{m}_1\end{aligned}$$

Then

$$\begin{aligned}300 \text{ kg/s} &= \dot{m}_1 + 2\dot{m}_1 \\ \dot{m}_1 &= 100 \text{ kg/s and } \dot{m}_2 = 200 \text{ kg/s}\end{aligned}$$

x**-momentum**

$$\begin{aligned}\sum F_x &= \sum v_x \rho \mathbf{v} \cdot \mathbf{A} \\ T &= \sum v_B \rho (v_B A_B) + v_A \rho (v_A A_A) \\ &= 600(200) + 1,000(100) - (300)(300) \\ T &= \underline{\underline{130,000 \text{ N}}}\end{aligned}$$

6.85 **Information and assumptions**

provided in problem statement

Find

initial mass to establish rocket in orbit

Solution

$$M_0 = M_f \exp(V_{b0}\lambda/T) = 50 \exp(7,200/3,000) = \underline{\underline{551.2 \text{ kg}}}$$

6.86 **Information and assumptions**

provided in problem statement

Find

maximum velocity

x-momentum

$$T - W = ma$$

where T =thrust and W =weight

$$\begin{aligned}
T &= \dot{m}v_e \\
\dot{m}v_e - mg &= m dv_R/dt \\
dv_R/dt &= (T/m) - g \\
&= (T/(m_i - \dot{m}t)) - g \\
dv_R &= ((Tdt)/(m_i - \dot{m}t)) - gdt \\
v_R &= (-T/\dot{m})\ell n(m_i - \dot{m}t) - gt + \text{const.}
\end{aligned}$$

where $v_R = 0$ when $t = 0$. Then

$$\begin{aligned}
\text{const.} &= (T/\dot{m})\ln(m_i) \\
v_R &= (T/\dot{m})\ln((m_i)/(m_i - \dot{m}t)) - gt \\
v_{R\max} &= (T/\dot{m})\ln(m_i/m_f) - gt_f \\
T/\dot{m} &= \dot{m}v_e/\dot{m} = v_e
\end{aligned}$$

Bernoulli equation

(neglecting hydrostatic pressure)

$$p_i + \rho_f v_i^2/2 = p_e + \rho_f v_e^2/2$$

The exit pressure is zero (gage) and the inlet kinetic pressure is negligible. So

$$\begin{aligned}
v_e^2 &= 2p_i/\rho_f = 2 \times 100 \times 10^3/998 = 200 \text{ m}^2/\text{s}^2 \\
v_e &= 14.14 \text{ m/s} \\
\dot{m} &= \rho_e v_e A_e = 1000 \times 14.14 \times 0.1 \times 0.05^2 \times \pi/4 \\
&= 2.77 \text{ kg/s}
\end{aligned}$$

Calculations

Time for water to exhaust: $t = m_w/\dot{m} = 0.10/2.77 = 0.036$ s

$$\therefore v_{\max} = 14.14\ln((100 + 50)/50) - (9.81)(0.036) = \underline{\underline{15.2 \text{ m/s}}}$$

6.87 **Information and assumptions**

provided in problem statement

Find

thrust of rocket

***z*-momentum**

$$\begin{aligned}
\sum F_z &= \dot{m} v_z[\text{per engine}] \\
T - p_a A_e \cos 30^\circ + p_e A_e \cos 30^\circ &= -v_e \cos 30^\circ \rho v_e A_e \\
T &= -1 \times 0.866 \times (50,000 - 10,000 + 0.3 \times 2,000 \ \times 2,000) \\
&= -1.074 \times 10^6 \text{ N}
\end{aligned}$$

Thrust of four engines

$$T = 4 \times 1.074 \times 10^6 = 4.3 \times 10^6 N = \underline{\underline{4.3 \text{ MN}}}$$

6.88 **Information and assumptions**

provided in problem statement

Find

force on nozzle-chamber connection

x-momentum

$$\begin{aligned}
\sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\
F + p_1 A_1 - p_e A_e &= \dot{m}(v_e - v_i) \\
F &= 220(2000 - 100) - (1.5 \times 10^6 - 10^5) \times 1 + (8 \times 10^4 - 10^5) \times 2 \\
&= -1.022 \times 10^6 \text{ N } = -1.022 \text{ MN}
\end{aligned}$$

The force on the connection will be

$$F = \underline{\underline{1.022 \text{ MN}}}$$

6.89 Derive an equation for thrust of conical nozzle

x-**momentum**

$$\begin{aligned}
\sum \mathbf{F} &= \int \mathbf{v}\rho\mathbf{v}\cdot\mathbf{dA} \\
T &= \int_0^\alpha v_e\cos\theta\rho v_e \int_0^{2\pi} \sin\theta r d\phi r d\theta \\
T &= 2\pi r^2\rho v_e^2 \int_0^\alpha \cos\theta\sin\theta d\theta \\
&= 2\pi r^2\rho v_e^2 \sin^2\alpha/2 \\
&= \rho v_e^2 2\pi r^2(1-\cos\alpha)(1+\cos\alpha)/2
\end{aligned}$$

Exit Area

$$A_e = \int_0^\alpha \int_0^{2\pi} \sin\theta r d\phi r d\theta = 2\pi r^2(1-\cos\alpha)$$

$$T = \rho v_e^2 A_e(1+\cos\alpha)/2 = \underline{\underline{\dot{m}v_e(1+\cos\alpha)/2}}$$

6.90 **Information and assumptions**

provided in problem statement

Find

water hammer pressure rise

Solution

$$
\begin{aligned}
\Delta p &= \rho v c \\
c &= \sqrt{E_v/\rho} = ((715)(10^6)/(680))^{0.5} = 1,025 \text{ m/s} \\
\Delta p &= (680)(10)(1,025) = \underline{\underline{6.97 \text{ MPa}}}
\end{aligned}
$$

6.91 **Information and assumptions**

provided in problem statement

Find

water hammer pressure

Solution

$$\begin{aligned} c &= (2.2 \times 10^9/1,000)^{1/2} = 1,483 \text{ m/s} \\ t_{\text{crit}} &= 2L/c = 2 \times 10,000/1,483 = 13.5 \text{ s} > 10 \text{ s} \end{aligned}$$

Then

$$\Delta p = \rho v c = 1,000 \times 3 \times 1,483 = 4,449,000 \text{ Pa} = \underline{\underline{4.45 \text{ MPa}}}$$

6.92 **Information and assumptions**

provided in problem statement

Find

pipe length

Solution

From solution to problem 6.88 $c = 1,483$ m/s

$$
\begin{aligned}
t &= 4L/c \\
3 &= 4L/1,483 \\
& \underline{\underline{L = 1,112 \text{ m}}}
\end{aligned}
$$

6.93 **Information and assumptions**

provided in problem statement

Find

maximum water hammer pressure

Solution

$$\begin{aligned} c &= (320,000 \times 144/1.94)^{1/2} = 4,874 \text{ ft/s} \\ t_{\text{crit}} &= 2L/c = 2 \times 5 \times 5,280/4,874 = 10.83 \text{ s} > 10 \text{ s} \end{aligned}$$

Then

$$\Delta p_{\max} = \rho v c = 1.94 \times 8 \times 4,874 = \underline{\underline{75{,}644 \text{ psf} = 525 \text{ psi}}}$$

6.94 **Information and assumptions**

provided in problem statement

Find

maximum force exerted on valve

Solution

$$
\begin{aligned}
t_{\text{crit}} &= 2L/c = 2 \times 4,000/1,485.4 = 5.385 \text{ s} > 3 \text{ s} \\
F_{\text{valve}} &= A\Delta p = A\rho(Q/A)c = \rho Qc = 998 \times 0.025 \times 1,483 = \underline{\underline{37{,}000 \text{ N}{=}37.0 \text{ kN}}}
\end{aligned}
$$

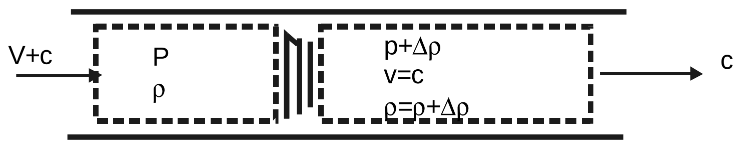
6.95 Information and assumptions

provided in problem statement

Find

Derivation of equation for pressure rise

Solution



From continuity equation

$$\begin{aligned}(v+c)\rho &= c(\rho+\Delta\rho) \\ \therefore \quad & \Delta\rho = v\rho/c\end{aligned}$$

x**-momentum**

$$\begin{aligned}\sum F_x &= \sum v_x \rho \mathbf{v}\cdot\mathbf{A} \\ pA-(p+\Delta p)A &= -(V+c)\rho(V+c)A + c^2(\rho+\Delta\rho)A \\ \Delta p &= 2\rho vc - c^2\Delta\rho + v^2\rho = 2\rho vc - c^2 v\rho/c + v^2\rho = \rho vc + \rho v^2\end{aligned}$$

Here ρv^2 is very small compared to ρvc

$$\underline{\underline{\therefore \Delta p = \rho vc}}$$

6.96 **Information and assumptions**

provided in problem statement

Find

pressure-time trace

Solution

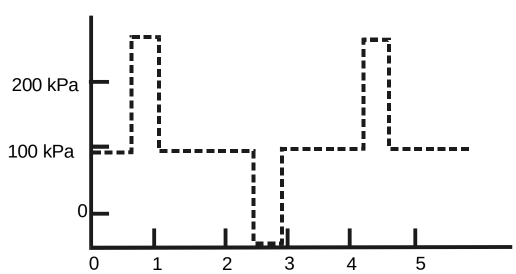
From the solution to Problem 6.88 $v = 0.1$ m/s; $c = 1,483$ m/s

$$\begin{aligned} p_{\text{pipe}} &= 10\gamma - \rho v_{\text{pipe}}^2/2 \approx 98,000 \text{ Pa} \\ \Delta p &= \rho vc = 1,000 \times 0.10 \times 1,483 \\ \Delta p &= 148,000 \text{ Pa} \end{aligned}$$

Thus $p_{\max} = p + \Delta p = 98,000 + 148,000 = 246$ kPa gage
$p_{\min} = p - \Delta p = -50$ kPa gage
The sequence of events are as follows:

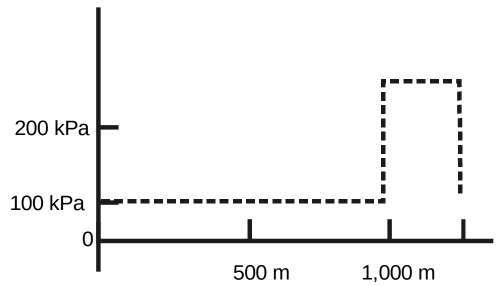
			Δt		$\Sigma\Delta t$	
Pressure wave reaches pt.	B at $t = 1,000$ m/1,483 m/s	=	0.674	s	0.67	s
Time period of high pressure at	$B = 600/1,483$	=	0.405	s	1.08	s
Time period of static pressure at	$B = 2,000/1,483$	=	1.349	s	2.43	s
Time period of negative pressure at	$B = 600/1,483$	=	0.405	s	2.83	s
Time period of static pressure at	$B = 2,000/1,483$	=	1.349	s	4.18	s
Time period of high pressure at	$B = 600/1,483$	=	0.405	s	4.59	s
Time period of static pressure at	$B = 2,000/1,483$	=	1.349	s	5.94	s

Results are plotted below:



At $t = 1.5$ s high pressure wave will have travelled to reservoir and static wave will be travelling toward valve.

Time period for wave to reach reservoir $= 1,300/1,483 = 0.877$ s. Then static wave will have travelled for $1.5 - 0.877$ s $= 0.623$ s. Distance static wave has travelled $= 0.623$ s $\times 1,483$ m/s $= 924$ m. The pressure vs. position plot is shown below:



6.97 **Information and assumptions**

provided in problem statement

Find

initial discharge and length from A to B

Solution

$$
\begin{aligned}
c &= 1,483 \text{ m/s} \\
\Delta p &= \rho \Delta v c \\
t &= L/c \\
L &= tc = 1.46 \text{ s } \times 1,483 = \underline{\underline{2,165 \text{ m}}} \\
\Delta v &= \Delta p/\rho c \\
&= (2.5 - 0.2) \times 10^6 \text{ Pa}/1.483 \times 10^6 \text{ kg/m}^2\text{s} = 1.551 \text{ m/s} \\
Q &= vA = 1.551 \times \pi/4 = \underline{\underline{1.22 \text{ m}^3/\text{s}}}
\end{aligned}
$$

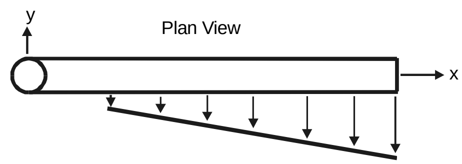
6.98 **Information and assumptions**

provided in problem statement

Find

reaction at station $A-A$

Solution



$$v_y = -(3.1 + 3x) \text{ m/s}$$

y**-momentum**

$$\begin{aligned} \sum F_y &= \int v_y \rho \mathbf{v} \cdot \mathbf{dA} \\ F_y &= -\int_{0.3}^{1.3} (3.1+3x) \times 1,000 \times (3.1+3x) \times 0.015 dx = -465 \text{ N} \\ R_y &= 465 \text{ N} \end{aligned}$$

Discharge

$$\begin{aligned} Q &= \int v dA = 0.015 \int_{0.3}^{1.3} (3.1+3x) dx = 0.0825 \text{ m}^3/\text{s} \\ v_1 &= Q/A = 0.0825/(\pi \times 0.04^2) = 16.4 \text{ m/s} \end{aligned}$$

where section 1 is the inlet

z**-momentum**

$$\sum F_z = -\dot{m}_1 v_1$$

$$F_z - p_A A_A - W_f = -\dot{m} v_1$$

$$F_z = 30,000 \times \pi \times 0.04^2 + 0.08 \times \pi \times 0.04^2 \times 9,810$$

$$+1.3 \times \pi \times 0.025^2 \times 9,810 + 1,000 \times 0.0825 \times 16.4$$

$$= 1,530 \text{ N}$$

$$R_z = -1530 \text{ N}$$

z**-moment-of-momentum**

$$\begin{aligned} T_z &= \int_{cs} r v \rho \mathbf{v} \cdot \mathbf{dA} \\ &= 15 \int_{0.3}^{1.3} (3.1+3r)^2 r dr = 413.2 \text{ N} \cdot \text{m} \end{aligned}$$

y-moment-of-momentum

$$T_y + W r_{cm} = 0$$

where W=weight, r_{cm}=distance to center of mass

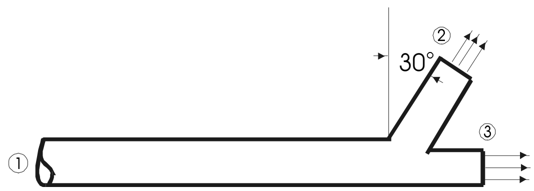
$$T_y = -1.3\pi \times 0.025^2 \times 9,810 \times 0.65 = -16.28 \text{ N} \cdot \text{m}$$

Net Reaction at A-A

$$\begin{aligned} \mathbf{F} &= (465\mathbf{j} - 1,530\mathbf{k}) \text{ N} \\ \mathbf{T} &= (16.3\mathbf{j} - 413\mathbf{k}) \text{ N} \cdot \text{m} \end{aligned}$$

6.99 **Information and assumptions**

provided in problem statement



Find

reaction at section 1

Continuity equation

$$v_1 = (0.1 \times 50 + 0.2 \times 50)/0.6 = 25 \text{ ft/s}$$

x-momentum

$$\sum F_x = \dot{m}_3 v_{3x} + \dot{m}_2 v_{2x}$$

$$F_x = -20 \times 144 \times 0.6 - 1.94 \times 25^2 \times 0.6 + 1.94 \times 50^2 \times 0.2$$
$$+1.94 \times 50^2 \times 0.1 \times \cos 60^\circ = -1{,}243 \text{ lbf}$$

y-momentum

$$\begin{aligned}\sum F_y &= \dot{m}_2 v_{2y} \\ F_y &= 1.94 \times 50 \times 50 \times 0.1 \times \cos 30^\circ = 420 \text{ lbf}\end{aligned}$$

z-moment-of-momentum

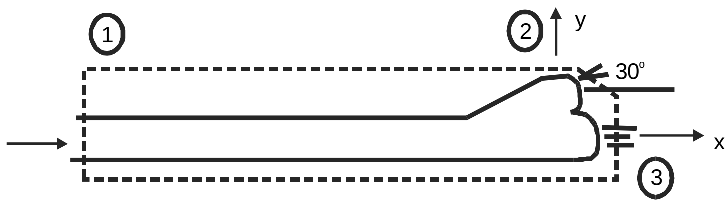
$$r_2 \dot{m}_2 v_{2y} = (36/12)(1.94 \times 0.1 \times 50)50 \sin 60^\circ = 1,260 \text{ ft-lbf}$$

Reaction at section 1

$$\begin{aligned}\mathbf{F} &= (1,243\mathbf{i} - 420\mathbf{j}) \text{ lbf} \\ \mathbf{M} &= (-1,260\mathbf{k}) \text{ ft-lbf}\end{aligned}$$

6.100 **Information and assumptions**

provided in problem statement



Find

reaction at section 1

Continuity equation

$$V_1 = (0.01 \times 20 + 0.02 \times 20)/0.1 = 6 \text{ m/s}$$

***x*-momentum**

$$\begin{aligned}
\sum F_x &= \sum \dot{m}_o v_{ox} - \sum \dot{m}_i v_{ix} \\
F_x + p_1 A_1 &= \dot{m}_3 v_3 + \dot{m}_2 v_2 \cos 30 - \dot{m}_1 v_1 \\
F_x &= -200,000 \times 0.1 - 1,000 \times 6^2 \\
&\quad \times 0.1 + 1,000 \times 20^2 \times 0.02 \\
&\quad +1,000 \times 20^2 \times 0.01 \times \cos 30^\circ \\
&= \underline{\underline{-12,135 \text{ N}}}
\end{aligned}$$

***y*-momentum**

$$F_y - W = \dot{m}_2 v_2 \sin 30^\circ$$

Weight

$$\begin{aligned}
W &= W_{\mathrm{H_2O}} + W_{\text{pipe}} \\
&= (0.1)(1)(9,800) + 90 \\
&= 1,071 \text{ N}
\end{aligned}$$

thus

$$F_y = 1,000 \times 20^2 \times 0.01 \times \sin 30^\circ + 1,071 = \underline{\underline{3,071 \text{ N}}}$$

***z*-moment-of-momentum**

$$\begin{aligned}
M_z - W r_{cm} &= r_2 \dot{m}_2 v_{2y} \\
M_z &= (1,071 \times 0.5) + (1.0)(1,000 \times 0.01 \times 20)(20 \sin 30^\circ) \\
&= 2,535 \text{ N} \cdot \text{m}
\end{aligned}$$

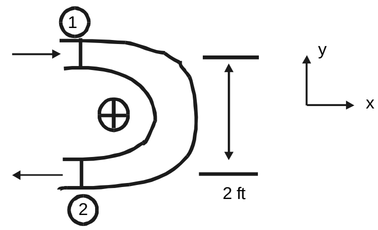
Reaction at section 1

$$\begin{aligned}\mathbf{F} &= (12.1\mathbf{i} - 3.1\mathbf{j}) \text{ kN} \\ \mathbf{M} &= (-2.54\mathbf{k}) \text{ kN} \cdot \text{m}\end{aligned}$$

6.101 **Information and assumptions**

A reducing pipe bend held in place by a pedestal. Water flow. No force transmission through the pipe at sections 1 and 2.

Assume irrotational flow. Neglect weight



Find

Force needed to hold bend stationary: **F**

Moment needed to hold bend stationary: **M**

Bernoulli equation

$$
\begin{aligned}
\frac{p_1}{\gamma}+\frac{v_1^2}{2g} &= \frac{p_2}{\gamma}+\frac{v_2^2}{2g} \\
v_1 &= Q/A_1 = 2/(\pi/4\times 0.5^2) = 10.19 \text{ ft/s} \\
v_2 &= Q/A_1 = 2/(\pi/4\times (4/12)^2) = 22.92 \text{ ft/s} \\
p_1 &= 20\times 144 = 2,880 \text{ psf} \\
p_2 &= p_1+\rho(v_1^2-v_2^2)/2 \\
&= 2,880+1.94(10.19^2-22.92^2)/2 \\
&= 2,471 \text{ psf}
\end{aligned}
$$

x**-momentum**

$$
\begin{aligned}
F_x+p_1A_1+p_2A_2 &= \dot{m}v_{2_x}-\dot{m}v_{1_x} \\
F_x &= -p_1A_1-p_2A_2-\dot{m}(v_2+v_1)
\end{aligned}
$$

where

$$
\begin{aligned}
A_1 &= \pi/4\times 0.52 = 0.196 \text{ ft}^2 \\
A_2 &= \pi/4\times 0.333^2 = 0.0753 \text{ ft}^2 \\
\dot{m} &= \rho A_1 v_1 = 1.94\times 0.196\times 10.19 = 3.875 \text{ slug/s}
\end{aligned}
$$

thus

$$
F_x = -2,880\times 0.196 - 2,471\times 0.0873 - 3.875(10.19+22.92) = -909.6 \text{ lbf}
$$

z-moment-of-momentum

$$\begin{aligned} m_z - rp_1A_1 + rp_2A_2 &= -r\dot{m}v_2 + r\dot{m}v_1 \\ m_z &= r(p_1A_1 - p_2A_2) - r\dot{m}(v_2 - v_1) \end{aligned}$$

where $r = 1.0$ ft.

$$\begin{aligned} M_z &= 1.0(2,880 \times 0.196 - 2,471 \times 0.08753) - 1.0 \times 3.875(22.92 - 10.19) \\ &= 300.4 \text{ ft-lbf} \end{aligned}$$

y-moment-of-momentum

$$M_y + p_1A_1r_3 + p_2A_2r_3 = -r_3\dot{m}v_2 - r_3\dot{m}_1v_1$$

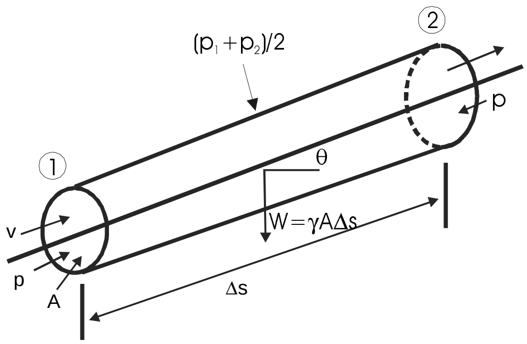
where $r_3 = 2.0$ ft.

$$\begin{aligned} M_y &= -r_3[p_1A_1 + p_2A_2 + \dot{m}(v_1 + v_2)] \\ &= -2.0 \times 909.6 \\ M_y &= -1,819 \text{ ft-lbf} \end{aligned}$$

Net force and moment at 3

$$\begin{aligned} \mathbf{F} &= -910\mathbf{i} \text{ lbf} \\ \mathbf{M} &= (-1,819\mathbf{j} + 300\mathbf{k}) \text{ ft-lbf} \end{aligned}$$

6.102 Derive Euler's equation using momentum equation



Continuity equation

$$\frac{d}{dt}\int \rho dV + \dot{m}_o - \dot{m}_i = 0$$

For a control volume which is fixed in space, the continuity equation can be written as

$$\int \frac{\partial \rho}{\partial t} dV + \dot{m}_o - \dot{m}_i = 0$$

For the control volume shown above the continuity equation is expressed as

$$\frac{\partial \rho}{\partial t}\bar{A}\Delta s + (\rho v A)_2 - (\rho v A)_1 = 0$$

where $\bar{A}$ is the average cross-sectional area between 1 and 2 and the volume of the control volume is $\bar{A}\Delta s$. Dividing by Δs and taking the limit as $\Delta s \to 0$ we have

$$A\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial s}(\rho v A) = 0$$

In the limit the average area becomes the local area of the stream tube.
The momentum equation for the control volume is

$$\sum F_s = \frac{d}{dt}\int \rho v dV + \dot{m}_o v_o - \dot{m}_i v_i$$

For a control volume fixed in space, the accumulation term can be written as

$$\frac{d}{dt}\int \rho v dV = \int \frac{\partial}{\partial t}(\rho v) dV$$

The forces are due to pressure and weight

$$\sum F_s = p_1 A_1 - p_2 A_2 + (\frac{p_1 + p_2}{2})(A_2 - A_1) - \gamma \bar{A}\Delta s \sin\theta$$

where the third term on the right is the pressure force on the sloping surface and θ is the orientation of control volume from the horizontal. The momentum equation for the control volume around the stream tube becomes

$$\frac{\partial}{\partial t}(\rho v)\bar{A}\Delta s + \rho A v_2 v_2 - \rho A v_1 v_1 = (p_1 - p_2)\bar{A} - \gamma \bar{A}\Delta s \sin\theta$$

Dividing by Δs and taking limit as $\Delta s \to 0$, we have

$$A\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial s}(\rho A v^2) = -\frac{\partial p}{\partial s}A - \gamma A \sin\theta$$

By differentiating product terms the left side can be written as

$$A\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial s}(\rho A v^2) = v[A\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial s}(\rho v A)] + A\rho\frac{\partial v}{\partial t} + A\rho v\frac{\partial v}{\partial s}$$

The first term on the right is zero because of the continuity equation. Thus the momentum equation becomes

$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial s} = -\frac{\partial p}{\partial s} - \gamma \sin\theta$$

But $\sin\theta = \partial z/\partial s$ and $\partial v/\partial t + v\partial v/\partial s = a_s$, the acceleration along the path line. Thus the equation becomes

$$\rho a_s = -\frac{\partial}{\partial s}(p + \gamma z)$$

which is Euler's equation.

6.103 **Information and assumptions**

provided in problem statement

Find

power provided by rocket motors

Analysis

Select a control volume that encloses one motor

Select a stationary reference frame

x-momentum

$$F_x = \dot{m}v_0 - \dot{m}v_i$$

Velocity analysis

$$\begin{aligned} v_i &= 0 \\ V_i &= rw = 4 \times 2\pi = 25.13 \text{ m/s} \\ V_0 &= 500 \text{ m/s} \\ v_0 &= (500 - 25.13) \text{ m/s} = 474.9 \text{ m/s} \\ \dot{m} &= \rho A_i V_i = 1.2 \times 20 \times 10^{-4} \times 25.13 = 0.0603 \text{ kg/s} \end{aligned}$$

Calculate force

$$\begin{aligned} F_x &= \dot{m}v_0 \\ &= 0.0603 \times 474.9 = 28.64 \text{ N} \end{aligned}$$

Power

$$\begin{aligned} P &= 2Frw = 2 \times 28.64 \times 4 \times 2\pi \\ P &= \underline{\underline{1.44 \text{ kW}}} \end{aligned}$$

6.104 It is necessary to do some initial calculations based on the information provided. To supply water to a circle 50 ft. in diameter at a 1/4 inch per hour requires a discharge of

$$Q = \dot{h}A = (1/48)\pi(50^2/4)/3600 = 0.011 \text{ cfs}$$

Assuming no losses between the supply pressure and the sprinkler head would give and exit velocity at the head of

$$V = \sqrt{\frac{2p}{\rho}} = \sqrt{\frac{2 \times 50 \times 144}{1.94}} = 86 \text{ ft/s}$$

If the water were to exit the sprinkler head at the angle for the optimum trajectory (45^o), the distance traveled by the water would be

$$s = \frac{V_e^2}{2g}$$

The velocity necessary for a 25 ft distance (radius of the spray circle) would be

$$\begin{aligned} V_e^2 &= 2gs = 2 \times 32.2 \times 25 = 1610 \\ V_e &= 40 \text{ ft/s} \end{aligned}$$

This means that there is ample pressure available to do the design. There will be losses which will affect the design. As the water spray emerges from the spray head, atomization will occur which produce droplets. These droplets will experience aerodynamic drag which will reduce the distance of the trajectory. The size distribution of droplets will lead to small droplets moving shorted distances and larger droplets farther which will contribute to a uniform spray pattern.

The sprinkler head can be set in motion by having the water exit at an angle with respect to the radius. For example if the arm of the sprinkler is 4 inches and the angle of deflection at the end of the arm is 10 degrees, the torque produced is

$$\begin{aligned} M &= \rho Q r V_e \sin\theta \\ &= 1.94 \times 0.011 \times 40 \times \sin 10 = 0.148 \text{ ft-lbf} \end{aligned}$$

The downward load on the head due to the discharge of the water is

$$\begin{aligned} F_y &= \rho Q V_e \sin 45 = 1.94 \times 0.011 \times 40 \times \sin 45 \\ &= 0.6 \text{ lbf} \end{aligned}$$

The moment necessary to overcome friction on a flat plate rotating on another flat plate is

$$M = (2/3)\mu F_n r_o$$

where μ is the coefficient of friction and r_o is the radius of the plate. Assuming a 1/2 inch radius, the limiting coefficient of friction would be

$$\mu = \frac{3}{2}\frac{M}{F_n r_o} = \frac{3}{2}\frac{0.148}{0.6 \times (1/24)} = 8.9$$

This is very high, which means there is adequate torque to overcome friction.

These are initial calculations showing the feasibility of the design. A more detailed design would now follow.

6.105 Following the same development in the text done for the planar case, there will be another term added for the two additional faces in the z-direction. The rate of change of momentum in the control volume plus the net afflux through the surfaces becomes

$$\frac{1}{\Delta x \Delta y \Delta z}\int_{cv}\frac{\partial}{\partial t}(\rho u)dV + \frac{\rho \mathrm{u}\, u_{x+\Delta x/2} - \rho \mathrm{u}\, u_{x-\Delta x/2}}{\Delta x} + \frac{\rho u v_{y+\Delta y/2} - \rho u v_{y-\Delta y/2}}{\Delta y} + \frac{\rho u w_{z+\Delta z/2} - \rho u w_{z-\Delta z/2}}{\Delta z}$$

where w is the velocity in the $z-$direction and Δz is the size of the control volume in the $z-$direction. Taking the limit as Δx, Δy, and $\Delta z \to 0$ results in

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho \mathrm{u}\, u) + \frac{\partial}{\partial y}(\rho u v) + \frac{\partial}{\partial z}(\rho u w)$$

In the same way, accounting for the pressure and shear stress forces on the three-dimensional control volume leads to an additional shear stress term on the z-face. There is no additional pressure force because there can only be a force due to pressure on the faces normal to the x-direction. The force terms on the control volume become

$$\frac{p_{x-\Delta x/2} - p_{x+\Delta x/2}}{\Delta x} + \frac{\tau_{xx}\mid_{x+\Delta x/2} - \tau_{xx}\mid_{x-\Delta x/2}}{\Delta x} + \frac{\tau_{yx}\mid_{y+\Delta y/2} - \tau_{yx}\mid_{y-\Delta y/2}}{\Delta y} + \frac{\tau_{zx}\mid_{z+\Delta z/2} - \tau_{zx}\mid_{z-\Delta z/2}}{\Delta z}$$

Taking the limit as Δx, Δy, and $\Delta z \to 0$ results in

$$-\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

The body force in the x-direction is

$$\frac{\rho g_x \cancel{\Delta V}}{\Delta x \Delta y \Delta z} = \rho g_x$$

6.106 Substituting in the constitutive relations gives

$$\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}=2\mu\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial}{\partial y}(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x})+\mu\frac{\partial}{\partial z}(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x})$$

This can be written as

$$\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}=\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2})+\mu\frac{\partial}{\partial x}(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z})$$

The last term is equal to zero from the continuity equation for an incompressible flow, so

$$\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}=\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2})$$

Chapter Seven

7.1 **Information and assumptions**

provided in problem statement

Find

Power Output

Energy equation

$$\dot{Q} - \dot{W}_s = \dot{m}(h_2 + V_2^2/2 - h_1 - V_1^2/2)$$
$$-2,500 - \dot{W}_s = 5,800[1,098 + 200^2/(2 \times 778 \times 32.2)]$$
$$-1,268 - [50^2/(2 \times 778 \times 32.2)] \quad \text{BTU/hr}$$
$$\dot{W}_s = 9.79 \times 10^5 \text{ BTU/hr} = \underline{\underline{384 \text{ hp}}}$$

7.2 **Information and assumptions**

provided in problem statement

Find

power output

Energy equation

$$
\begin{aligned}
\dot{Q} - \dot{W}_s &= \dot{m}[(h_2 - h_1) + (V_2^2 - V_1^2)/2] \\
-10 - \dot{W}_s &= 4,000[(2,621 - 3,062) + (50^2 - 10^2)/(2 \times 1,000)] \ \ \text{kJ/hr} \\
\dot{W}_s &= \underline{\underline{489 \ \ \text{kW}}}
\end{aligned}
$$

7.3 **Information and assumptions**

provided in problem statement

Find

rate of heat loss

Solution

$$
\begin{aligned}
\dot{Q} - \dot{W}_s &= \dot{m}[(h_2 - h_1) + (V_2^2 - V_1^2)/2] \\
&= -1[(2,630 - 3,470) + (70^2 - 5^2)/(2 \times 1,000)] \\
&= -837.6 \text{ kJ/s} \\
\dot{Q} &= \dot{W}_s - 837.6 = 830 - 837.6 = \underline{\underline{-7.6 \text{ kJ/s}}}
\end{aligned}
$$

7.4 **Information and assumptions**

provided in problem statement

Find

power required to operate compressor

Solution

Energy equation:

$$\dot{W} = \dot{Q} + \dot{m}(V_1^2/2 - V_2^2/2 + h_1 - h_2)$$

The inlet kinetic energy is negligible so

$$\begin{aligned} \dot{W} &= \dot{m}(-V_2^2/2 + h_1 - h_2) \\ &= 1.5(-200^2/2 + 300 \times 10^3 - 500 \times 10^3) \\ \dot{W} &= \underline{\underline{-220 \ \text{kW}}} \end{aligned}$$

Note: Work done to system is indicated by the negative sign.

7.5 **Information and assumptions**

provided in problem statement

Find

velocity and temperature at outlet

Solution

$$h_1 + V_1^2/2 = h_2 + V_2^2/2 \tag{1}$$
$$h_1 - h_2 = V_2^2/2 - V_1^2/2$$
$$\dot{m} = \rho_1 V_1 A = (p_1/RT_1)V_1 A$$

or

$$T_1 = p_1 V_1 A/(R\dot{m})$$

where

$$\begin{aligned} A &= (\pi/4) \times (0.08)^2 = 0.005024 \text{ m}^2 \\ h_1 - h_2 &= c_p(T_1 - T_2) = [c_p p_1 V_1 A/(R\dot{m})] - [c_p p_2 V_2 A/(R\dot{m})] \\ c_p p_1 A(R\dot{m}) &= 1,004 \times 150 \times 10^3 \times 0.005024/(287 \times 0.5) \\ &= 5,272 \text{ m/s} \end{aligned} \tag{2}$$

and

$$c_2 p_2 A/(R\dot{m}) = (100/150) \times (5,272) = 3,515 \text{ m/s}$$

Continuity equation

$$V_1 = \dot{m}/\rho_1 A$$

where

$$\rho_1 = 150 \times 10^3/(287 \times 298) = 1.784 \text{ kg/m}^3$$

Then

$$V_1 = 0.50/(1.784 \times 0.005024) = 55.8 \text{ m/s} \tag{3}$$

Utilizing Eqs. (1), (2) and (3), we have

$$55.8 \times 5,272 - 3,515V_2 = (V_2^2/2) - (55.8^2/2) \tag{4}$$

Solving Eq. (4) yields $V_2 = \underline{\underline{83.15 \text{ m/s}}}$

$$\begin{aligned} c_p(T_1 - T_2) &= (83.15^2 - 55.8^2)/2 = 1,900 \text{ m}^2/\text{s}^2 \\ T_2 &= T_1 - (1,900/c_p) \\ &= 20°C - 1,900/1,004 = \underline{\underline{18.1°C}} \end{aligned}$$

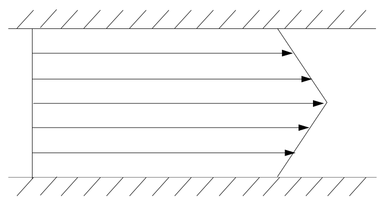
7.6 **Information and assumptions**

provided in problem statement

Find

α and mean velocity in terms of $V_{\max}$

Solution



$$\bar{V} = Q/A = \int_A V dA/A$$

where

$$\begin{aligned} V &= V_{\max} - 0.3 V_{\max} r/r_0 \\ \bar{V} &= V_{\max}(1 - 0.3\, r/r_0) \end{aligned}$$

Then

$$\begin{aligned} \bar{V} &= (V_{\max}/\pi r_0^2) \int_0^{r_0} (1 - 0.3\ r/r_0) 2\pi r dr \\ &= (2\pi V_{\max}/\pi r_0^2) \int_0^{r_0} (1 - 0.3\ r/r_0) r dr \\ &= (2V_{\max}/r_0^2) \int_0^{r_0} (rdr - (0.3\ r^2/r_0)) dr \\ &= (2V_{\max}/r_0^2)(r_0^2/2 - 0.3 r_0^2/3) = \underline{\underline{0.800 V_{\max}}} \\ \alpha &= (1/\pi r_0^2) \int_0^{r_0} [(1 - 0.3\ r/r_0) V_{\max}/0.800 V_{\max}]^3 2\pi r dr \\ \alpha &= 2\pi/((0.800)^3 \pi r_0^2) \int_0^{r_0} (1 - 0.3\ r/r_0)^3 r dr \end{aligned}$$

Integrating yields $\underline{\underline{\alpha = 1.022}}$

7.7 **Information and assumptions**

provided in problem statement

Find

kinetic energy correction factor α

Solution

$$\bar{V} = V_{\max}/2 \text{ and } V = V_{\max}y/d$$

Then

$$\alpha = (1/d)\int_0^d (V_{\max}y/((V_{\max}/2)d))^3 dy = (1/d)\int_0^d (2y/d)^3 dy = \underline{\underline{2}}$$

7.8 a) $\underline{\underline{\alpha = 1.0}}$; b) $\underline{\underline{\alpha > 1.0}}$; c) $\underline{\underline{\alpha > 1.0}}$; d) $\underline{\underline{\alpha > 1.0}}$

7.9 **Information and assumptions**

provided in problem statement

Find

kinetic energy correction factor α

Solution

$$
\begin{aligned}
\alpha &= (1/A)\int_A (V/\bar{V})^3 dA \\
V &= V_{\max} - (r/r_0)V_{\max} \\
V &= V_{\max}(1-(r/r_0)) \\
Q &= \int V dA = \int_0^{r_0} V(2\pi r dr) = \int_0^{r_0} V_m(1-r/r_0)2\pi r dr \\
&= 2\pi V_m \int_0^{r_0} [r-(r^2/r_0)]dr
\end{aligned}
$$

Integrating yields

$$
\begin{aligned}
Q &= 2\pi V_m[(r^2/2)-(r^3/(3r_0))]_0^{r_0} \\
Q &= 2\pi V_m[(1/6)r_0^2] \\
Q &= (1/3)V_m A
\end{aligned}
$$

or

$$V = Q/A = 1/3V_m$$

Then

$$
\begin{aligned}
\alpha &= (1/A)\int_0^{r_0} [V_m(1-r/r_0)/((1/3)V_m)]^3 2\pi r dr \\
\alpha &= (54\pi/\pi r_0^2)\int_0^{r_0} (1-(r/r_0))^3 r dr = \underline{\underline{2.7}}
\end{aligned}
$$

7.10 **Information and assumptions**

provided in problem statement

Find

kinetic energy correction factor α

Solution

$$
\begin{aligned}
V &= kr \\
Q &= \int_0^{r_0} V(2\pi r dr) = \int_0^{r_0} 2\pi k r^2 dr = 2\pi k r_0^3/3 \\
\bar{V} &= Q/A = ((2/3)k\pi r_0^3)/\pi r_0^2 = 2/3\ k\ r_0
\end{aligned}
$$

Then

$$
\begin{aligned}
\alpha &= (1/A)\int_A (V/\bar{V})^3 dA \\
\alpha &= (1/A)\int_0^{r_0} (kr/(2/3\ kr_0))^3 2\pi r dr \\
\alpha &= ((3/2)^3 2\pi/(\pi r_0^2))\int_0^{r_0} (r/r_0)^3 r dr \\
\alpha &= ((27/4)/r_0^2)(r_0^5/(5r_0^3)) = \underline{\underline{27/20}}
\end{aligned}
$$

7.11 b) turbulent

7.12 Derive formula for kinetic energy correction factor

$$\begin{aligned}
u/u_{\max} &= (y/r_0)^n = ((r_0 - r)/r_0)^n = (1 - r/r_0)^n \\
Q &= \int_A u dA = u_{\max}(1 - r/r_0)^n 2\pi r dr \\
&= 2\pi u_{\max} \int_0^{r_0} (1 - r/r_0)^n r dr
\end{aligned}$$

Upon integration

$$Q = 2\pi u_{\max} r_0^2 [(1/(n+1)) - (1/(n+2))]$$

Then

$$\begin{aligned}
\bar{V} &= Q/A = 2u_{\max}[(1/(n+1)) - (1/(n+2))] \\
&= 2u_{\max}/[(N+1)(n+2)] \\
\alpha &= \frac{1}{A}\int_0^{r_0} [u_{\max}(1 - r/r_0)^n / (2u_{\max}/((N+1)(n+2)))]^3 2\pi r dr
\end{aligned}$$

Upon integration one gets

$$a = (1/4)[((n+2)(n+1))^3/((3n+2)(3n+1))]$$

If $n = 1/6$, then

$$\alpha = (1/4)[((1/6+2)(1/6+1))^3/((3 \times 1/6 + 2)(3 \times 1))]$$
$$\underline{\underline{\alpha = 1.078}}$$

7.13 Derive formula for kinetic energy correction factor.

$$u/u_{\max} = (y/d)^n$$

Solve for q first in terms of $u_{\max}$ and d

$$q = \int_0^d u dy = \int_0^d u_{\max}(y/d)^n dy = u_{\max}/d^n \int_0^d y^n dy$$

Integrating:

$$\begin{aligned} q &= (u_{\max}/d^n)[y^{n+1}/(n+1)]_0^d = u_{\max}d^{n+1}d^{-n}/(n+1) \\ q &= u_{\max}d/(n+1) \end{aligned}$$

Then

$$\begin{aligned} \bar{u} &= q/d = u_{\max}/(n+1) \\ \alpha &= (1/A)\int_A (u/\bar{u})^3 dA = 1/d \int_0^d [u_{\max}(y/d)^n/(u_{\max}/(n+1))]^3 dy \\ &= ((n+1)^3/d^{3n+1}) \int_0^d y^{3n} dy \end{aligned}$$

Integrating

$$\alpha = ((n+1)^3/d^{3n+1})[d^{3n+1}/(3n+1)] = \underline{\underline{(n+1)^3/(3n+1)}}$$

When $n = 1/7$

$$\alpha = (1+1/7)^3/(1+3/7) = \underline{\underline{1.045}}$$

7.14 The kinetic energy correction factor is defined as

$$\alpha = \frac{1}{A}\int_A \left(\frac{V}{\bar{V}}\right)^3 dA$$

The integral is evaluated using

$$\int_A \left(\frac{V}{\bar{V}}\right)^3 dA \simeq \frac{1}{\bar{V}^3}\sum_i \pi(r_i^2 - r_{i-1}^2)(\frac{v_i + v_{i-1}}{2})^3$$

The mean velocity is 24.32 m/s and the kinetic energy correction factor is $\underline{\underline{1.187}}$.

7.15 **Information and assumptions**

provided in problem statement

Find

find value for K_L

Solution

Write the energy equation from water surface in tank to outlet:

$$\begin{aligned}(p_1/\gamma) + (V_1^2/2g) + z_1 &= (p_2/\gamma) + (V_2^2/2g) + z_2 + h_L \\ (100,000/9,810) + 0 + 12 &= 0 + (10^2/2g) + 0 + K_L V^2/2g \\ K_L &= \underline{\underline{3.35}}\end{aligned}$$

7.16 **Information and assumptions**

provided in problem statement

Find

pressure in tank

Solution

Write the energy equation from the water surface in the tank to the outlet:

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
p_1/\gamma &= V_2^2/2g + h_L - z_1 = 6V_2^2/2g - 10 \\
V_2 &= Q/A_2 = 0.1/((\pi/4)(1/12)^2) = 18.33 \text{ ft/s} \\
p_1/\gamma &= (6(18.33^2)/64.4) - 10 = 2.13 \text{ ft} \\
p_1 &= 62.4 \times 21.3 = 1{,}329 \text{ psfg} = \underline{\underline{9.23 \text{ psig}}}
\end{aligned}
$$

7.17 **Information and assumptions**

Assume $\alpha_2 = 1$
provided in problem statement

Find

pressure at point A and velocity at exit

Solution

$$p_A = -\gamma y = -62.4 \times 4 = \underline{\underline{-250 \text{ lb/ft}^2}}$$

Apply energy equation between top of tank and exit.

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_t + h_L \\ z_1 &= V_2^2/2g + z_2 \\ V_2 &= \sqrt{2g(z_1 - z_2)} = \sqrt{2 \times 32.2 \times 14} \\ &= \underline{\underline{30.0}} \text{ ft/s} \end{aligned}$$

7.18 70pt**Information and assumptions**

Assume $\alpha_1 = 1$.
provided in problem statement

Find

pressure at point A and velocity at exit

Solution

$$p_A = 9,810(-3) = \underline{\underline{-29.43 \text{ kPa}}}$$

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_t + h_L \\ z_1 &= V_2^2/2g + z_2 \\ V_2 &= \sqrt{2g(z_1 - z_2)} = \sqrt{2 \times 9.81 \times 7} \\ &= \underline{\underline{11.7}} \text{ m/s} \end{aligned}$$

7.19 **Information and assumptions**

provided in problem statement

Find

pressure difference between A and B

Solution

$$\begin{aligned} p_A - p_B &= 1\gamma + (\rho/2)(V_B^2 - V_A^2)\ ;\ V_A = Q/A_1 = 1.910 \text{ m/s} \\ V_B &= \left(\frac{20}{12}\right)^2 \times V_A = 5.31 \text{ m/s} \end{aligned}$$

Then

$$p_A - p_B = 1 \times 9810 \times 0.9 + (900/2)(5.31^2 - 1.91^2) = \underline{\underline{19.88 \text{ kPa}}}$$

7.20 **Information and assumptions**

provided in problem statement

Find

discharge in pipe

Solution

Write the energy equation from the water surface in the reservoir (pt. 1) to the outlet end of the pipe (pt. 2). Also assume $\alpha = 1$.

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ p_1 &= 0; \quad p_2 = 0; \quad z_2 = 0; \quad V_1^2/2g \simeq 0 \end{aligned}$$

$$\begin{aligned} z_1 &= V_2^2/2g + h_L \\ 11 &= V_2^2/2g + 10V_2^2/2g \\ V_2^2 &= (2g/11)(11) \\ V_2 &= 4.429 \text{ m/s} \\ Q &= V_2 A_2 = (4.429 \text{ m/s})(5 \text{ cm}^2)(10^{-4} \text{ m}^2/\text{cm}^2) \\ Q &= \underline{\underline{2.21 \times 10^{-3} \text{ m}^3/\text{s}}} \end{aligned}$$

7.21 The pipe will have to decrease in elevation at a rate greater than the head loss per given length of pipe.

7.22 **Information and assumptions**

provided in problem statement

Find

pressure at station 2

Solution

$$\begin{aligned}
V_1 &= Q/A_1 = 6/0.8 = 7.5 \text{ ft/s} \\
V_1^2/2g &= 0.873 \text{ ft} \\
V_2 &= Q/A_2 = 6/0.2 = 30 \text{ ft/s} \\
V_2^2/2g &= 13.98 \text{ ft} \\
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + 4 \\
(10 \times 144)/(0.8 \times 62.4) + 0.873 + 12 &= p_2/\gamma + 13.98 + 0 + 4 \\
p_2/\gamma &= 23.74 \text{ ft} \\
p_2 &= 23.74 \times 0.8 \times 62.4 = 1,185 \text{ psfg} \\
&= \underline{\underline{8.23 \text{ psig}}}
\end{aligned}$$

7.23 **Information and assumptions**

provided in problem statement

Find

discharge in pipe and pressure at point A

Solution

$$\begin{aligned} p_{\text{reser.}}/\gamma + V_r^2/2g + z_r &= p_{\text{outlet}}/\gamma + V_0^2/2g + z_0 \\ 0 + 0 + 5 &= 0 + V_0^2/2g;\ V_0 = 9.90 \text{ m/s} \\ Q &= V_0 A_0 = 9.90 \times (\pi/4) \times 0.20^2 = \underline{\underline{0.311 \text{ m}^3/\text{s}}} \end{aligned}$$

Write energy equation from reservoir surface to point B:

$$0 + 0 + 5 = p_B/\gamma + V_B^2/2g + 3.5$$

where

$$\begin{aligned} V_B &= Q/V_B = 0.311/(\pi/4) \times 0.4^2 = 2.48 \text{ m/s} \\ V_B^2/2g &= 0.312 \text{ m} \end{aligned}$$

Assuming $\gamma = 9810$ N/m

$$p_B/\gamma - 5 - 3.5 = 0.312;\ p_B = \underline{\underline{11.7 \text{ kPa}}}$$

7.24 **Information and assumptions**

provided in problem statement

Find

pressure in syring pump

Solution

Applying the energy equation to this problem gives

$$\frac{p_1}{\gamma} = h_L + \alpha_2 \frac{V^2}{2g} = \frac{32\mu L V}{D^2} + 2\frac{V^2}{2g}$$

The cross-sectional area of the channel is 3.14×10^{-8} m^2. A flow rate of 0.1 μl/s is 10^{-7} l/s or 10^{-10} m^3/s. The flow velocity is

$$V = \frac{Q}{A} = \frac{10^{-10}}{3.14 \times 10^{-8}} = 0.318 \times 10^{-2} \text{ m/s} = 3.28 \text{ mm/s}$$

Substituting the velocity and other parameters in the above equation gives

$$\frac{p_1}{\gamma} = \frac{32 \times 1.2 \times 10^{-3} \times 0.05 \times 0.318 \times 10^{-2}}{7,850 \times 4 \times (10^{-4})^2} + 2 \times \frac{\left(0.318 \times 10^{-2}\right)^2}{2 \times 9.81} = 0.0194 \text{ m}$$

The pressure is

$$p_1 = 799 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \times 0.0194 \text{ m } = \underline{\underline{152.1 \text{ Pa}}}$$

7.25 **Information and assumptions**

provided in problem statement

Find

pressure at hydrant

Solution

Applying the energy equation to this problem we have

$$\frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + z_2 + h_L$$

where the kinetic energy of the fluid feeding the hydrant is neglected. Because of the contraction at the exit, the outlet velocity is 4 times the velocity in the pipe, so the energy equation becomes

$$\frac{p_1}{\gamma}_1 = \frac{V_2^2}{2g} + z_2 - z_1 + 10\frac{V^2}{16 \times 2g} = \frac{1.625}{2g}V^2 + 50$$

For an exit velocity of 40 m/s, the pressure at the hydrant must be

$$p_1 = 9810 \times (132.5 + 50) = \underline{\underline{1,790 \text{ kPa}}}$$

7.26 **Information and assumptions**

provided in problem statement

Find

pressure at point B.

Solution

First solve for h_L from reservoir to C. Let point 1 be reservoir surface.

$$\begin{aligned}
p_1/\gamma + V_1^2/g + z_1 &= p_c/\gamma + V_c^2/2g + z_c + h_L;\ V_c = Q/A_2 \\
0 + 0 + 3 &= 0 + 8.02^2/64.4 + 0 + h_L \\
V_c &= 2.8/((\pi/4) \times (8/12)^2) = 8.02 \text{ ft/s} \\
h_L &= \underline{\underline{2.00 \text{ ft}}}
\end{aligned}$$

Now get p_B be writing energy equation from reservoir surface to B.

$$\begin{aligned}
0 + 0 + 3 &= p_B/\gamma + V_B^2/2g + 6 + (3/4) \times 2\ ;\ V_B = V_C = 8.02 \text{ ft/s} \\
p_B/\gamma &= 3 - 1 - 6 - 1.5 = -5.5 \text{ ft} \\
p_B &= -5.5 \times 62.4 = -343 \text{ psfg} = -\underline{\underline{2.38 \text{ psig}}}
\end{aligned}$$

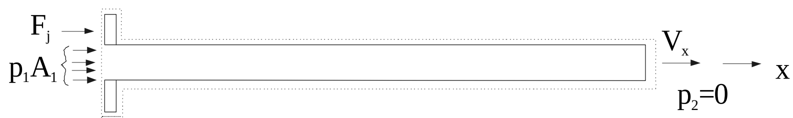
7.27 **Information and assumptions**

provided in problem statement

Find

force on pipe joint

Solution



Momentum Eq.

$$\begin{aligned}\sum F_x &= \dot{m}V_{o,x} - \dot{m}V_{i,x} \\ F_j + p_1 A_1 &= -\rho V_x^2 A + \rho V_x^2 A \\ F_j &= -p_1 A_1\end{aligned}$$

Energy Eq.

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + h_L$$

$$p_1 - p_2 = \gamma h_L$$

$$p_1 = \gamma(1) = 62.4 \text{ psfg}$$

$$F_j = -62.4 \times (10/144)$$

$$\underline{\underline{F_j = -4.33 \text{ lbf}}}$$

7.28 **Information and assumptions**

provided in problem statement

Find

discharge and pressure at point B

Solution

$$\begin{aligned} h_{\ell_{\text{pipe}}} &= V^2/2g \\ h_{\text{total}} &= h_{\ell_{\text{pipel}}} + h_{\ell_{\text{outlet}}} = 2V_p^2/2g \end{aligned}$$

Write the energy equation from A to C:

$$\begin{aligned} 0+0+30 &= 0+0+27+2V_p^2/2g \\ V_p &= 5.42 \text{ m/s} \\ Q &= V_pA_p = 5.42 \times (\pi/4) \times 0.30^2 \\ Q &= \underline{\underline{0.383 \text{ m}^3/\text{s}}} \end{aligned}$$

Write the energy equation to pint B:

$$\begin{aligned} 30 &= p_B/\gamma + V_p^2/2g + 32 + 0.75V_p^2/2g;\ p_B/\gamma = -2 - 1.5 \times 1.75 \text{ m} \\ p_B &= \underline{\underline{-45.3 \text{ kPa, gage}}} \end{aligned}$$

7.29 **Information and assumptions**

provided in problem statement

Find

depth of water in upper reservoir for incipient cavitation

Solution

Write energy equation from point A to point B.

$$\begin{aligned} p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/2g + z_B + h_L \\ V &= Q/A = (8)(10^{-4})\ \mathrm{m}^3/\mathrm{s}/((10^{-4})\ \mathrm{m}^2) = 8\ \mathrm{m/s} \end{aligned}$$

Then

$$\begin{aligned} V^2/2g &= 8^2/(2 \times 9.81) = 3.262\ \mathrm{m} \\ h_{L,A \to B} &= 2V^2/2g = 6.524\ \mathrm{m} \end{aligned}$$

Let $z = 0$ at bottom of reservoir. Then

$$\begin{aligned} 100,000/9,810 + 0 + z_A &= 1,230/9,810 + 2.497 + 10 + 6.524 \\ z_A &= \text{depth } = \underline{\underline{8.95\ \mathrm{m}}} \end{aligned}$$

7.30 **Information and assumptions**

provided in problem statement

Find

Assume the flow is from A to B. Then write the energy equation from A to B:

Solution

$$\begin{aligned} p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/g + z_B + h_L \\ (10,000/9,810) + 10 &= (98,100/9,810) + 0 + h_L \\ h_L &= 1.02 + 10 = 10.0 = +1.02 \end{aligned}$$

Because the value for head loss is positive it verifies our assumption of downward flow. Correction selection is b)

7.31 **Information and assumptions**

provided in problem statement

Find

pressures at points A and B.

Solution

Applying the energy equation between the top of the tank and the exit results in

$$z_1 + h_p = \frac{V_2^2}{2g} + z_2$$

assuming the machine is a pump. If the machine is a turbine, then h_p will be negative. The velocity at the exit is

$$V_2 = \frac{Q}{A_2} = \frac{10}{\frac{\pi}{4}0.5^2} = 50.95 \text{ ft/s}$$

Solving for h_p and taking the pipe exit as zero elevation we have

$$h_p = \frac{50.95^2}{2 \times 32.2} - (6 + 12) = 22.31 \text{ ft}$$

Therefore the machine is a pump.

Applying the energy equation between point B and the exit gives

$$\frac{p_B}{\gamma} + z_B = z_2$$

Solving for p_B we have

$$p_B = \gamma(z_2 - z_B) = -6 \times 62.4 = -374 \text{ psfg } = \underline{\underline{-2.6 \text{ psig}}}$$

The velocity at point A is

$$V_A = \left(\frac{6}{12}\right)^2 \times 50.95 = 12.74 \text{ ft/s}$$

Applying the energy equation between point A and the exit gives

$$\frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g} = \frac{V_2^2}{2g}$$

so

$$\begin{aligned} p_A &= \gamma(\frac{V_2^2}{2g} - z_A - \frac{V_A^2}{2g}) = 62.4 \times (\frac{50.95^2 - 12.74^2}{2 \times 32.2} - 18) \\ &= 1235 \text{ psfg } = \underline{\underline{8.58 \text{ psig}}} \end{aligned}$$

7.32 **Information and assumptions**

provided in problem statement

Find

pressure at point 2

Solution

Let V_n = velocity of jet from nozzle:

$$\begin{aligned}
V_n &= Q/A_n = 0.20/((\pi/4) \times 0.15^2) = 11.32 \text{ m/s} \\
V_n^2/2g &= 6.53 \text{ m} \\
V_2 &= Q/A_2 = 0.20/((\pi/4) \times 0.3^2) = 2.83 \text{ m/s} \\
V_2^2/2g &= 0.408 \text{ m} \\
p_2/\gamma + 0.408 + 2 &= 0 + 6.53 + 7 \\
p_2/\gamma &= \underline{\underline{11.1 \text{ m}}}
\end{aligned}$$

7.33 **Information and assumptions**

provided in problem statement

Find

height above water surface

Solution

Write the energy equation from the reservoir water surface to the outlet:

$$0 + 0 + 0 + h_p = 0 + h + V_c^2/2g + 2.0V_c^2/2g$$

where

$$\begin{aligned} V_c^2/2g &= 12^2/64.4 = 2.24 \text{ ft} \\ P(\text{hp}) &= Q\gamma h_p/(550 \times 0.7) \end{aligned}$$

Then

$$h_p = (30 \times 550 \times 0.7)/(62.4 \times 12 \times (\pi/4) \times 0.5^2) = 78.56 \text{ ft}$$

Solve energy equation for h:

$$h = 78.56 - 3.0 \times 2.24 = \underline{\underline{71.8 \text{ ft}}}$$

7.34 **Information and assumptions**

provided in problem statement

Find

height above water surface.

Solution

Follow the same solution process as in Problem 7.33.

$$
\begin{aligned}
0+0+0+h_p &= 0+h+3.0V_c^2/2g \\
V_C^2/2g &= 4^2/(2\times 9.81) = 0.815 \text{ m} \\
p &= Q\gamma h_p/0.7 \\
h_p &= 35,000\times 0.7/((4\times \pi/4\times 0.15^2)(9,810)) = 35.3 \text{ m} \\
h &= 35.3 - 3.0\times 0.815 = \underline{\underline{32.9 \text{ m}}}
\end{aligned}
$$

7.35 **Information and assumptions**

provided in problem statement

Find

horsepower delivered by pump

Solution

$$\begin{aligned} V_A &= Q/A_A = 3.92/((\pi/4)\times 1^2) = 4.99 \text{ ft/sec} \\ V_A^2/2g &= 0.387 \text{ ft} \\ V_B &= Q/A_B = 3.92/((\pi/4)\times 0.5^2) = 19.96 \text{ ft/s} \\ V_B^2/2g &= 6.19 \text{ ft} \end{aligned}$$

Write the energy equation from A to B:

$$p_A/\gamma + V_A^2/2g + z_A + h_p = p_B/\gamma + V_B^2/2g + z_B$$
$$10\times 144/62.4 + 0.387 + 0 + h_p = 40\times 144/62.4 + 6.19 + 0$$
$$h_p = 75.04 \text{ ft}$$
$$P(\text{hp}) = Q\gamma h_p/550 = 3.92\times 62.4\times 75.04/550$$
$$P = \underline{\underline{33.4 \text{ hp}}}$$

7.36 **Information and assumptions**

provided in problem statement

Find

power supplied to flow.

Solution

Write energy equation from reservoir surface to end of pipe:

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
0 + 0 + 40 + h_p &= 0 + V^2/2g + 20 + 7V^2/2g \\
V &= Q/A = 7.85/((\pi/4) \times 1^2) = 10.0 \ \text{ m/s} \\
V^2/2g &= 10^2/(2 \times 9.81) = 5.09 \text{ m}
\end{aligned}$$

Then

$$\begin{aligned}
h_p &= 8 \times 5.1 + 20 - 40 = 20.8 \text{ m} \\
P &= Q\gamma h_p = 7.85 \times 9,810 \times 20.8 \\
&= \underline{\underline{1.60 \text{ MW}}}
\end{aligned}$$

7.37 **Information and assumptions**

provided in problem statement

Find

power pump must supply.

Solution

$$\begin{aligned} V &= Q/A = 0.25/((\pi/4) \times 0.3^2) = 3.54 \text{ m/s} \\ V^2/2g &= 0.638 \text{ m} \end{aligned}$$

Write energy equation from reservoir surface to 10 m elevation:

$$\begin{aligned} 0 + 0 + 6 + h_p &= 100,000/9,810 + V^2/2g + 10 + 2.0V^2/2g \\ h_p &= 10.19 + 10 - 6 + 3.0 \times 0.638 \\ h_p &= 16.1 \text{ m} \\ P &= Q\gamma h_p = 0.25 \times 9.180 \times 16.1 = \underline{\underline{39.5 \text{ kW}}} \end{aligned}$$

7.38 50pt**Information and assumptions**

provided in problem statement

Find

horsepower pump supplies

Solution

$$
\begin{aligned}
V_{12} &= Q/A_{12} = 6/((\pi/4) \times 1^2) = 7.643 \text{ ft/sec} \\
V_{12}^2/2g &= 0.907 \text{ ft} \\
V_6 &= 4V_{12} = 30.57 \text{ ft/sec} \\
V_6^2/2g &= 14.51 \text{ ft} \\
(p_6/\gamma + z_6) - (p_{12}/\gamma + z_{12}) &= (13.55 - 0.88)(46/12)/0.88 \\
(p_{12}/\gamma + z_{12}) + V_{12}^2/2g + h_p &= (p_6/\gamma + z_6) + V_6^2/2g \\
h_p &= (13.55/0.88 - 1) \times 3.833 + 14.51 - 0.907 \\
h_p &= 68.8 \text{ ft} \\
P(\text{hp}) &= Q\gamma h_p/550 \\
P &= 6 \times 0.88 \times 62.4 \times 68.8/550 = \underline{\underline{41.2 \text{ hp}}}
\end{aligned}
$$

7.39 **Information and assumptions**

provided in problem statement

Find

power output from turbine

Solution

Write the energy equation form the upstream water surface to the downstream water surface:

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2/2g + z_2 + h_L + h_T \\
0 + 0 + 35 &= 0 + 0 + 0 + 1.5V^2/2g + h_T \\
V &= Q/A_6 = 250/((\pi/4) \times 6^2) = 8.84 \text{ ft/s} \\
& \quad V^2/2g \;\; 1.21 \text{ ft} \\
h_t &= 35 - 1.82 = 33.19 \text{ ft} \\
P(\text{hp}) &= Q\gamma h_t \times 0.8/550 \\
P &= \underline{\underline{753 \text{ hp}}}
\end{aligned}$$

7.40 **Information and assumptions**

provided in problem statement

Find

power produced by turbine.

Solution

Write the energy equation from the upstream water surface to the downstream water surface. Assume all head loss is expansion loss. Assume 100% efficiency.

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_t + h_L \\
0 + 0 + 12 \text{ m} &= 0 + 0 + 0 + h_t + V^2/2g \\
h_t &= 12 \text{ m} - (5^2/2g) = 10.725 \text{ m} \\
P &= Q\gamma h_t = (1 \text{ m}^3/\text{s})(9{,}810 \text{ N/M}^3)(10.725 \text{ m}) \\
P &= \underline{\underline{105.2 \text{ kW}}}
\end{aligned}$$

7.41 **Information and assumptions**

provided in problem statement

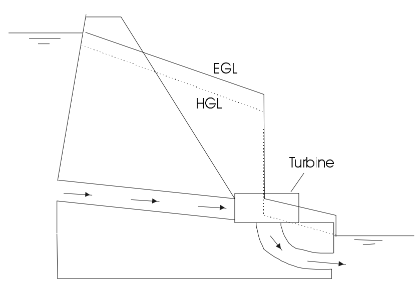
Find

power generated by turbine.

Solution

Write the energy equation from the upper water surface to the lower water surface:

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L + h_t \\
0 + 0 + 100 \text{ ft} &= 0 + 0 + 4 \text{ ft } + h_t \\
h_t &= 96 \text{ ft} \\
P &= (Q\gamma h_t)(\text{eff.}) \\
P(\text{hp}) &= Q\gamma h_t(\text{eff.})/550 = 1,000 \times 62.4 \times 96 \times 0.85/550 \\
P &= \underline{\underline{9,258 \text{ hp}}}
\end{aligned}
$$



7.42 **Information and assumptions**

provided in problem statement

Find

power delivered by pump.

Solution

Write the energy equation from the reservoir water surface to point B:

$$
\begin{aligned}
p/\gamma + V^2/2g + z + h_p &= p_B/\gamma + V_B^2/2g + z_B \\
0 + 0 + 40 + h_p &= 0 + 0 + 64;\ h_p = 25 \text{ m} \\
P &= Q\gamma h_p \\
Q &= V_j A_j = 25 \times 10^{-4} \text{ m}^2 \times V_j \\
\text{where } V_j &= \sqrt{2g \times (65 - 35)} = 24.3 \text{ m/s};\ Q = 25 \times 10^{-4} \times 24.3 = 0.0607 \text{ m}^3/\text{s} \\
\text{Then Power} &= 8.0607 \times 9,810 \times 25W = \underline{\underline{14.89 \text{ kW}}}
\end{aligned}
$$

7.43 **Information and assumptions**

provided in problem statement

Find

power delivered by pump.

Solution

Solution procedure is the same as for Problem 7.42.

$$\begin{aligned}
0+0+110+h_p &= 0+0+200;\ h_p = 90 \text{ ft} \\
P(\text{hp}) &= Q\gamma h_p/550 \\
Q &= V_j A_j = 0.10\ V_j \\
V_j &= \sqrt{2g \times (200-100)} = 76.13 \text{ ft/s} \\
Q &= 7.613 \text{ ft}^3/\text{s}
\end{aligned}$$

Then

$$P = 7.613 \times 62.4 \times 90/550 = \underline{\underline{77.7 \text{ hp}}}$$

7.44 **Information and assumptions**

provided in problem statement

Find

power required for pump.

Solution

Applying the energy equation with equal pressures at the inlet and outlet and then recognizing that there is no change in kinetic energy gives

$$h_p = z_2 - z_1 + h_L$$

Expressing this equation in terms of pressure

$$\gamma h_p = \gamma z_2 - \gamma z_1 + \Delta p_{loss}$$

Thus pressure rise across the pump is

$$\gamma h_p = 53 \text{ lbf/ft}^3 \times 200 \text{ ft } + 60 \times 144 \text{ lbf/ft}^2 = 19,240 \text{ psf}$$

The discharge is

$$Q = 3500 \text{ gpm} \times 0.002228 \, \frac{\text{ft}^3/\text{s}}{\text{gpm}} = 7.80 \text{ cfs}$$

The power required is

$$\dot{W} = Q\gamma h_p = 150 \times 10^3 \text{ ft} \cdot \text{lbf/s} = \underline{\underline{273 \text{ hp}}}$$

7.45 **Information and assumptions**

provided in problem statement

Find

time required to transfer oil.

Solution

Applying the energy equation between the top of the fluid in tank A to that in tank B, we have

$$h_p + z_A = z_B + h_L$$

or

$$h_p + z_A = z_B + 20\frac{V^2}{2g} + \frac{V^2}{2g}$$

Solving for the velocity we have

$$V^2 = \frac{2g}{21}(h_p + z_A - z_B)$$

or

$$V = 0.966(60 + z_A - z_B)^{1/2}$$

However the sum of the elevations of the liquid surfaces in the two tanks is $z_A + z_B = 21$. So the energy equation becomes

$$V = 0.966(81 - 2z_B)^{1/2}$$

The rate at which the liquid level increases in tank B is

$$\frac{dz_B}{dt} = V\frac{A_p}{A_T} = 4 \times 10^{-4}V = 3.864 \times 10^{-4}(81 - 2z_B)^{1/2}$$

or

$$\frac{dz_B}{(81 - 2z_B)^{1/2}} = 3.864 \times 10dt$$

Integrating this equation gives

$$\Delta t = 2588 \times (81 - 2z_B)^{1/2} \mid_1^{20} = 6430 \text{ s or } \underline{\underline{1.79 \text{ hrs}}}$$

7.46 **Information and assumptions**

provided in problem statement

Find

Applying the energy equation between the water surface at the intake and the water surface inside the tank we have

Solution

$$h_p + z_1 = \frac{p_2}{\gamma} + z_2 + h_L$$

Expressing the head loss in terms of the velocity allows one to solve for the velocity in the form

$$V^2 = \frac{2g}{10}(h_p + z_1 - z_t - \frac{p_t}{\gamma})$$

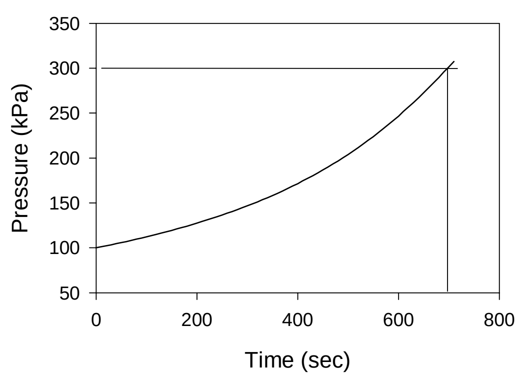
Substituting in values

$$V = 1.401(46 - z_t - 10.19\frac{3}{4 - z_t})^{1/2}$$

The equation for the water surface elevation in the tank is

$$\Delta z_t = V\frac{A_p}{A_t}\Delta t = \frac{V}{2500}\Delta t$$

A computer program can be written taking time intervals and finding the fluid level and pressure in the tank at each time step. The time to reach a pressure of 300 kPa abs in the tank is 698 seconds or 11.6 minutes. A plot of how the pressure varies with time is provided.



7.47 **Information and assumptions**

provided in problem statement

Find

discharge between two tanks.

Solution

Write the energy equation from water surface in A to water surface in B.

$$\begin{aligned} p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/2g + z_B + \sum h_L \\ p_A &= p_B = p_{\text{atm}} \text{ and } V_A = V_B = 0 \end{aligned}$$

Let the pipe from A be called pipe 1. Let the pipe from B be called pipe 2
Then

$$\sum h_L = (V_1 - V_2)^2/2g + V_2^2/2g$$

But

$$\begin{aligned} V_1 A_1 &= V_2 A_2 \\ V_1 &= V_2(A_2/A_1) \end{aligned}$$

However $A_2 = 2A_1$ so $V_1 = 2V_2$. Then the energy equation gives

$$\begin{aligned} z_A - z_B &= (2V_2 - V_2)^2/2g + V_2^2/2g \\ &= 2V_2^2/2g \\ V_2 &= \sqrt{g(z_A - z_B)} \\ &= \sqrt{10g} \text{ m/s} \end{aligned}$$

Then

$$\begin{aligned} Q &= V_2 A_2 \\ &= \left(\sqrt{10g} \text{ m/s}\right)(20 \text{ cm}^2)(10^{-4} \text{ m}^2/\text{cm}^2) \\ &= \underline{\underline{0.0198 \text{ m}^3/\text{s}}} \end{aligned}$$

7.48 **Information and assumptions**

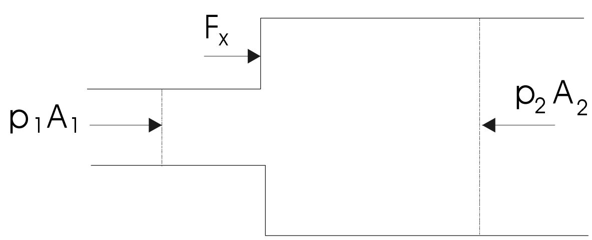
provided in problem statement

Find

horizontal force required to hold transition in place and head loss.

Solution

$$
\begin{aligned}
V_{40} &= Q/A_{40} = 1.0/((\pi/4)\times 0.40^2) = 7.962 \text{ m/s} \\
V_{40}^2/2g &= 3.231 \text{ m} \\
V_{60} &= V_{40}\times(4/6)^2 = 3.539 \text{ m/s} \\
V_{60}^2/2g &= 0.638 \text{ m} \\
h_L &= (V_{40}-V_{60})^2/2g = \underline{\underline{0.997 \text{ m}}} \\
p_{40}/\gamma + V_{40}^2/2g &= p_{60}/\gamma + V_{60}^2/2g + h_L \\
p_{60} &= 70,000 + 9,810(3.231 - 0.638 - 0.997) = 85,657 \text{ Pa}
\end{aligned}
$$



Momentum equation:

$$\sum F_x = \dot{m}_o V_{x,o} - \dot{m}_i V_{x,i}$$

$$
\begin{aligned}
70,000\times\pi/4\times 0.4^2 - 85,657\times\pi/4\times(0.6^2) + F_x &= 1,000\times 1.0\times(3.539 - 7.962) \\
F_x &= -8,796 + 24,219 - 4,423 \\
&= 10,993 \text{ N} = \underline{\underline{11.0 \text{ kN}}}
\end{aligned}
$$

7.49 **Information and assumptions**

provided in problem statement

Find

head loss

Solution

$$
\begin{aligned}
V_{10}A_{10} &= V_{15}A_{15} \\
V_{15} &= V_{10}A_{10}/A_{15} = 7 \times (10/15)^2 = 3.11 \text{ m/s} \\
h_L &= (7 - 3.11)^2/(2 \times 9.81) = \underline{\underline{0.771 \text{ m}}}
\end{aligned}
$$

7.50 **Information and assumptions**

provided in problem statement

Find

head loss

Solution

$$
\begin{aligned}
V_6 &= Q/A_6 = 5/((\pi/4)\times(1/2)^2) = 25.46 \text{ ft/s;} \\
V_{12} &= (1/4)V_6 = 6.37 \text{ ft/s} \\
h_L &= (25.46 - 6.37)^2/(2\times 32.2) = \underline{\underline{5.66 \text{ ft}}}
\end{aligned}
$$

7.51 **Information and assumptions**

provided in problem statement

Find

a) horsepower lost, b) pressure at section 2, c) force needed to hold expansion.

Solution

$$\begin{aligned} h_L &= (V_1 - V_2)^2/(2g) \\ V_2 &= V_1(A_1/A_2) = 25(1/4) = 6.25 \text{ ft/s} \\ h_L &= (25 - 6.25)^2/64.4 = 5.46 \text{ ft} \end{aligned}$$

a)

$$\begin{aligned} P(\text{hp}) &= Q\gamma h/550 \\ Q &= VA = 25(\pi/4)(5^2) = 490.9 \text{ ft}^3/\text{s} \\ P &= (490.9)(62.4)(5.46)/550 = \underline{\underline{304 \text{ hp}}} \end{aligned}$$

b)

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ (5 \times 144)/62.4 + 25^2/64.4 &= p_2/\gamma + 6.25^2/64.4 + 5.46 \\ p_2/\gamma &= 15.17 \text{ ft} \\ p_2 &= 15.17 \times 62.4 = 946.6 \text{ psfg} = \underline{\underline{6.57 \text{ psig}}} \end{aligned}$$

c)

$$\begin{aligned} \sum F_x &= \dot{m}_o V_{x,o} - \dot{m}_i V_{x,i} \\ p_1 A_1 - p_2 A_2 + F_x &= (25)(1.94)((-25)(\pi/4)(5^2) + (6.25)(1.94)(\pi/4)(10^2)) \\ (5)(14)\pi/4)(5^2) - (6.57)(144)(\pi/4)(10^2) + F_x &= -23,807 + 5952 \\ F_x &= 74,305 - 14,137 - 23,807 + 5,952 \\ F_x &= \underline{\underline{42,313 \text{ lbf}}} \end{aligned}$$

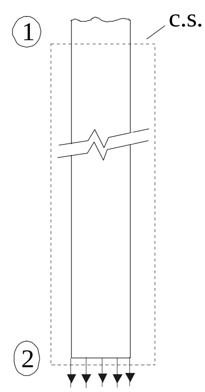
7.52 **Information and assumptions**

provided in problem statement

Find

longitudional force transmitted through pipe wall.

Solution



$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + h_L$$

but $V_1 = V_2$ and $p_2 = 0$. Therefore

$$\begin{aligned}
p_1/\gamma &= -50 + 10; p_1 = -2,496 \text{ lbf/ft}^2 \\
\sum F_y &= \dot{m}V_{y,o} - \dot{m}V_{y,i} = \rho Q(V_{2y} - V_{1y}) \\
-p_1A_1 - \gamma AL - 2L + F_{\text{wall}} &= 0 \\
F_{\text{wall}} &= 1.5L + \gamma A_1 L - p_1 A_1 \\
&= 75 + (\pi/4) \times 0.5^2(62.4 \times 50 - 2,496) \\
&= 75 + 122.5 \\
F_{\text{wall}} &= \underline{\underline{197.5 \text{ lbf}}}
\end{aligned}$$

7.53 **Information and assumptions**

provided in problem statement

Find

pressure at outlet of bend and force on anchor block in the x-direction

Solution

$$p_{50}/\gamma + V_{50}^2/2g + z_{50} = p_{80}/\gamma + V_{80}^2/2g + z_{80} + h_L$$

where $p_{50} = 650,000$ Pa and $z_{50} = z_{80}$

$$\begin{aligned} V_{80} &= Q/A_{80} = 5/((\pi/4) \times 0.8^2) = 9.947 \text{ m/s} \\ V_{80}^2/2g &= 5.04 \text{ m} \\ V_{50} &= V_{80} \times (8/5)^2 = 25.46 \text{ m/s} \\ V_{50}^2/2g &= 33.05 \text{ m} \\ h_L &= 10 \text{ m} \end{aligned}$$

Then

$$\begin{aligned} p_{80}/\gamma &= 650,000/\gamma + 33.05 - 5.04 - 10 \\ p_{80} &= 650,000 + 9,810(33.05 - 5.04 - 10) = 826,700 \text{ Pa} \\ &= \underline{\underline{826.7 \text{ kPa}}} \end{aligned}$$

$$\begin{aligned} \sum F_x &= \dot{m}V_o - \dot{m}V_i = \rho Q(V_{80,x} - V_{50,x}) \\ p_{80}A_{80} + p_{50}A_{50} \times \cos 60^\circ + F_x &= 1,000 \times 5(-9.947 - 0.5 \times 25.46) \\ F_x &= -415,540 - 63,814 - 113,385 = -592,700 \,\text{N} \\ &= \underline{\underline{-592.7 \text{ kN}}} \end{aligned}$$

7.54 **Information and assumptions**

provided in problem statement

Find

head loss at pipe outlet.

Solution

$$\begin{aligned} V &= Q/A = 10((\pi/4) \times 1^2) = 12.73 \text{ ft/sec} \\ h_L &= V^2/2g = \underline{\underline{2.52 \text{ ft}}} \end{aligned}$$

7.55 **Information and assumptions**

provided in problem statement

Find

head loss at pipe outlet.

Solution

$$
\begin{aligned}
V &= Q/A = 0.50/((\pi/4) \times 0.5^2) = 2.546 \text{ m/s} \\
h_L &= V^2/2g = (2.546)^2/(2 \times 9.81) = \underline{\underline{0.330 \text{ m}}}
\end{aligned}
$$

7.56 **Information and assumptions**

provided in problem statement

Find

maximum allowable discharge before cavitation

Solution

Take section 1 at reservoir surface and section 2 at section of d diameter. From Table A.5 $p_v = 2340$ Pa, abs

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 \\ 0 + 0 + 5 &= p_{2,\text{vapor}}/\gamma + V_2^2/2g + 0 \\ p_{2,\text{vapor}} &= 2340 - 100,000 = -97,660 \text{ Pa gage} \end{aligned}$$

Then

$$\begin{aligned} V_2^2/2g &= 5 + 97,660/9,790 = 14.97 \text{ m}; \; V_2 = 17.1 \text{ m/s} \\ Q &= V_2 A_2 = 17.1 \times \pi/4 \times 0.15^2 = \underline{\underline{0.303 \text{ m}^3/\text{s}}} \end{aligned}$$

7.57 **Information and assumptions**

provided in problem statement

Find

discharge at incipient cavitation

Solution

First write the energy equation from the Venturi section to the end of the pipe: From Table A.5 $p_v = 2340$ Pa, abs

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
p_{\text{vapor}}/\gamma + V_1^2/2g &= 0 + V_2^2/2g + 0.9V_2^2/2g \\
p_{\text{vapor}} &= 2,340 \text{ Pa abs.} = -97,660 \text{ Pa gage} \\
V_1 A_1 &= V_2 A_2; V_1 = V_2 A_2/A_1 = 2.56V_2; V_1^2/2g = 6.55V_2^2/2g \\
\text{Then } -97,660/9,790 + 6.55V_2^2/2g &= 1.9V_2^2/2g; V_2 = 6.49 \text{ m/s} \\
Q &= V_2 A_2 = 6.49 \times \pi/4 \times 0.4^2 = \underline{\underline{0.815\ m^3/s}}
\end{aligned}$$

Now write the energy equation from reservoir water surface to outlet:

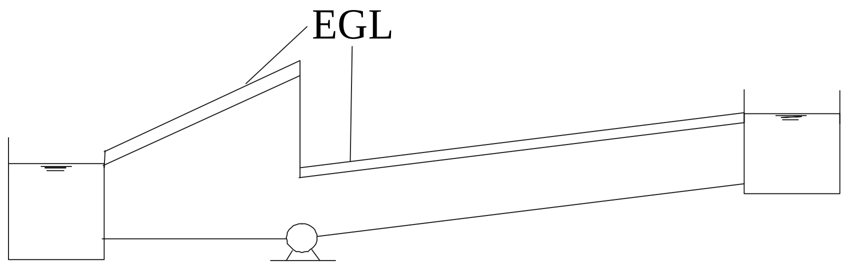
$$\begin{aligned}
z_1 &= V_2^2/2g + h_L \\
H &= 1.9V_2^2/2g = \underline{\underline{4.08 \text{ m}}}
\end{aligned}$$

7.58 **a)** Flow is from right to left.

b) Machine is a pump.

c) Pipe CA is smaller because of steeper H.G.L.

d)



e) No vacuum in the system

7.59 **Information and assumptions**

provided in problem statement

Find

discharge for water

Solution

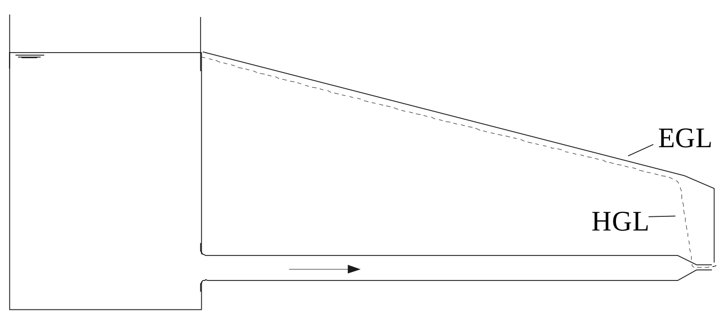
Write the energy equation from the reservoir water surface to the jet from the nozzle.

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ 0 + 0 + 100 &= 0 + V_6^2/2g + 60 + 0.02(1{,}000/1)V_{12}^2/2g \end{aligned}$$

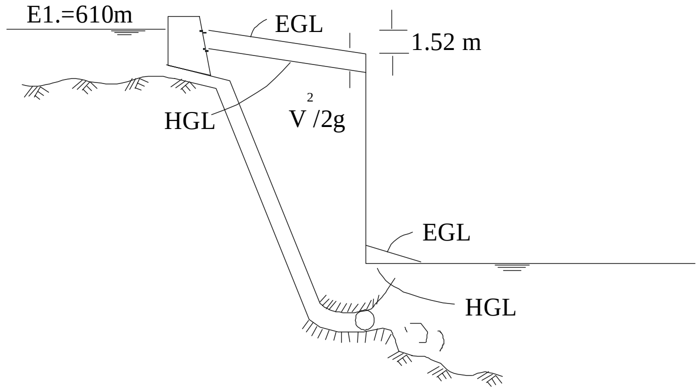
Continuity equation:

$$\begin{aligned} V_6 A_6 &= V_{12} A_{12} \\ V_6 &= V_{12}(A_{12}/A_6) \\ V_6 &= 4V_{12} \\ V_6^2/2g &= 16V_{12}^2/2g \end{aligned}$$

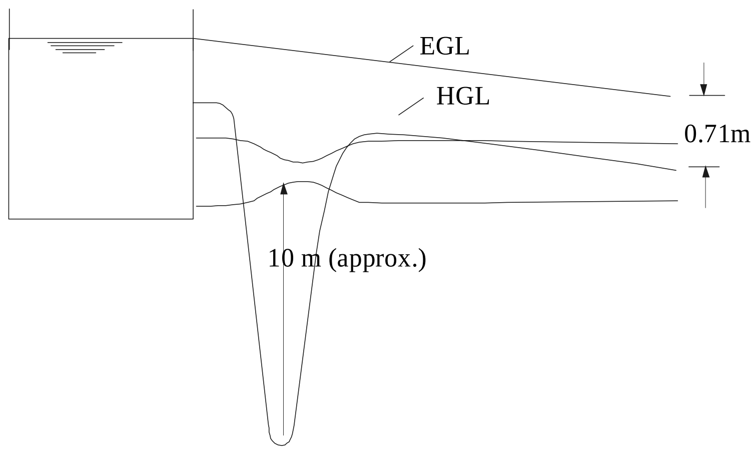
$$\begin{aligned} 40 &= (V_{12}^2/2g)(16+20) \\ V_{12}^2 &= (40/36)2g \; ; \; V_{12} = 8.46 \text{ ft/s} \\ Q &= V_{12}A_{12} = (8.46)(\pi/4)(1^2) = \underline{\underline{6.64 \text{ ft}^3/\text{s}}} \end{aligned}$$



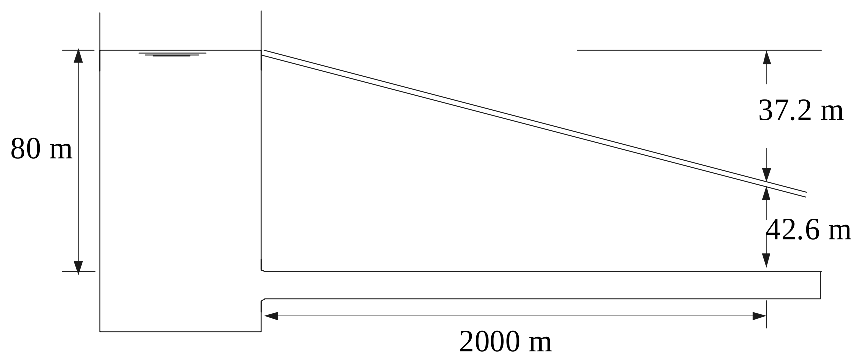
7.60



7.61



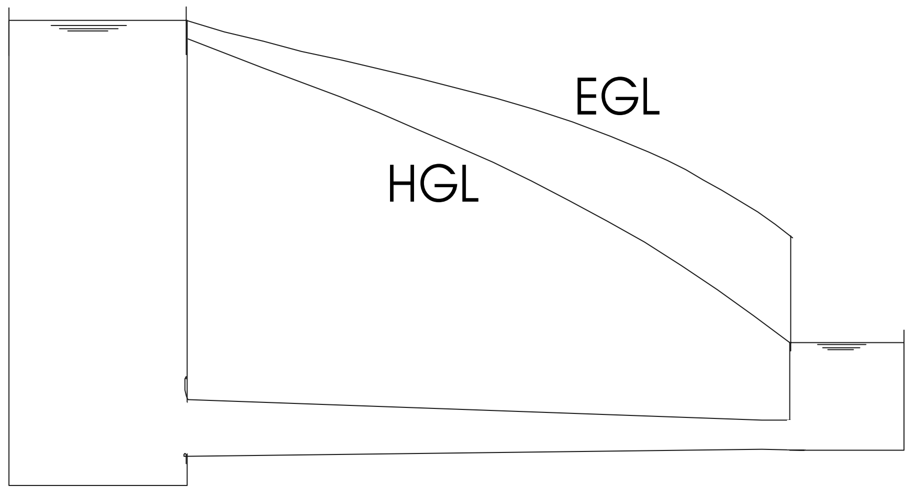
7.62



7.63 Because the E.G.L. slopes downward to the left, it is obvious that the flow is from right to left and that in the "black box" there could either be a turbine, an abrupt expansion or a partially closed valve. Circle $\underline{\underline{\text{b, c, d}}}$.

7.64 This is possible if the fluid is being accelerated to the left.

7.65



7.66 **a)** Solid line is EGL, dashed line is HGL

b) No; AB is smallest.

c) from B to C

d) p_{max} is at the bottom of the tank

e) p_{min} is at the bend C

f) A nozzle

g) above atmospheric pressure

h) abrupt expansion

7.67 **Information and assumptions**

provided in problem statement

Find

discharge of water in system

Solution

Write energy equation from upper to lower reservoir:

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 100 &= 0 + 0 + 070 + \sum h_L \\ \sum h_L &= 30 \text{ m} \end{aligned}$$

$$0.02 \times (200/0.3)(V_u^2/2g) + (0.02(100/0.15) + 1.0)V_d^2/2g = 30 \tag{1}$$

$$\text{but } V_u = Q/A_u = Q/((\pi/4) \times 0.3^2) \tag{2}$$

$$V_d = Q/A_d = Q/((\pi/4) \times 0.15^2) \tag{3}$$

Substituting Eq. (2) and Eq. (3) into (1) and solving for Q yields:

$$Q = \underline{\underline{0.110 \text{ m}^3/\text{s}}}$$

7.68 **Information and assumptions**

provided in problem statement

Find

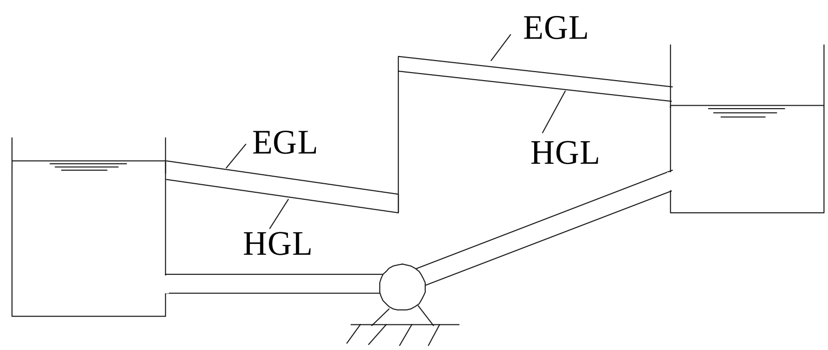
power supplied to water pump

Solution

$$
\begin{aligned}
V &= Q/A = 3.0/((\pi/4)\times(2/3)^2) = 8.59 \text{ ft/sec} \\
h_L &= 0.15\times 3,000\times(8.59)^2/((2/3)\times(2\times 32.2)) + (8.59)^2/(2\times 32.2) \\
&= 78.5 \text{ ft}
\end{aligned}
$$

Write the energy equation from water surface to water surface:

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
0 + 0 + 90 + h_p &= 0 + 0 + 140 + 78.5 \\
h_p &= 128.5 \text{ ft} \\
P &= Q\gamma h_p = 3.0\times 62.4\times 128.5 = 24,055 \text{ ft-lbf/s} \\
P &= \underline{\underline{43.7 \text{ hp}}}
\end{aligned}
$$



7.69 **Information and assumptions**

provided in problem statement

Find

discharge in pipe and pressure halfway between two reservoirs.

Solution

Write the energy equation from the water surface in A to the water surface in B:

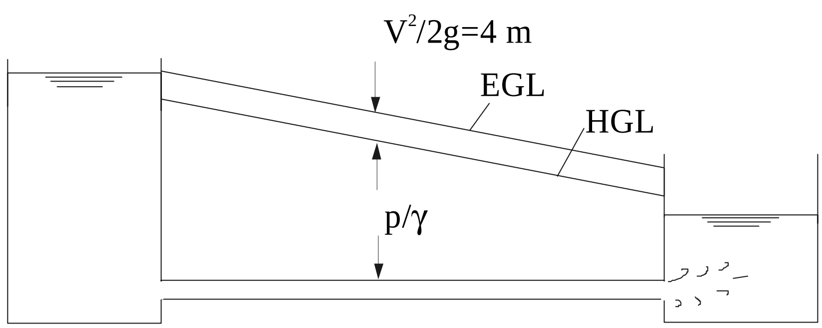
$$
\begin{aligned}
p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/2g + z_B + h_L \\
0 + 0 + H &= 0 + 0 + 0 + 0.01 \times (300/1)V_P^2/2g + V_p^2/2g \\
16 &= 4V_p^2/2g \\
V_p &= \sqrt{4 \times 2 \times 9.81} = 8.86 \text{ m}^3/\text{s} \\
Q &= VA = 8.86 \times (\pi/4) \times 1^2 = \underline{\underline{6.96 \text{ m}^3/\text{s}}}
\end{aligned}
$$

To determine p_p write the energy equation between the water surface in A and point P:

$$
\begin{aligned}
0 + 0 + H &= p_p/\gamma + V_p^2/2g - h + 0.01 \times (150/1)V_p^2/2g \\
16 &= p_p/\gamma - 2 + 2.5V_p^2/2g
\end{aligned}
$$

where $V_p^2/2g = 4$ m. Then

$$
p_p = 9.810(16 + 2 - 10) = \underline{\underline{78.5 \text{ kPa}}}
$$



7.70 **Information and assumptions**

provided in problem statement

Find

elevation in left reservoir.

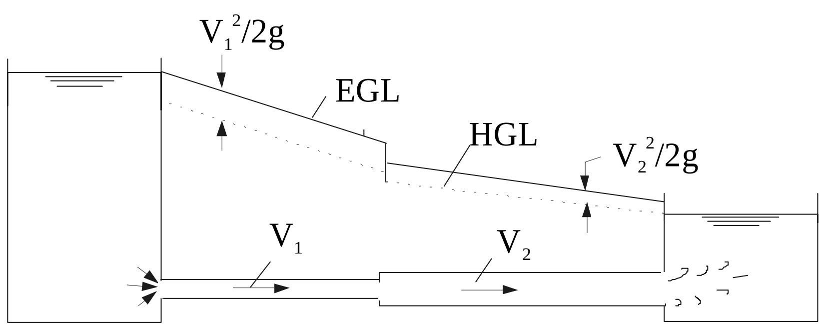
Solution

Write the energy equation from the left reservoir to the right reservoir:

$$\begin{aligned} p_L/\gamma + V_L^2/2g + z_L &= p_R/\gamma + V_R^2/2g + z_R + h_L \\ 0 + 0 + z_L &= 0 + 0 + 110 + 0.02(200/1.128)(V_1^2/2g) \\ &\quad +0.02(300/1.596)(V_2^2/2g) + (V_1 - V_2)^2/2g + V_2^2/2g \end{aligned}$$

when

$$\begin{aligned} V_1 &= Q/A_1 = 16/1 = 16 \text{ ft/s} \\ V_2 &= 8 \text{ ft/s} \\ z_L &= 110 + (0.02/2g)((200/1.238)(16^2) + (300/1.596)(8^2)) + ((16-8)^2/64.4) + 8^2/64.4 \\ &= 110 + 17.83 + 0.99 + 0.99 \\ z_L &= \underline{\underline{129.8 \text{ ft}}} \end{aligned}$$



7.71 **Information and assumptions**

provided in problem statement

Find

pump power

Solution

Write the energy equation from the lower reservoir surface to the upper reservoir surface:

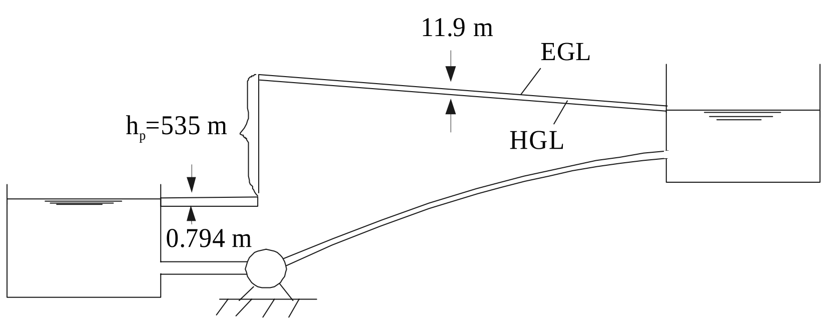
$$\begin{aligned} p_2/\gamma + V_2^2/2g + z_2 + h_p &= p_1/\gamma + V_1^2/2g + z_1 + h_L \\ 0 + 0 + 150 + h_p &= 0 + 0 + 250 + \sum 0.018(L/D)(V^2/2g) + V^2/2g \end{aligned}$$

where

$$\begin{aligned} V_1 &= Q/A_1 = 3/((\pi/4) \times 1^2) = 3.82 \text{ m/s} \\ V_1^2/2g &= 0.744 \text{ m} \\ V_2 &= Q/A_2 = 4V_1 = 15.28 \text{ m/s} \\ V_2^2/2g &= 11.9 \text{ m} \end{aligned}$$

Then

$$\begin{aligned} h_p &= 250 - 150 + 0.018[(100/1) \times 0.744 + (1,000/0.5) \times 11.9] + 11.9 = 541.6 \text{ m} \\ P &= Q\gamma h_p/\text{eff.} = 3 \times 9,810 \times 541.6/0.74 = \underline{\underline{21.54 \text{ MW}}} \end{aligned}$$



7.72 **Information and assumptions**

provided in problem statement

Find

water discharge in pipe and highest point in pipe.

Solution

First write energy equation from reservoir water surface to end of pipe:

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
0 + 0 + 200 &= 0 + V^2/2g + 185 + 0.02(200/0.30)V^2/2g \\
14.33V^2/2g &= 15 \\
V^2/2g &= 1.047 \\
V &= 4.53 \text{ m/s} \\
Q &= VA = 4.53 \times (\pi/4) \times 0.30^2 = \underline{\underline{0.320 \text{ m}^3/\text{s}}}
\end{aligned}$$

To solve for the pressure midway along pipe, write the energy equation to the midpoint:

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_m/\gamma + V_m^2/2g + z_m + h_L \\
0 + 0 + 200 &= p_m/\gamma + V_m^2/2g + 200 + 0.02(100/0.30)V^2/2g \\
p_m/\gamma &= -(V^2/2)(1 + 6.667) \\
&= (-1.047)(7.667) = -8.027 \text{ m} \\
p_m &= -8.027\gamma = -78,745 \text{ Pa } = \underline{\underline{-78.7 \text{ kPa}}}
\end{aligned}$$

7.73 **Information and assumptions**

provided in problem statement

Find

time required to fill tank to depth of 10 m.

Solution

Write the energy equation between the water surface in the river and water surface in the tank. Let point 1 be the river water surface.

$$p_1/\gamma + V_1^2/2g + z_1 + h_p = p_2/\gamma + V_2^2/2g + z_2 + h_L$$

but $p_1 = p_2 = 0$, $z_1 = 0$, $V_1 = 0$, $V_2 \simeq 0$. The energy equation reduces to

$$0 + 0 + 0 + h_p = 0 + 0 + (2 \text{ m} + h) + h_L$$

where h =depth of water in the tank

$$20 - (4)(10^4)Q^2 = h + 2 + V^2/2g + 10V^2/2g$$

where $V^2/2g$ is the head loss due to the abrupt expansion. Then

$$\begin{aligned}
18 &= (4)(10^4)Q^2 + 11(V^2/2g) + h \\
V &= Q/A \\
(11V^2)/2g &= (11/2g)(Q^2/A^2) = (1.46)(10^5)Q^2 \\
18 &= 1.86 \times 10^5 Q^2 + h \\
Q^2 &= (18 - h)/((1.86)(10^5)) \\
Q &= (18 - h)^{0.5}/431
\end{aligned}$$

But $Q = A_T dh/dt$ where A_T = tank area, so

$$\begin{aligned}
&\therefore \;\; dh/dt = (18 - h)^{0.5}/((431)(\pi/4)(5)^2) = (18 - h)^{0.5}/8,458 \\
dh/(18 - h)^{0.5} &= dt/8,458
\end{aligned}$$

Integrate:

$$-2(18 - h)^{0.5} = (t/8,458) + \text{ const.}$$

But $t = 0$ when $h = 0$ so const. $= -2(10)^{0.5}$. Then

$$t = (18^{0.5} - (18 - h)^{0.5})(16,916)$$

For $h = 10$ m

$$t = (18^{0.5} - 8^{0.5})(16,916) = 23,993 \text{ s } = \underline{\underline{6.65 \text{ hrs}}}$$

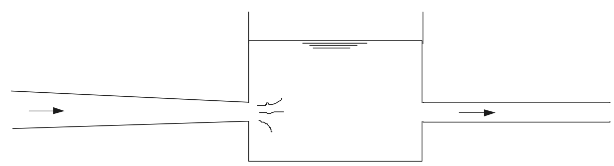
7.74 **a)** Flow is from A to E because EGL slopes downward in that direction.

b) Yes, at D, because EGL and HGL are coincident there.

c) Uniform diameter because $V^2/2g$ is constant (EGL and HGL uniformly spaced).

d) No, because EGL is always dropping (no energy added).

e)



1. **f)** Nothing else.

7.75 **Information and assumptions**

provided in problem statement

Find

discharge, HGL and EGL, locations of maximum and minimum pressure, values for maximum and minimum pressure

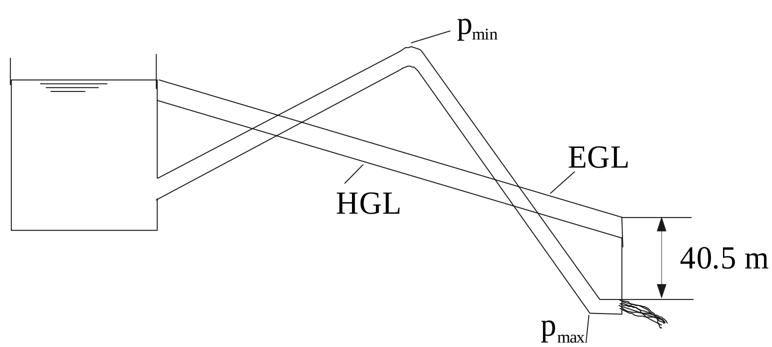
Solution

Write energy equation from reservoir water surface to jet:a)

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
0 + 0 + 100 &= 0 + V_2^2/2g + 30 + 0.014(L/D)(V_p^2/2g) \\
100 &= 0 + V_2^2/2g + 30 + 0.014(500/0.60)V_p^2/2g \\
V_2 A_2 &= V_p A_p \\
V_2 &= V_p A_p/A_L = 4V_p
\end{aligned}
$$

Then

$$
\begin{aligned}
V_p^2/2g(16 + 11.67) &= 70;\ V_p = 7.046 \text{ m/s};\ V_p^2/2g = 2.53 \text{ m} \\
V_p^2/2g &= 2.53 \text{ m} \\
Q &= V_p A_p = 7.045 \times (\pi/4) \times 0.60^2 = \underline{\underline{1.992 \text{ m}^3/\text{s}}}
\end{aligned}
$$



$$
\begin{aligned}
p_{\min} &: \quad 100 = p_{\min}/\gamma + V_p^2/2g + 100 + 0.014(100/0.60)V_p^2/2g \\
100 &= p_{\min}/\gamma + 100 + 3.33 \times 2.53 \\
p_{\min} &= \underline{\underline{-82.6 \text{ kPa, gage}}} \\
p_{\max}/\gamma &= 40.5 - 2.53 \text{ m} \\
p_{\max} &= \underline{\underline{372.5 \text{ kPa}}}
\end{aligned}
$$

7.76 **Information and assumptions**

Assume negligible head loss:

Find

power developed by windmill

Solution

$$p_1/\gamma + V_1^2/2g = p_2/\gamma + V_2^2/2g + h_t;\ h_t = V_1^2/2g - V_2^2/2g$$

where

$$\begin{aligned} V_2 &= V_1 A_1/A_2 = V_1(3/4.5)^2 = 0.444 V_1 \\ V_2^2/2g &= 0.197 V_1^2/2g \end{aligned}$$

Then

$$\begin{aligned} h_t &= 10^2/(2 \times 9.81)[1 - 0.197] = 4.09 \text{ m} \\ P &= Q\gamma h_t = 10(\pi/4) \times 3^2 \times 1.2 \times 9.81 \times 4.09 = \underline{\underline{3.40 \text{ kW}}} \end{aligned}$$

7.77 **Information and assumptions**

provided in problem statement

Find

power required

Solution

Write energy equation from upstream end to downstream end:

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
0 + 0 + 0 + h_p &= 0 + V_2^2/2g + 0 + 0.02V_T^2/2g \\
V_T A_T &= V_2 A_2; V_2 = V_T A_T / A_2 = V_T \times 0.4 \\
V_2^2/2g &= 0.16 V_T^2/2g \\
h_p &= V_T^2/2g(0.18) = (60^2/(2 \times 9.81))(0.18) \\
h_p &= 33.03 \text{ m} \\
P &= Q\gamma h_p = 60 \times 4 \times 1.2 \times 9.81 \times 33.03 = \underline{\underline{93.3 \text{ kW}}}
\end{aligned}
$$

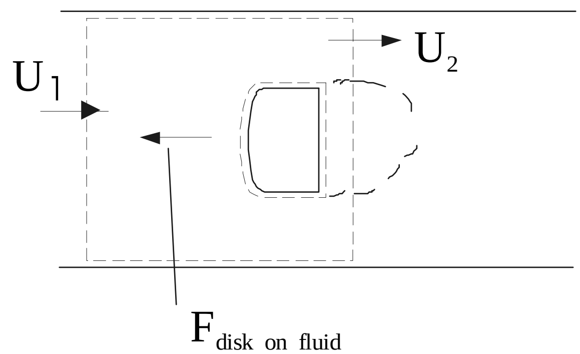
7.78 **Information and assumptions**

provided in problem statement

Find

Write energy equation from section (1) to section (2):

Solution



$$\begin{aligned} p_1 + \rho U_1^2/2 &= p_2 + \rho U_2^2/2 \\ p_1 - p_2 &= \rho U_2^2/2 - \rho U_1^2/2 \end{aligned}$$

but

$$\begin{aligned} U_1 A_1 &= U_2(\pi/4)(D^2 - d^2) \\ U_2 &= U_1 D^2/(D^2 - d^2) \end{aligned} \quad (1)$$

Then

$$p_1 - p_2 = (\rho/2)U_1^2[(D^4/(D^2 - d^2)^2 - 1] \quad (2)$$

Now write the momentum equation for the C.V.

$$\begin{aligned} \sum F_x &= \dot{m}_o U_o - \dot{m}_i U_i = \rho Q(U_{2x} - U_{1x}) \\ p_1 A - p_2 A + F_{\text{disk on fluid}} &= \rho Q(U_2 - U_1) \\ F_{\text{fluid on disk}} &= F_d = \rho Q(U_1 - U_2) + (p_1 - p_2)A \end{aligned}$$

Eliminate $p_1 - p_2$ by Eq. (2), and U_2 by Eq. (1):

$$\begin{aligned} F_d &= \rho U A(U_1 - U_1 D^2/(D^2 - d^2)) + (\rho U^2/2)[(D^4/(D^2 - d^2)^2 - 1]A \\ &\underline{\underline{F_d = \rho U^2 \pi D^2/8[1/(D^2/d^2 - 1)^2]}} \end{aligned}$$

When $U = 10$ m/s, $D = 5$ cm, $d = 4$ cm and $\rho = 1.2$ kg/m^3

$$F_d = (1.2 \times 10^2 \pi \times (0.05)^2/8)[1/((0.05/0.04)^2 - 1)^2] = \underline{\underline{0.372 \text{ N}}}$$

7.79 **Information and assumptions**

provided in problem statement

Find

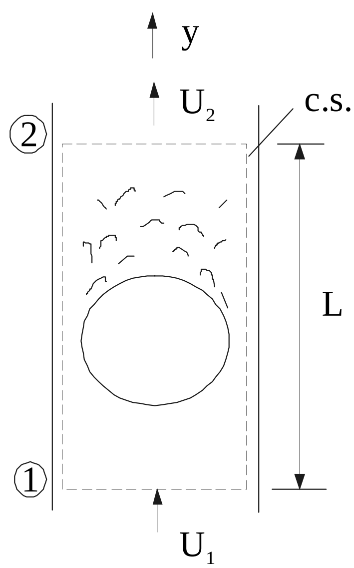
specific gravity of sphere

Solution

Let the control volume include the sphere and fluid in a given length of tube. Also let the control volume move with the sphere; thus, steady flow conditions will prevail. First write the momentum equation for this control volume. Neglect viscous forces in the solution:

$$\sum F_y = \dot{m}(U_2 - U_1)$$

Because $U_1 = U_2$ this equation reduces to



$$\begin{aligned} p_1A_1 - p_2A_2 - W_{\text{water}} - W_{\text{sphere}} &= 0 \\ (p_1 - p_2)A &= \gamma_f(LA_c - V_s) - \gamma_s V_s = 0 \end{aligned}$$

where A_c =cross-sectional area of cylinder, γ_f =specific wgt. of fluid, γ_s =specific wgt. of sphere and V_s =sphere volume.

$$p_1 - p_2 = \gamma_f(LA_c - V_s)/A_C + \gamma_s V_s/A_C \tag{1}$$

Now write the energy equation from section (1) to section (2)

$$p_1/\gamma + U_1^2/2g + z_1 = p_2/\gamma + U_2^2/2g + z_2 + h_L$$

But $U_1^2/2g = U_2^2/2g$ and $z_2 - z_1 = L$. Also $h_L = (U_a - U_c)^2/2g$. where V_a =velocity in annulus between the sphere and cylinder and V_C =velocity in unobstructed cylinder. Then

$$p_1 - p_2 = \gamma_f(L + ((U_a - U_C)^2/2g)) \tag{2}$$

Now eliminate $p_1 - p_2$ between Eqs. (1) and (2) yielding:

$$\begin{aligned}
\gamma_s V_s/A_C + (\gamma_f(LA_C - V_s)/A_C) &= \gamma_f(L + ((U_a - U_C)^2/2g)) \\
\gamma_s/\gamma_f &= ((A_C/V_s)(U_a - U_C)^2/2g) + 1 \\
U_C &= 0.50 \text{ ft/s} \\
U_a A_a &= U_C A_C \\
U_a &= U_C(A_C/A_a) \\
&= 0.5(1.05^2/(1.05^2 - 1) \\
U_a &= 5.38 \text{ ft/s}
\end{aligned}$$

$$\begin{aligned}
A_C/V_s &= (\pi D_C^2/4)/(\pi D_s^3/6) \\
&= (3/2)(D_C^2/D_S^3) \\
&= (3/2)(1.05^2/1^3) \\
&= 1.654 \text{ ft}^{-1}
\end{aligned}$$

Then

$$\begin{aligned}
\gamma_s/\gamma_f &= ((1.654 \text{ ft}^{-1})(5.38 - 0.50)^2 \text{ ft}^2/\text{s}^2/(64.4 \text{ ft/s}^2)) + 1 \\
\gamma_s/\gamma_f &= S = \underline{\underline{1.612}}
\end{aligned}$$

Chapter Eight

8.1 **a**

$$Q = (2/3)CL\sqrt{2g}H^{3/2}$$
$$[Q] = L^3/T = L(L/T^2)^{1/2}L^{3/2}$$
$$L^3/T = L^3/T \quad \underline{\underline{\text{homogeneous}}}$$

b

$$V = (1.49/n)R^{2/3}S^{1/2}$$
$$[V] = L/T = L^{-1/6}L^{2/3} \quad \underline{\underline{\text{not homogeneous}}}$$

c

$$h_f = f(L/D)V^2/2g$$
$$[h_f] = L = (L/L)(L/T)^2/(L/T^2)\underline{\underline{\text{homogeneous}}}$$

d

$$D = 0.074R_e^{-0.2}Bx\rho V^2/2$$
$$[D] = ML/T^2 = L \times L \times (M/L^3)(L/T)^2\underline{\underline{\text{homogeneous}}}$$

8.2 (a) $[T] = ML/T^2 \times L = \underline{\underline{ML^2/T^2}}$

(b) $[\rho V^2/2] = (M/L^3)(L/T)^2 = \underline{\underline{M/LT^2}}$

(c) $[\sqrt{\tau/\rho}] = \sqrt{(ML/T^2)/L^2)/(M/L^3)} = \underline{\underline{L/T}}$

(d) $[Q/ND^3] = (L^3/T)/(T^{-1}L^3) = 1 \rightarrow \underline{\underline{\text{Dimensionless}}}$

8.3 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the step-by-step method

Δh	L	$\frac{\Delta h}{d}$	0	$\frac{\Delta h}{d}$	0	$\frac{\Delta h}{d}$	0
t	T	t	T	t	T		
ρ	$\frac{M}{L^3}$	ρd^3	M				
D	L	$\frac{D}{d}$	0	$\frac{D}{d}$	0	$\frac{D}{d}$	0
d	L						
γ	$\frac{M}{L^2T^2}$	γd^2	$\frac{M}{T^2}$	$\frac{\gamma}{\rho d}$	$\frac{1}{T^2}$	$\frac{\gamma t^2}{\rho d}$	0
h	L	$\frac{h}{d}$	0	$\frac{h}{d}$	0	$\frac{h}{d}$	0

In the first step, length is taken out with d. In the second step, mass is taken out with ρd^3. In the third step, time is taken out with t. The functional relationship is

$$\frac{\Delta h}{d} = f(\frac{D}{d}, \frac{\gamma t^2}{\rho d}, \frac{h}{d})$$

This can also be written as

$$\frac{\Delta h}{d} = f(\frac{d}{D}, \frac{gt^2}{d}, \frac{h}{d})$$

8.4 **Information and Assumptions**

provided in problem statement

Find

nondimensional function form for wave celerity

Solution

Using the exponent method

$$\begin{aligned}
V &= f(h,\sigma,\gamma,g) \\
\text{where } [V] &= L/T, [h]=L, [\sigma]=M/T^2, [\gamma]=M/(L^2T^2), [g]=L/T^2 \\
[V] &= [h^a\sigma^b\gamma^c g^d] \\
L/T &= (L^a)(M^b/T^{2b})(M^c/(L^{2c}T^{2c})(L^d/T^{2d})
\end{aligned}$$

$$\begin{aligned}
L &: 1 = a - 2c + d \\
M &: 0 = b + c \\
T &: 1 = 2b + 2c + 2d
\end{aligned}$$

Determine the exponents b, c & d in terms of a

$$\begin{aligned}
0 - 2c + d &= 1 - a \\
b + c + 0 &= 0 \\
2b + 2c + 2d &= 1
\end{aligned}$$

Solution yields: $b = -c,\ d = 1/2$

$$\begin{aligned}
-2c + 1/2 &= 1 - a \Longrightarrow -2c = 1/2 - a \Longrightarrow c = -1/4 + a/2 \\
b &= 1/4 - a/2
\end{aligned}$$

Thus

$$\begin{aligned}
V &= h^a\sigma^{(1/4-a/2)}\gamma^{(-1/4+a/2)}g^{1/2} \\
&= g^{1/2}\sigma^{1/4}/\gamma^{1/4})(h\gamma^{1/2}/\sigma^{1/2})^a
\end{aligned}$$

Which can also be written as

$$V^4\gamma/(g^2\sigma) = f(h^2\gamma/\sigma)$$

Alternate forms:

$$\begin{aligned}
(V^4\gamma/(g^2\sigma))(\sigma/h^2\gamma) &= f(h^2\gamma/\sigma) \\
V^2/(gh)^2 &= f(h^2\gamma/\sigma)
\end{aligned}$$

or

$$V/\sqrt{gh} = f(h^2\gamma/\sigma)$$

8.5 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the step-by-step method

$$\begin{array}{llllll} h & L & \frac{h}{d} & 0 & \frac{h}{d} & 0 \\ d & L & & & & \\ \sigma & \frac{M}{T^2} & \sigma & \frac{M}{T^2} & \frac{\sigma}{\gamma d^2} & 0 \\ \gamma & \frac{M}{L^2T^2} & \gamma d^2 & \frac{M}{T^2} & & \end{array}$$

In the first step, d was used to remove length and in the second γd^2 was used to remove both length and time. The final functional form is

$$\frac{h}{d} = f(\frac{\sigma}{\gamma d^2})$$

8.6 **Information and Assumptions**

provided in problem statement

Find

the dimensionless relationship

Solution

Using the step-by-step method

$$\begin{array}{llllllll}
F_D & \frac{ML}{T^2} & \frac{F_D}{d} & \frac{M}{T^2} & \frac{F_D}{\mu d^2} & \frac{1}{T} & \frac{F_D}{\mu V d} & 0 \\
V & \frac{L}{T} & \frac{V}{d} & \frac{1}{T} & \frac{V}{d} & \frac{1}{T} & & \\
\mu & \frac{M}{LT} & \mu d & \frac{M}{T} & & & & \\
d & L & & & & & &
\end{array}$$

In the first step, length is removed with d. In the second, mass is removed with μd and in the third time is removed with V/d. Finally

$$\frac{F_D}{\mu V d} = C$$

8.7 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the step-by-step method

$$
\begin{array}{cccccccc}
F_D & \frac{ML}{T^2} & \frac{F_D}{D} & \frac{M}{T^2} & \frac{F_D}{\rho D^4} & \frac{1}{T^2} & \frac{F_D}{\rho V^2 D^2} & 0 \\
D & L & & & & & & \\
\rho & \frac{M}{L^3} & \rho D^3 & M & & & & \\
\mu & \frac{M}{LT} & \mu D & \frac{M}{T} & \frac{\mu}{\rho D^2} & \frac{1}{T} & \frac{\mu}{\rho V D} & 0 \\
V & \frac{L}{D} & \frac{V}{D} & \frac{1}{T} & \frac{V}{D} & \frac{1}{T} & & \\
k & L & \frac{k}{D} & 0 & \frac{k}{D} & 0 & \frac{k}{D} & 0
\end{array}
$$

In the first step, length is removed with D. In the second step, mass is removed with ρD^3 and in the final step time removed with V/D. The final functional form is

$$\frac{F_D}{\rho V^2 D^2} = f(\frac{\rho V D}{\mu}, \frac{k}{D})$$

Other forms are possible.

8.8 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the step-by-step method

$$
\begin{array}{llllllll}
F & \frac{ML}{T^2} & \frac{F}{D} & \frac{M}{T^2} & \frac{F}{\rho D^4} & \frac{1}{T^2} & \frac{F}{\rho V^2 D^2} & 0 \\
D & L & & & & & & \\
V & \frac{L}{T} & \frac{V}{D} & \frac{1}{T} & \frac{V}{D} & \frac{1}{T} & & \\
\rho & \frac{M}{L^3} & \rho D^3 & M & & & & \\
\mu & \frac{M}{LT} & \mu D & \frac{M}{T} & \frac{\mu}{\rho D^2} & \frac{1}{T} & \frac{\mu}{\rho V D} & 0 \\
k_s & L & \frac{k_s}{D} & 0 & \frac{k_s}{D} & 0 & & \\
\omega & \frac{1}{T} & \omega & \frac{1}{T} & \omega & \frac{1}{T} & \frac{\omega D}{V} & 0
\end{array}
$$

Length is removed in the first step with D, mass in the second step with ρD^3 and time in the third step with V/D. The functional form is

$$\frac{F}{\rho V^2 D^2} = f(\frac{\rho V D}{\mu}, \frac{k_s}{D}, \frac{\omega D}{V})$$

There are other possible forms.

8.9 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the step-by-step method

F_D	$\frac{ML}{T^2}$	$\frac{F_D}{B}$	$\frac{M}{T^2}$	$\frac{F_D}{\rho B^4}$	$\frac{1}{T^2}$	$\frac{F_D}{\rho V^2 B^2}$	0
V	$\frac{L}{T}$	$\frac{V}{B}$	$\frac{1}{T}$	$\frac{V}{B}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρB^3	M				
B	L						
μ	$\frac{M}{LT}$	μB	$\frac{M}{T}$	$\frac{\mu}{\rho B^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho VB}$	0
u'	$\frac{L}{T}$	$\frac{u'}{B}$	$\frac{1}{T}$	$\frac{u'}{B}$	$\frac{1}{T}$	$\frac{u'}{V}$	0
L_x	L	$\frac{L_x}{B}$	0	$\frac{L_x}{B}$	0	$\frac{L_x}{B}$	0

Length is removed in first step with B, mass is removed in second with ρB^3 and time is removed in the third with V/B. The function form is

$$\frac{F_D}{\rho V^2 B^2} = f\left(\frac{\mu}{\rho VB}, \frac{u'}{V}, \frac{L_x}{B}\right)$$

Other forms are possible.

8.10 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the step-by-step method

$$\begin{array}{llllllll}
\frac{\Delta p}{\Delta \ell} & \frac{M}{L^2T^2} & \frac{\Delta p}{\Delta \ell}D^2 & \frac{M}{T^2} & \frac{\Delta p}{\Delta \ell}\frac{D}{\mu} & \frac{1}{T} & \frac{\Delta p}{\Delta \ell}\frac{D^2}{\mu V} & 0 \\
\mu & \frac{M}{LT} & \mu D & \frac{M}{T} & & & & \\
V & \frac{L}{T} & \frac{V}{D} & \frac{1}{T} & \frac{V}{D} & \frac{1}{T} & & \\
D & L & & & & & &
\end{array}$$

Length is removed in the first step with D, mass is removed in the second with μD and time is removed in the third with V/D. Finally we have

$$\frac{\Delta p}{\Delta \ell}\frac{D^2}{\mu V} = C$$

or

$$\frac{\Delta p}{\Delta \ell} = C\frac{\mu V}{D^2}$$

8.11 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the step-by-step method

$$\begin{array}{llllllll}
\Delta p & \frac{M}{LT^2} & \Delta pD & \frac{M}{T^2} & \frac{\Delta p}{\rho D^2} & \frac{1}{T^2} & \frac{\Delta p}{n\rho D^2} & 0 \\
D & L & & & & & & \\
n & \frac{1}{T} & n & \frac{1}{T} & n & \frac{1}{T} & & \\
Q & \frac{L^3}{T} & \frac{Q}{D^3} & \frac{1}{T} & \frac{Q}{D^3} & \frac{1}{T} & \frac{Q}{nD^3} & 0 \\
\rho & \frac{M}{L^3} & \rho D^3 & M & & & &
\end{array}$$

In the first step, length is removed with D. In the second step, mass is removed with ρD^3 and time is removed in the third step with n. The functional form is

$$\frac{\Delta p}{n\rho D^2} = f(\frac{Q}{nD^3})$$

8.12 **Information and Assumptions**

provided in problem statement

Find

the π−groups

Solution

Using the step-by-step method

$$\begin{array}{llllllll}
f & \frac{1}{T} & f & \frac{1}{T} & f & \frac{1}{T} & f\sqrt{\frac{\rho R^2}{p}} & 0 \\
p & \frac{M}{LT^2} & \frac{p}{\rho} & \frac{L^2}{T^2} & \frac{p}{\rho R^2} & \frac{1}{T^2} & & \\
R & L & R & L & & & & \\
\rho & \frac{M}{L^3} & & & & & & \\
k & 0 & k & 0 & k & 0 & k & 0
\end{array}$$

In the first step, mass is removed with ρ. In the second step, length is removed with R and, finally, in third step time is removed with $p/\rho R^2$. The final functional form is

$$fR\sqrt{\frac{\rho}{p}} = f(k)$$

8.13 **Information and Assumptions**

provided in problem statement

Find

the nondimensional form of equation

Solution

$$\begin{aligned} F &= \lambda^a \rho^b D^c c^d \\ ML/T^2 &= L^a (M/L^3)^b L^c (L/T)^d \\ &= L^{a-3b+c+d} M^b T^{-d} \end{aligned}$$

Equating powers of $M,\ L$ and T, we have

$$\begin{aligned} T : d &= 2 \\ M : b &= 1 \\ L : 1 &= a - 3 + c + c \\ 1 &= a - 3 + c + 2 \\ a + c &= 2 \\ a &= 2 - c \end{aligned}$$

Therefore,

$$F = \lambda^{(2-c)} \rho D^c c^2$$

$$\underline{\underline{F/(\rho c^2 \lambda^2) = f(D/\lambda)}}$$

Another valid answer would be

$$\underline{\underline{F/(\rho c^2 D^2) = f(D/\lambda)}}$$

8.14 **Information and Assumptions**

provided in problem statement

Find

an expression for V

Solution

Using the step-by-step method

$$\begin{array}{llllllll} V & \frac{L}{T} & V & \frac{L}{T} & \frac{V}{\ell} & \frac{1}{T} & \frac{V\ell^{1/2}\rho^{1/2}}{\sigma^{1/2}} & 0 \\ \ell & L & \ell & L & & & & \\ \rho & \frac{M}{L^3} & \frac{\rho}{\sigma} & \frac{T^2}{L^3} & \frac{\rho}{\sigma}\ell^3 & T^2 & & \\ \sigma & \frac{M}{T^2} & & & & & & \end{array}$$

In the first step, mass is removed with σ. In the second step, length is removed with ℓ and in the third step, time is removed with $\rho\ell^3/\sigma$. The functional form is

$$V\sqrt{\frac{\ell\rho}{\sigma}} = C$$

or

$$V = C\sqrt{\frac{\sigma}{\rho\ell}}$$

8.15 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the step-by-step method

$$\begin{array}{llllllll}
T & \frac{ML^2}{T^2} & \frac{T}{D^2} & \frac{M}{T^2} & \frac{T}{\mu D^3} & \frac{1}{T} & \frac{T}{\mu D^3\omega} & 0 \\
\mu & \frac{M}{LT} & \mu D & \frac{M}{T} & & & & \\
\omega & \frac{1}{T} & \omega & \frac{1}{T} & \omega & \frac{1}{T} & & \\
S & L & \frac{S}{D} & 0 & \frac{S}{D} & 0 & \frac{S}{D} & 0 \\
D & L & & & & & &
\end{array}$$

In the first step, length is removed with D. In the second step, mass is removed with μD and in the last step, time is removed with ω. The final functional form is

$$\frac{T}{\mu D^3\omega} = f(\frac{S}{D})$$

8.16 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the step-by-step method

$$
\begin{array}{llllllll}
h & L & \frac{h}{d} & 0 & \frac{h}{d} & 0 & \frac{h}{d} & 0 \\
t & T & t & T & t & T & & \\
\sigma & \frac{M}{T^2} & \sigma & \frac{M}{T^2} & \frac{\sigma}{\rho d^3} & \frac{1}{T^2} & \frac{\sigma t^2}{\rho d^3} & 0 \\
\rho & \frac{M}{L^3} & \rho d^3 & M & & & & \\
\gamma & \frac{M}{L^2T^2} & \gamma d^2 & \frac{M}{T^2} & \frac{\gamma}{\rho d} & \frac{1}{T^2} & \frac{\gamma t^2}{\rho d} & 0 \\
\mu & \frac{M}{LT} & \mu d & \frac{M}{T} & \frac{\mu}{\rho d^2} & \frac{1}{T} & \frac{\mu t}{\rho d^2} & 0 \\
d & L & & & & & &
\end{array}
$$

In the first step, length is removed with d. In the second step, mass is removed with ρd^3 and in the final step, time is removed with t. The final functional form is

$$\frac{h}{d} = f(\frac{\sigma t^2}{\rho d^3}, \frac{\gamma t^2}{\rho d}, \frac{\mu t}{\rho d^2})$$

8.17 First, establish the dimensions of the variables:

$$
\begin{aligned}
[\Delta p] &= F/L^2 = (ML/T^2)/L^2 = M/(T^2L) \\
[\rho] &= M/L^3; [\mu] = FT/L^2 = (ML/T^2)(T)/L^2 = M/LT \\
[V] &= L/T; [E_V] = F/L^2 - (ML/T^2)/L^2 = M/LT^2 \\
[\sigma] &= F/L = (ML/T^2)/L = M/T^2 \\
[\Delta\gamma] &= F/L^3 = (ML/T^2)/L^3 = M/L^2T^2
\end{aligned}
$$

Assume

$$
f(\rho, L, V, \mu, E_v, \sigma, \Delta\gamma) = \rho^a L^b V^c \mu^d E_V^e \sigma^f \Delta\gamma^g
$$
$$
\therefore [\Delta p] = M/T^2L = (M/L^3)^a L^b (L/T)^c (M/LT)^d (M/LT^2)^e (M/T^2)^f (M/L^2T^2)^g
$$

Equating powers of $M,\ L$ and T, we have

$$
\begin{aligned}
L &: 3a + b + c - d - e - 2g = -1 \\
M &: a + d + e + f + g = 1 \\
T &: -c - d - 2e - 2f - 2g = -2 \\
-3a + b + c &= d + e + 2g - 1 \\
a &= 1 - d - e - f - g \\
c &= 2 - d - 2e - 2f - 2g
\end{aligned}
$$

Solving the above three equations for b yields

$$
b = -d - f + g
$$
$$
\Delta p = \rho^{1-d-e-f-g} L^{-d-f+g} V^{2-d-2e-2f-2g} \mu^d E_V^e \sigma^f \Delta\gamma^g
$$
$$
\Delta p = \rho V^2 (\mu/\rho LV)^d (E_v/\rho V^2)^e (\sigma/\rho LV^2)^f (\Delta\gamma L/\rho V^2)^g
$$
$$
\text{or } \Delta p/\rho V^2 = (\mu/\rho LV)^d (E_v/\rho V^2)^e (\sigma/\rho LV^2)^f (\Delta\gamma L/\rho V^2)^g
$$

Arbitrary values can be given to $d,\ e,\ f$ and g and the combinations of variables will still be dimensionless. Therefore, let $d = -1, e = -1/2, f = -1$ and $g = -1$ to yield

$$
\underline{\underline{\Delta p/\rho V^2 = f(\rho LV/\mu), V/\sqrt{E_v/\rho}, \rho LV^2/\sigma, \rho V^2/\Delta\gamma)}}
$$

8.18 **Information and Assumptions**

provided in problem statement

Find

dimensional analysis for erosion rate, e

Solution

Use exponent method

$$e = f(Br, \sigma, E, V, d, \dot{M}_p, D)$$

where

$$\begin{aligned}
[e] &= M/(L^2T) \; ; \; [Br] = \text{ dimensionless} \\
[E] &= M/(LT^2) \; ; \; [\sigma] = M/(LT^2) \\
[V] &= L/T \; ; \; [d] = L \; ; \; [\dot{M}_p] = M/T \; ; \; [D] = L \\
\therefore [e] &= [E^\alpha \sigma^\beta V^\gamma d^\delta \dot{M}_p^\varepsilon D^\lambda] \\
M(L^2T) &= (M/(LT^2))^\alpha (M/(LT^2))^\beta (L/T)^\gamma L^\delta (M/T)^\varepsilon L^\lambda
\end{aligned}$$

$$\begin{aligned}
M &: \; 1 = \alpha + \beta + \varepsilon \\
L &: \; 2 = \alpha + \beta - \gamma - \delta - \lambda \\
T &: \; 1 = 2\alpha + 2\beta + \gamma + \varepsilon
\end{aligned}$$

Use α, γ and ε as unknowns

$$\alpha + 0 + \varepsilon = 1 - \beta \tag{5}$$

$$\alpha - \gamma + 0 = 2 - \beta + \delta + \lambda \tag{6}$$

$$2\alpha + \gamma + \varepsilon = 1 - 2\beta \tag{7}$$

$$\begin{aligned}
(1) &: \; \alpha + \varepsilon = 1 - \beta \\
(2) + (3) &: \; 3\alpha + \varepsilon = 3 - 3\beta + \delta + \lambda \\
(2) + (3) - (1) &: \; 2\alpha = 2 - 2\beta + \delta + \lambda
\end{aligned}$$

$$\begin{aligned}
\alpha &= 1 - \beta + (\delta + \lambda)/2 \\
\varepsilon &= -\alpha + 1 - \beta = -1 + \beta - ((\delta + \lambda)/2) + 1 - \beta = -(\delta + \lambda)/2 \\
&= \alpha - 2 + \beta - \delta - \lambda \\
&= 1 - \beta + ((\delta + \lambda)/2) - 2 + \beta - (\delta + \lambda) = -1 - ((\delta + \lambda)/2) \\
e &= f(E^{(1-\beta+((\delta+\lambda)/2)} \alpha^\beta V^{-1-((\delta+\lambda)/2} d^\delta \dot{M}_p^{-((\delta+\lambda)/2)} D^\lambda, \text{ Br}
\end{aligned}$$

or

$$\underline{\underline{eV/E = f(\sigma/E, Ed^2/(V\dot{M}_p), ED^2/(\dot{M}_p V),\ Br)}}$$

Alternate form:

$$\underline{\underline{eV/E = f(\sigma/E, Ed^2/V\dot{M}_p, d/D, Br)}}$$

8.19 **Information and Assumptions**

provided in problem statement

Find

dimensionless relationship for pressure change

Solution

Using the step-by-step method

Δp	$\frac{M}{LT^2}$	$\Delta p d$	$\frac{M}{T^2}$	$\frac{\Delta p}{\rho d^2}$	$\frac{1}{T^2}$	$\frac{\Delta p d^4}{\rho Q^2}$	0
Q	$\frac{L^3}{T}$	$\frac{Q}{d^3}$	$\frac{1}{T}$	$\frac{Q}{d^3}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρd^3	M				
μ	$\frac{M}{LT}$	μd	$\frac{M}{T}$	$\frac{\mu}{\rho d^2}$	$\frac{1}{T}$	$\frac{\mu d}{\rho Q}$	0
D	L	$\frac{D}{d}$	0	$\frac{D}{d}$	0	$\frac{D}{d}$	0
d	L						

Length is removed with d in the first step, mass with ρd^3 in the second step and time with Q/d^3 in the third step. The final form is

$$\frac{\Delta p d^4}{\rho Q^2} = f(\frac{\mu d}{\rho Q}, \frac{D}{d})$$

8.20 The viscous forces are relatively small.

8.21 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the exponent method

$$V^a = \rho_f^b \rho_p^c \mu^d D^e g^f$$

Writing out the dimensions

$$\left(\frac{L}{T}\right)^a = \left(\frac{M}{L^3}\right)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT}\right)^d (L)^e \left(\frac{L}{T^2}\right)^f$$

Setting up the equations for dimensional homogeneity

$$\begin{array}{ll} L: & a = -3b - 3c - d + e + f \\ M: & 0 = b + c + d \\ T: & a = d + 2f \end{array}$$

Substituting the equation for T into the one for L gives

$$\begin{aligned} 0 &= -3b - 3c - 2d + e - f \\ 0 &= b + c + d \end{aligned}$$

Solving for e from the first equation and c from the second equation

$$\begin{aligned} e &= 3b + 3c + 2d + f \\ c &= -d - b \end{aligned}$$

and the equation for e becomes

$$e = -d + f$$

Substituting into the original equation

$$V^{d+2f} = \rho_f^b \rho_p^{-d-b} \mu^d D^{-d+f} g^f$$

Collecting terms

$$\left(\frac{V\rho_p D}{\mu}\right)^d = \left(\frac{Dg}{V^2}\right)^f \left(\frac{\rho_f}{\rho_p}\right)^b$$

The functional equation can be written as

$$\frac{V}{\sqrt{gD}} = f(\frac{V\rho_p D}{\mu}, \frac{\rho_f}{\rho_p})$$

8.22 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

The functional relationship is

$$V = f(\rho_l, \mu_l, D, \sigma, g)$$

Using the step-by-step method

V	$\frac{L}{T}$	$\frac{V}{D}$	$\frac{1}{T}$	$\frac{V}{D}$	$\frac{1}{T}$	$\frac{V}{\sqrt{gD}}$	0
ρ_l	$\frac{M}{L^3}$	$\rho_l D^3$	M				
μ_l	$\frac{M}{LT}$	$\mu_l D$	$\frac{M}{T}$	$\frac{\mu_l}{\rho_l D^2}$	$\frac{1}{T}$	$\frac{\mu_l}{\rho_l D^{3/2} g^{1/2}}$	0
D	L						
σ	$\frac{M}{T^2}$	σ	$\frac{M}{T^2}$	$\frac{\sigma}{\rho_l D^3}$	$\frac{1}{T^2}$	$\frac{\sigma}{\rho_l D^2 g}$	0
g	$\frac{L}{T^2}$	$\frac{g}{D}$	$\frac{1}{T^2}$	$\frac{g}{D}$	$\frac{1}{T^2}$		

In the first step, D was used to remove the length dimension. In the second step, $\rho_l D^3$ was used to remove the mass dimension and finally, in the third step, $\sqrt{g/D}$ was used to remove the time dimension. The final functional form can be expressed as

$$\frac{V}{\sqrt{gD}} = f(\frac{\mu_l^2}{\rho_l^2 D^3 g}, \frac{\sigma}{\rho_l D^2 g})$$

8.23 **Information and Assumptions**

provided in problem statement

Find

the π−group

Solution

The functional relationship is

$$\dot{m} = f(D, \mu, \Delta p, \rho)$$

Using the exponent method, we have

$$\dot{m}^a = D^b \mu^c \Delta p^d \rho^e$$

Writing out the dimensional equation

$$\left(\frac{M}{T}\right)^a = L^b \left(\frac{M}{LT}\right)^c \left(\frac{M}{LT^2}\right)^d \left(\frac{M}{L^3}\right)^e$$

and the equations for the dimensions are

$$\begin{aligned} L: &\quad 0 = b - c - d - 3e \\ M: &\quad a = c + d + e \\ T: &\quad a = c + 2d \end{aligned}$$

Substituting the equation for time into the equation for mass yields two equations

$$\begin{gathered} 0 = b - c - d - 3e \\ 0 = -d + e \quad \text{or} \quad d = e \end{gathered}$$

and the first equation becomes

$$0 = b - c - 4d \quad \text{or} \quad b = c + 4d$$

Substituting back into the original equation

$$\dot{m}^{c+2d} = D^{c+4d} \mu^c \Delta p^d \rho^d$$

Collecting like powers gives

$$\left(\frac{\dot{m}^2}{D^4\rho\Delta p}\right)^d = \left(\frac{\mu D}{\dot{m}}\right)^c$$

A functional relationship is

$$\frac{\dot{m}}{\sqrt{\rho\Delta p}D^2} = f(\frac{\mu D}{\dot{m}})$$

The functions can be combined to form

$$\frac{\dot{m}}{\sqrt{\rho\Delta p}D^2} = f(\frac{\mu}{\sqrt{\rho\Delta p}D})$$

8.24 The significant dimensionless numbers would be Reynolds number and Froude number because of the viscous drag forces on the body and the waves that would be produced by the proximity of the body to the water surface

8.25 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

The functional relationship is

$$\Delta p = f(D, L, \alpha, \mu, \rho)$$

Use the step-by-method

Δp	$\frac{M}{LT^2}$	$\Delta p \Delta s$	$\frac{M}{T^2}$	$\frac{\Delta p}{\rho \Delta s^2}$	$\frac{1}{T^2}$	$\frac{\Delta p \rho \Delta s^2}{\mu^2}$	0
D	L	$\frac{D}{\Delta s}$	0	$\frac{D}{\Delta s}$	0	$\frac{D}{\Delta s}$	0
Δs	L						
α	0	α	0	α	0	α	0
μ	$\frac{M}{LT}$	$\mu \Delta s$	$\frac{M}{T}$	$\frac{\mu}{\rho \Delta s^2}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	$\rho \Delta s^3$	M				

In the first step, the length was removed with D. In the second step, the mass was removed with $\rho \Delta s^2$. In the third step, time was removed with $\mu / \rho \Delta s^2$. Finally the function form is

$$\frac{\sqrt{\rho \Delta p} \Delta s}{\mu} = f(\frac{D}{\Delta s}, \alpha)$$

8.26 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

The functional relationship is

$$F_D = f(\rho, V, S, \omega)$$

Writing out the dimensional parameters using the exponent method

$$F_D^a = \rho^b V^c S^d \omega^e$$

Including the dimensions

$$\left(\frac{ML}{T^2}\right)^a = \left(\frac{M}{L^3}\right)^b \left(\frac{L}{T}\right)^c L^{2d} \left(\frac{1}{T}\right)^e$$

Writing the equations for dimensional homogeneity,

$$\begin{array}{ll} M: & a = b \\ L: & a = -3b + c + 2d \\ T: & 2a = c + e \end{array}$$

Solving for a, b and c in terms of d, and e gives

$$\begin{aligned} a &= d - e/2 \\ b &= d - e/2 \\ c &= 2d - 2e \end{aligned}$$

Substituting into the original equation

$$F_D^{d-e/2} = \rho^{d-e/2} V^{2d-2e} S^d \omega^e$$

$$\left(\frac{F_D}{\rho V^2 S}\right)^d = \left(\frac{F_D^{1/2}\omega}{\rho^{1/2} V^2}\right)^e$$

so

$$\frac{F_D}{\rho V^2 S} = f(\frac{F_D^{1/2}\omega}{\rho^{1/2}V^2})$$

Two variables can be combined to give

$$\frac{F_D}{\rho V^2 S} = f\left(\frac{\omega^2 S}{V^2}\right)$$

8.27 **Information and Assumptions**

provided in problem statement

Find

the functional relationship for Q

Solution

$$Q = f(N, D, h_p, \mu, \rho, g)$$

Use the step-by-step method

Q	$\frac{L^3}{T}$	$\frac{Q}{D^3}$	$\frac{1}{T}$	$\frac{Q}{D^3}$	$\frac{1}{T}$	$\frac{Q}{ND^3}$	0
N	$\frac{1}{T}$	N	$\frac{1}{T}$	$\frac{1}{T}$	$\frac{1}{T}$		
D	L						
h_p	L	$\frac{h_p}{D}$	0	$\frac{h_p}{D}$	0	$\frac{h_p}{D}$	0
μ	$\frac{M}{LT}$	μD	$\frac{M}{T}$	$\frac{\mu}{\rho D^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho N D^2}$	0
ρ	$\frac{M}{L^3}$	ρD^3	M				
g	$\frac{L}{T^2}$	$\frac{g}{D}$	$\frac{1}{T^2}$	$\frac{g}{D}$	$\frac{1}{T^2}$	$\frac{g}{N^2 D}$	0

The functional relationship is

$$\frac{Q}{ND^3} = f(\frac{h_p}{D}, \frac{\mu}{\rho N D^2}, \frac{g}{N^2 D})$$

Some dimensionless variables can be combined to yield a different form

$$\frac{Q}{ND^3} = f(\frac{h_p g}{N^2 D^2}, \frac{\mu}{\rho N D^2}, \frac{g}{N^2 D})$$

8.28 **Information and Assumptions**

provided in problem statement

Find

speed in water tunnel for dynamic similitude and the ratio of drag forces

Solution

The Reynolds number is the significant dimensionless parameter so

$$\mathrm{Re}_m = \mathrm{Re}_p$$

Therefore the speed of the model for similitude is

$$\begin{aligned} V_m &= \frac{L_p}{L_m}\frac{\nu_m}{\nu_p}V_p \\ V_m &= 20 \times \frac{1 \times 10^{-6}}{1.4 \times 10^{-6}} \times 2 = \underline{\underline{28.6}} \text{ m/s} \end{aligned}$$

The ratio of the drag force on the model to that on the prototype is

$$\begin{aligned} \frac{F_{D,m}}{F_{D,p}} &= \frac{\rho_m}{\rho_p}\left(\frac{V_m}{V_p}\right)^2\left(\frac{l_m}{l_p}\right)^2 \\ &= \frac{1000}{1015}\left(\frac{28.6}{2}\right)^2\left(\frac{1}{20}\right)^2 \\ &= \underline{\underline{0.504}} \end{aligned}$$

8.29 **Information and Assumptions**

provided in problem statement

Find

velocity of water for dynamic similarity

Solution

Dynamic similarity when Reynolds numbers are the same

$$
\begin{aligned}
\mathrm{Re}_w &= \mathrm{Re}_0 \\
V_w d/\nu_w &= V_0 d/\nu_0 \\
V_w &= V_0 \nu_w/\nu_0 = 2 \text{ m/s } (10^{-6}/10^{-5}) = \underline{\underline{0.20 \text{ m/s}}}
\end{aligned}
$$

8.30 **Information and Assumptions**

provided in problem statement

Find

velocity of water for dynamic similarity

Solution

Dynamic similarity when Reynolds numbers are the same.

$$\begin{aligned}
\mathrm{Re}_5 &= \mathrm{Re}_{20} \\
V_5 D_5/\nu_5 &= V_{20} D_{20}/\nu_{20} \\
V_5 &= V_{20}(D_{20}/D_5)(\nu_5/\nu_{20}) \\
&= (5 \text{ m/s})(20/5)(10^{-6})/((4)(10^{-6})) \\
V_5 &= \underline{\underline{5 \text{ m/s}}}
\end{aligned}$$

8.31 **Information and Assumptions**

provided in problem statement

Find

the discharge ratio and pressure difference in prototype

Solution

Dynamic similarity when Reynolds numbers are the same.

$$
\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
V_m L_m/\nu_m &= V_p L_p/\nu_p \\
V_m/V_p &= (L_p/L_m)(\nu_m/\nu_p) \qquad (1)
\end{aligned}
$$

Multiply both sides of Eq. (1) by $A_m/A_p = L_m^2/L_p^2$:

$$
\begin{aligned}
(V_m A_m)/(V_p A_p) &= (L_p/L_m) \times (1) \times L_m^2/L_p^2 \\
Q_m/Q_p &= L_m/L_p \\
Q_m/Q_p &= \underline{\underline{1/10}} \\
C_{p_m} &= C_{p_p} \\
(\Delta p/\rho V^2)_m &= (\Delta p/\rho V^2)_p \\
\Delta p_p &= \Delta p_m(\rho_p/\rho_m)(V_p/V_m)^2 \\
&= \Delta p_m(1)(L_m/L_p)^2 \\
&= 300 \times (1/10)^2 = \underline{\underline{3.0 \text{ kPa}}}
\end{aligned}
$$

8.32 **Information and Assumptions**

provided in problem statement

Find

the $\pi-$groups

Solution

Using the step-by-step method

$$\begin{array}{llllllll}
n & \frac{1}{T} & n & \frac{1}{T} & n & \frac{1}{T} & \frac{nd}{V} & 0 \\
V & \frac{L}{T} & V & \frac{L}{T} & \frac{V}{d} & \frac{1}{T} & & \\
d & L & d & L & & & & \\
\rho & \frac{M}{L^3} & \frac{\rho}{\mu} & \frac{T}{L^2} & \frac{\rho d^2}{\mu} & T & \frac{Vd\rho}{\mu} & 0 \\
\mu & \frac{M}{LT} & & & & & &
\end{array}$$

Mass is removed with μ in the first step, length with d in the second step and time with V/d in the last step. The final functional form is

$$\frac{nd}{V} = f(\frac{Vd\rho}{\mu})$$

8.33 **Information and Assumptions**

provided in problem statement

Find

the towing speed relative to the speed of the prototype.

Solution

Dynamic similarity based on Reynolds number

$$\text{Re}_m = \text{Re}_p$$

$$V_m L_m/\nu_m = V_p L_p/\nu_p; \text{ Assume } \nu_m = \nu_p$$

$$\underline{\underline{V_m = V_p L_p/L_m = 4V_c}}$$

8.34 **Information and Assumptions**

provided in problem statement

Find

drag force on prototype in air

Solution

Given

$$
\begin{aligned}
D_m &= 1 \text{ ft; } D_p = 3 \text{ ft; } \nu_p = 1.58 \times 10^{-4} \text{ ft}^2/\text{sec;} \\
\nu_m &= 1.22 \times 10^{-5} \text{ ft}^2/\text{sec; } V_m - 5 \text{ ft/sec; } F_m = 20 \text{ lb.}
\end{aligned}
$$

Reynolds model law must be applied, so:

$$
\text{Re}_m = \text{Re}_p; V_m D_m/\nu_m = V_p D_p/\nu_p
$$

or

$$
V_p V_m = (D_m/D_p)(\nu_p/\nu_m) = (1/3)(1.58 \times 10^{-4}/1.22 \times 10^{-5}) \tag{1}
$$

Also $C_{p_m} = C_{p_p}$ for dynamic similitude; thus, $\Delta p_m/(\rho_m V_m^2/2) = \Delta p_p/(\rho_p V_p^2/2)$

$$
\begin{aligned}
\Delta p_p/\Delta p_m &= (\rho_p/\rho_m)(V_p^2/V_m^2) \\
F_p/F_m &= (\Delta p_p A_p)/(\Delta p_m A_m) = (A_p/A_m)(\rho_p/\rho_m)(V_p^2/V_m^2) \qquad (2)
\end{aligned}
$$

Combine Eq. (1) and (2)

$$
\begin{aligned}
F_p/F_m &= (\rho_p/\rho_m)(\nu_p/v_m)^2 = (0.00237/1.94)(1.58 \times 10^{-4}/1.22 \times 10^{-5})^2 \\
&= 0.2049 \\
F_p &= 15 \times 0.2049 = \underline{\underline{3.07 \text{ lbf}}} \text{ or } = \underline{\underline{13.7 \text{ N}}}
\end{aligned}
$$

8.35 **Information and Assumptions**

provided in problem statement

Find

diameter of model for dynamic similarity.

Solution

Dynamic similarity based on Reynolds number

$$
\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
V_m D_m/\nu_m &= V_p D_p/\nu_p \\
D_m/D_p &= (\nu_m/\nu_p)(V_p/V_m) \\
&= ((1.66\times10^{-5})/(1.5\times10^{-3}))(100/10) = 0.111 \\
D_m &= \underline{\underline{0.11 D_p = 0.111 \text{ in.}}}
\end{aligned}
$$

8.36 **Information and Assumptions**

provided in problem statement

Find

the density of the air in tunnel

Solution

$$\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p; (VD/\nu)_m = (VD/\nu)_p \\
(V_m/V_p) &= (D_p/D_m)(\nu_m/\nu_p); \nu_m/\nu_p = (V_mD_m/V_pD_p) \\
(\mu_m\rho_p/\mu_p0_m) &= (V_mD_m/V_pD_p) \text{ or } \rho_m = (\mu_m/\mu_p)(V_p/V_m)(D_p/D_m) \qquad (1) \\
M_m &= M_p; (V/c)_m = (V/c)_p \\
(V_m/V_p) &= c_m/c_p = ((\sqrt{kRT})_m/(\sqrt{kRT})_p) = \sqrt{T_m/T_p} = (298/283)^{1/2} \qquad (2)
\end{aligned}$$

Combining Eqs. (1) and (2):

$$\rho_m = 1.23(1.83 \times 10^{-5}/1.76 \times 10^{-5})(283/298)^{1/2}(6) = \underline{\underline{7.48}} \text{ kg/m}^3$$

8.37

$$\mathrm{Re}_A = \mathrm{Re}_W$$
$$V_A L_A/\nu_A = V_W L_W/\nu_W \ \ ; \ \ \text{but } L_A/L_W = 1$$
$$\therefore V_A V_W = \nu_A/\nu_W \approx (1.6)(10^{-4})/(1.2)(10^{-5})(\text{at } 60^\circ F)$$
$$V_A/V_W > 1$$

The correct choice is $\underline{\underline{\text{c)}}}$

8.38 **Information and Assumptions**

provided in problem statement

Find

mean velocity of water in model to insure dynamic similarity

Solution

$$
\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
V_m d_m \rho_m/\mu_m &= V_p d_p \rho_p/\mu_p \\
V_m &= V_p(d_p/d_m)(\rho_p/\rho_m)(\mu_m/\mu_p) \\
V_m &= (8 \text{ ft/s})(48/4)(1.75/1.94)((2.36 \times 10^{-5})/(4 \times 10^{-4})) \\
V_m &= \underline{\underline{5.11 \text{ ft/s}}}
\end{aligned}
$$

8.39 **Information and Assumptions**

provided in problem statement

Find

pressure change in air flow for same length of pipe.

Solution

Following same procedure as in solution to Problem 8.38

$$V_{\text{air}} = 3(20/10)(1.6 \times 10^{-5}/1.00 \times 10^{-6}) = \underline{\underline{96 \text{ m/s}}}$$

The pressure coefficients will be the same or:

$$\begin{aligned} C_{p_a} &= C_{p_W} \\ (\Delta p/(\rho V^2))_{\text{air}} &= (\Delta p/(\rho V^2))_{\text{water}} \\ \text{or } \Delta p_a &= \Delta p_w(\rho_a/\rho_w)(V_a^2/V_w^2) \\ \Delta p_a &= 2.0 \text{ kPa } (1.17/998)(96/3)^2 \\ \Delta p_a &= 2.40 \text{ kPa} \end{aligned}$$

The $\Delta p_a = 2.40$ kPa is for the pressure difference in an air pipe that is geometrically similar to the water pipe. Therefore, this air pipe would be half as long as the water pipe because the $D_a = 1/2 D_w$. Consequently, the Δp_a as obtained from the pressure coefficient similarity relationship will have to be multiplied by two to obtain the Δp_a for an air pipe that is the same length as the water pipe:

$$\Delta p_a = 2 \times 2.40 \text{ kPa } = \underline{\underline{4.80 \text{ kPa}}}$$

8.40 **Information and Assumptions**

provided in problem statement

Find

the air velocity in wind tunnel.

Solution

Following the same basic procedures in the solution to Prob. 8.38.

$$V_{\text{air}} = (10)(1/1)(1.41 \times 10^{-5}/1.31 \times 10^{-6}) = \underline{\underline{107.6 \text{ m/s}}}$$

8.41 **Information and Assumptions**

provided in problem statement

Find

kinematic viscosity of fluid for model on earth

Solution

Dynamic similarity based on Froude number

$$
\begin{aligned}
Fr_{\text{moon}} &= Fr_{\text{earth}} \\
(V/\sqrt{gL})m &= (V/\sqrt{gL})e \\
V_e/V_m &= (g_e/g_m)^{0.5}(L_e/L_m)^{0.5} \\
&= (5)^{0.5}(1) \\
\mathrm{Re}_m &= \mathrm{Re}_e \\
(VL/\nu)_m &= (VL/\nu)_e \\
\nu_e &= (V_e/V_m)\nu_m = (5)^{0.5}0.5 \times (10^{-5}) \text{ m}^2/\text{s} \\
\nu_e &= \underline{\underline{1.19 \times 10^{-5} \text{ m}^2/\text{s}}}
\end{aligned}
$$

8.42 **Information and Assumptions**

provided in problem statement

Find

entry velocity of water

Solution

$$
\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
V_m L_m/\nu_m &= V_p L_p/\nu_p \\
V_m &= (L_p/L_m)(\nu_m/\nu_p)V_p \\
&= (10)(10^{-6}/((4)(10^{-5})))(10 \text{ m/s}) \\
V_m &= \underline{\underline{2.50 \text{ m/s}}}
\end{aligned}
$$

8.43 **Information and Assumptions**

provided in problem statement

Find

velocity in prototype and pressure difference in prototype

Solution

Dynamic similarity based on Reynolds number

$$\begin{aligned}
\mathrm{Re}_{\mathrm{prot.}} &= \mathrm{Re}_{\mathrm{model}} \\
V_{\mathrm{prot.}} &= V_{\mathrm{model}}(L_{\mathrm{model}}/L_{\mathrm{prot.}})(\nu_{\mathrm{prot.}}/\nu_{\mathrm{model}}) \\
V_{\mathrm{prot.}} &= 1(1/5)(10^{-5}/10^{-6}) = \underline{\underline{2.0 \text{ m/s}}}
\end{aligned}$$

Pressure coefficients are the same.

$$\begin{gathered}
C_{p,m} = C_{p,p} \\
(\Delta p/\rho V^2)_m = (\Delta p/\rho V^2)_p \\
\Delta p_p = \Delta p_m(\rho_p/\rho_m)(V_p/V_m)^2 \\
= 2.0 \times (860/998) \times (2.0/1.0)^2 \\
= \underline{\underline{6.90}} \text{ kPa}
\end{gathered}$$

8.44 **Information and Assumptions**

provided in problem statement

Find

air velocity for dynamic similarity and pressure difference for water flow

Solution

Dynamic similitude based on Reynolds number.

$$\begin{aligned}
\mathrm{Re}_{\mathrm{air}} &= \mathrm{Re}_{\mathrm{water}} \\
(VD\rho/\mu)_{\mathrm{air}} &= (VD\rho/\mu)_{\mathrm{water}} \\
V_a &= V_w(D_w/D_a)(\rho_w/\rho_a)(\mu_a/\mu_w) \\
\rho_w &= 1,000 \text{ kg/m}^3 \\
\rho &= \rho_{a,\text{ std. atm.}} \times (150 \text{ kPa}/101 \text{ kPa}) \\
&= 1.20 \times (150/101) = 1.78 \text{ kg/m}^3 \\
\mu_a &= 1.81 \times 10^{-5} \text{ N} \cdot \text{s/m}^2 \\
\mu_w &= 1.31 \times 10^{-3} \text{ N} \cdot \text{s/m}^2
\end{aligned}$$

Then

$$\begin{aligned}
V_a &= 1.5 \text{ m/s } (1{,}000/1.78)(1.81 \times 10^{-5}/1.31 \times 10^{-3}) \\
V_a &= \underline{\underline{11.6 \text{ m/s}}}
\end{aligned}$$

The pressure coefficients are equal so

$$\begin{aligned}
C_{p_w} &= C_{p_a} \\
(\Delta p/\rho V^2)_w &= (\Delta p/\rho V^2)_a \\
\Delta p_w &= \Delta p_a(\rho_w/\rho_a)(V_w/V_a)^2 \\
&= 780 \times (1,000/1.78)(1.5/11.6)^2 \\
&= 7,330 \text{ Pa } = \underline{\underline{7.33 \text{ kPa}}}
\end{aligned}$$

8.45 **Information and Assumptions**

provided in problem statement

Find

velocity in water tunnel and force on prototype.

Solution

Dynamic similarity based on Reynolds number

$$\begin{aligned}
\text{Re}_{\text{tunnel}} &= \text{Re}_{\text{prototype}} \\
V_{\text{tunnel}} &= V_{\text{prot.}}(4/1)(\nu_{\text{tunnel}}/\nu_{\text{prot.}}) \\
V_{\text{tunnel}} &= 3(5/1)(1) = 15 \text{ m/s}
\end{aligned}$$

For dynamic similarity, the pressure coefficients are the same.

$$\begin{aligned}
C_{p_{\text{tunnel}}} &= C_{p_{\text{prototype}}} \\
(\Delta p/\rho V^2)_{\text{tunnel}} &= (\Delta p/\rho V^2)_{\text{prototype}} \\
(\Delta p_{\text{tunnel}}/\Delta p_{\text{prot.}}) &= (\rho_{\text{tunnel}}/\rho_{\text{prot.}})(V_{\text{tunnel}}^2/V_{\text{prot.}}^2)
\end{aligned}$$

Multiply both sides of the equation by $A_{\text{tunnel}}/A_{\text{prot.}} = L_t^2/L_p^2$

$$\begin{aligned}
(\Delta p \times A)_{\text{tunnel}}/(\Delta p \times A)_{\text{prot.}} &= (\rho_{\text{tunnel}}/\rho_{\text{prot.}}) \times (V_{\text{tunnel}}^2/V_{\text{prot.}}^2) \times (L_t/L_p)^2 \\
F_{\text{tunnel}}/F_{\text{prot.}} &= (1/1)(5)^2(1/5)^2 \\
F_{\text{tunnel}} &= F_{\text{prot.}} = \underline{\underline{868\ N}}
\end{aligned}$$

8.46 **Information and Assumptions**

provided in problem statement

Find

density needed for wind tunnel and force on the prototype

Solution

Dynamic similarity by matching Reynolds number.

$$\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
(\rho VL/\mu)_m &= (\rho VL/\mu)_p \\
\rho_m/\rho_p &= (V_p/V_m)(L_p/L_m)(\mu_m/\mu_p) \\
&= (25/300)(100)(1) \\
&= 8.33 \\
\rho_m &= 8.33\rho_p = \underline{\underline{0.020}} \text{ slugs/ft}^3
\end{aligned}$$

$$F_m/F_p = (\Delta p_m/\Delta p_p)(A_m/A_p) \tag{1}$$

$$\begin{aligned}
\frac{C_{p,m}}{C_{p,p}} &= \left(\frac{\Delta p_m}{\rho_m V_m^2}\right)\left(\frac{\rho_p V_p^2}{\Delta p_p}\right) \\
1 &= \left(\frac{\Delta p_m}{\Delta p_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p^2}{V_m^2}\right) = \left(\frac{\Delta p_m}{\Delta p_p}\right)\left(\frac{1}{8.33}\right)\left(\frac{25}{300}\right)^2
\end{aligned}$$

Then

$$\Delta p_m/\Delta p_p = 1,200 \tag{2}$$

solve Eqs. (1) and (2) for F_m/F_p

$$\begin{aligned}
F_m/F_p &= 1,000A_m/A_p = 1,200(1/10^4) = 0.12 \\
F_p &= F_m \times 10 = \underline{\underline{417}} \text{ lbf}
\end{aligned}$$

8.47 **Information and Assumptions**

provided in problem statement

Find

flow rate of dynamic similarity and pressure coefficient for the prototype

Solution

Dynamic similarity through matching Reynolds number

$$\mathrm{Re}_m = \mathrm{Re}_p \text{ or } (VD\rho/\mu)_m = (VD\rho/\mu)_p$$

Then

$$V_m/V_p = (D_p/D_m)(\rho_p/\rho_m)(\mu_m/\mu_p)$$

Multiply both sides of above equation by $A_m/A_p = (D_m/D_p)^2$

$$\begin{aligned}
(A_m/A_p)(V_m/V_p) &= (D_p/D_m)(D_m/D_p)^2(\rho_p/\rho_m)(\mu_m/\mu_p) \\
Q_m/Q_p &= (D_m/D_p)(\rho_p/\rho_m)(\mu_m/\mu_p) \\
&= (1/3)(0.82)(10^{-3}/(3\times10^{-3})) \\
Q_m/Q_p &= 0.0911 \\
\text{or } Q_m &= Q_p \times 0.0911 \\
Q_m &= 0.50 \times 0.0911 \text{ m}^3/\text{s} \ = \underline{\underline{0.0455 \text{ m}^3/\text{s}}} \\
C_p &= \underline{\underline{1.07}}
\end{aligned}$$

8.48 **Information and Assumptions**

provided in problem statement

Find

corresponding moment and speed for the prototype

Solution

$$\begin{aligned} C_{p_m} &= C_{p_p} \\ (\Delta p/\rho V^2)_m &= (\Delta p/\rho V^2)_p \end{aligned}$$

or

$$\Delta p_m/\Delta p_p - (\rho_m V_m^2)/(\rho_p V_p^2) \tag{1}$$

Multiply both sides of Eq. (1) by $(A_m/A_p) \times (L_m/L_p) = (L_m/L_p)^3$ and obtain

$$\text{Mom.}_m/\text{Mom.}_p = (\rho_m/\rho_p)(V_m/V_p)^2(L_m/L_p)^3 \tag{2}$$

Also $\text{Re}_m = \text{Re}_p$ or

$$\begin{gathered} V_m L_m/\nu_m = V_p L_p/\nu_p \\ V_m/V_p = (L_p/L_m)(\nu_m/\nu_p) \end{gathered} \tag{3}$$

Substitute Eq. (3) into Eq. (2) to obtain

$$\begin{aligned} M_m/M_p &= (\rho_m/\rho_p)(\nu_m/\nu_p)^2(L_m/L_p) \\ M_p &= M_m(\rho_p/\rho_m)(\nu_p/\nu_m)^2(L_p/L_m) = 2(1,026/1,000)(1.4/1.31)^2(60 = \underline{\underline{141}} \text{ N} \cdot \text{m} \end{aligned}$$

Also,

$$V_p = 10(1/60)(1.41/1.310) = \underline{\underline{0.179}} \text{ m/s}$$

8.49 **Information and Assumptions**

provided in problem statement

Find

lift force on the prototype

Solution

$$\begin{aligned} C_{p_m} &= C_{p_p} \\ (\Delta_p/\rho V^2)_m &= (\Delta p/\rho V^2)_p \\ \Delta p_m/\Delta p_p &= (\rho_m/\rho_p)(V_m^2/V_p^2) \end{aligned}$$

Multiply both sides of the above equation by $A_m/A_p = (L_m/L_p)^2$

$$(\Delta p_m/\Delta p_p)(A_m/A_p) = (\rho_m/\rho_p)(V_m^2/V_p^2)(L_m^2/L_p^2) = F_m/F_p \tag{1}$$

For dynamic similitude $\text{Re}_m = \text{Re}_p$ or $(VL\rho/\mu)_m = (VL\rho/\mu)_p$ or

$$(V_p/V_m)^2 = (L_m/L_p)^2(\rho_m/\rho_p)^2(\mu_m/\mu_m)^2 \tag{2}$$

Eliminating $(V_p/V_m)^2$ between Eq. (1) and Eq. (2) yields

$$F_p/F_m = (\rho_m/\rho_p)(\mu_p/\mu_m)^2$$

Then if the same fluid is used for models and prototype, we have

$$F_p/F_m = 1$$

or

$$F_p = \underline{\underline{20 \text{ kN}}}$$

8.50 **Information and Assumptions**

provided in problem statement

Find

pressure in tunnel test section

Solution

$$\begin{aligned} M_m &= M_p \\ V_m/c_m &= V_p/c_p; V_m/V_p = c_m/c_p \end{aligned} \tag{1}$$

Also $\mathrm{Re}_m = \mathrm{Re}_p$

$$V_m L_m \rho_m/\mu_m = V_p L_p \rho_p/\mu_p$$

or

$$V_m/V_p = (L_p/L_m)(\rho_p/\rho_m)(\mu_m/\mu_p) \tag{2}$$

Eliminate V_m/V_p between Eqs. (1) and (2) to obtain

$$c_m/c_p = (L_p/L_m)(\rho_p/\rho_m)(\mu_m/\mu_p) \tag{3}$$

But

$$c = \sqrt{E_V/\rho} = \sqrt{kp/\rho} = \sqrt{kp/(p/RT)} = \sqrt{kRT}$$

Therefore $c_m/c_p = 1$, then from Eq. (3)

$$1 = (8)(\rho_p/\rho_m)(1)$$

or

$$\rho_m = 8\rho_p$$

But

$$\rho = p/RT$$

so

$$(p/RT)_m = 8(p/RT)_p; p_m = 8p_p = 8 \text{ atm } = \underline{\underline{0.808 \text{ MPa abs.}}}$$

8.51 **Information and Assumptions**

provided in problem statement

Find

speed needed to achieve same Reynolds number and importance of Mach number effects.

Solution

$$\begin{aligned} \mathrm{Re}_m &= \mathrm{Re}_p \\ V_m L_m \rho_m/\mu_m &= V_p L_p \rho_p/\mu_p; \text{ But } \rho_m/\mu_m = \rho_p/\mu_p \end{aligned}$$

so

$$V_m = V_p(L_p/L_m) = 80 \times 10 = 800 \text{ km/hr } = \underline{\underline{222 \text{ m/s}}}$$

$$M = V/c = 222/345 = 0.644$$

Mach number effects would be important.

8.52 **Information and Assumptions**

provided in problem statement

Find

if rarefied flow

Solution

$$M/\,\mathrm{Re} = (V/c)(\mu/\rho V D) = (\mu)/(\rho c D)$$

where

$$\rho = p/RT = 22/(1716 \times 393) = 3.26 \times 10^{-5} \text{ slugs/ft}^3$$

and $c = 975$ ft/s and $\mu = 3.0 \times 10^{-7}$ lbf-s/ft^2 so

$$\begin{aligned} M/\,\mathrm{Re} &= 3.0 \times 10^{-7}/(3.26 \times 10^{-5} \times 975 \times 2) = 4.72 \times 10^{-6} < 1 \\ & \underline{\underline{\text{Not rarefied}}}. \end{aligned}$$

8.53 **Information and Assumptions**

provided in problem statement

Find

droplet diameter for break up

Solution

$$\begin{aligned}
W/(\mathrm{Re})^{0.5} &= (\rho g V^2 d/\sigma)(\mu_g/(V d\rho_g))^{0.5} = 0.5 \\
&= \rho_g^{0.5} V^{1.5} \mu^{0.5} d^{0.5}/\sigma = 0.5\ ;\ \rho g = 1.2\ \mathrm{kg/m^3} \\
d^{0.5} &= 0.5\sigma/(\rho_g^{0.5} V^{1.5} \mu^{0.5}) = 0.5(\sigma^2/(\rho_g \mu V^3))^{0.5} \\
&= (0.5)((7.3\times 10^{-2})^2/((1.2)(1.81)(10^{-5})(25^3)))^{0.5} \\
d &= 0.00393\ \mathrm{m} = \underline{\underline{3.93}}\ \mathrm{mm}
\end{aligned}$$

8.54 **Information and Assumptions**

from Table A.3, $\rho = 0.95 \text{ kg/m}^3$
provided in problem statement

Find

expected size of droplets

Solution

$$W = 6.0 = \rho D V^2/\sigma$$

$$D = 6\sigma/\rho V^2 = 6 \times 0.02/(0.95 \times (30)^2) = 1.40 \times 10^{-4} \text{ m} = \underline{\underline{140 \ \mu\text{m}}}$$

8.55 **Information and Assumptions**

provided in problem statement

Find

estimated size of droplets

Solution

$$W = 6.0 = \rho D V^2/\sigma$$

From Table A.3 $\rho = 1.20$ kg/m^3 and from Table A.5 $\sigma = 0.073$ N/m so

$$D = 6\sigma/\rho V^2 = 6 \times 0.073/(1.2 \times (15)^2) = 1.62 \times 10^{-3} \text{ m} = \underline{\underline{1.62 \text{ mm}}}$$

8.56 **Information and Assumptions**

Find

relationship between kinematic viscosity ratio and scale ratio

Solution

$$
\begin{aligned}
F_m &= F_p;\ (V/\sqrt{gL})_m = (V/\sqrt{gL})_p \\
\text{or } V_m/V_p &= \sqrt{g_m L_m / g_p L_p} \qquad (1) \\
\mathrm{Re}_m &= \mathrm{Re}_p;\ (VL/\nu)_m = (VL/\nu)_p \text{ or } V_m/V_p = (L_p/L_m)(\nu_m/\nu_p) \qquad (2)
\end{aligned}
$$

Eliminate V_m/V_p between Eqs. (1) and (2) to obtain:

$$\sqrt{g_m L_m / g_p L_p} = (L_p/L_m)(\nu_m/\nu_p),\ \text{but } g_m = g_p$$

Therefore: $\underline{\underline{\nu_m/\nu_p = (L_m/L_p)^{3/2}}}$

8.57 **Information and Assumptions**

provided in problem statement

Find

wave period in prototype

Solution

Dynamic similarity based on Froude number

$$t_p/t_m = (L_p/L_m)^{1/2}$$

Then wave period$_{\text{prot}} = 1 \times (10)^{1/2} = \underline{\underline{3.16 \text{ s}}}$ and the wave height$_{\text{prot}} = 12 \text{ cm} \times 10 = \underline{\underline{1.2 \text{ m}}}$

8.58 **Information and Assumptions**

provided in problem statement

Find

discharge in prototype

Solution

Dynamic similarity based on Froude number.

$$\begin{aligned}
Fr_m &= Fr_D \\
V_m/((g_m)(L_m))^{0.5} &= V_p/((g_p)(L_p))^{0.5} \\
V_p/V_m &= (L_p/L_m)^{0.5} = 5 \\
V_p &= (2.5)(5) \text{ m/s} = \underline{\underline{12.5 \text{ m/s}}} \\
Q_p/Q_m &= (V_p/V_m)(A_p/A_m) \\
&= (5)(625) \\
Q_p &= (5)(625)(0.10) = \underline{\underline{312.5 \text{ m}^3/\text{s}}}
\end{aligned}$$

8.59 **Information and Assumptions**

provided in problem statement

Find

corresponding model speed.

Solution

Assume Froude model law applies, then following the procedure of solution to Prob. 8.58 we have:

$$V_m = V_p\sqrt{L_m/L_p} = 100\sqrt{1/10} = \underline{\underline{31.6 \text{ m/s}}}$$

8.60 **Information and Assumptions**

provided in problem statement

Find

model discharge.

Solution

From solution to Prob. 8.58 we have:

$$V_m/V_p = \sqrt{L_m/L_p} \tag{1}$$

or for this case

$$V_m/V_p = \sqrt{1/36} = \underline{\underline{1/6}}$$

Multiply both sides of Eq. (1) by $A_m/A_p = (L_m/L_p)^2$

$$V_m A_m/V_p A_p = (L_m/L_p)^{1/2}(L_m/L_p)^2$$
$$Q_m/Q_p = (L_m/L_p)^{5/2}$$

or for this case

$$Q_m/Q_p = (1/36)^{5/2} = \underline{\underline{1/7,776}}$$
$$Q_m = 3,000/7,776 = \underline{\underline{0.386 \text{ m}^3/\text{s}}}$$

8.61 **Information and Assumptions**

provided in problem statement

Find

velocity and depth in model at corresponding point.

Solution

Dynamic similarity using Froude number

$$
\begin{aligned}
Fr_m &= Fr_p \\
V_m/((g_m)(L_m))^{0.5} &= V_p/((g_p)(L_p))^{0.5} \\
V_m &= V_p(L_m/L_p)^{0.5} = V_p(1/8) = \underline{\underline{1.875 \text{ ft/s}}} \\
d_m/d_p &= 1/64 \\
d_m &= (1/64)d_p \\
&= (1/64)(20) = \underline{\underline{0.312 \text{ ft}}}
\end{aligned}
$$

8.62 **Information and Assumptions**

provided in problem statement

Find

velocity and discharge for prototype

Solution

Froude model law applies:

$$V_p = V_m\sqrt{L_p/L_m} = 7.87\sqrt{36} = \underline{\underline{47.2 \text{ ft/s}}}$$

From solution to Prob. 8.60:

$$Q_p/Q_m = (L_p/L_m)^{5/2};\ Q_p = 3.53 \times (36)^{5/2} = \underline{\underline{27,450 \text{ ft}^3/\text{s}}}$$

8.63 **Information and Assumptions**

provided in problem statement

Find

velocity and wave height in prototype

Solution

Froude model law applies:

$$V_p = V_m \sqrt{L_p/L_m} = 0.90\sqrt{10} = \underline{\underline{2.85 \text{ m/s}}}$$

$L_p/L_m = 10$; therefore, wave height$_{\text{prot.}} = 10 \times 2.5 \text{ cm} = \underline{\underline{25 \text{ cm}}}$

8.64 **Information and Assumptions**

provided in problem statement

Find

time for particle to move along corresponding path in the prototype

Solution

Froude model law applies:

$$V_p/V_m = \sqrt{L_p/L_m}$$

or

$$(L_p/t_p)/(L_m/t_m) = (L_p/L_m)^{1/2}$$

Then

$$\begin{aligned} t_p/t_m &= (L_p/L_m)(L_m/L_p)^{1/2} \\ t_p/t_m &= (L_p/L_m)^{1/2} \\ t_p &= 1 \times \sqrt{25} = \underline{\underline{5 \text{ min}}} \end{aligned}$$

Also

$$\begin{aligned} Q_p/Q_m &= (L_p/L_m)^{5/2} \\ Q_p &= 0.10 \times (25)^{5/2} = \underline{\underline{312.5 \text{ m}^3/\text{s}}} \end{aligned}$$

8.65 **Information and Assumptions**

provided in problem statement

Find

velocity and period observed in the model

Solution

Froude number applies:

$$F_m = F_p$$

or

$$\begin{aligned} (V/\sqrt{gL})_m &= (V/\sqrt{gL})_p \\ V_m/V_p &= (L_m/L_p)^{1/2} \end{aligned} \tag{1}$$

because $g_m = g_p$. Then

$$(L_m/t_m)/(L_p/t_p) = (L_m/L_p)^{1/2}$$

or

$$t_m/t_p = (L_m/L_p)^{1/2} \tag{2}$$

Then from Eq. (1)

$$V_m = V_p(L_m/L_p)^{1/2} = 4.0 \times (1/300)^{1/2} = \underline{\underline{0.231 \text{ m/s}}}$$

From Eq. (2)

$$t_m = 12.5hr(1/300)^{1/2} = 0.722\,\text{hr} = \underline{\underline{43.3 \text{ min}}}$$

8.66 **Information and Assumptions**

provided in problem statement

Find

full scale force

Solution

$$\begin{aligned} C_{p_m} &= C_{p_p};\ (\Delta p/\rho V^2)_m = (\Delta p/\rho V^2)_p \\ \Delta p_m/\Delta p_p &= (\rho_m/\rho_p)(V_m/V_p)^2 \end{aligned} \tag{1}$$

Multiply both sides of Eq. (1) by $A_m/A_p = L_m^2/L_p^2$

$$(\Delta p_m A_m)/(\Delta p_p A_p) = (\rho_m/\rho_p)(L_m/L_p)^2(V_m/V_p)^2$$

Also from the Froude model law

$$V_m/V_p = \sqrt{L_m/L_p} \tag{2}$$

Eliminating V_m/V_p from Eqs. (1) and (2) yields

$$\begin{aligned} F_m/F_p &= (\rho_m/\rho_p)(L_m/L_p)^2(L_m/L_p) \\ F_m/F_p &= (\rho_m/\rho_p)(L_m/L_p)^3 \\ F_p &= F_m(\rho_p/\rho_m)(L_p/L_m)^3 = 80(1,026/1,000)(36)^3 = \underline{\underline{3.83\text{ MN}}} \end{aligned}$$

8.67 **Information and Assumptions**

provided in problem statement

Find

water discharge in model for dynamic similarity and force on prototype

Solution

From Problem 8.64

$$\begin{aligned} Q_m/Q_p &= (L_m/L_p)^{5/2} \\ Q_m &= 150 \times (1/20)^{5/2} = \underline{\underline{0.084 \text{ m}^3/\text{s}}} \end{aligned}$$

From solution to Prob. 8.66 we have:

$$F_p = F_m(\rho_p/\rho_m)(L_p/L_m)^3 = 22(1/1)(20)^3 = \underline{\underline{176 \text{ kN}}}$$

8.68 **Information and Assumptions**

provided in problem statement

Find

the largest feasible scale ratio

Solution

Check the scale ratio as dictated by Q_m/Q_p (see Problem 8.64)

$$Q_m/Q_p = 0.90/5,000 = (L_m/L_p)^{5/2}$$

or

$$L_m/L_p = 0.0318$$

Then with this scale ratio

$$\begin{aligned} L_m &= 0.0318 \times 1,200 \text{ m} = 38.1 \text{ m} \\ W_m &= 0.0318 \times 300 \text{ m} = 9.53 \text{ m} \end{aligned}$$

Therefore, model will fit into the available space, so use

$$\underline{\underline{L_m/L_p = 0.0318}}$$

8.69 **Information and Assumptions**

provided in problem statement

Find

speed for the prototype

Solution

The Froude number applies.

$$V_m/\sqrt{g_m L_m} = V_p/\sqrt{g_p L_p}$$

$$V_p = V_m \sqrt{L_p}/\sqrt{L_m}$$

$$= (4 \text{ ft/s}) (150/4)^{1/2}$$

$$\underline{\underline{V_p = 24.5 \text{ ft/s}}}$$

8.70 **Information and Assumptions**

provided in problem statement

Find

velocity and wave resistance of the prototype.

Solution

Froude model law applies, so we follow the solution procedure of Prob. 8.66:

$$
\begin{aligned}
V_m/V_p &= \sqrt{L_m/L_p};\ V_p = 5 \times \sqrt{25} = \underline{\underline{25 \text{ ft/s}}} \\
F_m/F_p &= (L_m/L_p)^3;\ F_p = 2(25)^3 = \underline{\underline{31,250 \text{ lbf}}}
\end{aligned}
$$

8.71 **Information and Assumptions**

provided in problem statement

Find

corresponding velocity and wave resistance of the prototype

Solution

Dynamic similarity by equating Froude numbers

$$\begin{aligned} Fr_m &= Fr_p \\ V_m/(g_m L_m)^{0.5} &= V_p/(g_p L_p)^{0.5} \\ V_p &= V_m(L_p/L_m)^{0.5} = \underline{\underline{12}} \text{ m/s} \\ F_p &= (12 \text{ N})(L_p/L_m)^3 = (12)(16)^3 = \underline{\underline{49,152 \text{ N}}} \end{aligned}$$

8.72 Assume $C_{p_m} = C_{p_p}$

$$\begin{aligned}(\Delta p/(\rho V^2/2)_m &= (\Delta p/(\rho V^2/2)_p \\ \Delta p_m/\Delta p_p &= (\rho_m/\rho_p)(V_m^2/V_p^2)\end{aligned}$$

Assume $\rho_m = \rho_p$

$$\begin{aligned}F_m/F_p &= (\Delta p_m/\Delta p_p)(A_m/A_p) = (V_m/V_p)^2(L_m/L_p)^2 \\ (F_p/F_m) &= (40/20)^2(20)^2 \\ F_p &= (200\text{ N})(4)(400) = 320,000\text{ N} = 320\text{ kN}\end{aligned}$$

Choice $\underline{\underline{\text{(d)}}}$ is the correct.

8.73 **Information and Assumptions**

provided in problem statement

Find

lateral force on the prototype

Solution

Assume

$$C_{p,\text{model}} = C_{p,\text{prot.}}$$

then

$$\Delta p_p/((1/2)\rho_p V_p^2) = C_{p_p} = C_{p_m}$$

or

$$\begin{aligned} \Delta p_p &= C_{p_m}((1/2)\rho_p V_p^2) \\ &= C_{p_m} \times (1/2) \times 1.25 \times (150,000/3,600)^2 \\ p - p_0 &= 1,085.6 C_{p_m} \end{aligned}$$

but $p_0 = 0$ gage so

$$p = 1,085.6 C_{p_m} \text{ Pa}$$

Extremes of pressure are therefore:

$$\begin{aligned} p_{\text{windward wall}} &= \underline{\underline{1.085 \text{ kPa}}} \\ p_{\text{side wall}} &= 1,085.6 \times (-2.7) = \underline{\underline{-2.93 \text{ kPa}}} \\ p_{\text{leeward wall}} &= 1,085 \times (-0.8) = \underline{\underline{-868 \text{ Pa}}} \end{aligned}$$

Lateral Force:

$$\Delta p_m/\Delta p_p = ((1/2)\rho_m V_m^2)/((1/2)\rho_p V_p^2)$$

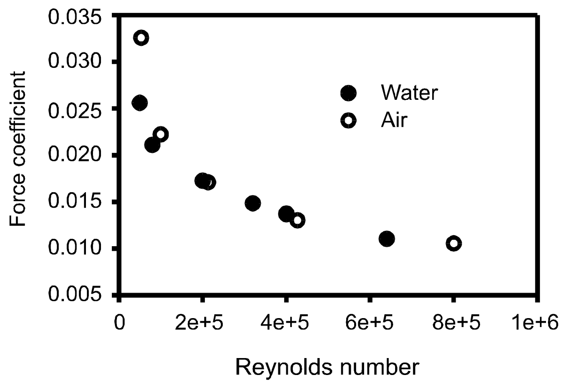
Multiply both sides of equation by $A_m/A_p = L_m^2/L_p^2$

$$\begin{aligned} (\Delta p_m A_m)/(\Delta p_p A_p) &= (\rho_m/\rho_p)(V_m^2/V_p^2)(L_m^2/L_p^2) = F_m/F_p \\ F_p/F_m &= (\rho_p/\rho_m)(V_p^2/V_m^2)(L_p^2/L_m^2) \\ F_p &= 20(1.25/1.20)((15,000/3,600)^2/(20)^2)(250)^2 \\ F_p &= \underline{\underline{5.65 \text{ MN}}} \end{aligned}$$

8.74 Performing a dimensional analysis shows that

$$\frac{F_D}{\rho V^2 D^2} = f(\frac{\rho V D}{\mu})$$

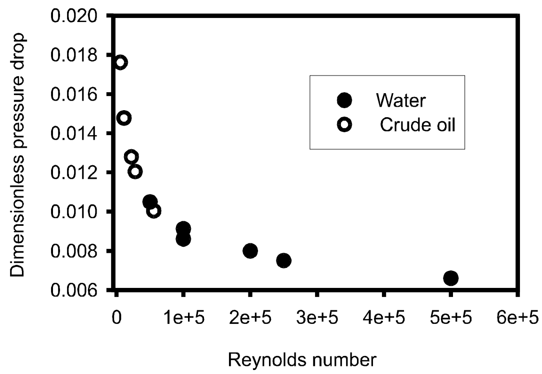
The independent variable is the Reynolds number. Plotting the data using the dimensionless numbers gives the following graph.



8.75 Performing an dimensional analysis on the equation for pressure drop shows

$$\frac{\Delta p}{L}\frac{D}{\rho V^2} = f(\frac{\rho VD}{\mu})$$

where the independent parameter is Reynolds number. Plotting the data using the dimensionless parameters gives the following graph.



Chapter Nine

9.1 **Information and assumptions**

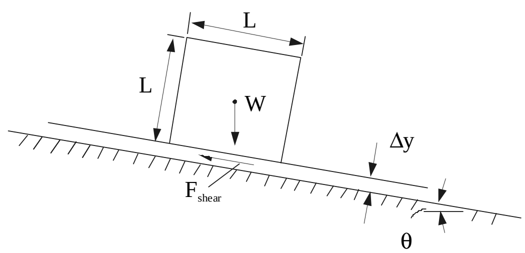
provided in problem statement

Find

terminal velocity of block

Solution

$$
\begin{aligned}
F_{\text{shear}} &= W \ \sin\theta \\
\tau &= F_{\text{shear}}/A_s = W\sin\theta/L^2 \\
\tau &= \mu dV/dy = \mu \times V/\Delta y
\end{aligned}
$$



or

$$V = \tau\Delta y/\mu$$

Then

$$
\begin{aligned}
V &= (W\sin\theta/L^2)\Delta y/\mu \\
V &= (200\sin 10^\circ/0.30^2) \times 1 \times 10^{-4}/10^{-2} \\
V &= \underline{\underline{3.86 \text{ m/s}}}
\end{aligned}
$$

9.2 **Information and assumptions**

provided in problem statement

Find

dynamic viscosity of oil

Solution

Same solution procedure applies as in Prob. 9.1. From the solution to Prob. 9.1, we have

$$\begin{aligned}
\mu &= (W \sin\theta / L^2)\Delta y / V \\
\mu &= (40 \times (5/13)/3^2) \times (0.02/12)/0.5 \\
\mu &= \underline{\underline{5.70 \times 10^{-3} \text{ lbf-s/ft}^2}}
\end{aligned}$$

9.3 **Information and assumptions**

provided in problem statement

Find

dynamic viscosity of oil

Solution

Same type of solution procedure applies as in Prob. 9.1 and 9.2. Then

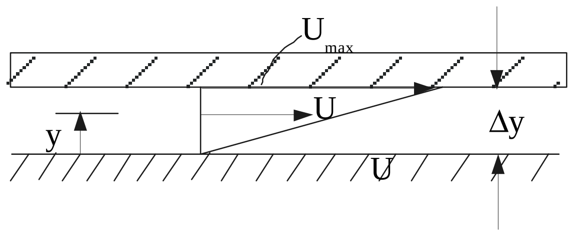
$$\begin{aligned} \mu &= (20 \times (5/13)/1^2) \times 5 \times 10^{-4}/0.12 \\ \mu &= \underline{\underline{3.20 \times 10^{-2} \text{ N} \cdot \text{s/m}^2}} \end{aligned}$$

9.4 **a)** Upper plate is moving to the left relative to the lower plate.

b) Minimum shear stress occurs where the maximum velocity occurs (where $du/dy = 0$).

9.5 a). $\underline{\underline{\text{True}}}$ b). $\underline{\underline{\text{False}}}$ c). $\underline{\underline{\text{False}}}$ d). $\underline{\underline{\text{False}}}$ e). $\underline{\underline{\text{True}}}$

9.6 **a)** By similar triangles $u/y = u_{\max}/\Delta y$



or

$$
\begin{aligned}
u &= (u_{\max}/\Delta y)y \\
u &= (0.3/0.002)y \ \underline{\underline{\text{m/s} = \ 150\ y\ \text{m/s}}} \\
v &= 0
\end{aligned}
$$

b) For flow to be irrotational $\partial u/\partial y = \partial V/\partial x$ here $\partial u/\partial y = 150$ and $\partial V/\partial x = 0$. The equation is not satisfied; $\underline{\underline{\text{flow is rotational}}}$.

c) $\partial u/\partial x + \partial v/\partial y = 0$ (continuity equation) $\partial u/\partial x = 0$ and $\partial v/\partial y = 0$ so $\underline{\underline{\text{continuity is satisfied}}}$.

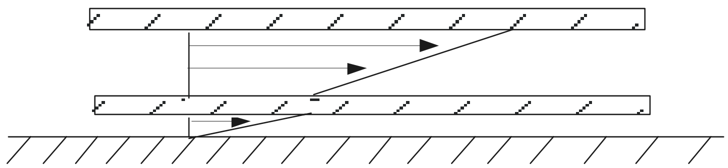
d) Use the same formula as developed for solution to Prob. 9-1, but $W \sin\theta = F_{\text{shear}}$. Then

$$
\begin{aligned}
V &= (F_s/LW)\Delta y/\mu \\
F_s &= VLW\mu/\Delta y \\
F_s &= 0.3 \times (1 \times 0.3) \times 4/0.002 \\
F_s &= 180 \text{ N}
\end{aligned}
$$

9.7 Valid statements are (c), (e).

9.8 The shear force is the same on the wire and tube wall; however, there is less area in shear on the wire so there will be a greater shear stress on the wire.

9.9 Assume a linear velocity distribution within the oil. The velocity distribution will appear as below:



Because the lower plate is moving at a constant speed, the shear stresses on the top and bottom of it will be the same, or

$$\begin{aligned}
\tau_1 &= \tau_2 \\
\mu_1 dV_1/t_1 &= \mu_2 dV_2/t_2 \\
\mu_1 \times (V - V_{\text{lower}})/t_1 &= \mu_2 V_{\text{lower}}/t_2 \\
V\mu_1/t_1 - \mu_1 V_{\text{lower}}/t_1 &= \mu_2 V_{\text{lower}}/t_2 \\
V_{\text{lower}}(\mu_2/t_2 + \mu_1/t_1) &= V\mu_1/t_1 \\
V_{\text{lower}} &= \underline{\underline{(V\mu_1/t_1)/(\mu_2/t_2 + \mu_1/t_1)}}
\end{aligned}$$

9.10 **Information and assumptions**

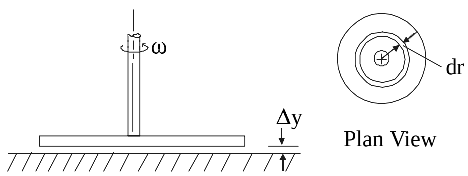
provided in problem statement

Find

torque required to rotate disc.

Solution

$$\begin{aligned}\tau &= \mu dv/dy\\ \tau &= \mu r\omega/\Delta y\\ dT &= rdF\\ dT &= r\delta dA\\ dT &= r(\mu r\omega/\Delta y)2\pi r dr\end{aligned}$$



Then

$$\begin{aligned}T &= \int_0^r dT = \int_0^{r_0}(\mu\omega/\Delta y)2\pi r^3 dr\\ T &= (2\pi\mu\omega/\Delta y)r^4/4|_0^{r_0} = 2\pi\mu\omega r_0^4/(4\Delta y)\end{aligned}$$

For

$$\begin{aligned}\Delta y &= 0.001 \text{ ft};\ r_0 = 6\text{”} = 0.50 \text{ ft};\ \omega = 180\times 2\pi/60 = 6\pi \text{ rad/s}\\ \mu &= 0.12 \text{ lbf-s/ft}^2\\ T &= (2\times 0.12\times 6\pi/0.001)(0.5^4/4)\\ \underline{\underline{T}} &= \underline{\underline{222 \text{ ft-lbf}}}\end{aligned}$$

9.11 **Information and assumptions**

provided in problem statement

Find

torque required to rotate disc.

Solution

The problem is the same type as Prob. 9.10; therefore,

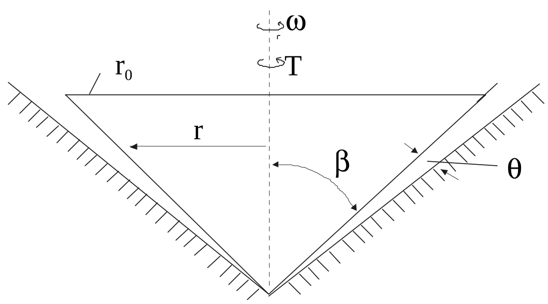
$$T - 2\pi\mu\omega r_0^4/(4\Delta y)$$

where

$$\begin{aligned}
r &= 0.10\text{ m; } \Delta y = 2\times 10^{-3}\text{ m; } \omega = 110\text{ rad/s; } \mu = 6\text{ N}\cdot\text{s/m}^2 \\
T &= 2\pi\times 8\times 10\times 10^{-4}/(4\times(2\times 10^{-3})) = \underline{\underline{6.28\text{ N}\cdot\text{m}}}
\end{aligned}$$

9.12

$$\begin{aligned}
dT &= (\mu u/s)dA \times r \\
&= \mu r\omega \sin\beta 2\pi r^2 dr/(r\theta\sin\beta) \\
&= 2\pi\mu\omega r^2 dr/\theta \\
T &= (\mu\omega/\theta)(2\pi r^3/3)|_0^{r_0} = \underline{\underline{(2/3)\pi r_0^3 \mu\omega/\theta}}
\end{aligned}$$



9.13 **Information and assumptions**

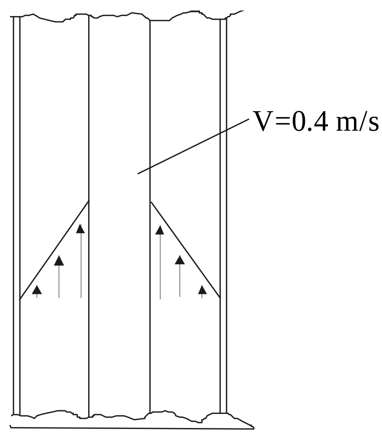
provided in problem statement

Find

force required to pull plate.

Solution

Velocity distribution:



Force

$$
\begin{aligned}
&= \tau A \\
&= \mu(dV/dy)A \\
&= (0.62 \times (0.4/0.002) \times 1 \times 2) \times 2 \\
F &= \underline{\underline{496 \text{ N}}}
\end{aligned}
$$

9.14 **Information and assumptions**

provided in problem statement

Find

torque required to turn bearing.

Solution

$$
\begin{aligned}
\tau &= \mu V/\delta \\
T &= \tau A r
\end{aligned}
$$

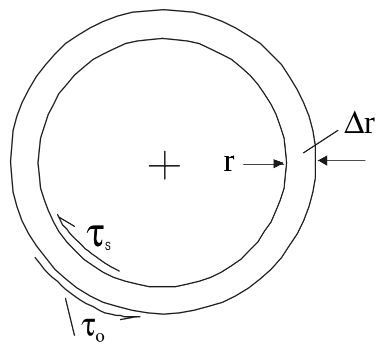
where T = torque, A = bearing area = $2\pi rb$

$$
\begin{aligned}
T &= \tau 2\pi rbr = \tau 2\pi r^2 b \\
&= (\mu V/\delta)(2\pi r^2 b)
\end{aligned}
$$

where V=rω. Then

$$
\begin{aligned}
&= (\mu/\delta)(r\omega)(2\pi r^2 b) \\
&= (\mu/\delta)(2\pi\omega)r^3 b \\
&= (0.1/0.001)(2\pi)(200)(0.015)^3(0.1) \\
T &= \underline{\underline{42.4 \times 10^{-4}\mathrm{N \cdot m}}}
\end{aligned}
$$

9.15 Subscript s refers to inner cylinder. Subscript o refers to outer cylinder. The cylinder is unit length into page.



$$
\begin{aligned}
T_s &= \tau(2\pi r)(r) \\
T_o &= \tau(2\pi r)(r) + d/dr(\tau 2\pi r \cdot r)\Delta r \\
T_s - T_o &= 0 \\
d/dr(\tau 2\pi r^2 \ell)\Delta r &= 0; d/dr(\tau r^2) = 0
\end{aligned}
$$

Since there is no angular acceleration, the sum of the torques must be zero. Therefore

$$
\begin{aligned}
T_s - T_o &= 0 \\
d/dr(\tau 2\pi r^2)\Delta r &= 0 \\
d/dr(\tau r^2) &= 0
\end{aligned}
$$

Then

$$
\begin{aligned}
\tau r^2 &= C_1 \\
\tau &= \mu r(d/dr)(v/r)
\end{aligned}
$$

So

$$
\begin{aligned}
\mu r^3(d/dr(V/r)) &= C_1 \\
\mu(d/dr(V/r)) &= C_1 r^{-3}
\end{aligned}
$$

Integrating,

$$\mu v/r = (-1/2)C_1 r^{-2} + C_2$$

At $r = r_o$, $v = 0$ and at $r = r_s$, $v = r_s\omega$ so

$$
\begin{aligned}
C_1 &= 2C_2 r_0^2 \\
\mu\omega &= C_2(1 - r_0^2/r_s^2) \\
C_2 &= \mu\omega/(1 - r_0^2/r_s^2)
\end{aligned}
$$

Then

$$\tau_s = C_1 r_s^{-2} = 2C_2(r_0/r_s)^2 = 2\mu\omega r_0^2/(r_s^2 - r_0^2) = 2\mu\omega/((r_s^2/r_0^2) - 1)$$

So

$$T_s = \tau 2\pi r_s^2 = 4\pi\mu\omega r_s^2/((r_s^2/r_0^2) - 1)$$

which is the torque on the fluid. Torque on shaft per unit length

$$T = \underline{\underline{4\pi\mu\omega r_s^2/(1 - (r_s^2/r_0^2))}}$$

9.16 **Information and assumptions**

provided in problem statement

Find

power necessary to rotate shaft.

Solution

$$\begin{aligned} P &= T\omega \\ T &= 4\pi\mu\omega\ell r_s^2/(1-(r_s^2/r_0^2)) \\ &= 4\pi \times 0.1 \times (50)(0.01)^2 0.04/(1-(1/1.1)^2) = 0.00145 \text{ N}\cdot\text{m} \\ P &= 0.00145(50) = \underline{\underline{0.0725 \text{ W}}} \end{aligned}$$

9.17 **Information and assumptions**

provided in problem statement

Find

viscosity of fluid

Solution

$$
\begin{aligned}
T &= 0.6(0.02) = 0.012 \text{ N} \cdot \text{m} \\
\mu &= T(1 - r_s^2/r_0^2)/(4\pi\omega\ell r_s^2) = 0.012(1 - 2^2/2.25^2)/(4\pi(20)(2\pi/60)(0.1)(0.02)^2) \\
&= \underline{\underline{2.39 \text{ N} \cdot \text{s/m}^2}}
\end{aligned}
$$

9.18 **Information and assumptions**

provided in problem statement

Find

maximum and mean velocity of flow

Solution

$$u = (g \sin\theta/2\nu) y(2d - y)$$

$u_{\max}$ occurs at the liquid surface where $y = d$, so

$$\begin{aligned}
u_{\max} &= (g \sin\theta(2\nu))d^2 \\
\text{where } \theta &= 30^\circ, \nu = 10^{-3} \text{ m}^2/\text{s and d=}3.0 \times 10^{-3} \text{ m} \\
u_{\max} &= (9.81 \times \sin 30^\circ/(2 \times 10^{-3})) \times (3.0 \times 10^{-3})^2 \\
u_{\max} &= 22 \times 10^{-3} \text{ m/s} = \underline{\underline{0.022 \text{ m/s}}} \\
V &= (gd^2 \sin\theta)/(3\nu) \\
&= 9.81 \times (3.0 \times 10^{-3})^2 \sin 30^\circ/(3 \times 10^{-3}) \\
V &= 0.015 \text{ m/s} \\
V &= \underline{\underline{(2/3)u_{\max}}}
\end{aligned}$$

9.19 **Information and assumptions**

provided in problem statement

Find

depth and discharge per unit depth

Solution

$$
\begin{aligned}
\text{Re} &= Vd/\nu = q/\nu \\
q &= 200 \times 1.2 \times 10^{-3} = \underline{\underline{0.240 \text{ cfs/ft}}} \\
q &= (1/3)\gamma d^3 \sin\theta/\mu \\
d^3 &= 3\mu g/\gamma \sin\theta = 3\nu q/g \sin\theta = \\
& \quad 3 \times 1.2 \times 10^{-3} \times 0.24/(32.2 \times 0.707) = 0.3795 \times 10^{-4} \text{ ft}^3 \\
d &= 0.0336 \text{ ft } = \underline{\underline{0.403 \text{ in}}}.
\end{aligned}
$$

9.20 **Information and assumptions**

provided in problem statement

Find

depth and average velocity

Solution

Total discharge per unit width of roof is:

$$q = L \times 1 \times R_r \tag{1}$$

where R_r = rainfall rate. But

$$q = (1/3)(\gamma/\mu)d^3 \sin\theta$$

or

$$d = (3q\mu/(\gamma \sin\theta))^{1/3}$$

Combining equations 1 and 2, gives

$$d = (3LR_r\mu/(\gamma \sin\theta))^{1/3}$$

In this problem L = 15 ft; R_r = 0.4 in./hr. = 9.26×10^{-6} ft/s, $\mu = 2.73 \times 10^{-5}$ lb-s/ft^2; $\gamma = 62.4$ lbf/ft^3; $\theta = 10°$. Then

$$\begin{aligned} d &= (3 \times 15 \times 9.26 \times 10^{-6} \times 2.73 \times 10^{-5}/(62.4 \times \sin 10°))^{1/3} \\ d &= 1.02 \times 10^{-3} \text{ ft} = \underline{\underline{0.012 \text{ in}}}. \end{aligned}$$

Using Eq. 9.9a

$$\underline{\underline{V = 0.137 \text{ ft/s}}}$$

9.21 **Information and assumptions**

provided in problem statement

Find

shearing force on upper plate.

Solution

$$u = -(\gamma/2\mu)(By - y^2)dh/ds$$

$u_{\max}$ occurs at $y = B/2$ so

$$u_{\max} = -(\gamma/2\mu)(B^2/2 - B^2/4)dh/ds = -(\gamma/2\mu)(B^2/4)dh/ds$$

From problem statement $dp/ds = -1,200$ Pa/m and $dh/ds = (1/\gamma)dp/ds$. Also $B = 2$ mm= 0.002 m and $\mu = 10^{-1}$N·s/m^2. Then

$$\begin{aligned}
u_{\max} &= -(\gamma/2\mu)(B^2/4)((1/\gamma)(-1,200)) \\
&= (B^2/8\mu)(1,200) \\
&= (0.002^2/(8 \times 0.1))(1,200) = 0.006 \text{ m/s} = \underline{\underline{6.0 \text{ mm/s}}} \\
F_s &= \tau A = \mu(du/dy) \times 2 \times 1.5 \\
\tau &= \mu \times [-(\gamma/2\mu)(B - 2y)dh/ds]
\end{aligned}$$

but τ_{plate} occurs at $y = 0$. So

$$\begin{aligned}
F_s &= -\mu \times (\gamma/2\mu) \times B \times (-1,200/\gamma) \times 3 = (B/2) \times 1,200 \times 3 \\
F_s &= (0.002/2) \times 1,200 \times 3 = \underline{\underline{3.6 \text{ N}}}
\end{aligned}$$

9.22 **Information and assumptions**

provided in problem statement

Find

maximum fluid velocity in $x-$direction.

Solution

From the solution to problem 9-21 we have

$$u_{\max} = -(\gamma B^2/8\mu)((1/\gamma)(dp/ds))$$

so

$$u_{\max} = -(0.01^2/(8 \times 10^{-3}))(-12) = \underline{\underline{0.150 \text{ ft/s}}}$$

9.23

$$
\begin{aligned}
h_A &= (p_A/\gamma) + z_A = (150/000) + 0 = 1.5 \text{ (Assume } z_A = 0) \\
h_B &= (p_B/\gamma) + z_B = (100/100) + 1 = 2 \\
h_B &> h_A;
\end{aligned}
$$

Therefore flow is from B to A: ($\underline{\underline{\text{downward}}}$)

9.24 **Information and assumptions**

from Table A.4 $\mu = 6.2 \times 10^{-1}$ N·s/m^2, $\nu = 5.1 \times 10^{-4}$ m^2/s
provided in problem statement

Find

discharge per unit width

Solution

Assume the flow will be laminar.

$$q = -\frac{B^3\gamma}{12\mu}\frac{dh}{ds}$$

$$\begin{aligned} dh/ds &= d/ds(p/\gamma + z) \\ &= (1/\gamma)dp/ds + dz/ds \\ &= -1 \end{aligned}$$

since $dp/ds = 0$. Then

$$\begin{aligned} q &= -\frac{B^3\gamma}{12\mu}\frac{dh}{ds} = -(-1)(0.005^3 \times 12,300)/(12 \times 6.2 \times 10^{-1}) \\ &= \underline{\underline{0.00021 \text{ m}^2/\text{s}}} \end{aligned}$$

Now check to see if the flow is laminar (Re $< 1,000$) :

$$\begin{aligned} \text{Re} &= VB/\nu = qB/\nu \\ &= (0.00021 \text{ m}^2/\text{s})(0.005 \text{ m})/(5.1 \times 10^{-4} \text{ m}^2/\text{s}) \\ \text{Re} &= 0.002 \leftarrow \text{ Laminar} \end{aligned}$$

Therefore our original assumption of laminar flow was correct

9.25 **Information and assumptions**

provided in problem statement

Find

maximum fluid velocity in $z-$direction

Solution

The expression for $u_{\max}$ is

$$u_{\max} = -\frac{B^2\gamma}{8\mu}\frac{dh}{ds}$$

where

$$\begin{aligned} dh/ds &= dh/dz = d/dz(p/\gamma + z) \\ &= (1/\gamma)dp/dz + 1 \\ &= (1/(0.8 \times 62.4))(-8) + 1 = -0.16 + 1 = 0.840 \end{aligned}$$

Then

$$\begin{aligned} u_{\max} &= -((0.8 \times 62.4 \times 0.01^2)/(8 \times 10^{-3})(0.840) \\ u_{\max} &= \underline{\underline{-0.524 \text{ ft/s}}} \end{aligned}$$

Flow is downward.

9.26 **Information and assumptions**

provided in problem statement

Find

maximum fluid velocity in z-direction.

Solution

$$u_{\max} = -\frac{B^2\gamma}{8\mu}\frac{dh}{ds}$$

where

$$\begin{aligned} dh/ds = dh/dz &= d/dz(p/\gamma + z) \\ &= (1/\gamma)dp/dz + 1 \\ &= (1/(0.85 \times 9,810)(-10,000) + 1 \\ &= -0.199 \end{aligned}$$

Then

$$\begin{aligned} u_{\max} &= -(0.85 \times 9,810 \times 0.002^2)/(8 \times 0.1)(-0.199) \\ &= 0.00830 \text{ m/s} = \underline{\underline{3.31 \text{ mm/s}}} \end{aligned}$$

Flow is upward.

9.27 **Information and assumptions**

provided in problem statement

Find

maximum fluid velocity in z-direction

Solution

From solution to Prob. 9-21 we have

$$u_{\max} = -\frac{B^2\gamma}{8\mu}\frac{dh}{ds}$$

where

$$\begin{aligned} dh/ds &= dh/dz = d/dz(p/\gamma + z) \\ &= (1/\gamma)dp/dz + 1 \\ &= (1/(0.8 \times 62.4))(-60)) + 1 = -0.202 \end{aligned}$$

Then

$$\begin{aligned} u_{\max} &= -(0.8 \times 62.4 \times 0.01^2)/(8 \times 0.001)(-0.202) \\ &= \underline{\underline{+0.126 \text{ ft/s}}} \end{aligned}$$

The flow is upward.

9.28 **Information and assumptions**

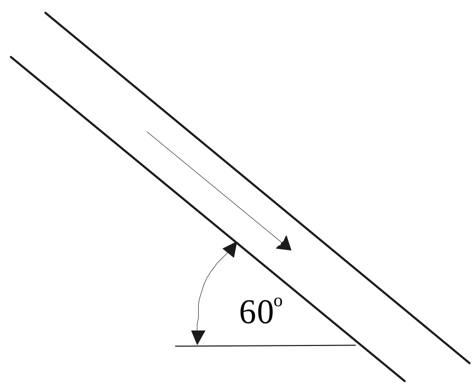
from Table A.4 $\mu = 2 \times 10^{-3}$ lbf·s/ft^2; $\gamma = 55.1$ lbf/ft^3
provided in problem statement

Find

pressure gradient in the direction of flow.

Solution

$$\begin{aligned} \bar{V} &= q/B = 0.009/(0.10/12) = 1.08 \text{ ft/s} \\ u_{\max} &= (3/2)\bar{V} = 1.62 \text{ ft/s} \end{aligned}$$



$$\begin{aligned} u_{\max} &= -\frac{B^2\gamma}{8\mu}\frac{dh}{ds} \\ \frac{dh}{ds} &= -\frac{8\mu u_{\max}}{\gamma B^2} \\ &= -(8 \times 2 \times 10^{-3} \times 1.62)/(55.1 \times (0.1/12)^2) = -6.77 \end{aligned}$$

But

$$dh/ds = (1/\gamma)dp/ds + dz/ds$$

where $dz/ds = -0.866$. Then

$$\begin{aligned} -6.77 &= (1/\gamma)dp/ds - 0.866 \\ dp/ds &= \gamma - (6.77 + 0.866) = \underline{\underline{-325 \text{ psf/ft}}} \end{aligned}$$

9.29 **Information and assumptions**

provided in problem statement

Find

pressure gradient in direction of flow.

Solution

$$\begin{aligned}\bar{V} &= q/B = 24 \times 10^{-4}/(0.002) = 1.2 \text{ m/s} \\ u_{\max} &= (3/2)\bar{V} = 1.8 \text{ m/s}\end{aligned}$$

From solution to Prob. 9-28

$$\begin{aligned}\frac{dh}{ds} &= -\frac{8\mu u_{\max}}{\gamma B^2} \\ dh/ds &= -8 \times 0.1 \times 1.8/(0.8 \times 9,810 \times 0.002^2) = -45.87\end{aligned}$$

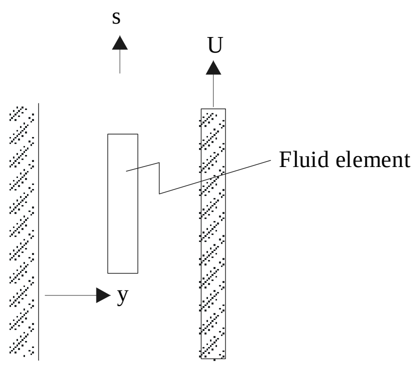
But

$$dh/ds = (1/\gamma)dp/ds + dz/ds$$

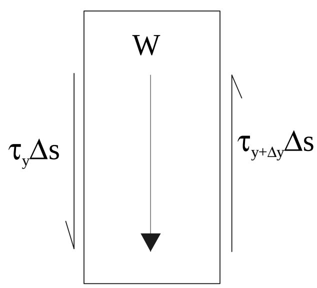
where $dz/ds = -0.866$.Then

$$\begin{aligned}-45.87 &= (1/\gamma)dp/ds - 0.866 \\ dp/ds &= \gamma(-45.87 + 0.866) = \underline{\underline{-353 \text{ kPa/m}}}\end{aligned}$$

9.30 Consider the fluid element between the plates



Consider the forces on the fluid element



$$-\tau_y \Delta s + \gamma_{y+\Delta s} y \Delta s - \gamma \Delta s \Delta y = 0$$

Divide by $\Delta s \Delta y$

$$\frac{-\tau_y}{\Delta y} + \frac{\tau_{y+\Delta y}}{\Delta y} - \gamma = 0$$

Take the limit as Δy approaches zero

$$d\tau/dy = \gamma$$

But

$$\tau = \mu du/dy$$

So

$$\frac{d}{dy}(\mu du/dy) = \gamma$$

Integrate

$$
\begin{aligned}
\mu du/dy &= \gamma y + C_1 \\
du/dy &= \frac{\gamma}{\mu} y + C_1
\end{aligned}
$$

Integrate again

$$u = \frac{\gamma}{\mu}\frac{y^2}{2} + C_1 y + C_2$$

Boundary Conditions: At $y = 0, u = 0$ and at $v = L, u = U$. Therefore,

$$C_2 = 0 \text{ and } C_1 = \frac{U}{L} - \frac{\gamma}{\mu}\frac{L}{2}$$

$$\underline{\underline{u = \frac{\gamma}{\mu}\frac{y^2}{2} + \left(\frac{U}{L} + \frac{\gamma}{\mu}\frac{L}{2}\right) y}}$$

The discharge per unit dimension (normal to page) is given by

$$
\begin{aligned}
q &= \int_0^L u dy \\
&= \int_0^L \left[\frac{\gamma}{\mu}\frac{y^2}{2} + \left(\frac{U}{L} - \frac{\gamma}{\mu}\frac{L}{2}\right) y\right] dy \\
&= \frac{\gamma}{\mu}\frac{y^3}{6} + \frac{Uy^2}{2L} - \frac{\gamma}{\mu}\frac{Ly^2}{4}|_0^L \\
&= \frac{\gamma}{\mu}\frac{L^3}{6} + \frac{UL}{2} - \frac{\gamma}{\mu}\frac{L^3}{4}
\end{aligned}
$$

For zero discharge

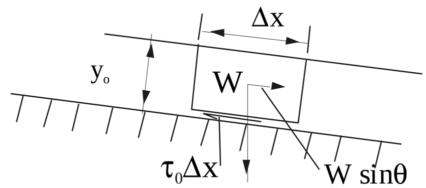
$$\frac{UL}{2} = \frac{\gamma L^3}{4\mu} - \frac{\gamma}{\mu}\frac{L^3}{6}$$

or

$$\underline{\underline{U = \frac{1}{6}\frac{\gamma}{\mu} L^2}}$$

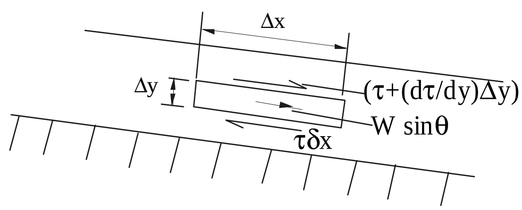
9.31

a) First consider the forces on an element of mud Δx long and y_0 deep as shown below.



$$\underline{\underline{\text{There will be no motion if } \gamma y_0 \sin\theta < \tau_0}}$$

b) Determine the velocity field when there is flow. Consider forces on the element of mud shown below. Assume unit dimension normal to page.



$$\sum F_x = 0$$
$$-\tau\Delta x + (\tau + (d\tau/dy)\Delta y)\Delta x = 0$$
$$(d\tau/dy)\Delta y - \gamma\sin\theta\Delta y = 0$$
$$d\tau/dy = -\gamma\sin\theta$$
$$\tau = -\int \gamma\sin\theta dy + C$$
$$= -\gamma\sin\theta y + C$$

when $y = 0$, $\tau = 0$ so

$$\begin{aligned} C &= \gamma\sin\theta y_0 \\ \tau &= -\gamma\sin\theta y + \gamma\sin\theta y_0 \qquad (1) \end{aligned}$$

and

$$\tau = \gamma\sin\theta(y_0 - y)$$

But for the mud

$$\tau = \tau_0 + \eta du/dy \tag{2}$$

Eliminate τ between equations (1) and (2)

$$\begin{aligned} \tau_0 + \eta du/dy &= \gamma \sin\theta(y_0 - y) \\ du/dy &= \left[\gamma \sin\theta(y_0 - y) - \tau_0\right]/\eta \end{aligned} \tag{3}$$

Upon integration

$$u = (1/\eta)\left[\gamma \sin\theta(y_0 y - y^2/2) - \tau_0 y\right] + C$$

when

$$y = 0,\ u = 0 \implies C = 0$$

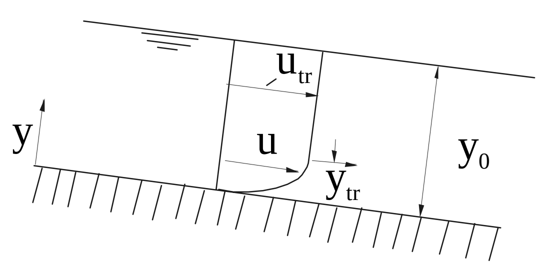
If $\tau < \tau_0$, $du/dy = 0$. Transition point is obtained from Eq. (3)

$$\begin{aligned} 0 &= (\gamma \sin\theta(y_0 - y) - \tau_0) \\ \tau_0 &= \gamma \sin\theta(y_0 - y) \\ \tau_0 &= \gamma \sin\theta y_0 - \gamma \sin\theta y \\ y &= \frac{\gamma \sin\theta y_0 - \tau_0}{\gamma \sin\theta} && (4) \\ y_u &= y_0 - (\tau_0/\gamma \sin\theta) && (5) \end{aligned}$$

When $0 < y < y_{tr}, \tau > \tau_0$ and

$$u = \left[\gamma \sin\theta(yy_0 - y^2/2) - \tau_0 y\right]/\eta \tag{6}$$

When $y_{tr} < y < y_0, \tau < \tau_0$ so $u = u_{\max} = u_{tr}$ and the velocity distribution is shown on the figure.



9.32 **Information and assumptions**

provided in problem statement

Find

the discharge

Solution

$$\begin{aligned} q &= (2/3)u_{\max}B;\ dh/ds = -1 \\ u &= -(\gamma/2\mu)(By - y^2)dh/ds \end{aligned}$$

and $u = u_{\max}$ when $y = B/2$ so

$$\begin{aligned} u_{\max} &= -(\gamma/2\mu)((B^2/2) - (B^2/4))dh/ds \\ &= -(\gamma/2\mu)(B^2/4)dh/ds \end{aligned}$$

Thus

$$q = (2/3)(-\gamma/2\mu)(B^2/4)(dh/ds)(B)$$

or

$$\begin{aligned} Q &= -(2/3)(\gamma/2\mu)(B^3/4)(dh/ds)(\pi D) \\ &= (-2/3)(12,300/(2 \times 6.2 \times 10^{-1}))((1.5 \times 10^{-3})^3/4)(-1)(\pi \times 0.0185) \\ Q &= \underline{\underline{3.24 \times 10^{-7}\ \mathrm{m^3/s}}} \end{aligned}$$

9.33 **Information and assumptions**

provided in problem statement

Find

amount of oil pumped per hour

Solution

$$\begin{aligned} F &= p_{\text{avg.}} \times A \\ &= 1/2\, p_{\max} \times A \\ &= 1/2\, p_{\max} \times 0.3 \text{ m} \times 1 \text{ m} \end{aligned}$$

or

$$\begin{aligned} p_{\max} &= 2F/0.3 \text{ m}^2 = 2 \times 50,000/0.30 \\ &= 333,333 \text{ N/m}^2 \end{aligned}$$

Then $dp/ds = -333,333 \text{ N/m}^2/0.15 \text{ m} = -2,222,222 \text{ N/m}^3$. For flow between walls where $\sin\theta = 0$, we have

$$\begin{aligned} u_{\max} &= -(\gamma/2\mu)(BxB/2 - B^2/4)(d/ds(p/\gamma)) \\ u_{\max} &= -(B^2 8\mu)dp/ds \\ V_{\text{avg.}} &= 2/3 u_{\max} \\ &= -(1/12)(B^2/\mu)dp/ds \end{aligned}$$

Then

$$q_{\text{per side}} = VB = -(1/12)(B^3/\mu)dp/ds$$

and

$$\begin{aligned} q_{\text{total}} &= 2VB = -(1/6)(B^3/\mu)dp/ds \\ &= -(1/6) \times ((6 \times 10^{-4} \text{ m})^3/(20^{-1} \text{ N} \cdot \text{ s/m}^2)) \times -2,222,222 \text{ N/m}^3) \\ &= 4.00 \times 10^{-4} \text{ m}^3/\text{s} \\ q &= \underline{\underline{1.44 \text{ m}^3/\text{hr}}} \end{aligned}$$

9.34 The flow is steady and incompressible. There is no pressure gradient in the flow direction. Let x be in the flow direction and y is the cross-stream direction. In the Couette flow problem

$$\frac{\partial u}{\partial x} = 0$$

so from the continuity equation

$$\frac{\partial v}{\partial y} = 0$$

or $v =$constant. The constant must be zero to satisfy the boundary conditions.
The x-component of the Naiver Stokes equation reduces to

$$\frac{d^2 u}{dy^2} = 0$$

Integrating twice gives

$$u = C_1 y + C_2$$

Applying the boundary conditions that $u(0) = 0$ and $u(L) = U$ gives

$$\underline{\underline{u = U\frac{y}{L}}}$$

9.35 For flow down an inclined plane, the flow is steady and uniform (no change in the flow direction). The fluid is also incompressible. Let x be in the flow direction and y in the cross-stream direction. Since the flow is uniform

$$\frac{\partial u}{\partial x} = 0$$

so the continuity equation gives

$$\frac{\partial v}{\partial y} = 0$$

Thus v must be equal to a constant and the constant must be zero to satisfy the boundary conditions. The Naiver-Stokes equation in the x direction reduces to

$$0 = \mu \frac{d^2 u}{dy^2} + \rho g \sin\theta$$

since u is only a function of y and where θ is the angle of inclination. Thus the differential equation can be written as

$$\frac{d^2 u}{dy^2} = -\frac{g}{\nu} \sin\theta$$

The boundary conditions are $u(0) = 0$ and $u'(d) = 0$. Integrating once gives

$$\frac{du}{dy} = -\frac{g}{\nu} \sin\theta y + C_1$$

Applying the boundary condition at L gives

$$C_1 = \frac{g}{\nu} \sin\theta d$$

so

$$\frac{du}{dy} = \frac{g}{\nu} \sin\theta (d - y)$$

Integrating again

$$u = \frac{g}{\nu} \sin\theta (dy - \frac{y^2}{2}) + C_2$$

Applying the boundary condition at 0 gives $C_2 = 0$ so the final result is

$$\underline{\underline{u = \frac{g}{\nu} \sin\theta (dy - \frac{y^2}{2})}}$$

9.36

$$
\begin{aligned}
q &= -(B^3/(12\mu))dp/dr \\
Q &= 2\pi r q = \text{ constant} \\
Q &= 2\pi r(-B^3/(12\mu))dp/dr
\end{aligned}
$$

Separate variables:

$$((-\pi B^3)/(6\mu))dp = Q(dr/r)$$

Integrate:

$$-\pi B^3 p/(6\mu) = Q\ln r + C$$

Use boundary conditions to determine $\Delta p:$ p_1 at r_1 and p_2 at r_2

$$
\begin{gathered}
\therefore (-\pi B^3/(6\mu))p_1 = Q\ln r_1 + C \\
(\pi B^3/(6\mu))p_2 = Q\ln r_2 + C \\
(p_2 - p_1)(\pi B^3/(6\mu)) = Q(\ln r_1 - \ln r_2) \\
\underline{\underline{\Delta p = (6\mu/(\pi B^3))Q\ln(r_1/r_2)}}
\end{gathered}
$$

For $r_1 = 0.01$ m, $r_2 = 0.1$ m, $B = 0.005$ m and $Q = 0.0003$ m^3 /s we have:

$$
\begin{aligned}
\Delta p &= ((6 \times 3.6 \times 10^{-2})/(\pi \times 0.005^3))(0.0003)\ln(0.01/0.1) \\
\Delta p &= -380 \text{ Pa}
\end{aligned}
$$

Pressure drop $= \underline{\underline{p_1 - p_2 = 380 \text{ Pa}}}$

9.37 **Information and assumptions**

provided in problem statement

Find

thickness of boundary layer, distance from leading edge and shear stress.

Solution

$$
\begin{aligned}
\mathrm{Re} &= U_0 x/\nu \\
x &= \mathrm{Re}\,\nu/U_0 = 500,000 \times 1.22 \times 10^{-5}/5 \\
x &= \underline{\underline{1.22 \text{ ft}}} \\
\delta &= 5x/\mathrm{Re}_x^{1/2} = 5 \times 1.22/(500,000)^{1/2} \\
\delta &= 0.0086 \text{ ft } = \underline{\underline{0.103 \text{ in.}}} \\
\tau_0 &= 0.332\mu(U_0/x)\,\mathrm{Re}_x^{1/2} \\
\tau_0 &= 0.332 \times 2.36 \times 10^{-5}(6/1.22) \times (500,000)^{1/2} = \underline{\underline{0.273 \text{ lbf/ft}^2}}
\end{aligned}
$$

9.38 **Information and assumptions**

provided in problem statement

Find

ratio of the boundary layer thickness to the distance from leading edge just before transition.

Solution

$$\delta/x = 5/\mathrm{Re}_x^{1/2} = 5/(500,000)^{1/2} = 0.0071$$

9.39 **Information and assumptions**

provided in problem statement

Find

shearing force due to laminar part of boundary layer.

Solution

$$F_{\mathrm{s,lam}} = C_f A \rho U_0^2/2$$

where $C_f = 1.33/\mathrm{Re}^{1/2}$.

$$\begin{aligned} C_f &= 1.33/(500,000)^{1/2} = 0.00188 \\ F_{\mathrm{s,lam}} &= (0.00188 \times 3 \times 1.017) \times 1.94 \times 6^2/2 \\ &= \underline{\underline{0.200 \text{ lbf}}} \end{aligned}$$

9.40 At the edge of the boundary layer the shear stress, τ, is approximately zero. Therefore, $\tau/\tau_0 \approx 0$. Choice $\underline{\underline{\text{(a)}}}$ is the correct one.

9.41 **Information and assumptions**

provided in problem statement

Find

force due to shear stress on element.

Solution

This is a turbulent boundary layer; therefore,

$$
\begin{aligned}
c_f &= 0.058\,\mathrm{Re}_x^{-1/5} \\
\mathrm{Re}_x &= Ux/\nu \\
&= 25 \times 1/(1.5 \times 10^{-5}) \\
&= 16.6 \times 10^5 \\
&= 1.66 \times 10^6 \\
\mathrm{Re}_x^{-1/5} &= 0.0570 \\
c_f &= 0.058 \times 0.0570 \\
&= 0.00330 \\
F &= C_f A \times (\rho U^2/2) \\
&= 0.00330 \times (0.01)^2 (1.2)(25)^2/2 \\
&= \underline{\underline{1.24 \times 10^{-4}\,\mathrm{N}}}
\end{aligned}
$$

9.42

$$u/U_0 = (y/\delta)^{1/2} \tau_0 = 1.66 U_0 \mu/\delta$$

$$\tau_0 = \rho U_0^2 d/dx \int_0^\delta (u/U_0(1 - u/U_0))dy$$

$$= \rho u_0^2 d/dx \int_0^\delta ((y/\delta)^{1/2} - (y/\delta))dy$$

$$= \rho U_0^2 d/dx[(2/3)(y/\delta)^{3/2} - 1/2(y/\delta)^2]_0^\delta$$

$$1.66 U_0 \mu/\delta = (1/6)\rho U_0^2 d\delta/dx$$

$$\delta d\delta/dx = 9.96\mu/(\rho U_0)$$

$$\delta^2/2 = 9.96\mu x/(\rho U_0) = 9.96x^2/\mathrm{Re}_x$$

$$\delta = \underline{\underline{4.45x/\mathrm{Re}_x^{1/2}}}$$

For the Blasius solution $\delta = 5x/\mathrm{Re}^{1/2}$

9.43 **Information and assumptions**

provided in problem statement

Find

liquid velocity 1 m from leading edge and 1 mm from surface.

Solution

$$\mathrm{Re}_x = Vx/\nu = 1 \times 1/2 \times 10^{-5} = 50,000$$

The boundary layer is laminar. Use Fig. 9-6 to obtain u/U_0

$$y\mathrm{Re}_x^{0.5}/x = 0.001(5x10^4)^{0.5}/1 = 0.224$$

Then from Fig. 9.6 $u/U_0 \approx 0.075$; $u = \underline{\underline{0.075 \text{ m/s}}}$

9.44 **Information and assumptions**

provided in problem statement

Find

skin friction drag on one side of plate.

Solution

$$
\begin{aligned}
\mathrm{Re}_L &= 1.5 \times 10^5 \\
C_f &= 1.33/\mathrm{Re}_L^{0.5} = 0.00343 \\
F_x &= C_f B L \rho U^2/2 = .00343 \times 1 \times 3 \times 1,000 \times 1^2/2 = \underline{\underline{5.15 \text{ N}}}
\end{aligned}
$$

9.45 **Information and assumptions**

provided in problem statement

Find

velocity 1 m downstream and 10 mm from plate.

Solution

$$\mathrm{Re}_x = Ux/\nu = 6 \times 1/10^{-4} = 6 \times 10^4$$

Therefore the boundary layer is laminar.

$$y\mathrm{Re}_x^{0.5}/x = (0.010)(6 \times 10^4)^{0.5}/1 = 2.45$$

From Fig. 9-6 $u/U_0 = 0.72$. Therefore

$$u = 6 \times 0.72 = \underline{\underline{4.3 \text{ m/s}}}$$

9.46 **Information and assumptions**

provided in problem statement

Find

oil velocity 1 m from leading edge and 10 cm from surface

Solution

$$\mathrm{Re}_x = 1 \times 1/10^{-4} = 10^4$$

Therefore the boundary layer is laminar.

$$y\mathrm{Re}_x^{0.5}/x = 0.10 \times 10^2/1 = 10$$

Therefore the point is outside the boundary layer so $u = U_0 = \underline{\underline{1 \text{ m/s}}}$.

9.47 **Information and assumptions**

provided in problem statement

Find

thickness of boundary layer, distance from leading edge and local shear stress

Solution

$$Re_x = U_0 x/\nu = 500,000$$

$$x = 500,00\nu/U_0 = 500,000 \times 1.31 \times 10^{-6}/1 = \underline{\underline{0.655 \text{ m}}}$$

$$\delta = 5x/\mathrm{Re}_x^{1/2} = 5 \times 0.655/(500,000)^{1/2} = 4.63 \times 10^{-3} \text{ m } = \underline{\underline{4.63 \text{ mm}}}$$

$$\tau_0 = 0.332\mu(U_0/x)\mathrm{Re}_x^{1/2}$$

$$\tau_0 = 0.332 \times 1.31 \times 10^{-3}(2/0.655) \times (500,000)^{1/2} = \underline{\underline{0.94 \text{ N/m}^2}}$$

9.48 **Information and assumptions**

provided in problem statement

Find

shearing resistance on one side of plate and ratio of laminar shearing force to total shearing force.

Solution

$$F_s = 0.664 B\mu U_0 \mathrm{Re}_L^{1/2} = 0.664 \times 1 \times 1.31 \times 10^{-3} \times 2 \times (500,000)^{1/2}$$

$$F_s = \underline{\underline{1.23 \text{ N}}}$$

$$F_{s_{\text{total}}} = C_f A \rho U_0^2/2$$

$$\mathrm{Re}_L = U_0 \times 1/\nu = 2 \times 1/(1.31 \times 10^{-6}) = 1.53 \times 10^6$$

From Fig. $C_f = 0.0030$ from Fig. 9-14

$$\begin{aligned} F_{s_{\text{total}}} &= 0.0030 \times 1 \times 500 \times 4 = 6.0 \text{ N} \\ F_{s_{\text{lam.}}}/F_{s_{\text{total}}} &= 1.23/6.0 = \underline{\underline{0.205}} \end{aligned}$$

9.49 **Information and assumptions**

From Table A.3 $\nu = 1.6 \times 10^{-5}$ m^3/s and $\rho = 1.17$ kg/m^3.
provided in problem statement

Find

a) friction drag on wing, b) power to overcome friction drag, c) fraction of chord which is laminar flow and d) change in drag is boundary tripped at leading edge.

Solution

$$\begin{aligned} U_0 &= (200 \text{ km/hr})(1{,}000 \text{ m/km})/(3{,}600 \text{ s/hr}) \\ U_0 &= 55.56 \text{ m/s} \end{aligned}$$

Then

$$\text{Re} = U_0 L/\nu = (55.56)(2)/(1.6 \times 10^{-5}) = 6.9 \times 10^6$$

From Fig. 9.14, the flow is combined laminar and turbulent

$$\begin{aligned} C_f &= \frac{0.523}{\ln^2(0.06\text{Re})} - \frac{1520}{\text{Re}} = 0.00290 \\ F_s &= C_f B L \rho U_0^2/2 \end{aligned}$$

Wing has two surfaces so

$$\begin{aligned} F_{s,\text{wing}} &= C_f B L \rho U_0^2/2 \\ &= (0.00290)(11)(1.17)(55.56)^2 = \underline{\underline{230 \text{ N}}} \text{ (a)} \end{aligned}$$

Power

$$P = F_{\text{s,wing}} U_0 = 230 \times 55.56 = \underline{\underline{12.78 \text{ kW}}} \text{ (b)}$$

Critical laminar Re $= 5 \times 10^{-5} = U_0 x/\nu$

$$\begin{aligned} x_{cr} &= 5 \times 10^5 \nu/U_0 \\ x_{cr} &= (5 \times 10^5)(1.6 \times 10^{-5})/55.56 \\ x_{cr} &= \underline{\underline{14 \text{ cm}}} \text{ (c)} \end{aligned}$$

If all of boundary layer is turbulent then

$$\begin{aligned} C_f &= 0.074/\text{Re}^{0.2} \\ C_f &= 0.00317 \end{aligned}$$

Then $F_{\text{tripped B.L.}}/F_{\text{normal}} = 0.00317/0.00290 = 1.086$ Change in drag with tripped B.L. is $\underline{\underline{\text{8.6 increase}}}$

9.50 **Information and assumptions**

From Table A.5 $\rho = 998$ kg/m^3; $\nu = 10^{-6}$ m^2/s.
provided in problem statement

Find

velocity 1 cm above plate surface.

Solution

$$\begin{aligned} u_* &= (\tau_0/\rho)^{0.5} = (0.1/998)^{0.5} = 0.01 \text{ m/s} \\ u_* y/\nu &= (0.01)(0.01)/(10^{-6}) = 10^2 \end{aligned}$$

From Fig. 9-10 for $u_* y/\nu = 100$ it is seen that Eq. 9-34 applies

$$\begin{aligned} u/u_* &= 5.57 \log(y u_*/\nu) + 5.56 \\ &= 5.75 \log(100) + 5.56 = 17.06 \\ u &= u_*(17.06) = 0.01(17.06) = \underline{\underline{0.171 \text{ m/s}}} \end{aligned}$$

9.51 **Information and assumptions**

provided in problem statement

Find

resistance of plate and boundary layer thickness at trailing edge.

Solution

$$\mathrm{Re}_L = U_0 L/\nu = 0.20 \times 1.5/(10^{-6}) = 3.0 \times 10^5$$

Re_L is less than 500,000; therefore, laminar boundary layer

$$\begin{aligned} \delta &= 5x/\mathrm{Re}_x^{1/2} = 5 \times 1.5/(3.0 \times 10^5)^{1/2} = 0.0137 \text{ m} = \underline{\underline{13.7 \text{ mm}}} \\ C_f &= 1.33/\mathrm{Re}_L^{1/2} = 1.33/(3.0 \times 10^5)^{1/2} = 0.00243 \\ F_s &= C_f A \rho U_0^2/2 = 0.00243 \times 1.0 \times 1.5 \times 2 \times 1,000 \times 0.20^2/2 = \underline{\underline{0.146 \text{ N}}} \end{aligned}$$

9.52 **Information and assumptions**

provided in problem statement

Find

skin friction drag per unit width of plate and velocity gradient at surface 1 m downstream from leading edge.

Solution

$$
\begin{aligned}
\mathrm{Re}_L &= U_0 L\rho/\mu = 20 \times 2 \times 1.5/10^{-5} = 6 \times 10^6 \\
C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re})} - \frac{1520}{\mathrm{Re}} = 0.00294 \\
F_s &= C_f(2BL)\rho U_0^2/2 \\
&= 0.00294 \times (2 \times 1 \times 2)(1.5 \times 20^2/2) \\
&= \underline{\underline{3.53 \text{ N}}} \\
\mathrm{Re}_{1m} &= 6 \times 10^6 \times (1/2) = 3 \times 10^6 \\
c_f &= 0.058/(3 \times 10^6)^{1/5} = 0.00294 \\
\tau_0 &= c_f \rho U_0^2/2 = 0.00294 \times 1.5 \times 20^2/2 = 0.881 \text{ N/m}^2
\end{aligned}
$$

But

$$\tau_0 = \mu du/dy$$

or

$$
\begin{aligned}
du/dy &= \tau_0/\mu = 0.881/10^{-5} \\
&= \underline{\underline{8.81 \times 10^4 \text{ s}^{-1}}}
\end{aligned}
$$

9.53 Equation 9.44 is

$$\frac{\tau_0}{\rho} = \frac{7}{72}U_0^2\frac{d\delta}{dx}$$

Substituting in Eq. 9.46 gives

$$0.010U_0^2\left(\frac{\nu}{U_0\delta}\right)^{1/6} = \frac{7}{72}U_0^2\frac{d\delta}{dx}$$

Cancelling the U_0's and rearranging gives

$$\frac{72}{7}\times 0.010\left(\frac{\nu}{U_0}\right)^{1/6} = \delta^{1/6}\frac{d\delta}{dx}$$

Separating variables

$$0.1028\left(\frac{\nu}{U_0}\right)^{1/6}dx = \delta^{1/6}d\delta$$

Integrating

$$\frac{6}{7}\delta^{7/6} = 0.1028\left(\frac{\nu}{U_0}\right)^{1/6}x + C$$

But $\delta(0) = 0$ so the constant is zero. Solving for δ gives

$$\delta = (\frac{7}{6}\times 0.1028)^{6/7}\left(\frac{\nu}{U_0}\right)^{1/7}x^{6/7}$$

Dividing through by x results in

$$\underline{\underline{\frac{\delta}{x} = \frac{0.16}{\mathrm{Re}_x^{1/7}}}}$$

9.54 **Information and assumptions**

provided in problem statement

Find

speed at which turbulent boundary layer appears and total drag at this speed

Solution

$$\mathrm{Re}_{\mathrm{turb}} = 5 \times 10^5 = Uc/\nu$$

$$U = (5 \times 10^5)v/c = (5 \times 10^5)(1.58 \times 10^{-4})/0.5 = \underline{\underline{158 \text{ ft/s}}}$$

$$C_f = 1.33/(5 \times 10^5)^{0.5} = 0.00188$$

$$\begin{aligned} F = C_f(\rho U^2/2)A &= (0.00188)((0.00237)(158)^2/(2))(2)(3)(0.5) \\ &= \underline{\underline{0.167 \text{ lbf}}} \end{aligned}$$

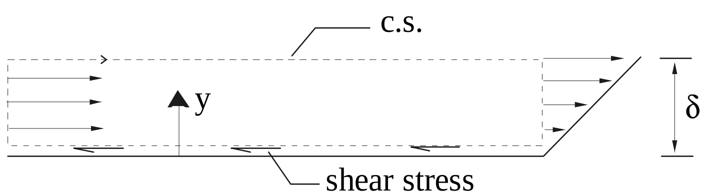
9.55 **Information and assumptions**

provided in problem statement

Find

skin friction drag on top per unit width and shear stress on plate at downstream end.

Solution



Apply the momentum equation to the c.v. shown above.

$$
\begin{aligned}
\sum F_x &= \int_{\text{c.v.}} V_x \rho \mathbf{V} \cdot d\mathbf{A} \\
F_{s,\text{plate on c.v.}} &= -\rho V_1^2 \delta + \int \rho V_2^2 dA + \rho V_1 q_{\text{top}}
\end{aligned}
$$

where

$$
\begin{aligned}
V_2 &= (V_{\max}/\delta) y = V_1 y/\delta \\
q_{\text{top}} &= V_1 \delta - \int_0^\delta V_2 dy = V_1 \delta - \int_0^\delta V_1 y/\delta dy \\
q_{\text{top}} &= V_1 \delta - V_1 y^2/2\delta|_0^\delta = V_1 \delta - 0.5 V_1 \delta = 0.5 V_1 \delta
\end{aligned}
$$

Then

$$
\begin{aligned}
F_s &= -\rho V_1^2 \delta + \int_0^\delta \rho (V_1 y/\delta)^2 dy + 0.5 \rho V_1^2 \delta \\
&= -\rho V_1^2 \delta + \rho V_1^2 \delta/3 + 0.5 \rho V_1^2 \delta \\
&= \rho V_1^2 \delta(-1 + (1/3) + (1/2)) = -0.1667 \rho V_1^2 \delta
\end{aligned}
$$

For $V_1 = 40$ m/s, $\rho = 1.2$ kg/m^3, and $\delta = 3 \times 10^{-3}$ m we have

$$
F_s = -0.1667 \times 1.2 \times 40^2 \times 3 \times 10^{-3} = -0.960 N
$$

or the skin friction drag on top side of plate is $F_s = \underline{\underline{+0.960 \text{ N}}}$. The shear stress at the downstream end of plate is

$$
\tau_0 = \mu dV/dy = 1.8 \times 10^{-5} \times 40/(3 \times 10^{-3}) = \underline{\underline{0.24 \text{ N/m}^2}}
$$

9.56 Equation 9.43 is

$$\frac{\tau_0}{\rho} = U_0^2 \frac{d}{dx} \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7} \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy$$

Changing the variable of integration to

$$\eta = \left(\frac{y}{\delta}\right)$$

the integral becomes

$$\begin{aligned} \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7} \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy &= \delta \int_0^1 \eta^{1/7} \left[1 - \eta^{1/7}\right] d\eta \\ &= \delta \int_0^1 [\eta^{1/7} - \eta^{2/7}] d\eta \end{aligned}$$

Integrating we have

$$\delta \int_0^1 [\eta^{1/7} - \eta^{2/7}] d\eta = \delta[\frac{7}{8}\eta^{8/7} - \frac{7}{9}\eta^{9/7}]_0^1 = \frac{7}{72}\delta$$

The equation then becomes

$$\frac{\tau_0}{\rho} = \frac{7}{72} U_0^2 \frac{d\delta}{dx}$$

9.57

$$\dot{m} = \int_0^\delta \rho u dy = \int_{\delta^*}^\delta \rho_\infty U_\infty dy = \rho_\infty U_\infty(\delta - \delta^*)$$

$$\rho_\infty U_\infty \delta^* = \rho_\infty U_\infty \delta - \int_0^\delta \rho u dy$$

$$= \rho_\infty U_\infty \int_0^\delta (1 - (\rho u)/\rho_\infty U_\infty) dy$$

$$\therefore \delta^* = \int_0^\delta (1 - (\rho u)/(\rho_\infty U_\infty)) dy$$

9.58 **Information and assumptions**

provided in problem statement

Find

magnitude of displacement thickness.

Solution

The streamlines will be displaced a distance $\delta^* = q_{\text{defect}}/V_1$ where

$$q_{\text{defect}} = \int_0^\delta (V_1 - V_2)dy = \int_0^\delta (V_1 - V_1 y/\delta)dy$$

Then

$$\delta^* = [\int_0^\delta (V_1 - V_1 y/\delta)dy]/V_1 = \int_0^\delta (1 - y/\delta)dy = \delta - \delta/2 = \underline{\underline{\delta/2}}$$

9.59 **Information and assumptions**

provided in problem statement

Find

Evaluating the integral for the 1/7th power profile gives

Solution

$$\frac{\tau_0}{\rho} = \frac{7}{72} U_0^2 \frac{d\delta}{dx}$$

Substituting in the expression for shear stress gives

$$\frac{0.0225\nu^{1/4}}{U_0^{1/4}} = \frac{7}{72}\delta^{1/4}\frac{d\delta}{dx}$$

Integrating and using the initial condition at $\delta(0) = 0$ gives

$$\frac{\delta}{x} = \frac{0.37}{\mathrm{Re}_x^{1/5}}$$

Substituting the equation for δ into the equation for shear stress gives

$$c_f = \frac{0.058}{\mathrm{Re}_x^{1/5}}$$

Integrating this over a plate for the average shear stress coefficient gives

$$C_f = \frac{1}{L}\int_0^L c_f dx = \underline{\underline{\frac{0.072}{\mathrm{Re}_L^{1/5}}}}$$

9.60 **Information and assumptions**

provided in problem statement

Find

ratio of skin friction drag on two plates.

Solution

$$F_s = C_f B L \rho U_0^2/2$$

where $C_f = \frac{0.523}{\ln^2(0.06\times\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L}$

$$\begin{aligned}\mathrm{Re}_{L,30} &= 30\times 10/10^{-6} = 3\times 10^8\\ \mathrm{Re}_{L,10} &= 10^8\end{aligned}$$

Then

$$\begin{aligned}C_{f,30} &= 0.00187\\ C_{f,10} &= 0.00213\end{aligned}$$

Then

$$\begin{aligned}F_{s,30}/F_{s,10} &= (0.00187/0.00213)\times 3\\ &= \underline{\underline{2.59}}\end{aligned}$$

9.61 **Information and assumptions**

From Table A.3 $\nu = 1.41 \times 10^{-5}$ m^2/s and $\rho = 1.25$ kg/m^3.
provided in problem statement

Find

power required to pull sign.

Solution

$$
\begin{aligned}
F_x &= C_f B L \rho U_0^2/2 \\
\mathrm{Re}_L &= V_0 L/\nu = 30 \times 30/(1.41 \times 10^{-5}) \\
\mathrm{Re}_L &= 6.38 \times 10^7
\end{aligned}
$$

Then form Fig. 9-14 $C_f = 0.00225$

$$
\begin{aligned}
F_s &= 0.00225 \times 2 \times 30 \times 1.5 \times 1.25 \times 30^2/2 = 113.9 \text{ N} \\
P &= FV = 113.9 \times 30 = \underline{\underline{3.42 \text{ kW}}}
\end{aligned}
$$

9.62 **Information and assumptions**

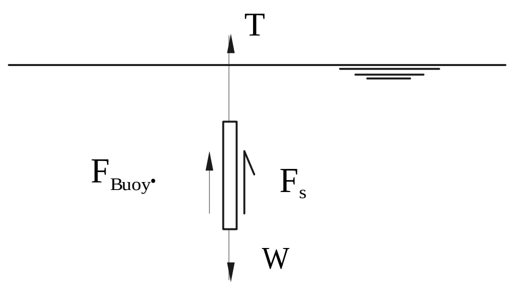
provided in problem statement

Find

tension in cable.

Solution

$$\sum F_z = 0$$
$$T + F_s = F_{\text{Buoy.}} - W = 0$$
$$T = W - F_s - F_{\text{Buoy.}}$$



where $W = 200$ N

$$\begin{aligned} F_{\text{Buoy.}} &= \gamma_{\text{water}} = 0.003 \times 3 \times 9,810 = 88.3 \text{ N} \\ F_s &= C_f A \rho U_0^2/2;\ R_{e_L} = VL/\nu = 2 \times 1/(1.31 \times 10^{-6}) = 1.53 \times 10^6 \end{aligned}$$

Therefore, from Fig. 9-14, $C_f = 0.0030$
Then

$$F_s = 0.0030 \times 2 \times 3 \times 1,000 \times 4/2 = 36.0 \text{ N}$$

Solving for tension in the cable.

$$T = 200 - 36.0 - 88.3 = \underline{\underline{75.7 \text{ N}}}$$

9.63 **Information and assumptions**

provided in problem statement

Find

falling speed in fresh water

Solution

Equating the weight, buoyancy and shearing resistance gives

$$\begin{aligned} W-B &= F_s \\ 23.5-1000\times 9.81\times 0.002 &= \frac{1}{2}\times 1000\times 2\times 2\times C_f\times U_0^2 \end{aligned}$$

or

$$U_0^2=\frac{0.001962}{C_f}$$

Using the equations for C_f for the normal boundary layer and using the direct substitution method gives

$$U_0=\underline{\underline{0.805\text{ m/s}}}$$

9.64 **Information and assumptions**

From Table A.5 $\nu = 10^{-6}$ m^2/s
provided in problem statement

Find

thickness of viscous sublayer 1 m downstream from leading edge.

Solution

$$\delta' = 5\nu/u_*$$

where $u_* = (\tau_0/\rho)^{0.5}$ and $\tau_0 = c_f \rho U^2/2$

$$\begin{aligned} \tau_0/\rho &= (0.058/\,\mathrm{Re}_x^{0.2})U_0^2/2 \\ \mathrm{Re}_x &= U_0 x/\nu \\ &= (5)(1)/10^{-6} = 5 \times 10^6 \\ \mathrm{Re}_x^{0.2} &= 21.87 \end{aligned}$$

Then

$$\begin{aligned} \tau_0/\rho &= (0.058/21.87)(25/2) \\ \tau_0/\rho &= 0.0332 \text{ m}^2/\text{s}^2 \\ u_* &= (\tau_0/\rho)^{0.5} = 0.1822 \text{ m/s} \end{aligned}$$

Finally

$$\delta' = 5\nu/u_* = (5)(10^{-6})/(0.1822)$$

$$\delta' = \underline{\underline{27.4 \times 10^{-6} \text{ m}}}$$

Roughness element size of 100 microns is about 4 times greater than the thickness of the viscous sublayer; therefore, it would definitely affect the skin friction coefficient.

9.65 **Information and assumptions**

From Table A.3 $\rho = 1.2$ kg/m^3; $\nu = 1.51 \times 10^{-5}$ m^2/s^2
provided in problem statement

Find

falling speed

Solution

$$
\begin{aligned}
W. &= F_s = C_f \rho (U_0^2/2) A \\
C_f &= 0.074/\operatorname{Re}^{0.2} \\
W &= 3 = 2(0.074/(\&_0 \times 0.1/(1.51 \times 10^{-5}))^{0.2})(1.2)(U_0^2/2)(1 \times 0.1)
\end{aligned}
$$

Solving for U_0 yields $U_0 = \underline{\underline{67.6 \text{ m/s}}}$.

9.66 **Information and assumptions**

provided in problem statement

Find

total drag force on plate.

Solution

The force due to shear stress is

$$F_s = C_f \frac{1}{2} \rho U_o^2 BL$$

The density and kinematic viscosity of air at 20°C and atmospheric pressure is 1.2 kg/m^3 and 1.5×10^{-5} N·s/m^2, respectively. The Reynolds number based on the plate length is

$$\text{Re}_L = \frac{15 \times 1}{1.5 \times 10^{-5}} = 10^6$$

The average shear stress coefficient on the "tripped" side of the plate is

$$C_f = \frac{0.074}{(10^6)^{1/5}} = 0.0047$$

The average shear stress on the "untripped" side is

$$C_f = \frac{0.523}{\ln^2(0.06 \times 10^6)} - \frac{1520}{10^6} = 0.0028$$

The total force is

$$F_s = \frac{1}{2} \times 1.2 \times 15^2 \times 1 \times 0.5 \times (0.0047 + 0.0028) = \underline{\underline{0.506 \text{ N}}}$$

9.67 **Information and assumptions**

provided in problem statement

Find

length where boundary layers merge and the shearing force per unit depth.

Solution

The density and kinematic viscosity of water at these conditions are 1000 kg/m^3 and 10^{-6} m^2/s. The boundary layer growth is given by

$$\delta = \frac{0.37x}{\mathrm{Re}_x^{1/5}} = \frac{0.37x^{4/5}}{\left(\frac{U_o}{\nu}\right)^{1/5}}$$

Setting $\delta = 0.002$ m and $x = L$, we have

$$L^{4/5} = \frac{0.002}{0.37}\left(\frac{10}{10^{-6}}\right)^{1/5} = 0.135$$

or

$$L = \underline{\underline{0.0824 \text{ m}}}$$

Check the Reynolds number

$$\mathrm{Re}_x = \frac{0.0824 \times 10}{10^{-6}} = 8.24 \times 10^5$$

so the equations for the tripped boundary layer ($Re_x < 10^7$) are valid.
The average shear stress coefficient is

$$C_f = \frac{0.074}{\left(\frac{0.0824\times10}{10^{-6}}\right)^{1/5}} = 0.00485$$

The force due to shear stress on both plates is

$$\frac{F_s}{B} = 2 \times \frac{1}{2}\rho U_o^2 C_f L = 1000 \times 10^2 \times 0.00485 \times 0.0824 = \underline{\underline{40.0 \text{ N/m}}}$$

9.68 Typical results from program.

Normal

Reynolds number	δ/x	c_f	C_f
5×10^5	0.00707	0.000939	0.001881
1.0×10^6	0.0222	0.00376	0.002801
1.0×10^7	0.01599	0.00257	0.002803

Tripped

Reynolds number	δ/x	c_f	C_f
1.0×10^6	0.0233	0..336	.004669
1.0×10^8	0.0115	0.00186	0.00213

9.69 **Information and assumptions**

From Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s
provided in problem statement

Find

power required

Solution

$$\mathrm{Re}_L = U_0 L/\nu = 50 \times (44/30) \times 8/(1.22 \times 10^{-5}) = 4.8 \times 10^7$$

From Fig. 9-14 $C_f = 0.00233$. Then

$$\begin{aligned} F_D &= C_f A \rho U_0^2/2 = 0.00233 \times 32 \times 1.94 \times (73.33)^2/2 = 389 \text{ lbf.} \\ P &= FV = 389 \times 73.33 = 28,525 \text{ ft-lbf/sec} = \underline{\underline{51.9 \text{ hp}}} \end{aligned}$$

9.70 **Information and assumptions**

From Table A.3 $\nu = 1.51 \times 10^{-5}$ m^2/s and $\rho = 1.20$ kg/m^3
provided in problem statement

Find

deceleration, drag and acceleration in head and tail wind and maximum distance.

Solution

$$F_s = C_f A_s \rho U_0^2/2$$

Assume turbulent boundary layer where $A_s = \pi DL = \pi \times 0.025 \times 2.65 = 0.208$ m^2;

$$\begin{aligned}\mathrm{Re}_L &= U_0 L/\nu = 30 \times 2.65/(1.51 \times 10^{-5}) \\ &= 5.3 \times 10^6\end{aligned}$$

Then from Fig. 9-14, $C_f = 0.00297$. Then

$$\begin{aligned}F_s &= 0.00297 \times 0.208 \times 1.2 \times 30^2/2 = 0.334 \text{ N} \\ F &= ma\end{aligned}$$

or

$$a = F/m = 0.334/(8.0/9.81) = \underline{\underline{0.410 \text{ m/s}^2}}$$

With tailwind or headwind C_f will still be about the same value: $C_f \approx 0.00297$. Then

$$\begin{aligned}F_{s,\text{headwind}} &= 0.334 \times (35/30)^2 = \underline{\underline{0.455 \text{ N}}} \\ F_{s,\text{tailwind}} &= 0.334 \times (25/30)^2 = \underline{\underline{0.232 \text{ N}}}\end{aligned}$$

As a first approximation for maximum distance, assume no drag or lift. So for maximum distance, the original line of flight (from release point) will be at 45° with the horizontal–this is obtained from basic mechanics. Also, from basic mechanics:

$$y = -gt^2/2 + V_0 t \sin\theta$$

and

$$x = V_0 t \cos\theta$$

or upon eliminating t from the above with $y = 0$, we get

$$x = 2V_0^2 \sin\theta\cos\theta/g = 2 \times 32^2 \times 0.707^2/9.81 = \underline{\underline{104.4 \text{ m}}}$$

Then

$$t = x/V_0 \cos\theta = 104.4/(32 \times 0.707) = 4.61 \text{ s}$$

Then the total change in velocity over 4.6 s$\approx 4.6 x a_s = 4.6 \times (-0.41) = -1.89$ m/s and the average velocity is $V = (32 + 30.1)/2 = 31$ m/s. Then, a better estimate of distance of throw is: $x = 31^2/9.81 = \underline{\underline{98.0 \text{ m}}}$

9.71 **Information and assumptions**

From table A.5 1.31×10^{-6} m^2/s
provided in problem statement

Find

force required to overcome surface resistance.

Solution

$$\begin{aligned} F_s &= C_f A_s \rho V_0^2/2 \\ \mathrm{Re}_L &= 1.7 \times 50/(1.31 \times 10^{-6}) = 6.49 \times 10^7 \end{aligned}$$

From Fig. 9-14 $C_f = 0.00225$

$$F_s = 0.00225 \times \pi \times 0.5 \times 50 \times 1,000 \times 1.7^2/2 = \underline{\underline{255 \text{ N}}}$$

9.72 **Information and assumptions**

From Table A.3 $\nu = 1.41 \times 10^{-5}$ m^2/s
provided in problem statement

Find

power required.

Solution

$$\begin{aligned}
F_s &= C_f A \rho U_0^2/2 \\
\mathrm{Re}_L &= U_0 L/\nu = (100,000/3,600) \times 150/(1.41 \times 10^{-5}) \\
\mathrm{Re}_{100} &= 2.95 \times 10^8 \\
\mathrm{Re}_{200} &= 5.9 \times 10^8 \\
C_{f_{100}} &= 0.00187 \\
C_{f_{200}} &= 0.00173
\end{aligned}$$

Then

$$\begin{aligned}
F_{s_{100}} &= 0.00187 \times 10 \times 150 \times 1.25 \times (100,000/3,600)^2/2 \\
F_{s_{100}} &= \underline{\underline{1,353 \text{ N}}} \\
P_{100} &= 1,353 \times (100,000/3,600) = \underline{\underline{37.6 \text{ kW}}} \\
F_{s_{200}} &= \underline{\underline{5,006 \text{ N}}} \\
P_{200} &= 5,006 \times (200,000/3,600) = \underline{\underline{278 \text{ kW}}}
\end{aligned}$$

9.73 **Information and assumptions**

From Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s and $\rho = 1.94$ slug/ft^3.
provided in problem statement

Find

a) thickness of boundary layer at 100 m downstream, b) velocity of water at $y/\delta = 0.5$ and c) shear stress on hull.

Solution

$$
\begin{aligned}
\mathrm{Re}_x &= Ux/\nu = (30)(100)/(1.22 \times 10^{-5}) = 2.46 \times 10^8 \\
c_f &= \frac{0.455}{\ln^2(0.06\,\mathrm{Re}_x)} \\
&= 0.00167 \\
\tau_0 &= c_f \rho U_0^2/2 = (0.00167)(1.94)(30^2)/2 = \underline{\underline{1.456 \text{ lbf/ft}^2 \text{ (c)}}} \\
u_* &= (\tau_0/\rho)^{0.5} = (1.456/1.94)^{0.5} = 0.866 \text{ ft/s} \\
\delta/x &= 0.16\,\mathrm{Re}_x^{-1/7} = 0.010 \\
\delta &= (0.010)(100) = \underline{\underline{1.0 \text{ ft}}} \text{ (a)} \\
\delta/2 &= 0.5 \text{ ft}
\end{aligned}
$$

From Fig. 9-12 at $y/\delta = 0.50$, $(U_0 - u)u_* \approx 3.0$. Then

$$(30 - u)/0.866 = 3.0$$

$$\underline{\underline{u_{\delta/2}{=}27.4 \text{ ft/s}}} \text{ (b)}$$

9.74 **Information and assumptions**

From Table A.5 $\nu = 1.41 \times 10^{-5}$ ft^2/s and $\rho = 1.94$ slugs/ft^3.
provided in problem statement

Find

skin friction drag on ship.

Solution

$$F_s = C_f A_s \rho U_0^2/2$$

where $C_f = f(\text{Re}_L)$

$$\text{Re}_L = U_0 L/\nu = (30)(600)/(1.41 \times 10^{-5}) = 1.28 \times 10^9$$

From Fig. 9-14 $C_f == 0.00158$. Then

$$F_s = (0.00158)(50,000)(1.94)(30)^2/2 = \underline{\underline{68,967 \text{ lbf}}}$$

9.75 **Information and assumptions**

Assume $\nu = 1.2 \times 10^{-5}$ ft^2/s and $\rho = 1.94$ slugs/ft^3
provided in problem statement

Find

the shearing force

Solution

$$\begin{aligned} F_s &= C_f B L \rho V_0^2/2 \\ \text{Re}_L &= VL/\nu = 10 \times 208/(1.2 \times 10^{-5}) = 1.73 \times 10^8 \end{aligned}$$

From Fig. 9-14 $C_f = 0.00199$. Then

$$F_s = (0.00199)(44)(208)(1.94/2)(10^2) = \underline{\underline{1,767 \ \text{ lbf}}}$$

9.76 **Information and assumptions**

From Table A.4 $\nu = 1.4 \times 10^{-6}$ m^2/s and $\rho = 1026$ kg/m^3.
provided in problem statement

Find

skin friction drag, power required and boundary layer thickness 300 m from bow.

Solution

$$\mathrm{Re}_L = U_0 L/\nu = (15 \times 0.515) \times 325/(1.4 \times 10^{-6}) = 1.79 \times 10^9$$

From Fig. 9-14 $C_f = 0.00153$. Then

$$F_s = C_f A \rho U_0^2/2$$

$$\begin{aligned} F_s &= 0.00153 \times 325(48 + 38) \times 1,026 \times (15 \times 0.515)^2/2 = \underline{\underline{1.309 \text{ MN}}} \\ P &= 1.309 \times 10^6 \times 15 \times 0.515 = \underline{\underline{10.1 \text{ MW}}} \end{aligned}$$

To find δ at $x = 300$ m

$$\begin{aligned} \mathrm{Re}_{300} &= U_0 x/\nu = 15 \times 0.515 \times 0.515 \times 300/(1.4 \times 10^{-6}) \\ &= 1.66 \times 10^9 \\ \delta/x &= 0.16/\mathrm{Re}_x^{1/2} = 0.0077 \\ \delta &= 300 \text{ m} \times .0077 \\ \delta &= \underline{\underline{2.31 \text{ m}}} \end{aligned}$$

9.77 **Information and assumptions**

From Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s and $\rho = 1.94$ slugs/ft^3
provided in problem statement

Find

wave drag on actual ship.

Solution

$$\begin{aligned}
Fr_m &= Fr_p \\
L_m/L_p &= 1/100 \\
V_m/(gL_m)^{0.5} &= V_p/(gL_p)^{0.5} \\
V_m/V_p &= (L_m/L_p)^{0.5} = 1/10 \\
V_m^2/V_p^2 &= 1/100 \\
V_m &= (1/10)(30 \text{ ft/s}) = 3 \text{ ft/s}
\end{aligned}$$

Viscous drag on model:

$$\begin{aligned}
\mathrm{Re}_L &= VL/\nu = (3)(5)/(1.22 \times 10^{-5}) = 1.23 \times 10^6 \\
&= 0.00293 \text{ from Fig. 9-14} \\
F_{s,m} &= C_f(1/2)\rho V^2 A = (0.00293)(1/2)(1.94)(3^2)(2.5) \\
&= 0.0639 \text{ lbf} \\
&\therefore \; F_{\text{wave,m}} = 0.1 - 0.0639 = 0.0361 \text{ lbf}
\end{aligned}$$

From Fig. 9-14, $C_f = 0.00293$ so

$$\begin{aligned}
F_{s,m} &= C_f(1/2)\rho V^2 A = (0.00293)(1/2)(1.94)(3^2)(2.5) \\
&= 0.0639 \text{ lbf} \\
F_{\text{wave,m}} &= 0.1 - 0.0639 = 0.0361 \text{ lbf}
\end{aligned}$$

Assume, for scaling up wave drag, that

$$\begin{aligned}
(C_p)_m &= (C_p)_p \\
(\Delta p/(\rho V^2/2))_m &= (\Delta p/(\rho V^2/2))_p \\
\Delta p_m/\Delta p_p &= (\rho_m/\rho_p)(V_m^2/V_p^2)
\end{aligned}$$

But

$$\begin{aligned}
F_m/F_p &= (\Delta p_m/\Delta p_p)(A_m/A_p) = (\rho_m/\rho_p)(V_m^2/V_p^2)(A_m/A_p) \\
&= (\rho_m/\rho_p)(L_m/L_p)^3 = (1.94/1.99)(1/100)^3 \\
F_p &= F_m(1.99/1.94)(100)^3 = 0.0361(1.99/1.94)(10^6) \\
F_p &= \underline{\underline{3.70 \times 10^4 \text{ lbf}}}
\end{aligned}$$

9.78 **Information and assumptions**

From Table A.5 $\nu_m = 1.00 \times 10^{-6}$ m^2/s and $\rho_m = 998$ kg/m^3
From Table A.4 $\nu_p = 1.4 \times 10^{-6}$ m^2/s and $\rho_m = 1026$ kg/m^3.
provided in problem statement

Find

a) speed of prototype, b) model skin friction and wave drag and c) ship drag in salt water.

Solution

$$
\begin{aligned}
V_m &= 1.45 \text{ m/s} \\
V_p &= (L_p/L_m)^{1/2} \times V_m = \sqrt{30} \times 1.45 = \underline{\underline{7.94 \text{ m/s}}} \\
\mathrm{Re}_m &= 1.45(250/30)/(1.00 \times 10^{-6}) = 1.2 \times 10^7 \\
\mathrm{Re}_p &= 7.94 \times 250/1.4 \times 10^{-6} = 1.42 \times 10^9 \\
C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re})} - \frac{1520}{\mathrm{Re}} \\
C_{fm} &= 0.00275 \\
C_{fp} &= 0.00157 \\
F_{sm} &= C_{fm}A\rho V^2/2 = 0.00275(8,800/30^2)998 \times 1.45^2/2 = \underline{\underline{28.21 \text{ N}}} \\
F_{\mathrm{wave}_m} &= 38.00 - 28.21 = \underline{\underline{9.79 \text{ N}}} \\
F_{\mathrm{wave}_p} &= (\rho_p/\rho_m)(L_p/L_m)^3 F_{\mathrm{wave}_m} = (1,026/998)30^3(9.79) = 9,040 \text{ N} \\
F_{sp} &= C_{f_p}A\rho V^2/2 = 0.00157(8,800)1,026 \times 7.94^2/2 = 446,000 \text{ N} \\
F_p &= F_{\mathrm{wave}_p} + F_{sp} = 9,040 + 444,000 = \underline{\underline{453,000 \text{ N}}}
\end{aligned}
$$

9.79 **Information and assumptions**

provided in problem statement

Find

minimum shear stress on smooth bottom.

Solution

Minimum τ_0 occurs where c_f is minimum. Two points to check: (1) where Re_x is highest; i.e., $\mathrm{Re}_x = \mathrm{Re}_L$ and (2) end of laminar sublayer where c_f reaches minimum value for the laminar part.

(1)

$$\begin{aligned} \mathrm{Re}_L &= V_0 L/\nu = 20 \times 4/10^{-6} = 8 \times 10^7 \\ c_f &\approx \frac{0.455}{\ln^2(0.06\,\mathrm{Re}_x)} = 0.00192 \end{aligned}$$

(2)

$$\mathrm{Re}_x = 5 \times 10^5$$

at the end of laminar boundary layer.

$$c_f = 0.664/\mathrm{Re}_x^{1/2} = \underline{\underline{0.00094}}$$

which is the minimum. So

$$\tau_{0_{\min}} = c_{f_{\min}} \rho V_0^2/2 = 0.00094 \times 998 \times 20^2/2 = \underline{\underline{188\ \mathrm{N/m^2}}}$$

9.80 **Information and assumptions**

From Table A.5 $\nu = 1.2 \times 10^{-5}$ ft^2/s and $\rho = 1.94$ slugs/ft^3.
provided in problem statement

Find

power to overcome surface resistance.

Solution

$$\text{Re}_L = VL/\nu = 44 \times 4/1.2 \times 10^{-5} = 144(10^5) = 1.44(10^7)$$

From Fig. 9.14 $C_f = 0.0027$. Then

$$\begin{aligned} F_D \text{ (per ski)} &= 0.0027(4)(1/2)(1.94)(44^2/2) = 10.135 \text{ lbf} \\ F_D \text{ (2 skis)} &= 20.27 \text{ lbf} \\ P(\text{hp}) &= 20.27 \times 44/550 = \underline{\underline{1.62 \text{ hp}}} \end{aligned}$$

9.81 **Information and assumptions**

From Table A.4 $\nu = 1.4 \times 10^{-6}$ m^2/s
provided in problem statement

Find

surface drag and thickness of boundary layer at stern.

Solution

$$\begin{aligned}
\mathrm{Re}_L &= U_0 L/\nu = 10 \times 80/(1.4 \times 10^{-6}) \\
\mathrm{Re}_L &= 5.7 \times 10^8
\end{aligned}$$

From Fig. 9-14 $C_f = 0.00173$. Then

$$\begin{aligned}
F_D &= C_f A \rho U_0^2/2 = 0.00173 \times 1,500 \times 1,026 \times 10^2/2 = \underline{\underline{133 \text{ kN}}} \\
\delta/x &= \frac{0.16}{\mathrm{Re}_x^{1/7}} \\
\delta/x &= 0.0089 \\
\delta &= 80 \times 0.0089 = \underline{\underline{0.712 \text{ m}}}
\end{aligned}$$

Chapter Ten

10.1 a. $\underline{\underline{(3)}}$ b. $\underline{\underline{(1)}}$ c. $\underline{\underline{(2)}}$ d. $\underline{\underline{(1)}}$ e. $\underline{\underline{(3)}}$ f. $\underline{\underline{(2)}}$

10.2 **Information and Assumptions**

provided in problem statement

Find

mean velocity in pipe.

Energy equation from the 0 elevation to the 10-ft elevation:

$$
\begin{aligned}
p_0/\gamma + V_0^2/2g + z_0 &= p_{10}/\gamma + V_{10}^2/2g + z_{10} + h_L \\
200,000/10,000 + 0 &= 110,000/10,000 + 10 + h_L \\
h_L &= 10 - 11 - 10 = -1 \text{ m}
\end{aligned}
$$

Because h_L is negative, the flow must be downward.

$$
\begin{aligned}
32\mu LV/\gamma D^2 &= h_L = 1 \text{ m} \\
V &= \gamma D^2/(32\mu L) \\
&= 8,000 \times 0.01^2/(32 \times 3.0 \times 10^{-3} \times 10) \\
V &= \underline{\underline{0.83 \text{ m/s}}}
\end{aligned}
$$

10.3 Valid statements are (a), (d) and (e).

10.4 **Information and Assumptions**

provided in problem statement

Find

pressure drop per 100 feet of level pipe

Solution

$$\begin{aligned} V &= Q/A = 0.25/((\pi/4) \times (1/6)^2) = 11.46 \text{ ft/sec} \\ \text{Re} &= VD\rho/\mu = 11.46 \times (1/6) \times 0.97 \times 1.94/10^{-2} = 360 \text{ (laminar)} \\ \Delta p &= 32\mu LV/D^2 = 32 \times 10^{-2} \times 100 \times 11.46/(144 \times (1/6)^2) = \underline{\underline{91.7 \text{ psi/100 ft}}} \end{aligned}$$

10.5 **Information and Assumptions**

provided in problem statement

Find

pressure at section 10 feet below

Solution

First, check Re:

$$\text{Re} = VD\rho/\mu = 2 \times 0.01 \times 1,000/0.06 = 333 \text{ (laminar)}$$

Energy equation from the point of highest elevation to a point of 10 meters below.

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
600,000/(9.81 \times 1,000) + 10 &= p_2/\gamma + 0 + 32\mu LV/\gamma D^2 \\
p_2/\gamma &= 600,000/\gamma + 10 - 32 \times 0.06 \times 10 \times 2/(\gamma(0.01)^2) \\
p_2 &= 600,000 + 10 \times 9,810 - 384,000 = \underline{\underline{314 \text{ kPa}}}
\end{aligned}$$

10.6 **Information and Assumptions**

provided in problem statement

Find

distribution logarithmic or parabolic and ratio of shear stress 1 mm from wall to that at wall.

Solution

Check Reynolds number

$$\text{Re} = VD\rho/\mu = (1)(0.01)(1,000)/10^{-1} = 100 \text{ (laminar)}$$

Because the flow is laminar the velocity distribution will be $\underline{\underline{\text{parabolic}}}$.

For a parabolic velocity distribution

$$V = V_c(1 - r^2/R^2)$$

The velocity gradient is

$$dV/dr = -2rV_c/R^2$$

The shear stress is proportional to the velocity gradient. One millimeter from the wall is 4 millimeter from the centerline. Therefore

$$\frac{\tau_{4\text{mm}}}{\tau_{5\text{mm}}} = \frac{4}{5} = \underline{\underline{0.8}}$$

10.7 **Information and Assumptions**

provided in problem statement

Find

pressure drop in Pa per 10 m.

Solution

$$\begin{aligned}
\mu &= 3.8 \times 10^{-1} \text{ N} \cdot \text{s/m}^2 \\
\nu &= 2.2 \times 10^{-4} \text{ m}^2/\text{s} \\
V &= Q/A = 8 \times 10^{-6}/((\pi/4) \times 0.030^2) = 0.0113 \text{ m/s} \\
\text{Re} &= VD/\nu = 0.0113 \times 0.030/(2.2 \times 10^{-4}) = 1.54 \text{ (laminar)}
\end{aligned}$$

Then

$$\Delta p_f = 32\mu LV/D^2 = 32 \times 0.38 \times 10 \times 0.0113/0.030^2 = \underline{\underline{1{,}527 \text{ Pa/10 m}}}$$

10.8 **Information and Assumptions**

provided in problem statement

Find

mean velocity in tube and discharge

Energy equation from reservoir liquid surface to outlet of pipe. Assume laminar flow so $\alpha = 2$.

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + 2V^2/2g + z_2 + 32\mu LV/(\gamma D^2) \\
0 + 0 + 0.50 &= 0 + V^2/g + 32\mu LV/(\gamma D^2) \\
V^2/g + 32\mu LV/(\gamma D^2) - 0.50 &= 0 \\
V^2/32.2 + 32(4\times 10^{-5})(10)V/(0.80\times 62.4\times(1/32)^2) - 0.50 &= 0 \\
V^2 + 8.45V - 16.1 &= 0
\end{aligned}$$

Solving the above quadratic equation for V yields:

$$\underline{\underline{V = 1.60 \text{ ft/s}}}$$

Check Reynolds number to see if flow is indeed laminar

$$\begin{aligned}
\text{Re} &= VD\rho/\mu = 1.602\times(1/48)(1.94\times 0.8)/(4\times 10^{-5}) \\
\text{Re} &= 1294 \text{ (laminar)} \\
Q &= VA = 1.602\times(\pi/4)(1/32)^2 = \underline{\underline{1.23\times 10^{-3} \text{ cfs}}}
\end{aligned}$$

10.9 **Information and Assumptions**

provided in problem statement

Find

pressure drop per 10 m of pipe

$$\begin{aligned} V &= Q/A = 2 \times 10^{-3}/(\pi/4 \times (0.05)^2) \\ V &= 1.019 \text{ m/s} \\ Re &= VD\rho/\mu = 1.019 \times 0.05 \times 940/0.048 = 997 \end{aligned}$$

Energy equation:

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + 32\mu LV/\gamma D^2$$

$$\begin{aligned} p_1 - p_2 &= 32\mu LV/D^2 = 32 \times 0.048 \times 10 \times 1.019/\delta/(0.05)^2 \\ p_1 - p_2 &= \underline{\underline{6.26 \text{ kPa}}} \end{aligned}$$

10.10 **Information and Assumptions**

provided in problem statement

Find

power to operate the pump

Energy equation from (1) to (2):

$$
\begin{aligned}
p_1/\gamma + z_1 + V_1^2/2g + h_p &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
h_p &= h_L = f(L/D)(V^2/2g) \\
V &= Q/A = 7.85 \times 10^{-4}/((\pi/4)(0.01)^2) = 10 \text{ m/s} \\
\text{Re} &= VD/\nu = (10)(0.01)/(7.6 \times 10^{-5}) = 1,316 \text{ (laminar)} \\
f &= 64/\text{Re} = 64/1,316 = 0.0486 \\
h_p &= 0.0486(8/0.01)(10^2/((2)(9.81)) = 198 \text{ m} \\
P &= \gamma Q h_p = (8,630)(7.85 \times 10^{-4})(198) = \underline{\underline{1,341 \text{ Watts}}}
\end{aligned}
$$

10.11 **Information and Assumptions**

provided in problem statement

Find

pressure gradient along pipe

Solution

$$
\begin{aligned}
V &= Q/A = 0.0157/((\pi/4)(0.1^2)) = 2.0 \text{ ft/s} \\
\text{Re} &= VD/\nu = (2)(0.10)/(0.0057) = 35.1 \text{ (laminar)} \\
d/ds(p+\gamma z) &= 32\mu V/D^2 \\
-dp/ds - \gamma dz/ds &= (32)(10^{-2})(2)/0.1^2 \\
-dp/ds - \gamma(-0.5) &= 64 \\
dp/ds &= (0.5)(0.9)(62.4) - 64 \\
dp/ds &= 28.08 - 64 = \underline{\underline{-35.9 \text{ psf/ft}}}
\end{aligned}
$$

10.12 **Information and Assumptions**

provided in problem statement

Find

magnitude of maximum velocity, resistance coefficient, shear velocity and shear stress 25 mm from pipe center.

Solution

First check Re:

$$\mathrm{Re} = VD\rho/\mu = 0.1 \times 0.1 \times 800/0.1 = 800$$

Therefore, the flow is laminar

$$\begin{aligned}
V_{\max} &= 2V = \underline{\underline{20 \text{ cm/s}}} \\
f &= 64/\mathrm{Re} = 64/800 = \underline{\underline{0.080}} \\
u_*/V &= \sqrt{f/8} \\
u_* &= \sqrt{0.08/8} \times 0.1 \\
&= \underline{\underline{0.010 \text{ m/s}}} \\
\tau_0 &= \rho u_*^2 = 800 \times 10^{-4} = 0.08 \text{ N/m}^2
\end{aligned}$$

Get $\tau_{r=0.025}$ by proportions:

$$\begin{aligned}
0.025/0.05 = \tau/\tau_0;\ \tau &= 0.50\tau_0 \\
\tau &= 0.50 \times 0.080 = \underline{\underline{0.040 \text{ N/m}^2}}
\end{aligned}$$

10.13 **Information and Assumptions**

provided in problem statement

Find

if flow laminar or turbulent

Solution

$$\begin{aligned}
\text{Re} &= VD\rho/\mu = (Q/A)D/\nu = 4QD/(\pi D^2\nu) = 4Q/(\pi D\nu) \\
\text{Re} &= 4 \times 0.02/(\pi \times 0.2 \times 2.37 \times 10^{-6}) = 53,723
\end{aligned}$$

Flow is $\underline{\underline{\text{turbulent}}}$

10.14 **Information and Assumptions**

provided in problem statement

Find

ratio of head loss

Solution

$$
\begin{aligned}
h_{L,1}/h_{L,2} &= (f_1/f_2)((L/D)_1/(L/D_2))(V_1^2/V_2^2) \\
&= (D_2/D_1)(V_1^2/V_2^2) \\
V_1 A_1 &= V_2 A_2 \\
V_1/V_2 &= A_2/A_1 = (D_2/D_1)^2 \\
(V_1/V_2)^2 &= (D_2/D_1)^4 \\
h_{L,1}/h_{L,2} &= (D_2/D_1)(D_2/D_1)^4 = (D_2/D_1)^5 \\
&= 2^5 = \underline{\underline{32}}
\end{aligned}
$$

Correct choice of answers is $\underline{\underline{\text{(d)}}}$.

10.15 **Information and Assumptions**

provided in problem statement

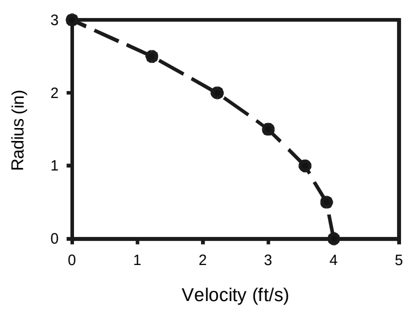
Find

if flow laminar or turbulent

Solution

$$
\begin{aligned}
\text{Re} &= VD/\nu = 2 \times 0.5/(5.3 \times 10^{-3}) = 188 \text{ (laminar)} \\
h_L/L &= 32\mu\bar{V}/\gamma D^2 = dh/ds \\
V &= (\gamma/4\mu)(r_0^2 - r^2)(32\mu\bar{V}/(\gamma D^2) \\
V &= (1/4)(r_0^2 - r^2)(32\bar{V}/4r_0^2) \\
V &= (32/16)\bar{V}(1-(r/r_0)^2) = [1-(r/r_0)^2]2\bar{V} = 4[1-(r/r_0)^2]
\end{aligned}
$$

r (in)	r/r_0	V (ft/s)
0	0	4
0.5	1/6	3.89
1.0	1/3	3.56
1.5	1/2	3.00
2	2/3	2.22
2.5	5/6	1.22
3	1	0



10.16 **Information and Assumptions**

from Table A.4, $\mu = 0.62$ N·s/m^2 $\gamma = 12,300$ N/m^3
provided in problem statement

Find

mean velocity of glycerine

Energy equation from surface in funnel to outlet

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
0 + 0 + 0.30 &= 0 + 2V_2^2/2g + 0 + 32\mu L V_2/(\gamma D^2) \\
(0.30) \times (g) &= V_2^2 + V_2((g)(32)(0.62)(0.2)/(12,300 \times .01^2) \\
2.943 &= V_2^2 + 31.69V_2 \\
V_2^2 + 31.69V_2 - 2.943 &= 0 \\
V_2 &= \underline{\underline{0.93 \text{ m/s}}}
\end{aligned}
$$

10.17 **Information and Assumptions**

from Table A.2, $\mu = 8.5 \times 10^{-3}$ lbf-s/ft^2
provided in problem statement

Find

size of steel pipe.

Solution

Assume laminar flow, so

$$\Delta p_f = 32\mu LV/D^2$$

or

$$\Delta p_f = 32\mu LQ/((\pi/4) \times D^4)$$

Then

$$\begin{aligned} D^4 &= 128\mu LQ/(\pi\Delta p_f) = 128 \times 8.5 \times 10^{-3} \times 3,168 \times 0.1/(\pi \times 10 \times 144) \\ D^4 &= 0.0762 \\ D &= 0.525 \text{ ft} \end{aligned}$$

An 8-in. pipe should be used. Check Re in 8" pipe:

$$\begin{aligned} V &= Q/A = 0.1/((\pi/4) \times 0.67^2) = 0.284 \text{ ft/sec.} \\ \text{Re} &= VD\rho/\mu = 0.284 \times 0.67 \times 0.85 \times 1.94/(8.5 \times 10^{-3}) = \underline{\underline{37 \text{ (laminar)}}} \end{aligned}$$

10.18 **Information and Assumptions**

from Table A.4 for mercury at 20°C, $\mu = 1.5 \times 10^{-3}$ Ns/m^2, $\nu = 1.2 \times 10^{-7}$ m^2/s, $\gamma = 133,000$ N/m^3
provided in problem statement

Find

tube diameter at which flow just becomes turbulent

Solution

Energy equation

$$p_1/\gamma + \alpha_1 V_1^2/2g + z_1 = p_2/\gamma + \alpha_2 V_2^2/2g + z_2$$

But $p_1 = p_2$, $V_1 = V_2$ and $\alpha_1 = \alpha_2$

$$\begin{aligned}
z_1 - z_2 &= L = h_f \\
\cancel{L} &= 32\mu \cancel{L} V/\gamma D^2 \\
D^2 &= 32\mu V/\gamma && (1) \\
\text{Re} &= VD/\nu = 2,000 \\
V &= 2,000\nu/D = 2,000 \times 1.2 \times 10^{-7}/D \\
V &= 2.4 \times 10^{-4}/D && (2)
\end{aligned}$$

Solve Equations (1) and (2) for D:

$$\begin{aligned}
D^2 &= 32\mu(2.4 \times 10^{-4})/\gamma D \\
D^3 &= 32(1.5 \times 10^{-3})(2.4 \times 10^{-4})/133,000 \\
D &= \underline{\underline{4.42 \times 10^{-4} \text{ m}}}
\end{aligned}$$

10.19 **Information and Assumptions**

provided in problem statement

Find

if flow laminar or turbulent and pressure change in direction of flow

Solution

$$\text{Re} = VD/\nu = 0.40 \times 0.04/5.3 \times 10^{-3} = \underline{\underline{3.02 \text{ (laminar)}}}$$

From solution to Problem 10-11

$$dh/ds = -32\mu V/(\gamma D^2)$$

$$\begin{aligned} dh/ds &= -32 \times 6.2 \times 10^{-1} \times 0.40/(12,300 \times (0.04)^2) \\ d/ds(p/\gamma + z) &= -0.403 \end{aligned}$$

or

$$(1/\gamma)dp/ds + dz/ds = -0.403$$

Because flow is downward, $dz/dz = -1$. Then

$$dp/ds = \gamma[1 - 0.403] = \underline{\underline{7.34}} \text{ kPa/m}$$

and $\underline{\underline{\text{pressure increases downward}}}$

From Eq. 10-3

$$\tau = \gamma(r/2)[-dh/ds]$$

or

$$\tau = 12,300(r/2) \times 0.403$$

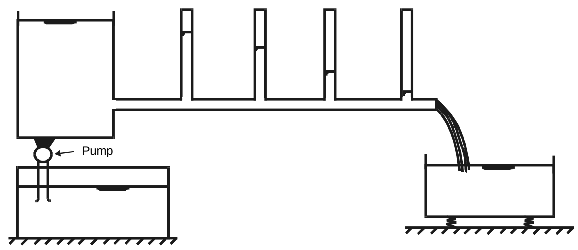
Then at

$$r = 0, \ \tau = 0$$

and

$$\begin{aligned} \tau_{\text{wall}} &= \tau_0 = 12,300(0.02/2) \times 0.403 \\ \tau_{\text{wall}} &= \underline{\underline{49.6 \text{ N/m}^2}} \end{aligned}$$

10.20 The design might have a physical configuration as shown below. The design should be based upon solving Eq. 10.17 ($h_f = 32\mu LV/(\gamma D^2)$) for the viscosity μ. Since this is for laminar flow, the size of pipe and depth of liquid in the tank should be such that laminar flow will be assured ($R_e < 1,000$). For the design suggested here, the following measurements, conditions, and calculations would have to be made:



A. Measure tube diameter by some means.

B. Measure γ or measure temperature and get γ from a handbook.

C. Establish steady flow by having a steady supply source (pump liquid from a reservoir).

D. Measure Q. This could be done by weighing amount of flow for a given period of time or by some other means.

E. Measure h_f/L by the slope of the piezometric head line as obtained from piezometers. This could also be obtained by measuring Δp along the tube by means of pressure gages or pressure transducers from which h_f/L could be calculated.

F. Solve for μ with Eq. 10.17.

10.21 **Information and Assumptions**

provided in problem statement

Find

kinematic viscosity of fluid

Solution

Since the velocity distribution is parabolic, the flow is laminar. Then

$$
\begin{aligned}
\Delta p_f &= 32\mu LV/D^2 \\
\nu &= \mu/\rho = \Delta p_f D^2/(32LV\rho) \\
\nu &= 16 \times 1^2/(32 \times 100 \times 2/2 \times 0.9 \times 1.94) = \underline{\underline{0.00286 \text{ ft}^2/\text{s}}}
\end{aligned}
$$

10.22 **Information and Assumptions**

provided in problem statement

Find

kinematic viscosity of fluid.

Solution

Following the solution for Problem 10.21,

$$\begin{aligned}\nu &= \Delta p_f D^2/(32LV\rho) = 1,900 \times (0.3)^2/(32 \times 100 \times 0.75 \times 800) \\ &= \underline{\underline{8.91 \times 10^{-5}\ \mathrm{m^2/s}}}\end{aligned}$$

10.23 **Information and Assumptions**

provided in problem statement

Find

pressure difference across heat exchanger

Solution

Check *Re*:

$$\mathrm{Re}_{20^\circ} = VD/\nu = 0.12 \times 0.005/10^{-6} = 600$$

So the flow is laminar and

$$\Delta p = 32\mu LV/D^2$$

Assume linear variation in μ and use the temperature at 25^oC. From Table A.5

$$\mu_{\text{avg.}} = \mu_{25^\circ} = 8.91 \times 10^{-4} \mathrm{N \cdot s/m^2}$$

and

$$\Delta p = 32 \times 8.91 \times 10^{-4} \times 5 \times 0.12/(0.005)^2 = \underline{\underline{684 \text{ Pa}}}$$

10.24 **Information and Assumptions**

provided in problem statement

Find

flow direction, resistance coefficient, nature of the flow and viscosity of oil.

Solution

The flow is $\underline{\underline{\text{downward (from right to left)}}}$.

$$\Delta h((13.55-0.8)/0.8)\times \text{ deflection} = 5.312 \text{ ft of oil } = h_f$$

Then

$$\begin{aligned} h_f &= f(L/D)(V^2/2g) \\ f &= 5.312\times((1/6)/(30))\times 2\times 32.2/15^2) \\ &= \underline{\underline{0.076}} \end{aligned}$$

Assume flow is laminar. Then $h_f = 32\mu LV/\gamma D^2$ or

$$\mu = h_f\gamma D^2/(32LV) \tag{1}$$

and

$$\text{Re} = VD\rho/\mu$$

so

$$\begin{aligned} \text{Re} &= VD\rho/[h_f\gamma D^2/(32LV)] = 32LV^2/(gh_fD) \\ \text{Re} &= 32\times 30\times 5^2/(32.2\times 5.312\times(1/6)) = \underline{\underline{842\text{ (laminar)}}} \end{aligned}$$

From Eq. (1):

$$\begin{aligned} \mu &= 5.312\times 0.8\times 62.4\times(1/6)^2/(32\times 30\times 5) \\ \mu &= \underline{\underline{1.53\times 10^{-3}}}\text{ lbf-s/ft}^2 \end{aligned}$$

10.25 **Information and Assumptions**

provided in problem statement

Find

flow direction, resistance coefficient, nature of flow and viscosity of oil

Solution

The flow is $\underline{\underline{\text{downward (from right to left)}}}$.

$$
\begin{aligned}
\Delta h &= [(13.55 - 0.8)/0.8] \times \text{deflection } = 1.91 \text{ m } = h_f \\
h_f &= f(L/D)(V^2/2g);\ f = 1.59 \times (0.05/10) \times 2 \times 9.81/1.2^2 = \underline{\underline{0.108}}
\end{aligned}
$$

Assume flow is laminar. From Problem P10-24

$$
\begin{aligned}
\text{Re} &= 32LV^2/(gh_f D) \\
&= 32 \times 10 \times 1.2^2/(9.81 \times 1.59 \times 0.05) \\
&= 590\underline{\underline{\text{(laminar)}}}
\end{aligned}
$$

Then

$$
\begin{aligned}
\mu &= h_f \gamma D^2/(32LV) \\
&= 1.91 \times 9.81 \times 800 \times 0.05^2/(32 \times 10 \times 1.2) \\
&= \underline{\underline{0.0976 \text{ N}\cdot\text{s/m}^2}}
\end{aligned}
$$

10.26 **Information and Assumptions**

provided in problem statement

Find

head loss change

Solution

$$\begin{aligned} h_f/L &= 2 = (f/D)(V^2/2g) = (f/0.03(1/(2 \times 9.81)) \\ f &= 1.177 \end{aligned}$$

Assume laminar flow:

$$f = 64/\mathrm{Re}$$

or

$$\mathrm{Re} = 64/1.177 = 54.4 \text{ (laminar}$$

Indeed, the flow is laminar and it will be laminar if the flow rate is doubled. The head loss varies directly with V (and Q); therefore, the head loss will also be doubled when the flow rate is doubled.

10.27 **Information and Assumptions**

provided in problem statement

Find

viscous shear stress on wall.

Solution

For a straight pipe there is no momentum change, so the forces on the column of fluid in the pipe are zero. For a horizontal pipe

$$\begin{aligned}
\tau_0 \pi D L &= -\Delta p A \\
\tau_0 \pi D L &= \gamma h_f A \\
\tau_0 \pi D L &= \gamma f (L/D)(V^2/2g)(\pi D^2/4) = f \pi D \rho V^2/8
\end{aligned}$$

Thus,

$$\tau_0 = f \rho V^2/8 = 0.017 \times 0.82 \times 1.94 \times 6^2/8 = \underline{\underline{0.122 \text{ psf}}}$$

10.28

$$\begin{aligned} \mathrm{Re}_{\mathrm{oil}} &= VD\rho/\mu \\ &= (1)(900)(0.1)/(10^{-1}) \\ &= 900 \\ V_{\max} &= 2\bar{V} \\ \mathrm{Re}_{\mathrm{gas}} &= (1.0)(0.1)(1)/(10^{-5}) = 10^{4} \\ V_{\max} &< 2\bar{V} \end{aligned}$$

Therefore

$$(V_{\max,\mathrm{oil}}/V_{\max,\mathrm{gas}}) > 1$$

or case $\underline{\underline{\text{(a)}}}$ is correct answer.

10.29 **Information and Assumptions**

provided in problem statement

Find

thickness of viscous sublayer and resistance coefficient

Solution

$$\begin{aligned} u_*\delta'_N/\nu &= 11.6 \\ \delta'_N &= 11.6\nu/u_* \end{aligned}$$

From solution to Problem $\tau_0 = (f/8)\rho V^2$ so

$$u_* = \sqrt{\tau_0/\rho} = V\sqrt{f/8}$$

Also $Re = VD/\nu$ or $\nu = VD/\text{Re}$. So

$$\delta'_N = 11.6(VD/\text{Re})/(V\sqrt{f/8})$$

From Fig. 10-8 $f = \underline{\underline{0.0185}}$. So

$$\delta'_N = 11.6(0.12/80,000)/\sqrt{0.0185/8} = 3.62 \times 10^{-4} \text{ m} = \underline{\underline{0.362 \text{ mm}}}$$

10.30 Shear stress decreases with x near the entrance; therefore, case (a) is correct choice.

10.31 **Information and Assumptions**

from Fig. A.3 $\nu(70^{\circ}\text{F}) = 1.06 \times 10^{-5}$ ft^2/s
provided in problem statement

Find

the resistance coefficient

Solution

Calculate Reynolds number

$$\begin{aligned}\text{Re} &= 4Q/\pi D\nu = 4 \times 2/(\pi \times (6/12) \times 1.06 \times 10^{-5}) \\ &= 4.8 \times 10^5\end{aligned}$$

From Fig. 10.8 $\underline{\underline{f = 0.013}}$

10.32 **Information and Assumptions**

from Table A.5, $\nu(10^{\circ}\mathrm{C}) = 1.31 \times 10^{-6}$ m^2/s.
provided in problem statement

Find

the resistance coefficient

Solution

Calculate Reynolds number

$$\mathrm{Re} = 4 \times 0.06/(\pi \times 0.25 \times 1.31 \times 10^{-6}) = 2.33 \times 10^5$$

From Fig. 10.8 $\underline{\underline{f = 0.015}}$

10.33 **Information and Assumptions**

from Table A.3 $\mu(20^{\text{o}}) = 1.81 \times 10^{-5}$ N·s/m^2
provided in problem statement

Find

the pressure drop per meter

Solution

$$\begin{aligned} V &= Q/A = 0.012 \times 4/(\pi \times 0.02^2) = 38.2 \text{ m/s} \\ \rho &= p/RT = 110,000/(287 \times 293) = 1.31 \text{ kg/m}^3 \\ \text{Re} &= VD\rho/\mu = 38.2 \times 0.02 \times 1.31/(1.81 \times 10^{-5}) = 5.5 \times 10^4 \end{aligned}$$

Then from Fig. 10-8

$$\begin{aligned} f &= 0.0205 \\ \Delta p &= (0.0205 \times 38.2^2 \times 1.31/(0.02 \times 2) \\ &= \underline{\underline{980 \text{ Pa/m}}} \end{aligned}$$

10.34 **Information and Assumptions**

from Table A.4, $\gamma = 12,300$ N/m^3 and $\mu = 6.2 \times 10^{-1}$ N·s/m^2
provided in problem statement

Find

height differential between two sandpipes

Energy equation from one standpipe to the other:

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ p_1/\gamma + z_1 &= p_2/\gamma + z_2 + h_L \\ \Delta h = ((p_1/\gamma) + z_1)) - ((p_2/\gamma) + z_2) &= h_L \\ \mathrm{Re} = VD/\nu = (0.6)(0.02)/(5.1 \times 10^{-4}) &= 23.5 \text{ (laminar)} \end{aligned}$$

Laminar head loss equation

$$\begin{aligned} \Delta h &= h_1 - h_2 = 32\mu LV/(\gamma D^2) \\ \Delta h &= (32)(6.2 \times 10^{-1})(1)(0.6)/(12,300 \times 0.02^2) \\ \Delta h &= \underline{\underline{2.42 \text{ m}}} \end{aligned}$$

10.35 **Information and Assumptions**

from Table A.3 $\mu(80^{\circ}\mathrm{F}) = 3.85 \times 10^{-7}$ lbf-s/ft^2
provided in problem statement

Find

pressure drop per foot

Solution

$$\begin{aligned}
\Delta p &= f(L/D)\rho V^2/2 \\
V &= Q/A = 25 \times 4/(60 \times \pi \times (1/12)^2) = 76.4 \text{ ft/s} \\
\rho &= p/(RT) = 15 \times 144/(1,716 \times 540) = 0.00233 \text{ slugs/ft}^3 \\
\mathrm{Re} &= VD\rho/\mu = 76.4 \times (1/12) \times 0.00233/(3.85 \times 10^{-7}) = 3.9 \times 10^4
\end{aligned}$$

Then $f = 0.022$

$$\Delta p = 0.022 \times 1 \times 12 \times 76.4^2 \times 0.00233/2 = \underline{\underline{1.8 \text{ psf/ft}}}$$

10.36 **Information and Assumptions**

provided in problem statement

Find

kinematic viscosity

Solution

$$\begin{aligned}
h_f &= f(L/D)(V^2/2g) \\
0.50 &= f(1/0.01)(3^2/(2\times 9.81)) \\
f &= 0.0109
\end{aligned}$$

From Fig. 10-8 Re $= 1.5\times 10^6$ so

$$\nu = VD/\,\mathrm{Re} = (3)(0.01)/(1.5\times 10^6) = \underline{\underline{2.0{\times}10^{-8}\ \mathrm{m^2/s}}}$$

10.37 **Information and Assumptions**

provided in problem statement

Find

resistance coefficient

Solution

$$\begin{aligned}
\Delta h &= h_f = 0.80(2.5-1) = 1.2 \text{ ft of water} \\
h_f &= f(L/D)V^2/2g \\
f &= 1.2 \times (0.05/4) \times 2 \times 9.81/3^2 \\
&= \underline{\underline{0.033}}
\end{aligned}$$

10.38 **Information and Assumptions**

from Table A.5 $\nu(10^o\text{C})=1.31\times10^{-6}$ m^2/s
provided in problem statement

Find

velocity distribution

Solution

$$\begin{aligned}\text{Re} &= VD/\nu = 4(0.3)/(1.31\times10^{-6}) = 9.16\times10^5 \\ k_s/D &= 0.00026/0.30 = 0.00087\end{aligned}$$

From Fig. 10-8 $f = 0.0195$

$$u/u_* = 5.75 \log{(y/k_s)} + 8.5$$

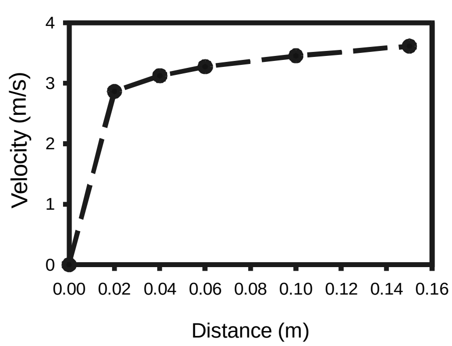
where

$$u_* = V\sqrt{f/8} = 3\sqrt{0.0195/8} = 0.148 \text{ m/s}$$

Then

$$u = 0.148[5.75\log(y/0.00026) + 8.5]$$

y(m) $\rightarrow$	0.02	0.04	0.06	0.10	0.15
u(m/s) $\rightarrow$	2.86	3.12	3.27	3.45	3.61



10.39 **Information and Assumptions**

provided in problem statement

Find

resistance coefficient

Solution

$$\begin{aligned} \mathrm{Re} &= Vd/\nu = (1)(0.10)/(10^{-4}) = 10^3 \text{ (laminar)} \\ f &= 64/\mathrm{Re} = 64/1,000 = 0.064 \; . \end{aligned}$$

Case $\underline{\underline{\text{(a)}}}$ is correct

10.40

$$V = Q/A = 0.002/[(\pi/4) \times (0.06)^2] = 0.707 \text{ m/s}$$
$$\mathrm{Re} = VD/\nu = 0.707 \times 0.06/10^{-6} = 4.24 \times 10^4$$

From Fig. 10-8 $\underline{\underline{f = 0.021}}$

10.41 **Information and Assumptions**

assume $T = 60°F$
from Table A.3 $\gamma = 0.0764$ lb/ft^3 and $\nu = 1.58 \times 10^{-4}$ ft^2/s
provided in problem statement

Find

difference in pressure across the train and power required

Energy equation from front of train to outlet of tunnel.

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
p_1/\gamma + V_1^2/2g &= 0 + 0 + 0 + V_2^2/2g + f(L/D)V_2^2/2g \\
p_1/\gamma &= f(L/D)V^2/2g
\end{aligned}$$

$$\begin{aligned}
k_s/D &= 0.05/10 = 0.005 \\
\text{Re} &= VD/\nu = (50)(10)/(1.58 \times 10^{-4}) = 3.2 \times 10^6
\end{aligned}$$

From Fig. 10.8

$$f = 0.030$$

and

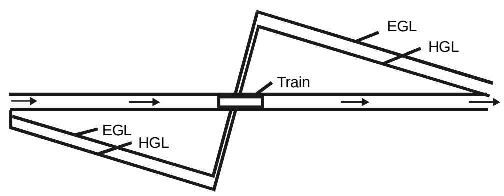
$$\begin{aligned}
p_1 &= \gamma f(L/D)(V^2/2g) \\
&= (0.0764)(0.03)(2,500/10)(50^2/(64.4)) \\
p_1 &= 22.24 \text{ psfg}
\end{aligned}$$

Energy equation from outside entrance to rear of train

$$\begin{aligned}
p_3/\gamma + V_3^2/2g + z_3 &= p_4/\gamma + V_4^2/2g + z_4 + \sum h_L \\
0 + 0 + 0 &= p_4/\gamma + V_4^2/2g + 0 + (K_e + f(L/D))V^2/2g \\
p_4/\gamma &= -(V^2/2g)(1.5 + f(L/D)) \\
&= -(50^2/2g)(1.5 + 0.03(2,500/10)) \\
p_4 &= -\gamma(349.4) = -26.69 \text{ psf} \\
\Delta p &= p_1 - p_4 = 22.24 - (-26.69) = \underline{\underline{48.93 \text{ psf}}}
\end{aligned}$$

Power requirements

$$\begin{aligned} P &= FV = (\Delta p A)(50) = (48.93)(\pi/4)(10^2)(50) \\ &= 192,158 \text{ ft-lbf/s} = \underline{\underline{349 \text{ hp}}} \end{aligned}$$



10.42 **Information and Assumptions**

assume $T \simeq 60^{\circ}$F with $\nu = 1.2 \times 10^{-5}$ ft^2/s
assume no bend losses
provided in problem statement

Find

time to fill cylinder

$$\begin{aligned} d_{\text{tube}} &= 3/16 \text{ in.} = 0.01562 \text{ ft} \\ L_{\text{tube}} &= 50 \text{ in.} \end{aligned}$$

Energy equation from the jug to the cylinder

$$p_j/\gamma + V_j^2/2g + z_j = p_c/\gamma + V_c^2/2g + z_c + \sum h_L \tag{1}$$

Assume that the entrance loss coefficient is equal to 0.5. It would actually be somewhat larger than 0.5 but this should yield a reasonable approximation. Therefore

$$\sum h_L = (0.5 + fL/D + K_E)V^2/2g$$

The exit loss coefficient, K_E, is equal to 1.0. Therefore, Eq. 1 becomes

$$\begin{aligned} \Delta z &= z_j - z_c = (V^2/2g)(1.5 + fL/D) \\ \text{or } V &= \sqrt{2g\Delta z/(1.5 + fL/D)} \qquad (1) \\ &= \sqrt{2g\Delta z/(1.5 + f \times 267)} \end{aligned}$$

First assume $f = 0.03$ and initial $\Delta z = (21 - 2.5)/12 = 1.54$ ft. Then

$$\begin{aligned} V &= \sqrt{(2g)(1.54)/(1.5 + 8)} = 3.23 \text{ ft/s} \\ \text{Re} &= VD/\nu = 3.23 \times .01562/(1.2 \times 10^{-5}) = 4205 \end{aligned}$$

From Fig. 10.8 $f = 0.038$. Repeat with new value of friction factor.

$$\begin{aligned} V &= \sqrt{2g \times 1.54/(1.5 + 8)} = 3.23 \text{ ft/s} \\ \text{Re} &= VD/\nu = 3.23 \times 0.01562/(1.2 \times 10^{-5}) = 4205 \end{aligned}$$

and $f = 0.040$

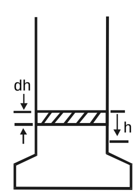


Fig. A

Use $f = 0.040$ for final solution. As a simplifying assumption assume that as the cylinder fills the level of water in the jug has negligible change. As the cylinder is being filled one can visualize (see figure) that in time dt a volume of water equal to Qdt will enter the cylinder and that volume in the cylinder can be expressed as $A_c dh$, that is

$$\begin{aligned} Qdt &= A_c dh \\ dt &= (A_c/Q)dh \end{aligned}$$

But

$$Q = V_t A_t \tag{2}$$

so

$$dt = ((A_c/A_t)/V)dh$$

Substitute V of Eq. (1) into Eq. (2):

$$\begin{aligned} dt &= (A_c/A_t)/(2g\Delta z/(1.5 + 267f))^{1/2}dh \\ \forall_c &= .500 \text{ liter } = 0.01766 \text{ ft}^3 \end{aligned}$$

or

$$\begin{aligned} 0.01766 &= A_c \times (11.5 \text{ in.}/12) \\ A_c &= 0.01842 \text{ ft}^2 \\ A_{\text{tube}} &= (\pi/4)((3/16)/12)^2 = 0.0001917 \text{ ft}^2 \\ A_c/A_t &= 96.1 \end{aligned}$$

The differential equation becomes

$$dt = 96.1/(2g\Delta z/(1.5 + 10.7))^{1/2}dh$$

Let h be measured from the level where the cylinder is 2 in full. Then

$$\begin{aligned} \Delta z &= ((21 \text{ in} - 2.5 \text{ in})/12) - h \\ \Delta z &= 1.542 - h \end{aligned}$$

Now we have

$$\begin{aligned} dt &= (96.1/(2g(1.54 - h)/12.2^{1/2})dh \\ dt &= 41.8/(1.54 - h)^{1/2})dh \\ dt &= -41.8(1.4 - h)^{1/2})(-dh) \end{aligned}$$

Integrate:

$$\begin{aligned}
t &= -41.8(1.54-h)^{1/2}/(1/2)|_0^h \\
&= -83.6(1.54-h)|_0^{0.75} \\
&= -83.6[(0.79)^{1/2}-(1.54)^{1/2}] \\
&= -83.6(0.889-1.241) \\
&= \underline{\underline{29.4 \text{ s}}}
\end{aligned}$$

Possible problems with this solution: The Reynolds number is very close to the point where turbulent flow will occur and this would be an unstable condition. The flow might alternate between turbulent and laminar flow.

10.43 **Information and Assumptions**

provided in problem statement

Find

elevation of upper reservoir

Energy equation between water surfaces of the reservoirs:

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + z_1 &= 0 + 0 + 100 + \sum h_L \end{aligned}$$

where

$$\sum h_L = (K_e + 2K_b + K_E + fL/D)(V^2/2g)$$

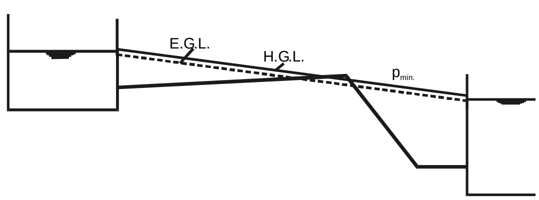
and $K_e = 0.50;\ K_b = 0.40$ (assumed); $K_E = 1.0;\ fL/D = 0.025 \times 430/1 = 10.75$

$$V = Q/A = 10.0/((\pi/4) \times 1^2) = 12.73 \text{ ft/s}$$

then

$$\begin{aligned} z_1 &= 100 + (0.5 + 2 \times 0.40 + 1.0 + 10.75)(12.73^2)/64.4 \\ &= 133 \text{ ft} \end{aligned}$$

The point of minimum pressure will occur just downstream of the first bend as shown by the hydraulic grade line (below).



To determine the magnitude of the minimum pressure, write the **energy equation** from the upstream reservoir to just downstream of bend:

$$\begin{aligned} z_1 &= z_b + p_b/\gamma + V^2/2g + (fL/D)V^2/2g + K_eV^2/2g + K_bV^2/2g \\ p_b/\gamma &= 133 - 110.70 - (12.73^2/64.4)(1.9 + 0.025 \times 300/1) = -1.35 \text{ ft} \\ p_B &= -1.35 \times 62.4 = \underline{\underline{\text{-84 psfg}}} = \underline{\underline{\text{-0.59 psig}}} \\ \text{Re} &= VD/\nu = 12.73 \times 1/(1.41 \times 10^{-5}) = 9.0 \times 10^5 \end{aligned}$$

With an f of 0.025 at a Reynolds number of 9×10^5 a value for k_s/D of 0.0025 (approx) is read from Fig. 10-8. From Table 10.2 the pipe appears to be fairly rough concrete pipe.

10.44 **Information and Assumptions**

from Table A.5 $\nu(70^{\circ}\mathrm{F}) = 1.06 \times 10^{-5}$ ft^2/s
provided in problem statement

Find

power delivered by turbine.

Energy equation from the reservoir water surface to the jet at the end of the pipe.

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_T + \sum h_L \\
0 + 0 + z_1 &= 0 + V_2^2/2g + z_2 + h_T + (K_e + fL/D)V^2/2g \\
z_1 - z_2 &= h_T + (1 + 0.5 + fL/D)V^2/2g \\
100 \text{ ft} &= h_T + (1.5 + fL/D)V^2/2g
\end{aligned}
$$

But

$$
\begin{aligned}
V &= Q/A = 5/((\pi/4)1^2) = 6.37 \text{ ft/s} \\
V^2/2g &= 0.629 \text{ ft} \\
\mathrm{Re} &= VD/\nu = 6.0 \times 10^5
\end{aligned}
$$

From Fig. 10.8 $f = 0.0140$ for $k_s/D = 0.00015$. Then

$$
\begin{aligned}
100 \text{ ft} &= h_T + (1.5 + 0.0140 \times 1,000/1)(0.629) \\
h_T &= (100 - 9.74) \text{ ft} \\
P &= Q\gamma h_T \times \text{eff} \\
&= 5 \times 62.4 \times 90.26 \times 0.80 \\
&= 22,529 \text{ ft} \cdot \text{lbf/s} \\
&= \underline{\underline{40.96 \text{ horsepower}}}
\end{aligned}
$$

10.45 **Information and Assumptions**

provided in problem statement

Find

maximum velocity, resistance coefficient, shear velocity, shear stress 25 mm from pipe center and head loss if discharge doubled.

Solution

Reynolds number

$$\mathrm{Re} = VD\rho/\mu = (0.1)(0.1)(800)/(10^{-2}) = 800$$

Because $\mathrm{Re} < 2,000$ the flow is laminar. Therefore,

a)

$$V_{\max} = 2\bar{V} = 2 \times 0.1 \text{ m/s } = \underline{\underline{0.2 \text{ m/s}}}$$

b)

$$f = 64/\mathrm{Re} = 64/800 = 0.080$$

c)

$$\begin{aligned}(u_*/V) &= (f/8)^{0.5} = (0.08/8)^{0.5} = 0.10 \\ u_* &= (0.1)(0.1) = \underline{\underline{0.01 \text{ m/s}}}\end{aligned}$$

d)

$$\begin{aligned}(\tau_0/\rho)^{0.5} &= 0.01 \Longrightarrow \tau_0/\rho = 10^{-4} \\ \tau_0 &= (10^{-4})(800) = 0.08 \text{ N/m}^2 \\ \tau_{25\mathrm{mm}} &= 0.5\mathrm{t}_0 = \underline{\underline{0.04 \text{ N/m}^2}}\end{aligned}$$

e) Yes <u>it will be doubled</u> because flow will still be laminar and head loss is linear with V (and Q) in laminar range.

10.46 The valid statements are: a, b, d. For cases c & e:

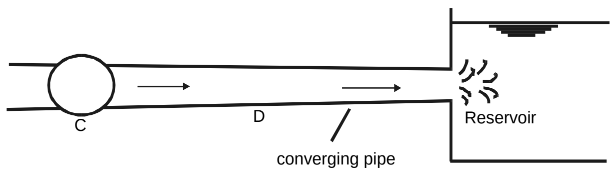
$$\text{Re} = VD/\nu = (1)(1)/(10^{-6}) = 10^6$$

The flow at 1 m/s is in the turbulent range; therefore, the head loss will be more than doubled with a doubling of the velocity.

10.47 a) Pumps are at A and C

b) A contraction, such as a Venturi meter or orifice, must be at B.

c)



d) Other information:

(1) Flow is from left to right

(2) The pipe between AC is smaller than before or directly after it.

(3) The pipe between BC is probably rougher than AB.

10.48 **Information and Assumptions**

from Table A.5 $\nu(20^o\text{C}) = 10^{-6}$ m^2/s
provided in problem statement

Find

shear stress at wall, shear stress 1 cm from wall and velocity 1 cm from wall

Solution

$$\begin{aligned} V &= Q/A = 0.05/((\pi/4)\times 0.15^2) = 2.83 \text{ m/s} \\ \text{Re} &= VD/\nu = 2.83 \times 0.15/(10^{-6}) = 4.2 \times 10^5 \end{aligned}$$

From Fig. 10.8 $k_s/D = 0.26/.50 = 0.0017$ and $f = 0.023$. From Eq. (10-21)

$$\begin{aligned} \tau_0 &= f\rho V^2/8 \\ \tau_0 &= 0.023 \times 998 \times 2.262^2/8 = \underline{\underline{14.7 \text{ N/m}^2}} \end{aligned}$$

Assume a linear shear stress variation; thus,

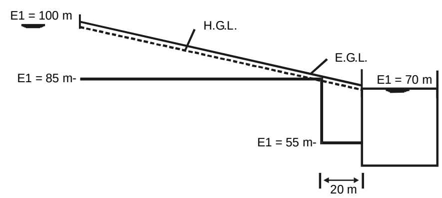
$$\tau_1 = (6.5/7.5) \times \tau_0 = \underline{\underline{12.7 \text{ N/m}^2}}$$

Assume logarithmic velocity distribution. Thus,

$$\begin{aligned} u/u_* &= 5.75\log(y/k_s) + 8.5 \\ u_* &= \sqrt{\tau_0/\rho} = 0.113 \text{ m/s} \end{aligned}$$

$$u = 0.113[5.75\log(0.01/(0.00026)) + 8.51] = \underline{\underline{1.99 \text{ m/s}}}$$

10.49 One possibility is shown below:



Assume that the pipe diameter is 0.50 m. Also assume $K_b = 0.20$, and $f = 0.015$. Then

$$\begin{aligned} 100 - 70 &= (0.5 + 2 \times 0.20 + 1 + 0.015 \times 130/0.5)V^2/2g \\ V^2/2g &= 5.17 \end{aligned}$$

The minimum pressure will occur just downstream of the first bend and its magnitude will be as follows:

$$\begin{aligned} p_{\min}/\gamma &= 100 - 85 - (0.5 + 0.20 + 1 + ((0.015 \times 80/0.5) + 1)V^2/2g \\ &= -6.20 \text{ m} \\ p_{\min} &= -6.20 \times 9,810 = \underline{\underline{\text{-60.8 kPa gage}}} \end{aligned}$$

10.50 **Information and Assumptions**

provided in problem statement

Find

pressure at point 80 m above pump

Solution

$$\begin{aligned}
\text{Re} &= 4Q/(\pi D\nu) \\
&= 4 \times 0.02/(\pi \times 0.10 \times 10^{-6}) = 2.55 \times 10^5 \\
k_s/D &= 4.6 \times 10^{-2}/100 = 4.6 \times 10^{-4}
\end{aligned}$$

From Fig. 10.8 $f = 0.0185$ Then

$$h_f = (f(L/D)V^2/2g$$

where

$$\begin{aligned}
V &= 0.02/((\pi/4) \times 0.1^2) = 2.546 \text{ m/s} \\
h_f &= 0.0185 \times (80/0.10) \times 2.546^2/(2 \times 9.81) = 4.89 \text{ m}
\end{aligned}$$

Energy equation. from pump to point 80 m. higher:

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_f \\
1.6 \times 10^6/9,790 + V_1^2/2g &= p_2/\gamma + V_2^2/2g + 80 + 4.89 \\
V_1 &= V_2 \\
p_2 &= \underline{\underline{769 \text{ kPa}}}
\end{aligned}$$

10.51 **Information and Assumptions**

from Table 10.3 $K_e = 0.5$
provided in problem statement

Find

velocity in pipe

Energy equation from the water surface in the tank to the outlet of the pipe.

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 12 &= 0 + V_2^2/2g + 0 + (K_e + fL/D)V^2/2g \\ 14 &= V_2^2/2g(1 + K_e + fL/D) \end{aligned} \tag{1}$$

The relative roughness is $5{\times}10^{-4}/(1/12) = 0.006$. The equation for velocity becomes

$$V^2 = \frac{2 \times 32.2 \times 14}{1.5 + 120 \times f}$$

Using equation 10.26 for f and solving on a programmable calculator gives

$$V = 12.80 \text{ ft/s}$$

10.52 **Information and Assumptions**

assume the pipe is galvanized iron
assume water temperature is 20°C so $\nu = 10^{-6}$ m^2/s
from Table 10.2 $k_s = 0.15$ mm $= 0.015$ cm
from Table 10.3 $K_b = 0.9$ and $K_e = 0.5$
provided in problem statement

Find

exit velocity of water and height of water jet

Solution

Relative roughness $k_s/D = .015/2 = .0075$. From Fig. 10.8 $f = 0.035$

Energy equation from the water surface in the tank to the pipe outlet

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_q &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 5 &= 0 + V_2^2/2g + 0 + (K_e + 2K_b + fL/D)V_2^2/2g \\
5 &= (V_2^2/2g)(1 + 0.5 + 2 \times 0.9 + .035 \times 10/.02) \\
5 &= (V_2^2/2g)(2.08) \\
V_2 &= 2.17 \text{ m/s}
\end{aligned}$$

Check Re and new f. $Re = VD/\nu = 2.17 \times 0.02/10^6 = 4.34 \times 10^4$. From Fig. 10.8 $f = 0.036$. With new f:

$$\begin{aligned}
V_2 &= \underline{\underline{2.15 \text{ m/s}}} \\
h &= V^2/2g = (2.15)^2/(2 \times 9.81) \\
&= \underline{\underline{0.24 \text{ m} = 24 \text{ cm}}}
\end{aligned}$$

10.53 **Information and Assumptions**

from Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s
from Table 10.2 $k_s = 1.5 \times 10^{-4}$ ft
from Table 10.2 $K_e = 0.03$
provided in problem statement

Find

the power to operate pump

Solution

$$
\begin{aligned}
V &= Q/A = 1.0/((\pi/4)D^2) \\
&= 1.0/((\pi/4)(1/3)^2) \\
&= 11.46 \text{ ft/s}
\end{aligned}
$$

Then

$$
\begin{aligned}
\text{Re} &= 11.46 \times (1/3)/(1.22 \times 10^{-5}) = 3.13 \times 10^5 \\
k_s/D &= 4.5 \times 10^{-4}
\end{aligned}
$$

From Fig. 10.8 $f = 0.0165$ and

$$
fL/D = 0.0165 \times 300/(1/3) = 14.86
$$

Energy equation from water surface A to water surface B

$$
\begin{aligned}
p_A/\gamma + V_A^2/2g + z_A + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 0 + h_p &= (10 \times 144/62.4) + 0 + (K_e + K_E + fL/D)V^2/2g
\end{aligned}
$$

$$
\begin{aligned}
h_p &= 23.08 + (0.03 + 1 + 14.86)(11.46^2/64.4) \\
&= 55.48 \text{ ft}
\end{aligned}
$$

Pump power

$$
\begin{aligned}
P &= Q\gamma h_p/eff \\
&= 1.0 \times 62.4 \times 55.48/0.85 \\
&= \underline{\underline{4{,}073 \text{ ft}\cdot\text{lbf/s}}} \\
&= \underline{\underline{7.41 \text{ horsepower}}}
\end{aligned}
$$

10.54 **Information and Assumptions**

from Table 10.3 $K_e = 0.5$ and $K_E = 1.0$.
provided in problem statement

Find

time to fill tank

Energy equation from the reservoir water surface to the tank water surface. The head losses will be due to entrance, pipe resistance, and exit.

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + z_1 + h_p &= 0 + 0 + z_2 + (K_e + fL/D + K_E)V^2/2g \\
h_p &= (z_2 - z_1) + (0.5 + (0.018 \times 30/0.9) + 1.0)V^2/2g \\
h_p &= h + (2.1)V^2/2g
\end{aligned}$$

But the head supplied by the pump is $h_o(1 - (Q^2/Q_{\max}^2))$ so

$$\begin{aligned}
h_o(1 - Q^2/Q_{\max}^2)) &= h + 1.05V^2/g \\
50(1 - Q^2/4) &= h + 1.05Q^2/(gA^2) \\
50 - 12.5Q^2 &= h + 1.05Q^2/(gA^2)
\end{aligned}$$

However $A = (\pi/4)D^2 = (\pi/4)(0.9^2) = 0.63$ m^2 and $g = 9.81$ m/s^2 so

$$\begin{aligned}
50 - 12.5Q^2 &= h + 0.790Q^2 \\
50 - h &= 13.29Q^2 \\
\sqrt{50 - h} &= 3.646Q
\end{aligned}$$

The discharge into the tank and the rate of water level increase is related by

$$Q = A_{\text{tank}}dh/dt$$

so

$$\sqrt{50 - h} = 3.646A_{\text{tank}}dh/dt$$

or

$$dt = 3.646A_{\text{tank}}(50 - h)^{-1/2}dh$$

Integrating

$$t = 2 \times 3.646 A_{\text{tank}} (50 - h)^{1/2} + C$$

when $t = 0$, $h = 0$ and $A_{\text{tank}} = 100 \text{ m}^2$ so

$$t = 729.1(7.071 - (50 - h)^{1/2})$$

When $h = 40$ m

$$t = 2850 \text{ s} = \underline{\underline{47.5}} \text{ min}$$

10.55 **Information and Assumptions**

provided in problem statement

Find

ratio of head loss for laminar to turbulent flow

Solution

$$\begin{aligned}\mathrm{Re} &= VD/\nu = 4 \times 0.03/(2 \times 10^{-6}) = 6 \times 10^4 \\ f_{\mathrm{lam}} &= 65/\mathrm{Re} = 64/(6 \times 10^4)\end{aligned}$$

From Fig.10-8 $f_{\mathrm{turb}} = 0.020$. Then

$$h_{f_{\mathrm{lam}}}/h_{f_{\mathrm{turb}}} = 64/((6 \times 10^4) \times (0.020)) = \underline{\underline{0.0533}}$$

10.56 **Information and Assumptions**

from Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s
from Table 10.2 $k_s = 8.5 \times 10^{-4}$ in
provided in problem statement

Find

resistance coefficient f

Solution

$$\begin{aligned} \text{Re} &= 4Q/\pi D\nu = 4 \times 0.02/(\pi \times (4/12) \times (1.22 \times 10^{-5})) \\ &= 6.3 \times 10^3 \\ k_s/D &= 8.5 \times 10^{-4}/(4/12) = 0.0025 \end{aligned}$$

Then from Fig. 10.8, $\underline{\underline{f = 0.038}}$

10.57 **Information and Assumptions**

provided in problem statement

Find

the head loss

Solution

$$
\begin{aligned}
\text{Re} &= 4Q/(\pi D\nu) = 4(1.0)/(\pi(1/2)3.33\times 10^{-3}) = 765 \text{ (laminar)} \\
\nu &= Q/(\pi D^2/4) = 1.0/(\pi/4\times 0.5^2) = 5.09 \text{ ft/s} \\
\mu &= \rho\nu = 0.005 \text{ lbf-s/ft}^2 \\
h_f &= 32\mu LV/(\gamma D^2) = 32(5\times 10^{-3})1,500(5.09)/(1.5\times 32.2\times (1/2)^2) \\
&= \underline{\underline{101.2 \text{ ft}}}
\end{aligned}
$$

10.58 **Information and Assumptions**

from Table 10.2 $k_s = 4.6 \times 10^{-5}$ m
provided in problem statement

Find

pressure at point A

Solution

$$\begin{aligned} \text{Re} &= VD/\nu = 4Q/(\pi D\nu) = 4 \times 0.03/(\pi \times 0.15 \times (10^{-2}/820)) \\ \text{Re} &= 2.09 \times 10^4 \text{ (turbulent)} \\ k_s/D &= 4.6 \times 10^{-5}/0.15 = 3.1 \times 10^{-4} \\ V &= Q/A = 0.03/(\pi \times 0.15^2/4) = 1.698 \text{ m/s} \end{aligned}$$

From Fig. 10.8 $f = 0.027$. Then

$$h_f = f(L/D)(V^2/2g) = 0.027(1,000/0.15)(1.698^2/(2 \times 9.81)) = 26.4 \text{ m}$$

Energy equation

$$p_A/\gamma + V_A^2/2g + z_A + p_B/\gamma + V_B^2/2g + z_B + h_f$$

$$p_A = 0.82 \times 9,810[(250,000/(0.82 \times 9,810)) + 20 + 26.41] = \underline{\underline{623 \text{ kPa}}}$$

10.59 **Information and Assumptions**

provided in problem statement

Find

time required to reduce water level in tank from 10 m to 2 m.

Energy equation for this problem gives

$$h = \frac{V^2}{2g}(K_E + K_{\text{valve}} + f\frac{L}{D})$$

Substituting in values and expressing V in terms of h we have

$$V = \sqrt{\frac{2gh}{6 + 100 \times f}}$$

The relative roughness is

$$\frac{k_s}{D} = \frac{0.15}{26} = 5.8 \times 10^{-3}$$

The Reynolds number in terms of V is

$$\text{Re} = \frac{V \times 0.026}{10^{-6}} = 2.6 \times 10^4 V$$

The rate of decrease of height in the tank is

$$\frac{dh}{dt} = -\frac{Q}{A} = -\frac{0.000531}{3.14}V = -0.000169V$$

A program was written to first find V iteratively for a given h using Eq. 10.26 for the friction factor. Then a new h was found by

$$h_n = h_{n-1} - 0.000169V\Delta t$$

where Δt is the time step. The result was 1424 sec or $\underline{\underline{23.7}}$ minutes.

When valves are tested to evaluate K_{valve} the pressure taps are usually connected to pipes both upstream and downstream of the valve. Therefore, the head loss in this problem may not actually be $5V^2/2g$. Also, the velocity exiting the valve will probably be highly non-uniform; therefore, this solution should be considered a gross approximation only.

10.60 **Information and Assumptions**

from Table A.5 $\nu = 1.41 \times 10^{-5}$ ft^2/s
from Table 10.2 $k_s = 0.00085$ ft
provided in problem statement

Find

direction and rate of flow

Solution

$$h_f = \Delta(p/\gamma + z) = (-20 \times 144/62.4) + 30 = -16.2 \text{ ft}$$

Therefore, flow is from B to A

$$\begin{aligned}
\operatorname{Re} f^{1/2} &= (D^{3/2}/\nu)(2gh_f/L)^{1/2} = (2^{3/2}/(1.41 \times 10^{-5}) \times 64.4 \times 16.2/(3 \times 5,280))^{1/2} = 5.14 \times 10^4 \\
k_s/D &= 0.0004
\end{aligned}$$

From Fig. 10.8 $f = 0.0175$. Then

$$\begin{aligned}
V &= \sqrt{h_f 2gD/fL} = \sqrt{(16.2 \times 64.4 \times 2)/(0.0175 \times 3 \times 5,280)} = 2.74 \text{ ft/s} \\
q &= VA = 2.74 \times (\pi/4) \times 2^2 = \underline{\underline{8.60 \text{ cfs}}}
\end{aligned}$$

10.61 **Information and Assumptions**

from Table 10.2 $k_s = 0.046$ mm
provided in problem statement

Find

power provided by pump

Solution

$$\begin{aligned}
V &= Q/A = 0.10/((\pi/4) \times 0.15^2) = 5.66 \text{ m/s} \\
V^2/2g &= 1.63 \text{ m} \\
k_s/D &= 0.0046/15 = 0.0003 \\
\text{Re} &= VD/\nu = 5.66 \times 0.15/(1.3 \times 10^{-6}) \\
&= 6.4 \times 10^5
\end{aligned}$$

From Fig. 10.8 $f = 0.016$.

Energy equation between the two reservoirs:

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
h_p &= z_2 - z_1 + V^2/2g(K_e + f(L/D) + K_0) \\
&= 13 - 10 + 1.63 \times (0.1 + 0.016 \times 80/(0.15) + 1) \\
&= 3 + 15.7 = 18.7 \text{ m} \\
p &= Q\gamma h_p = 0.10 \times 9,810 \times 18.7 \\
&= 18,345 \text{ W} = \underline{\underline{18.3 \text{ kW}}}
\end{aligned}$$

10.62 **Information and Assumptions**

from Table 10.2 concrete $k_s = 0.3$ mm, riveted steel $k_s = 0.9$ mm
from Table A.5 $\nu = 1.31 \times 10^{-6}$ m^2/s
provided in problem statement

Find

pump power

Energy equation from upstream reservoir water surface to downstream water surface.

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ 0 + 0 + z_1 &= 0 + 0 + z_2 + h_f \\ 100 \text{ m} &= (fL/D)V^2/2g \\ k_s/D &= 0.3/10^3 = 0.00035 \end{aligned}$$

From Fig. 10.8 $f = 0.016$. Then

$$100m = (0.016 \times 10,000/1)V^2/2g$$

$$\begin{aligned} V &= (100(2g)/(160))^{1/2} = 3.50 \text{ m/s} \\ \text{Re} &= VD/\nu = (3.50)(10)/(1.31 \times 10^{-6}) \\ &= 2.67 \times 10^6 \end{aligned}$$

Check f from Fig. 10.8 ($f = 0.0155$) and solve again:

$$\begin{aligned} V &= 3.55 \text{ m/s} \\ Q &= VA = (3.55)(\pi/4)D^2 = \underline{\underline{2.79 \text{ m}^3/\text{s}}} \end{aligned}$$

For riveted steel: $k_s/D = 0.9/1000 \simeq 001$ and from Fig. 10.8 $f = 0.0198$.

$$\begin{aligned} Q_{R.S}/Q_c &= \sqrt{0.0155/0.0198} = 0.885 \\ Q_{R.S} &= \underline{\underline{2.47 \text{ m}^3/\text{s}}} \end{aligned}$$

Pump power

$$h_p = (z_1 - z_2) + h_L$$

$$\begin{aligned} &= 100 \text{ m} + 100(2.8/2.79)^2 \\ &= 201 \text{ m} \\ P &= Q\gamma h_p = (2.8)(9,810)(201) = \underline{\underline{5.52 \text{ MW}}} \end{aligned}$$

10.63 **Information and Assumptions**

from Table 10.2 $k_s = 0.15$ mm
Energy equation between stations
provided in problem statement

Find

an estimate of flow rate

Solution

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_f \\ 150,000/(800 \times 9.81) + V_1^2/2g + 0 &= 120,000/(800 \times 9.81) + V_2^2/2g + 3 + h_f \\ h_f &= 0.823 \\ ((D^{3/2})/(\nu)) \times (2gh_f/L)^{1/2} &= ((0.8)^{3/2}/10^{-6}) \times (2 \times 9.81 \times 0.0823/30)^{1/2} \\ &= 1.66 \times 10^4 \end{aligned}$$

Relative roughness

$$k_s/D = 1.5 \times 10^{-4}/0.08 = 1.9 \times 10^{-3}$$

From Fig. 10-8 $f = 0.025$. Then

$$h_f = f(L/D)(V^2/2g);\ V = \sqrt{(h_f/f)(D/L)2g}$$

$$\begin{aligned} V &= \sqrt{(0.823/0.025)(0.08/30) \times 2 \times 9.81} = 1.312 \text{ m/s} \\ Q &= VA = 1.312 \times (\pi/4) \times (0.08)^2 = \underline{\underline{6.59\ \times 10^{-3}\ \text{m}^3/\text{s}}} \end{aligned}$$

10.64 **Information and Assumptions**

from Table 10.2 $k_s = 0.046$ mm
provided in problem statement

Find

pump power

Energy equation between reservoir surfaces

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 100 + h_p &= 0 + 0 + 112 + V^2/2g(K_e + fL/D + K_0) \\ h_p &= 12 + (V^2/2g)(0.03 + fL/D + 1 \end{aligned}$$

Here

$$\begin{aligned} V &= Q/A = 0.20/((\pi/4) \times 0.30^2) = 2.83 \text{ m/s} \\ V^2/2g &= 0.408\,\text{m} \end{aligned}$$

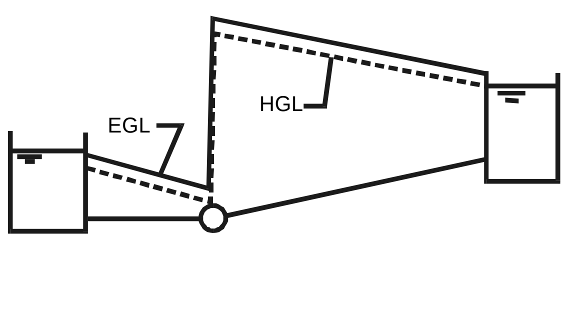
$$\begin{aligned} \text{Re} &= VD/\nu = 2.83 \times 0.30/(10^{-5}) = 8.5 \times 10^4 \\ k_s/D &= 4.6 \times 10^{-5}/0.3 = 1.5 \times 10^{-4} \end{aligned}$$

From Fig. 10.8 $f = 0.019$. Then

$$h_p = 12 + 0.408(0.03 + (0.019 \times 150/0.3) + 1.0) = 16.3 \text{ m}$$

Finally

$$\begin{aligned} P &= Q\gamma h_p = 0.20 \times (940 \times 9.81) \times 16.3 = 2.67 \times 10^4 W \\ &= \underline{\underline{30.1 \text{ kW}}} \end{aligned}$$



10.65 **Information and Assumptions**

provided in problem statement

Find

change in head loss

Solution

$$\text{Re} = VD/\nu = 3 \times 0.40/10^{-5} = 1.2 \times 10^{5}$$

The flow is turbulent and obviously the conduit is very rough ($f = 0.06$); therefore, one would expect f to be virtually constant. Thus, $h_f \sim V^2$, so if the velocity is doubled, the head loss will be $\underline{\underline{\text{quadrupled}}}$.

10.66 **Information and Assumptions**

From Table 10.2 $k_s = 8.5 \times 10^{-4}$ ft
assume water temperature of 60°F and $\nu = 1.22 \times 10^{-5}$ ft^2/s
provided in problem statement

Find

discharge in pipe

Energy equation from the water surface in the upper reservoir to the water surface in the lower reservoir:

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 100 &= 0 + 0 + 40 + (K_e + 2K_v + +K_E + fL/D)V^2 2/g \\ 100 &= 40 + (0.5 + 2 \times 0.2 + 1.0 + f \times 200/1)V^2/2g \end{aligned}$$

The relative roughness is $8.5 \times 10^{-4}/1 = 0.00085$. The equation for V becomes

$$\frac{V^2}{2g} = 1.9 + 200f$$

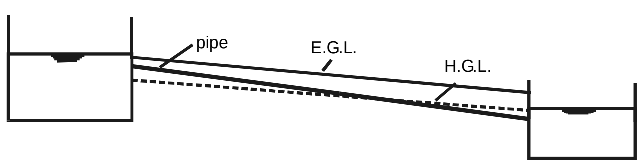
and the Reynolds number is

$$\text{Re} = 8.2 \times 10^4 \times V$$

Using Eq. 10.26 for f and solving on a programmable calculator gives

$$V = 14.3 \text{ m/s}$$

$$Q = VA = 14.30 \times (\pi/4) \times 1^2 = \underline{\underline{11.22 \text{ cfs}}}$$



10.67 **Information and Assumptions**

from Table 10.2 $k_s = 8.5 \times 10^{-4}$ ft
provided in problem statement

Find

diameter of cast iron pipe

Solution

Energy equation between points

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + z_1 &= 0 + 0 + z_2 + \sum h_L \\
20 &= (V^2/2g)(K_e + f(L/D) + K_0) \\
&= V^2/2g(0.5 + f(L/D) + 1.0) \\
[(\pi/4)^2 \times 2g \times 20/Q^2] &= [1.5 + f(L/D)] \times D^{-4};\ 7.94D^4 = (1.5 + fL/D)
\end{aligned}
$$

For first trial assume $f = 0.02$ and neglect the entrance loss:

$$
\begin{aligned}
D^5 &= 0.02 \times 2 \times 5,280/7.94 \\
D &= 1.93 \text{ ft}
\end{aligned}
$$

Then

$$
V = Q/A = 10/((\pi/4) \times (1.93)^2) = 3.43 \text{ ft/s}
$$

and

$$
\begin{aligned}
\text{Re} &= 3.43 \times 1.93/(1.2 \times 10^{-5}) = 5.5 \times 10^5 \\
k_s/D &= 8.5 \times 10^{-4}/1.93 \simeq 0.00045
\end{aligned}
$$

From Fig. 10.8 $f = 0.0175$
2nd trial:

$$
\begin{aligned}
D^5 &= 0.0175 \times 2 \times 5,280/7.94 \\
D &= 1.88 ft = \underline{\underline{22.5 \text{ in.}}}
\end{aligned}
$$

Use next commercial size larger; $\underline{\underline{D = 24 \text{ in.}}}$

10.68 **Information and Assumptions**

from Table A.5 $\nu = 1.22 \times 10^{-5} \text{ft}^2/\text{s}$
from Table 10.2 $k_s = 1.5 \times 10^{-4}$ ft
provided in problem statement

Find

diameter of commercial steel pipe

Solution

First assume

$$f = 0.015$$

Then

$$\begin{aligned} h_f &= (fL/D)V^2/2g \\ &= (0.015 \times 1,000/D)(Q^2/((\pi/4)^2 D^4)/2g) \\ 1 \text{ ft} &= 33,984/D^5 \\ D &= 8.06 \text{ ft} \end{aligned}$$

Relative roughness $k_s/D = 1.5 \times 10^{-4}/8.06 \simeq 0.00002$.
Now get better estimate of f:

$$\text{Re} = 4Q/(\pi D\nu) = 3.9 \times 10^6$$

From Fig. 10.8 $f \approx 0.010$
Compute D again:

$$\begin{aligned} 1 &= 22,656/D^5 \\ D &= 7.43 \text{ ft} = 89 \text{ in.} \end{aligned}$$

Use 90 in. pipe.

10.69 **Information and Assumptions**

from Table 10.2 $k_s = 0.046$ mm
provided in problem statement

Find

diameter of pipe and pump power

Solution

$$
\begin{aligned}
h_f &= f(L/D)V^2/2g = f(L/D)(Q^2/(2gA^2)) \\
&= f(L/D)(Q^2/(2g(\pi/4)^2 \times D^4)) \\
&= fLQ^2/(2g(\pi/4)^2 D^5) \\
D &= [8fLQ^2/(g\pi^2 h_f)]^{1/5}
\end{aligned}
$$

Assume $f = 0.015$

$$D = [8 \times 0.015 \times 1,000 \times 0.1^2/(9.81 \times \pi^2 \times 50)]^{1/5} = 0.19 \text{ m}$$

Then

$$
\begin{aligned}
k_s/D &= 0.046/190 \simeq 0.00025 \\
\text{Re} &= 4Q/(\pi D\nu) = 4 \times 0.1/(\pi \times 0.19 \times 10^{-5}) \\
&= 6.7 \times 10^4
\end{aligned}
$$

From Fig. 10.8 $f = 0.021$. Try again

$$D = (0.021/0.015)^{1/5} \times 0.19 = 0.203 \text{ m} = 20.3 \text{ cm}$$

Use next commercial size larger; $\underline{\underline{D = 22 \text{ cm}}}$. Still assume $h_L \approx 50$ m/1,000 m
Then

$$P = Q\gamma h_f = 0.1 \times 0.93 \times 9,810 \times 50 = 45,620 \text{ W/km} = \underline{\underline{45.6 \text{ kW/km}}}$$

10.70 **Information and Assumptions**

assume $T = 60^{\circ}\text{F}$, $\nu = 1.22 \times 10^{-5}$ ft^2/s
assume commercial steel pipe $k_s = 0.00015$ ft
provided in problem statement

Find

pipe diameter

Energy equation

$$30 = (K_e + K_E + fL/D)(Q^2/A^2)/2g$$

Assume $f = 0.015$. Then

$$\begin{aligned} 30 &= (1.5 + 0.015 \times 3 \times 5,280/D)(Q^2/((\pi/4)^2 D^4)/2g \\ 30 &= (1.5 + 237.6/D)(15^2/(0.617D^4)/64.4 \\ 30 &= (1.5 + 237.6/D)(5.66/D^4) \end{aligned}$$

Neglect the entrance and exit losses and solve

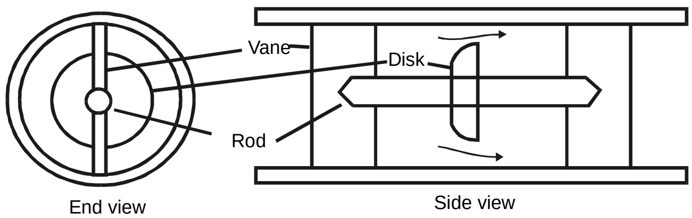
$$\begin{aligned} D &= 2.15 \text{ ft} \\ Re &= 4Q/(\pi D\nu) = 7.3 \times 10^5 \\ k_s/D &= 0.00015/2.15 \simeq 0.00007 \end{aligned}$$

From Fig. 10.8 $f = 0.0135$. Solve again

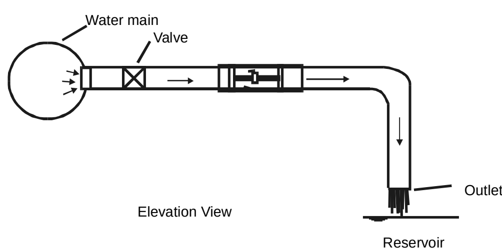
$$\begin{aligned} 30 &= (1.5 + 214/D)(5.66/D^4) \\ D &= 2.10 \text{ ft} = 25.2 \text{ in.} \end{aligned}$$

Use 26 in. steel pipe. (one possibility)

10.71 First you might consider how to physically hold the disk in the pipe. One way to do this might be to secure the disk to a rod and then secure the rod to streamlined vanes in the pipe such as shown below. The vanes would be attached to the pipe.



To establish cavitation around the disk, the pressure in the water at this section will have to be equal to the vapor pressure of the water. The designer will have to decide upon the pipe layout in which the disk is located. It might be something like shown below. By writing the energy equation from the disk section to the pipe outlet one can determine the velocity required at the disk to create vapor pressure at that sectional. This calculation will also establish the disk size relative to the pipe diameter. Once these calculations are made, one can calculate the required discharge, etc. Once that calculation is made, one can see if there is enough pressure in the water main to yield that discharge with the control valve wide open. If not, re-design the system. If it is OK, then different settings of the control valve will yield different degrees of cavitation.



10.72 **Information and Assumptions**

from Table 10.2 $k_s = 4 \times 10^{-4}$ ft
from Table A.5 $\nu = 1.41 \times 10^{-5}$ ft^2/s
from Table 10.3 $K_e = 0.5$
provided in problem statement

Find

discharge

Energy equation from water surface in reservoir to the outlet:

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
0 + 0 + 120 &= 0 + V^2/2g + 70 + (K_e + +K_E + f(L/D))V^2/2g \\
(V^2/2g)(1.5 + f(L/D)) &= 50 \text{ ft} \\
k_s/D &= 4 \times 10^{-4}/0.5 = 0.0008
\end{aligned}$$

The equation for velocity is

$$\frac{V^2}{2g} = 1.5 + 200f$$

The equation for Reynolds number is

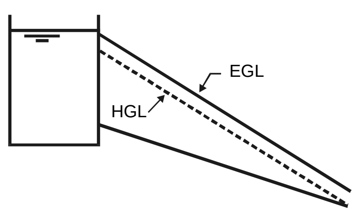
$$\text{Re} = 3.54 \times 10^4 \times V$$

Using a programmable calculator and Eq. 10.26 for f gives

$$V = 24.6 \text{ ft/s}$$

Discharge

$$Q = VA = 24.6(\pi/4)(0.5^2) = \underline{\underline{4.83}} \text{ cfs}$$



10.73 **Information and Assumptions**

assume $K_e = 0.10$
from Table A.5 $\nu = 1.41 \times 10^{-5}$ ft^2/s
provided in problem statement

Find

minimum pressure in pipe

Energy equation from water surface in reservoir to the outlet.

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ V &= Q/A = 50 \text{ ft/s} \end{aligned}$$

$$\text{Re} = VD/\nu = (50)(2)/(1.41 \times 10^{-5}) = 7.1 \times 10^6$$

$$\begin{aligned} 0 + 0 + 600 &= 0 + V_2^2/2g + 200 + (K_e + f(L/D))V^2/2g \\ 400 &= (V^2/2g)(1.10 + f(1,200/2)) \\ 400 &= (50^2/64.4)(1.10 + 600f) \\ f &= 0.0153 \end{aligned}$$

From Fig. 10.8 $k_s/D = 0.00035$ so $\underline{\underline{k_s = 0.00070 \text{ ft}}}$
The minimum pressure in the pipe is at the pipe outlet.

10.74 **Information and Assumptions**

from Table 10.2 $k_s = 0.15$ mm
provided in problem statement

Find

power required to operate heat exchanger with a) clean tubes and b) scaled tubes

Solution

$$\dot{m}/\text{tube} = 0.50 \text{ kg/s}$$

$$\begin{array}{c} Q/\text{tube } = 0.50/860 = 5.8139 \times 10^{-4} \text{ m}^3\text{/s} \\ V = Q/A = 5.8139{\times}10^{-4}/((\pi/4) \times (2 \times 10^{-2})^2) = 1.851 \text{ m/s} \\ \text{Re} = VD\rho/\mu = 1.851 \times 0.02 \times 860/(1.35 \times 10^{-4}) = 2.35 \times 10^5 \\ k_s/D = 0.15/20 \approx 0.007 \end{array}$$

From Fig. 10.8 $f = 0.034$. Then

$$\begin{array}{c} h_f = f(L/D)V^2/2g = 0.034(5/0.02) \times (1.851^2/2 \times 9.81) = 1.48 \text{ m} \\ \text{a)} \;\; P = Q\gamma h_f = 5.8139 \times 10^{-4} \times 860 \times 9.81 \times 1.48 \times 100 = \underline{\underline{728 \text{ W}}} \\ \text{b)} \;\; k_s/D = 0.5/16 = 0.031 \end{array}$$

so from Fig. 10.8 $f = 0.058$

$$P = 728 \times (0.058/0.034) \times (20/16)^4 = \underline{\underline{3.03 \text{ kW}}}$$

10.75 **Information and Assumptions**

provided in problem statement

Find

pump power required

Solution

Examination of the data given indicates that the tubing in the exchanger has an $r/d \approx 1$. Assuming smooth bends of $180°, K_b \approx 0.7$

Energy equation

$$p_1/\gamma + V_1^2/2g + z_1 + h_p = p_2/\gamma + V_2^2/2g + z_2 + h_L$$

But $V_1 = V_2$ and $p_1 = p_2$ so

$$z_2 - z_1 = 0.8 \text{ m}$$

The average temperature $= 50°C$ so $\nu = 0.58 \times 10^{-6}$ m^2/s

$$\begin{aligned} V &= Q/A = 3 \times 10^{-4}/(\pi/4(0.02)^2) = 0.955 \text{ m/s} \\ \text{Re} &= VD/\nu = 0.955(0.02)/(0.58 \times (10^{-6}) = 3.3 \times 10^4 \\ f &= 0.023 \\ h_L &= (fL/D + 19K_b)V^2/2g = (0.023(20)/0.02 + 19 \times 0.7)0.955^2/(2 \times 9.81) \\ &= 1.69 \text{ m} \end{aligned}$$

$$\begin{aligned} h_p &= z_2 - z_1 + h_L = 0.8 + 1.69 = 2.49 \text{ m} \\ P &= \gamma h_p Q = 9,685(2.49)3 \times 10^{-4} = \underline{\underline{7.23 \text{ W}}} \end{aligned}$$

10.76 **Information and Assumptions**

from Table A.5 $\nu = 6.58 \times 10^{-7}$ m^2/s
from Table 10.2 $k_s = 0.0015$ mm
provided in problem statement

Find

power required to operate pump

Solution

The Reynolds number of the water in the pipe is

$$\text{Re} = \frac{0.02 \times 10}{6.58 \times 10^{-7}} = 3.04 \times 10^5$$

and the discharge is

$$Q = \frac{\pi}{4} \times 0.02^2 \times 10 = 0.00314 \text{ m}^3/\text{s}$$

The relative roughness of the copper tubing is

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-3} \text{ mm}}{20 \text{ mm}} = 7.5 \times 10^{-5}$$

The Darcy-Weisbach friction factor is

$$f = 0.0155$$

Energy equation for this problem gives

$$h_p = \frac{V^2}{2g}(f\frac{L}{D} + \sum K_L)$$

or

$$h_p = \frac{10^2}{2 \times 9.81}(0.0155 \times \frac{10 \text{ m}}{0.02 \text{ m}} + 14 \times 2.2) = 196 \text{ m}$$

The power required is

$$\dot{W} = \gamma Q h_p = 9732z \times 0.00314 \times 196 = 5989 \text{ W}$$

or

$$\dot{W} = 5.99 \text{ kW}$$

Since the pump is 70% efficient, the actual power required is

$$\dot{W} = \frac{2.94}{0.7} = \underline{\underline{8.56}} \text{ kW}$$

10.77 **Information and Assumptions**

from Table 10.2 $k_s = 1.5 \times 10^{-3}$ mm
provided in problem statement

Find

system operating points

Energy equation for this problem reduces to

$$h_p = \frac{V^2}{2g}(\sum K_L + f\frac{L}{D})$$

Substituting in the values for loss coefficients, L/D and the equation for h_p we have

$$h_{p0}\left[1-\left(\frac{Q}{Q_{\max}}\right)^3\right] = \frac{V^2}{2g}(14 \times 2.2 + f \times 1000)$$

The velocity and discharge are related by

$$Q = VA = 1.767 \times 10^{-4} V$$

Substituting into the energy equation to give an equation for Q we have

$$h_{p0}\left[1-\left(\frac{Q}{Q_{\max}}\right)^3\right] = 1.632 \times 10^6 Q^2 (30.8 + f \times 1000)$$

The relative roughness is

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-3}}{15} = 10^{-4}$$

The Reynolds number in terms of Q is

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{V \times 0.015}{6.58 \times 10^{-7}} = 2.28 \times 10^4 V = 1.29 \times 10^8 Q$$

The equation for discharge was written in the form

$$F(Q) = h_{p0}\left[1-\left(\frac{Q}{Q_{\max}}\right)^3\right] - 1.632 \times 10^6 Q^2 (30.8 + f \times 1000)$$

A program was written to evaluate $F(Q)$ by inputting a value for Q and trying different Q's until $F(Q) = 0$. The results are

h_{p0} (m)	Q (m^3/s)
2	$\underline{\underline{0.000356}}$
10	$\underline{\underline{0.000629}}$
20	$\underline{\underline{0.000755}}$

10.78 **Information and Assumptions**

assume $T = 60^{\circ}\text{F}$ and $\nu = 1.22 \times 10^{-5}$ ft^2/s
assume $r/d = 2$ and $K_b = 0.2$
provided in problem statement

Find

the discharge, points of maximum and minimum pressures

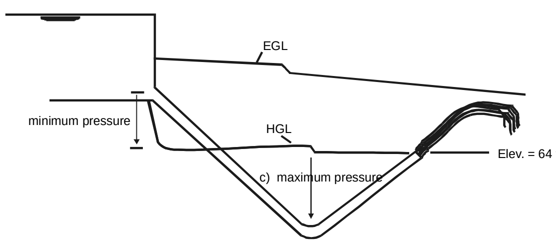
$$k_s/D = 0.004$$

Assume $f = 0.028$

Energy equation

$$\begin{aligned}
p_1/\gamma + z_1 + V_1^2/2g &= p_2/\gamma + z_2 + V_2^2/2g + \sum h_L \\
100 &= 64 + (V^2/2g)(1 + 0.5 + K_b + f \times L/D) \\
&= 64 + (V^2/2g)(1 + 0.5 + 0.2 + 0.028 \times 100/1) \\
36 &= (V^2/2g)(4.5) \\
V^2 &= 72g/4.5 = 515 \text{ ft}^2/\text{s}^2 \\
V &= 22.7 \text{ ft/s} \\
\text{Re} &= 22.7(1)/(1.22 \times 10^{-5}) = 1.9 \times 10^6 \\
f &= 0.028 \\
Q &= 22.7(\pi/4)1^2 = \underline{\underline{17.8 \text{ cfs}}}
\end{aligned}$$

$$V^2/2g = 36/4.5 = 8.0 \text{ ft}$$



$$\begin{aligned}
p_{\min}/\gamma &= 100 - 95 - (V^2/2g)(1 + 0.5) = 5 - 8(1.5) = -7 \text{ ft} \\
p_{\min} &= -7(62.4) = -437 \text{ psfg} = \underline{\underline{-3.03 \text{ psig}}} \\
p_{\max}/\gamma + V_m^2/2g + z_m &= p_2/\gamma + z_2 + V_2^2/2g + \sum h_L \\
p_{\max}/\gamma &= 64 - 44 + 8.0(0.2 + 0.028(28/1)) = 27.9 \text{ ft} \\
p_{\max} &= 27.9(62.4) = 1,739 \text{ psfg} = \underline{\underline{12.1 \text{ psig}}}
\end{aligned}$$

10.79 **Information and Assumptions**

from Fig. A.2 $S = 0.68$, $\nu = 5.5 \times 10^{-6}$ ft^2/sec
provided in problem statement

Find

pump power

Solution

$$\begin{aligned}
Q &= 0.12 \text{ gpm} = 2.68 \times 10^{-4} \text{ cfs} \\
d_1 &= (1/4)(1/12) = 0.0208 \text{ ft} \\
d_2 &= (1/32)(1/12) = 0.0026 \text{ ft} \\
d_2/d_1 &= (1/32)/(1/4) = 0.125 \\
\gamma &= 62.4(0.68) = 42.4 \text{ lbf/ft}^3 \\
V_1 &= Q/A = 2.68 \times 10^{-4}/(\pi/4(1/48)^2) = 0.786 \text{ ft/s} \\
V_1^2/2g &= 0.00959 \text{ ft} \\
V_2 &= (32/4)^2 \times 0.786 = 50.3 \text{ ft/s} \\
V_2^2/2g &= 39.3 \text{ ft} \\
\text{Re}_1 &= V_1 D_1/\nu = 0.786(0.0208)/(5.5 \times 10^{-6}) = 2,972
\end{aligned}$$

From Fig. 10.8 $f \approx 0.040$

$$\begin{aligned}
p_1 &= 14.7 \text{ psia; } z_2 - z_1 = 2 \text{ ft; } p_2 = 14.0 \text{ psia} \\
h_L &= (fL/D + 5K_b)V_1^2/2g \\
&= (0.040 \times 10/0.0208 + 5 \times 0.21)0.00959 = 0.194 \text{ ft} \\
h_p &= (p_2 - p_1)/\gamma + z_2 - z_1 + V_2^2/2g + h_L \\
&= (14.0 - 14.7)144/42.4 + 2 + 39.3 + 0.194 = 39.1 \text{ ft}
\end{aligned}$$

Pump power

$$\begin{aligned}
P &= \gamma h_p Q/(550e) = 42.4(39.1)0.000268/(550 \times 0.8) \\
&= \underline{\underline{10.1 \times 10^{-4}}} \text{ hp}
\end{aligned}$$

10.80 **Information and Assumptions**

from Table 10.2 $k_s = 0.046$ mm
provided in problem statement

Find

loss coefficient for valve
First find Q for valve wide open. Assume valve is a gate valve.

Energy equation

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
2 &= 0 + 0 + 0 + (V^2/2g)(0.5 + 0.9 + 0.2 + 0.9 + 1 + fL/D) \\
V^2 &= 4g/(3.5 + fL/D)
\end{aligned}$$

Assume $f = 0.015$. Then

$$\begin{aligned}
V &= [4 \times 9.81/(3.5 + 0.15 \times 14/0.1)]^{1/2} = 2.65 \text{ m/s} \\
k_s/D &\simeq 0.0005 \\
\text{Re} &= 2.65 \times 0.10/(1.3 \times 10^{-6}) = 2.0 \times 10^5
\end{aligned}$$

From Fig. 10.8 $f = 0.019$. Then

$$\begin{aligned}
V &= [4 \times 9.81/(3.5 + 0.019 \times 14/0.10)]^{1/2} = 2.52 \text{ m/s} \\
\text{Re} &= 2.0 \times 10^5 \times 2.52/2.65 = 1.9 \times 10^5;\ \text{O.K.}
\end{aligned}$$

This is close to 2.0×10^5 so no further iterations are necessary. For one-half the discharge

$$\begin{aligned}
V &= 1.26 \text{ m/s} \\
Re &= 9.5 \times 10^4
\end{aligned}$$

and from Fig. 10.8 $f = 0.021$. So

$$\begin{aligned}
V^2 &= 1.588 = 4 \times 9.81/(3.3 + K_v + 0.021 \times 14/0.1) \\
3.3 + K_v + 2.94 &= 24.7 \\
& \underline{\underline{K_v = 18.5}}
\end{aligned}$$

10.81 **Information and Assumptions**

from Table 10.2 $k_s = 0.15$ mm
provided in problem statement

Find

the pipe size

Energy equation

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_f \\
(300,000/9,810) + 0 &= (60,000/9,810) + 10 + h_f \\
h_f &= 14.45 \text{ m} \\
f(L/D)(Q^2/A^2)/2g &= 14.46 \\
f(L/D)[Q^2/((\pi/4)D^2)^2/2g] &= 14.46 \\
(4^2 fLQ^2/\pi^2)/2gD^5 &= 14.46 \\
D &= [(8/14.46)fLQ^2/(\pi^2 g)]^{1/5}
\end{aligned}$$

Assume $f = 0.020$. Then

$$D = [(8/14.46) \times 0.02 \times 140 \times (0.025)^2/(\pi^2 \times 9.81)]^{1/5} = 0.100 \text{ m}$$

This corresponds to a relative roughness of

$$k_s/D = 0.15/100 = 0.0015$$

and from Fig. 10.8 $f = 0.022$. Try again

$$D = 0.100 \times (0.022/0.020)^{1/5} = \underline{\underline{0.102 \text{ m}}}$$

Use a $\underline{\underline{\text{12-cm pipe}}}$

10.82 **Information and Assumptions**

from Table 10.3 $K_{bl} = 0.35$; $K_{b2} = 0.16$; $K_c = 0.39$, $K_e = 0.5$ and $K_E = 1.0$
from Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s
from Table 10.2 $k_s = 1.5 \times 10^{-4}$ ft
provided in problem statement

Find

the discharge

Energy equation

$$\begin{aligned}
p_1/\gamma + z_1 + V_1^2/2g &= p_2/\gamma + z_2 + V_2^2/2g + \sum h_L \\
11 &= \sum h_L = (V_1^2/2g)(K_e + 3K_{bl} + f_1 \times 50/1) \\
&\quad +(V_2^2/2g)(K_c + 2K_{b2} + K_E + f_2 \times 30/(1/2))
\end{aligned}$$

Assume $f_1 = 0.015$; $f_2 = 0.016$

$$\begin{aligned}
11 \times 2g &= V_1^2(0.5 + 3 \times 0.35 + 0.015(50)) + V_2^2(0.39 + 2 \times 0.16 + 1.0 + 0.016(60)) \\
708 &= V_1^2(2.3) + V_2^2(2.67) = Q^2(2.3/((\pi/4)^2(1)^4) + 2.67/((\pi/4)^2(1/2)^4)) = 73.0Q^2 \\
Q^2 &= 708/73.0 = 9.70 \\
Q &= 3.11 \text{ cfs} \\
\text{Re} &= 4Q/(\pi D \nu) \\
\text{Re}_1 &= 4(3.111)/(\pi(1.22 \times 10^{-5})) = 3.2 \times 10^5 \\
k_s/D_1 &= 1.5 \times 10^{-4}/1 = 0.00015 \\
\text{Re}_2 &= 6.5\ 10^5;\ k_s/D_2 = 0.0003
\end{aligned}$$

From Fig. 10.8 $f_1 = 0.016$ and $f_2 = 0.016$. No further iterations are necessary so

$$Q = \underline{\underline{3.11 \text{ cfs}}}$$

10.83 **Information and Assumptions**

from Table A.5 $\nu = 1.31 \times 10^{-6}$ m^2/s
provided in problem statement

Find

the discharge and pressure at point A

Energy equation

$$\begin{aligned} p_1/\gamma + z_1 + V_1^2/2g &= p_2/\gamma + z_2 + V_2^2/2g + \sum h_L \\ 0 + 12 + 0 &= 0 + 0 + (V^2/2g)(1 + K_e + K_v + 4K_b + f \times L/D) \end{aligned}$$

Using a pipe diameter of 10 cm and assuming $f = 0.025$

$$\begin{aligned} 24g &= V^2(1 + 0.5 + 10 + 4(0.9) + 0.025 \times 1{,}000/(0.10)) \\ V^2 &= 24g/265.1 = 0.888 \text{ m}^2/\text{s}^2 \\ V &= 0.942 \text{ m/s} \\ Q &= VA = 0.942(\pi/4)(0.10)^2 = \underline{\underline{0.0074}} \text{ m}^3/\text{s}; \\ \text{Re} &= 0.942 \times 0.1/1.31 \times 10^{-6} = 7 \times 10^4 \end{aligned}$$

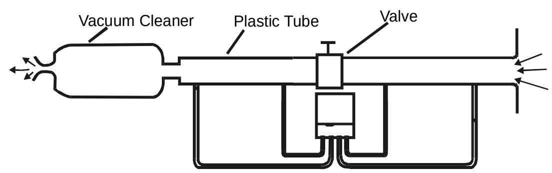
From Fig. 10.8 $f \approx 0.025$

$$\begin{aligned} p_A/\gamma + z_A + V^2/2g &= p_2/\gamma + z_2 + V^2/2g + \sum h_L \\ p_A/\gamma + 15 &= V^2/2g(2K_b + f \times L/D) \\ p_A/\gamma &= (0.888/2g)(2 \times 0.9 + 0.025 \times 500/0.1) - 15 = -9.26 \text{ m} \\ p_A &= 9810 \times (-9.26) = \underline{\underline{-90.8}} \text{ kPa} \end{aligned}$$

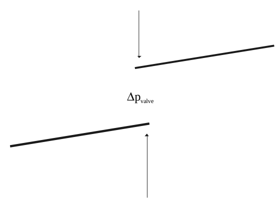
Note that this is not a good installation because the pressure at A is near cavitation level.

10.84 Assume that your vacuum cleaner is a tank type (the type with a hose). One might consider a setup as shown below.

Vacuum Cleaner



By measuring the height of liquid in each piezometer one may calculate the pressure at each section. One could also measure the pressure at each section by a pressure gage or pressure transducer. Once the pressures at all sections are obtained, one can plot pressure vs. distance along the tube. It might appear as shown below.



By extending the slopes of p vs. distance one can determine the Δp for the valve.

The air density can be determined by measuring the temperature and then picking ρ from the appropriate table or by solving for it by the equation of state. The discharge can be determined by taking a Pitot tube velocity traverse across the section of the pipe and integrating that to obtain Q. Once the discharge is determined, the mean velocity can be calculated from $V = Q/A$. Then the loss coefficient can be calculated using Eq. (10.29):

$$\begin{aligned} \gamma h_L &= K\gamma V^2/2g \\ \Delta p_f &= K\rho V^2/2 \\ \text{or } K &= 2\Delta p/\rho V^2 \end{aligned}$$

10.85 The design problem is specific house design by the architect.

10.86 **Information and Assumptions**

From Table 10.3 $K_e = 0.03; K_b = 0.35; K_E = 1.0$
from Table A.5 $\nu = 10^{-6}$ m^2/s
from Table 10.2 $k_s = 0.046$ mm
provided in problem statement

Find

the pump power

Energy equation from the water surface in the lower reservoir to the water surface in the upper reservoir.

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 200 \text{ m} + h_p &= 0 + 0 + 235 \text{ m } + (V^2/2g)(K_e + K_b + K_E + fL/D) \\
V &= Q/A = 0.314/((\pi/4) \times 0.3^2) = 4.44 \text{ m/s} \\
V^2/2g &= 1.01 \text{ m} \\
\text{Re} &= VD/\nu = 4.44 \times 0.3/10^{-6} = 1.33 \times 10^6 \\
k_s/D &\approx 0.00015
\end{aligned}$$

so from Fig. 10.8 $f = 0.00014$

$$\begin{aligned}
fL/D &= 0.014 \times 140/0.3 = 6.53 \\
h_p &= 235 - 200 + 1.01(0.03 + 0.35 + 1 + 6.53) = 42.0 \text{ m} \\
P &= Q\gamma h_p = 0.314 \times 9,790 \times 42.0 = \underline{\underline{129 \text{ kW}}}
\end{aligned}$$

10.87 **Information and Assumptions**

provided in problem statement

Find

rate of discharge

Solution

From the solution to 10.85

$$h_p = 35 + 8.38V^2/2g$$

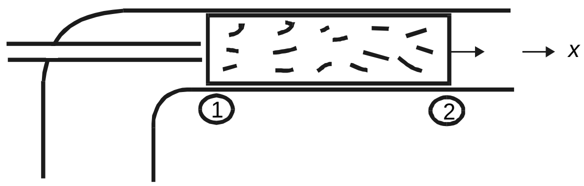
$$h_p = 35 + 8.38[(Q/((\pi/4) \times 0.3^2)^2/2g] = 35 + 85.6Q^2$$

System data computed and shown below:

$Q(\mathrm{m^3 s})$	$\rightarrow$	0.05	0.10	0.15	0.20	.30
$h_p(\mathrm{m})$	$\rightarrow$	35.2	35.8	36.9	38.4	42.7

Then, plotting the system curve on the pump performance curve of Fig.10-16 yields $\underline{\underline{Q = 0.25 \text{ m}^3/\text{s}}}$ for the operating point.

10.88 For the system to operate as a pump, the increase in head produced by the jet must be greater than 9 ft (the difference in elevation between the lower and upper reservoir). Consider the head change between a section just to the right of the jet and far to the right of it with zero flow in the lower pipe. Determine this head change by applying the momentum equation.



$$
\begin{aligned}
V_1 &= 60 \text{ ft/s} \\
Q &= V_1 A_1 = 2.94 \text{ cfs} \\
V_2 &= Q/A_2 = (60)(\pi/4)(3^2)/((\pi/4)(12^2)) \\
V_2 &= 60(3^2/12^2) = 3.75 \text{ ft/s} \\
\sum F_x &= \dot{m}_o V_o - \dot{m}_i V_i \\
p_1 A_1 - p_2 A_2 &= (3.75)(1.94)(3.75 \times (\pi/4)(1^2)) - (60)(1.94)(60 \times (\pi/4)(1/4)^2) \\
A(p_1 - p_2) &= 1.94(-176.7 + 11.04) \\
p_2 - p_1 &= 409.2 \text{ psf} \\
h_2 - h_1 &= (409.2 \text{ lbf/ft}^2)/(62.4 \text{ lb/ft}^3) = 6.56 \text{ ft}
\end{aligned}
$$

The change in head of 6.56 ft is not enough to overcome the static head of 9.0 ft.; therefore, the system will not act as a pump.

10.89 **Information and Assumptions**

provided in problem statement

Find

the discharge

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 10 + h_p &= 0 + 0 + 20 + V_2^2/2g(K_e + fL/D + k_0) \\ h_p &= 10 + (Q^2/(2gA^2))(0.1 + 0.02 \times 1,000/(10/12) + 1) \\ A &= (\pi/4) \times (10/12)^2 = 0.545 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} h_p &= 10 + 1.31 Q_{\text{cfs}}^2 \\ 1 \text{ cfs} &= 449 \text{ gpm} \\ h_p &= 10 + 1.31 Q_{\text{gpm}}^2/(449)^2 \\ h_p &= 10 + 6.51 \times 10^{-6} Q_{\text{gpm}}^2 \end{aligned}$$

$Q \rightarrow$	1,000	2,000	3,000
$h \rightarrow$	16.5	36.0	68.6

Plotting this on pump curve figure yields $Q \approx \underline{\underline{2,950 \text{ gpm}}}$

10.90 **Information and Assumptions**

provided in problem statement

Find

the pumping rate

Solution

$h_p = 20$ ft - 10 ft $= 10$ ft

Then from the pump curve for 10.89 one finds $\underline{\underline{Q = 4,700 \text{ gpm}}}$.

10.91 **Information and Assumptions**

from Table 10.2 $k_s = 0.046$ mm
assume from Table A.5 $\nu = 1.31 \times 10^{-6}$ mm
provided in problem statement

Find

pump power

Energy equation

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 100 + h_p &= 0 + V_2^2/2g + 150 + V_2^2/2g(0.03 + fL/D) \\
V_2 &= Q/A_p = 20/((\pi/4) \times 1.5^2) = 11.32 \text{ m/s} \\
\text{Re} &= VD/\nu = 11.32 \times 1.5/(1.31 \times 10^{-6}) = 1.3 \times 10^7 \\
k_s/D &= 0.046/1500 = 0.00003
\end{aligned}$$

From Fig. 10.8 $f = 0.010$. Then

$$\begin{aligned}
h_p &= 140 - 100 + V_2^2/2g(1.03 + 0.010 \times 300/1.5) \\
h_p &= 40 + 19.8 = 59.8 \text{ m}
\end{aligned}$$

Pump power

$$P = Q\gamma h_p = 15 \times 9,810 \times 59.8 = \underline{\underline{8.80 \text{ MW}}}$$

10.92 **Information and Assumptions**

provided in problem statement

Find

difference in water surface between two reservoirs

Solution

assume $T = 20^{\circ}\text{C}$ so $\nu = 10^{-6}\ \text{ft}^2/\text{s}$

$$\begin{aligned}
k_s/D_{15} &= 0.1/150 = 0.00067 \\
k_s/D_{30} &= 0.1/300 = 0.00033 \\
V_{15} &= Q/A_{15} = 0.1/((\pi/4) \times 0.15^2) = 5.659\ \text{m/s} \\
V_{30} &= 1.415\ \text{m/s} \\
\text{Re}_{15} &= VD/\nu = 5.659 \times 0.15/10^{-6} = 8.49 \times 10^5 \\
\text{Re}_{30} &= 1.415 \times 0.3/10^{-6} = 4.24 \times 10^5
\end{aligned}$$

From Fig. 10-8 $f_{15} = 0.0185;\ f_{30} = 0.0165$

Energy equation

$$z_1 - z_2 = \sum h_L$$

$$\begin{aligned}
z_1 - z_2 &= (V_{15}^2/2g)(0.5 + 0.0185 \times 50/0.15) \\
&\quad +(V_{30}^2/2g)(1 + 0.0165 \times 160/0.30) + (V_{15} - V_{30})^2/2g \\
z_1 - z_2 &= (5.659^2/(2 \times 9.81))(6.67) \\
&\quad +((1.415^2/(2 \times 9.81))(9.80) + (5.659 - 1.415)^2/(2 \times 9.81) \\
z_1 - z_2 &= 1.632(6.67) + 1.00 + 0.918 = \underline{\underline{12.80\ \text{m}}}
\end{aligned}$$

10.93 **Information and Assumptions**

from Table 10.3 $K_e = 0.5$ and $K_E = 1.0$
assume $T = 68°F$ so $\nu = 1.1 \times 10^{-5}$ ft^2/s.
provided in problem statement

Find

difference in water surface elevation between two reservoirs

Energy equation from the water surface in the tank at the left to the water surface in the tank on the right

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + \sum h_L$$

$$z_1 = z_2 + (K_e + f_1 L_1/D_1)V_1^2/2g + (V_1 - V_2)^2/2g + ((f_2 L_2/D_2) + K_E)V_2^2/2g$$

Calculate velocities and Reynolds number

$$\begin{aligned} V_1 &= Q/A_1 = Q/((\pi/4)(1/2)^2) = 25.48 \text{ ft/s} \\ \text{Re}_1 &= 25.48 \times (1/2)/1.1 \times 10^{-5}) = 1.16 \times 10 \\ V_1^2/2g &= 10.1 \text{ ft} \\ V_2 &= V_1/4 = 6.37 \text{ ft/s} \\ Re_2 &= 6.37 \times 1/1.1 \times 10^{-5} = 5.8 \times 10^6 \\ V_2^2/2g &= 0.630 \\ k_s/D_1 &= 4 \times 10^{-4}/0.5 = 8 \times 10^{-4} \\ k_s/D_2 &= 4 \times 10^{-4} \end{aligned}$$

From Fig. 10.8 $f_1 = 0.019$ and $f_2 = 0.016$

$$\begin{aligned} z_1 - z_2 &= h = (0.5 + .019 \times 150/(1/2))10.1 + (25.48 - 6.37)^2/64.4 \\ &\quad +((0.016 \times 500/1) + 1)0.630 \\ &= 62.6 + 5.7 + 5.7 \\ &= \underline{\underline{74.0 \text{ ft}}} \end{aligned}$$

10.94 **Information and Assumptions**

from Table 10.3 $K_e = 0.50; K_v = 5.6$
from Table 10.2 $k_s = 1.5 \times 10^{-4}$ ft
provided in problem statement

Find

discharge of oil

Energy equation from reservoir water surface to pipe outlet:

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 100 \text{ ft} &= 0 + V_2^2/2g + 64 + (V^2/2g)(K_e + K_v + fL/D)
\end{aligned}
$$

Assume $f = 0.015$ for first trial. Then

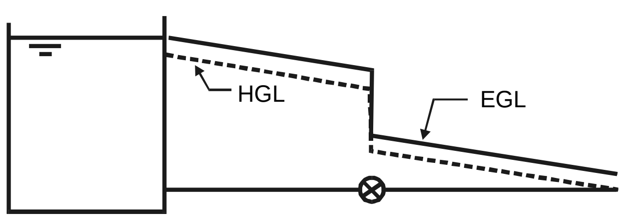
$$
\begin{gathered}
(V^2/2g)(0.5 + 5.6 + 1 + 0.015 \times 300/1) = 36 \\
V = 14.1 \text{ ft/s} \\
\text{Re} = VD/\nu = 14.1 \times 1/10^{-4} = 1.4 \times 10^5 \\
k_s/D = 0.00015
\end{gathered}
$$

From Fig. 10.8 $f \approx 0.0175$.
Second Trial:

$$
\begin{aligned}
V &= 13.7 \text{ ft/s} \\
\text{Re} &= 1.37 \times 10^5
\end{aligned}
$$

From Fig. 10.8 $f = 0.0175$.so

$$
Q = VA = 13.7 \times (\pi/4) \times 1^2 = \underline{\underline{10.8 \text{ ft}^3/\text{s}}}
$$



10.95 **Information and Assumptions**

from Table 10.3 $K_b = 0.19$
from Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s
provided in problem statement

Find

pump horsepower and pressure at midpoint of long pipe.

Energy equation

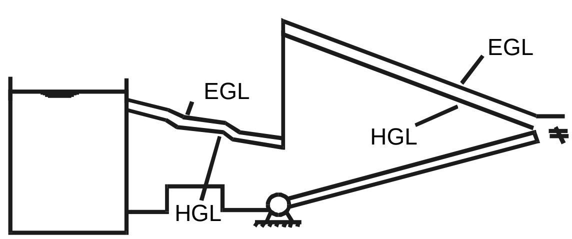
$$p_1/\gamma + V_1^2/2g + z_1 + h_p = p_2/\gamma + V_2^2/2g + z_2 + \sum h_L$$
$$0 + 30 + 0 + h_p = 0 + 60 + (V^2/2g)(1 + 0.5 + 4K_b + fL/D)$$
$$V = Q/A = 2.0/((\pi/4) \times (1/2)^2) = 10.18 \text{ ft/sec}$$
$$V^2/2g = 1.611 \text{ ft}$$
$$\text{Re} = 4Q/(\pi D\nu) = 4 \times 2/(\pi \times (1/2) \times 1.22 \times 10^{-5})$$
$$= 4.17 \times 10^5$$

From Table 10.8 $f = 0.0135$ so

$$h_p = 30 + 1.611(1 + 0.5 + 4 \times 0.19 + 0.0135 \times 1,700/(1/2)) = 107.6 \text{ ft}$$
$$P = Q\gamma h_p/550 = \underline{\underline{24.4 \text{ horsepower}}}$$

Pressure at midpoint of long pipe

$$p_m/\gamma + z_m = z_2 + h_L$$
$$p_m = \gamma[(z_2 - z_m) + h_L]$$
$$p_m = 62.4[(60 - 35) + 0.0135 \times (600/0.5) \times 1.611]$$
$$p_m = 3,189 \text{ psf} = \underline{\underline{22.1 \text{ psig}}}$$



10.96 **Information and Assumptions**

from Table 10.2 $k_s = 0.046$ mm
provided in problem statement

Find

pump power

Energy equation

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 20 + h_p &= 0 + 0 + 40 + V^2/2g(K_e + 2K_b + K_0 + fL/D) \\
h_p &= 20 + V^2/2g(0.5 + 2 \times 0.19 + 1 + fL/D) \\
V &= Q/A = 1.2/((\pi/4 \times 0.6^2) = 4.25 \text{ m/s} \\
V^2/2g &= 0.921 \text{ m} \\
\text{Re} &= VD/\nu = 4.25 \times 0.6/(5 \times 10^{-5}) = 5.1 \times 10^4 \\
k_s/D &= 0.00008 \\
f &= 0.021 \text{ (from Fig. 10-8);}
\end{aligned}$$

From Fig. 10.8 $f = 0.021$. Then

$$h_p = 20 + 0.921(0.5 + 0.38 + 1 + 6.65) = 27.9 \text{ m}$$

Pump power

$$P = Q\gamma h_p = 1.2 \times 0.94 \times 9,810 \times 27.9/0.70 = \underline{\underline{441 \text{ kW}}}$$

10.97 **Information and Assumptions**

provided in problem statement

Find

elevation of water surface in upstream reservoir

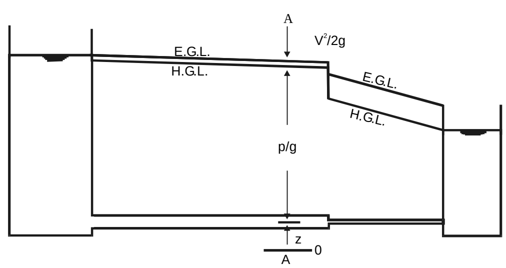
Energy equation

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + z_1 &= 0 + 0 + 12 + (V_{30}^2/2g)(0.5 + fL/D) + (V_{15}^2/2g)(K_c + f(L/D) + 1.0) \qquad (8) \\
V_{30} &= Q/A_{30} = 0.15/((\pi/4) \times 0.30^2) = 2.12 \text{ m/s} \\
V_{30}^2/2g &= 0.229 \text{ m} \qquad (9)
\end{aligned}
$$

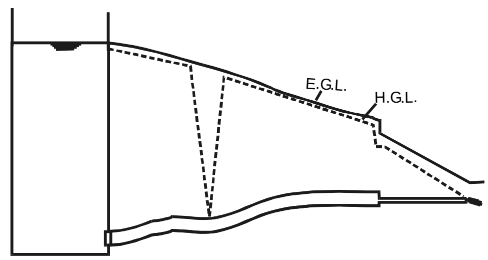
$$
\begin{aligned}
V_{15} &= 4V_{30} = 8.488 \text{ m/s} \\
V_{15}^2/2g &= 3.67 \text{ m} \\
D_2/D_1 &= 15/30 = 0.5 \rightarrow K_C = 0.37
\end{aligned}
$$

Then

$$
\begin{aligned}
z_1 &= 12 + 0.229[0.5 + 0.02 \times (20/0.3)] + 3.67[0.37 + 0.02(10/0.15) + 1.0] \\
z_1 &= \underline{\underline{22.3 \text{ m}}}
\end{aligned}
$$



10.98



Cavitation could occur in the venturi throat section or just downstream of the abrupt contraction (where there will be a contraction of the flow area).

10.99 **Information and Assumptions**

from Table 10.3 $K_b = 0.9,\ K_v = 10$
from Table 10.2 $k_s = 5 \times 10^{-4}$ ft
from Table A.5 $\nu = 1.41 \times 10^{-5}$ ft^2/s
provided in problem statement

Find

pressure at point A

Energy equation

$$
\begin{aligned}
p_A/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + z_2 + V_2^2/2g + \sum h_L \\
p_A/\gamma + 20 + 0 &= 0 + 90 + 0 + V^2/2g(0.5 + 2K_b + K_v + f(L/D) + 1) \\
V &= Q/A = (50/449)/((\pi/4)(2/12)^2) = 5.1 \\
V^2/2g &= 5.1^2/64.4 = 0.404 \\
\text{Re} &= 5.1(2/12)/(1.41 \times 10^{-5}) = 6 \times 10^4 \\
k_s/D &= 5 \times 10^{-4} \times 12/2 = 0.003 \\
p_1 &= \gamma[70 + 0.404(0.5 + 2 \times 0.9 + 10 + (0.028 \times 240/(2/12)) + 1.0)]
\end{aligned}
$$

From Fig. 10.8 $f = 0.028$. Then

$$
\begin{aligned}
p_A &= \gamma[70 + 0.404(0.5 + 2 \times 0.9 + 10 + (0.028 \times 240/(2/12)) + 1.0)] \\
&= 62.4 \times 91.7 = 5722 \text{ psfg} = \underline{\underline{39.7}} \text{ psig}
\end{aligned}
$$

10.100 **Information and Assumptions**

from Table 10.2 $k_s = 0.26$ mm
from Table A.5 $\nu = 1.3 \times 10^{-6}$ m^2/s
provided in problem statement

Find

water surface elevation in reservoir A.

Solution

$$\begin{aligned} k_s/D_{20} &= 0.26/200 = 0.0013 \\ k_s/D_{15} &= 0.0017 \\ V_{20} &= Q/A_{20} = 0.03/((\pi/4) \times 0.20^2) = 0.955 \text{ m/s} \\ Q/A_{15} &= 1.697 \text{ m/s} \\ \text{Re}_{20} &= VD/\nu = 0.955 \times 0.2/(1.3 \times 10^{-6}) = 1.5 \times 10^5 \\ \text{Re}_{15} &= 1.697 \times 0.15/1.3 \times 10^{-6} = 1.9 \times 10^5 \end{aligned}$$

From Fig. 10-8: $f_{20} = 0.022$; $f_{15} = 0.024$

$$\begin{aligned} z_1 &= z_2 + \sum h_L;\ z_1 = 100 + \sum h_L \\ z_1 &= 110 + V_{20}^2/2g(0.5 + 0.022 \times 100/0.2 + 0.19) + V_{15}^2/2g[(0.024 \times 150/0.15) \\ +1.0 + 0.19)] &= 110 + 0.0465(11.5) + 0.1468(25.19 \\ &= 110 + 0.535 + 3.70 = \underline{\underline{114.2 \text{ m}}} \end{aligned}$$

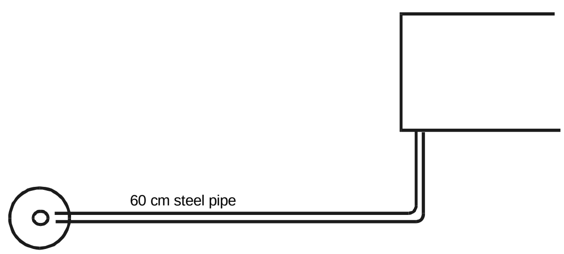
10.101 One possible design given below:

$$\begin{aligned}
L &\approx 300 + 50 + 50 = 400 \text{ m}; \ K_b = 0.19 \\
50 &= \sum h_L = V^2/2g(K_e + 2K_b + f(L/D) + 1.0) = V^2/2g(1.88 + f(L/D)) \\
50 &= [Q^2/(2gA^2)](f(L/D) + 1.88) = [2.5^2/(2 \times 9.81 \times A^2)]((400\ f/D) + 1.88)
\end{aligned}$$

Assume $f = 0.015$. Then $50 = [0.318/((\pi/4)^2 \times D^4)](0.015 \times (400/D)) + 1.88)$
Solving, one gets $D \approx 0.59$ m = 59 cm. Try commercial size $D = 60$ cm.
Then

$$\begin{aligned}
V_{60} &= 2.5/((\pi/4) \times 0.6^2) = 8.84 \text{ m/s} \\
\text{Re} &= 8.8 \times 0.6/10^{-6} = 5.3 \times 10^6; \ k_s/D = 0.0001 \text{ and } f \approx 0.013
\end{aligned}$$

Since $f = 0.13$ is less than originally assumed f, the design is conservative. So use $D = \underline{\underline{60 \text{ cm}}}$ and $\underline{\underline{L \approx 400 \text{ m}}}$.



10.102 First write the energy equation from the reservoir to the tank and assume that the same pipe configuration as used in the solution to P10-99 is used. Also a pump, two open gate valves, and two bends will be in the pipe system. Assume steel pipe will be used.

Assume $L \approx 400$ ft.

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 450 + h_p &= 0 + 0 + 500 + (V^2/2g)(K_e + 2K_b + 2K_v + K_E + fL/D) \end{aligned}$$

Assume $V \approx 2$ m/s; $A = Q/V = 1.0/2 = 0.50 \text{ m}^2$

$$A = (\pi/4)D^2 = 0.50 \text{ or } D = .799 \text{ m} \text{ Choose a pipe size of 0.80 m}$$

Then

$$\begin{aligned} V &= Q/A = 1.0/((\pi/4) \times 0.8^2) = 1.99 \text{ m/s and } V^2/2g = 0.202 \text{ m} \\ k_s/D &= 0.00006; \text{ Re} = VD/\nu = 1.6/10^{-6} = 1.6 \times 10^6 \end{aligned}$$

Then $f = 0.012$ (from Fig. 10-8)

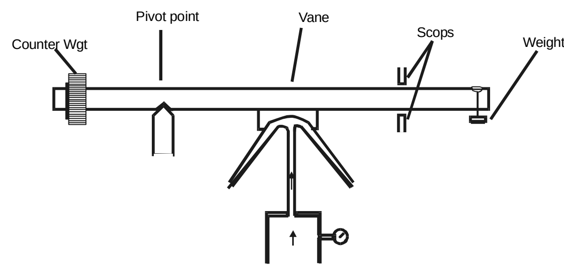
$$\begin{aligned} h_p &= 50 + (V^2/2g)(0.5 + 2 \times 0.2 + 2 \times 0.19 + 1.0 + 0.012 \times 400/1) \\ &= 50 + 1.43 = 51.43 \text{ m} \\ P &= Q\gamma h_p = 2.0 \times 9,810 \times 51.43 \\ &= 1.01 \text{ MW} \end{aligned}$$

Design will include 0.80 m steel pipe and a pump with output of 1.01 MW

Note: An infinite number of other designs is possible. Also, a design solution would include the economics of the problem to achieve the desired result at minimum cost.

10.103 Because you want to design equipment to illustrate cavitation, it would be desirable to make the flow restriction device from clear plastic so that one may observe the formation of cavitation bubbles. The design calculation for pressure and discharge would be the same as given for 10.71.

10.104 The equipment for the momentum experiment could be shown below:



Necessary measurements and calculations:

a) Discharge. This could be done by using a scale and tank to weigh the flow of water that has occurred over a given period of time.

b) The velocity in the jet could be measured by means of a stagnation tube or solving for the velocity by using Bernoulli's equation given the pressure in the nozzle from which the jet issues.

c) Initially set the counter balance so that the beam is in its horizontal equilibrium position. By opening a valve establish the jet of water. Apply necessary weight at the end of the beam balance to bring the beam back to horizontal equilibrium. By calculation (using moment summation) determine the force that the jet is exerting on the vane. Compare this force with the calculated force from the momentum equation (using measured Q, V, and vane angle).

10.105 **Information and Assumptions**

provided in problem statement

Find

ratio of discharge in line A to B

Solution

$$\begin{aligned} h_{LA} &= h_{LB} \\ 0.2V_A^2/2g &= 10V_B^2/2g \qquad (1) \\ V_A &= \sqrt{50}V_B \\ Q_B/Q_A &= V_B A_B/V_A A_A \\ &= V_B A_B/V_A((1/2)A_B) \qquad (2) \\ Q_B/Q_A &= 2V_B/V_A \end{aligned}$$

Solve Eqs. (1) and (2) for Q_B/Q_A:

$$\begin{aligned} Q_B/Q_A &= 2 \times V_B/\sqrt{50}V_B \\ &= 0.283 \end{aligned}$$

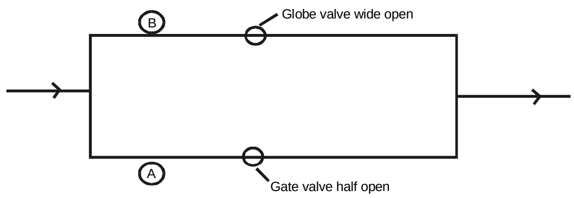
10.106 **Information and Assumptions**

provided in problem statement

Find

ratio of velocity in line A to B

Solution



$$
\begin{aligned}
\sum h_{LB} &= \sum h_{LA} \\
h_{L,\text{globe}} + 2h_{L,\text{elbow}} &= h_{L,\text{gate}} + 2h_{L,\text{elbow}} \\
10V_B^2/2g + 2(0.9V_B^2/2g) &= 5.6V_A^2/2g + 2(0.9V_A^2/2g) \\
11.8V_B^2/2g &= 7.4V_A^2/2g \\
V_A/V_B &= \underline{\underline{1.26}}
\end{aligned}
$$

10.107 **Information and Assumptions**

provided in problem statement

Find

division of flow of water

Solution

$$V_1/V_2 = [(f_2/f_1)(L_2/L_1)(D_1/D_2)]^{1/2}$$

Initially assume $f_1 = f_2$

Then

$$V_1/V_2 = [(1,500/1,000)(0.50/0.40)]^{1/2} = 1.369;\ V_1 = 1.369V_2$$

$$V_1A_1 + V_2A_2 = 1.2$$

$$1.369V_2 \times (\pi/4) \times 0.5^2 + V_2 \times (\pi/4) \times 0.4^2 = 1.2$$

$$V_2 = \underline{\underline{3.038 \text{ m/s}}};\text{ Then } V_1 = 1.369 \times 3.038 = \underline{\underline{4.16 \text{ m/s}}}$$

$$Q_1 = V_1A_1 = 4.16(\pi/4) \times 0.5^2 = \underline{\underline{0.816\ m^3/s}};\ Q_2 = \underline{\underline{0.384\ m^3/s}}$$

10.108 **Information and Assumptions**

provided in problem statement

Find

discharge in pipe 1

Solution

$$h_{f,1} = h_{f,2}$$

$$\begin{aligned} f(L/D)(V_1^2/2g) &= f(4L/D)(V_2^2/2g) \\ V_1^2 &= 4V_2^2 \\ V_1 &= 2V_2 \end{aligned}$$

Thus

$$Q_1 = 2Q_2 = \underline{\underline{2 \text{ cfs}}}$$

10.109

$$
\begin{aligned}
h_{p,A} &= h_{f,B} = h_{f,C} \\
f(L/D)(V^2/2g)_A &= f(L/D)(V^2/2g)_B = f(L/D)(V^2/2g)_C \\
0.012(6,000/1.5)V_A^2 &= 0.02(2,000/.5)V_B^2 = .015(5,000)V_C^2 \\
48V_A^2 &= 80V_B^2 = 75V_C^2
\end{aligned}
$$

Therefore, V_A will have the greatest velocity. Correct choice is $\underline{\underline{\text{a}}}$)

10.110 **Information and Assumptions**

provided in problem statement

Find

ratio of discharges in two pipes

Solution

$$(V_1/V_2) = [(f_2/f_1)(L_2/L_1)(D_1/D_2)]^{1/2}$$

Let pipe 1 be large pipe and pipe 2 be smaller pipe. Then

$$\begin{aligned}
(V_1/V_2) &= [(0.014/0.01)(L/3L)(2D/D)]^{1/2} = 0.966 \\
(Q_1/Q_2) &= (V_1/V_2)(A_1/A_2) = 0.966 \times (2D/D)^2 = 3.86 \\
(Q_{\text{large}}/Q_{\text{small}}) &= \underline{\underline{3.86}}
\end{aligned}$$

10.111 **Information and Assumptions**

provided in problem statement

Find

division of flow and head loss

Solution

$$\begin{aligned} Q_{18} + Q_{12} &= 14 \text{ cfs} \\ h_{L_{18}} &= h_{L_{12}} \\ f_{18}(L_{18}/D_{18})(V_{18}^2/2g) &= f_{12}(L_{12}/D_{12})(V_{12}^2/2g) \\ f_{18} &= 0.018 = f_{12} \end{aligned}$$

so

$$\begin{aligned} L_{18}Q_{18}^2/D_{18}^5 &= L_{12}Q_{12}^2/D_{12}^5 \\ Q_{18}^2 &= (D_{18}/D_{12})^5(L_{12}/L_{18})Q_{12}^2 = (18/12)^5(2,000/6,000)Q_{12}^2 = 2.53Q_{12}^2 \\ Q_{18} &= 1.59Q_{12} \\ 1.59Q_{12} + Q_{12} &= 14 \\ 2.59Q_{12} &= 14 \\ \text{Q}_{12} &= \underline{\underline{5.4 \text{ cfs}}} \\ Q_{18} &= 1.59Q_{12} = 1.59(5.4) = \underline{\underline{8.6 \text{ cfs}}} \\ V_{12} &= 5.4/((\pi/4(1)^2) = 6.88 \\ V_{18} &= 8.6/((\pi/4)(18/12)^2) = 4.87 \\ h_{L_{12}} &= 0.018((2,000)/1)(6.88)^2/64.4 = 26.5 \\ h_{L_{18}} &= 0.018(6,000/1.5)(4.87^2/64.4) = 26.5 \\ \text{Thus, } h_{L_{A-B}} &= \underline{\underline{26.5 \text{ ft}}} \end{aligned}$$

10.112 **Information and Assumptions**

provided in problem statement

Find

division of flow and head loss

Solution

$$\begin{aligned}
Q &= Q_{14} + Q_{12} + Q_{16} \\
25 &= V_{14} \times (\pi/4) \times (14/12)^2 + V_{12} \times (\pi/4) \times 1^2 + V_{16} \times (\pi/4) \times (16/12)^2; \qquad (1)
\end{aligned}$$

Also, $h_{f_{14}} = h_{f_{12}} = h_{f_{16}}$ and assuming $f = 0.03$ for all pipes

$$(3,000/14)V_{14}^2 = (2,000/12)V_{12}^2 = (3,000/16)V_{16}^2 \qquad (2)$$

$$V_{14}^2 = 0.778V_{12}^2 = 0.875V_{16}^2$$

From Eq(1)

$$\begin{aligned}
25 &= 1.069V_{14} + 0.890V_{14} + 1.49V_{14} \\
V_{14} &= 7.74 \text{ ft/s}
\end{aligned}$$

and $V_{12} = 1.134$, $V_{14} = 8.21$ ft/s; $V_{16} = 7.74$ ft/s

$$\begin{aligned}
Q_{12} &= \underline{\underline{6.44}} \text{ ft}^3/\text{s};\ Q_{14} = \underline{\underline{7.74}} \text{ ft}^3/\text{s};\ Q_{16} = \underline{\underline{10.8}} \text{ ft}^3/\text{s} \\
V_{24} &= Q/A_{24} = 25/(\pi/4 \times 2^2) = 7.96 \text{ ft/s}; \\
V_{30} &= 5.09 \text{ ft/s} \\
h_{LAB} &= (0.03/64.4)[(2,000/2.00)(7.96)^2 + (2,000/1) \times (8.21)^2 \\
&\quad +(3,000/(30/12) \times (5.09)^2] = \underline{\underline{106.8}} \text{ ft}
\end{aligned}$$

10.113 **Information and Assumptions**

from Table 10.2 $k_s = 0.046$ mm
provided in problem statement

Find

division of flow between pipes and head loss

Solution

Call pipe A-B pipe and pipe ACB pipe 2. Then

$$\begin{aligned} h_{f,1} + h_p &= h_{f,2} \\ k_s/D &= 0.046/500 \simeq 0.0001 \end{aligned}$$

Assume $f_1 = f_2 = 0.013$ (guess from Fig. 10-8)

$$\begin{aligned} f(L_1/D_1)(V_1^2/2g) + h_p &= f(L_2/D_2)(V_2^2/2g) \\ 0.013(2,000/0.5)(V_1^2/2g) + h_p &= 0.013(6,000/0.5)(V_2^2/2g) \\ 2.65V_1^2 + h_p &= 7.951V_2^2 \qquad (1) \end{aligned}$$

Continuity

$$\begin{aligned} (V_1 + V_2)A &= 0.60 \text{ m}^3/\text{s} \\ V_1 + V_2 &= 0.60/A = 0.6/((\pi/4)(0.5^2)) = 3.0558 \\ V_1 &= 3.0558 - V_2 \qquad (2) \end{aligned}$$

By iteration (Eqs. (1), (2) and pump curve) one can solve for the division of flow:

$$Q_1 = \underline{\underline{0.27}} \text{ m}^3/\text{s} \quad Q_2 = \underline{\underline{0.33}} \text{ m}^3/\text{s}$$

Head loss determined along pipe 1

$$\begin{aligned} h_L &= f(L/D)(V_1^2/2g) \\ V_1 &= Q_1/A = 0.27/((\pi/4)(0.5^2)) = 1.38 \text{ m/s} \\ h_l &= 0.013(2000/0.5)(1.38^2/(2 \times 9.81)) = \underline{\underline{5.05 \text{ m}}} \end{aligned}$$

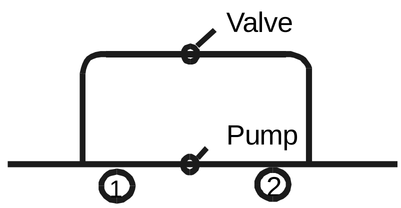
10.114 **Information and Assumptions**

provided in problem statement

Find

discharge through and bypass line

Solution



$$
\begin{aligned}
Q_p &= Q_v + 0.2 \\
(p_2 - p_1)/\gamma &= h_p \\
A &= (\pi/4)(0.1^2) = 0.00785 \text{ m}^2 \\
K_v V_v^2/2g &= K_v Q_v^2/(2gA^2) = h_p \\
h_p &= 100 - 100(Q_v + 0.2) \\
(0.2)(Q_v^2)/2 \times 9.81 \times (0.00785)^2) &= 100 - 100Q_v - 20 \\
165Q_v^2 + 100Q_v - 80 &= 0
\end{aligned}
$$

Solve by quadratic formula

$$
Q_v = \underline{\underline{0.456}} \text{ m}^3/\text{s} \ \ ; \ \ Q_p = 0.456 + 0.2 = \underline{\underline{0.656}} \text{ m}^3/\text{s}
$$

10.115

$$
\begin{aligned}
R_h &= A/P \\
R_{h,A} &= (A/P)_A = 16/16 = 1 \\
R_{h,W} &= (A/P)_W = 8/8 = 1 \quad \therefore \underline{\underline{R_{h,A} = R_{h,W}}}
\end{aligned}
$$

The correct choice is $\underline{\underline{\text{(a)}}}$.

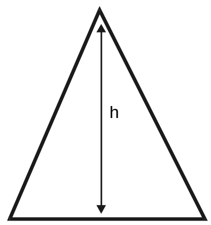
10.116 **Information and Assumptions**

from Table A.3 $\nu = 1.58 \times 10^{-4}$ ft^2/s and $\rho = 0.00237$ slug/ft^3
from Table 10.2 $k_s = 0.0005$ ft
provided in problem statement

Find

pressure drop over 100 ft length

Solution



$$
\begin{aligned}
h &= (6 \text{ in})(\cos\ 30°) = 5.20 \\
A &= (6)(5.20)/2 = 15.6 \text{ in}^2 = 0.108 \text{ ft}^2 \\
R_h &= A/P = 15.6 \text{ in}^2/(3 \times 6) = 0.867 \text{ in.} \\
4R_h &= 3.47 \text{ in.} = 0.289 \text{ ft.} \\
k_s/4R_h &= 0.0005/0.289. = 0.00173 \\
\text{Re} &= (V)(4R_h)/\nu = (2)(2.89)/(1.58 \times 10^{-4}) = 2.2 \times 10^4
\end{aligned}
$$

From Fig. 10.8 $f = 0.030$ so the pressure drop is

$$
\begin{aligned}
\Delta p_f &= (f(L/4R_h)(\rho V^2/2) \\
\Delta p_f &= 0.030(100/0.289)(0.00237 \times 12^2/2) \\
\Delta p_f &= \underline{\underline{1.77 \text{ lbf/ft}^2}}
\end{aligned}
$$

10.117

$$
\begin{aligned}
Q &= (1.49/n)AR_h^{2/3}S^{1/2} \\
Q_A/Q_B &= R_{h,A}^{2/3}/R_{h,B}^{2/3} = (R_{h,A}/R_{h,B})^{2/3} \\
\text{where } R_{h,A} &= 50/20 = 2.5;\ R_{h,B} = 50/(3 \times 7.07) = 2.36 \\
R_{h,A} &> R_{h,B} \therefore \underline{\underline{Q_A > Q_B.}}
\end{aligned}
$$

The correct choice is $\underline{\underline{\text{(c)}}}$.

10.118 **Information and Assumptions**

assume $k_s = .15$ mm$= 1.5 \times 10^{-4}$ m
from Table A.3 $\nu = 1.46 \times 10^{-5}$
provided in problem statement

Find

power loss in duct

Solution

$$\begin{aligned} A &= 0.15 \text{ m}^2 \\ P &= 2.30 \\ R &= A/P = 0.0652 \text{ m} \\ 4R &= 0.261 \text{ m} \end{aligned}$$

$$\begin{aligned} k_s/4R &= 1.5 \times 10^{-4}/0.261 = 5.7 \times 10^{-4}; \\ V &= Q/A = 5/0.15 = 33.3 \text{ m/s} \end{aligned}$$

and

$$\text{Re} = V \times 4R/\nu = 33.3 \times 0.261/(1.46 \times 10^{-5}) = 5.95 \times 10^5$$

Then from Fig. 10.8 $f = 0.018$

$$\begin{aligned} h_f &= f(L/D)(V^2/2g) \\ &= 0.018 \times (100/0.26)(33.3^2/(2 \times 9.81)) = 391 \text{ m} \\ P_{\text{loss}} &= Q\gamma h_f = 5 \times 12.0 \times 391 = \underline{\underline{23.5 \text{ kW}}} \end{aligned}$$

10.119 **Information and Assumptions**

provided in problem statement

Find

ratio of velocity in trapezoidal to rectangular duct

Solution

$$\Delta h_{\text{rect}} = \Delta h_{\text{trap}}$$
$$\therefore h_{f,\text{rect}} = h_{f,\text{trap}}$$
$$(f_b L/4R_b)V_b^2/2g = (f_a L/4R_a)V_a^2/2g$$
$$R_b = A_b/P_b = 2/6 = 0.333 \text{ ft}$$
$$R_a = A_a/P_a = 1.4/6 = 0.233 \text{ ft}$$
$$V_1^2/V_b^2 = R_a/R_b = 0.70$$
$$\underline{\underline{V_{\text{trap}}/V_{\text{rect}} = 0.84}}$$

10.120

$$
\begin{aligned}
f &= f(\mathrm{Re}, k_s/4R) \;\; R = A/P = 0.7 \text{ m}^2/3.4 \text{ m} = 0.206 \text{ m} \\
\mathrm{Re} &= V(4R)/\nu = 10 \times 4 \times .206 \times 10^6 = 8.2 \times 10^6 \\
k_s/4R &= 10^{-3} \text{ m}/0.824 \text{ m} = 1.2 \times 10^{-3} = .0012
\end{aligned}
$$

From Fig. 10.8: $\underline{\underline{f \approx 0.020}}$ Choice $\underline{\underline{\text{(b)}}}$ is the correct one.

10.121 **Information and Assumptions**

assume $n = 0.012$
provided in problem statement

Find

discharge of water

Solution

Use Manning's formula

$$\begin{aligned}
Q &= (1/n)AR_h^{2/3}S_0^{1/2} \\
A &= (1)(2)/2 = 1m^2 \\
Q &= (1/0.012)(1)(0.35)^{2/3}(0.0015)^{0.5} \\
R_h &= A/P \\
&= 1/2(1^2+1^2)^{0.5} = 0.35 \text{ m} \\
Q &= \underline{\underline{1.60}} \text{ m}^3/\text{s}
\end{aligned}$$

10.122 **Information and Assumptions**

assume $k_s = 30$ cm
provided in problem statement

Find

discharge

Solution

from Fig. 10.8 $f \approx 0.060$

$$\begin{aligned} R &= A/P \approx 2.21 \text{ m} \\ k_s/4R &= 0.034 \end{aligned}$$

from Fig. 10.8 $f \approx 0.060$

$$C = \sqrt{8g/f} = 36.2 \text{ m}^{1/2}\text{s}^{-1}$$

$$Q = CA\sqrt{RS} = \underline{\underline{347 \text{ m}^3/\text{s}}}$$

10.123 **Information and Assumptions**

assume $k_s = 10^{-3}$ m
provided in problem statement

Find

discharge

Solution

$$\begin{aligned} A &= 4.5m^2 \\ P &= 6m \\ R &= A/P = 0.75m \\ k_s/4R &= 0.333 \times 10^{-3} \end{aligned}$$

From Fig. 10.8 $f = 0.016$

$$\begin{aligned} h_f/L &= fV^2/(2g4R) \\ V &= \sqrt{(8g/f)RS} = 1.92 \text{ m/s} \\ \text{Re} &= 1.92 \times 3/(1.31 \times 10^{-6}) = 4.4 \times 10^6 \ f = 0.015 \end{aligned}$$

From Fig. 10.8 $f = 0.015$
Then

$$V = 1.92 \times \sqrt{0.016/0.015} = 1.98 \text{ m/s}$$

Finally,

$$Q = 1.98 \times 4.5 = \underline{\underline{8.73 \text{ m}^3/\text{s}}}$$

10.124 **Information and Assumptions**

assume $k_s = 0.003$
provided in problem statement

Find

the discharge

Solution

$$R = A/P = 4 \times 12/(12 + 2 \times 4) = 2.4$$
$$k_s/(4R) = 0.003/(4 \times 2.4) = 0.00031$$
$$\text{Re} f^{1/2} = ((4R)^{3/2}/\nu)(2gS)^{1/2} = ((4 \times 2.4)^{3/2}/(1.22 \times 10^{-5}))$$
$$\times (2g \times 5/8,000)^{1/2} = 4.9 \times 10^5;$$

From Fig. 10.8 $f = 0.015$

$$\begin{aligned} V &= \sqrt{8gRS/f} = \sqrt{8g(0.20)5/(0.015(8,000))} = 5.07 \\ Q &= 5.07(4)12 = \underline{\underline{243}} \text{ cfs} \end{aligned}$$

Alternate solution:
Assume $n = 0.015$

$$\begin{aligned} Q &= (1.49/n)AR^{2/3}S^{1/2} \\ &= (1.49/0.015)4 \times 12(2.4)^{2/3}\ (5/8,000)^{1/2} = \underline{\underline{214 \text{ cfs}}} \end{aligned}$$

10.125 **Information and Assumptions**

assume $n = 1$
provided in problem statement

Find

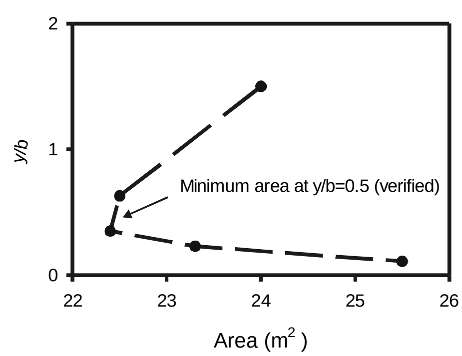
cross-sectional areas for various widths

Solution

$$\begin{aligned} Q &= 100 \text{ cfs} \\ S &= 0.001 \\ Q &= (1.49/n)AR^{0.667}S^{0.5} \\ \text{or } Qn/(1.49S^{0.5}) &= AR^{0.667} \\ 31.84 &= AR^{0.667} \\ 31.84 &= (by)(by/(b+2y))^{0.667} \end{aligned}$$

For different values of b one can compute y and the area by. The following table results

b (ft)	y (ft)	A (ft^2)	y/b
2	16.5	33.0	8.2
4	6.0	24.0	1.5
6	3.8	22.5	0.63
8	2.8	22.4	0.35
10	2.3	23.3	0.23
15	1.7	25.5	0.11



10.126 **Information and Assumptions**

assume $n = 0.013$
assume $k_s = 10^{-3}$ ft
assume $\nu = 1.2 \times 10^{-5}$ ft^2/s
provided in problem statement

Find

the discharge

Solution

$$\begin{aligned} Q &= (1.49/n)AR^{2/3}S^{1/2} \\ R &= A/P = (\pi D^2/8)/(\pi D/2) = D/4;\ A = \pi D^2/8 \end{aligned}$$

$$\begin{aligned} Q &= (1.49/0.013) \times (\pi D^2/8)(D/4)^{2/3} \times (1.0/1,000)^{1/2} \\ Q &= \underline{\underline{22.8 \text{ ft}^3/\text{s}}} \end{aligned}$$

Alternate solution:

$$Q = CA\sqrt{RS}$$

where $C = \sqrt{8g/f}$.

$$k_s/4R = 10^{-3}/(4 \times D/4) = 2.5 \times 10^{-4}$$

Assume $f = 0.016$. Then

$$\begin{aligned} C &= \sqrt{8 \times 32.2/0.016} = 127 \\ Q &= 127 \times (\pi \times 4^2/8)\sqrt{1 \times 1.0/1,000} = 25.2 \text{ ft}^3/\text{s} \\ V &= 4.00 \text{ ft/s} \\ \text{Re} &= V \times 4R/\nu = 4.00 \times 4 \times 1/(1.2 \times 10^{-5}) = 1.37 \times 10^6 \end{aligned}$$

From Fig. 10.8 $f = 0.015$. Solve again

$$C = \sqrt{8g/0.015} = 131; Q = \underline{\underline{26.0 \text{ ft}^3/\text{s}}}$$

10.127 **Information and Assumptions**

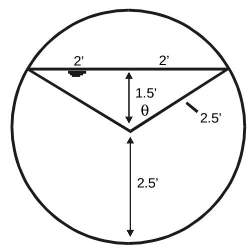
assume $n = 0.012$
provided in problem statement

Find

the discharge

Solution

$$Q = (1.49/n)AR_h^{0.667}S_0^{0.5}$$



$$\begin{aligned}
\cos\theta &= 1.5 \text{ ft}/2.5 \text{ ft} \\
\theta &= 53.13^\circ \\
A &= \pi r^2((360^\circ - 2 \times 53.13^\circ)/360) + 0.5 \times 4 \text{ ft} \times 1.5 \text{ ft} \\
A &= 16.84 \text{ ft}^2 \\
P &= \pi D((360^\circ - 2 \times 53.13^\circ)/360) = 11.071 \text{ ft} \\
R_h &= A/P = 1.521 \text{ ft} \\
R_h^{0.667} &= 1.323 \\
\text{Then } Q &= (1.49/0.012)(16.84)(1.323)(0.001)^{0.5} \\
Q &= \underline{\underline{87.5 \text{ cfs}}}
\end{aligned}$$

10.128 **Information and Assumptions**

assume $k_s = 0.003$ ft
assume $\nu = 1.41 \times 10^{-5}$ ft^2/s\
provided in problem statement

Find

average velocity and discharge

Solution

$$R = A/P = (10+12)6/(10+6\sqrt{5}\times 2) = 132/36.8 = 3.58$$

$$\begin{aligned} (k_s/4R) &= 0.003/(4\times 3.58) = 0.00021 \\ \mathrm{Re}f^{1/2} &= ((4R)^{3/2}/\nu)(2gS)^{1/2} = [(4\times 3.58)^{3/2}/1.41\times 10^{-5}](2g/2000)^{1/2} \\ &= 6.9\times 10^5 \end{aligned}$$

From Fig. 10.8 $f = 0.014$. Then

$$\begin{aligned} V &= \sqrt{8gRS/f} = \sqrt{8g\times 3.58/(2000\times 0.014)} = \underline{\underline{5.74}} \text{ ft/s} \\ Q &= VA = 5.74\times 132 = \underline{\underline{758}} \text{ cfs} \end{aligned}$$

Alternate method, assuming $n = 0.015$

$$\begin{aligned} V &= (1.49/n)R^{2/3}S^{1/2} = (1.49/0.015)(3/3.58)^{2/3}(1/2,000)^{1/2} = \underline{\underline{5.18 \text{ fps}}} \\ Q &= 5.18(132) = \underline{\underline{684 \text{ cfs}}} \end{aligned}$$

10.129 **Information and Assumptions**

assume $n = 0.012$
provided in problem statement

Find

depth of flow in trapezoidal channel

Solution

$$\begin{aligned} Q &= (1.49/n)AR^{0.667}S_0^{0.5} \\ 1200 &= (1.49/0.012)(10d + d^2)((10d + d^2)/(10 + 2\sqrt{2}d))^{0.667}(1/500)^{0.5} \\ 1200 &= (124.2/0.012)(10d + d^2)^{5/3}(10 + 2\sqrt{2}d)^{-0.667}(0.0447) \end{aligned}$$

Solving for d yields $d =$ $\underline{\underline{5.85 \text{ ft}}}$

10.130 **Information and Assumptions**

assume $n = 0.012$
provided in problem statement

Find

discharge in trapezoidal channel

Solution

$$\begin{aligned} Q &= (1.49/n)AR^{2/3}S^{1/2} \\ A &= 10 \times 5 + 5^2, P = 10 + 2\sqrt{5^2 + 5^2} = 24.14 \text{ ft} \\ R &= A/P = 75/24.14 = 3.107 \text{ ft} \end{aligned}$$

Then

$$Q = (1.49/0.012)(75)(3.107)^{2/3}(4/5,280)^{1/2} = \underline{\underline{546 \text{ cfs}}}$$

10.131 **Information and Assumptions**

assume $n = 0.015$
provided in problem statement

Find

the uniform flow depth

Solution

$$\begin{aligned} Q &= (1/n)AR^{2/3}S^{1/2} \\ 25 &= (1.0/0.015)4d(4d/(4+2d))^{2/3} \times 0.004^{1/2} \end{aligned}$$

Solve by trial and error: $d = \underline{\underline{1.6 \text{ m}}}$

10.132 **Information and Assumptions**

assume $n = 0.015$
provided in problem statement

Find

the depth of flow

Solution

$$\begin{aligned} Q &= (1.49/n)AR^{2/3}S^{1/2} \\ 500 &= (1.49/0.012)12d(12d/(12+2d))^{2/3} \times (10/8,000)^{1/2} \end{aligned}$$

Solve by trial by trial and error. $d =$ $\underline{\underline{4.92 \text{ ft}}}$

10.133 **Information and Assumptions**

assume $n = 0.015$
provided in problem statement

Find

depth of flow in channel

Solution

$$\begin{aligned} Q &= (1.49/n)A\, R_h^{2/3} S_0^{1/2} \\ 3,000 &= ((1.49)/(0.015))(10d + 2d^2)((10d + 2d^2)/(10 + 2\sqrt{5}d))^{2/3}(0.001)^{1/2} \\ 955 &= (10d + 2d^2)((10d + 2d^2)/(10 + 2\sqrt{5}d))^{2/3} \end{aligned}$$

Solve for d by trial and error $\underline{\underline{d = 10.1 \text{ ft}}}$

10.134 **Information and Assumptions**

provided in problem statement

Find

the flood discharge

Solution

Divide the channel into parts 1, main, & 2. With these divisions we compute $A_{1,}$ A_{main} & A_2 and also $R_{1,}$ R_{main} & R_2. These results of these computations along with $R^{2/3}$ values shown in tabular form below:

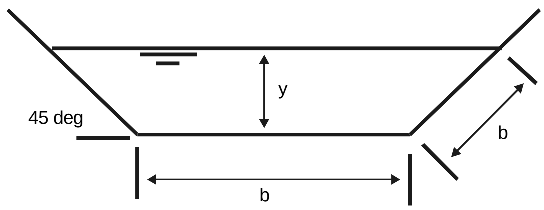
Channel Part	$A(\text{ft}^2)$	$P(\text{ft})$	$R_h = A/P(\text{ft})$	$R_h^{2/3}(\text{ft})^{2/3}$
1	1,218	208.5	5.842	3.244
main	5,600	228.3	24.53	8.443
2	1,218	208.5	5.842	3.244

From Table 10.5 assume: $n_1 = n_2 = 0.035$. From Table 10.4 $n_{\text{main}} = 0.025$. Then using Eq. 10.47 we have:

$$\begin{aligned} Q &= 1.49\sqrt{0.0015}[(2(1,218 \times 3.244)/.035) + (5,600 \times 8.443/.025)] \\ Q &= \underline{\underline{122{,}170 \text{ ft}^3/\text{s}}} \end{aligned}$$

10.135 For best hydraulic section, the shape will be a half hexagon as depicted below

assume $n = 0.015$ (concrete, wood forms unfinished - Table 10.3)



Use the Manning equation to solve this problem:

$$Q = (1.49/n)AR_h^{0.667}S_0^{0.5}$$

Then

$$900 = (1.49/0.015)AR_h^{0.667}(0.002)^{0.5}$$

$$AR_h^{0.667} = 202.6$$

But $A =$ by $+y^2$ where $y = b\cos 45° = 0.707\text{b}$

$$\begin{aligned} A &= 0.707\text{b}^2 + 0.50b^2 = 1.207\text{b}^2 \\ R_h &= A/P = 1.207\text{b}^2/3b = 0.4024b \end{aligned}$$

Thus

$$AR_h^{0.667} = 202.6 = 1.207b^2(0.4024b)^{0.667}$$

$$b^{2.667} = 308;\ b = \underline{\underline{8.57\text{ ft}}}$$

10.136 **Information and Assumptions**

provided in problem statement

Find

Manning's n

Solution

From Eq. 10.50

$$Q = (1.49/n)[A_1 R_1^{2/3} + A_2 R_2^{2/3}]S^{1/2}$$

Let A_1 & R_1 denote the main channel and A_2 & R_2 denote the total overbank channel

$$\begin{aligned}
A_1 &= 10(200) = 2,000 \text{ ft}^2 \\
P_1 &= 200 + 2(5) = 210 \\
R_1 &= 2,000/210 = 9.523 \\
R_1^{2/3} &= 4.493 \\
A_2 &= 2(200)5 = 2,000 \text{ ft}^2 \\
P_2 &= 200 + 200 + 2(5) = 410 \\
R_2 &= 2,000/410 = 4.878 \\
R_2^{2/3} &= 2.876 \\
n &= (1.49/60,000)[2,000(4.493) + 2,000(2.876)](.0040)^{1/2} \\
n &= \underline{\underline{0.023}}
\end{aligned}$$

10.137 **Information and Assumptions**

provided in problem statement

Find

velocity in main channel and overbank areas

Solution

$$
\begin{aligned}
Q &= 1.49[A_1R_1^{2/3}/n_1 + A_2R_2^{2/3}/n_2]S^{1/2} \\
60,000 &= 1.49[2,000(4.493)/0.02 + 2,000(2.876)/n_2](.0040)^{1/2} \\
636,700 &= 449,300 + 5,752/n_2 \\
n_2 &= 5,752(636,700 - 449,300) \\
&= \underline{\underline{0.0307}} \\
V_1 &= (1.49/n_1)R_1^{2/3}S^{1/2} \\
&= (1.49/0.02)(4.493)(0.06325) = \underline{\underline{21.17}}\text{ ft/s} \\
Q_1 &= V_1A_1 = 21.17 \times 2,000 = \underline{\underline{42,343\text{ cfs}}} \\
V_2 &= (1.49/0.0307)(2.879)(0.06325) \\
&= \underline{\underline{8.838\text{ ft/s}}} \\
Q_2 &= V_2A_2 = 8.838 \times 2,000 = \underline{\underline{17,676\text{ cfs}}}
\end{aligned}
$$

10.138 **Information and Assumptions**

assume $y > 10$ but < 18
provided in problem statement

Find

depth and flow rate in main channel, left overbank area and right overbank area

Solution

Let subscripts 1 and 2 refer to the main channel and overbank channels respectively.
Main Channel:

$$\begin{aligned} A_1 &= 102.5(10) + [105 + (y-10)/8](y-10) = 1025 + (103.75 + 0.13y)(y-10) \\ P_1 &= 100 + (10^2 + 2.5^2)^{1/2} + [y^2 + (y/4)^2]^{1/2} \\ &= 100 + 10.3 + 1.03y \\ &= 110.3 + 1.03y \end{aligned}$$

Overbank:

$$\begin{aligned} A_2 &= 200(y-10) \\ P_2 &= 200 + y - 10 = 190 + y \end{aligned}$$

From Eq. 10.50

$$Q_T = 1.49\sqrt{S_o}\sum \frac{1}{n} A R_n^{2/3}$$

or for this problem

$$\frac{Q_T}{1.49 \times (0.004)^{1/2}} = \frac{1}{n_1} A_1 R_1^{2/3} + \frac{1}{n_2} A_2 R_2^{2/3}$$

$$\begin{aligned} \frac{45,500}{1.49(.004)^{1/2}} &= \frac{1,025 + (103.75 + .13y)(y-10)}{0.025} \times \\ & \left[\frac{1,025 + (103.75 + .13y)(y-10)}{110.3 + 1.03y}\right]^{2/3} \\ & + \frac{200(y-10)}{0.07}\left[\frac{200(y-10)}{190 + y}\right]^{2/3} \end{aligned}$$

Solving for y by trial and error yields: $y = \underline{\underline{17.1 \text{ ft}}}$
Thus for the main channel: $y = 17.1$ ft. The discharge is

$$Q_{\text{main}} = Q_1 = \frac{1.49(0.004)^{1/2}}{0.025}\left[A_1\left(\frac{A_1}{R_1}\right)^{2/3}\right]$$

$$= \underline{\underline{38{,}700 \text{ cfs}}}$$

$$Q_{\text{overbank}} = 45,400 - 38,700 = \underline{\underline{6,700 \text{ cfs}}}$$

10.139 **Information and Assumptions**

from Table 10.4: $n(\text{overbank}) = 0.06 = n_2$
from Table 10.3: $n(\text{main channel}) = 0.025 = n_1$
provided in problem statement

Find

depth of flow for summer flood

Solution

The solution for y must be made differently for $y < 20$ and $y > 20$. Initially we assume $y < 20$.

$$A_1 = (500 + y)y, P_1 = 500 + 2.828y$$

From Eq. 10.45

$$Q = 80,000 = (1.49/0.025)(500 + y)y[(500 + y)y/(500 + 2.828y)]^{2/3}(0.0009)^{1/2}$$

Solve for y by trial and error yields $y = \underline{\underline{14.8 \text{ ft}}}$

10.140 **Information and Assumptions**

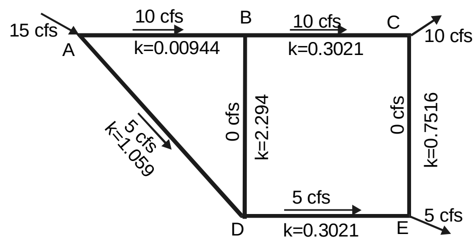
provided in problem statement

Find

load distribution and pressure at load points.

Solution

An assumption is made for the discharge in all pipes making certain that the continuity equation is satisfied at each junction. The following figure shows the network with assumed flows.



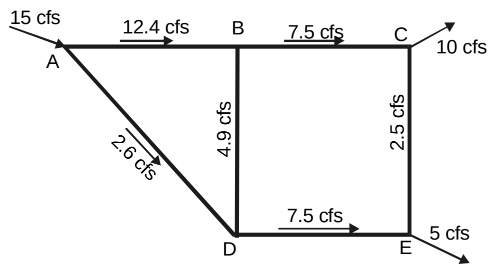
The Darcy-Weisbach equation is used for computing the head loss; therefore, we have

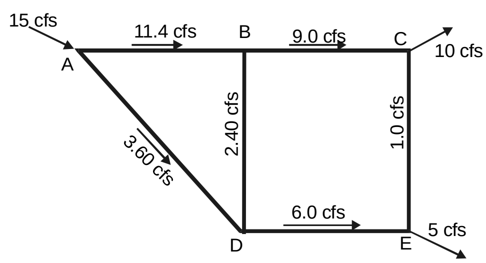
$$\begin{aligned} h_f &= f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) \\ &= 8\left(\frac{fL}{gD^5\pi^2}\right)Q^2 \\ &= kQ^2. \end{aligned}$$

where $k = 8\left(\frac{fL}{gD^5\pi^2}\right)$. The loss coefficient, k, for each pipe is computed and shown in Fig. A. Next, the flow corrections for each loop are calculated as shown in the accompanying table. Since $n = 2$ (exponent on Q), $nkQ^{n-1} = 2kQ$. When the correction obtained in the table are applied to the two loops, we get the pipe discharges shown in Fig. B. Then with additional iterations, we get the final distribution of flow as shown in Fig. C. Finally, the pressures at the load points are calculated.

	Loop ABC	
Pipe	$h_f = kQ^2$	$2kQ$
AB	+0.944	0.189
AD	-26.475	10.590
BD	0	0
$\sum kQ_c^2 - \sum kQ_{cc}^2$	-25.53	$\sum 2KQ = 10.78$
	ΔQ=-22.66/9.062=2.50 cfs	

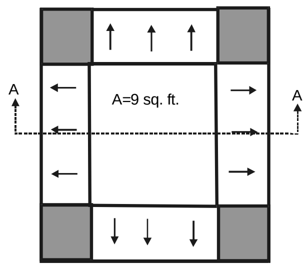
Loop $BCDE$		
Pipe	h_f	$2kQ$
BC	+30.21	6.042
BD	0	0
CE	0	0
DE	-7.55	3.02
	+22.66	9.062
ΔQ=-25.53/10.78=-2.40 cfs		

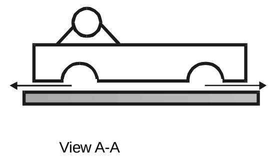




$$
\begin{aligned}
p_C &= p_A - \gamma(k_{AB}Q_{AB}^2 + k_{BC}Q_{BC}^2) \\
&= 60 \text{ psi } \times 144 \text{ psf/psi} - 62.4(0.00944 \times 11.4^2 + 0.3021 \times 9.0^2) \\
&= 8640 \text{ psf } - 1603 \text{ psf} \\
&= 7037 \text{ psf} \\
&= \underline{\underline{48.9 \text{ psi}}} \\
p_E &= 8640 - \gamma(k_{AD}Q_{AD}^2 + k_{DE}Q_{DE}^2) \\
&= 8640 - 62.4(1.059 \times 3.5^2 + 0.3021 \times 6^2) \\
&= 7105 \text{ psf} \\
&= \underline{\underline{49.3 \text{ psi}}}
\end{aligned}
$$

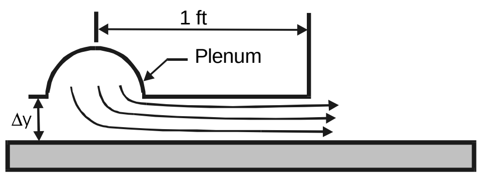
10.141 Assume that the equipment will have a maximum weight of 1,000 lbf and assume that the platform itself weighs 200 lbf. Assume that the platform will be square and be 5 ft on a side. The plan and elevation view are shown below:





Assume that a plenum 1 ft inside the perimeter of the platform will be the source of air for the underside of the platform.

Now develop the relationship for pressure distribution from plenum to edge of platform. The flow situation is shown below.



Determine the h_f from the plenum to the edge of the platform:

$$h_f = f(L/D)V^2/2g$$

Assume $f = 0.02$, $R = A/P = \Delta y B/2B = \Delta y/2$ and $L = 1$ ft.

$$\begin{aligned} h_f &= (0.02 \times 1/(\Delta y/2))V^2/2g \\ &= (0.02/\Delta y)V^2/g \\ &= 0.02V^2/(\Delta y g) \end{aligned}$$

Multiply both sides by γ

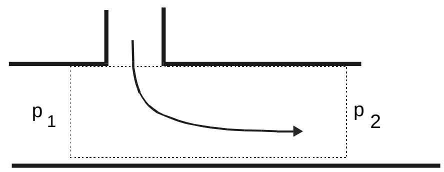
$$\Delta p_f = \gamma h_f = (0.02/\Delta y)\rho V^2$$

Assume $\rho = 0.0023$ slugs/ft^3.Then

$$\begin{aligned} \Delta p_f &= (0.02/\Delta y)(.0023)V^2 \\ &= (46V^2/\Delta y) \times 10^{-6} \end{aligned}$$

$$p_{\text{avg.}}(\text{over 4 ft}^2\text{ area}) = (23\ V^2/\Delta y) \times 10^{-6}$$

Also determine the Δp due to the change in momentum as the flow discharges from the plenum.



Apply the momentum equation in the x-direction:

$$\begin{aligned} \sum F_x &= \dot{m}_o V_o - \dot{m}_i V_i \\ B\Delta y(p_1 - p_2) &= V(\rho V B \Delta y) \\ \Delta p_{\text{mom}} &= \rho V^2 \end{aligned}$$

The pressure force on the platform is given by
The pressure within the 9 ft^2 interior area of the platform will be

$$\Delta p_{\text{mom}} + \Delta p_f = V^2(.0023 + (46/\Delta y) \times 10^{-6})$$

The pressure force on platform is given by

$$\begin{aligned} F &= 9\,\text{ft}^2 \times (\Delta p_{\text{mom}} + \Delta p_f) + \Delta p_{f,\text{avg.}} \times 12\ \text{ft}^2 \\ F &= 9 \times V^2[.0023 + (46/\Delta y) \times 10^{-6})] + 12V^2[(23V^2/\Delta y) \times 10^{-6}] \\ F &= V^2[9 \times .0023 + (9 \times 46/\Delta y) \times 10^{-6} + 12 \times 23 \times 10^{-6}/\Delta y] \\ F &= V^2[9 \times .0023 + 690 \times 10^{-6}/\Delta y] \end{aligned}$$

Let $\Delta y = 1/8$ in.$= 0.01042$ ft

$$\begin{aligned} f &= V^2[9 \times .0023 + 690 \times 10^{-6}/.01042] \\ &= V^2[0.0207 + 0.662] \\ F &= .0869V^2 \\ 1200 &= .0869V^2 \\ V^2 &= 13,809\ \text{ft}^2/\text{s}^2 \\ V &= 117.5\ \text{ft/s} \\ Q &= 117.5 \times \Delta y \times 12 = 14.69\ \text{ft}^3/\text{s} \\ \Delta p &= V^2(.0023 + 46 \times 10^{-6}/\Delta y) \\ &= V^2(.0023 + 46 \times 10^{-6}/0.01042) \\ &= V^2(.0023 + .00441) \\ &= 92.7\ \text{psf} \end{aligned}$$

Power

$$
\begin{aligned}
P &= Q\Delta p/550 \\
&= 14.69 \times 92.7/550 \\
&= 2.48 \text{ hp}
\end{aligned}
$$

Assume 50% efficiency for blower, so required power $\approx$ 5 horsepower. Blower could be driven by gasoline engine and also be located on the platform.

10.142

There are two design constraints; 1) the Reynolds number in the tube should be less than 1000 to insure that the flow in laminar and a closed form expression is available to the viscosity and 2) the pressure differential along the tube should sufficiently low that compressibility effects on the gas will not be important yet large enough that a measurement can be made with acceptable accuracy. Although not stated in the problem assume that the density of the gases ranges from 0.8 kg/m^3 to 1.5 kg/m^3. As a start assume the tube has a 1 mm internal diameter. The Reynolds number corresponding to the highest density and lowest viscosity would be

$$\mathrm{Re} = \frac{V \times 10^{-3} \times 1.5}{10^{-5}} = 150V$$

The maximum velocity should not exceed 6 m/s. The pressure drop for laminar flow in a pipe is

$$\Delta p = 32\frac{\mu LV}{D^2}$$

Assume the length of the tube is 500 mm (0.5 m), the pressure drop for the largest viscosity would be given by

$$\Delta p = 32\frac{1.5 \times 10^{-5} \times 0.5V}{10^{-6}} = 240V$$

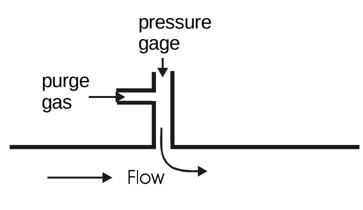
For a velocity of 6 m/s, the pressure drop would be 1,440 Pa or 0.2 psid. or about 5 in of water. If the initial pressure were atmospheric, this would represent about a 1% change in pressure which would be acceptable to avoid compressibility effects. Compressibility effect could also be reduced by operating at a higher pressure where the percentage change in pressure would be even smaller.

This design could now be refined to conform with the equipment available for measuring pressure. Another issue to consider is the design of the entrance to the tube to minimize entrance losses and exit losses such as a sudden expansion. There is also the problem of measuring a small discharge. An idea to consider would be attaching the end of the tube to an inflatable bag immersed in water and measuring the displacement of the water with time. Another idea is measuring the pressure drop in a tank supplying the tube and calculating the mass change with time.

10.143 One idea is to use a purge line as shown in the figure. There is a continuous flow of gas out the pressure tap which keeps the tap clean. The flow rate should be high enough to keep the tap clean and low enough not of affect the readings. The purge gases would be introduced close to the tap so the head loss associated with friction would be minimized. The largest pressure drop would be the sudden expansion loss at the tap exit. If p_o is the nominal pressure being measured at the tap, then the ratio of the sudden expansion losses to the nominal pressure is

$$\frac{\rho V^2}{2p_o}$$

and this ratio should be kept as small as possible. If the ratio is 0.01 then an error of 1% would be produced in the pressure measurement. The flow rate should be just sufficient to keep the taps clean. This value will depend on the experimental conditions.



Chapter Eleven

11.1 **Information and assumption**

Assume viscous effects are negligible.
provided in problem statement

Find

coefficient of drag

Solution

Force normal to plate

$$\begin{aligned} F_n &= \Delta p_{\text{average}} \times A \\ &= C_{p,ave}\rho V_0^2/2 \times c \times 1 \\ &= 1.5 \times \rho V_0^2/2 \times c \times 1 \end{aligned}$$

for unit depth of plate and a length c. Force parallel to free stream direction is the drag force and is equal to

$$\begin{aligned} F_D &= F_{\text{normal}} \cos 60^\circ \\ &= (1.5\rho V_0^2/2) \times c \times 1/2 \end{aligned}$$

The drag coefficient is defined as

$$\begin{aligned} C_D &= \frac{F_D}{\frac{1}{2}\rho V_0^2 A} = \frac{(1.5\rho V_0^2/2) \times c \times 1/2}{\frac{1}{2}\rho V_0^2 \times c \times 1} \\ &= \underline{\underline{1.5}} \end{aligned}$$

11.2 Flow is from the N.E. direction. Correct choice is $\underline{\underline{\text{d)}}}$

11.3 **Information and assumption**

provided in problem statement

Find

drag coefficient for rod.

Solution

The drag coefficient is based on the projected area of the block from the direction of the flow which is the area of each face of the block. The force contributing to drag on the downstream face is

$$F_D = 0.5 A_p \rho V^2/2$$

The force on each side face is

$$F_s = 0.5 A_p \rho V^2/2$$

Then the drag force on one side is

$$F_s \sin\alpha = 0.5 A_p \rho V_0^2/2 \times 0.5$$

The total drag force is

$$F_D = 2((0.5 A_p \rho V_0^2/2) \times 0.5) + 0.5 A_p \rho V_0^2/2 = C_D A_p \rho V_0^2/2$$

Solving for C_D one gets $\underline{\underline{C_D = 1.0}}$

11.4 **Information and assumption**

provided in problem statement

Find

drag coefficient for the block

Solution

The drag coefficient is based on the projected area of the block from the flow direction, A_p. The drag force on the windward side is

$$F_w = 0.8 \times \frac{1}{2}\rho V_0^2 A_p$$

The force on each of the two sloping sides is

$$F_s = -1.2 \times \frac{1}{2}\rho V_0^2 A_p$$

The total drag force on the rod is

$$\begin{aligned} F_D &= 0.8 \times \frac{1}{2}\rho V_0^2 A_p - 2(-1.2 \times \frac{1}{2}\rho V_0^2 A_p)\sin 30^o \\ &= \frac{1}{2}\rho V_0^2 A_p(0.8 + 1.2) \end{aligned}$$

The drag coefficient is

$$C_D = \frac{F_D}{\frac{1}{2}\rho V_0^2 A_p} = \underline{\underline{2.0}}$$

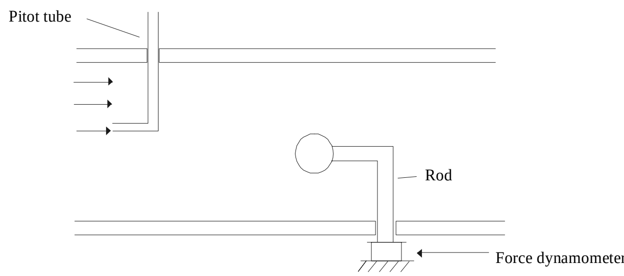
11.5 The design objective is to design an experiment to measure the drag coefficient of spheres of varying surface roughness.

The relevant equation is

$$
\begin{aligned}
F_D &= C_D A_p \rho V^2/2 \\
\text{or } C_D &= F_D/(A_p \rho V^2/2)
\end{aligned}
$$

Thus F_D, A_p, and V will have to be measured. The air density ρ can be obtained by measuring the air temperature with a thermometer and the air pressure with a barometer and solving for ρ by the equation of state.

You will need to decide how to position the sphere in the wind tunnel so that its support does not have an influence on flow past the sphere. One possible setup might be as shown below.



The sphere is attached to a rod and the rod in turn is attached to a force dynamometer as shown. Of course the rod itself will produce drag, however; its drag can be minimized by enclosing the vertical part of the rod in a streamlined housing. The horizontal part of the rod would have negligible drag because much of it would be within the low velocity wake of the sphere and the drag would be skin friction drag which is very small. The air velocity approaching the sphere could be measured by a pitot tube inserted into the wind tunnel. It would be removed when the drag of the sphere is being measured. The projected area of the sphere would be obtained by measuring the sphere diameter and then calculating the area. The pressure transducer is placed outside the wind tunnel. Blockage effects could also be addressed in the design of this experiment.

Another design consideration that could be addressed is size of sphere. It should be large enough to get measurable drag readings but not so large as to produce significant blockage.

11.6 The objective is to design an experiment to measure the drag coefficient of square rod.

Much of the design for determining the drag coefficient of the square rod would be the same as for determining the drag coefficient of a sphere (Prob. 11.5). However, it would be very difficult to measure the drag of a square rod supported like a sphere (as suggested in the design of problem 11.5) because end effects of the rod as well as the velocity distribution across the tunnel would present serious problems. A better way to determine the drag would be to measure the pressure distribution on the upstream and downstream faces of the rod and evaluate the force on the rod by integrating the pressure over these faces. The pressure could be measured by drilling small holes through the faces and connecting a tube to each hole inside the square rods. The pressure in each tube could be measured by a pressure transducer placed outside the wind tunnel. Blockage effects could also be addressed in the design of this experiment.

11.7 **Information and assumption**

From Table A.3 $\nu = 1.51 \times 10^{-5}$ m^2/s and $\rho = 1.20$ kg/m^3
provided in problem statement

Find

overturning moment on a smokestack.

Solution

$$\mathrm{Re} = VD/\nu = 40 \times 3/1.51 \times 10^{-5} = 7.95 \times 10^6$$

From Fig. 11.5 $C_D \approx 0.60$ so

$$F_D = C_D A_p \rho V_0^2/2 = 0.60 \times (3 \times 90) \times 1.20 \times (40)^2/2 = 146 \text{ kN}$$

The overturning moment is

$$F_D \times H/2 \text{ or M } = 146 \times 90/2 = \underline{\underline{7.02 \text{ MN}\cdot\text{m}}}$$

11.8 **Information and assumption**

From Table A.3 $\nu = 1.51 \times 10^{-5}$ m^2/s and $\rho = 1.20$ kg/m^3
provided in problem statement

Find

moment at bottom of flag pole.

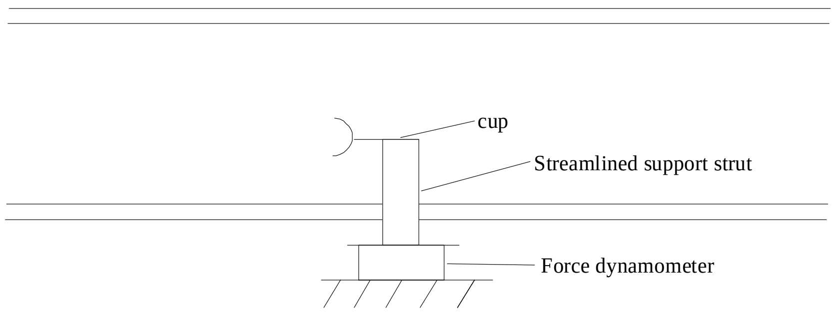
Solution

$$\text{Re} = VD/\nu = 25 \times 0.10/(1.51 \times 10^{-5}) = 1.66 \times 10^5$$

From Fig. 11-5: $C_D = 0.95$ so the moment is

$$\begin{aligned} M &= F_D H/2 = C_D A_p \rho (V_0^2/2) \times H/2 \\ &= 0.95 \times 0.10 \times (35^2/2) \times 1.2 \times 25^2/2 \\ &= \underline{\underline{21.8 \text{ kN} \cdot \text{m}}} \end{aligned}$$

11.9 The cup, sphere or disk should probably be located at the center of the pipe (as shown below) because the greatest velocity of the air stream in the pipe will be at the center.



You want to correlate V and Q with the force acting on your device. First, neglecting the drag of the support device, the drag is given as

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ \text{or } V_0 &= (2F_D/(C_D A_p))^{1/2} \end{aligned}$$

you can measure temperature, barometric pressure, and gage pressure in the pipe. Therefore, with these quantities the air density can be calculated by the equation of state. Knowing the diameter of the cup, sphere or disk you can calculate A_p. Assume that C_D will be obtained from Table 11.1 or Fig. 11.11. Then the other quantity that is needed to estimate V_0 is the drag F_D. This can be measured by a force dynamometer as indicated on the sketch of the device. However, the support strut will have some drag so that should be considered in the calculations. Another possibility is to minimize the drag of the support strut by designing a housing to fit around, but be separate from the vertical part of the strut thus eliminating most of the drag on the strut. This was also suggested for Problem 11.5.

Once the centerline velocity is determined it can be related to the mean velocity in the pipe by Table 10.1 from which the flow rate can be calculated. For example, if the Reynolds number is about 10^5 then $\bar{V}/V_{\max} \approx 0.82$ (from Table 10.1) and

$$\begin{aligned} Q &= \bar{V} A \\ Q &= 0.82 V_{\max} A \end{aligned}$$

There may be some uncertainty about C_D as well as the drag of the support rod; therefore, the device will be more reliable if it is calibrated. This can be done as follows. For a given flow make a pitot-tube-velocity-traverse across the pipe from which Q can be calculated. Also for the given run measure the force on the force dynamometer. Then plot F vs. Q. Do this for several runs so that a curve of F vs. Q is developed (calibration completed).

11.10 **Information and assumption**

From Table A.3 $\rho = 0.00237$ slugs/ft^3; $\nu = 1.58 \times 10^{-4}$ ft^2/s;
provided in problem statement

Find

drag on cooling tower.

Solution

$$
\begin{aligned}
V &= 150 \text{ mph} = 220 \text{ ft/s} \\
\text{Re} &\approx V_0 d/\nu = 220 \times 250/(1.58 \times 10^{-4}) = 3.5 \times 10^8
\end{aligned}
$$

From Fig. 11-5 (extrapolated) $C_D \approx 0.70$. Then

$$
\begin{aligned}
F_D &= C_D A_p \rho V^2/2 \\
&= 0.70 \times 250 \times 350 \times 0.00237 \times 220^2/2 \\
F_D &= \underline{\underline{3.51 \times 10^6 \text{ lbf}}}
\end{aligned}
$$

11.11 **Information and assumption**

provided in problem statement

Find

a) equation for power to rotate rod and b) the power.

Solution

For an infinitesimal element, dr,of the rod

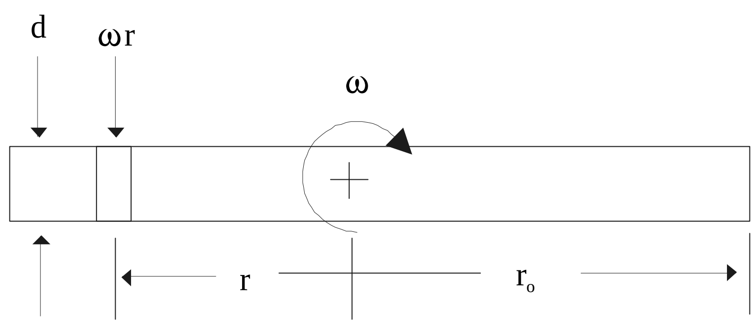
$$dF_D = C_D(dr)d\rho V_{\text{rel.}}^2/2$$

where $V_{\text{rel.}} = r\omega$. Then

$$\begin{aligned} dT &= rdF_D = C_D\rho d(V_{\text{rel.}}^2/2)rdr \\ T_{\text{total}} &= 2\int_0^{r_0} dT = 2\int_0^{r_0} C_D d\rho((r\omega)^2/2)rdr \\ T_{\text{total}} &= C_D d\rho\omega^2 \int_0^{r_0} r^3 dr = C_D d\rho\omega^2 r_0^4/4 \end{aligned}$$

but $r_0 = L/2$ so

$$T_{\text{total}} = C_D d\rho\omega^2 L^4/64 \text{ or } P = T\omega = \underline{\underline{C_D d\rho\omega^3 L^4/64}} \quad \text{(a)}$$



Then for the given conditions:

$$P = 1.2 \times 0.02 \times 1.2 \times (100)^3 \times 1^4/64 = \underline{\underline{450 \text{ W}}}$$

11.12 **Information and assumption**

provided in problem statement

Find

frequency at which vortices will be shed from smokestack.

Solution

From Problem 11.7 $Re = 7.95 \times 10^6$. From Fig. 11-10 $St = 0.25$

$$St = nd/V_0$$

or

$$\begin{aligned} n &= StV_0/d \\ &= 0.25 \times 40/3 = \underline{\underline{3.3 \text{ Hz}}} \end{aligned}$$

11.13 **Information and assumption**

provided in problem statement

Find

frequency at which vortices will be shed from flag pole.

Solution

From Problem 11.8 $Re = 1.66 \times 10^5$. From Fig. 11-10 $St = 0.21$

$$St = nd/V_0$$

or

$$\begin{aligned} n &= StV_0/d \\ &= 0.21 \times 25/0.1 = \underline{\underline{52.5 \text{ Hz}}} \end{aligned}$$

11.14 **Information and assumption**

provided in problem statement

Find

wind force on billboard.

Solution

From Table A.3 $\nu = 1.58 \times 10^{-4}$ ft^2/s; $\rho = 0.00237$ slugs/ft^3

$$
\begin{aligned}
V_0 &= 50 \text{ mph} = 73 \text{ ft/s} \\
\text{Re} &= V_0 b/\nu = 73 \times 10/(1.58 \times 10^{-4}) = 4.6 \times 10^6
\end{aligned}
$$

From Table 11-1 $C_D = 1.19$. Then

$$
\begin{aligned}
F_D &= C_D A_p \rho V_0^2/2 \\
&= 1.19 \times 300 \times 0.00237 \times 73^2/2 = \underline{\underline{2{,}250 \text{ lbf}}}
\end{aligned}
$$

11.15 **Information and assumption**

From Table A.3 $\nu = 1.58 \times 10^{-4}$ ft^2/s; $\rho = 0.00237$ slugs/ft^3
provided in problem statement

Find

drag force on a 4 ft by 4 ft plate.

Solution

$$R_e = VL/\nu = (100)(4)/(1.58 \times 10^{-4}) = 2.5 \times 10^6$$

From Table 11-1 $C_D = 1.18$ so

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ F_D &= (1.18)(4 \times 4)(0.00237)(100^{\bar{2}})/2 = \underline{\underline{224 \text{ lbf}}} \end{aligned}$$

11.16 **Information and assumption**

From Table A.5 $\nu = 1.31 \times 10^{-6}$ m^2/s
provided in problem statement

Find

ratio of drag on 2m by 2m square from normal to edgewise orientation.

Solution

$$\begin{aligned} F_{\text{edge}} &= 2C_f A\rho V^2/2 \\ F_{\text{normal}} &= C_D A\rho V^2/2 \end{aligned}$$

Then

$$\begin{aligned} F_{\text{normal}}/F_{\text{edge}} &= C_D/2C_f \\ \text{Re} &= \text{Re}_L = VB/\nu = 1 \times 2/(1.31 \times 10^{-6}) \\ &= 1.53 \times 10^6 \end{aligned}$$

From Fig. 9-13 $C_f = 0.0030$ and from Table 11-1 $C_D = 1.18$. So

$$F_{\text{normal}}/F_{\text{edge}} = 1.18/(2 \times 0.0030) = \underline{\underline{197}}$$

11.17 **Information and assumption**

provided in problem statement

Find

drag on a disk

Solution

From Table 11.1 $C_D = 1.17$.

$$F_D = C_D A_p \rho V^2/2 = 1.17 \times (\pi/4) \times 1^2 \times 1,000 \times 5^2/2 = \underline{\underline{11.5 \text{ kN}}}$$

11.18 **Information and assumption**

From Table A.3 $\rho = 1.25$ kg/m^3.
provided in problem statement

Find

force on circular billboard.

Solution

From Table 11.1 $C_D = 1.17$

$$\begin{aligned} F_D &= C_D A_p \rho V^2/2 \\ &= 1.17 \times (\pi/4) \times 6^2 \times 1.25 \times 30^2/2 = \underline{\underline{18{,}608 \text{ N}}} \\ &= \underline{\underline{18.6 \text{ kN}}} \end{aligned}$$

11.19 **Information and assumption**

From Table A.3 $\rho = 1.25 \text{ kg/m}^3$
provided in problem statement

Find

moment at ground level on a sign post.

Solution

From Table 1.1 $C_D = 1.18$ Then

$$M = 3 \times F_D = 3 \times C_D A_p \rho V^2/2 = 3 \times 1.18 \times 2^2 \times 1.25 \times 40^2/2 = \underline{\underline{14.16 \text{ kN}\cdot\text{m}}}$$

11.20 **Information and assumption**

Assume $\rho = 1.2\ \text{kg/m}^3$;
provided in problem statement

Find

additional power required to carry sign.

Solution

From Table 11-1 $C_D = 1.20$. Then

$$\begin{aligned} F_D &= C_D A_p \rho V^2/2 = 1.2 \times 1.83 \times 0.46 \times 1.2 \times 20^2/2 = 242\ \text{N} \\ P &= FV = 242 \times 20 = \underline{\underline{4.85\ \text{kW}}} \end{aligned}$$

11.21 **Information and assumption**

Assume $\rho = 1.20$ kg/m^3
provided in problem statement

Find

added power required for car

Solution

Assume C_D will be like that for a rectangular plate: $\ell/b = 1.5/0.2 = 7.5$
Then from Table 11-1 $C_D \approx 1.25$

$$\begin{aligned} V &= 105 \text{ km/hr} = 29.17 \text{ m/s} \\ \Delta P &= C_D A_p(\rho V^2/2)V \\ &= 1.25 \times 1.5 \times 0.2 \times 1.2 \times 29.17^2/2 \times 80,000/3,600 = \underline{\underline{4.25 \text{ kW}}} \end{aligned}$$

11.22 **Information and assumption**

provided in problem statement

Find

percentage savings in gas mileage when travelling a 55 mph instead of 65 mph

Solution

The energy required per distance of travel = $F \times s$ (distance). Thus, the energy, E, per unit distance is simply the force or

$$E/s = F = \mu \times W + C_D A_p \rho V^2/2 = 0.02 \times 3,000 + 0.3 \times 20 \times (0.00237/2)V^2$$

For

$$\begin{aligned} V &= 55 \text{ mph} = 80.67 \text{ ft/sec} \\ E/s &= 106.3 \text{ ft-lbf} \end{aligned}$$

For

$$\begin{aligned} V &= 65 \text{ mph} = 95.33 \text{ ft/sec} \\ E/s &= 124.6 \text{ ft-lbf} \end{aligned}$$

Then energy savings = (124.6 - 106.3)/124.6 = 0.147 or $\underline{\underline{14.7\%}}$

11.23 **Information and assumption**

provided in problem statement

Find

maximum coasting speed

Solution

Equating forces

$$F_D + F_r = W \times \sin 6^{\mathrm{o}}$$

where F_D =drag force, F_r =rolling friction and W =weight of car

$$\begin{aligned}
C_D A_p \rho V^2/2 + W \times 0.02 &= W \times \sin 6^{\mathrm{o}} \\
V^2 &= 2W(\sin 6^\circ - 0.02)/(C_D A_p \rho) \\
V^2 &= 2 \times 2,500(0.105 - 0.02)/(0.32 \times 25 \times 0.0024) \\
&= 22,135 \text{ ft}^2/\text{s}^2 \\
V &= 148.8 \text{ ft/s} = \underline{\underline{101 \text{ mph}}}
\end{aligned}$$

11.24 **Information and assumption**

provided in problem statement

Find

power required

Solution

The power required is the product of the forces acting on the automobile in the direction of travel and the speed. The drag force is

$$F_D = \frac{1}{2}\rho V^2 C_D A = \frac{1}{2} \times 1.2 \times 30^2 \times 0.4 \times 4 = 864 \text{ N}$$

The force due to gravity is

$$F_g = Mg \sin 3^\text{o} = 1000 \times 11.81 \times \sin 3^\text{o} = 513 \text{ N}$$

The force due to rolling friction is

$$F_r = \mu Mg \cos 3^\text{o} = 0.02 \times 1000 \times 11.81 \times \cos 3^\text{o} = 196 \text{ N}$$

The power required is

$$P = (F_D + F_r + F_f)V = 1573 \times 30 = \underline{\underline{47.2 \text{ kW}}}$$

11.25 **Information and assumption**

provided in problem statement

Find

power required

Solution

$$P = FV$$

where $F = F_D + F_r$.

$$\begin{aligned}
F_D &= C_D A_p \rho V_0^2/2 = 0.4 \times 2 \times 1.2 \times 40^2/2 = 768 \text{ N} \\
F_r &= 0.02\ W = 0.02 \times 10,000 \text{ N} = 200 \text{ N} \\
P &= (768 + 200) \times 30 = \underline{\underline{29.04 \text{ kW}}}
\end{aligned}$$

11.26 **Information and assumption**

provided in problem statement

Find

wind force that would act on you.

Solution

Ideal gas law

$$\rho = p/RT = 96,000/(287 \times (273 + 20)) = 1.14 \text{ kg/m}^3$$

Assume C_D is like a rectangular plate: $C_D \approx 1.20$. Then

$$F_D = C_D A_p \rho V^2/2 = 1.2 \times 1.83 \times 0.30 \times 1.14 \times 30^2/2 = \underline{\underline{338 \text{ N}}}$$

Note: F_D will depend upon C_D and dimensions assumed.

11.27 **Information and assumption**

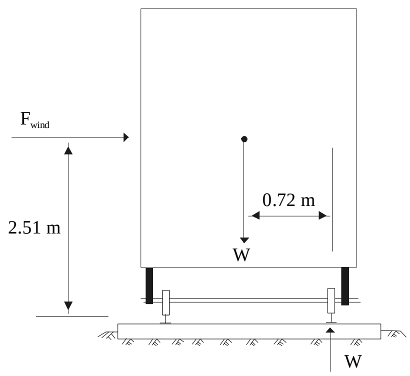
Assume $T = 10°C$; $\rho = 1.25$ kg/m^3
provided in problem statement

Find

wind required to blow boxcar over.

Solution

Take moments about one wheel for impending tipping. $\sum M = 0$



$$\begin{aligned} W \times 0.72 - F_D \times 2.51 &= 0 \\ F_D = (190,000 \times 1.44/2)/2.51 = 54,500 \text{ N} &= C_D A_p \rho V^2/2 \end{aligned}$$

From Table 11-1 assume $C_D = 1.20$. Then

$$\begin{aligned} V^2 &= 54,500 \times 2/(1.2 \times 12.5 \times 3.2 \times 1.25) \\ V &= \underline{\underline{42.6 \text{ m/s}}} \end{aligned}$$

11.28 **Information and assumption**

provided in problem statement

Find

speed of the bicycle

Solution

Consider force balance parallel to direction of motion of the bicyclist:

$$\begin{aligned}
\sum F &= 0 \\
+F_{\text{wgt. comp.}} - F_D - F_{\text{rolling resist.}} &= 0 \\
W \sin 8^\circ - C_D A_p \rho V_{R.}^2/2 - 0.02\ W \cos 8^\circ &= 0 \\
W \sin 8^\circ - 0.5 \times 0.5 \times 1.2 V_R^2/2 - 0.02 W \cos 8^\circ &= 0
\end{aligned}$$

$$\begin{aligned}
W &= 80g = 784.8 \text{ N} \\
W \sin 8^\circ &= 109.2 \text{ N} \\
W \cos 8^\circ &= 777.2 \text{ N}
\end{aligned}$$

Then

$$109.2 - 0.15 V_R^2 - .02 \times 777.2 = 0$$

$$V_R = 25.0 \text{ m/s} = V_{\text{bicycle}} + 5 \text{ m/s}$$

Note that 5 m/s is the head wind so the relative speed is $V_{bicycle} + 5$.

$$V_{\text{bicycle}} = \underline{\underline{20.0 \text{ m/s}}}$$

11.29 **Information and assumption**

provided in problem statement

Find

speed in a 5 m/s head wind.

Solution

$$
\begin{aligned}
P &= F_D V = C_D A_p \rho (V_R^2/2) V \\
V_R &= (V+5) \\
100 &= 0.3 \times 0.5 \times 1.2 (V+5)^2 \times V/2
\end{aligned}
$$

Solving the cubic equation for speed gives

$$\underline{\underline{V = 7.3 \text{ m/s}}}$$

11.30 **Information and assumption**

From Table A.3 $\rho = 1.2\ \mathrm{kg/m^3}$
provided in problem statement

Find

maximum speed with roof closed and open.

Solution

$$\begin{aligned} P &= FV = (\mu_{\mathrm{roll}} Mg + C_D A_p \rho V_0^2/2)V \\ P &= \mu_{\mathrm{roll}} Mg\ V_0 + C_D A_p \rho V_0^3/2 \end{aligned}$$

Then

$$\begin{aligned} 80,000 &= 0.05 \times 800 \times 9.81V + C_D \times 4 \times (1.2/2)V^3 \\ 80,000 &= 392.4V + 2.40 C_D V^3 \end{aligned}$$

Solving with $C_D = 0.30$ (roof closed) one finds

$$V = \underline{\underline{44.3\ \mathrm{m/s}}}$$

Solving with $C_D = 0.42$ (roof open) one finds

$$V = \underline{\underline{40.0\ \mathrm{m/s}}}$$

11.31 **Information and assumption**

provided in problem statement

Find

velocity of the head wind.

Solution

Assume gas consumption is proportional to power.

Then gas consumption is proportional to $F_D V$ where V is the speed of the automobile and F_D is the total drag of the auto (including rolling friction).

$$
\begin{aligned}
F_D &= C_D A_p \rho V_0^2/2 + 0.1Mg \\
&= 0.3 \times 2 \times 1.2 V_0^2/2 + 0.1 \times 500 \times 9.81 \\
&= 0.360 V_0^2 + 490.5 \text{ N} \\
V_{0,\text{still air}} &= (90,000/3,600) = 25.0 \text{ m/s}
\end{aligned}
$$

Then

$$
\begin{aligned}
F_{D,\text{still air}} &= 0.36 \times 25^2 = 490.5 = 715.5 \text{ N} \\
P_{\text{still air}} &= 715.5 \times 25 = 17.89 \text{ kW} \\
P_{\text{head wind}} &= 17,890 \times 1.20 = (0.36 V_0^2 + 490.5)(25)
\end{aligned}
$$

where

$$
\begin{aligned}
V_0 &= V_{\text{headwind}} + 25 = 32 \text{ m/s} \\
V_{\text{headwind}} &= \underline{\underline{7 \text{ m/s}}}
\end{aligned}
$$

11.32 **Information and assumption**

provided in problem statement

Find

maximum speed of "souped up" Balillo.

Solution

From Table 11.2 $C_D = 0.60$

$$\begin{aligned}
P &= (F_D + F_r)V \\
V &= 60 \text{ mph} = 88 \text{ ft/s} \\
F_r &= (P/V) - F_D = (P/V) - C_D A_p \rho V^2/2 \\
&= ((40)(550)/88) - (0.60)(30)(0.00237)(88^2)/2 \\
&= 250 - 165 = 85 \text{ lbf}
\end{aligned}$$

"Souped up" version:

$$\begin{aligned}
(F_D + 85)V &= (220)(550) \\
((C_D A_p \rho V^2/2) + 85)V &= (220)(550) \\
(C_D A_p \rho V^3/2) + 85V &= (220)(550) \\
0.0213V^3 + 85V - 121,000 &= 0
\end{aligned}$$

Solve for V:

$$V = 171.0 \text{ ft/s} = \underline{\underline{117 \text{ mph}}}$$

11.33 **Information and assumption**

Assume $\rho = 1.2 \text{ kg/m}^3$
provided in problem statement

Find

reduction in drag force.

Solution

$$\begin{aligned} F_D &= C_D A_p \rho V^2/2 \\ F_{D_{\text{reduction}}} &= 0.25 \times 0.78 \times 8.36 \times 1.2(100,000/3,600)^2/2 \\ F_{D_{\text{reduction}}} &= \underline{\underline{755 \text{ N}}} \end{aligned}$$

11.34 **Information and assumption**

provided in problem statement

Find

power required for dirigible.

Solution

$$\mathrm{Re} = V_0 d/\nu = (25)(100)/(1.3 \times 10^{-4}) = 2.3 \times 10^7$$

From Fig. 11.11 (extrapolated) $C_D = 0.05$

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 = (0.05)(\pi/4)(100^2)(0.07/32.2)(25^2)/2 \\ &= 267 \text{ lbf} \\ P &= F_D V_0 = (267)(25) = 6{,}670 \text{ ft-lbf/s} = \underline{\underline{\underline{12.1 \text{ hp}}}} \end{aligned}$$

11.35 **Information and assumption**

provided in problem statement

Find

percentage in fuel savings.

Solution

Assume that the fuel savings are directly proportional to power savings.

$$
\begin{aligned}
P &= FV \\
P &= C_D \times 8.36 \times 1.2V^3/2 + 450V
\end{aligned}
$$

At 80 km/hr:

$$
\begin{aligned}
P_{\text{w/o vanes}} &= 0.78 \times 8.36 \times 1.2V^3/2 + 450V = 52.9 \text{ kW} \\
P_{\text{with vanes}} &= 42.2 \text{ kW}
\end{aligned}
$$

which corresponds to a $\underline{\underline{20.2\%}}$ savings.

At 100 km/hr:

$$
\begin{aligned}
P_{\text{w/o vanes}} &= 96.4 \text{ kW} \\
P_{\text{with vanes}} &= 75.4 \text{ kW}
\end{aligned}
$$

which corresponds to a $\underline{\underline{21.8\%}}$ savings.

11.36 **Information and assumption**

Assume $\rho = 1.25$ kg/m^3 and $\nu = 1.41 \times 10^{-5}$ m^2/s
provided in problem statement

Find

percentage of resistance due to bearing resistance, form drag and skin friction drag.

Solution

$$\begin{aligned}
F_{D_{\text{form}}} &= C_D A_p \rho V_0^2/2 \\
F_{D_{\text{form}}} &= 0.80 \times 9 \times 1.25 \times V_0^2/2 = 4.5V_0^2 \\
F_{D_{\text{skin}}} &= C_f A \rho V_0^2/2 \\
\text{Re}_L &= VL/\nu = V \times 150/(1.41 \times 10^{-5}) \\
\text{Re}_{L,100} &= (100,000/3,600) \times 150/(1.41 \times 10^{-5}) = 2.9 \times 10^8 \\
\text{Re}_{L,200} &= 5.8 \times 10^8
\end{aligned}$$

From Fig. 9-14, $C_{f,100} = 0.00188$; $C_{f,200} = 0.00173$.

$V = 100$ km/hr	$V = 200$ km/hr
$F_{D,\text{form},100} = 3,472$ N	$F_{D,\text{form},200} = 13,889$ N
$F_{D,\text{skin},100} = 1,360$ N	$F_{D,\text{skin},200} = 5006$ N
$F_{\text{bearing}} = 3,000$ N	$F_{\text{bearing}} = 3,000$ N
$F_{\text{total}} = 7,832$ N	$F_{\text{total}} = 21,895$ N
44% form, 17% skin, 39% bearing	63% form, 23% skin, 14% bearing

11.37 Stoke's law is the equation of drag for a sphere for a Reynolds number less than 0.5:

$$\begin{aligned} F_D &= 3\pi\mu V_0 d \\ \text{or } \mu &= F_D/(3\pi V_0 d) \end{aligned}$$

One can use this equation to determine the viscosity of a liquid by measuring the fall velocity of a sphere in a liquid. Thus one needs a container to hold the liquid (for instance a long tube vertically oriented). The spheres could be ball bearings, glass or plastic spheres. Then one needs to measure the time of fall between two points. This could be done by measuring the time it takes for the sphere to drop from one level to a lower level. The diameter could be easily measured by a micrometer and the drag, F_D, would be given by

$$F_D = W_{\text{sphere}} = F_{\text{body of sphere}}$$

If the specific weight of the material of the sphere is known then the weight of the sphere can be calculated. Or one could actually weigh the sphere on an analytic balance scale. The buoyant force can be calculated if one knows the specific weight of the liquid. If necessary the specific weight of the liquid could be measured with a hydrometer.

To obtain a reasonable degree of accuracy the experiment should be designed so that a reasonable length of time (not too short) elapses for the sphere to drop from one level to the other. This could be assured by choosing a sphere that will yield a fairly low velocity of fall which could be achieved by choosing to use a small sphere over a large one or by using a sphere that is near the specific weight of the liquid (for instance, plastic vs. steel).

Other items that should be or could be addressed in the design are:

A. Blockage effects if tube diameter is too small.
B. Ways of releasing sphere and retrieving it.
C. Possibly automating the measurement of time of fall of sphere.
D. Making sure the test is always within Stoke's law range (Re < 0.5)
E. Making sure the elapsed time of fall does not include the time when the sphere is accelerating.

11.38 **Information and assumption**

provided in problem statement

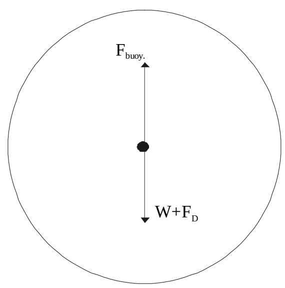
Find

terminal velocity

Solution

$$\begin{aligned} F_{\text{buoy.}} &= V\gamma_{\text{oil}} \\ &= (4/3)\pi \times (1/2)^3 \times 0.85 \times 62.4 \\ &= 27.77 \text{ lbf} \end{aligned}$$

Under non-accelerating conditions, the buoyancy is equal to the drag force plus the weight.



$$\begin{aligned} F_D &= -W + F_{\text{buoy.}} \\ &= -27.0 + 27.77 \text{ lbf} \\ &= 0.77 \text{ lbf upward} \end{aligned}$$

Assume laminar flow. hen

$$\begin{aligned} F_D &= 0.77 = 3\pi\mu D V_0 \\ V_0 &= 0.77/(3\pi D\mu) \\ V_0 &= 0.77/(3\pi \times 1 \times 1) \\ V_0 &= \underline{\underline{0.082 \text{ ft/s upward}}} \end{aligned}$$

$$\begin{aligned} \text{Re} &= V_0 d\rho/\mu = 0.082 \times 1 \times 1.94 \times 0.85/1 \\ &= 0.14 < 0.5 \end{aligned}$$

Therefore the laminar assumption is valid.

11.39 **Information and assumption**

provided in problem statement

Find

specific weight of sphere

Solution

$$\mathrm{Re} = VD\rho/\mu = 0.03 \times 0.02 \times 900/0.096 = 5.63$$

Then from Fig. 11.11 Then $C_D \approx 7.0$

$$\begin{aligned}
\sum F &= 0 = -F_D - W + F \\
F_D &= F_{\text{buoy.}} - W \\
C_D A_p \rho V_0^2/2 &= V(\gamma_{\text{oil}} - \gamma_{\text{sphere}}) \\
V &= (4/3)\pi r^3 = 4.19 \times 10^{-6} \text{ m}^3 \\
7 \times \pi \times 0.01^2 \times 900 \times 0.03^2/2 &= 4.19 \times 10^{-6}(900g - g\rho_{\text{sphere}}) \\
\rho_{\text{sphere}} &= 878.3 \text{ kg/m}^3 \\
\gamma_{\text{sphere}} &= \underline{\underline{8{,}620 \text{ N/m}^3}}
\end{aligned}$$

11.40 **Information and assumption**

provided in problem statement

Find

terminal velocity of a 2.5-mm sphere.

Solution

Because the viscosity is large, it is expected that the sphere will fall according to Stoke's law. Thus,

$$F_D = 3\pi\mu V_0 D = 3\pi\nu\rho V_0 D$$

where

$$\begin{aligned} F_D &= (1/6)\pi D^3(\gamma_{\text{sphere}} - \gamma_{\text{oil}}) \\ &= (1/6)\pi(0.0025)^3 \times 9,810(1.03 - 0.95) = 6.42 \times 10^{-6}\text{ N} \end{aligned}$$

Then

$$\begin{aligned} V_0 &= F_D/(3\pi\mu\rho D) = 6.42 \times 10^{-6}/(3\pi \times 10^{-4} \times 950 \times 0.0025) \\ V_0 &= 0.0029\text{ m/s} = \underline{\underline{2.9\text{ mm/s}}} \end{aligned}$$

Check Re:

$$\text{Re} = V_0 D/\nu = 0.0029 \times 0.0025/10^{-4} = 0.0725$$

Within Stokes' range so solution is correct.

11.41 The equation of motion for the plastic sphere is

$$m\frac{dv}{dt} = -F_D + W - F_B$$

The drag force can is expressed as

$$F_D = \frac{1}{2}\rho v^2 C_D \frac{\pi}{4} d^2 = \frac{C_D \mathrm{Re}}{24} 3\pi\mu d v$$

The equation of motion becomes

$$m\frac{dv}{dt} = -\frac{C_D \mathrm{Re}}{24} 3\pi\mu d v + \rho_b \forall g - \rho_w g \forall$$

Dividing through by the mass of the ball gives

$$\frac{dv}{dt} = -\frac{C_D \mathrm{Re}}{24}\frac{18\mu}{\rho_b d^2} v + g(1 - \frac{\rho_w}{\rho_b})$$

Substituting in the values

$$\frac{dv}{dt} = -0.0375\frac{C_D \mathrm{Re}}{24} v + 1.635$$

Eq. 11.10 can be rewritten as

$$\frac{C_D \mathrm{Re}}{24} = 1 + 0.15\mathrm{Re}^{0.687} + \frac{0.0175\mathrm{Re}}{1 + 4.25\times 10^4 \mathrm{Re}^{-1.16}}$$

This equation can be integrated using the Euler method

$$\begin{aligned} v_{n+1} &= v_n + \left(\frac{dv}{dt}\right)_n \Delta t \\ s_{n+1} &= s_n + 0.5(v_n + v_{n+1})\Delta t \end{aligned}$$

The terminal velocity is 0.362 m/s. The time to reach 99% of the terminal velocity is 0.54 seconds and travels 14.2 cm.

11.42 Equating the drag force and the buoyant force.

$$F_D = C_1 \gamma_{\text{liq.}} D^3 = C_2 D^3$$

Also

$$F_D = C_D A_p \rho V^2/2 = C_3 D^2 V^2$$

Eliminating F_D between these two equations yields

$$V^2 = C_4 D \text{ or } V = \sqrt{C_4 D}$$

As the bubble rises it will expand because the pressure decreases with an increase in elevation; thus, the bubble will accelerate as it moves upward. The drag will be form drag because there is no solid surface to the bubble for viscous shear stress to act on. As a matter of interest, the surface tension associated with contaminated fluids creates a condition which acts like a solid surface.

11.43 **Information and assumption**

provided in problem statement

Find

kinematic viscosity of fluid

Solution

$$\begin{aligned} F_D &= (1/6)\pi D^3(\gamma_{\text{sphere}} - \gamma_{\text{fluid}}) = (1/6)\pi(0.05)^3(9,810)(0.66 - 0.20) \\ F_D &= 0.295 \text{ N} \end{aligned}$$

Assume Stokes' law applies. Then

$$\begin{aligned} 0.295 &= 3\pi\mu V_0 D \\ \mu &= 0.295/(3\pi(0.06)(0.05)) = 10.4 \text{ N}\cdot\text{s/m}^2 \end{aligned}$$

Check Re:

$$Re = VD\rho/\mu = 0.06 \times 0.05 \times 1,000 \times 0.66/10.4 = 0.19$$

So the velocity is well into Stokes' range (Re < 0.5). Thus

$$\nu = \mu/\rho = 10.4 \text{ N}\cdot\text{s/m}^2/(660 \text{ kg/m}^3) = \underline{\underline{0.0158 \text{ m}^2/\text{s}}}$$

11.44 **Information and assumption**

Assume $T_{\text{air}} = 60° F$; $\rho_{\text{air}} = 0.00237$ slugs/ft^3; $\mu_{\text{air}} = 3.74 \times 10^{-7}$ lbf-sec/ft^2 provided in problem statement

Find

largest raindrop that will fall in the Stokes' flow regime.

Solution

$$
\begin{aligned}
F_D &= 3\pi\mu V_0 D \\
(1/6)\pi D^3 \gamma_{\text{water}} &= 3\pi\mu_{\text{air}} V_0 D \\
D^2 \gamma_{\text{water}} &= 18\mu_{\text{air}} V_0
\end{aligned}
$$

Also

$$
\begin{aligned}
V_0 D/\nu &= 0.5 \\
V_0 &= 0.5\, \nu_{\text{air}}/D
\end{aligned}
$$

Solving for D:

$$
\begin{aligned}
D^3 &= 9\mu_{\text{air}}^2/(\rho_{\text{air}}/\gamma_{\text{water}}) = 9 \times (3.74 \times 10^{-7})/(0.00237 \times 62.4) \\
&= 8.51 \times 10^{-12} \text{ ft}^3 \\
D &= 2.042 \times 10^{-4} \text{ ft } = 0.000204 \text{ ft} = \underline{\underline{0.0024 \text{ in.}}}
\end{aligned}
$$

11.45 **Information and assumption**

provided in problem statement

Find

terminal velocity of hail stone.

Solution

$$\begin{aligned} \rho &= p/RT = 96,000/(287 \times 273) = 1.23 \text{ kg/m}^3 \\ F_D &= V \times 6,000 = C_D A_p \rho V^2/2 \end{aligned}$$

Assume $C_D = 0.5$

$$\begin{aligned} (1/6)\pi d^3 \times 6,000 &= 0.5 \times (\pi d^2/4) \times 1.23V^2/2 \\ V &= \sqrt{d \times 1,000 \times 16/1.23} = \sqrt{10 \times 16/1.23} = 11.4 \text{ m/s} \end{aligned}$$

Check Re and drag coefficient

$$\text{Re} = 11.4 \times 0.01/(1.3 \times 10^{-5}) = 8.8 \times 10^3;\ C_D = 0.4 \text{ (Fig. 11-11)}$$

From Fig. 11-11 $C_D = 0.4$ so

$$V = 11.4 \times (0.5/0.4)^{1/2} = \underline{\underline{12.7 \text{ m/s}}}$$

The drag coefficient will not change with further iterations.

11.46 **Information and assumption**

provided in problem statement

Find

terminal velocity of rock falling in water

Solution

$$\begin{aligned} W_{\text{air}} &= V\gamma_{\text{rock}} \\ 45 &= V\gamma_{\text{rock}} \\ F_{\text{buoy}} &= (45-10) = V\gamma_{\text{water}} = V \times 9,790 \end{aligned}$$

Solving for γ_{rock} and d: $\gamma_{\text{rock}} = 12,587 \text{ N/m}^3$ and $d = 0.190$ m. Under terminal velocity conditions

$$\begin{aligned} F_D + F_{\text{buoy}} &= W \\ F_D &= 45 - 35 = 10 \text{ N} \end{aligned}$$

$$\begin{aligned} F_D = C_D A_p \rho V_0^2/2 \text{ or } V_0^2 = 2F_D/(C_D A_p \rho);\ V_0^2 &= 2 \times 10/(C_D \times 0.0283 \times 998) \\ V_0 &= 0.841/\sqrt{C_D} \end{aligned}$$

or

$$\begin{aligned} V_0^2 &= 2F_D/(C_D A_p \rho) \\ V_0^2 &= 2 \times 10/(C_D \times 0.0283 \times 998) \end{aligned}$$

where $A_p = \pi d^2 = 0.0283 \text{ m}^2$. Assume $C_D = 0.4$ so

$$V_0 = 1.33 \text{ m/s}$$

Calculate the Reynolds number

$$\text{Re} = (VD/\nu) = 1.33(0.190)/10^{-6} = 2.53 \times 10^5$$

Try $C_D = 0.41,\ V_0 = 1.31$ m/s, Re $= 2.45 \times 10^5$. There will be no change with further iterations so

$$V = \underline{\underline{1.31 \text{ m/s}}}$$

11.47 **Information and assumption**

Assume $\rho = 1.2$ kg/m^3
provided in problem statement

Find

initial deceleration of aircraft.

Solution

$$F_D = C_D A_p \rho V_0^2/2 = Ma$$

then

$$a = C_D A_p \rho V_0^2/(2M)$$

where $M = 20,000/32.2 = 621.1$ slugs. From Table 11.1 $C_D = 1.20$.

$$A_p = (\pi/4)D^2 = 113.1 \text{ ft}^2$$

Then

$$\begin{aligned} a &= 1.20 \times 113.1 \times 0.0024 \times 200^2/(2 \times 621.1) \\ &= \underline{\underline{10.5 \text{ ft/s}^2}} \end{aligned}$$

11.48 **Information and assumption**

Assume $\rho = 1.2$ kg/m^3
provided in problem statement

Find

descent rate of paratrooper.

Solution

From Table 11.1 $C_D = 1.20$

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ V_0 &= \sqrt{2F_D/(C_D A_p \rho)} \\ &= \sqrt{2 \times 800/(1.2 \times (\pi/4) \times 49 \times 1.2)} = \underline{\underline{5.37 \text{ m/s}}} \end{aligned}$$

11.49 **Information and assumption**

Assume $\rho = 1000 \text{ kg/m}^3$
provided in problem statement

Find

terminal velocity of a cylinder of wood.

Solution

$$F_{\text{buoy}} = V\gamma_{\text{water}} = 0.80 \times (\pi/4) \times 0.20^2 \times 9,810 = 246.5 \text{ N}$$

Then the drag force is

$$F_D = F_{\text{buoy}} - W = 246.5 - 200 = 46.5 \text{ N}$$

From Table 11-1 $C_D = 0.87$. Then

$$\begin{aligned} 46.5 &= C_D A_p \rho V_0^2/2 \\ V_0 &= \sqrt{(46.50 \times 2)/(0.87 \times (\pi/4) \times 0.2^2 \times 1,000} \\ &= \underline{\underline{1.84 \text{ m/s upward}}} \end{aligned}$$

11.50 **Information and assumption**

Assume $\rho = 1000$ kg/m^3
provided in problem statement

Find

terminal velocity in water.

Solution

From Table 11-1, $C_D = 0.81$

$$\begin{aligned}
F_D &= C_D A_p \rho V_0^2/2 \\
A_p &= (2)(L\cos 45^\circ)(L) = 1.414L^2 \\
F_D &= W - F_{\text{buoy}} \\
&= 19.8 - 9,810L^3 = 19.8 - 9,810 \times (10^{-1})^3 = 10 \text{ N} \\
10 &= (0.81)(1.414 \times 10^{-2})(1,000)(V_0^2)/2 \\
V_0 &= \underline{\underline{1.32 \text{ m/s}}}
\end{aligned}$$

11.51 **Information and assumption**

Assume $T = 15°C$ so $\rho_{\text{air}} = 1.22$ kg/m^3; $\rho_{\text{He}} = 0.169$ kg/m^3
provided in problem statement

Find

terminal velocity in air.

Solution

$$\begin{aligned} V_0 &= (2F_D/(C_D A\rho))^{1/2} \\ F_{\text{net}} &= F_D - W_{\text{balloon}} - W_{\text{helium}} + F_{\text{buoy}} = 0 \\ F_D &= +0.05 - (1/6)\pi D^3(\gamma_{\text{air}} - \gamma_{\text{He}}) \\ &= +0.5 - (1/6)\pi \times (0.30)^3 9.81(\rho_{\text{air}} - \rho_{\text{He}}) \end{aligned}$$

$$F_D = +0.05 - (1/6)\pi(0.30)^3 \times 9.81(1.22 - 0.169) = 0.099 \text{ N}$$

Assume $C_D \approx 0.40$ Then

$$V_0 = ((2 \times 0.099/(0.40 \times (\pi/4) \times 0.3^2 \times 1.3))^{1/2} = 2.32 \text{ m/s}$$

Check Re and C_D:

$$\text{Re} = VD/\nu = 2.32 \times 0.3/(1.46 \times 10^{-5}) = 5 \times 10^4$$

From Table 11-11, $C_D \approx 0.40$ so no further iterations are necessary.

$$V_0 = \underline{\underline{2.32 \text{ m/s upward}}}$$

11.52 The equation of motion is obtained by equating the mass times acceleration to the forces acting on the balloon.

$$m\frac{dv}{dt} = -F_D - W + F_B$$

The mass of the balloon is the sum of the mass associated with the "empty" weight, W_0, and the helium.

$$\begin{aligned} m &= \frac{W_0}{g} + \rho_H \forall \\ &= \rho_H \forall (1 + \frac{W}{\rho_H \forall g}) \end{aligned}$$

The drag force can be expressed as

$$F_D = \frac{1}{2}\rho v^2 C_D \frac{\pi}{4} d^2 = \frac{C_D \text{Re}}{24} 3\pi\mu d v$$

The buoyant force is

$$F_B = \rho_a g \forall$$

Substituting the values into the equation of motion, we have

$$m\frac{dv}{dt} = -\frac{C_D \text{Re}}{24} 3\pi\mu d v - mg + \rho_a g \forall$$

Dividing through by the mass, we get

$$\frac{dv}{dt} = -\frac{C_D \text{Re}}{24} \frac{18\mu}{\rho_H d^2} \frac{1}{F} v - g + \frac{\rho_a}{\rho_H} g \frac{1}{F}$$

where

$$F = 1 + \frac{W}{\rho_H \forall g}$$

The density of helium at 23°C and atmospheric pressure is 0.1643 kg/m^3. Substituting in the values, the equation becomes

$$\begin{aligned} \frac{dv}{dt} &= -\frac{C_D \text{Re}}{24} \frac{0.0219}{3.19} v - 11.81(1 - \frac{\rho_a}{0.1643 \times 3.19}) \\ &= -\frac{C_D \text{Re}}{24} 0.00686 v - 11.81(1 - \frac{\rho_a}{0.524}) \end{aligned}$$

The value for $C_D\text{Re}/24$ is obtained from Eq. 11.10.

$$\frac{C_D\text{Re}}{24} = 1 + 0.15\text{Re}^{0.687} + \frac{0.0175\text{Re}}{1 + 4.25 \times 10^4\text{Re}^{-1.16}}$$

The value for the air density is obtained from the relations for a standard atmosphere.

$$T = 296 - 5.87 \times 10^{-3}h$$

and

$$p = 101.3(1 - \frac{T}{294})^{5.823}$$

and the density is obtained from the ideal gas law.
This equation can be integrated using the Euler method

$$\begin{aligned} v_{n+1} &= v_n + \left(\frac{dv}{dt}\right)_n \Delta t \\ h_{n+1} &= h_n + 0.5(v_n + v_{n+1})\Delta t \end{aligned}$$

The time to climb to 5000 m is 3081 seconds or $\underline{\underline{51.3}}$ minutes. Other methods may lead to slightly different answers.

11.53 **Information and assumption**

provided in problem statement

Find

terminal velocity of balloon.

Solution

As in solution to Prob. 11.49:

$$
\begin{aligned}
F_D &= +W_{\text{balloon}} + W_{\text{He}} - F_{\text{buoy}} \\
F_D &= +0.01 - (1/6)\pi \times 1^3(\gamma_{\text{air}} - \gamma_{\text{air}} \times 1,716/12,419) \\
F_D &= +0.01 - (1/6)\pi \times 1^3 \times 0.0764(1 - 0.138) \\
F_D &= +0.010 - 0.0345 = 0.0245 \text{ lbf} \\
V_0 &= \sqrt{2F_D/(C_D A_p \rho)} = \sqrt{2 \times 0.0245/((\pi/4) \times 0.00237 C_D)} \\
&= \sqrt{26.3/C_D}
\end{aligned}
$$

Assume $C_D = 0.40$ Then

$$V_0 = \sqrt{26.3/0.4} = 8.1 \text{ ft/s upward}$$

Check Reynolds number and C_D.

$$\text{Re} = VD/\nu = 8.1 \times 1/(1.58 \times 10^{-4}) = 5.2 \times 10^4;\ C_D = 0.50$$

From Fig. 11-11, $C_D = 0.50$. Recalculate velocity

$$V_0 = \sqrt{26.3/0.5} = \underline{\underline{7.25 \text{ ft/s}}}$$

No further iterations are necessary.

11.54 **Information and assumption**

Assume $\rho = 1000 \text{ kg/m}^3$
provided in problem statement

Find

tension in rope to pull up anchor.

Solution

$$\begin{aligned} \sum F_y &= 0 \\ T - W - F_D - F_{\text{buoy.}} &= 0 \end{aligned}$$

Then

$$\begin{aligned} T &= W + F_D - F_{\text{buoy.}} \\ T &= (\pi/4) \times 110 \times 0.3(15,000 - 9,810) + C_D(\pi/4) \times 0.3^2 \times 1,000 \times 1.0^2/2 \end{aligned}$$

From Table 11-1 $C_D = 0.90$. Then

$$T = 110 + 31.8 = \underline{\underline{141.8 \text{ N}}}$$

11.55 **Information and assumption**

Assume $\nu = 10^{-5}$ ft^2/s
provided in problem statement

Find

terminal velocity of spherical pebble.

Solution

Assume $C_D = 0.5$

$$\begin{aligned} V_0 &= [(\gamma_s - \gamma_w)(4/3)D/(C_D\rho_w)]^{1/2} \\ V_0 &= [62.4(2.94 - 1)(4/3) \times (1/(4 \times 12))/(0.5 \times 1.94)]^{1/2} \\ V_0 &= 1.86 \text{ ft/s} \end{aligned}$$

Check Reynolds number and C_D.

$$\text{Re} = 1.86 \times (1/48)/10^{-5} = 3.8 \times 10^3$$

From Fig. 11-11 $C_D = 0.4$. Then

$$V_D = 1.86 \times (0.5/0.4)^{1/2} = \underline{\underline{2.08 \text{ ft/s}}}$$

No further iterations are necessary.

11.56 **Information and assumption**

provided in problem statement

Find

terminal velocity of a ball in water

Solution

$$
\begin{aligned}
F_D &= W - F_{\text{buoy}} \\
F_D &= 15 - 9,810 \times (1/6)\pi D^3 = 15 - 9,810 \times (1/6)\pi \times 0.1^3 \\
&= 9.86 \text{ N}
\end{aligned}
$$

Buoyant force is less than weight, so ball will drop.

$$
\begin{aligned}
9.86 &= C_D(\pi D^2/4) \times 1,000V^2/2 \\
V &= \sqrt{9.86 \times 8/(\pi C_D \times 1,000 \times 0.1^2)} = 1.58/\sqrt{C_D}
\end{aligned}
$$

Assume $C_D = 0.4$. Then

$$V = 2.50 \text{ m/s}$$

Check Reynolds number and C_D.

$$\text{Re} = VD/\nu = 2.50 \times 0.1/(1.3 \times 10^{-6}) = 1.9 \times 10^5$$

From Fig. 11-11 $C_D = 0.48$. So

$$V = 1.58/\sqrt{0.48} = \underline{\underline{2.28 \text{ m/s downward}}}$$

11.57 **Information and assumption**

provided in problem statement

Find

ascent velocity of balloon.

Solution

$$
\begin{aligned}
0 &= -W_{\text{balloon}} - W_{\text{He}} + F_{\text{buoy}} + F_D \\
F_D &= +3 - (1/6)\pi D^3(\gamma_{\text{air}} - \gamma_{\text{He}}) \\
&= +3 - (1/6)\pi \times 2^3 \times \gamma_{\text{air}}(1 - 287/2,077) \\
&= +3 - (1/6)\pi \times 8 \times 1.225(1 - 0.138) \\
&= +3 - 4.422 = -1.422 \text{ N}
\end{aligned}
$$

Then

$$
\begin{aligned}
F_D &= C_D A_p \rho V_0^2/2 \\
V_0 &= \sqrt{1.422 \times 2/((\pi/4) \times 2^2 \times 1.225 C_D)} \\
&= \sqrt{0.739/C_D}
\end{aligned}
$$

Assume $C_D = 0.4$ then

$$V_0 = \sqrt{0.739/0.4} = 1.36 \text{ m/s}$$

Check Reynolds number and C_D

$$\text{Re} = VD/\nu = 1.36 \times 2/(1.46 \times 10^{-5}) = 1.86 \times 10^5$$

From Fig. 11-11 $C_D = 0.42$ so

$$V_0 = \sqrt{0.739/0.42} = \underline{\underline{1.33 \text{ m/s upward}}}$$

No further iterations are necessary.

11.58 **Information and assumption**

provided in problem statement

Find

diameter of meteor

Solution

$$\begin{aligned} F_D &= W \\ C_D A_p \rho V_0^2/2 &= W \\ C_D A_p k p M^2/2 &= W \\ \therefore A_p &= W \times 2/(C_D k p M^2) \end{aligned}$$

From Fig. 11-12 $C_D = 0.80$

$$\begin{aligned} \pi D^2/4 &= (3,000\pi D^3/6)(9.81)(2)/((0.8)(20 \times 10^3)(1.4)(1)) \\ D &= \underline{\underline{0.80 \text{ m}}} \end{aligned}$$

11.59 **Information and assumption**

provided in problem statement

Find

characteristics of sphere falling in water.

Solution

$$F_D = C_D A_p \rho V_0^2/2$$
$$(\gamma_s - \gamma_w)\pi d^3/6 = C_D(\pi/4)d^2 \times 998 V_0^2/2$$

Assume $C_D = 0.50$. Then

$$\gamma_s = (93.56/d) + \gamma_w$$

Now determine values of γ_s for different d values. Results are shown below for a C_D of 0.50

d(cm)	10	15	20
γ_s(N/m^3)	10,725	10,413	10,238

$Re = VD/\nu = 0.5 \times 0.1/10^{-6} = 5 \times 10^4$
$C_D = 0.5$ O.K.

11.60 **Information and assumption**

provided in problem statement

Find

lift force on sphere

Solution

$$\begin{aligned} F_L &= C_L A_p \rho V_0^2/2 \\ r\omega/V_0 &= (0.15)(60)/3 = 3 \end{aligned}$$

From Fig. 11-17 $C_L = 0.45$

$$\begin{aligned} F_L &= (0.45)(\pi/4)(0.3^2)(1.94)(3^2)/2 \\ F_L &= \underline{\underline{0.28 \text{ lbf}}} \end{aligned}$$

11.61 It will "break" toward the north. $\underline{\underline{\text{a)}}}$ is the correct answer

11.62 **Information and assumption**

Assume $T = 70°F$; then $\rho = 0.0023$ slugs/ft^3
provided in problem statement

Find

deflection of ball from original path.

Solution

$$\begin{aligned} V_0 &= 85 \text{ mph} = 125 \text{ ft/s} \\ r\omega/V_0 &= (9/(12 \times 2\pi)) \times 35 \times 2\pi/125 = 0.21 \end{aligned}$$

From Fig. 11-17 $C_L = 3 \times 0.05 = 0.15$

$$F_L = C_L A \rho V_0^2/2 = 0.15 \times (9/12\pi)^2 \times (\pi/4) \times 0.0023 \times 125^2/29 = \underline{\underline{0.121 \text{ lbf}}}$$

Deflection will be $\delta = 1/2\ at^2$ where a is the acceleration

$$\begin{aligned} a &= F_L/M \\ t &= L/V_0 = 60/125 = 0.48 \text{ s} \\ a &= F_L/M = 0.100/((5/16)/(32.2)) = 12.4 \text{ ft/s}^2 \end{aligned}$$

Then

$$\delta = (1/2) \times 12.4 \times 0.48^2 = \underline{\underline{1.43 \text{ ft}}}$$

11.63 Correct choice is force vector $\underline{\underline{\text{a)}}}$.

11.64 **Information and assumption**

From Table A.3 $\rho = 0.83$ kg/m^3 and $\nu = 2.8 \times 10^{-5}$ m^2/s
provided in problem statement

Find

range of airspeeds for popper operation.

Solution

Range: Before corn is popped, it should not be thrown out by the air, so let

$$V_{\max} = (2F_D/C_D A_p \rho_{\text{air}})^{1/2}$$

where F_D =weight of unpopped corn$= 0.15 \times 10^{-3} \times 9.81$ N=0.00147 N. The cross-section area of the kernels is

$$A_p = (\pi/4) \times (0.006)^2 \text{ m}^2$$

Assume $C_D \backsimeq 0.4$. Then

$$V_{\max} = [2 \times 0.15 \times 10^{-3} \times 9.81/(0.40 \times (\pi/4)(0.006)^2 \times 1.2)]^{1/2} = \underline{\underline{18 \text{ m/s}}}$$

Check Re and C_D:

$$\text{Re} = VD/\nu = 14.7 \times 0.006/() = 3 \times 10^3$$

From Fig. 11-11 $C_D \backsimeq 0.4$ so solution is valid.

For minimum velocity let popped corn be suspended by stream of air. Assume only that diameter changes. So

$$V_{\min} = V_{\max} \times (A_u/A_p)^{1/2} = V_{\max}(D_u/D_p)$$

where D_p = diameter of popped corn and D_u = diameter of unpopped corn

$$V_{\min} \backsimeq (6/18) \times V_{\max} = \underline{\underline{6 \text{ m/s}}}$$

11.65 An American flag is 1.9 times as long as it is high. Thus $A = 6^2 \times 1.9 = 68.4$ ft^2.Assume

$$\begin{aligned} T &= 60^\circ F, \rho = 0.00237 \text{ slugs/ft}^3 \\ V_0 &= 100 \text{ mph } = 147 \text{ ft/s} \end{aligned}$$

Compute drag force on flag

$$\begin{aligned} F_D &= C_D A \rho V_0^2/2 \\ &= 0.14 \times 68.4 \times 0.00237 \times 147^2/2 \\ &= 244 \text{ lbf} \end{aligned}$$

Make the flag pole of steel using one size for the top half and a larger size for the bottom half. To start the determination of d for the top half, assume that the pipe diameter is 6 in. Then

$$\begin{aligned} F_{\text{on pipe}} &= C_D A_p \rho V_0^2/2 \\ \text{Re} = VD/\nu &= 147 \times 0.5/(1.58 \times 10^{-4}) \\ &= 4.7 \times 10^5 \end{aligned}$$

With an Re of $4.7 \times 10^5, C_D$ may be as low as 0.3 (Fig. 11-5); however, for conservative design purposes, assume $C_D = 1.0$. Then

$$\begin{aligned} F_{\text{pipe}} &= 1 \times 50 \times 0.5 \times 0.00237 \times 147^2/2 = 640 \text{ lbf} \\ M &= 244 \times 50 \times 12 + 640 \times 25 \times 12 = 338,450 \text{ in.-lbf} \\ I/c &= M/\sigma \\ I/c &= 338,450/30,000 = 11.28 \text{ in}^3 \end{aligned}$$

Assume allowable stress is $30,000$ psi.

$$I/C = 338,450/30,000 = 11.28 \text{ in}^3$$

From a handbook it is found that a 6 in. double extra-strength pipe will be adequate.
Bottom half, Assume bottom pipe will be 12 in. in diameter.

$$\begin{aligned} F_{\text{flag}} &= 224 \text{ lbf} \\ F_{6 \text{ in.pipe}} &= 640 \text{ lbf} \\ F_{12 \text{ in.pipe}} &= 1 \times 50 \times 1 \times 0.00237 \times 147^2/2 = 1,280 \text{ lbf} \end{aligned}$$

$$\begin{aligned} M &= 12(244 \times 100 + 640 \times 75 + 1,280 \times 25) = 1,253,000 \text{ in.-lbf} \\ M_s &= 41.8 \text{ in.}^3 = I/c \end{aligned}$$

Handbook shows that 12 in. extra-strenght pipe should be adequate. Note: Many other designs are possible.

11.66 **Information and assumption**

provided in problem statement

Find

lift coefficient on plate.

Solution

Force normal to plate will be based upon the $C_{p,\text{net}}$, where $C_{p,\text{net}}$ is the average net C_p producing a normal pressure on the plate. For example, at the leading edge of the plate the $C_{p,\text{net}} = 2.0 + 1.0 = 3.0$. Thus, for the entire plate the average net $C_p = 1.5$.

Then

$$\begin{aligned} F_{\text{normal to plate}} &= C_{p,\text{net}} A_{\text{plate}} \rho V_0^2/2 \\ &= 1.5 A_{\text{plate}} \rho V_0^2/2 \end{aligned}$$

The force normal to V_0 is the lift force.

$$\begin{aligned} F_L &= (F_{\text{normal to plate}})(\cos 30^\circ) \\ C_L S \rho V_0^2/2 &= (1.5)(A_{\text{plate}})(\rho V_0^2/2)\cos 30^\circ \\ C_L &= 1.5\cos 30^\circ = \underline{\underline{1.30}} \end{aligned}$$

based on plan form area. However if C_L is to be based upon projected area where

$$\begin{aligned} A_{\text{proj}} &= A_{\text{plate}} \sin 30^\circ \text{ then} \\ C_L &= \underline{\underline{2.60}} \end{aligned}$$

11.67 **Information and assumption**

provided in problem statement

Find

the span of the wing

Solution

From Fig. 11-23 assume $C_L \approx 0.60$

$$\begin{aligned} F_L &= C_L A \rho V_0^2/2 \\ 2,000 &= (0.60)(4b)(0.0024)(200^2)/2 \\ b &= 17.4 \text{ ft} \\ b/c &= 17.4/4 = 4.34 \end{aligned}$$

From Fig. 11-23 $C_L = 0.59$. Recalculate the span

$$b = 17.4(0.60/0.59) = \underline{\underline{17.7 \text{ ft}}}$$

11.68 **Information and assumption**

provided in problem statement

Find

foil dimensions to support boat.

Solution

Use Fig. 11-23 for characteristics; $b/c = 4$ so $C_L = 0.55$

$$\begin{aligned}
F_L &= C_L A \rho V_0^2/2 \\
10,000 &= 0.55 \times 4c^2 \times (1.94/2) \times 3,600 \\
c^2 &= 1.30 \text{ ft} \\
c &= 1.14 \text{ ft} \\
b &= 4c = 4.56 \text{ ft}
\end{aligned}$$

Use a foil $\underline{\underline{1.14 \text{ ft wide} \times 4.56 \text{ ft long}}}$

11.69 Correct choice is (d) because C_L increases with increase in aspect ratio.

11.70 **Information and assumption**

provided in problem statement

Find

$$C_{Di} = C_L^2/(\pi(b^2/S))$$

Solution

In the equation for the induced drag coefficient (see above) the only variable for a given airplane is C_L; therefore, one must determine if C_L varies for the given conditions. If the airplane is in level flight the lift force must be constant. Because $F_L = C_L A\rho V^2/2$ it is obvious that C_L must decrease with increasing V. This would be accomplished by decreasing the angle of attack. If C_L decreases, then Eq. (11.19) shows that C_{Di} also must decrease. The correct answer is $\underline{\underline{\text{(b)}}}$.

11.71

$$W/S = \frac{1}{2}\rho C_L V^2$$

or

$$\begin{aligned}
C_L &= (2/\rho)(1/V^2)(W/S) \\
P &= F_D V = (C_{Do} + C_L^2/\pi\Lambda)(1/2)\rho V^3 S \\
P &= \frac{1}{2}\rho V^3 S C_{Do} + (4/\rho^2)(1/V^4)W^2S^2)(1/(\pi\Lambda))(\frac{1}{2}\rho V^3 S) \\
P &= \left[\frac{1}{2}V^3 C_{Do} + (2/\rho)(1/(\pi\Lambda V)(W^2/S^2)\right] S \\
dP/dV &= ((3/2)\rho V^2 C_{Do} - (2/\rho)(1(\pi\Lambda V^2))(W/S)^2)S
\end{aligned}$$

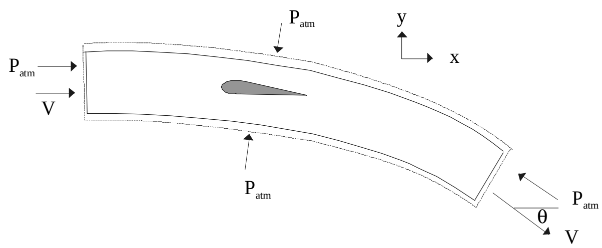
For minimum power $dP/dV = 0$ so

$$\begin{aligned}
(3/2)\rho V^2 C_{Do} &= (2/\rho)(1/(\pi\Lambda V^2)(W/S)^2 \\
V &= \left[\frac{4}{3}(W/S)^2(1/(\pi\Lambda\rho^2 C_{D0}))\right]^{1/4}
\end{aligned}$$

For $\rho = 1$ kg/m^3, $\Lambda = 10$, $W/S = 600$ and $C_{Do} = 0.2$

$$\begin{aligned}
V &= \left[\frac{4}{3}(600^2)(1/(\pi \times 10 \times 1^2 \times 0.02))\right]^{1/4} \\
&= \underline{\underline{29.6 \text{ ms}}}
\end{aligned}$$

11.72 Take the stream tube between sections 1 and 2 as a control volume and apply the momentum equation



For steady flow the momentum equation is

$$\sum F_y = \dot{m}_2 V_{2y} - \dot{m}_1 V_{1y}$$

Also $V_1 = V_2 = V$. The only F_y, is the force of the wing on the fluid in the control volume:

$$\begin{aligned} F_y &= (-V\sin\theta)\dot{m} = (-V\sin\theta)\rho VA \\ &= -\rho V^2 A\sin\theta \end{aligned}$$

But the fluid acting on the wing in the y direction is the lift F_L and it is the negative of F_y. So

$$\begin{aligned} F_L &= \rho V^2 A\sin\theta \\ C_L &= 2F_L/(\rho V^2 S) \end{aligned}$$

Eliminate F_L between the two equations yields

$$\begin{aligned} C_L &= 2\rho V^2 A\sin\theta/(\rho V^2 S) \\ C_L &= 2A\sin\theta/S \\ &= 2(\pi/4)b^2\sin\theta/S \\ C_L &= (\pi/2)\sin\theta(b^2/S) \end{aligned}$$

But $\sin\theta \approx \theta$ for small angles. Therefore

$$C_L = (\pi/2)\theta(b^2/S)$$

or

$$\begin{aligned} \theta &= 2C_L/(\pi b^2/S) \\ C_{Di}\rho V^2 S/2 &= (C_L\rho V^2 S/2)(\theta/2) \end{aligned}$$

Eliminating θ between the two equations gives

$$C_{Di}\rho V^2 S/2 = (C_L\rho V^2 S/2)(C_L/(\pi b^2/S))$$

$$\underline{\underline{C_{Di} = C_L^2/(\pi\Lambda)}}$$

11.73 **Information and assumption**

provided in problem statement

Find

landing speed and stalling speed.

Solution

$C_{L\,\max} = 1.40$ which is the C_L at stall so for stall

$$\begin{aligned} W &= C_{L\,\max} S \rho V_s^2/2 \\ &= 1.4 S \rho V_s^2/2 \end{aligned}$$

For landing

$$W = 1.2 S \rho V_L^2/2$$

But

$$V_L = V_s + 7$$

so

$$W = 1.2 A \rho (V_s + 7)^2/2$$

Therefore

$$1.2(V_s + 7)^2 = 1.4 V_s^2$$
$$V_s = \underline{\underline{87.6 \text{ m/s}}}$$
$$V_L = V_s + 7 = \underline{\underline{94.6 \text{ m/s}}}$$

11.74 **Information and assumption**

provided in problem statement

Find

total drag on wing and power to overcome drag

Solution

Calculate p and then ρ:

$$
\begin{aligned}
p &= p_0[T_0 - \alpha(z - z_0))/T_0]^{g/\alpha R} \\
p &= 101.3[(296 - (5.87 \times 10^{-3})(3,000))/296]^{(9.81/(5.87\times 10^{-3}\times 287))} = 70.1 \text{ kPa} \\
T &= 296 - 5.87 \times 10^{-3} \times 3,000 = 278.4 \text{ K}
\end{aligned}
$$

Then

$$\rho = p/RT = 70,100/(287 \times 278.4) = 0.877 \text{ kg/m}^3$$

$$C_L = (F_L/S)/(\rho V_0^2/2) = (1,200 \times 9.81/20)/(0.877 \times 60^2/2) = 0.373$$

Then

$$C_{D_i} = C_L^2/(\pi(b^2/S)) = 0.373^2/(\pi/(14^2/20)) = 0.0045$$

Then the total drag coefficient

$$C_D = C_{D_i} + 0.01 = 0.0145$$

Total wing drag

$$F_D == 0.0145 \times 20 \times 0.877 \times 60^2/2 = \underline{\underline{458 \text{ N}}}$$

Power

$$P = 60 \times 458 = \underline{\underline{27.5 \text{ kW}}}$$

11.75 **Information and assumption**

provided in problem statement

Find

speed at which cavitation begins and lift per unit length on foil

Solution

Cavitation will start at point where C_p is minimum, or in this case, where

$$\begin{aligned} C_p &= -1.95 \\ C_p &= (p - p_0)/(\rho V_0^2/2) \end{aligned}$$

Also

$$p_0 = 0.70 \times 9,810 \text{ Pa gage}$$

and for cavitation

$$\begin{aligned} p &= p_{\text{vapor}} = 1,230 \text{ Pa abs} \\ p_0 &= 0.7 \times 9,810 + 101,300 \text{ Pa abs} \end{aligned}$$

So

$$\begin{aligned} -1.95 &= [1,230 - (0.7 \times 9,810 + 101,300)]/(1,000 V_0^2/2) \\ V_0 &= \underline{\underline{10.5 \text{ m/s}}} \end{aligned}$$

By approximating the C_p diagrams by triangles, it is found that $C_{p_{\text{avg.}}}$ on the top of the lifting vane is approx. -1.0 and $C_{p_{\text{avg.,bottom}}} \approx +0.45$

Thus, $\Delta C_{p_{\text{avg.}}} \approx 1.45$. Then

$$\begin{aligned} F_{L/\text{length}} &= 1.45 \times 0.20 \times 1,000 \times (10.5)^2/2 \\ F_{L/\text{length}} &= \underline{\underline{16,000 \text{ N/m}}} \end{aligned}$$

11.76 The correct choice is $\underline{\underline{\text{(b)}}}$.

11.77

$$C_D/C_L = (C_{D_0}/C_L) + (C_L/(\pi\Lambda))$$
$$d/dC_L(C_D/C_L) = (-C_{D_0}/C_L^2) + (1/(\pi\Lambda)) = 0$$
$$C_L = \underline{\underline{\sqrt{\pi\Lambda C_{D_0}}}}$$
$$C_D = C_{D_0} + \pi\Lambda C_{D_0}/(\pi\Lambda) = 2C_{D_0}$$

Then

$$C_L/C_D = \underline{\underline{(1/2)\sqrt{\pi\Lambda/C_{D_0}}}}$$

11.78 **Information and assumption**

provided in problem statement

Find

time to reach sea level

Solution

$$
\begin{aligned}
\ell &= 1,000/(\sin 1.7^\circ) = 33,708 \text{ m} \\
F_L &= W = (1/2)\rho V^2 C_L S \\
200 \times 9.81 &= 0.5 \times 1.2 \times V^2 \times 0.8 \times 2.0;\ V = 14.3 \text{ m/s}
\end{aligned}
$$

Then

$$
t = 33,708 \text{ m}/(14.3 \text{ m/s}) = 2357 \text{ s} = \underline{\underline{39.3 \text{ min}}}
$$

11.79 **Information and assumption**

provided in problem statement

Find

the drag force

Solution

$$
\begin{aligned}
F_L &= C_L S \rho V_0^2/2 \\
F_L/S &= C_L \rho V_0^2/2 \\
\rho V_0^2/2 &= (2,000/0.4) = 5,000 \text{ N/m}^2 \\
\text{At } C_L &= 0.40,\ C_D = 0.02
\end{aligned}
$$

From Fig. 11-24 at $C_L = 0.40,\ C_D = 0.02$

$$F_D = C_D S \rho V_0^2/2 = (0.02)(5,000)(10) = \underline{\underline{1,000 \text{ N}}}$$

11.80 **Information and assumption**

provided in problem statement

Find

angle of attack and drag force on wing

Solution

$$
\begin{aligned}
W &= C_L S \rho V_0^2 / 2 \\
C_L &= W/(S \rho V_0^2/2) = (400)/((200)(0.002)(50^2)/2) = 0.80
\end{aligned}
$$

From Fig. 11-23 $C_D = 0.06$ and $\underline{\underline{\alpha = 7^\circ}}$

$$F_D = C_D S \rho V_0^2/2 = (0.06)(200)(0.002)(50^2)/2 = \underline{\underline{30 \text{ lbf}}}$$

11.81 There are several ways to address this design problem. One approach would be to consider the wing area and velocities necessary to meet the power constraint. That is,

$$225 = (0.05 + C_D)\frac{1}{2}(0.00238 \text{ slugs/ft}^3)V_0^3 S$$

Make plots of V_0 versus S with C_D as a parameter. Then use the constraint of the lift equaling the weight.

$$40 + 0.12 \times S = C_L\frac{1}{2}(0.00238 \text{ slugs/ft}^3)V_0^2 S$$

Make plots of V_0 versus S with C_L as a parameter. Where these curves intersect would give values where both constraints are satisfied. Next you can plot the curve for the pairs of C_D and C_L where the curves cross. You can also plot C_D versus C_L (drag polar) for the airfoil and see if there is a match. If there is no match, the airfoil will not work. If there is a match, you should try to find the configuration that will give the minimum weight.

Chapter Twelve

12.1 **Information and Assumptions**

provided in problem statement

Find

speed of sound in methane

Solution

$$c = \sqrt{kRT} = \sqrt{1.31 \times 518 \times 293} = \underline{\underline{446}} \text{ m/s}$$

12.2 **Information and Assumptions**

provided in problem statement

Find

speed of sound in helium

Solution

$$c = \sqrt{kRT} = \sqrt{1.66 \times 2077 \times (50 + 273)} = \underline{\underline{1,055}} \text{ m/s}$$

12.3 **Information and Assumptions**

provided in problem statement

Find

speed of sound in hydrogen

Solution

$$c=\sqrt{kRT}=\sqrt{1.41\times 24,677\times(460+68)}=\underline{\underline{4286}}\text{ ft/s}$$

12.4 **Information and Assumptions**

provided in problem statement

Find

difference in speed of sound

Solution

$$
\begin{aligned}
c_{\text{He}} &= \sqrt{(kR)_{\text{He}}T} = \sqrt{1.66 \times 2077 \times 293} = 1005 \text{ m/s} \\
c_{\text{N}_2} &= \sqrt{(kR)_{\text{N}_2}T} = \sqrt{1.40 \times 297 \times 293} = 349 \text{ m/s} \\
c_{\text{He}} - c_{\text{N}_2} &= \underline{\underline{656}} \text{ m/s}
\end{aligned}
$$

12.5 **Information and Assumptions**

provided in problem statement

Find

speed of sound for an isothermal process

Solution

$$c^2 = \partial p/\partial \rho;\ p = \rho RT$$

If isothermal, T=const.

$$\begin{aligned} \therefore \quad & \partial p/\partial \rho = RT \\ \therefore \quad & c^2 = RT \\ & \underline{\underline{c = \sqrt{RT}}} \end{aligned}$$

12.6 **Information and Assumptions**

provided in problem statement

Find

speed of sound in water

Solution

$$
\begin{aligned}
p - p_o &= E_V \ln(\rho/\rho_o) \\
c^2 &= \frac{\partial p}{\partial \rho} = \frac{E_v}{\rho} \\
c &= \sqrt{E_v/\rho} \\
c &= \sqrt{2.20 \times 10^9/10^3} = \underline{\underline{1,483}} \text{ m/s}
\end{aligned}
$$

12.7 **Information and Assumptions**

provided in problem statement

Find

surface temperature and airspeed behind shock

Solution

Total temperature will develop at exposed surface.
At $M_1 = 2.5$, $T/T_t = 0.444$ (from Table A.1)

$$T_t = (273 + 30)/0.444 = 547 \text{ K} = \underline{\underline{274\ ^{\circ}\text{C}}}$$

Airspeed behind the shock. From Table A.1 $M_2 = 0.513$, $T_2/T_1 = 2.138$. Then

$$\begin{aligned} V_2 &= M_2 c_2 \\ c_2 &= (kRT_2)^{0.5} \end{aligned}$$

where $k = 1.4$ and $R = 287$ J/kg/K

$$\begin{aligned} T_2 &= (273 + 30)(2.138) = 520 \text{ K} \\ c_2 &= (1.4(287)(520))^{0.5} = 457 \text{ m/s} \\ V_2 &= 0.513(475) = \underline{\underline{234}} \text{ m/s } = \underline{\underline{844}} \text{ km/hr.} \end{aligned}$$

12.8 **Information and Assumptions**

provided in problem statement

Find

temperature on nose

Solution

From Table A.1 $T/T_t = 0.5556$ at $M = 2.0$

$$T_t = (1/0.5556)(273) = \underline{\underline{491}}\text{ K} = 218\ ^{\text{o}}\text{C}$$

12.9 **Information and Assumptions**

provided in problem statement

Find

speed of aircraft, total temperature and speed for $M = 1$.

Solution

At 10,000 m, $c = \sqrt{(1.40)(287)(229)} = 303.3$ m/s

1. **a)** $V = (1.8)(303.3)(3,600/1,000) = \underline{\underline{1,965 \text{ km/hr}}}$

1. **b)** $T_t = 229(1 + ((1.4 - 1)/2) \times 1.8^2) = 377 \text{ K} = \underline{\underline{104\ ^{\circ}\text{C}}}$

   c) $p_t = (30.5)(1 + 0.2 \times 1.8^2)^{(1.4/(1.4-1))} = \underline{\underline{175 \text{ kPa}}}$

   d) $M = 1;\ V = c$

   $V = (303.3)(3,600/1,000) = \underline{\underline{1,092 \text{ km/hr}}}$

12.10 **Information and Assumptions**

provided in problem statement

Find

speed of aircraft at $T = -40^{\rm o}$C

Solution

$$
\begin{aligned}
c &= \sqrt{kRT} = \sqrt{(1.4)(287)(288)} = 340.2 \text{ m/s} \\
V &= 800 \text{ km/hr } = 222.2 \text{ m/s} \\
M &= 222.2/340.2 = 0.653 \\
c \text{ at altitude} &= \sqrt{(1.4)(287)(233)} = 306.0 \text{ m/s} \\
V &= \underline{\underline{199.8 \text{ m/s}{=}719.3 \text{ km/hr}}}
\end{aligned}
$$

At altitude

$$
\begin{aligned}
c &= \sqrt{(1.4)(287)(233)} = 306.0 \text{ m/s} \\
V &= \underline{\underline{200}} \text{ m/s}{=}\underline{\underline{719}} \text{ km/hr}
\end{aligned}
$$

12.11 **Information and Assumptions**

provided in problem statement

Find

wing loading

Solution

$$
\begin{aligned}
q &= (k/2)pM^2 = (1.4/2)(30)(0.95)^2 = 18.95 \text{ kPa} \\
W &= C_L q S = L \\
W &= L/S = C_L q = (0.05)(18.95) = 0.947 \text{ kPa} = \underline{\underline{947 \text{ Pa}}}
\end{aligned}
$$

12.12 **Information and Assumptions**

provided in problem statement

Find

pressure and temperature at stagnation point

Solution

$$\begin{aligned}
c &= \sqrt{kRT} = \sqrt{(1.4)(287)(293)} = 343 \text{ m/s} \\
M &= 250/343 = 0.729 \\
T_t &= (293)(1 + 0.2 \times (0.729)^2) = 293 \times 1.106 = 324 \text{ K} = \underline{\underline{51^\circ\text{C}}} \\
p_t &= (200)(1.106)^{3.5} = \underline{\underline{284.6 \text{ kPa}}}
\end{aligned}$$

12.13 **Information and Assumptions**

provided in problem statement

Find

mass flow rate through conduit

Solution

$$
\begin{aligned}
T_t &= T(1+(k-1)/2)M^2 \\
T &= 283/(1+0.2\times 0.5^2) = 270 \text{ K} \\
p &= 360/(1.05)^{3.5} = 303 \text{ kPa} \\
c &= [(1.4)(287)(270)]^{1/2} = 329.4 \text{ m/s} \\
V &= (0.5)(329.4) = 165.7 \text{ m/s} \\
\rho &= p/RT = 303\times 10^3/(287\times 270) = 3.91 \text{ kg/m}^3 \\
\dot{m} &= (164.7)(3.91)(0.0065) = \underline{\underline{4.18 \text{ kg/s}}}
\end{aligned}
$$

12.14 **Information and Assumptions**

provide in problem statement

Find

velocity, pressure and temperature

Solution

$$
\begin{aligned}
p_t &= 300 \text{ kPa} \\
T_t &= 200^\circ C = 473 \text{ K} \\
T &= 473/(1 + 0.2 \times 0.9^2) = 473/1.162 = \underline{\underline{407 \text{ K}}} \\
p &= 300/(1.162)^{3.5} = \underline{\underline{177.4 \text{ kPa}}} \\
c &= [(1.4)(260)(407)]^{1/2} = 384.9 \text{ m/s} \\
V &= (0.9)(384.9) = \underline{\underline{346.4 \text{ m/s}}}
\end{aligned}
$$

12.15 **Information and Assumptions**

provided in problem statement

Find

Mach number condensation will occur

Solution

$$\begin{aligned} T_t &= 300 \text{ K} \\ T &= 50 \text{ K} \\ T_0/T &= 1+((k-1)/2)M^2 \\ 300/50 &= 6 = 1+0.2\ M^2 \\ M &= 5 \end{aligned}$$

12.16 **Information and Assumptions**

provided in problem statement

Find

temperature, pressure, Mach number and mass flow rate.

Solution

$$\begin{aligned}
T_t &= 20^{\circ}\mathrm{C} = 293\ \mathrm{K} \\
P_t &= 500\ \mathrm{kPa} \\
V &= 250\ \mathrm{m/s} \\
c_pT + V^2/2 &= c_pT_0 \\
T &= T_t - V^2/(2c_p) = 293 - (250)^2/((2)(14,223)) = \underline{\underline{290.8\ \mathrm{K}}} \\
c &= \sqrt{kRT} = \sqrt{(1.41)(4,127)(290.8)} = 1,301\ \mathrm{m/s} \\
M &= 300/1,299 = \underline{\underline{0.231}} \\
p &= 500/[1 + (0.41/2) \times 0.231^2]^{(1.41/0.41)} = \underline{\underline{481.6\ \mathrm{kPa}}} \\
\rho &= p/RT = (481.6)(10^3)/(4,127 \times 289.8) = 0.403\ \mathrm{kg/m^3} \\
\dot{m} &= \rho AV = (0.403)(0.02)^2(\pi/4)(300) = \underline{\underline{0.038\ \mathrm{kg/s}}}
\end{aligned}$$

12.17 **Information and Assumptions**

provided in problem statement

Find

drag on the sphere

Solution

$$
\begin{aligned}
M &= 2 \\
p_t &= 600 \text{ kPa} \\
F_D &= C_D(1/2)\rho U^2 A \\
p &= p_t/[1+((k-1)/2)M^2]^{k/(k-1)} = 600/[1+0.2(2.5)^2]^{3.5} = 35.1 \text{ kPa} \\
(1/2)\rho U^2 &= kpM^2/2 = 1.4 \times 35.1 \times 2.5^2/2 = 153.6 \text{ kPa} \\
F_D &= (0.95)(153.6 \times 10^3)(0.01)^2(\pi/4) = \underline{\underline{11.45 \text{ N}}}
\end{aligned}
$$

12.18 **Information and Assumptions**

provided in problem statement

Find

values for pressure coefficient

Solution

$$
\begin{aligned}
C_p &= (p_t - p)\rho U^2/2 = (p_t - p)/kpM^2/2 = (2/kM^2)[(p_t/p) - 1] \\
&= \underline{\underline{2/(kM^2)[(1 + (k - 1/2)M^2)^{(k/(k-1))} - 1]}} \\
C_p(2) &= \underline{\underline{2.43}} \\
C_p(4) &= \underline{\underline{13.47}} \\
C_{p_{\text{inc.}}} &= \underline{\underline{1.0}}
\end{aligned}
$$

12.19

$$
\begin{aligned}
p_t/p &= [1+(k-1)M^2/2]^{k/(k-1)} \\
M &= \sqrt{(2/(k-1))[(p_t/p)^{(k=1)/k}-1]} \\
p_t/p &= 1+\varepsilon; (p_t/p)^{(k-1)/k} = (1+\varepsilon)^{(k-1)/k} = 1+((k-1)/k)\varepsilon + 0(\varepsilon^2) \\
(p_t/p)^{(k-1)/k}-1 &\backsimeq ((k-1)/k)\varepsilon + 0(\varepsilon^2)
\end{aligned}
$$

Neglecting higher order terms

$$
\begin{aligned}
M &= [(2/(k-1))((k-1)/k)\varepsilon]^{1/2} \\
M &= \underline{\underline{[(2/k)((p_t)/p)-1)]^{1/2} \text{ as } \varepsilon \to 0}}
\end{aligned}
$$

12.20 **Information and Assumptions**

provided in problem statement

Find

Mach number, pressure and temperature downstream of wave and entropy increase

Solution

$$
\begin{aligned}
c_1 &= \sqrt{kRT} = \sqrt{(1.4)(297)(223)} = 304.5 \text{ m/s} \\
M_1 &= 500/304.8 = 1.64 \\
M_2^2 &= [(k-1)M_1^2 + 2]/[2kM_1^2 - (k-1)] = [(0.4)(1.64)^2 + 2]/[(2)(1.4)(1.64)^2 - 0.4] \\
M_2 &= \underline{\underline{0.657}} \\
p_2 &= p_1(1 + k_1 M_1^2)/[(1 + k_1 M_2^2)] = (70)(1 + 1.4 \times 1.64^2)/(1 + 1.4 \times 0.657^2) \\
&= \underline{\underline{208 \text{ kPa}}} \\
T_2 &= T_1(1 + ((k-1)/2)M_1^2)/(1 + ((k-1)/2)M_2^2 \\
&= 223[1 + 0.2 \times 1.64^2]/[1 + 0.2 \times 0.657^2] = 315 \text{ K} = \underline{\underline{43^\circ\text{C}}} \\
\Delta s &= R\ell n[(p_1/p_2)(T_2/T_1)^{k/(k-1)}] \\
&= R[\ell n(p_1/p_2) + (k/(k-1))\ell n(T_2/T_1)] \\
&= 297[\ell n(70/208) + 3.5\ell n(315/223)] = \underline{\underline{38.9 \text{ J/kg K}}}
\end{aligned}
$$

12.21 **Information and Assumptions**

provided in problem statement

Find

Mach number, pressure and temperature downstream of shock wave

Solution

$$\begin{aligned}
M_2^2 &= [(k-1)M_1^2+2]/[2kM_1^2-(k-1)];\ M_2 = \underline{\underline{0.475}} \\
(T_2/T_1) &= [1+((k-1)/2)M_1^2]/[1+((k-1)/2)M_2^2] \\
&= (1+(0.2)(9))/(1+(0.2)(0.475)^2) = 2.679 \\
T_2 &= 505 \times 2.679 = 1,353^\circ R = \underline{\underline{893^\circ \mathrm{F}}} \\
p_2/p_1 &= (1+kM_1^2)/(1+kM_2^2) = (1+1.4\times 9)/(1+1.4\times(0.475)^2) = 10.33 \\
p_2 &= (10.33)(30) = \underline{\underline{\text{310 psia}}}
\end{aligned}$$

12.22 **Information and Assumptions**

provided in problem statement

Find

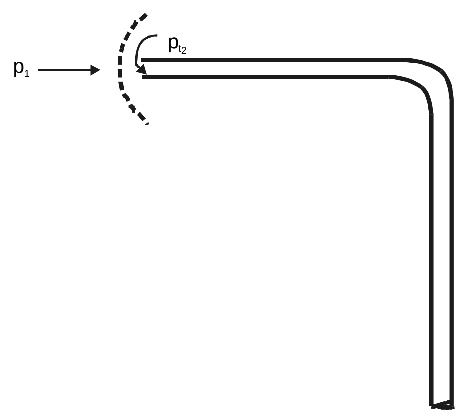
the Mach number

Solution

$$p_{t_2}/p_1 = 150/40 = 3.75 = (p_{t_2}/p_{t_1})(p_{t_1}/p_1)$$

Using compressible flow tables:

M	p_{t_2}/p_{t_1}	p_1/p_{t_1}	p_{t_2}/p_1
1.60	0.8952	0.2353	3.80
1.50	0.9278	0.2724	3.40
1.40	0.9582	0.3142	3.04
1.35	0.9697	0.3370	2.87



Therefore, interpolating, $M = \underline{\underline{1.59}}$

12.23 **Information and Assumptions**

from Table A.2 $k = 1.31$
provided in problem statement

Find

the downstream Mach number, static pressure, static temperature and density.

Solution

$$
\begin{aligned}
T &= 20^{\circ}C = 293 \text{ K;} \\
M_2^2 &= [(k-1)M_1^2 + 2]/[2kM_1^2 - (k-1)] = ((0.31)(9) + 2)/((2)(1.31)(9) - 0.31) = 0.2058 \\
M_2 &= 0.454 \\
p_2/p_1 &= (1 + kM_1^2)/(1 + kM_2^2) = (1 + 1.31 \times 9)/(1 + 1.31 \times 0.2058) = 10.07 \\
p_2 &= \underline{\underline{\text{1,007 kPa, abs}}} \\
T_2/T_1 &= [1 + ((k-1)/2)M_1^2]/[1 + ((k-1)/2)M_2^2] = 2.32 \\
T_2 &= (293)(2.32) = \underline{\underline{680}} \text{ K}=\underline{\underline{407}}^{\circ}\text{C} \\
\rho_2 &= p_2/(RT_2) = (1,007)(10^3)/((518)(680)) = \underline{\underline{2.86 \text{ kg/m}^3}}
\end{aligned}
$$

12.24 **Information and Assumptions**

from Table A.2 $k = 1.66$; $R = 2,077$ J/kg/K
provided in problem statement

Find

velocity upstream of wave

Solution

$$
\begin{aligned}
T_2 &= 100^\circ\text{C} = 373\text{ K} \\
M_1^2 &= [(k-1)M_2^2 + 2]/[2kM_2^2 - (k-1)] = 1.2949;\ M_1 = 1.12 \\
T_1/T_2 &= [1 + ((k-1)/2)M_2^2]/[1 + ((k-1)/2)M_1^2] = 0.897; T_1 = (0.897)(373) = 335\text{ K} \\
c_1 &= (1.66 \times 2,077 \times 335)^{1/2} = 1,075\text{ m/s} \\
V_1 &= (1,075)(1.12) = \underline{\underline{1,204}}\text{ m/s}
\end{aligned}
$$

12.25

$$M_2^2 = ((k-1)M_1^2 + 2)/(2kM_1^2 - (k-1))$$

Because

$$\begin{aligned} M_1 >> 1, (k-1)M_1^2 &> > 2 \\ 2kM_1^2 &> > (k-1) \end{aligned}$$

So in limit

$$\begin{aligned} M_2^2 &\rightarrow ((k-1)M_1^2)/2kM_1^2 = (k-1)/2k \\ \therefore &\quad M_2 \rightarrow \sqrt{(k-1)/2k} \end{aligned}$$

$$\begin{aligned} \rho_2/\rho_1 &= (p_2/p_1)(T_1/T_2) \\ &= ((1+kM_1^2)/(1+kM_2^2))(1+((k-1)/2)M_2^2)/(1+((k-1)/2)M_1^2) \end{aligned}$$

in limit $M_2^2 \rightarrow (k-1)/2k$ and $M_1 \rightarrow \infty$

$$\begin{aligned} \therefore \quad & \rho_2/\rho_1 \rightarrow [(kM_1^2)/((k-1)/2)M_1^2][(1+(k-1)^2/4k)/(1+k(k-1)/2k)] \\ & \rho_2/\rho_1 \rightarrow (k+1)/(k-1) \\ & M_2(\text{air}) = 0.378 \\ & \rho_2/\rho_1(\text{air}) = 6.0 \end{aligned}$$

12.26

$$
\begin{aligned}
M_2^2 &= [(k-1)M_1^2+2]/[2kM_1^2-(k-1)] \\
&= [(k-1)(1+\varepsilon)+2]/[2k(1+\varepsilon)-(k-1)] = [k+1+(k-1)\varepsilon]/[k+1+2k\varepsilon] \\
&= [1+(k-1)\varepsilon/(k+1)]/[1+(2k\varepsilon)/(k+1)] \\
&\approx [1+(k-1)\varepsilon/(k+1)][1-(2k\varepsilon)/(k+1)] \\
&\approx 1+(k-1-2k)\varepsilon/(k+1) \\
&\approx 1-\varepsilon \\
&\approx 1-(M_1^2-1) \\
&\approx \underline{\underline{2-M_1^2}}
\end{aligned}
$$

M_1	M_2	M_2 (Table A-1)
1.0	1.0	1.0
1.05	0.947	0.953
1.1	0.889	0.912
1.2	0.748	0.842

12.27 The computer program shows the flow is subsonic at the exit and the mass flow rate is 0.239 kg/s. The flow rate as a function of back pressure is given in the following table.

Back pressure, kPa	Flow rate, kg/s
80	0.243
90	0.242
100	0.239
110	0.229
120	0.215
130	0.194

One notes that the mass flow rate begins to decrease more quickly as the back pressure approaches the total pressure.

12.28 **Information and Assumptions**

provided in problem statement

Find

the mass flow rate

Solution

$$\begin{aligned} A_T &= 3 \text{ cm}^2 = 3 \times 10^{-4} \text{ m}^2 \\ p_t &= 300 \text{ kPa; } T_t = 20^\circ = 293 \text{ K} \\ p_b &= 90 \text{ kPa} \\ p_b/p_t &= 90/300 = 0.3 \end{aligned}$$

Because $p_b/p_t < 0.528$, sonic flow at exit.

$$\therefore \dot{m} = 0.685 p_t A_* / \sqrt{RT_t} = (0.685)(2 \times 10^5)(3 \times 10^{-4})/\sqrt{(287)(293)} = \underline{\underline{0.212 \text{ kg/s}}}$$

12.29 **Information and Assumptions**

from Table A.2 $k = 1.31$; $R = 518$ J/kgK
provided in problem statement

Find

mass flow rate of methane

Solution

$$\begin{aligned}
A_T &= 3\text{ cm}^2 = 3\times 10^{-4} m^2\\
A_p &= 12\text{ cm}^2 = 12\times 10^{-4} m^2\\
p_t &= 150\text{ kPa; } T_t = 303\text{ K}\\
p_b &= 100\text{ kPa;}\\
p_b/p_t &= 100/150 = 0.667\\
p_*/p_{t|_{\text{methane}}} &= (2/(k+1))^{k/(k-1)} = 0.544\\
p_b &> p_*,\ \text{subsonic flow at exit}
\end{aligned}$$

1)

$$\begin{aligned}
M_e &= \sqrt{(2/(k-1))[(p_t/p_b)^{(k-1)/k}-1]} = \sqrt{6.45[(1.5)^{0.2366}-1]} = 0.806\\
T_e &= 303\text{ K}/(1+(0.31/2)\times(0.806)^2) = 275\text{ K}\\
c_e &= \sqrt{(1.31)(518)(275)} = 432\text{ m/s}\\
\rho_e &= p_b/(RT_e) = 100\times 10^3/(518\times 275) = 0.702\text{ kg/m}^3\\
\dot{m} &= \rho_e V_e A_T = (0.702)(0.806)(432)(3\times 10^{-4}) = \underline{\underline{0.0733\text{ kg/s}}}
\end{aligned}$$

2) Assume the Bernoulli equation is valid, $p_t - p_b = (1/2)\rho V_e^2$

$$\begin{aligned}
V_e &= \sqrt{2(150-100)10^3/0.702} = 377\text{ m/s}\\
\dot{m} &= (0.702)(377)(3\times 10^{-4}) = \underline{\underline{0.0794\text{ kg/s}}}\\
\text{Error} &= 8.3\%\ \text{(too high)}
\end{aligned}$$

12.30 **Information and Assumptions**

provided in problem statement

Find

the total pressure

Solution

$$c_e = \sqrt{kRT_e} = \sqrt{(1.4)(287)(283)} = 337 \text{ m/s}$$

Assuming sonic flow at exit and exhausting to 100 kPa, one finds

$$\begin{aligned} \rho_e &= p/RT_e = 100 \times 10^3/(287)(283) = 1.23 \text{ kg/m}^3 \\ \dot{m} &= (1.23)(4 \times 10^{-4})(337) = 0.166 \text{ kg/s} \end{aligned}$$

Because the mass flow is too low, flow must exit sonically at pressure higher than the back pressure.

$$\begin{aligned} &\therefore \quad \rho_e = \dot{m}/(c_e A_e) = 0.25/((337)(4 \times 10^{-4})) = 1.85 \text{ kg/m}^3 \\ &\therefore \quad p_e = \rho_e RT_e = 1.50 \times 10^5 \text{ Pa} \\ &\therefore \quad p_t/p_e = ((k+1)/2)^{k/(k-1)} = (1.2)^{3.5} = 1.893 \\ &\therefore \quad p_t = 2.83 \times 10^5 \text{ Pa} = \underline{\underline{283 \text{ kPa}}} \end{aligned}$$

12.31 **Information and Assumptions**

from Table A.2 $k = 1.66$
provided in problem statement

Find

mass flow rate of helium

Solution

a) $p_t = 130$ kPa

If sonic at exit, $p_* = [2/(k+1)]^{k/(k-1)} p_t = 0.487 \times 130 \text{ kPa} = 63.3 \text{ kPa}$

$\therefore$ Flow must exit subsonically

$$\begin{aligned}
M_e^2 &= (2/(k-1))[(p_t/p_b)^{(k-1)/k} - 1] \\
&= 3.03[(130/100)^{0.4} - 1] = 0.335 \\
M_e &= 0.579 \\
\therefore T_e &= T_t/(1 + ((k-1)/2)M^2) = 301/(1 + (1/3)(0.335)) = 271 \text{ K} \\
\rho_e &= 100 \times 10^3/[(2,077)(271)] = 0.178 \text{ kg/m}^3 \\
\dot{m} &= \rho_e A_e V_e = (0.178)(12 \times 10^{-4})(0.579)\sqrt{(1.66)(2,077)(271)} \\
\dot{m} &= \underline{\underline{0.120 \text{ kg/s}}}
\end{aligned}$$

b)

$$\begin{aligned}
p_t &= 350 \text{ kPa} \\
&\therefore \; p_* = (0.487)(350) = 170 \text{ kPa} \\
&\therefore \; \text{Flow exits sonically} \\
\dot{m} &= 0.727 p_t A_*/\sqrt{RT_t} = (0.727)(350)10^3(12 \times 10^{-4})/\sqrt{2,077 \times 301} \\
\dot{m} &= \underline{\underline{0.386 \text{ kg/s}}}
\end{aligned}$$

12.32 **Information and Assumptions**

from Table A.2 $R = 287$ J/kgK; $k = 1.4$
provided in problem statement

Find

pressure required for isokinetic sampling

Solution

$$\begin{aligned}
\rho &= p/RT = 100 \times 10^3/(287)(873) = 0.399\ \text{kg/m}^3 \\
\dot{m} &= (0.399)(60)(\pi/4)(4 \times 10^{-3})^2 = \underline{\underline{0.000301\ \text{kg/s}}} \\
M &= 60/\sqrt{(1.4)(287)(873)} = 0.101 \\
p_t &= (100)[1 + (0.2)(0.101)^2]^{3.5} = 100.7\ \text{kPa} \\
T_t &= 875\ \text{K}
\end{aligned}$$

If sonic flow at constriction, then

$$\dot{m} = 0.685(100.7 \times 10^3)(\pi/4)(2 \times 10^{-3})^2\sqrt{(287)(875)} = \underline{\underline{0.000432\ \text{kg/s}}}$$

$\therefore$ flow must be subsonic at constriction.
Solution must be found iteratively.
Assume M at constriction:

$$\begin{aligned}
\rho_e &= \rho_t(1 + ((k-1)/2)M^2)^{(-1/(k-1))} = \rho_t(1 + 0.2M^2)^{-2.5} \\
c_e &= c_t(1 + ((k-1)/2)M^2)^{-1/2} = c_t(1 + 0.2M^2)^{-0.5} \\
\dot{m} &= \rho_e A_e c_e M_e = A_e M_e \rho_t c_t (1 + 0.2M^2)^{-3} \\
\rho_t &= (0.399)[1 + (0.2)(0.101)^2]^{2.5} = 0.401\ \text{kg/m}^3 \\
c_t &= \sqrt{(1.4)(287)(875)} = 593\ \text{m/s} \\
\therefore \dot{m} &= 7.47 \times 10^{-4} M(1 + 0.2M^2)^{-3}
\end{aligned}$$

M	$\dot{m} \times 10^4$
0.5	3.22
0.4	2.71
0.45	2.98
0.454	3.004
0.455	3.01 (correct flow rate)

$$\therefore p_b = (100.7)(1 + 0.2 \times 0.455^2)^{-3.5} = \underline{\underline{87.4\ \text{kPa}}}$$

12.33 The following results are obtained from the computer program for a Mach number of 2:

A/A_*	1.69	1.53	1.88
T/T_t	0.555	0.427	0.714
p/p_t	0.128	0.120	0.132
ρ/ρ_t	0.230	0.281	0.186
M_2	0.577	0.607	0.546
p_2/p_1	4.5	4.75	4.27

12.34 The following results are obtained for an area ratio of 5:

k	M_{subsonic}	$M_{\text{supersonic}}$
1.4	0.117	3.17
1.67	0.113	3.81
1.31	0.118	2.99

12.35 **Information and Assumptions**

from Table A.2 $k = 1.4$
provided in problem statement

Find

the area ratio and reservoir conditions

Solution

$$\begin{aligned} A/A_* &= (1/M)[(1 + ((k-1)/2)M^2)/((k+1)/2)]^{(k+1)/(2(k-1))} \\ &= (1/2.5)[(1 + 0.2 \times 2.5^2)/1.2]^3 = \underline{\underline{2.64}} \end{aligned}$$

From Table A.1, $p/p_t = 0.0585$; $T/T_t = 0.444$

$$\begin{aligned} \therefore \quad p_t &= 1.5 \text{ psia } /0.0585 = \underline{\underline{25.6 \text{ psia}}} \\ T_t &= 450^\circ R/0.444 = \underline{\underline{1{,}013^\circ\text{R}}} = \underline{\underline{553^\circ\text{F}}} \end{aligned}$$

12.36 **Information and Assumptions**

from Table .2 $k = 1.4$; $R = 297\,\mathrm{J/kgK}$
provided in problem statement

Find

the nozzle throat area

Solution

$$\begin{aligned}
M_e &= \sqrt{(2/(k-1))[(p_t/p_e)^{(k-1)/k}1]} = \sqrt{5[(1,000/30)^{0.286}-1]} = 2.94 \\
A_e/A_* &= (1/2.94)[(1+(0.2)(2.94)^2)/1.2]^3 = \underline{\underline{4.00}} \\
\dot{m} &= 0.685 p_t A_T/\sqrt{RT_t} \\
A_T &= \dot{m}\sqrt{RT_t}/(0.685 \times p_t) = 5 \times \sqrt{(297)(550)}/((0.685)(10^6)) \\
&= 0.00295 \text{ m}^2 = \underline{\underline{29.5}} \text{ cm}^2
\end{aligned}$$

12.37 **Information and Assumptions**

provided in problem statement

Find

the state of exit conditions

Solution

$$A/A_* = 4; p_t = 1.3 \text{ MPa } = 1.3 \times 10^6 \text{ Pa}; p_b = 35 \text{ kPa}; k = 1.4$$

From Table A1:

$$\begin{aligned} M_e &\approx 2.94 => p_e/p_t \approx 0.030 \\ \therefore\ & p_e = 39 \text{ kPa} \\ \therefore\ & p_e > p_b \underline{\underline{\text{(under expanded)}}} \end{aligned}$$

12.38 **Information and Assumptions**

provided in problem statement

Find

state of exit conditions

Solution

Running the program from Problem 12.33 with $k = 1.2$ and $A/A_* = 4$ gives $p_t/p = 23.0$. Thus the exit pressure is

$$p_e = \frac{1.3 \text{ MPa}}{23} = 56 \text{ kPa}$$

Therefore the nozzle is underexpanded.

12.39 **Information and Assumptions**

provided in problem statement

Find

static pressure and temperature at throat, exit conditions and pressure for normal shock at exit

Solution

a) $p = p_t$ in reservoir because $V = 0$ in reservoir
$p/p_t = 0.1278$ for $A/A_* = 1.688$ and $M_2 = 2$ (Table A.1)

$$p_t = p/0.1278 = 100/0.1278 = \underline{\underline{782.5 \text{ kPa}}}$$

b) Throat conditions for $M = 1$:

$$\begin{aligned} p/p_t &= 0.5283 \;\; T/T_t = 0.8333 \\ p &= 0.5283(782.5) = \underline{\underline{413.4}} \text{ kPa} \\ T &= 0.8333(17 + 273) = 242 \text{ K } = \underline{\underline{-31^\circ\text{C}}} \end{aligned}$$

c) Conditions for $p_t = 700$ kPa:

$$\begin{aligned} p/p_t &= 0.1278 \\ p &= 0.1278(700) = 89.5 \text{ kPa} \Longrightarrow 89.5 \text{ kPa} < 100 < \text{kPa} \end{aligned}$$

$\underline{\underline{\text{overexpanded}}}$ exit condition
d) p_t for normal shock at exit:
Assume shock exists at $M = 2$; we know $p_2 = 100$ kPa.
From table A.1: $p_2/p_1 = 4.5$

$$\begin{aligned} p_1 &= p_2/4.5 = 22.2 \text{ kPa} \\ p/p_t &= 0.1278 \\ p_t &= p/0.1278 = 22.2/0.1278 = \underline{\underline{173.9 \text{ kPa}}} \end{aligned}$$

12.40

$$
\begin{aligned}
q &= (k/2)pM^2 = (k/2)p_t[1 + ((k - 1\_/2)M^2]^{-k/(k-1)}M^2 \\
\ell nq &= \ell n(kp_t/2) - (k/(k-1))\ell n(1 + ((k-1)/2)M^2 + 2\ell nM \\
(\partial/\partial M)\ell nq &= (1/q)(\partial q/\partial M) = (-k/(k-1))[1/(1 + ((k-1)/2)M^2)][(k-1)M] + 2/M \\
0 &= [-kM]/[1 + ((k-1)/2)M^2] + (2/M) = [-kM^2 + 2 + (k-1)M^2/[(1 + ((k-1)/2)M^2)M] \\
0 &= 2 - M^2 \rightarrow M = \underline{\underline{\sqrt{2}}} \\
A/A_* &= (1/M)[1 + ((k-1)/2)M^2]/[(k+1)/2]^{(k+1)/2(k-1)} \\
&= (1/\sqrt{2})[(1 + 0.2(2))/1.2]^3 = \underline{\underline{1.123}}
\end{aligned}
$$

12.41 **Information and Assumptions**

provided in problem statement

Find

a) Mach number, pressure and density at exit, b) mass flow rate, c) thrust and d) chamber pressure for ideal expansion

Solution

$$A/A_* = (1/M_e)((1 + 0.1 \times M_e^2)/1.1)^{5.5} = 4$$

a) Solve for M by iteration:

M_e	A/A_*
3.0	6.73
2.5	3.42
2.7	4.45
2.6	3.90
2.62	4.0

$$\begin{aligned}
& \therefore \quad M_e = \underline{\underline{2.62}} \\
p_e/p_t &= (1 + 0.1 \times 2.62^2)^{-6} = 0.0434 \\
& \therefore \quad p_e = (0.0434)(1.2 \times 10^6) = \underline{\underline{52.1 \times 10^3 \text{ Pa}}} \\
T_e/T_t &= (1 + 0.1 \times 2.62^2)^{-1} = 0.593 \\
T_e &= (3,273 \times 0.593) = 1,941 \text{ K} \\
\rho_e &= p_e/(RT_e) = (52.1 \times 10^3)/(400 \times 1,941) = \underline{\underline{0.0671 \text{ kg/m}^3}} \\
c_e &= \sqrt{(1.2 \times 400 \times 1,941)} = 965 \text{ m/s} \\
V_e &= (965)(2.62) = \underline{\underline{2{,}528 \text{ m/s}}}
\end{aligned}$$

b)

$$\dot{m} = \rho_e A_e V_e = (0.0671)(4)(10^{-2})(2,528) = \underline{\underline{6.78 \text{ kg/s}}}$$

c)

$$T = (6.78)(2,528) + (52.1 - 25) \times 10^3 \times 4 \times 10^{-2} = \underline{\underline{18.22 \text{ kN}}}$$

d)

$$\begin{aligned}
p_t &= 25/0.0434 = \underline{\underline{576 \text{ kPa}}} \\
\dot{m} &= (25/52.1)(6.78) = 3.25 \text{ kg/s} \\
T &= (3.25)(2,528) = \underline{\underline{8.22 \text{ kN}}}
\end{aligned}$$

12.42 **Information and Assumptions**

provided in problem statement

Find

a) nozzle expansion ratio for ideal expansion and b) thrust if expansion ratio reduced by 10%

Solution

a)

$$\begin{aligned}
p_t/p_e &= (1+((k-1)/2)M^2)^{k/(k-1-} = (1+0.1M^2)^6 \\
M_e &= \sqrt{10[(p_t/p_e)^{1/6}-1]} = \sqrt{10[(2,000/100)^{1/6}-1]} = 2.54 \\
A_e/A_* &= (1/M_e)[(1+0.1M_e^2)/1.1]^{5.5} = \underline{\underline{3.60}} \\
T_e &= 3,300/(1+(0.1)(2.54)^2) = 2,006 \text{ K} \\
\rho_e &= 100\times 10^3/(400\times 2,006) = 0.125 \text{ kg/m}^3 \\
c_e &= \sqrt{(1.2)(400)(2006)} = 981 \text{ m/s} \\
\dot{m} &= \rho_e A_e V_e = (0.125)(3.38)(10^{-3})(981)(2.54) = 1.053 \text{ kg/s} \\
T &= (1.053)((51)(2.54) = \underline{\underline{2624 \text{ N}}}
\end{aligned}$$

b)

$$\begin{aligned}
A_e/A_* &= (0.9)(3.60) = 3.24 \\
3.42 &= (1/M_e)((1+0.1M_e^2)/1.1)^{5.5}
\end{aligned}$$

Solve by iteration:

M_e	A/A_*
2.4	3.011
2.5	3.420
2.45	3.204
2.455	3.228
2.458	3.241

$$\begin{aligned}
M_e &= 2.46 \\
p_e/p_t &= (1+0.1M_e^2)^{-6} = 0.0587 \\
p_e &= (0.0587)(2.0\times 10^6) = 117.4 \text{ kPa} \\
T_e &= 3,300/(1+0.1\times 2.46^2) = 2,056 \text{ K} \\
c_e &= \sqrt{(1.2)(400)(2056)} = 993 \text{ m/s} \\
T &= (1.053)(993)(2.46) + (117.4-100)\times 10^3\times 3.0432\times 10^{-3} \\
T &= \underline{\underline{2{,}573 \text{ N}}}
\end{aligned}$$

12.43 **Information and Assumptions**

provided in problem statement

Find

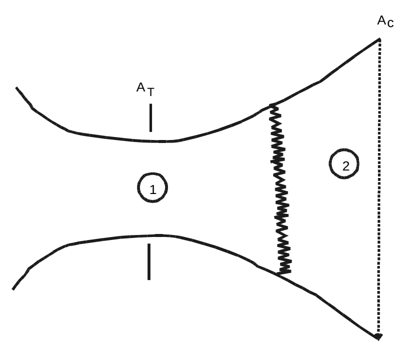
area ratio where shock occurs in nozzle

Solution

$$p_b/p_t = 0.5$$

Solution by iteration:

1. Choose M
2. Determine A/A^*
3. Find $p_{t_2}/p_{t_1} = A_{*_1}/A_{*_2}$
4. $(A_e/A_*)_2 = 4(A_{*_1}/A_{*_2})$
5. Find M_e
6. $p_e/p_{t_1} = (p_e/p_{t_2})(p_{t_2}/p_{t_1})$ and converge on $p_e/p_{t_1} = 0.5$



M	A/A_*	P_{t_2}/p_{t_1}	(A_e/A_*)	M_e	p_e/p_{t_1}	
2	1.69	0.721	2.88	0.206	0.7	
2.5	2.63	0.499	2.00	0.305	0.468	
2.4	2.40	0.540	2.16	0.28	0.511	$\therefore A/A_* = \underline{\underline{2.46}}$
2.43	2.47	0.527	2.11	0.287	0.497	
2.425	2.46	0.530	2.12	0.285	0.50	

12.44 **Information and Assumptions**

provided in problem statement

Find

area ratio and location of shock wave

Solution

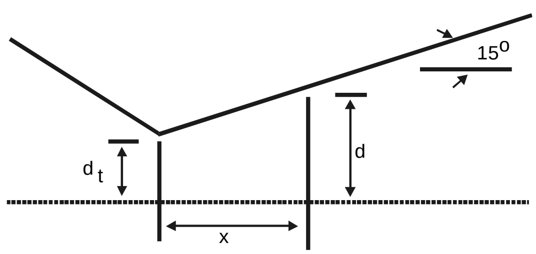
Use same iteration scheme as problem 12-43 but with $k = 1.2$ to find A/A_* of shock:

$$p_b/p_t = 100/250 = 0.4 \quad A_e/A_T(8/4)^2 = 4$$

M	A/A_*	P_{t_2}/p_{t_1}	$(A_e/A_*)_2$	M_e	p_e/p_{t_1}	
2.0	1.88	0.671	2.68	0.227	0.568	
2.4	3.01	0.463	1.85	0.341	0.432	
2.5	3.42	0.416	1.65	0.385	0.380	$\therefore A/A_* = \underline{\underline{3.25}}$
2.46	3.25	0.434	1.74	0.366	0.400	

From geometry: $d = d_t + 2 \times \tan 15^\circ$

$$\begin{aligned}
d/d_t &= 1 + (2x/d_t)\tan 15^\circ \\
A/A_* &= (d/d_t)^2 = 3.25 \\
&= [1 + (2x/d_t)(0.268)]^2 \\
&= [1 + (0.536x/d_t)]^2 \\
\therefore x/d_t &= 1.498 \\
x &= (1.498)(4) = \underline{\underline{5.99\text{ cm}}}
\end{aligned}$$



12.45 **Information and Assumptions**

from Table A.2 $k = 1.41$
provided in problem statement

Find

entropy increase

Solution

$$3 = (1/M)((1 + 0.205 \times M^2)/1.205)^{2.939}$$

Solve iteratively for M:

M	A/A_*
2.5	2.61
2.8	3.451
2.7	3.143
2.65	3.0

$$\begin{aligned}
M_1 &= 2.65 \\
M_2^2 &= ((k-1)M_1^2 + 2)/(2kM_1^2 - (k-1)) \\
M_2 &= 0.501 \\
p_1/p_1 &= (1 + kM_1^2)/(1 + kM_2^2) = 8.05 \\
p_t/p|_1 &= (1 + ((k-1)/2)M_1^2)^{k/(k-1)} = 21.48 \\
p_t/p|_2 &= 1.188 \\
p_{t_2}/p_{t_1} &= (p_{t_2}/p_2)(p_2/p_1)(p_1/p_{t_1}) = 0.445 \\
\Delta s &= R\ell n(p_{t_1}/p_{t_2}) = 4,127\ell n(1/0.445) = \underline{\underline{3{,}341 \text{ J/kgK}}}
\end{aligned}$$

12.46 **Information and Assumptions**

provided in problem statement

Find

Mach number, static pressure and stagnation pressure at station 3

Solution

$$\begin{aligned}
\text{from Table A-1 } M &= 2.1,\ A/A_* = 1.837,\ p/p_t = 0.1094 \\
A_* &= 100/1.837 = 54.4 \\
p_t &= 65/0.1094 = 594 \text{ kPa} \\
A_2/A_* &= 75/54.4 = 1.379 \rightarrow p_2/p_t = 0.1904 \rightarrow p_2 = 0.1904(594) = 113 \text{ kPa}
\end{aligned}$$

after shock, $M_2 = 0.630$; $p_2 = 3.377(113) = 382$ kPa

$$\begin{aligned}
A_2/A_* &= (1/M)((1+0.2M^2)/1.2)^3 = 1.155;\ p_t/p_2 = (1+0.2M^2)^{3.5} = 1.307 \\
A_* &= 75/1.155 = 64.9;\ p_t = 382(1.307) = \underline{\underline{499 \text{ kPa}}} \\
A_3/A_* &= 120/64.9 = 1.849;\ \text{from Table A.1, } M_3 = \underline{\underline{0.336}} \\
p_3/p_t &= 0.9245;\ p_3 = 0.9245(499) = \underline{\underline{461 \text{ kPa}}}
\end{aligned}$$

12.47 **Information and Assumptions**

provided in problems statement

Find

atmospheric pressure for shock position

Solution

$$\begin{aligned} M_1 &= 0.3;\ A/A_* = 2.0351;\ A_* = 200/2.0351 = 98.3 \text{ cm}^2 \\ p/p_t &= 0.9395;\ p_t = 400/0.9395 = 426 \text{ kPa};\ A_s/A_* = 120/98.3 = 1.2208 \end{aligned}$$

By interpolation from Table A-1:

$$\begin{aligned} M_{s1} &= 1.562;\ p_1/p_t = 0.2490 \rightarrow p_1 = 0.249(426) = 106 \text{ kPa} \\ M_{s2} &= 0.680;\ p_{s2}/p_1 = 2.679 \rightarrow p_{s2} = 2.679(106) = 284 \text{ kPa} \\ A_s/A_{*2} &= 1.1097 \rightarrow A_{*2} = 120/1.1097 = 108 \text{ cm}^2 \\ p_{s2}/p_{t2} &= 0.7338;\ p_{t2} = 284/0.7338 = 387 \text{ kPa} \\ A_2/A_{*2} &= 140/108 = 1.296 \rightarrow M_2 = 0.525 \\ p_2/p_{t2} &= 0.8288;\ p_2 = 0.8288(387) = \underline{\underline{321 \text{ kPa}}} \end{aligned}$$

12.48 Running the program for initial Mach number given a value of $\bar{f}\,(x_* - x)\,/D$ results in

$\bar{f}\,(x_* - x)\,/D$	$k = 1.4$		$k = 1.31$	
	M	p_M/p_*	M	p_M/p_*
1	0.508	2.10	0.520	2.02
10	0.234	4.66	0.241	4.44
100	0.0825	13.27	0.0854	12.57

12.49 **Information and Assumptions**

provided in problem statement

Find

pipe diameter

Solution

Assume $M_e = 1;\ p_e = 100$ kPa; $T_e = 373(0.8333) = 311$ k

$$\begin{aligned} c_e &= \sqrt{1.4(287)311} = 353 \text{ m/s; } \rho_e = 100 \times 10^3/(287 \times 311) = 1.12 \text{ kg/m}^3 \\ A &= \dot{m}/(\rho V) = 0.2/(1.12 \times 353) = 5.06 \times 10^{-4} \text{ m}^2 = 5.06 \text{ cm}^2 \\ D &= ((4/\pi)A)^{1/2} = 2.54 \text{ cm} \\ \text{Re} &= (353 \times 0.0254)/(1.7 \times 10^{-5}) = 5.3 \times 10^5 \rightarrow f = 0.0132 \\ f\Delta x/D &= (0.0132 \times 10)/0.0254 = 5.20 \end{aligned}$$

from Fig. 12.19 $M_1 = 0.302$
from Fig. 12.20 $p/p_* = 3.6$

$$\begin{aligned} p_1 &= 100(3.6) = 360 \text{ kPa } > 240 \text{ kPa} \\ \therefore & \text{ Case B} \end{aligned}$$

Solve by iteration.

M_e	T_e	c_e	V_e	ρ_e	$A \times 10^4$	Re $\times 10^{-5}$	M_1	p_1/p_e
0.8	331	365	292	1.054	6.51	4.54	0.314	2.55
0.7	340	369	259	1.026	7.54	4.11	0.322	2.18

By interpolation, for $p_1/p_e = 2.4,\ M_e = 0.76$

$$\begin{aligned} T_e &= 334 \text{ K; } c_e = 367 \text{ m/s; } V_e = 279 \text{ m/s; } \rho_e = 1.042 \text{ kg/m}^3 \\ A &= 6.89 \times 10^{-4} \text{ m}^2; \ D = 0.0296 \text{ m} = \underline{\underline{2.96 \text{ cm}}} \end{aligned}$$

12.50 **Information and Assumptions**

from Table A.2 $R = 1,716$ ft-lbf/slug
provided in problem statement

Find

length of pipe for sonic flow, pressure at pipe exit

Solution

$$\begin{aligned}
T &= 67^\circ\text{F} = 527^\circ\text{R} \\
c &= \sqrt{(1.4)(1,716)(527)} = 1,125 \text{ ft/sec} \\
M_1 &= 120/1,125 = 0.107 \\
\rho &= p/RT = (30 \times 144)/(1,716 \times 527) = 0.00478 \text{ slug/ft}^3 \\
\mu &= 3.8 \times 10^{-7} \text{ lbf-sec/ft}^2 \\
\text{Re} &= (120 \times 1/12 \times 0.00478)/(3.8 \times 10^{-7}) = 1.25 \times 10^5
\end{aligned}$$

From Figs. 10-8 and Table 10.2, $f = 0.025$

$$\begin{aligned}
\bar{f}(x_* - x_M)/D &= (1 - M^2)/kM^2 + ((k+1)/2k)\ell n[(k+1)M^2/(2 + (k-1)M^2)] = 62.0 \\
\therefore x_* - x_M = L &= (62.0)(D/\bar{f}) = (62.0 \times 1/12)/0.025 = \underline{\underline{207 \text{ ft}}}
\end{aligned}$$

from Eq. 12.79

$$\begin{aligned}
p_M/p_* &= 10.2 \\
p_* &= 30/10.2 = \underline{\underline{2.94 \text{ psia}}}
\end{aligned}$$

12.51 **Information and Assumptions**

from Table A.2 $R = 287$ J/kgK
provided in problem statement

Find

distance upstream where $M = 0.2$

Solution

$$\begin{aligned}
T_e &= 373/(1 + 0.2 \times 0.9^2) = 321 \text{ K} \\
c_e &= \sqrt{(1.4)(287)(321)} = 359 \text{ m/s} \\
V_e &= (0.9)(359) = 323 \text{ m/s} \\
\mu_e &= 2.03 \times 10^{-5} \text{N} \cdot \text{s/cm}^2 \\
\rho_e &= (100 \times 10^3)/(287 \times 321) = 1.085 \text{ kg/m}^3 \\
\text{Re} &= (232)(1.085)(3 \times 10^{-2})/(2.03 \times 10^{-5}) = 5.18 \times 10^5
\end{aligned}$$

from Figs. 10-8 and Table 10.2 $f = 0.0145$

$$\begin{aligned}
\bar{f}(x_* - x_{0.9})/D &= 0.014 \\
\bar{f}(x_* - x_{0.2})/D &= 14.5 \\
\therefore \bar{f}(x_{0.8} - x_{0.2})/D = 14.49 &= \bar{f}L/D \\
\therefore L = (14.49)(3 \times 10^{-2})/0.0145 &= \underline{\underline{30.0 \text{ m}}}
\end{aligned}$$

12.52 **Information and Assumptions**

provided in problem statement

Find

friction factor $\bar{f}$

Solution

By Eq. (12-75)

$$
\begin{aligned}
M &= 0.2 \\
\bar{f}(x_I - x_{0.2})/D &= 14.53 \\
M &= 0.6 \\
\bar{f}(x_* - x_{0.7})/D &= 0.2 \\
\bar{f}(x_{0.6} - x_{0.2})/D &= 14.33 \\
\bar{f} = 14.33(0.5)/(20 \times 12) &= \underline{\underline{0.0298}}
\end{aligned}
$$

12.53 **Information and Assumptions**

from Table A.2 $k = 1.4$; $R = 260$ J/kgK
provided in problem statement

Find

mass flow rat in pipe

Solution

Assume sonic flow at exit.

$$\begin{aligned} T_e &= 293/1.2 = 244 = -29°C \\ c_e &= V_e = \sqrt{(1.4)(260)(244)} = 298 \text{ m/s} \\ \nu_e &\backsimeq 1 \times 10^{-5} \text{ m}^2/\text{s (Fig. A3)} \\ \text{Re} &= (298 \times 2.5 \times 10^{-2})/(1 \times 10^{-5}) = 7.45 \times 10^5 \end{aligned}$$

From Figs. 10-8 an Table 10.2, $f = 0.024$

$$f(x_* - x_M)/D = (10 \times 0.024)/0.025 = 9.6$$

From Fig. 12-19 M at entrance $= 0.235$

$$\begin{aligned} p_M/p_* &= 4.6 \\ p_1 &= 460 \text{ kPa} > 300 \text{ kPa} \end{aligned}$$

Therefore flow must be subsonic at exit so $p_e/p_1 = 100/300 = 0.333$.
Use iterative procedure:

M_1	$\frac{f(x_*-x_M)}{D}$	Re $\times 10^5$	f	fL/D	$\frac{f(x_*-x_e)}{D}$	M_e	p_e/p_1
0.20	14.5	6.34	0.024	9.6	4.9	0.31	0.641
0.22	11.6	6.97	0.024	9.6	2.0	0.42	0.516
0.23	10.4	7.30	0.024	9.6	0.8	0.54	0.416
0.232	10.2	7.34	0.024	9.6	0.6	0.57	0.396
0.234	10.0	7.38	0.024	9.6	0.4	0.62	0.366
0.2345	9.9	7.40	0.024	9.6	0.3	0.65	0.348

For M_1 near 0.234, $p_M/p_* = 4.65$

$$\begin{aligned} p_e/p_* &= (p_M/p_*)(p_e/p_M) \\ p_e/p_* &= (4.65)(0.333) = 1.55 \end{aligned}$$

which corresponds to $M_e = 0.68$

$$\begin{aligned} \therefore \quad & T_e = 293/(1 + (0.2)(0.68)^2) = 268 \text{ K} \\ c_e &= \sqrt{(1.4)(260)(268)} = 312 \text{ m/s} \\ V_e &= 212 \text{ m/s} \\ \rho_e &= 10^5/(260 \times 268) = 1.435 \text{ kg/m}^3 \\ \therefore \quad & \dot{m} = (1.435)(212)(\pi/4)(0.025)^2 = \underline{\underline{0.149 \text{ kg/s}}} \end{aligned}$$

12.54 **Information and Assumptions**

provided in problem statement

Find

mass flow rate in pipe

Solution

From Prob. 12.53, we know flow at exit must be sonic since $p_1 > 460$ kPa. Use an iterative solution. Assume $f = 0.025$

$$\begin{aligned} \bar{f}(x_* - x_M)/D &= 10 \\ M &= 0.23 \\ T_t &= 293/(1+0.2(0.23)^2 = 290 \text{ K} \\ c_1 &= \sqrt{(1.4)(290)(260)} = 325 \text{ m/s} \\ \rho_1 &= (500\times 10^3)/(260\times 290) = 6.63 \text{ kg/m}^3 \end{aligned}$$

Assuming μ not a function of pressure

$$\begin{aligned} \mu_1 &= 1.79\times 10^{-5}\text{N}\cdot\text{s/m}^2 \\ \text{Re} &= (0.23)(325)(6.63)(2.5\times 10^{-2})/(1.79\times 10^{-5}) = 6.9\times 10^5 \end{aligned}$$

From Fig. 10.8 and Table 10.2

$$f = 0.024$$

Try

$$\begin{aligned} f &= 0.024 \\ f(x_* - x_M)/D &= 9.6;\ M = 0.235;\ T_t \backsimeq 290 \text{ K} \\ c_1 &= 325 \text{ m/s};\ \rho_1 = 6.63 \text{ kg/m}^3;\ \mu_1 \backsimeq 1.79\times 10^{-5} \text{ N}\cdot\text{s/m}^2; \\ \text{Re} &= 7\times 10^5 \end{aligned}$$

gives same f of 0.024. For $M = 0.235,\ p_M/p_* = 4.64$

$$\begin{aligned} p_* &= 107.8 \text{ kPa} \\ T_e &= 293/1.2 = 244 \text{ K} \\ c_e &= 298 \text{ m/s} \\ \rho_e &= (107.8\times 10^3)/(260\times 244) = 1.70 \text{ kg/m}^3 \\ \therefore \dot{m} &= (1.70)(298)(\pi/4)(0.025)^2 = \underline{\underline{0.248 \text{ kg/s}}} \end{aligned}$$

12.55 **Information and Assumptions**

provided in problem statement

Find

hose diameter

Solution

Assume $M_e = 1;\ p_e = 7$ psia

$$
\begin{aligned}
T_e &= 560(0.8333) = 467^\circ R;\ c_e = \sqrt{1.4(1,776)467} = 1,077 \text{ ft/s} \\
\rho_e &= 7(144)32.2/(1,776 \times 467) = 0.039 \text{ lbm/ft}^3 \\
A &= \dot{m}/(\rho V) = 0.06/(0.039 \times 1,077) = 1.43 \times 10^{-3} \text{ ft}^2;\ D = 0.0425 \text{ ft} = 0.51 \text{ in.} \\
\text{Re} &= (1,077)(0.0425)(0.039)/(1.36 \times 10^{-7} \times 32.2) = 4.1 \times 10^5 \\
k_s/D &= 0.0117;\ f = 0.040 \\
f\Delta x/D &= (0.04 \times 10)/0.0426 = 9.40
\end{aligned}
$$

From Fig. 12-19 $M_1 = 0.24$. From Fig. 12-20, $p_1/p_* = 4.54$

$$p_1 = 31.8 \text{ psia} < 45 \text{ psia}$$

Therefore Case D applies so $M = 1$ at exit and $p_e > 7$ psia.
Solve by iteration:

M_1	T_1	V_1	ρ_1	D	Re $\times 10^{-5}$	f	M_1	p_e
0.24	553	281	0.212	0.0358	1.62	0.040	0.223	9.16
0.223	554	262	0.212	0.0371	1.56	0.040	0.223	9.16

$D = 0.0371$ ft $= \underline{\underline{0.445 \text{ inch}}}$

12.56 **Information and Assumptions**

provided in problem statement

Find

pressure,velocity and density at pipe inlet

Solution

Assuming viscosity of particle-laden flow is same as air,

$$
\begin{aligned}
c &= \sqrt{1.4(287)288} = 340 \text{ m/s} \\
M_e &= 50/340 = 0.147 \rightarrow \bar{f}(x_* - x_{0.147})/D = 29.2;\ p_e/p_* = 7.44 \\
\text{Re} &= 50(0.2)/(1.44 \times 10^{-5}) = 6.94 \times 10^5;\ k_s/D = 0.00025;\ f = 0.0158 \\
\bar{f}\Delta x/D &= [\bar{f}(x_* - x_M)/D] - [\bar{f}(x_* - x_{0.147})/D] \\
&= 0.0158 \times 120/0.2 = 9.48 \\
\bar{f}(x_* - x_M)/D &= 29.2 + 9.48 = 38.7 \rightarrow M_1 = 0.14;\ p_1/p_* = 7.81 \\
p_1/p_e &= (p_1/p_*)(p_*/p_e) = 7.81/7.44 = 1.050 \\
p_1 &= 1.05(100) = \underline{\underline{105 \text{ kPa}}} \\
V_1 &= 0.14(340) = \underline{\underline{47.6 \text{ m/s}}} \\
T_1 &= T_t/(1 + 0.2M_1^2) = 288/(1 + 0.2(0.14)^2) = 287 \\
\rho_1 &= (105 \times 10^3)/(287 \times 287) = \underline{\underline{1.27 \text{ kg/m}^3}}
\end{aligned}
$$

12.57 **Information and Assumptions**

provided in problem statement

Find

pressure 3 km downstream

Solution

$$\begin{aligned} c_1 &= \sqrt{1.31(518)320} = 466 \text{ m/s} \\ \rho_1 &= 10^6/(518 \times 320) = 6.03 \text{ kg/m}^3 \\ M_1 &= 20/466 = 0.043 \end{aligned}$$

By Eq. 12-75

$$\bar{f}(x_* - x_{0.043})/D = 407$$

and by Eq. 12-79

$$\begin{aligned} p_1/p_* &= 25.0 \\ \text{Re} &= 20(0.15)6.03/(1.5 \times 10^{-5}) = 1.2 \times 10^6;\ k_s/D = 0.00035 \\ f &= 0.0162;\ f\Delta x/D = 0.0162(3,000)/0.15 = 324 \\ [f(x_* - x_{0.043})/D] - [\bar{f}(x_* - x_M)/D] &= f\Delta x/D \\ f(x_* - x_M)/D &= 407 - 324 = 83 \rightarrow M_e = 0.093 \end{aligned}$$

By Eq. 12-79

$$\begin{aligned} p_e/p_* &= 11.5 \\ p_e &= (p_e/p_*)(p_*/p_1)p_1 = (11.5/25.0)10^6 = \underline{\underline{460 \text{ kPa}}} \end{aligned}$$

12.58 **Information and Assumptions**

from Table A.2 $R = 4,127$ J/kgK; $k = 1.41$; $\nu = 0.81 \times 10^{-4}\text{m}^2$
provided in problem statement

Find

pressure drop in pipe

Solution

Speed of sound at entrance $= \sqrt{(1.41)(4,127)(288)} = 1,294$ m/s

$$\therefore \quad M = 200/1,294 = 0.154$$
$$\therefore \quad kM^2 = .0334; \ \sqrt{k}M = 0.183$$

Reynolds number = (200) (0.1)/($0.81{\times}10^{-4}$) = $2.5{\times}10^5$
From Fig. 10-8 and Table 10.2 $f = 0.018$
At entrance $f(x_m - x_1)/D = \ell n(0.0334) + (1 - 0.0334)/0.0334 = 25.5$
At exit

$$\begin{aligned} f(x_m - x_2)/D &= f(x_M - x_1)/D + f(x_1 - x_2)/D = 25.5 - (0.018)(50)/0.1 \\ &= 25.5 - 9.0 = 16.5 \end{aligned}$$

From Fig. 12-22, $kM^2 = 0.05$ or $\sqrt{k}M = 0.2236$

$$\begin{aligned} p_2/p_1 &= (p_m/p_1)(p_2/p_m) = 0.183/0.2236 = 0.818 \therefore p_2 = 204.5 \text{ kPa} \\ \Delta p &= \underline{\underline{45.5 \text{ kPa}}} \end{aligned}$$

12.59 **Information and Assumptions**

from Table A.2 $R = 2,077$ J/kgK; $k = 1.66$; $\nu = 1.14 \times 10^{-4}$ m^2/s
provided in problem statement

Find

mass flow rate in pipe

Solution

$$\begin{aligned} c &= \sqrt{(1.66)(2.077)(288)} = 996 \text{ m/s} \\ p_2/p_1 &= 100/120 = 0.833 \end{aligned}$$

Iterative solution:

V_1	M_1	Re $\times 10^{-4}$	f	kM_1^2	$\frac{f(x_T - x_M)}{D}$	$\frac{f(x_T - x_e)}{D}$	kM_2^2	p_2/p_1
100	0.100	4.4	0.022	0.0166	55.1	11.1	0.0676	0.495
50	0.050	2.2	0.026	0.00415	234.5	182.5	0.0053	0.885
55	0.055	2.4	0.025	0.00502	192.9	149.2	0.006715	0.864
60	0.060	2.6	0.25	0.00598	161.1	111.1	0.008555	0.836
61	0.061	2.6	0.25	0.00618	155.8	105.8	0.00897	0.830
60.5	0.0605	2.6	0.25	0.006076	158.5	108.5	0.00875	0.833

$$\begin{aligned} \therefore \rho &= 120 \times 10^3/(2,077)(288) = 0.201 \text{ kg/m}^3 \\ \dot{m} &= (0.201)(60.6)(\pi/4)(0.05)^2 = \underline{\underline{0.0239 \text{ kg/s}}} \end{aligned}$$

12.60 The area of the test section is

$$A_T = 0.05 \times 0.05 = 0.0025 \text{ m}^2$$

From Table A.1, the conditions for a Mach number of 1.5 are

$$p/p_t = 0.2724, \quad T/T_t = 0.6897 \quad A/A_* = 1.176$$

The area of the throat is

$$A_* = 0.0025/1.176 = 0.002125 \text{ m}^2$$

Since the air is being drawn in from the atmosphere, the total pressure and total pressure are 293 K and 100 kPa. The static temperature and pressure at the test section will be

$$T = 0.6897 \times 293 = 202 \text{ K}, \;\; p = 0.2724 \times 100 = 27.24 \text{ kPa}$$

The speed of sound and velocity in the test section is

$$\begin{aligned} c &= \sqrt{kRT} = \sqrt{1.4 \times 287 \times 202} = 285 \text{ m/s} \\ v &= 1.5 \times 285 = 427 \text{ m/s} \end{aligned}$$

The mass flow rate is obtained using

$$\begin{aligned} \dot{m} &= 0.685 \frac{p_t A_*}{\sqrt{RT_t}} \\ &= 0.685 \frac{10^5 \times 0.002125}{\sqrt{287 \times 293}} \\ &= 0.502 \text{ kg/s} \end{aligned}$$

The pressure and temperature in the vacuum tank can be analyzed using the relationships for an open, unsteady system. The system consists of a volume (the vacuum tank) and an inlet coming from the test section. In this case, the first law of thermodynamics gives

$$m_2 u_2 - m_1 u_1 = m_{in}(h_{in} + v_{in}^2/2) +_1 Q_2$$

Assume that the heat transfer is negligible and that the tank is initially evacuated. Then

$$m_2 u_2 = m_2(h_{in} + v_{in}^2/2)$$

since $m_{in} = m_2$. Thus the temperature in the tank will be constant and given by

$$\begin{aligned} c_v T &= c_p T_{in} + v_{in}^2/2 \\ 717 \times T &= 1004 \times 202 + 427^2/2 \\ T &= 410 \text{ K} \end{aligned}$$

The continuity equation applied to the vacuum tank is

$$V\frac{d\rho}{dt} = \dot{m}$$

The density from the ideal gas law is

$$\rho = \frac{p}{RT}$$

which gives

$$V\frac{dp}{dt} = \dot{m}RT$$

or

$$V = \frac{\dot{m}RT}{dp/dt}$$

Assume the final pressure in the tank is the pressure in the test section. Thus the rate of change of pressure will be

$$\frac{dp}{dt} = \frac{27.24}{30} = 0.908 \text{ kPa/s}$$

The volume of the tank would then be

$$\begin{aligned} V &= \frac{0.502 \times 0.287 \times 410}{0.908} \\ &= 65 \text{ m}^3 \end{aligned}$$

This would be a spherical tank with a diameter of

$$D = \sqrt[3]{\frac{6V}{\pi}} = 5.0 \text{ m}$$

The tank volume could be reduced if the channel was narrowed after the test section to reduce the Mach number and increase the pressure. This would reduce the temperature in the tank and increase the required rate of pressure increase.

The tunnel would be designed to have a contour between the throat and test section to generate a uniform velocity profile. Also a butterfly valve would have to be used to open the channel in minium time.

12.61 A truncated nozzle is attached to a storage tank supplied by the compressor. The temperature and pressure will be measured in the tank. These represent the total conditions. The nozzles will be sonic provided that the tank pressure is greater than 14.7/0.528=33 psia (or 18 psig).

The density of air at standard conditions is

$$\rho = \frac{p}{RT} = \frac{14.7 \times 144}{1716 \times 520} = 0.00237 \text{ slugs/ft}^3$$

A mass flow rate of 200 scfm corresponds to

$$\dot{m} = 200 \times 0.00237/60 = 0.00395 \text{ slugs/s}$$

The flow rate is given by

$$\dot{m} = 0.685 \frac{p_t A_*}{\sqrt{RT_t}}$$

Using 120 psig and a flow rate of 200 scfm gives an throat area of

$$\begin{aligned} A_* &= \frac{\dot{m}\sqrt{RT_t}}{0.685 p_t} \\ &= \frac{0.00395 \times \sqrt{1716 \times 520}}{0.685 \times 134 \times 144} \\ &= 2.82 \times 10^{-4} \text{ ft}^2 \end{aligned}$$

This area corresponds to an opening of

$$\begin{aligned} D &= \sqrt{\frac{4}{\pi} \times 2.82 \times 10^{-4}} \\ &= 0.0189 \text{ ft} = 0.23 \text{ in} \end{aligned}$$

This would represent the maximum nozzle size. A series of truncated nozzles would be used which would yield mass flows of 1/4,1/2 and 3/4 of the maximum flow rate. The suggested nozzle diameters would be 0.11 in, 0.15 in and 0.19 in. Another point would be with no flow which represents another data point.

Each nozzle would be attached to the tank and the pressure and temperature measured. For each nozzle the pressure in the tank must exceed 18 psig to insure sonic flow in the nozzle. The mass flow rate would be calculated for each nozzle size and these data would provide the pump curve, the variation of pressure with flow rate. More data can be obtained by using more nozzles.

Chapter Thirteen

13.1 **Information and Assumptions**

A stagnation tube ($d = 1$ mm) is used to measure air speed

Find

Velocity such that the measurement error is $\leq 1\%$

Solution

Because $V = \sqrt{2\Delta p/(\rho C_p)}$, a 2% deviation in C_p from unity will yield a 1% error in V when the equation is applied by assuming $C_p = 1$.

Thus, find Re where $C_p = 1.02$. From Fig. 13-1, $\mathrm{Re} \approx 60$

Then

$$\begin{aligned} Vd/\nu &= 60 \\ V &= 60\nu/d \end{aligned}$$

where $\nu = 1.45 \times 10^{-5}$ m^2/s and $d = 0.001$ m. Thus

$$V = 60 \times 1.45 \times 10^{-5}/0.001 = \underline{\underline{0.87 \text{ m/s}}}$$

13.2 **Information and Assumptions**

A stagnation tube ($d = 1$ mm) is used to measure the speed of water

Find

Velocity such that the measurement error is $\leq 1\%$

Solution

From Prob. 13.1, Re $= 60$;

$$V = 60\nu/d$$

where $\nu = 10^{-6}$ m^2/s. Then

$$V = 60 \times 10^{-6}/0.001 = \underline{\underline{0.06 \text{ m/s}}}$$

13.3 **Information and assumptions**

A stagnation tube ($d = 2$ mm) is used to measure air speed. Manometer deflection is 1 mm-H_2O.

Find

Air Velocity: V

Solution

$$\rho_{\text{air}} = 1.25 \text{ kg/m}^3;\ \Delta h_{\text{air}} = 0.001 \times 1,000/1.25 = 0.80 \text{ m}$$

Then

$$V = \sqrt{2g\Delta h} = \underline{\underline{3.96 \text{ m/s}}}$$

Check C_p

$$\text{Re} = Vd/\nu = 3.96 \times 0.002/(1.41 \times 10^{-5}) = 563$$

$$\begin{aligned} C_p &\approx 1.001 \\ V &= 3.96/\sqrt{c_p} = 3.96/\sqrt{1.001} = \underline{\underline{3.96 \text{ m/s}}} \end{aligned}$$

13.4 **Information and assumptions**

A stagnation tube ($d = 2$ mm) is used to measure air speed ($V = 12$ m/s). For air, $\nu = 1.4 \times 10^{-5}$ m^2/s.

Find

Deflection on a water manometer: Δh

Determine C_p

$$\mathrm{Re} = Vd/\nu = 12 \times 0.002/(1.4 \times 10^{-5}) = 1,714$$

From Fig. 13.1 $C_p \approx 1.00$

Pressure drop calculation

$$\Delta p = \rho V^2/2$$

where $\rho = p/RT = 98,000/(287 \times (273 + 10)) = 1.21$ kg/m^3. Then

$$\begin{aligned}
\Delta p &= 9,810\Delta h = 1.21 \times 12^2/2 \\
\Delta h &= 8.88 \times 10^{-3} \text{ m} = \underline{\underline{8.88 \text{ mm}}}
\end{aligned}$$

13.5 **Information and assumptions**

A stagnation tube ($d = 2$ mm) is used to measure air speed. Stagnation pressure is $\Delta p = 5$ Pa. Air kinematic viscosity is 1.55×10^{-5}

Find

Error in velocity if $C_p = 1$ is used for the calculation

Calculate V

$$\begin{aligned} \rho &= p/RT = 100,000/(287 \times 298) = 1.17 \text{ kg/m}^3 \\ V &= \sqrt{2\Delta p/\rho} = \sqrt{2 \times 5/1.17} = 2.92 \text{ m/s} \end{aligned}$$

Check C_p

$$\text{Re} = Vd/\nu = 2.92 \times 0.002/(1.55 \times 10^{-5}) = 377$$

Thus, $C_p = 1.002$

$$\%error = (1 - 1/\sqrt{1.002}) \times 100 = \underline{\underline{0.1\%}}$$

13.6 **Information and assumptions**

A probe for measuring velocity of a stack gas is described in the textbook.
provided in problem statement

Find

Stack gas velocity: V_o

Solution

$$\begin{aligned} C_p &= 1.4 = \Delta p/(\rho V_0^2/2) \\ \text{Thus } V_0 &= \sqrt{2\Delta p/(1.4\rho)} \end{aligned}$$

where $\rho = p/RT = 100,000/(410 \times 573) = 0.426\ \text{kg/m}^3$

$$\Delta p = 0.01\ \text{ m} \times 9,810 = 98.1\ \text{Pa}$$

Then

$$V_0 = \sqrt{2 \times 98.1/(1.4 \times 0.426)} = \underline{\underline{18.1\ \text{m/s}}}$$

13.7 **Information and assumptions**

In 5 minutes, 10 kN of water flows into a weigh tank

Find

Discharge: Q

Solution

$$\gamma_{\text{water }20^{\circ}C} = 9,790 \text{ N/m}^3$$

Then

$$\dot{W} = W/\Delta t = 10,000/(4 \times 60) = 41.67 \text{ N/s}$$

But $\gamma = 9,790$ N/m^3 so

$$Q = \dot{W}/\gamma = 41.67/9,790 = \underline{\underline{4.26 \times 10^{-3} \text{ m}^3/\text{s}}}$$

13.8 **Information and assumptions**

In 5 minutes, 80 m^3 of water flows into a weigh tank
provided in problem statement

Find

Discharge: Q in units of m^3/s, gpm and cfs

Solution

$$
\begin{aligned}
Q &= \cancel{V}/t = 80/300 = \underline{\underline{0.267 \ \mathrm{m^3/s}}} \\
Q &= 0.267\ (\mathrm{m^3/s})/(0.02832\ \mathrm{m^3/s/cfs}) = \underline{\underline{9.42\ \mathrm{cfs}}} \\
Q &= 9.42\ \mathrm{cfs}\ \times 449\ \mathrm{gpm/cfs} = \underline{\underline{4,230\ \mathrm{gpm}}}
\end{aligned}
$$

13.9 **Information and assumptions**

Velocity data in a 24 inch oil pipe are given
provided in problem statement

Find

Discharge
Mean velocity
Ratio of maximum to minimum velocity

Numerical integration

r(m)	V(m/s)	$2\pi Vr$	area (by trapezoidal rule)
0	8.7	0	
0.01	8.6	0.54	0.0027
0.02	8.4	1.06	0.0080
0.03	8.2	1.55	0.0130
0.04	7.7	1.94	0.0175
0.05	7.2	2.26	0.0210
0.06	6.5	2.45	0.0236
0.07	5.8	2.55	0.0250
0.08	4.9	2.46	0.0250
0.09	3.8	2.15	0.0231
1.10	2.5	1.57	0.0186
0.105	1.9	1.25	0.0070
0.11	1.4	0.97	0.0056
0.115	0.7	0.51	0.0037
0.12	0	0	0.0013

Summing the values in the last column in the above table gives $Q = 0.196 \text{ m}^3/\text{s}$
Then,

$$\begin{aligned} V_{\text{mean}} &= Q/A = 0.196/(0.785(0.24)^2) = \underline{\underline{4.33 \text{ m/s}}} \\ V_{\max}/V_{\text{mean}} &= 8.7/4.33 = \underline{\underline{2.0}} \end{aligned}$$

This ratio indicates the flow is laminar. The discharge is

$$\underline{\underline{Q = 0.196 \text{ m}^3/\text{s}}}$$

13.10 **Information and assumptions**

Velocity data in a 16 inch circular air duct are given. $p = 14.3$ psia, $T = 70\ ^o$F provided in problem statement

Find

Flow rate: Q in cfs and cfm
Ratio of maximum to mean velocity
Is flow laminar or turbulent?
Mass flow rate: $\dot{m}$

Numerical integration

y(in.)	r(in.)	V(ft/s)	$2\pi rV$(ft^2/s)	area (ft^3/s)
0.0	8.0	0	0	
0.1	7.9	72	297.8	1.24
0.2	7.8	79	322.6	2.58
0.4	7.6	88	350.2	5.61
0.6	7.4	93	360.3	5.92
1.0	7.0	100	366.5	12.11
1.5	6.5	106	360.8	15.15
2.0	6.0	110	345.6	14.72
3.0	5.0	117	306.3	27.16
4.0	4.0	122	255.5	23.41
5.0	3.0	126	197.9	18.89
6.0	2.0	129	135.1	13.88
7.0	1.0	132	69.4	8.51
8.0	0.0	135	0	2.88

Summing values in the last column of the above table gives $\underline{\underline{Q = 152.1 \text{ ft}^3/\text{s} \ = \ 9,124 \text{ cfm}}}$

$$\begin{aligned} V_{\text{mean}} &= Q/A = 152.1/(0.785(1.33)^2) = 109 \text{ ft/s} \\ V_{\max}/V_{\text{mean}} &= 135/109 = \underline{\underline{1.24}} \end{aligned}$$

which suggests turbulent flow.

$$\begin{aligned} \rho &= 14.3(144)/((53.3(530)) = 0.0728 \text{ lbm/ft}^3 \\ \dot{m} &= 0.0728(152.1) = \underline{\underline{11.1 \text{ lbm/s}}} \end{aligned}$$

13.11 **Information and assumptions**

Information about gas flow in a cylindrical stack is given in the textbook

Find

ratio r_m/D, probe location and mass flow rate

Solution

a)

$$\begin{aligned}
\pi r_m^2 &= (\pi/4)[(D/2)^2 - r_m^2) \\
(r_m/D)^2 &= 1/16 - (r_m/D)^2(1/4) \\
5/4(r_m/D)^2 &= 1/16;\ 5(r_m/D)^2 = 1/4 \\
r_m/D &= \sqrt{1/20} = \underline{\underline{0.224}}
\end{aligned}$$

b)

$$r_c A = \int_{0.2236D}^{D/2} [r\sin(\alpha/2)/(\alpha/2)](\pi/4)2r dr = 0.9(\pi/2)(r^3/3)|_{0.2236D}^{0.5D}$$

$$(rc)(\pi/4)[(D/2)^2 - (0.2236D)^2] = 0.90(\pi/6)[(0.5D)^3 - (0.2236D)^3]$$

$$\underline{\underline{r_c/D = 0.341}}$$

c)

$$\begin{aligned}
\rho &= p/(RT) = 110 \times 10^3/(400 \times 573) = 0.480 \text{ kg/m}^3 \\
V &= \sqrt{2\Delta p/\rho_g} = \sqrt{(2)\rho_w g \Delta h/\rho_g} = \sqrt{(2)(1,000)(9.81)/0.48}\sqrt{\Delta h} = 202.2\sqrt{\Delta h}
\end{aligned}$$

Values for each section are

Station	Δh	V
1	0.012	7.00
2	0.011	6.71
3	0.011	6.71
4	0.009	6.07
5	0.0105	6.55

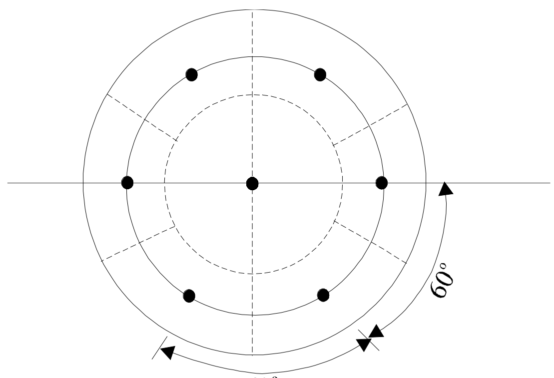
Mass flow rate is given by

$$\begin{aligned}
\dot{m} &= \sum A_{\text{sector}}\rho V_{\text{sector}} = A_T\rho(\sum V/5) \\
&= (\pi 2^2/4)(0.480)(6.61) = \underline{\underline{9.96 \text{ kg/s}}}
\end{aligned}$$

13.12 **Information and assumptions**

Information about gas flow in a cylindrical stack is given in the textbook

Schematic of measurement locations



Solution

a)

$$
\begin{aligned}
\pi r_m^2 &= (\pi/6)[(D/2)^2 - r_m^2] \\
7/6(r_m/D)^2 &= (1/6)(1/4) \\
(r_m/D)^2 &= 1/28 \\
r_m/D &= \underline{\underline{0.189}}
\end{aligned}
$$

b)

$$
\begin{aligned}
r_c A &= 1/6 \int_{0.189D}^{0.5D} [r \sin(\alpha/2)/(\alpha/2)] 2\pi r \, dr \\
(\pi r_c/6)[(D/2)^2 - (r_m)^2] &= 0.955(\pi/3)(r^3/3)|_{0.189D}^{0.50D} \\
r_c(0.5^2 - 0.189^2) &= 0.955(6/9)[0.5^3 - 0.189^3]D \\
r_c/D &= (0.955)6(0.118)/(9(0.2143)) = \underline{\underline{0.351}}
\end{aligned}
$$

c)

$$
\begin{aligned}
\rho &= p/RT = 115 \times 10^3/((420)(250+273)) = 0.523 \text{ kg/m}^3 \\
V &= \sqrt{2g\rho_w \Delta h/\rho_g} = \sqrt{(2)(9.81)(1,000)/0.523}\sqrt{\Delta h} = 193.7\sqrt{\Delta h}
\end{aligned}
$$

Calculating velocity from Δh data gives

Station	Δh(mm)	V
1	8.2	17.54
2	8.6	17.96
3	8.2	17.54
4	8.9	18.27
5	8.0	17.32
6	8.5	17.86
7	8.4	17.75

From the above table, $V_{avg} = 17.75$ m/s, Then

$$\dot{m} = (\pi D^2/4)\rho V_{\text{avg.}} = ((\pi)(1.5)^2/4)(0.523)(17.75) = \underline{\underline{16.4}} \text{ kg/s}$$

13.13 **Information and assumptions**

Velocity data for a river is given

Find

Discharge: Q

Solution

$$Q = \sum V_i A_i$$

V	A	VA
1.32 m/s	7.6 m^2	10.0
1.54	21.7	33.4
1.68	18.0	30.2
1.69	33.0	55.8
1.71	24.0	41.0
1.75	39.0	68.2
1.80	42.0	75.6
1.91	39.0	74.5
1.87	37.2	69.6
1.75	30.8	53.9
1.56	18.4	28.7
1.02	8.0	8.2

Summing the last column gives

$$\underline{\underline{Q = 549.1 \text{ m}^3/\text{s}}}$$

13.14 **Information and assumptions**

Velocity is measured with LDV. $\lambda = 4880$ Å, $2\theta = 20^o$. On the Doppler burst, 5 peaks occur in 12 μs.

Find

Air velocity: V

Fringe spacing

$$\begin{aligned}
\Delta x &= \lambda/2\sin\theta \\
&= (4,880 \times 10^{-10})/(2\sin 10^o) \\
&= 1.41 \times 10^{-6} \text{ m}
\end{aligned}$$

Velocity

$$\begin{aligned}
\Delta t &= 12\ \mu\text{s}/4 = 3\ \mu\text{s} \\
V &= \Delta x/\Delta t = 1.41 \times 10^{-6}/3 \times 10^{-6} = \underline{\underline{0.47 \text{ m/s}}}
\end{aligned}$$

13.15 **Information and assumptions**

A jet and orifice is described in the textbook

Find

Coefficients for an orifice: C_v, C_c, C_d

Solution

Assume $V_j = \sqrt{2g \times 1.90}$ Then

$$\begin{aligned}
C_v &= V_j/V_{\text{theory}} = \sqrt{2g \times 1.90}/\sqrt{2g \times 2} \\
C_v &= \sqrt{1.90/2.0} = \underline{\underline{0.975}} \\
C_c &= A_j/A_0 = (8/10)^2 = \underline{\underline{0.640}} \\
C_d &= C_v C_c = 0.975 \times 0.64 = \underline{\underline{0.624}}
\end{aligned}$$

13.16 **Information and assumptions**

A fluid jet discharges from a 3 inch orifice. At the vena contracta, $d = 2.6$ cm

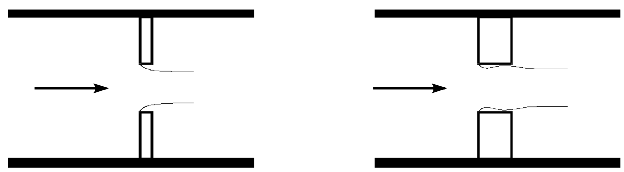
Find

Coefficient of contraction: C_c

Solution

$$C_c = A_j/A_0 = (2.6/3)^2 = \underline{\underline{0.751}}$$

13.17 If the angle is 90°, the orifice and expected flow pattern is shown below in Fig. A.



A   B

We presume that the flow would separate at the sharp edge just as it does for the orifice with a knife edge. Therefore, the flow pattern and flow coefficient K should be the same as with the knife edge.

However, if the orifice were very thick relative to the orifice diameter (Fig. B), then the flow may reattach to the metal of the orifice thus creating a different flow pattern and different flow coefficient K than the knife edge orifice.

13.18 Some of the possible changes that might occur are listed below:

a) Blunting (rounding) of the sharp edge might occur because of erosion or corrosion. This would probably increase the value of the flow coefficient because C_c would probably be increased.

b) Because of corrosion or erosion the face of the orifice might become rough. This would cause the flow next to the face to have less velocity than when it was smooth. With this smaller velocity in a direction toward the axis of the orifice it would seem that there would be less momentum of the fluid to produce contraction of the jet which is formed downstream of the orifice. Therefore, as in case A, it appears that K would increase but the increase would probably be very small.

c) Some sediment might lodge in the low velocity zones next to and upstream of the face of the orifice. The flow approaching the orifice (lower part at least) would not have to change direction as abruptly as without the sediment. Therefore, the C_c would probably be increased for this condition and K would also be increased.

13.19 **Information and assumptions**

Water (60 ^{o}F, $Q = 4.5$ cfs) flows through an orifice ($d = 6$ in.) in a pipe ($D = 10$ in.). A mercury manometer is connected across the orifice.

Find

Manometer deflection: h

Find K

$$\begin{aligned} d/D &= 0.60 \\ \text{Re}_d &= 4Q/(\pi d\nu) = 4 \times 4.5/(\pi \times 0.5 \times 1.22 \times 10^{-5}) = 9.45 \times 10^5 \end{aligned}$$

from Fig. 13.13: $K = 0.65$

Calculate deflection

$$A = (\pi/4) \times 0.5^2 = 0.196 \text{ ft}^2$$

Then

$$\begin{aligned} \Delta h &= (Q/KA)^2/2g = (4.5/(0.65 \times 0.196))^2/64.4 = 19.37 \text{ ft of water} \\ h &= \Delta h/12.6 = 1.538 \text{ ft } = \underline{\underline{18.5 \text{ in.}}} \end{aligned}$$

13.20 **Information and assumptions**

Water flows through a 6 inch orifice in a 12 inch pipe. Assume $T = 60$ °F, $\nu = 1.22 \times 10^{-5}$ ft^2/s

Find

Discharge: Q

Solution

Calculate head

$$\Delta h = (1.0)(13.55 - 1) = 12.55 \text{ ft}$$

Find K

$$\begin{aligned} (d/D) &= 0.50 \\ (2g\Delta h)^{0.5} d/\nu &= (2g \times 12.55)^{0.5}(0.5)/(1.22 \times 10^{-5}) = 1.17 \times 10^6 \end{aligned}$$

From Fig. 13.13 $K = 0.625$. Calculate discharge

$$\begin{aligned} Q &= KA_0(2g\Delta h)^{0.5} \\ Q &= 0.625(\pi/4 \times 0.5^2)(64.4 \times 12.55)^{0.5} = \underline{\underline{3.49 \text{ cfs}}} \end{aligned}$$

13.21 A rough pipe will have a greater maximum velocity at the center of the pipe relative to the mean velocity than would a smooth pipe. Because more flow is concentrated near the center of the rough pipe less radial flow is required as the flow passes through the orifice; therefore, there will be less contraction of the flow. Consequently the coefficient of contraction will be larger for the rough pipe. So, using K from Fig. 13.13 would probably result in an estimated discharge that is too small.

13.22 **Information and assumptions**

Water flows through a 3 inch orifice in a 6 inch pipe. $\Delta h = 5$ ft. Assume $T = 60$ °F, then $\nu = 1.22 \times 10^{-5}$ ft^2/s

Find

Discharge: Q

Find K

$$\begin{aligned} d/D &= 1/2 \\ \sqrt{2g\Delta h}d/\nu &= 3.7 \times 10^5 \end{aligned}$$

From Fig. 13.13$K = 0.63$

Orifice discharge expression

$$\begin{aligned} Q &= KA\sqrt{2g\Delta h} \\ A &= (\pi/4) \times (3/12)^2 = 0.0491 \text{ ft}^2 \end{aligned}$$

Thus

$$Q = 0.63 \times 0.0491\sqrt{2 \times 32.2 \times 5} = \underline{\underline{0.55 \text{ cfs}}}$$

13.23 **Information and assumptions**

Kerosene at 20 ^{o}C flows through an orifice. $D = 2$ cm, $d/D = 0.6$, $\Delta p = 10$ kPa At 20 °F, $\rho = 814$ kg/m^3, $\nu = 2.37 \times 10^{-6}$ m^2/s

Find

Mean velocity: V

Find K

$$\begin{aligned}
\mathrm{Re}_d/K &= (2\Delta p/\rho)^{0.5}(d/\nu) \\
&= (2 \times 10 \times 10^3/814)^{0.5}(0.6 \times 0.02/(2.37 \times 10^{-6})) \\
&= 4.96 \times 5,063 = 2.51 \times 10^4
\end{aligned}$$

$$\text{From Fig. 13.13 } K = 0.67$$

Orifice discharge expression

$$\begin{aligned}
Q &= VA_p = KA_0(2\Delta p/\rho)^{0.5} \\
V_{\text{pipe}} &= K(A_0/A_p)(2\Delta p/\rho)^{0.5} = (0.67)(0.60^2)(4.96) \\
&= \underline{\underline{1.20 \text{ m/s}}}
\end{aligned}$$

13.24 **Information and assumptions**

Water at 20 ^oC flows in a pipe containing two orifices, one that is horizontal and one that is vertical. For each orifice, $D = 30$ cm and $d = 10$ cm. $Q = 0.1$ m^3/s.

Find

Pressure differential across each orifice: Δp_C, Δp_F

Deflection for each mercury-water manometer: Δh_C, Δh_F

Solution

Find K

$$4Q/(\pi d\nu) = 4 \times 0.10/(\pi \times 0.10 \times 1.31 \times 10^{-6}) = 9.7 \times 10^5$$

$$\text{From Fig. 13.13 } K = 0.60$$

Find Δh

$$\begin{aligned} Q &= KA\sqrt{2g\Delta h} \\ A &= (\pi/4)(0.10)^2 = 7.85 \times 10^{-3} \text{ m}^2 \end{aligned}$$

Thus

$$\begin{aligned} \Delta h &= Q^2/(K^2A^22g) = 0.1^2/(0.6^2 \times (7.85 \times 10^{-3})^2 \times 2 \times 9.81) \\ &\Delta h_C = \Delta h_F = 22.97 \text{ m} - \text{H}_2\text{O} \end{aligned}$$

The deflection across the manometers is

$$h_C = h_F = 22.97/(S_{\text{Hg}} - S_{\text{water}}) = \underline{\underline{1.82 \text{ m}}}$$

The deflection will be the same on each manometer

Find Δp

$$\begin{aligned} p_A - p_B &= \gamma\Delta h = 9,790 \times 22.97 = 224.9 \text{ kPa} \\ &\underline{\underline{\Delta p_C = 225 \text{ kPa}}} \end{aligned}$$

For manometer F

$$((p_D/\gamma) + z_D) - ((p_E/\gamma) + z_E) = \Delta h = 22.97 \text{ ft}$$

Thus,

$$\begin{aligned} \Delta p_F &= p_D - p_E = \gamma\Delta h - \gamma(z_D - z_E) \\ &= 9,810(22.97 - 0.3) \\ &\underline{\underline{\Delta p_F = 222 \text{ kPa}}} \end{aligned}$$

Because of the elevation difference for manometer F, $\Delta p_C \neq \Delta p_F$

13.25 **Information and assumptions**

A pipe (D = 30 cm) is terminated with an orifice. The orifice size is increased from 15 to 20 cm with pressure drop ($\Delta p = 50$ kPa) held constant.

Find

Percentage increase in discharge

Find K values

Assume large Re, so K depends only on d/D. From Fig. 13.13

$$\begin{aligned} K_{15} &= 0.62 \\ K_{20} &= 0.685 \end{aligned}$$

Find expressions for discharge

$$\begin{aligned} Q_{15} &= K_{15}A_{15}\sqrt{2g\Delta h} \\ Q_{15} &= 0.62 \times (\pi/4)(0.15)^2\sqrt{2g\Delta h} \\ Q_{15} &= 0.01395(\pi/4)\sqrt{2g\Delta h} \end{aligned}$$

For the 20 cm orifice

$$\begin{aligned} Q_{20} &= 0.685 \times (\pi/4)(0.20)^2\sqrt{2g\Delta h} \\ Q_{20} &= 0.0274(\pi/4)\sqrt{2g\Delta h} \end{aligned}$$

Thus the % increase is (0.0274 - 0.01395)/0.01395)$\times$100 = $\underline{\underline{96\%}}$

13.26 **Information and assumptions**

Water flows through the orifice (vertical orientation) shown in the textbook. $D = 50$ cm, $d = 5$ cm, $\Delta p = 10$ kPa, $\Delta z = 30$ cm.

Find

Flow rate: Q

Calculate Δh

$$\begin{aligned}
\Delta h &= (p_1/\gamma + z_1) - (p_2/\gamma + z_2) \\
&= \Delta p/\gamma + \Delta z \\
&= 10,000/9,790 + 0.3 \\
&= 1.321 \text{ m of water}
\end{aligned}$$

Find K

$$\begin{aligned}
d/D &= 5/50 = 0.10 \\
\mathrm{Re}/K &= \sqrt{2 \times 9.81 \times 1.32} \times 0.05/10^{-6} = 2.5 \times 10^5
\end{aligned}$$

From Fig. 13.13 $K = 0.60$

Orifice discharge expression

$$\begin{aligned}
Q &= KA\sqrt{2g\Delta h} \\
&= 0.60 \times (\pi/4) \times (0.05)^2 \sqrt{2 \times 9.81 \times 1.321} = \underline{\underline{0.006 \text{ m}^3/\text{s}}}
\end{aligned}$$

13.27 **Information and assumptions**

Flow through an orifice is shown in the textbook

Goal

Show that the difference in piezometric pressure is given by the pressure difference across the transducer

Solution

Using the orifice elevation as our reference elevation and the hydrostation relation

$$\begin{aligned} p_{T,1} &= p_1 + \gamma\ell_1 \\ p_{T,2} &= p_2 - \gamma\ell_2 \end{aligned}$$

so

$$\begin{aligned} p_{T,1} - p_{T,2} &= p_1 + \gamma\ell_1 - p_2 + \gamma\ell_2 \\ &= p_1 - p_2 + \gamma(\ell_1 + \ell_2) \end{aligned}$$

But

$$\ell_1 + \ell_2 = z_1 - z_2$$

or

$$p_{T,1} - p_{T,2} = p_1 - p_2 + \gamma(z_1 - z_2)$$

Thus,

$$\underline{\underline{p_{T,1} - p_{T,2} = (p_1 + \gamma z_1) - (p_2 + \gamma z_2)}}$$

13.28 **Information and assumptions**

Water ($T = 50\ ^o$F, $Q = 20$ cfs) flows in the system shown in the textbook. $f = 0.015$

Find

Pressure change across the orifice
Power delivered to the flow by the pump
Sketch the HGL and EGL

Solution

$$\begin{aligned}
\text{Re} &= 4Q/(\pi d\nu) \\
&= 4 \times 20/(\pi \times 1 \times 1.41 \times 10^{-5}) = 1.8 \times 10^6
\end{aligned}$$

Then for $d/D = 0.50$, $K = 0.625$

Orifice discharge expression

$$Q = KA\sqrt{2g\Delta h} \text{ or } \Delta h = (Q/(KA))^2/2g$$

where $A = \pi/4 \times 1^2$. Then

$$\begin{aligned}
\Delta h &= (20/(0.625 \times (\pi/4)))^2/2g \\
\Delta h &= 25.8 \text{ ft} \\
\Delta p &= \gamma\Delta h = 62.4 \times 25.8 = \underline{\underline{1{,}610 \text{ psf}}}
\end{aligned}$$

Energy equation (section 1 is the upstream reservoir water surface, section 2 is the downstream reservoir surface)

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 10 + h_p &= 0 + 0 + 5 + \sum h_L \\
h_p &= 05 + V^2/2g(K_e + K_E + fL/D) + h_{L,\text{orifice}} \\
K_e &= 0.5;\ K_E = 1.0
\end{aligned}$$

The orifice head loss will be like that of an abrupt expansion:

$$h_{L,\text{ orifice}} = (V_j - V_{\text{pipe}})^2/(2g)$$

Here, V_j is the jet velocity as the flow comes from the orifice.

$$V_j = Q/A_j \text{ where } A_j = C_c A_0$$

Assume

$$C_c \approx 0.65 \text{ then } V_j = 20/((\pi/4) \times 1^2 \times 0.65) = 39.2 \text{ ft/s}$$

Also

$$V_p = Q/A_p = 20/\pi = 6.37 \text{ ft/s}$$

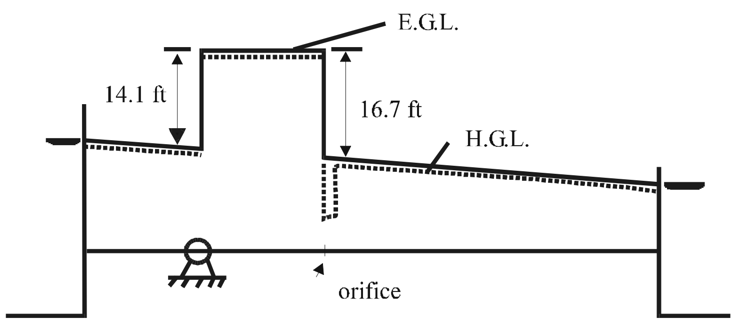
Then

$$h_{L,\text{orifice}} = (39.2 - 6.37)^2/(2g) = 16.74 \text{ ft}$$

Finally,

$$\begin{aligned}
h_p &= -5 + (6.37^2/(2g))(0.5 + 1.0 + (0.015 \times 300/2)) + 16.74 \\
h_p &= 14.10 \text{ ft} \\
P &= Q\gamma h_p/550 \\
&= 20 \times 62.4 \times 14.10/550 = \underline{\underline{32.0 \text{ hp}}}
\end{aligned}$$

The HGL and EGL are shown below:



13.29 **Information and assumptions**

Water flows ($Q = 0.03$ m^3/s) through an orifice. Pipe diameter, $D = 15$ cm. Manometer deflection is 1 m-Hg.

Find

Orifice size: d

Solution

$$\Delta h = 12.6 \times 1 = 12.6 \text{ m of water}$$

Orifice discharge expression

$$A = Q/(K\sqrt{2g\Delta h})$$

Guess $K = 0.7$, then

$$\begin{aligned} d^2 &= (4/\pi)Q/(K\sqrt{2g\Delta h}) \\ d^2 &= (4/\pi) \times 0.03/\left[0.7\sqrt{2g \times 12.6}\right] = 3.47 \times 10^{-3} \text{ m}^2 \\ d &= 5.89 \text{ cm} \\ d/D &= 0.39 \\ \mathrm{Re}_d &= 4 \times 0.03/(\pi \times 0.0589 \times 10^{-6}) = 6.5 \times 10^5 \\ K &= 0.62 \end{aligned}$$

so

$$d = \sqrt{(0.7/0.62)} \times 0.0589 = 0.0626 \text{ m}$$

Recalculate K to find that $K = 0.62$. Thus,

$$d = \underline{\underline{6.26 \text{ cm}}}$$

13.30 **Information and assumptions**

Gasoline ($S = 0.68$) flows through an orifice ($d = 6$ cm) in a pipe ($D = 10$ cm). $\Delta p = 50$ kPa. Assume $T = 20°C$; $\nu = 4 \times 10^{-7}$ m^2/s (Fig. A-3)

Find

Discharge: Q

Solution

$$\Delta h = \Delta p/\gamma = 50,000/(0.68 \times 9,810) = 7.50 \text{ m}$$

Find K

$$\begin{aligned} d/D &= 0.60 \\ \sqrt{2g\Delta h}d/\nu &= \sqrt{2 \times 9.81 \times 7.50} \times 0.06/(4 \times 10^{-7}) = 1.82 \times 10^6 \\ K &= 0.66 \end{aligned}$$

Orifice discharge expression

$$\begin{aligned} Q &= KA\sqrt{2g\Delta h} \\ &= 0.66 \times (\pi/4)(0.06)^2 \sqrt{2g \times 7.50} \\ Q &= \underline{\underline{0.0226 \text{ m}^3/\text{s}}} \end{aligned}$$

13.31 **Information and assumptions**

Water flows ($Q = 2$ m^3/s) through an orifice in a pipe ($D = 1$ m). $\Delta h = 8$ m-H_2O

Find

Orifice size: d

Solution

Follow same procedure as for Problem 13.29:

$$\Delta h = 8 \text{ m of water; } Q = 2 \text{ m}^3/\text{s; } D = 1 \text{ m; } K = 0.65 \text{ (assume)}$$

Then

$$\begin{aligned} d^2 &= (4/\pi) \times 2/((0.65\sqrt{2g \times 8})) = 0.313 \\ d &= 0.56 \text{ m} \end{aligned}$$

Try again:

$$\begin{aligned} d/D &= 0.56 \\ \text{Re} &= 4Q/(\pi d\nu) \simeq 4.5 \times 10^6 \end{aligned}$$

From Fig. 13.13 $K = 0.63$. Then

$$d = \sqrt{0.65/0.63} \times 0.56 = 0.569 \text{ m}$$

This gives the same value for K so

$$d = \underline{\underline{56.9 \text{ cm}}}$$

13.32 **Information and assumptions**

Water flows ($Q = 3$ m^3/s) through an orifice in a pipe ($D = 1.2$ m). $\Delta p = 50$ kPa

Find

Orifice size: d

Solution

Follow same procedure as for P13.29:

Assume $K = 0.65$; $T = 20°C$; $\Delta h = \Delta p/\gamma = 50,000/9,790 = 5.11$ m

$$\begin{aligned} d^2 &= (4/\pi) \times 3.0/(0.65\sqrt{2 \times 9.81 \times 5.11}) = 0.587 \\ d &= 0.766 \text{ m} \end{aligned}$$

Check K:

$$Re_d = 4Q/(\pi d\nu) = 4 \times 3.0/(\pi \times 0.766 \times 10^{-6}) = 5 \times 10^6$$

From Fig. 13.13 for $d/D = 0.766/1.2 = 0.64$, $K = 0.67$

Try again:

$$d = \sqrt{(0.65/0.67)} \times 0.766 = 0.754$$

Check K: $Re_d = 5 \times 10^6$ and $d/D = 0.63$. From Fig. 13.13 $K = 0.67$ so

$$d = \sqrt{(0.65/0.670)} \times 0.766 = \underline{\underline{0.754 \text{ m}}}$$

13.33 **Information and assumptions**

Water flows through a hemicircular orifice as shown in the textbook

Find

A formula for discharge

Estimate Q

Solution

$$\begin{aligned} p_1 + \rho V_1^2/2 &= p_2 + \rho V_2^2/2 \\ V_1 A_1 &= V_2 A_2;\ V_1 = V_2 A_2/A_1 \\ V_2 &= \sqrt{2(p_1 - p_2)/p}/\sqrt{1 - (A_2^2/A_1^2)} \end{aligned}$$

or

$$\begin{aligned} Q &= A_2 V_2 \\ & \left[A_2/\sqrt{1 - (A_2^2/A_1^2)}\right]\sqrt{2\Delta p/\rho} \end{aligned}$$

but $A_2 = C_c A_0$ where A_0 is area of orifice. Then

$$Q = \left[C_c A_0/\sqrt{1 - (A_2^2/A_1^2)}\right]\sqrt{2\Delta p/\rho}$$

or

$$Q = K A_0 \sqrt{2\Delta p/\rho}$$

where K is the flow coefficient. Assume $K = 0.65$; Also $A = (\pi/8) \times 0.30^2 = 0.0353$ m^2. Then

$$\begin{aligned} Q &= 0.65 \times 0.0353\sqrt{2 \times 80,000/1,000} \\ &= \underline{\underline{0.290 \text{ m}^3/\text{s}}} \end{aligned}$$

13.34 **Information and assumptions**

Water (20 ^{o}C, $Q = 0.75$ m^3/s) flows through a venturi meter ($d = 30$ cm) in a pipe ($D = 60$ cm)

Find

Deflection on a mercury manometer

Solution

$$\begin{aligned}
\text{Re}_d &= 4 \times 0.75/(\pi \times 0.30 \times 1 \times 10^{-6}) = 3.18 \times 10^6 \\
d/D &= 0.50;\ K = 1.02 \\
\Delta h &= [Q/(KA)]^2/(2g) \\
&= \left[.75/(1.02 \times (\pi/4) \times 0.3^2)\right]^2/(2 \times 9.81) = 5.52 \text{ m H}_2\text{O}
\end{aligned}$$

Manometer analysis yields

$$h_{H_g} = \Delta h_{H_2O}/\left(\frac{\gamma_{H_g}}{\gamma_{H_2O}} - 1\right)$$

$$h = 5.52/12.6 = \underline{\underline{0.44 \text{ m}}}$$

13.35 Information and assumptions

Water ($Q = 10$ m^3/s) flows through a venturi meter in a horizontal pipe ($D = 2$ m). $\Delta p = 200$ kPa. Assume $T = 20°C$.

Find

Venturi throat diameter

Solution

Guess that $K = 1.01$, and then proceed with calculations

$$Q = KA/\sqrt{2g\Delta h}$$

where $\Delta h = 200,000$ Pa/(9,790 N/m^3) = 20.4 m. Then

$$\begin{aligned}
A &= Q/(K\sqrt{2g\Delta h}) \text{ or } \pi d^2/4 = Q/(K\sqrt{2g\Delta h}) \\
d &= (4Q/(\pi K\sqrt{2g\Delta h}))^{1/2} \\
d &= (4 \times 10/(\pi \times 1.01\sqrt{2g \times 20.4}))^{1/2} = 0.794 \text{ m}
\end{aligned}$$

Calculate K and compare with the guessed value

$$\text{Re} = 4Q/(\pi d\nu) = 1.6 \times 10^7$$

Also $d/D = 0.4$ so from Fig. 13.13 $K \approx 1.0$. Try again:

$$d = (1.01/1.0)^{1/2} \times 0.794 = \underline{\underline{0.798 \text{ m}}}$$

Using this value, $K = 1$ from Fig. 13.13

13.36 **Information and assumptions**

A venturi meter is described in the textbook

Find

Rate of flow: Q

Solution

Find K

$$\begin{aligned}
\Delta h &= 4 \text{ ft and } d/D = 0.33 \\
\mathrm{Re}_d/K &= (1/3)\sqrt{2 \times 32.2 \times 4}/1.22 \times 10^{-5}) = 4.4 \times 10^5 \\
K &= 0.97 \text{ (Estimated from Fig. 13.13)}
\end{aligned}$$

Discharge expression

$$\begin{aligned}
Q &= KA\sqrt{2gh} \\
&= 0.97(\pi/4 \times 0.333^2)\sqrt{2 \times 32.2 \times 4} \\
Q &= \underline{\underline{1.36 \text{ cfs}}}
\end{aligned}$$

13.37 The answer is -10 psi $< \Delta$ p < 0 so the correct choice is $\underline{\underline{\text{b)}}}$

13.38 **Information and assumptions**

Water flows through a horizontal venturi meter. $\Delta p = 100$ kPa, $d = 1$ m, $D = 2$ m. Assume $T = 20°C$; so $\nu = 10^{-6}$ m^2/s

Find

Discharge: Q

Solution

$$\Delta p = 100 \text{ kPa so } \Delta h = \Delta p/\gamma = 100,000/9,790 = 10.2 \text{ m}$$

Find K

$$\begin{aligned} \sqrt{2g\Delta h}d/\nu &= \sqrt{2 \times 9.81 \times 10.2} \times 1/10^{-6} \\ &= 1.4 \times 10^7 \end{aligned}$$

Then $K \approx 1.02$ (extrapolated from Fig. 13.13). Applying the discharge equation

$$\begin{aligned} Q &= KA\sqrt{2g\Delta h} \\ &= 1.02 \times (\pi/4) \times 1^2 \sqrt{2g \times 10.2} \\ &\underline{\underline{= 11.3 \text{ m}^3/\text{s}}} \end{aligned}$$

13.39 Because of the streamline curvature (concave toward wall) near the pressure tap, the pressure at point 2 will be less than the average pressure across the section. Therefore, Q_0 will be too large as determined by the formula. Thus, $K < 1$.

13.40 **Information and assumptions**

Water (50 ^{o}F) flows through a vertical venturi meter. $\Delta p = 6.2$ psi, $d = 6$ in., $D = 12$ in., $\nu = 1.4 \times 10^{-5}$ ft^2/s

Find

Discharge: Q

Solution

$$\Delta p = 6.20 \text{ psi} = 6.2\ \times 144 \text{ psf}$$

Thus

$$\Delta h = 6.20 \times 144/62.4 = 14.3 \text{ ft}$$

Find K

$$\begin{aligned} & \sqrt{2g\Delta h}d/\nu \\ = & \sqrt{2 \times 32.2 \times 14.3} \times (6/12)/(1.4 \times 10^{-5}) = 10 \times 10^5 \end{aligned}$$

So $K = 1.02$. Discharge calculation

$$\begin{aligned} Q &= KA\sqrt{2g\Delta h} \\ &= 1.02 \times (\pi/4) \times (6/12)^2\sqrt{2 \times 32.2 \times 14.3} = \underline{\underline{6.08 \text{ cfs}}} \end{aligned}$$

13.41 **Information and assumptions**

Gasoline ($S = 0.69$) flows through a venturi meter. A differential pressure gage indicates $\Delta p = 45$ kPa. $d = 20$ cm, $D = 40$ cm. $\mu = 3 \times 10^{-4}$ N·s/m^2

Find

Discharge: Q

Solution

$$\begin{aligned} \Delta h &= 45,000/(0.69 \times 9,810) = 6.65 \text{ m} \\ \nu &= \mu/\rho = 3 \times 10^{-4}/690 = 4.3 \times 10^{-7} \text{ m}^2/\text{s} \end{aligned}$$

Then

$$\sqrt{2g\Delta h}d/\nu = \sqrt{2 \times 9.81 \times 6.65} \times 0.20/(4.3 \times 10^{-7}) = 5.3 \times 10^6$$

From Fig. 13.13 $K = 1.02$ so

$$Q = KA\sqrt{2g\Delta h} = 1.02 \times (\pi/4) \times (0.20)^2\sqrt{2 \times 9.81 \times 6.65} = \underline{\underline{0.366 \text{ m}^3/\text{s}}}$$

13.42 **Information and assumptions**

Water passes through a flow nozzle. $\Delta p = 8$ kPa. $d = 2$ cm, $d/D = 0.5$, $\nu = 10^{-6}$ m^2/s, $\rho = 1000$ kg/m^3.

Find

Discharge: Q

Solution

Find K

$$\mathrm{Re}_d/K = (2\Delta p/\rho)^{0.5}(d/\nu) = ((2 \times 8 \times 10^3)/(1,000))^{0.5}(0.02/10^{-6}) = 8.0 \times 10^4$$

From Fig. 13-13 with $d/D = 0.5$; $K = 0.99$. Calculate Discharge

$$\begin{aligned} Q &= KA(2\Delta p/\rho)^{0.5} \\ &= (0.99)(\pi/4)(0.02^2)(2 \times 8 \times 10^3/10^3)^{0.5} = \underline{\underline{0.00124 \text{ m}^3/\text{s}}} \end{aligned}$$

13.43 **Information and assumptions**

Water flows through the annular venturi that is shown in the textbook

Find

Discharge

Solution

Assume $C_d = 0.98$ Then from Eq. (13.5)

$$\begin{aligned} K &= C_d/\sqrt{1-(A_2/A_1)^2} \\ &= 0.98/\sqrt{1-0.75^2} \\ K &= 1.48 \end{aligned}$$

Calculating discharge

$$\begin{aligned} A &= 0.00147\,\mathrm{m}^2 \\ Q &= KA(2g\Delta h)^{0.5} \\ Q &= (1.48)(0.00147)(2.0 \times 9.81 \times 1)^{0.5} \\ &= \underline{\underline{0.00964\ \mathrm{m}^3/\mathrm{s}}} \end{aligned}$$

13.44 **Information and assumptions**

A flow nozzle with $d/D = 1.3$

Find

An expression for head loss

Solution

Formula for head loss in an abrupt expansion:

$$h_L = (V_j - V_0)^2/2g$$

Continuity equation:

$$\begin{aligned} V_0 A_0 &= V_j A_j \\ V_j &= V_0 A_0/A_j \\ &= V_0 \times (3/1)^2 = 9V_0 \end{aligned}$$

Then

$$\begin{aligned} h_L &= (9V_0 - V_0)^2/2g \\ &= \underline{\underline{64V_0^2/2g}} \end{aligned}$$

13.45 **Information and assumptions**

A vortex meter (1 cm shedding element) is used in a 5 cm diameter duct. For shedding on one side of the element, $S_t = 0.2$ and $f = 50$ Hz.

Find

Discharge: Q

Solution

$$
\begin{aligned}
St &= nD/V \\
V &= nD/St = (50)(0.01)/(0.2) = 2.5 \text{ m/s} \\
Q &= VA = (2.5)(\pi/4)(0.05^2) = \underline{\underline{0.0049 \text{ m}^3/\text{s}}}
\end{aligned}
$$

13.46 **Information and assumptions**

A rotometer is described in the text.

Goal

Describe how the reading on the rotometer would be corrected for nonstandard conditions

Solution

The deflection of the rotometer is a function of the drag on the rotating element. Equilibrium of the drag force with the weight of the float gives

$$\begin{aligned} F_D &= \text{Wgt} \\ C_D A \rho V^2/2 &= mg \end{aligned}$$

Thus

$$V = \sqrt{2gm/(\rho A C_D)}$$

Since all terms are constant except density

$$\begin{aligned} V/V_{\text{std.}} &= (\rho_{\text{std.}}/\rho)^{0.5} \text{ and } Q = VA \\ \therefore & \quad \underline{\underline{Q/Q_{\text{std.}} = (\rho_{\text{std.}}/\rho)^{0.5}}} \qquad (1) \end{aligned}$$

Correct by calculating ρ for the actual conditions and then use Eq. (1) to correct Q.

13.47 **Information and assumptions**

A rotometer is calibrated for gas with $\rho_{\text{standard}} = 1.2\ \text{kg/m}^3$, but is used for $\rho = 1.0\ \text{kg/m}^3$. The rotometer indicates $Q = 5\ \ell/\text{s}$.

Find

Actual gas flow rate: Q in ℓ/s.

Solution

The deflection of the rotometer is a function of the drag on the rotating element. Equilibrium of the drag force with the weight of the float gives

$$\begin{aligned} F_D &= W \\ C_D A \rho V^2/2 &= mg \end{aligned}$$

Using the above equation and deriving a ratio of standard to nonstandard conditions gives

$$V/V_{\text{std.}} = (\rho_{\text{std.}}/\rho)^{0.5}$$

also

$$Q = VA$$

Therefore

$$Q/Q_{\text{std.}} = (\rho_{\text{std.}}/\rho)^{0.5}$$

Thus

$$\begin{aligned} Q &= 5 \times (1.2/1.0)^{0.5} \\ &= \underline{\underline{5.48\ \ell/\text{s}}} \end{aligned}$$

13.48 a)

$$
\begin{aligned}
t_1 = L/(c+V);\ t_2 = L/(c-V) \\
\Delta t = t_2 - t_1 = L[(1/(c-V)) - (1/(c+V))] = +L(2V)/(c^2 - V^2) \\
\therefore (c^2 - V^2)\Delta t = 2LV \\
V^2\Delta t + 2LV - c^2\Delta t = 0 \\
V^2 + (2LV/\Delta t) - c^2 = 0
\end{aligned}
$$

Solving for V:

$$[(-2L/\Delta t) \pm \sqrt{(2L/\Delta t)^2 + 4c^2}]/2 = (-L/\Delta t) \pm \sqrt{(L/\Delta t)^2 + c^2}$$

Selecting the positive value for the radical

$$\underline{\underline{V = (L/\Delta t)[-1 + \sqrt{1 + (c\Delta t/L)^2}]}}$$

b) From above

$$
\begin{aligned}
\Delta t &= 2LV/c^2 \text{ for } c >> V \\
V &= \underline{\underline{c^2\Delta t/2L}}
\end{aligned}
$$

c)

$$V = (300)^2(10 \times 10^{-3})/((2)(20)) = \underline{\underline{22.5 \text{ m/s}}}$$

13.49 **Information and assumptions**

Water flows over a rectangular weir. $L = 4$ m; $H = 0.20$ m, $P = 0.3$ m

Find

Discharge: Q

Solution

$$Q = K\sqrt{2g}LH^{3/2}$$

where $H/P = 0.67$ so

$$K = 0.40 + 0.05 \times 0.67 = 0.434$$

Thus

$$\begin{aligned} Q &= 0.434 \times \sqrt{2 \times 9.81} \times 4 \times (0.2)^{3/2} \\ &= \underline{\underline{0.688 \text{ m}^3/\text{s}}} \end{aligned}$$

13.50 **Information and assumptions**

Water flows over a 60^o triangular weir. $H = 0.35$ m

Find

Discharge: Q

Solution

$$
\begin{aligned}
Q &= 0.179\sqrt{2g}H^{5/2} \\
Q &= 0.179\sqrt{2 \times 9.81}(0.35)^{5/2} = \underline{\underline{0.0575 \text{ m}^3/\text{s}}}
\end{aligned}
$$

13.51 Correct choice is $\underline{\underline{\text{c) } Q_A < Q_B}}$ because of the side contractions on A.

13.52 Correct choice is $\underline{\underline{\text{b)}}}$ $(H_1/H_2) < 1$ because K is larger for smaller height of weir as shown by Eq. (13-10); therefore, less head is required for the smaller P value.

13.53 **Information and assumptions**

A rectangular weir is being designed for $Q = 4\text{ m}^3/\text{s}$, $L = 3$ m, Water depth upstream of weir is 2 m.

Find

Weir height: P

Solution

First guess $H/P = 0.60$. Then

$$K = 0.40 + 0.05(0.60) = 0.43.$$

For a fully ventilated weir.

$$Q = K\sqrt{2g}LH^{3/2}$$

Solve for H:

$$\begin{aligned} H &= (Q/(K\sqrt{2g}L))^{2/3} \\ &= (4/(0.43\sqrt{(2)(9.81)}(3))^{2/3} = 0.788\text{ m} \end{aligned}$$

Iterate:

$$\begin{aligned} H/P &= 0.788/(2-0.788) = 0.65;\ K = 0.40 + .05(.65) = 0.433 \\ H &= 4/(0.433\sqrt{(2)(9.81}(3))^{2/3} = 0.785\text{ m} \end{aligned}$$

Thus:

$$P = 2.0 - H = 2.00 - 0.785 = \underline{\underline{1.215\text{ m}}}$$

13.54 Correct choice is c).

13.55 **Information and assumptions**

A basin is draining over a rectangular weir. $L = 2$ ft, $P = 2$ ft. Initially, $H = 12$ in.

Find

Time for the head to decrease from 12 inches to 1 inch

Solution

With a head of 1 ft. on the weir $H/P = 1/2$ thus $K_i = 0.40 + 0.05H/P = 0.425$. When $H = 1$ in., $H/P = (1/12)/2$ and $K_f \approx 0.400$. As a simplification, assume K is constant at $K = 0.413$ for the conditions of the problem.

$$Q = 0.413\sqrt{2g}LH^{3/2}$$

For a period of dt the volume of water leaving the basin is equal to $A_B dH$ where A_B is the plan area of the basin. Also this volume is equal to Qdt. Equating these two volumes yields:

$$Qdt = A_B dH$$

or

$$\begin{aligned} 0.413\sqrt{2g}LH^{3/2}dt &= A_B dH \\ dt &= (A_B/(0.413\sqrt{2g}LH^{3/2}))dH \\ dt &= (A_B/0.413\sqrt{2g}L)H^{-3/2}dH \end{aligned}$$

Integrate

$$\begin{aligned} \Delta t &= A_B/(0.413\sqrt{2g}L)\int H^{-3/2}dH \\ &= -(A_B/(0.413\sqrt{2g}L)2H^{-1/2}|_1^{1/12} \\ &= -(2A_B/(0.413\sqrt{2g}L)\left(\left(\frac{1}{1/12}\right)^{1/2} - 1\right) \\ &= (2 \times 2 \times 50/(0.413 \times 8.02 \times 2))(3.46 - 1) \\ &= \underline{\underline{74.3 \text{ s}}} \end{aligned}$$

13.56 **Information and assumptions**

A piping system and channel is described in the textbook. The channel empties over a rectangular weir.

Find

Water surface elevation in the channel

Discharge

Solution

Weir equation:

$$Q = K\sqrt{2g}LH^{3/2}$$

Assume $H = 1/2$ ft. Then $K = 0.4 + 0.05(\frac{1}{2}/3) = 0.41$, then

$$\begin{aligned} Q &= 0.41\sqrt{64.4} \times 2H^{3/2} \\ & Q = 6.58H^{3/2} \end{aligned} \tag{1}$$

Energy equation written from reservoir water surface to channel water surface:

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 100 &= 0 + 0 + 3 + H + \sum h_L \end{aligned} \tag{2}$$

where

$$\begin{aligned} \sum h_L &= (V^2/2g)(K_e + fL/D + 2K_b + K_E) \\ &= (V^2/2g)(0.5 + f(100/(1/3)) + 2 \times 0.35 + 1) \end{aligned}$$

Assume $f = 0.02$ (first try). Then

$$\sum h_L = 8.2V^2/2g$$

Eq. (2) then becomes

$$97 = H + 8.2V^2/2g \tag{3}$$

But $V = Q/A$ so Eq. (3) is written as

$$97 = H + 8.2Q^2/(2gA^2)$$

where

$$\begin{aligned} A^2 &= ((\pi/4)(1/3)^2)^2 = 0.00762 \text{ ft}^4 \\ 97 &= H + 8.2Q^2/(2g \times 0.00762) \\ 97 &= H + 16.72Q^2 \end{aligned} \tag{4}$$

Solve for Q and H between Eqs. (1) and (4)

$$\begin{aligned} 97 &= H + 16.72Q^2 \\ 97 &= H + 16.72(6.58H^{3/2})^2 \\ H &= 0.51 \text{ ft and } Q = 2.397 \text{ ft}^3/\text{s} \end{aligned}$$

Now check Re and f

$$\begin{aligned} V &= Q/A = 27.5 \text{ ft/s; Re} = VD/\nu = 27.5 \times (1/3)/(1.4 \times 10^{-5}) \\ \text{Re} &= 6.5 \times 10^5 \end{aligned}$$

From Figs. 10.8 and Table 10.2 $f = 0.017$. Then Eq. (3) becomes

$$97 = H + 7.3V^2/2g$$

and Eq. (4) is

$$97 = H + 14.88Q^2$$

Solve for H and Q again:

$$\underline{\underline{H = 0.53 \text{ ft and } Q = 2.54 \text{ ft}^3/\text{s}}}$$

13.57 **Information and assumptions**

Water flows into a tank at a rate $Q = 0.1$ m^3/s. The tank has two outlets: a rectangular weir ($P = 1$ m, $L = 1$ m) on the side, and an orifice ($d = 10$ cm) on the bottom.

Find

Water depth in tank

Solution

This involves a trial-and-error solution. First assume the head on the orifice is 1.05 m. Then

$$\begin{aligned} Q_{\text{orifice}} &= KA_0\sqrt{2gh};\ K \approx 0.595 \\ Q_{\text{orifice}} &= 0.595 \times (\pi/4) \times (0.10)^2\sqrt{2 \times 9.81 \times 1.05} = 0.0212 \text{ m}^3/\text{s} \end{aligned}$$

Then

$$\begin{aligned} Q_{\text{weir}} &= K\sqrt{2g}LH^{3/2};\ H_{\text{weir}} = (Q/(K\sqrt{2g}L)^{2/3} \text{ where } K \approx 0.405 \\ H_{\text{weir}} &= ((0.10 - 0.0212)/(0.405\sqrt{2 \times 9.81} \times 1))^{2/3} = 0.124 \text{ m} \end{aligned}$$

Try again:

$$\begin{aligned} Q_{\text{orifice}} &= (1.124/1.05)^{1/2} \times 0.0212 \text{ m}^3/\text{s} = 0.0219 \text{ m}^3/\text{s} \\ H_{\text{weir}} &= ((0.10 - 0.0219)/(0.405\sqrt{2 \times 9.81} \times 1))^{2/3} = \underline{\underline{0.124 \text{ m}}} \end{aligned}$$

H_{weir} is same as before, so iteration is complete. Depth of water in tank is $\underline{\underline{1.124 \text{ m}}}$

13.58 With a sharp edged weir, the flow will break free of the sharp edge and a definite (repeatable) flow pattern will be established. That assumes that the water surfaces both above and below the nappe are under atmospheric pressure. However, if the top of the weir was not sharp then the lower part of the flow may follow the rounded portion of the weir plate a slight distance downstream.

This would probably lessen the degree of contraction of the flow. With less contraction, the flow coefficient would be larger than given by Eq. (13.10).

If the weir is not ventilated below the Nappe, for example a weir that extends the full width of a rectangular channel (as shown in Fig. 13.18), then as the water plunges into the downstream pool air bubbles would be entrained in the flow and some of the air from under the Nappe would be carried downstream. Therefore, as the air under the Nappe becomes evacuated, a pressure less than atmospheric would be established in that region. This would draw the Nappe downward and cause higher velocities to occur near the weir crest. Therefore, greater flow would occur than indicated by use of Eqs. (13.9) and (13.10).

13.59 **Information and assumptions**

Water flows over a rectangular weir. $L = 10$ ft, $P = 3$ ft, and $H = 1.5$ ft

Find

Discharge: Q

Solution

$$\begin{aligned}
Q &= K(2g)^{0.5}LH^{3/2} \\
Q &= [0.40 + (1/2)(0.05)]\,[64.4]^{0.5} \times 10 \times 1.5^{3/2} \\
&= \underline{\underline{62.7 \text{ cfs}}}
\end{aligned}$$

13.60 **Information and assumptions**

Water (60 ^{o}F) flows into a reservoir through a venturi meter ($K = 1$, $A_o = 12$ in^2, $\Delta p = 10$ psi). Water flows out of the reservoir over a 60^o triangular weir.

Find

Head of weir: H

Solution

Discharge through the venturi meter.

$$\begin{aligned} Q &= KA_o\sqrt{2\Delta p/\rho} \\ &= 1 \times (12/144)\sqrt{2 \times 10 \times 144/1.94} \\ &= 3.21 \text{ ft}^3/\text{s} \end{aligned}$$

Weir equation.

$$\begin{aligned} Q &= 0.179\sqrt{2g}H^{5/2} \\ 3.21 &= 0.179\sqrt{64.4}H^{5/2} \\ H &= 1.38 \text{ ft} = \underline{\underline{16.5 \text{ in}}}. \end{aligned}$$

13.61 **Information and assumptions**

Water enters a tank through two pipes A and B. Water exits the tank through a rectangular weir.

Find

Is water level rising, falling or staying the same?

Solution

$$
\begin{aligned}
Q_{\text{out}} &= Q_{\text{weir}} = K(2g)^{0.5}LH^{3/2} \\
K &= 0.40 + 0.05(1/2) = 0.425 \\
Q_{\text{out}} &= 0.425(8.025)(2)(1) = 6.821 \text{ cfs} \\
Q_{\text{in}} &= V_A A_A + V_B A_B \\
&= 4(\pi/4)(1^2) + 4(\pi/4)(0.5^2) \\
&= \pi(1.25) = 3.927 \text{ cfs}
\end{aligned}
$$

$Q_{\text{in}} < Q_{\text{out}}$; therefore, water level is falling

13.62 **Information and assumptions**

Water exits an upper reservoir across a rectangular weir ($L/H_R = 3$, $P/H_R = 2$) and then into a lower reservoir. The water exits the lower reservoir through a 60^o triangular weir.

Assume steady flow.

Find

Ratio of head for the rectangular weir to head for the triangular weir: H_R/H_T

Solution

Weir equations

$$Q = (0.40 + .05(1/2))\sqrt{2g}(3H_R)H_R^{1.5} \tag{1}$$
$$Q = 0.179\sqrt{2g}H_T^{2.5} \tag{2}$$

Equate Eqs. (1) and (2)

$$\begin{aligned}(0.425\sqrt{2g}(3)H_R^{2.5} &= 0.179\sqrt{2g}H_T^{2.5} \\ (H_R/H_T)^{2.5} &= 0.179/(3 \times 0.425) \\ & \underline{\underline{H_R/H_T = 0.456}}\end{aligned}$$

13.63 **Information and assumptions**

For problem 13.62, the flow entering the upper reservoir is increased by 50%.

Goal

Describe what will happen both qualitatively and quantitatively

Solution

As soon as the flow is increased, the water level in the first reservoir will start to rise. It will continue to rise until the outflow over the rectangular weir is equal to the inflow to the reservoir. The same process will occur in the second reservoir until the outflow over the triangular weir is equal to the inflow to the first reservoir.

Calculations

Determine the increase in head on the rectangular weir with an increase in discharge of 50%.

Initial conditions: $H_R/P = 0.5$ so

$$K = 0.4 + .05 \times .5 = 0.425$$

Then

$$Q_{Ri} = 0.425\sqrt{2g}LH_{Ri}^{3/2} \tag{1}$$

Assume

$$K_f = K_i = 0.425 \text{ (first try)}$$

Then

$$Q_{Rf} = 0.425\sqrt{2g}LH_{Rf}^{3/2} \text{ (where } Q_{Rf} = 1.5Q_i) \tag{2}$$

Divide Eq. (2) by Eq. (1)

$$\begin{aligned} Q_{Rf}/Q_{Ri} &= (0.425L/0.425L)(H_{Rf}/H_{Ri})^{3/2} \\ H_{Rf}/H_{Ri} &= (1.5)^{2/3} = 1.31 \end{aligned}$$

Check K_i :

$$K = 0.40 + .05 \times 0.5 \times 1.31 = 0.433$$

Recalculate H_{Rf}/H_{Ri}.

$$H_{Rf}/H_{Ri} = ((0.425/0.433) \times 1.5)^{2/3} = 1.29$$

The final head on the rectangular weir will be 29% greater than the initial head. Now determine the increase in head on the triangular weir with a 50% increase in discharge.

$$\begin{aligned} Q_{Tf}/Q_{Ti} &= (H_{Tf}/H_{Ti})^{5/2} \\ \text{or } H_{Tf}/H_{Ti} &= (Q_{Tf}/Q_{Ti}) \\ &= (1.5)^{2/5} \\ &= \underline{\underline{1.18}} \end{aligned}$$

The head on the triangular weir will be 18% greater with the 50% increase in discharge.

13.64 **Information and assumptions**

A rectangular weir ($L = 3$ m) is situated in a canal. The water depth is 2 m and $Q = 6$ m^3/s.

Find

Necessary weir height: P

Solution

Weir equation

$$Q = K\sqrt{2g}LH^{3/2}$$

Assume $K \approx 0.41$ then

$$\begin{aligned} H &= (Q/(0.41\sqrt{2g} \times 3))^{2/3} \\ H &= (6/(0.41 \times \sqrt{2 \times 9.81} \times 3))^{2/3} = 1.10 \text{ m} \end{aligned}$$

Then

$$\begin{aligned} P &= 2.0 - 1.10 = 0.90 \text{ m} \\ \text{and } H/P &= 1.22 \end{aligned}$$

Check guessed K value:

$$K = 0.40 + 1.22 \times 0.05 = 0.461$$

Since this doesn't match, recalculate H:

$$H = (6/(0.461 \times \sqrt{2 \times 9.81} \times 3))^{2/3} = 0.986 \text{ m}$$

So height of weir

$$\begin{aligned} P &= 2.0 - 0.986 = 1.01 \text{ m} \\ H/P &= 0.976 \end{aligned}$$

Try again:

$$\begin{aligned} K &= 0.40 + 0.976 \times 0.05 = 0.449 \\ H &= (6/(0.449 \times \sqrt{2 \times 9.81} \times 3))^{2/3} = 1.00 \text{ m} \\ P &= 2.00 - 1.00 = \underline{\underline{1.00 \text{ m}}} \end{aligned}$$

13.65 **Information and assumptions**

Water flows over a 60^o triangular weir, $H = 1.5$ ft

Find

Discharge: Q

Solution

$$Q = 0.179\sqrt{2g}H^{5/2}$$

Then

$$Q = 0.179\sqrt{2 \times 32.2} \times (1.5)^{5/2} = \underline{\underline{3.96 \text{ ft}^3/\text{s}}}$$

13.66 **Information and assumptions**

Water flows over a 45^o triangular weir. $Q = 10$ cfm, discharge coefficient is 0.6. provided in problem statement

Find

Head on the weir: H

Solution

$$\begin{aligned} Q &= (8/15)C_d(2g)^{0.5}\tan(\theta/2)H^{5/2} \\ Q &= (8/15)(0.60)(64.4)^{0.5}\tan(22.5^\circ)H^{5/2} \\ Q &= 1.064H^{5/2} \\ H &= (Q/1.064)^{2/5} = (10/(60\times 1.064))^{2/5} = \underline{\underline{0.476 \text{ ft}}} \end{aligned}$$

13.67 **Information and assumptions**

A pump transports water from a well to a tank. The tank empties through a 60^o triangular weir. Details are provided in the textbook.

provided in problem statement

Find

Water level in the tank: h

Solution

$k_s/D = 0.001$; assume $f = 0.02$

Write the energy equation from well water surface to tank water surface.

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 0 + h_p &= 0 + 0 + (2 + h) + (V^2/2g)(k_e + (fL/D) + K_E) \end{aligned}$$

Inserting parameter values

$$\begin{aligned} 20 &= (2 + h) + (V^2/2g)(0.5 + (0.02 \times 2.5/0.05) + 1) \\ 18 &= h + 0.127V^2 \\ V &= ((18 - h)/0.127)^{0.5} \end{aligned}$$

$$\begin{aligned} Q &= VA = ((18 - h)/0.127)^{0.5}(\pi/4)(0.05)^2 \\ &= 0.00551(18 - h) \end{aligned} \quad (1)$$

Triangular Weir:

$$Q = 0.179\sqrt{2g}H^{2.5}$$

where $H = h - 1$. Then

$$Q = 0.179\sqrt{2g}(h - 1)^{2.5} = 0.793(h - 1)^{2.5} \quad (2)$$

To satisfy continuity, equate (1) and (2)

$$\begin{aligned} 0.00551(18 - h)^{0.5} &= 0.793(h - 1)^{2.5} \\ 0.00695(18 - h)^{0.5} &= (h - 1)^{2.5} \end{aligned}$$

Solve for h:

$$\underline{\underline{h = 1.24 \text{ m}}}$$

Also, upon checking Re we find our assumed f is OK.

13.68 **Information and assumptions**

A pitot tube is used to record data in subsonic flow. $p_t = 140$ kPa, $p = 100$ kPa, $T_t = 300$ K. provided in problem statement

Find

Mach number: M

Velocity: V

Solution

$$\begin{aligned}
p_t/p_1 &= (1 + (k - 1/2)M^2)^{k/(k-1)} \\
&= (1 + 0.2M^2)^{3.5} \text{ for air} \\
(140/100) &= (1 + 0.2M^2)^{3.5};\ M = \underline{\underline{0.710}} \\
T_t/T &= 1 + 0.2M^2 \\
T &= 300/1.10 = 273 \\
c &= \sqrt{(1.4)(287)(273)} = 331 \text{ m/s} \\
V &= Mc = (0.71)(331) = \underline{\underline{235 \text{ m/s}}}
\end{aligned}$$

13.69 **Goal**

Derive the Rayleigh supersonic Pitot tube formula

Solution

The purpose of the algebraic manipulation is to express p_1/p_{t_2} as a function of M_1 only. For convenience, express the group of variables below as

$$\begin{aligned} F &= 1 + ((k-1)/2)M^2 \\ G &= kM^2 - ((k-1)/2) \\ p_1/p_{t_2} &= (p_1/p_{t_1})(p_{t_1}/p_{t_2}) = (p_1/p_{t_1})(p_1/p_2)(F_1/F_2)^{k/k-1} \end{aligned}$$

From Eq. (12-38),

$$p_1/p_2 = (1 + kM_2^2)/(1 + kM_1^2)$$

So

$$p_1/p_{t_2} = (p_1/p_{t_1})((1 + kM_2^2)/(1 + kM_1^2))(F_1/F_2)^{k/k-1}$$

From Eq. (12-40), we have

$$(M_1/M_2) = ((1 + kM_1^2)/(1 + kM_2^2))(F_2/F_1)^{1/2}$$

Thus, we can write

$$(p_1/p_{t_2}) = (p_1/p_{t_1})(M_2/M_1)(F_1/F_2)^{k+1/(2(k-1))}$$

But, from Eq. (12-41)

$$M_2 = (F_1/G_1)^{1/2}$$

Also, $p_1/p_{t_1} = 1/(F_1^{k/k-1})$. So

$$\begin{aligned} p_1/p_{t_2} &= 1/(F_1^{k/k-1})(F_1^{1/2}/G_1^{1/2})(1/M_1)(F_1/F_2)^{k+1/(2(k-1))} \\ &= (G_1^{-1/2}/M_1)F_2^{-(k+1)/2(k-1)} \end{aligned}$$

However,

$$F_2 = 1 + ((k-1)/2)M_2^2 = 1 + ((k-1)/2)(F_1/G_1) = (((k+1)/2)M_1)^2/G_1$$

Substituting for F_2 in expression for p_1/p_{t_2} gives

$$p_1/p_{t_2} = (1/M_1)(G_1^{1/k-1})/((k+1)/2M_1)^{k+1/k-1}$$

Multiplying numerator and denominator by $(2/k+1)^{1/k-1}$ gives

$$p_1/p_{t_2} = \frac{\{[2kM_1^2/(k+1)] - (k-1)/(k+1)\}^{1/(k-1)}}{\{[(k+1)/2]M_1^2\}^{k/(k-1)}}$$

13.70 **Information and assumptions**

A pitot tube is used in supersonic airflow. $p = 54$ kPa, $p_t = 200$ kPa, $T_t = 350$ K.

Find

Mach number: M_1

Velocity: V_1

Solution

Rayleigh pitot tube formula

$$\begin{aligned} p_1/p_{t_2} &= \frac{\{[2kM_1^2/(k+1)] - (k-1)/(k+1)\}^{1/(k-1)}}{\{[(k+1)/2]M_1^2\}^{k/(k-1)}} \\ 54/200 &= (1.1667M_1^2 - 0.1667)^{2.5}/(1.2M_1^2)^{3.5} \end{aligned}$$

and solving for M_1 gives $M_1 = \underline{\underline{1.79}}$

$$\begin{aligned} T_1 &= T_t/\left[1 + 0.5(k-1)M_1^2\right] \\ T_1 &= 350/(1 + 0.2(1.79)^2) = 213 \text{ K} \\ c_1 &= \sqrt{kRT} = \sqrt{(1.4)(287)(213)} = 293 \text{ m/s} \\ V_1 &= M_1 c_1 = 1.79 \times 293 = \underline{\underline{521 \text{ m/s}}} \end{aligned}$$

13.71 **Information and assumptions**

A venturi meter is used to measure flow of helium. Details are provided in the textbook.

Find

Mass flow rate: $\dot{m}$

Solution

The conditions are $p_1 = 120$ kPa $p_2 = 80$ kPa $K = 1.66$ $D_2/D_1 = 0.5$, $T_1 = 17°C$ $R = 2,077$ J/kg·K

$$\begin{aligned}\rho_1 &= p_1/(RT_1) = 120 \times 10^3/(2,077 \times 290) = 0.199 \text{ kg/m}^3 \\ p_1/\rho_1 &= 6.03 \times 10^5\end{aligned}$$

Using Eq. (13-16),

$$\begin{aligned}V_2 &= ((5)(6.03 \times 10^5)(1 - 0.666^{0.4})/(1 - (0.666^{1.2} \times 0.54)))^{1/2} = 686 \text{ m/s} \\ \rho_2 &= (p_2/p_1)^{1/k}\rho_1 = (0.666)^{0.6}\rho_1 = 0.784\rho_1 = 0.156 \text{ kg/m}^3 \\ \dot{m} &= \rho_2 A_2 V_2 = (0.156)(\pi/4 \times 0.005^2)(686) = \underline{\underline{0.0021 \text{ kg/s}}}\end{aligned}$$

13.72 **Information and assumptions**

An orifice is used to measure the flow of methane. Details are provided in the textbook.

Find

Mass flow rate: $\dot{m}$

Solution

The conditions are $p_1 = 150$ kPa, $p_2 = 110$ kPa, $T = 300$ K, $R = 518$ J/kgK, $k = 1.31$, $d = 0.8$ cm, $d/D = 0.5$ and $\nu = 1.6 \times 10^{-5}$ m^2/s

$$\begin{aligned} \rho_1 &= 150 \times 10^3/(518 \times 300) = 0.965 \text{ kg/m}^3 \\ 2g\Delta h &= 2\Delta p/\rho_1 = (2(40 \times 10^3))/0.965 = 8.29 \times 10^4 \\ \text{Re}/K &= ((0.008)/(1.6 \times 10^{-5}))\sqrt{8.29 \times 10^4} = 1.43 \times 10^5 \end{aligned}$$

From Fig. 13.13 $K = 0.62$

$$\begin{aligned} V &= 1 - ((1/1.31)(1 - (110/150))(0.41 + 0.35(0.4)^4)) = 0.915 \\ \dot{m} &= (0.62)(0.915)(0.785)(0.008)^2\sqrt{(2)(0.965)(40 \times 10^3)} = \underline{\underline{0.00792 \text{ kg/s}}} \end{aligned}$$

13.73 **Information and assumptions**

Air flows through a 1 cm diameter orifice in a 2 cm pipe. The pressure readings for the orifice are 150 kPa (upstream) and 100 kPa (downstream). For air ρ(upstream) $= 1.8$ kg/m^3, $\nu = 1.8 \times 10^{-5}$ m^2/s, $k = 1.4$.

Find

Mass flow rate

Solution

$$A_0/A_1 = (1/2)^2 = 0.25; \quad A_0 = 7.85 \times 10^{-5} \text{ m}^2$$

Expansion factor:

$$\begin{aligned}
Y &= 1 - \{(1/k)(1-(p_2/p_1))(0.41 + 0.35(A_0/A_1)^2)\} \\
Y &= 1 - \{(1/1.4)(1-(100/150)(0.41 + 0.35(.25)^2)\} \\
&= 0.897 \\
\dot{m} &= Y A_0 K (2\rho_1(p_1 - p_2))^{0.5} \\
\text{Re}_d/K &= (2\Delta p/\rho)^{0.5} d/\nu = (2 \times 50 \times 10^3/1.8)^{0.5}(.01/(1.8 \times 10^{-5})) \\
&= 236 \times 556 = 1.31 \times 10^5
\end{aligned}$$

From Fig. 13.13 $K = 0.63$

$$\dot{m} = (0.897)(7.85 \times 10^{-5})(0.63)(2 \times 1.8 \times 50 \times 10^3)^{0.5} = \underline{\underline{1.88 \times 10^{-2} \text{ kg/s}}}$$

13.74 **Information and assumptions**

Hydrogen (100 kPa, 15 ^oC) flows through an orifice ($d/D = 0.5$, $K = 0.62$) in a 2 cm pipe. The pressure drop across the orifice is 1 kPa

Find

mass flow rate

Solution

$$\begin{aligned} d/D &= 0.50 \\ d &= 0.5 \times 0.02 \text{ m} = 0.01 \text{ m} \end{aligned}$$

From Table A.2 for hydrogen @ $T = 15°C = 288K, k = 1.41$, and $\rho = 0.0851$ kg/m^3.

$$\begin{aligned} A_0 &= (\pi/4)(0.01)^2 = 7.85 \times 10^{-5} \text{ m}^2 \\ \dot{m} &= YA_0K(2\rho_1\Delta p) \\ \dot{m} &= (1)(7.85 \times 10^{-5})(0.62)(2(0.0851)(1,000))^{0.5} \\ \dot{m} &= \underline{\underline{6.35 \times 10^{-4} \text{ kg/s}}} \end{aligned}$$

13.75 **Information and assumptions**

Natural gas (50 psig, 70 ^{o}F) flows in a pipe. A hole ($d = 0.25$ in) leaks gas, Assume that the hole shape is like a truncated nozzle. $p_{atm} = 14$ psia

Find

Rate of mass flow out of the leak: $\dot{m}$

Solution

Hole area

$$A = (\pi/4)(0.25)^2 = 0.049 \text{ in}^2 = 3.41 \times 10^{-4} \text{ ft}^2$$

For natural gas: $k = 1.31,\ R = 3,098$ ft-lbf/slug $^\circ R$. Also,

$$\begin{aligned} p_t &= (50 + 14) = 64 \text{ psia } = 9,216 \text{ psfa} \\ T &= (460 + 70) = 530\ ^\circ\text{R} \end{aligned}$$

Thus,

$$\begin{aligned} \dot{m} &= (p_t A_*/\sqrt{RT})(k^{0.5})(2/(k+1))^{(k+1/(2(k-1))} \\ &= ((9,216 \times 3.41 \times 10^{-4})/\sqrt{3,098 \times 530})(1.31)^{0.5}(2/2.31)^{3.726} \\ &= \underline{\underline{0.0164 \text{ slug/s}}} \\ &= \underline{\underline{0.0528 \text{ lbm/s}}} \end{aligned}$$

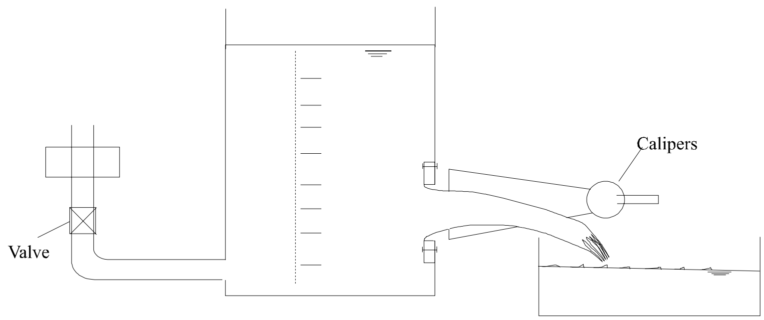
13.76 Some of the physical effects that might occur are:

a) Abrasion might cause the weir crest to be rounded and this would undoubtedly produce greater flow than indicated by Eqs. 13.9 and 13.10 (see the answer to problem 13.58)

b) If solid objects such as floating sticks come down the canal and hit the weir they may dent the weir plate. Such dents would be slanted in the downstream direction and may even cause that part of the weir crest to be lower than the original crest. In either case these effects should cause the flow to be contracted less than before thus increasing the flow coefficient.

c) Another physical effect that might occur in an irrigation canal is that sediment might collect upstream of the weir plate. Such sediment accumulation would force flow away from the bottom before reaching the weir plate. Therefore, with this condition less flow will be deflected upward by the weir plate and less contraction of the flow would occur. With less contraction the flow coefficient would be increased. For all of the physical effects noted above flow would be increased for a given head on the weir.

13.77 A jet to be studied can be produced by placing an orifice in the side of a rectangular tank as shown below.



The plate orifice could be machined from a brass plate so that the upstream edge of the orifice would be sharp. The diameter of the orifice could be measured by inside calipers and a micrometer. The contracted jet could be measured by outside calipers and micrometer. Thus the coefficient of contraction could be computed as $C_c = (d_j/d)^2$. However, there may be more than desired error in measuring the water jet diameter by means of a caliper. Another way to estimate d_j is to solve for it from A_j where A_j is obtained from $A_j = Q/V_j$. Then $d_j = (4A_j/\pi)^{\frac{1}{2}}$. The discharge, Q, could be measured by means of an accurate flow meter or by a weight measurement of the flow over a given time interval. The velocity at the vena contracta could be fairly accurately determined by means of the Bernoulli equation. Measure the head on the orifice and compute V_j from $V_j = \sqrt{2gh}$ where h is the head on the orifice. Because the flow leading up to the vena contracta is converging it will be virtually irrotational; therefore, the Bernoulli equation will be valid.

Another design decision that must be made is how to dispose of the discharge from the orifice. The could be collected into a tank and then discharged into the lab reservoir through one of the grated openings.

13.78 First, decisions have to be made regarding the physical setup. This should include:

a) How to connect the 2 in. pipe to the water source.

b) Providing means of discharging flow back into the lab reservoir. Probably have a pipe discharging directly into reservoir through one of the grated openings.

c) Locating control valves in the system

d) Deciding a length of 2" pipe on which measurements will be made. It is desirable to have enough length of pipe to yield a measurable amount of head loss.

To measure the head loss, one can tap into the pipe at several points along the pipe (six or eight points should be sufficient). The differential pressure between the upstream tap and downstream tap can first be measured. Then measure the differential pressure between the next tap and the downstream tap, etc., until the pressure difference between the downstream tap and all others has been completed. From all these measurements the slope of the hydraulic grade line could be computed.

The discharge could be measured by weighing a sample of the flow for a period of time and then computing the volume rate of flow. Or the discharge could be measured by an electromagnetic flow meter if one is installed in the supply pipe.

The diameter of the pipe should be measured by inside calipers and micrometer. Even though one may have purchased 2 inch pipe, the nominal diameter is usually not the actual diameter. With this diameter one can calculate the cross-sectional area of the pipe. Then the mean velocity can be computed for each run: $V = Q/A$.

Then for a given run, the resistance coefficient, f, can be computed with Eq. (10.22). Other things that should be considered in the design:

a) Make sure the pressure taps are far enough downstream of the control valve or any other pipe fitting so that uniform flow is established in the section of pipe where measurements are taken.

b) The differential pressure measurements could be made by either transducers or manometers or some combination.

c) Appropriate valving and manifolding could be designed in the system so that only one pressure transducer or manometer is needed for all pressure measurements.

d) The water temperature should be taken so that the specific weight of the water can be found.

e) The design should include means of purging the tubing and manifolds associated with the pressure differential measurements so that air bubbles can be eliminated from the measuring system. Air bubbles often produce erroneous readings.

13.79 Most of the design setup for this equipment will be the same as for Prob. (13.78) except that the valve to be tested would be placed about midway along the two inch pipe. Pressure taps should be included both upstream and downstream of the valve so that hydraulic grade lines can be established both upstream and downstream of the valve (see Fig. 10.15). Then as shown in Fig. (10.15) the head loss due to the valve can be evaluated. The velocity used to evaluate K_v is the mean velocity in the 2 in. pipe so it could be evaluated in the same manner as given in the solution for Prob. (13.78).

13.80 **Information and assumptions**

A stagnation tube is used to measure air speed $\rho_{\rm air} = 1.25$ kg/m^3, $d = 2$ mm, $C_p = 1.00$
Deflection on an air-water manometer, $h = 1$ mm. The only uncertainty in the manometer reading, $U_h = 0.1$ mm

Find

Air Speed: V
Uncertainty in air speed: U_V

Solution

$$
\begin{aligned}
V &= \left(\frac{2\Delta\rho}{\rho_{air}C_p}\right)^{1/2} \\
\Delta p &= h\gamma_w
\end{aligned}
$$

Combining equations

$$
\begin{aligned}
V &= \left(\frac{2\gamma_w h}{\rho_{air}C_p}\right)^{1/2} = \left(\frac{(2)(9,810)(0.001)}{(1.25)(1.00)}\right)^{1/2} \\
V &= \underline{\underline{3.96 \text{ m/s}}}
\end{aligned}
$$

Uncertainty formula
The derivative is

$$
\begin{aligned}
U_V &= \frac{\partial V}{\partial h}U_h \\
\frac{\partial V}{\partial h} &= \sqrt{\frac{2\gamma_w}{\rho_a C_p}}\frac{1}{2\sqrt{h}}
\end{aligned}
$$

Combining equations gives

$$
\begin{aligned}
\frac{U_V}{V} &= \frac{U_h}{2h} = \frac{0.1}{2\times 1.0} = 0.05 \\
\text{so } U_V &= 0.05V \\
&= 0.05\times 3.96 \\
&= \underline{\underline{0.198 \text{ m/s}}}
\end{aligned}
$$

13.81 **Information and assumptions**

Water flows through a 6 in. orifice situated in a 12 in. pipe. On a mercury manometer, $\Delta h = 1$ ft-Hg. The uncertainty values are $U_K = 0.03$, $U_H = 0.5$ in.-Hg, $U_d = 0.05$ in.

Find

Discharge: Q

Uncertainty in discharge: U_Q

Solution

Definition of piezometric head

$$\Delta h = \left(\frac{p_1}{\gamma_w} + z_1\right) - \left(\frac{p_2}{\gamma_w} + z_2\right)$$

Manometer equation

$$\begin{aligned} p_1 + \gamma_w z_1 - \gamma_{H_g} 1 \text{ ft} - \gamma_w (z_2 - 1 \text{ ft}) &= p_2 \\ \frac{p_1 - p_2}{\gamma_w} &= -(z_1 - z_2) + \left(\frac{\gamma_{Hg}}{\gamma_w}\right) 1 \text{ ft} - 1 \text{ ft} \end{aligned}$$

Combining equations

$$\begin{aligned} \Delta h &= (1.0 \text{ ft}) \left(\frac{\gamma_{Hg}}{\gamma_w} - 1\right) \\ &= 1.0(13.55 - 1) = 12.55 \text{ ft of water} \end{aligned}$$

Uncertainty in Δh

$$\begin{aligned} U_{\Delta h} &= \left(\frac{0.5}{12}\text{ft}\right) \left(\frac{\gamma_{Hg}}{\gamma_w} - 1\right) = \left(\frac{0.5}{12}\right) (13.55 - 1) \\ &= 0.523 \text{ ft of water} \end{aligned}$$

Orifice discharge expression

$$\begin{aligned} Q &= K \frac{\pi}{4} d^2 \sqrt{2g\Delta h} \\ \text{where } K &= 0.625 \text{ (from problem 13.20)} \\ \text{Thus, } Q &= 0.625 \times \frac{\pi}{4} \times 0.5^2 \sqrt{2 \times 32.2 \times 12.55} \\ &= \underline{\underline{3.49 \text{ cfs}}} \end{aligned}$$

Uncertainty equation applied to the discharge relationship

$$\begin{aligned}
\left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{\frac{\partial Q}{\partial K}U_K}{Q}\right)^2 + \left(\frac{\frac{\partial Q}{\partial d}U_d}{Q}\right)^2 + \left(\frac{\frac{\partial Q}{\partial_\Delta h}U_{\Delta h}}{Q}\right)^2 \\
\left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{U_K}{K}\right)^2 + \left(\frac{2U_d}{d}\right)^2 + \left(\frac{U_{\Delta h}}{2\Delta h}\right)^2 \\
\left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{.03}{0.625}\right)^2 + \left(\frac{2 \times 0.05}{6}\right)^2 + \left(\frac{.523}{2 \times 12.55}\right)^2 \\
\frac{U_Q}{Q} &= 0.055 \\
U_Q &= 0.055 \times 3.49 = \underline{\underline{0.192 \text{ cfs}}}
\end{aligned}$$

13.82 **Information and assumptions**

A rectangular weir ($L = 10$ ft, $P = 3$ ft, $H = 1.5$ ft) is used to measure discharge. The uncertainties are $U_k = 5\%$, $U_H = 3$ in., $U_L = 1$ in.

Find

Discharge: Q

Uncertainty in discharge:U_Q

Solution

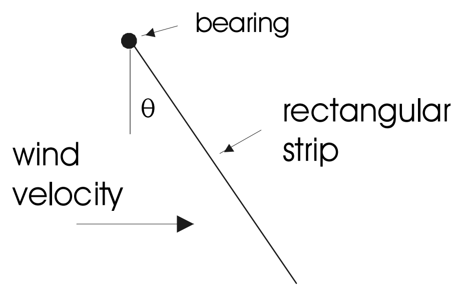
Rectangular Weir equation

$$
\begin{aligned}
K &= 0.4 + 0.05\frac{H}{P} = 0.4 + 0.05 \times \left(\frac{1.5}{3.0}\right) = 0.425 \\
Q &= K\sqrt{2g}LH^{3/2} = (0.625)\sqrt{2 \times 32.2}(10)(1.5)^{3/2} \\
Q &= \underline{\underline{62.7 \text{ cfs}}}
\end{aligned}
$$

Uncertainty equation

$$
\begin{aligned}
U_Q^2 &= \left(\frac{\partial Q}{\partial K}U_K\right)^2 + \left(\frac{\partial Q}{\partial L}U_L\right)^2 + \left(\frac{\partial Q}{\partial H}U_H\right)^2 \\
\left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{U_K}{K}\right)^2 + \left(\frac{U_L}{L}\right)^2 + \left(\frac{3}{2} \times \frac{U_H}{H}\right)^2 \\
&= (.05)^2 + \left(\frac{1/12}{10}\right)^2 + \left(\frac{3}{2} \times \frac{3/12}{1.5}\right)^2 \\
&= 0.255^2 \\
\text{Thus, } U_Q &= 0.255Q = (0.255)(62.7) \\
&= \underline{\underline{16.0 \text{ cfs}}}
\end{aligned}
$$

13.83 The are probably many different approaches to this design problem. One idea is to support a thin strip of material in an airstream from a low friction bearing as shown in the figure.



The drag force on the strip tends to rotate the strip and the angle of rotation will be related to the flow velocity. Assume the strip has an area S, a thickness δ and a material density of ρ_m. Also assume the length of the strip is L. Assume that the force normal to the strip is given by the drag force associated with the velocity component normal to the surface and that the force acts at the mid point of the strip. The moment produced by the flow velocity would be

$$Mom = F_D L/2 = C_D S(\rho_a V_0^2 \cos^2\theta/2)L/2$$

where θ is the deflection of the strip, ρ_a is the air density and V_0 is the wind velocity. This moment is balanced by the moment due to the weight of the strip

$$Mom = Mg(L/2)\sin\theta$$

Equating the two moments gives

$$Mg(L/2)\sin\theta = C_D S(\rho_a V_0^2 \cos^2\theta/2)L/2$$

Solving for V_0 gives

$$\begin{aligned} V_0^2 &= \frac{2Mg\sin\theta}{C_D S\rho_a \cos^2\theta} \\ V_0 &= \sqrt{\frac{2Mg\sin\theta}{C_D S\rho_a \cos^2\theta}} \end{aligned}$$

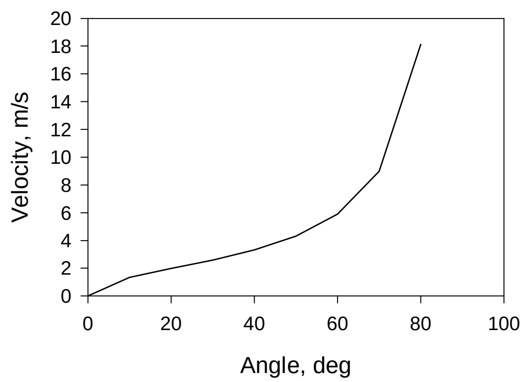
But the mass of the strip can be equated to $\rho_m S\delta$ so the equation for velocity reduces to

$$V_0 = \sqrt{\frac{2\rho_m \delta g\sin\theta}{C_D \rho_a \cos^2\theta}}$$

Assume the strip is a plastic material with a density of 800 kg/m^3 and a thickness of 1 mm. Also assume the drag coefficient corresponds to a rectangle with an aspect ratio of 10 which from Table 11.1 is 1.3. Assume also that a deflection of 10^o can be measured with reasonable accuracy. Assume also that the air density is 1.2 kg/m^3. The wind velocity would be

$$\begin{aligned} V_0 &= \sqrt{\frac{2 \times 800 \times 0.001 \times 9.81 \times 0.174}{1.3 \times 1.2 \times 0.985^2}} \\ &= 1.3 \text{ m/s} \end{aligned}$$

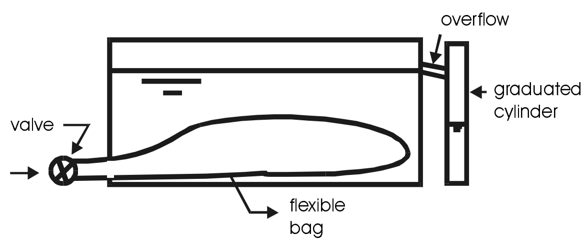
This is close to the desired lower limit so is a reasonable start. The lower limit can be extended by using a lighter material or possibly a wire frame with a thin film of material. The relationship between velocity and angle of deflection would be



This plot suggests that the upper range of 10 m/s could be reached with a deflection of about 70 degrees. The simple model used here is only an approximation for design purposes. An actual instrument would have to be calibrated.

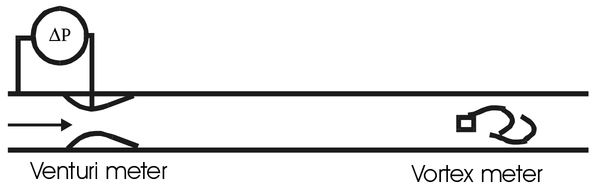
Other features to be considered would be a damping system for the bearing to handle flow velocity fluctuations and an accurate method to measure the deflection. The design calculations presented here show the concept is feasible. More detailed design considerations would then follow.

13.84 One approach may be to use a very small venturi meter but instrumentation would be difficult (installing pressure taps, etc.). A better approach may be the use of some volume displacement scheme. One idea may be to connect the flow to a flexible bag immersed in a water (or some liquid) bath as shown. As the gas enters the bag, the bag will expend displacing the liquid in the tank. The overflow of the tank would discharge into a graduated cylinder to measure the displacement as a function of time.



Features which must be considered are 1) the volume of the bag must be chosen such that pressure in the bag does not increase with increased displacement, 2) evaporation from the surface must be minimized and 3) a valve system has to be designed such that the flow can be diverted to the bag for a given time and then closed.

13.85 The two flow meters must be selected such that one depends on the density of the fluid and the other is independent of the fluid density. One such combination would be the venturi meter and the vortex meter as shown in the diagram.



The discharge in the venturi meter is given by

$$Q = KA_o\sqrt{\frac{2\Delta p}{\rho}}$$

while the velocity measured by a vortex meter is

$$V = \frac{nD}{St}$$

where D is the size of the element. For a calibrated vortex flow meter one has

$$Q = Cf$$

where C is a calibration constant and f is the shedding frequency. The calibration constant is essentially independent of Reynolds number over a wide range of Reynolds number. Thus we have

$$Cf = KA_o\sqrt{\frac{2\Delta p}{\rho}}$$

Solving for ρ

$$\rho = \frac{2\Delta p(KA_o)^2}{(Cf)^2}$$

The flow coefficient does depend weakly on Reynolds number so there may be a source of error if K is not known exactly. If the viscosity of the fluid is known, the Reynolds number could be calculated and the above equation could be used for an iterative solution.

Chapter Fourteen

14.1 **Information and Assumptions**

provided in problem statement

Find

thrust obtained from propeller

Solution

$$C_T = F_T/\rho D^4 n^2 = 0.048$$

Thus,

$$F_T = 0.048\rho D^4 n^2 = 0.048 \times 1.05 \times 3^4 \times (1,400/60)^2 = \underline{\underline{2,223}} \text{ N}$$

14.2 **Information and Assumptions**

provided in problem statement

Find

thrust obtained from propeller

Solution

$$V_0/nD = (80,000/3,600)/((1,400/60) \times 3) = 0.317$$

From Fig. 14-3, $C_T = 0.020$. Then

$$F_T = 0.020 \times \rho D^4 n^2 = 0.020 \times 1.05 \times 3^4 \times (1,400/60)^2 = \underline{\underline{926 \text{ N}}}$$

Also from Fig. 14-3, $C_p = 0.011$ and

$$P = 0.011 \rho D^5 n^3 = 0.011 \times 1.05 \times 3^5 \times (1,400/60)^3 = \underline{\underline{35.7 \text{ kW}}}$$

14.3 **Information and Assumptions**

provided in problem statement

Find

thrust produced by propeller

Solution

$$\begin{aligned} n &= 1,000/60 = 16.67 \text{ rev/sec} \\ V_0 &= 30 \text{ mph} = 44 \text{ fps} \\ V_0/nD &= 44/(16.67 \times 8) = 0.33 \end{aligned}$$

From Fig. 14-3 $C_T = 0.0182;\ C_p = 0.011$ so

$$\begin{aligned} F_T &= 0.0182 \times 0.0024 \times 8^4 \times 16.67^2 = \underline{\underline{49.7 \text{ lbf}}} \\ P &= 0.011 \times 0.0024 \times 8^5 \times 16.67^3 = 4,005 \text{ ft-lb/sec} = \underline{\underline{7.3 \text{ hp}}} \end{aligned}$$

If $V_0 = 0, C_T = 0.0475$ so

$$F_T = 0.0475 \times 0.0024 \times 8^4 \times 16.67^2 = \underline{\underline{130 \text{ lbf}}}$$

14.4 **Information and Assumptions**

provided in problem statement

Find

angular speed of propeller

Solution

$$V_0 = 30 \text{ mph} = 44 \text{ fps}$$

From Fig. 14-3, at maximum efficiency, $V_0/(nD) = 0.285$

$$\begin{aligned} n &= 44/(0.285 \times 8) = 19.30 \text{ rps} \\ N &= \underline{\underline{1,158 \text{ rpm}}} \end{aligned}$$

14.5 **Information and Assumptions**

provided in problem statement

Find

thrust and power output

Solution

At maximum efficiency $C_T = 0.023$ and $C_p = 0.012$

$$\begin{aligned} F_T &= 0.023 \times 0.0024 \times 6^4 \times 25.73^2 = \underline{\underline{47.4 \text{ lbf}}} \\ P &= 0.012 \times 0.0024 \times 6^5 \times 25.73^3 = 3,815 \text{ ft-lbf/s} = \underline{\underline{6.94 \text{ hp}}} \end{aligned}$$

14.6 **Information and Assumptions**

provided in problem statement

Find

diameter of propeller and speed of aircraft

Solution

$$
\begin{aligned}
P &= C_p \rho n^3 D^5 \\
\rho &= p/RT = 60 \times 10^3/((287)(273)) = 0.766 \text{ kg/m}^3 \\
T &= C_T \rho n^2 D^4 \\
T &= \text{Drag} = \text{Lift}/30 = (1,200)(9.81)/(30) = 392 \text{ N} \\
392 &= (0.025)(0.766)(3,000/60)^2 D^4 \\
D &= \underline{\underline{1.69 \text{ m}}} \\
L &= W = C_L (1/2) \rho V_0^2 S \\
L/(C_L S) &= (\rho V_0^2/2) = (1,200)(9.81)/((0.40)(10)) = 2,942 \\
V_0^2 &= (2,942)(2)/(0.766) = 7,681 \\
V_0 &= \underline{\underline{87.6 \text{ m/s}}}
\end{aligned}
$$

14.7 **Information and Assumptions**

provided in problem statement

Find

maximum allowable angular speed

Solution

$$\begin{aligned} V_{\text{tip}} &= 0.9c = 0.9 \times 335 = 301.5 \text{ m/s} \\ V_{\text{tip}} &= \omega r = n(2\pi)r \\ n &= 301.5/(2\pi r) = 301.5/(\pi D) \text{ rev/s} \\ N &= 60 \times n \text{ rpm} \end{aligned}$$

D (m)	N (rpm)
2	$\underline{\underline{2,879}}$
3	$\underline{\underline{1,919}}$
4	$\underline{\underline{1,440}}$

14.8 **Information and Assumptions**

provided in problem statement

Find

angular speed of propeller

At maximum efficiency $V_0/(nD) = 0.285$. Then

$$
\begin{aligned}
n &= V_0/(0.285D) = (40,000/3,600)/(0.285 \times 2) = 19.5 \text{ rev/s} \\
N &= 19.5 \times 60 = \underline{\underline{1,170 \text{ rpm}}}
\end{aligned}
$$

14.9 **Information and Assumptions**

provided in problem statement

Find

thrust and power input

Solution

At maximum efficiency $C_T = 0.023$ and $C_p = 0.012$

$$\begin{aligned} F_T &= 0.023 \times 1.1 \times 2^4 \times (19.5)^2 = \underline{\underline{154 \text{ N}}} \\ P &= 0.012 \times 1.1 \times 2^5 \times (19.5)^3 = \underline{\underline{3.13 \text{ kW}}} \end{aligned}$$

14.10 **Information and Assumptions**

provided in problem statement

Find

initial acceleration

Solution

$$C_T = 0.048$$

Thus

$$\begin{aligned} F_T &= 0.048\rho D^4 n^2 = 0.048 \times 1.1 \times 2^4 \times (1,000/60)^2 = 235 \text{ N} \\ a &= F/m = 235/300 = \underline{\underline{0.782 \text{ m/s}^2}} \end{aligned}$$

14.11 **Information and Assumptions**

provided in problem statement

Find

the discharge

Solution

$$\begin{aligned} n &= 1{,}000/60 = 16.67 \text{ rev/s} \\ C_H &= \Delta hg/D^2n^2 = 3 \times 9.81/((0.4)^2 \times (16.67)^2) = 0.662 \end{aligned}$$

From Fig. 14-6, $C_Q = Q/(nD^3) = 0.625$. Then

$$Q = 0.625 \times 16.67 \times (0.4)^3 = \underline{\underline{0.667 \text{ m}^3/\text{s}}}$$

14.12 **Information and Assumptions**

provided in problem statement

Find

discharge and power demand

Solution

$$\begin{aligned}
n &= 690/60 = 11.5 \text{ rev/s} \\
C_H &= \Delta hg/(n^2D^2) = 10 \times 9.81/((0.712)^2(11.5)^2) = 1.6 \\
\text{from Fig. 14-6, } C_Q &= 0.40 \text{ and } C_p = 0.76 \\
Q - C_Q n D^3 &= 0.40 \times 11.5 \times 0.712^3 = \underline{\underline{1.66 \text{ m}^3/\text{s}}} \\
\text{Power} &= C_p \rho D^5 n^3 = 0.76 \times 1,000 \times 0.712^5 \times 11.5^3 = \underline{\underline{211 \text{ kW}}}
\end{aligned}$$

From Fig. 14-6, $C_Q = 0.40$ and $C_p = 0.76$. Then

$$\begin{aligned}
Q &= C_Q n D^3 = 0.40 \times 11.5 \times 0.712^3 = \underline{\underline{1.66 \text{ m}^3/\text{s}}} \\
P &= C_p \rho D^5 n^3 = 0.76 \times 1,000 \times 0.712^5 \times 11.5^3 = \underline{\underline{211 \text{ kW}}}
\end{aligned}$$

14.13 **Information and Assumptions**

provided in problem statement

Find

discharge and power required

Solution

$$
\begin{aligned}
D &= 35.6 \text{ cm} \\
n &= 11.5 \text{ rev/s}
\end{aligned}
$$

Energy equation from the reservoir surface to the center of the pipe at the outlet,

$$p_1/\gamma + V_1^2/(2g) + z_1 + h_p = p_2/\gamma + V_2^2/(2g) + z_2 + \sum h_L$$

$$h_p = 21.5 - 20 + [Q^2/(A^2 2g)](1 + fL/D + k_e + k_b)$$

$$L = 64 \text{ m}$$

Assume $f = 0.014,\ r_b/D = 1$. From Table 10-3, $k_b = 0.35,\ k_e = 0.1$

$$
\begin{aligned}
h_p &= 1.5 + [Q^2((0.014(64)/0.356) + 0.35 + 0.1 + 1)]/[2(9.81)(\pi/4)^2(0.356)^4] = 1.5 + 20.42 \\
C_Q &= Q/(nD^3) = Q/[(11.5)(0.356)^3] = 1.93Q \\
h_p &= C_H n^2 D^2/g = C_H(11.5)^2(0.356)^2/9.81 = 1.71 C_H
\end{aligned}
$$

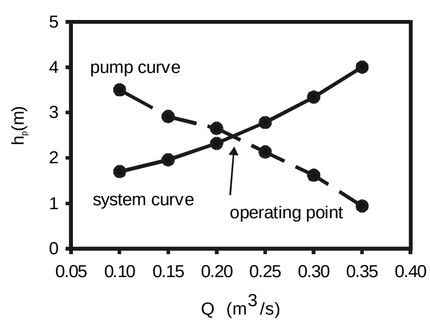
Q(m^3/s)	C_Q	C_H	h_{p1} (m)	h_{p2} (m)
0.10	0.193	2.05	1.70	3.50
0.15	0.289	1.70	1.96	2.91
0.20	0.385	1.55	2.32	2.65
0.25	0.482	1.25	2.78	2.13
0.30	0.578	0.95	3.34	1.62
0.35	0.675	0.55	4.00	0.94

Then plotting the system curve and the pump curve, we obtain the operating condition:

$$Q = 0.22 \text{ m}^3/\text{s}$$

From Fig. 14.7

$$P = \underline{\underline{6.5 \text{ kW}}}$$



14.14 **Information and Assumptions**

provided in problem statement

Find

discharge and power required

Solution

The system curve will be the same as in Prob. 14.13

$$\begin{aligned} C_Q &= Q/[nD^3] = Q/[15(0.356)^3] = 1.48Q \\ h_p &= C_H n^2 D^2/g = C_H (15)^2 (0.356)^2/9.81 = 2.91 C_H \end{aligned}$$

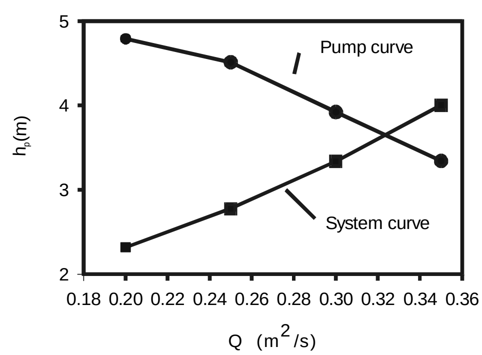
Q	C_Q	C_H	h_p
0.20	0.296	1.65	4.79
0.25	0.370	1.55	4.51
0.30	0.444	1.35	3.92
0.35	0.518	1.15	3.34

Plotting the pump curve with the system curve gives the operating condition;

$$\begin{aligned} Q &= \underline{\underline{0.32 \text{ m}^3/\text{s}}} \\ C_Q &= 1.48(0.32) = 0.474 \end{aligned}$$

Then from Fig. 14-6, $C_p = 0.70$

$$C_p n^3 D^3 \rho = 0.70(15)^3 (0.356)^5 1{,}000 = \underline{\underline{13.5 \text{ kW}}}$$



14.15 **Information and Assumptions**

provided in problem statement

Find

power required

Solution

At maximum efficiency, from Fig. 14-6, $C_Q = 0.64; C_p = 0.60$; and $C_H = 0.75$

$$
\begin{aligned}
D &= 1.67 \text{ ft} \\
n &= 1,100/60 = 18.33 \text{ rev/s} \\
Q &= C_Q n D^3 = 0.64 \times 18.33 \times 1.67^3 = \underline{\underline{54.6 \text{ cfs}}} \\
\Delta h &= C_H n^2 D^2/g = 0.75 \times 18.33^2 \times 1.67^2/32.2 = \underline{\underline{21.8 \text{ ft}}} \\
P &= C_p \rho D^5 n^3 = 0.60 \times 1.94 \times 1.67^5 \times 18.33^3 = 93,116 \text{ ft-lbf/sec} = \underline{\underline{169.3 \text{ hp}}}
\end{aligned}
$$

14.16 **Information and Assumptions**

provided in problem statement

Find

power required

Solution

At maximum efficiency, from Fig. 14-6, $C_Q = 0.64;\ C_p = 0.60;\ C_H = 0.75$

$$\begin{aligned}
Q &= C_Q n D^3 = 0.64 \times 45 \times 0.5^3 = \underline{\underline{3.60 \text{ m}^3/\text{s}}} \\
\Delta h &= C_H n^2 D^2/g = 0.75 \times 45^2 \times 0.5^2/9.81 = \underline{\underline{38.7 \text{ m}}} \\
P &= C_p \rho D^5 n^3 = 0.60 \times 1,000 \times 0.5^5 \times 45^3 = \underline{\underline{1,709 \text{ MW}}}
\end{aligned}$$

14.17

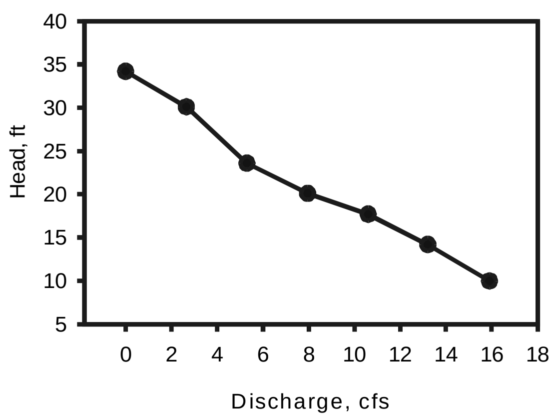
$$D = 14/12 = 1.167 \text{ ft}$$

$$n = 1,000/60 = 16.7 \text{ rev/s}$$

$$\Delta h = C_H n^2 D^2/g = C_H (16.7)^2 (1.167)^2/32.2 = 11.8 C_H \text{ ft}$$

$$Q = C_Q n D^3 = C_Q 16.7 (1.167)^3 = 26.5 C_Q \text{ cfs}$$

C_Q	C_H	Q(cfs)	Δh(ft)
0.0	2.9	0	34.2
0.1	2.55	2.65	30.1
0.2	2.0	5.3	23.6
0.3	1.7	7.95	20.1
0.4	1.5	10.6	17.7
0.5	1.2	13.2	14.2
0.6	0.85	15.9	10.0



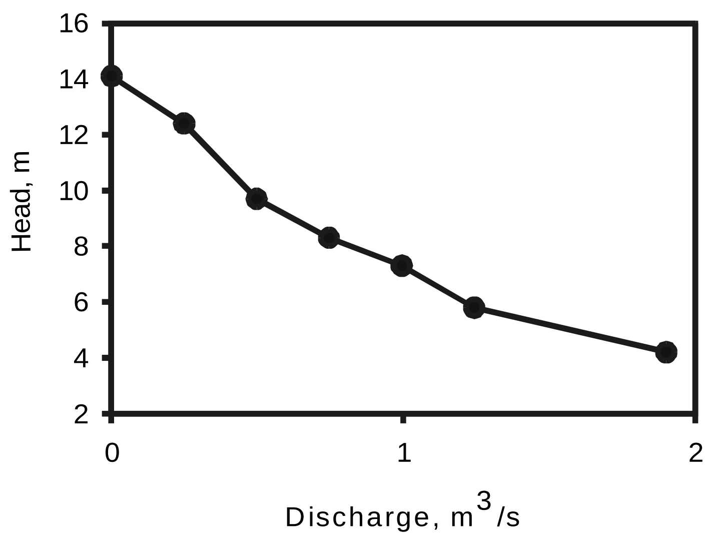
14.18

$$D = 60 \text{ cm} = 0.60 \text{ m}$$
$$N = 690 \text{ rpm} = 11.5 \text{ rev/s}$$

Then

$$\begin{aligned} \Delta h &= C_H D^2 n^2/g = 4.853 C_H \\ Q &= C_Q n D^3 = 2.484 C_Q \end{aligned}$$

C_Q	C_H	$Q(\text{m}^3/\text{s})$	$h(\text{m})$
0.0	2.90	0.0	14.1
0.1	2.55	0.248	12.4
0.2	2.00	0.497	9.7
0.3	1.70	0.745	8.3
0.4	1.50	0.994	7.3
0.5	1.20	1.242	5.8
0.6	0.85	1.490	4.2



14.19 **Information and Assumptions**

provided in problem statement

Find

head and discharge at maximum efficiency

Solution

$$D = 0.371 \times 2 = 0.742 \text{ m}$$
$$n = 2,133.5/(2 \times 60) = 17.77 \text{ rps}$$

From Fig. 14-10, at peak efficiency $C_Q = 0.121,\ C_H = 5.15$. Then

$$\begin{aligned}\Delta h &= C_H n^2 D^2/g = 5.15(17.77)^2(0.742)^2/9.81 = \underline{\underline{91.3 \text{ m}}} \\ Q &= C_Q n D^3 = 0.121(17.77)(0.742)^3 = \underline{\underline{0.878 \text{ m}^3/\text{s}}}\end{aligned}$$

14.20 **Information and Assumptions**

provided in problem statement

Find

power needed to operate fan

Solution

$$Q = VA = (60)(\pi/4)(1.2)^2 = 67.8 \text{ m}^3/\text{s}$$
$$C_Q = Q/(nD^3) = (67.8)/((1,800/60)(2)^3 = 0.282$$

From Fig. 14-16 $C_p = 2.6$. Then

$$P = C_p \rho D^5 n^3 = (2.6)(1.2)(2)^5(30)^3) = \underline{\underline{2.70 \text{ MW}}}$$

14.21 **Information and Assumptions**

provided in problem statement

Find

discharge through pipe

Solution

$$\Delta z = 450 - 366 = 84 \text{ m}$$

Assume $\Delta h = 90$ m[$>\Delta z$], then from Fig. 14-9, $Q = 0.24$ m^3/s

$$V = q/A = 0.24/[(\pi/4)(0.36)^2] = 2.36 \text{ m/s; } k_s/D = 0.00012$$

Assuming $T = 20°C$,

$$\text{Re} = VD/\nu = 2.36(0.36)/10^{-6} = 8.5 \times 10^5$$

from Fig. 10-8, $f = 0.014$

$$h_f = (0.014(610)/0.36)((2.36)^2/(2 \times 9.81)) = 6.73 \text{ m}$$
$$h \approx 84 + 6.7 = 90.7 \text{ m}$$

from Fig. 14-9

$$\begin{aligned} Q &= 0.23 \text{ m}^3\text{/s; } V = 0.23/((\pi/4)(0.36)^2) = 2.26 \text{ m/s} \\ h_f &= [0.014(610)/0.36](2.26)^2/(2 \times 9.81) = 6.18 \text{ m} \end{aligned}$$

so

$$\begin{aligned} \Delta h &= 84 + 6.2 = 90.2 \text{ m} \\ V &= 0.23/((\pi/4)(0.36)^2) = 2.26 \text{ m/s} \end{aligned}$$

and from Fig. 14-9

$$Q = \underline{\underline{0.225 \text{ m}^3\text{/s}}}$$

14.22 **Information and Assumptions**

provided in problem statement

Find

the discharge

Solution

$$\begin{aligned} D &= 0.371 \text{ m } = 1.217 \text{ ft} \\ n &= 1,600/60 = 26.7 \text{ rps} \\ \Delta h &= C_H n^2 D^2/g \end{aligned}$$

so

$$C_H = 150(32.2)/[(26.7)^2(1.217)^2] = 4.57$$

from Fig. 14-10 $C_Q = 0.145$ then

$$Q = C_Q n D^3 = 0.145(26.7)(1.217)^3 = \underline{\underline{6.98 \text{ cfs}}}$$

14.23 **Information and Assumptions**

provided in problem statement

Find

maximum possible head developed

Solution

$C_H = \Delta Hg/D^2n^2$ Since C_H will be the same for the maximum head condition, then

$$\Delta H \quad \alpha \quad n^2$$

or

$$H_{1,500} = H_{1,000} \times (1,500/1,000)^2$$

$$H_{1,500} = 102 \times 2.25 = \underline{\underline{229.5 \text{ ft}}}$$

14.24 **Information and Assumptions**

provided in problem statement

Find

shutoff head

Solution

$$H \ \alpha \ n^2$$

so

$$H_{30}/H_{35.6} = (30/35.6)^2$$

or

$$H_{30} = 104 \times (30/35.6)^2 = \underline{\underline{73.8 \text{ m}}}$$

14.25 **Information and Assumptions**

provided in problem statement

Find

discharge when head is 50 m

Solution

$$C_H = \Delta hg/(n^2D^2) = 50(9.81)/[(25)^2(0.40)^2] = 4.91$$

from Fig. 14-10 $C_Q = 0.136$ then

$$Q = C_Q nD^3 = 0.136(25(0.40)^3 = \underline{\underline{0.218 \text{ m}^3/\text{s}}}$$

14.26 **Information and Assumptions**

from table A.4 $\rho = 814 \text{ kg/m}^3$
provided in problem statement

Find

flow rate, pressure rise across pump and power required

Solution

$$N = 5,000 \text{ rpm} = 83.33 \text{ rps}$$

from Fig. 14-10 at maximum efficiency $C_Q = 0.125;\ C_H = 5.15;\ C_p = 0.69$

$$\begin{aligned} Q &= C_Q n D^3 = (0.125)(83.33)(0.20)^3 = \underline{\underline{0.0833 \text{ m}^3/\text{s}}} \\ \Delta h &= C_H D^2 n^2/g = (5.15)(0.20)^2(83.33)^2/9.81 = \underline{\underline{145.8 \text{ m}}} \\ P &= C_p \rho D^5 n^3 = (0.69)(814)(0.20)^5(83.33)^3 = \underline{\underline{104.0 \text{ kW}}} \end{aligned}$$

14.27 The discharge coefficient is defined as

$$C_Q = Q/nD^3$$

The rotational speed is 1750/60=29.2 rps. The diameter for each impeller is 0.4167 ft, 0.458 ft, 0.5 ft, 0.542 ft and 0.583 ft. One gallon per minute is 0.002228 ft^3/s. So for each impeller, the conversion factor to get the discharge coefficient is

5"	gpm	$\times\, 0.00105$
5.5"	gpm	$\times\, 0.000794$
6"	gpm	$\times\, 0.000610$
6.5"	gpm	$\times\, 0.000479$
7"	gpm	$\times\, 0.000385$

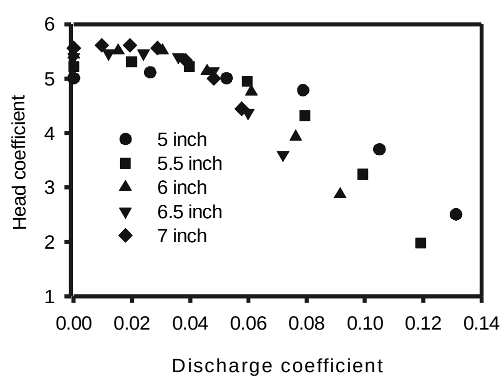
The head coefficient is

$$C_H = \frac{\Delta H g}{n^2 D^2}$$

The conversion factors to get the head coefficient are

5"	ft	$\times\, 0.2175$
5.5"	ft	$\times\, 0.1800$
6"	ft	$\times\, 0.1510$
6.5"	ft	$\times\, 0.1285$
7"	ft	$\times\, 0.1111$

The performance in terms of the nondimensional parameters is shown on the graph.



14.28 The rotational speed in rps is

$$n = 500/60 = 8.33 \text{ rps}$$

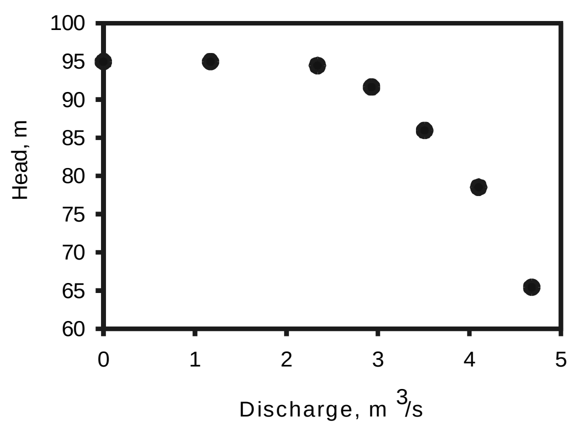
The discharge is given by

$$Q = C_Q n D^3 = C_Q(8.33)(1.52^3) = 29.27C_Q \text{ (m}^3\text{/s)}$$

The head is given by

$$\Delta h = C_H n^2 D^2/g = C_H(8.33^2)(1.52^2)/9.81 = 16.36C_H \text{ (m)}$$

C_Q	Q	C_H	Δh
0	0	5.8	94.9
0.04	1.17	5.8	94.9
0.08	2.34	5.75	94.1
0.10	2.93	5.6	91.6
0.12	3.51	5.25	85.9
0.14	4.10	4.8	78.5
0.16	4.68	4.0	65.4



14.29 **Information and Assumptions**

provided in problem statement

Find

suction specific speed

Solution

$$\begin{aligned}
N_{ss} &= NQ^{1/2}(NPSH)^{3/4} \\
N &= 690 \text{ rpm} \\
NPSH &\approx 14.7 \text{ psi } \times 2.31 \text{ ft/psi} - h_{\text{vap.press.}} \approx 33 \text{ ft} \\
Q &= 0.22 \text{ m}^3/s \times 264.2 \text{ gallons/s} \times 60 \text{ s/min} = 3,487 \text{ gpm} \\
N_{ss} &= 690 \times (3,487)^{1/2}/(33)^{3/4} = \underline{\underline{2,960}}
\end{aligned}$$

$\underline{\underline{N_{ss} \text{ is much below 8,500; therefore, it is in a safe operating range.}}}$

14.30 **Information and Assumptions**

provided in problem statement

Find

type of water pump

Solution

$$\begin{aligned} N &= 1,500 \text{ rpm so } n = 25 \text{ rps; } Q = 10 \text{ cfs; } h = 30 \text{ ft} \\ n_s &= n\sqrt{Q}/[g^{3/4}h^{3/4}] = (25)(10)^{1/2}/[(32.2)^{3/4}(30)^{3/4}] = 0.46 \end{aligned}$$

Then from Fig. 14-14, $n_s > 0.60$, so use a mixed flow pump.

14.31 **Information and Assumptions**

provided in problem statement

Find

type of pump

Solution

$$
\begin{aligned}
n &= 25 \text{ rps} \\
Q &= 0.30 \text{ m}^3/\text{sec} \\
h &= 8 \text{ meters} \\
n_s &= n\sqrt{Q}/[g^{3/4}h^{3/4}] = (25(0.3)^{1/2}/[(9.81)^{3/4}(8)^{3/4}] = 0.52
\end{aligned}
$$

Then from Fig. 14-14, $n_s < 0.60$ so use a mixed flow pump.

14.32 **Information and Assumptions**

provided in problem statement

Find

type of pump

Solution

$$
\begin{aligned}
N &= 1,100 \text{ rpm} = 18.33 \text{ rps} \\
Q &= 0.4 \text{ m}^3/\text{sec} \\
h &= 70 \text{ meters} \\
n_s &= n\sqrt{Q}/[g^{3/4}h^{3/4}] = (18.33)(0.4)^{1/2}/[(9.81)^{3/4}(70)^{3/4}] \\
&= (18.33)(0.63)/[(5.54)(24.2)] = 0.086
\end{aligned}
$$

Then from Fig. 14-14, $n_s < 0.23$ so $\underline{\underline{\text{use a radial flow pump}}}$.

14.33 **Information and Assumptions**

provided in problem statement

Find

maximum speed

Solution

The safe operating N_{ss} is 8,500, or

$$8,500 = NQ^{1/2}/(NPSH)^{3/4}$$

The suction head is given as 5 ft. Then assuming that the atmospheric pressure is 14.7 psia, and the vapor pressure is 0.256 psi, we commute $NPSH$ as

$$\begin{gathered} NPSH = 14.7 \text{ psi } \times 2.31 \text{ ft/psi} \\ +5 \text{ ft} - h_{\text{vap.press.}} = 38.4 \text{ ft} \\ Q = 0.40 \text{ m}^3/\text{s } \times 264.2 \text{ gals/m}^3 \times 60 \text{ s/min} \\ = 6,000 \text{ gpm} \end{gathered}$$

Then

$$\begin{aligned} N &= 8,500 \times (NPSH)^{3/4}/Q^{1/2} \\ &= 8,500 \times 15.4/77.5 \\ N &= \underline{\underline{1,689 \text{ rpm}}} \end{aligned}$$

14.34 **Information and Assumptions**

provided in problem statement

Find

type of pump

Solution

$$
\begin{aligned}
n_s &= n\sqrt{Q}/(g^{3/4}h^{3/4}) \\
n &= 10 \text{ rps} \\
Q &= 1.0 \text{ m}^3/\text{s} \\
h &= 3 + (1.5 + fL/D)V^2/(2g); \\
V &= 1.27 \text{ m/s}
\end{aligned}
$$

Assume $f = 0.01$, so

$$h = 3 + (1.5 + 0.01 \times 20/1)(1.27)^2/(2 \times 9.81) = 3.14 \text{ m}$$

Then

$$n_s = 10 \times \sqrt{1}/(9.81 \times 3.14)^{3/4} = 0.76$$

From Fig. 14-14, use axial flow pump.

14.35 **Information and Assumptions**

provided in problem statement

Find

diameter and power requirements for two blowers

Solution

Max. air speed = 40 m/s; Area = 0.36 m^2; $n = 2,000/60 = 33.3$ rps;

$$\begin{aligned} Q &= 40.0 \times 0.36 = 14.4 \text{ m}^3/\text{s} \\ \rho &= 1.2 \text{ kg/m}^3 \text{ at } 20°C \end{aligned}$$

From Fig. 14-6, at maximum efficiency, $C_Q = 0.63$ and $C_p = 0.60$

$$\begin{aligned} C_Q = Q/nD^{3,} \text{ so } D^3 = Q/(nC_Q) &= 14.4/(33.3 \times 0.63) = 0.686 \text{ m}^3 \\ D &= \underline{\underline{0.882 \text{ m}}} \\ C_p &= P/(\rho n^3 D^5) \end{aligned}$$

so

$$P = C_p \rho n^3 D^5 = 0.6(1.2)(33.3)^3(0.882)^5 = \underline{\underline{14.2 \text{ kW}}}$$

14.36 **Information and Assumptions**

provided in problem statement

Find

diameter and power requirements

Solution

Volume = 10^5 m^3; time for discharge = 15 min = 900 sec

$$\begin{aligned} N &= 600 \text{ rpm} = 10 \text{ rps} \\ \rho &= 1.22 \text{ kg/m}^3 \text{ at } 60^\circ F \\ Q &= (10^5 \text{ m}^3)/(900 \text{ sec}) = 111.1 \text{ m}^3/\text{sec} \end{aligned}$$

From Fig. 14-6, at maximum efficiency, $C_Q = 0.63$; $C_p = 0.60$

For two blowers operating in parallel, the discharge per blower will be one half so

$$Q = 55.55 \text{ m}^3/\text{sec}$$

then

$$\begin{aligned} D^3 &= Q/nC_Q = (55.55)/[10 \times 0.63] = 8.815 \\ D &= \underline{\underline{2.066 \text{ m}}} \\ P &= C_p \rho D^5 n^3 = (0.6)(1.22)(2.066)^5(10)^3 = \underline{\underline{27.6 \text{ kW}}} \end{aligned}$$

14.37 **Information and Assumptions**

from Table A-5 for methane $R = 518$ J/kg/K and $k = 1.26$
provided in problem statement

Find

shaft power to run compressor

Solution

$$
\begin{aligned}
P_{th} &= (k/(k-1))Qp_1[(p_2/p_1)^{(k-1)/k} - 1] = (k\dot{m}/(k-1))RT_1[(p_2/p_1)^{(k-1)/k} - 1] \\
&= (1.26/0.26)(1)518(300)[(1.5)^{0.26/1.26} - 1] = 65.6 \text{ kW} \\
P_{\text{ref}} &= P_{th}/e = 65.6/0.65 = \underline{\underline{101 \text{ kW}}}
\end{aligned}
$$

14.38 **Information and Assumptions**

provided in problem statement

Find

volume flow rate into the compressor

Solution

$$
\begin{aligned}
P_{th} &= 12 \text{ kW} \times 0.6 = 7.2 \text{ kW} \\
P_{th} &= (k/(k-1))Qp_1[(p_2/p_1)^{(k-1)/k} - 1] = (1.3/0.3)Q \times 9 \times 10^4[(140/90)^{0.3/1.3} - 1] \\
&= 4.18 \times 10^4 Q \\
Q &= 7.2/41.8 = \underline{\underline{0.172 \text{ m}^3/\text{s}}}
\end{aligned}
$$

14.39 **Information and Assumptions**

provided in problem statement

Find

the shaft power

Solution

$$
\begin{aligned}
P_{th} &= p_1 Q_1 \ell n(p_2/p_1) = \dot{m} R T_1 \ell n(p_2/p_1) = 1 \times 287 \times 288 \times \ell n 4 = 114.6 \text{ kW} \\
P_{\text{ref}} &= 114.6/0.5 = \underline{\underline{229 \text{ kW}}}
\end{aligned}
$$

14.40 **Information and Assumptions**

provided in problem statement

Find

diameter of turbine wheel

Solution

Assume $T = 10°C$

Writing the energy equation from reservoir to turbine jet,

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 650 &= 0 + V_{\text{jet}}^2/2g + 0 + (fL/D)(V_{\text{pipe}}^2/2g) \end{aligned}$$

but from continuity,

$$\begin{aligned} V_{\text{pipe}}A_{\text{pipe}} &= V_{\text{jet}}A_{\text{jet}} \\ V_{\text{pipe}} &= V_{\text{jet}}(A_{\text{jet}}/A_{\text{pipe}}) = V_{\text{jet}}(0.16) = 0.026V_{\text{jet}} \end{aligned}$$

so

$$\begin{aligned} (V_{\text{jet}}^2/2g)(1 + (fL/D)0.026^2) &= 650 \\ V_{\text{jet}} &= [(2 \times 9.81 \times 650)/(1 + (0.016 \times 10,000)/1)0.026^2)]^{1/2} = 107.3 \text{ m/s} \\ P &= Q\gamma V_{\text{jet}}^2 e = 107.3(\pi/4)(0.16)^2 9,810(107.3)^2 0.85/(2 \times 9.81) = \underline{\underline{10.55 \text{ MW}}} \\ V_{\text{bucket}} &= (1/2)V_{\text{jet}} = 53.7 \text{ m/s } = (D/2)\omega \\ D &= 53.7 \times 2/(360\times(\pi/30)) = \underline{\underline{2.85 \text{ m}}} \end{aligned}$$

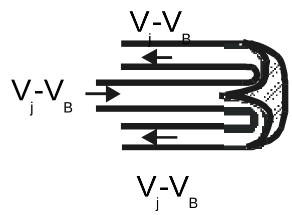
14.41 **Information and Assumptions**

provided in problem statement

Find

Referencing velocities to the bucket

Solution



$$\sum F_{\text{bucket on jet}} = \rho Q[-(V_j - V_B) - (V_j - V_B)]$$

Then

$$\sum F_{\text{on bucket}} = \rho V_j A_j 2(V_j - V_B)$$

assuming the combination of buckets to be intercepting flow at the rate of $V_j A_j$. Then

$$P$$

$$P = FV_B = 2\rho A_j[V_j^2 V_B - V_j V_B^2]$$

For maximum power production, $dP/dV_B = 0$, so

$$\begin{aligned} 0 &= 2\rho A(V_j^2 - V_j 2V_B) \\ 0 &= V_j - 2V_B \end{aligned}$$

or

$$V_B = 1/2V_j$$

14.42 Consider the power developed from the force on a single bucket. Referencing velocities to the bucket gives

$$\sum F_{\text{on bucket}} = \rho Q_{\text{rel. to bucket}}(-(1/2)V_j - (1/2)V_j)$$

Then

$$F_{\text{on bucket}} = \rho(V_j - V_B)A_j(V_j)$$

but

$$V_j - V_B = 1/2V_j$$

so

$$F_{\text{on bucket}} = 1/2\rho A V_j^2$$

Then

$$P = FV_B = (1/2)\rho Q V_j^3/2$$

The power is 1/2 that given by Eq. (14-20). The extra power comes from the operation of more than a single bucket at a time so that the wheel as a whole turns the full discharge; whereas, a single bucket intercepts flow at a rate of 1/2 $V_j A_j$.

14.43 **Information and Assumptions**

provided in problem statement

Find

α_1 for nonseparating flow conditions and maximum attainable power

Solution

$$\begin{aligned} V_{r_1} &= q/(2\pi r_1 B) = 126/(2\pi \times 5sx1) = 4.01 \text{ m/s} \\ \omega &= 60 \times 2\pi/60 = 2\pi \text{ rad/s} \end{aligned}$$

1. **a)**

$$\alpha_1 = \text{arc cot } ((r_1\omega/V_{r_1}) + \cot \beta_1) = \text{arc cot } ((5 \times 2\pi/4.01) + 0.577) = \underline{\underline{6.78^\circ}}$$
$$\alpha_2 = \text{arc tan } (V_{r_2}/\omega r_2) = \text{arc tan } ((4.01 \times 5/3)/(3 \times 2\pi)) = \text{arc tan } 0.355 = 15.5^\circ$$

b)

$$V_1 = V_{r_1}/\sin\alpha_1 = 4.01/0.118 = 39.97 \text{ m/s}; \; V_2 = V_{r_2}/\sin\alpha_2 = 20.0 \text{ m/s}$$
$$P = \rho Q\omega(r_1 V_1 \cos\alpha_1 - r_2 V_2 \cos\alpha_2)$$
$$P = 998 \times 126 \times 2\pi(5 \times 39.97 \times \cos\ 6.78^\circ - 3 \times 20.0 \times \cos\ 15.5^\circ) = \underline{\underline{111.1 \text{ MW}}}$$

c) $\underline{\underline{\text{Increase } \beta_2}}$

14.44 **Information and Assumptions**

provided in problem statement

Find

α_1 for nonseparating flow conditions and resulting power and torque

Solution

$$
\begin{aligned}
V_{r_1} &= 3(2\pi \times 1.5 \times 0.3) = 1.061 \text{ m/s} \\
V_{r_2} &= 3/(2\pi \times 1.2 \times 0.3) = 1.326 \text{ m/s}; \\
\omega &= (60/60)2\pi = 2\pi s^{-1} \\
\alpha_1 &= \text{arc cot }((r_1\omega/V_{r_1}) + \text{cot } \beta_1) = \text{arc cot }((1.5(2\pi)/1.415) + \text{cot } 85^\circ) \\
\alpha_1 &= \text{arc cot }(6.66 + 0.0875) = \underline{\underline{8^\circ 25'}} \\
V_{\tan_1} &= r_1\omega + V_{r_1} \text{ cot } \beta_1 = 1.5(2\pi) + 1.061(0.0875) = 9.518 \text{ m/s} \\
V_{\tan_2} &= r_2\omega + V_{r_2}\text{cot } \beta_2 = 1.2(2\pi) + 1.326(-3.732) = 2.591 \text{ m/s} \\
T &= \rho Q(r_1 V_{\tan_1} - r_2 V_{\tan_2}) = 1,000(4)(1.5 \times 9.518 - 1.2 \times 2.591) = \underline{\underline{44{,}671 \text{ N-m}}} \\
\text{Power} &= T\omega = 44,671 \times 2\pi = \underline{\underline{280.7 \text{ kW}}}
\end{aligned}
$$

14.45 **Information and Assumptions**

provided in problem statement

Find

α_1 for nonseparating flow conditions

Solution

$$\begin{aligned}
\omega &= 120/60 \times 2\pi = 4\pi \text{ s}^{-1} \\
V_{r_1} &= 113/(2\pi(2.5)0.9) = 7.99 \text{ m/s} \\
\alpha_1 &= \text{arc cot } ((r_1\omega/V_{r_1}) + \cot \beta_1) = \text{arc cot } ((2.5(4\pi)/7.99) + \cot 45^\circ) \\
&= \text{arc cot } (3.93 + 1) = \underline{\underline{11^\circ 28'}}
\end{aligned}$$

14.46 **Information and Assumptions**

Assume $k_e = 0.50;\ K_E = 1.0;\ K_b = 0.2;\ K_s/D = 0.00016$
provided in problem statement

Find

power output

Solution

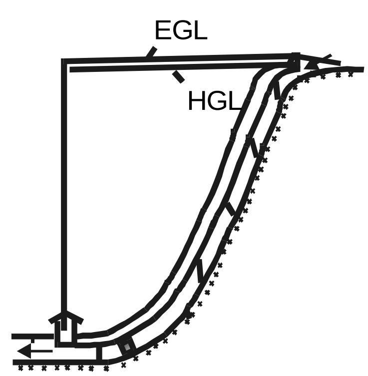
To get power write the energy equation

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L + h_t \\
0 + 0 + 3,000 &= 0 + 0 + 2,600 + \sum h_L + h_t \\
\sum h_L &= (V^2/2g)(f(L/D) + K_E + K_e + 2K_b)
\end{aligned}$$

$$\begin{aligned}
V &= Q/A = 8/((\pi/4)(1)^2) = 10.19 \text{ ft/s}; \\
\text{Re} &= VD/\nu = (10.19)(1)/(1.2 \times 10^{-5}) = 8.5 \times 10^5 \\
f &= 0.0145 \\
\sum h_L &= ((10.19)^2/(64.4))[(0.0145)(1,000/1) + 1.0 + 0.5 + 2 \times 0.2] \\
\sum h_L &= 1.612(16.4) = 26.44 \text{ ft} \\
h_t &= 3,000 - 2,600 - 26.44 = 373.6 \text{ ft}
\end{aligned}$$

$$\begin{aligned}
\text{Power to turbine} &= \gamma Q h_t/550 = (8)(62.4)(373.6)/550 \\
&= 339 \text{ horsepower}
\end{aligned}$$

Power output from the turbine = 339 × eff. = 339 × 0.8 = $\underline{\underline{271 \text{ hp}}}$



14.47 **Information and Assumptions**

provided in problem statement

Find

maximum power deliverable

Solution

$$p_{\max} = (16/54)\rho U^3 A = (16/54) \times 1.2 \times (50,000/3,600)^3 \pi \times 2^2/4 = \underline{\underline{2.99 \text{ kW}}}$$

14.48 **Information and Assumptions**

provided in problem statement

Find

width of wind turbine

Solution

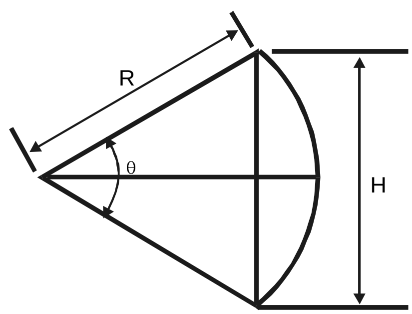
Each windmill must produce 2 MW/20 = 100,000 W. The maximum power produced by each windmill is given by

$$P_{\max} = \frac{16}{54}\rho V_o^3 A$$

In a 20 m/s wind with a density of 1.2 kg/m^3, the capture area is

$$A = \frac{54}{16}\frac{100000}{1.2 \times 20^3} = 35.26m^2$$

Consider the figure for the section of a circle.



The area of a sector is given by

$$A_s = \frac{1}{2}\theta R^2 - \frac{1}{2}RH\cos(\theta/2)$$

where θ is the angle subtended by the arc and H is the distance between the edges of the arc. But

$$R = \frac{H}{2\sin(\theta/2)}$$

so

$$\begin{aligned} A &= 2A_s = \frac{H^2}{4}[\frac{\theta}{\sin^2(\theta/2)} - 2\frac{\cos(\theta/2)}{\sin(\theta/2)}] \\ &= 56.2 \times [\frac{\theta}{\sin^2(\theta/2)} - 2\frac{\cos(\theta/2)}{\sin(\theta/2)}] \end{aligned}$$

Solving graphically gives $\theta = 52^o$. The width of the windmill is

$$W = H[\frac{1}{\sin(\theta/2)} - \frac{1}{\tan(\theta/2)}]$$

Substituting in the numbers gives W=3.45 m.

14.49 **Information and Assumptions**

provided in problem statement

Find

discharge of pump

Solution

$$\begin{aligned}
P &= (16/27/)(\rho A V^3/2) \\
&= (16/27)(0.07/32.2)(\pi/4)(10)^2(44)^{3/2} = 4,307 \text{ ft-lbf/s} \\
0.80 \times P &= \gamma Q h_p = (0.80)(4,307) = 3,446 \text{ ft-lbf/s} \\
Q &= (3,446)/((62.4)(10)) = 5.52 \text{ cfs } = 331 \text{ cfm} \\
&= \underline{\underline{\text{2,476 gpm}}}
\end{aligned}$$

14.50 Assume that this system will be used on a daily basis; therefore, some safety should be included in the design. That is, include more than one pump so that if one malfunctions there will be at least another one or two to satisfy the demand. Also, periodic maintenance may be required; therefore, when one pump is down there should be another one or two to provide service. The degree of required safety would depend on the service. For this problem, assume that three pumps will be used to supply the maximum discharge of 1 m^3/s. Then each pump should be designed to supply a flow of water of 0.333 m^3/s (5,278 gpm). Also assume, for the first cut at the design, that the head loss from reservoir to pump will be no greater than 1 meter and that each pump itself will be situated in a pump chamber at an elevation 1 m below the water surface of the reservoir. Thus, the $NPSH$ will be approximately equal to the atmospheric pressure head, or 34 ft.

Assume that the suction specific speed will be limited to a value of 8,500:

$$\begin{aligned} N_{ss} &= 8,500 = NQ^{1/2}/(NPSH)^{3/4} \\ \text{or } NQ^{1/2} &= 8,500 \times (34)^{3/4} \qquad (1) \\ &= 119,681 \end{aligned}$$

Assume that 60 cycle A.C. motors will be used to drive the pumps and that these will be synchronous speed motors. Common synchronous speeds in rpm are: 1,200, 1,800, 3,600; however, the normal speed will be about 97% of synchronous speed*. Therefore, assume we have speed choices of 1,160 rpm, 1,750 rpm and 3,500 rpm. Then from Eq. (1) we have the following maximum discharges for the different speeds of operation:

N(rpm)	Q(gpm)	Q(m/s)
1,160	10,645	0.672
1,750	1,169	0.295
3,500	1,169	0.074

Based upon the value of discharge given above, it is seen that a speed of 1,160 rpm is the choice to make if we use 3 pumps. The pumps should be completely free of cavitation.

Next, calculate the impeller diameter needed. From Fig. 14.10 for maximum efficiency $C_Q \approx 0.12$ and $C_H \approx 5.2$ or

$$\begin{aligned} 0.12 &= q/nD^3 \qquad (2) \\ \text{and } 5.2 &= \Delta H/(D^2n^2/g) \qquad (3) \end{aligned}$$

Then for $N = 1,160$ rpm ($n = 19.33$ rps) and $Q = 0.333$ m^3/s we can solve for D from Eq. (2).

$$\begin{aligned} D^3 &= q/(0.12 \text{ n}) \\ &= 0.333/(0.12 \times 19.33) \\ &= 0.144 \\ \text{or } D &= 0.524 \text{ m} \end{aligned}$$

Now with a D of 0.524 m the head produced will be

$$\begin{aligned} \Delta H &= 5.2D^2n^2/g \text{ (from Eq. (3))} \\ &= 5.2(0.524)^2(19.33)^2/(9.81) \\ &= 54.4 \text{ m} \end{aligned}$$

With a head of 54.4 m determine the diameter of pipe required to produce a discharge of 1 m^3/s. From the solution to Prob. 10.100 (as an approximation to this problem), we have

$$\begin{aligned} h_p &= 50 \text{ m } + (V^2/2g)(2.28 + fL/D) \text{ m} \\ \text{Assume } f &= 0.012 \\ L &= 400 \text{ m} \\ \text{so } h_p &= 50 \text{ m } + (V^2/2g)(2.28 + 4.8/D) \text{ m} \\ 54 \text{ m} &= 50 + (V^2/2g)(2.28 + 4.8/D) \quad (4) \end{aligned}$$

Equation (4) may be solved for D by an iteration process: Assume D, then solve for V and then see if Eq. (4) is satisfied, etc. The iteration was done for D of 60 cm, 70 cm and 80 cm and it was found that the closest match came with $\underline{\underline{D = 70 \text{ cm}}}$. Now compute the required power for an assumed efficiency of 92%.

$$\begin{aligned} P &= Q\gamma h_p/\text{eff.} \\ &= 0.333 \times 9,810 \times 54/0.92 \\ P &= 192 \text{ kW} = \underline{\underline{257 \text{ hp}}} \end{aligned}$$

In summary, $D = 70$ cm, $N = 1{,}160$ rpm,

$$Q \text{ per pump} = 0.333 \text{ m}^3/\text{s}, \ P = 192 \text{ kW}$$

The above calculations yield a solution to the problem. That is, a pump and piping system has been chosen that will produce the desired discharge. However, a truly valid design should include the economics of the problem. For example, the first cost of the pipe and equipment should be expressed in terms of cost per year based upon the expected life of the equipment. Then the annual cost of power should be included in the total cost. When this is done, the size of pipe becomes important (smaller size yields higher annual cost of power). Also, pump manufacturers have a multiple number of pump designs to choose from which is different than for this problem. We had only one basic design although considerable variation was available with different diameters and speed.

The design could also include details about how the piping for the pumps would be configured. Normally this would include 3 separate pipes coming from the reservoir, each going to a pump, and then the discharge pipes would all feed into the larger pipe that delivers water to the elevated tank. Also, there should be gate valves on each side of a pump so it could be isolated for maintenance purposes, etc. Check valves would also be included in the system to prevent back flow through the pumps in event of a power outage.

Chapter Fifteen

15.1 **Information and Assumptions**

provided in problem statement

Find

if flow subcritical or supercritical and alternate depth

Solution

Check Froude number

$$Fr = V/\sqrt{gy} = 28\sqrt{32.2 \times 0.333} = 8.55$$

The Froude number is greater than 1 so the flow is supercritical.

$$\begin{aligned} E &= y + V^2/g \\ E &= 0.333 + 28^2/(2 \times 32.2) = 12.51 \text{ ft} \end{aligned}$$

Solving for the alternative depth of an E of 12.51 yields $\underline{\underline{y_{\text{alt}} = 12.43 \text{ ft.}}}$

15.2 **Information and Assumptions**

provided in problem statement

Find

if flow subcritical or supercritical

Solution

$$\begin{aligned} V &= Q/A = 900/(16 \times 3) = 18.75 \text{ ft/s} \\ Fr &= V/\sqrt{gy} = 18.75/\sqrt{32.2 \times 3} = 1.91 \end{aligned}$$

Flow is $\underline{\underline{\text{supercritical}}}$

15.3 **Information and Assumptions**

provided in problem statement

Find

if flow subcritical or supercritical

Solution

$$
\begin{aligned}
Q &= VA \\
420 &= 8 \times 18 \times y \\
y &= 2.917 \text{ ft} \\
Fr &= V/\sqrt{gy} \\
&= 8 \text{ ft/s } /(\sqrt{32.2 \times 2.917}) \\
Fr &= 0.085
\end{aligned}
$$

Flow is $\underline{\underline{\text{subcritical}}}$

15.4 **Information and Assumptions**

provided in problem statement

Find

the Froude number, type of flow and critical depth

Solution

$$
\begin{aligned}
Q &= VA \\
12\text{ m}^3/\text{s} &= V(3\times y) \\
V_{0.30} &= 12\text{ m}^3/\text{s }/(3\text{ m}\times 0.30\text{ m}) = 13.33\text{ m/s}; \\
V_{1.0} &= 12\text{ m}^3/\text{s }/(3\text{ m }\ \times 1\text{ m}) = 4\text{ m/s} \\
V_{2.0} &= 12\text{ m}^3/\text{s}/(3\text{ m}\times 2\text{ m}) = 2\text{ m/s} \\
Fr_{0.3} &= 13.33\text{ m/s}/(9.81\text{ m/s}^2\times 0.30\text{ m})^{1/2} = \underline{\underline{7.77}}\text{ (supercritical)} \\
Fr_{1.0} &= 4\text{ m/s}/(9.81\text{ m/s}^2\times 1.0\text{ m})^{1/2} = \underline{\underline{1.27\ }}\text{(supercritical)} \\
Fr_{2.0} &= 2\text{ m/s }/9.81\text{ m/s}^2\times 1.0\text{ m})^{1/2} = \underline{\underline{0.452}}\text{ (subcritical)}
\end{aligned}
$$

The critical depth is

$$
\begin{aligned}
y_c &= (q^2/g)^{1/3} = ((4\text{ m}^2/\text{s})^2/(9.81\text{ m/s}^2))^{1/3} \\
&= \underline{\underline{1.18\text{ m}}}
\end{aligned}
$$

15.5 **Information and Assumptions**

provided in problem statement

Find

alternate depth and critical depth

Solution

$$\begin{aligned} V_{0.30} &= 13.33 \text{ m/s} \\ E &= y + V^2/2g \\ &= 0.30 + 9.06 \\ &= 9.36 \text{ m} \end{aligned}$$

Solving for alternate depth yields: $\underline{\underline{y = 9.35 \text{ m}}}$

15.6 **Information and Assumptions**

provided in problem statement

Find

depth of flow

Solution

$$\begin{aligned} Fr_c &= V/(gy_c) \\ 1 &= 6 \text{ m/s } /(9.81 \text{ m/s}^2 \times y_c)^{1/2} \\ y_c &= (6 \text{ m/s})^2/(9.81 \text{ m/s}^2) \\ &= \underline{\underline{3.67 \text{ m}}} \end{aligned}$$

15.7 **Information and Assumptions**

provided in problem statement

Find

if flow subcritical or supercritical

Solution

$$
\begin{aligned}
Q &= \frac{1.49}{n} A R^{2/3} S_o^{1/2} \\
&= \frac{1.49}{n} A (A/P)^{2/3} S_o^{1/2} \\
&= \frac{1.49}{n} By(By/(b+2y))^{2/3} S_o^{1/2} \\
&= \frac{1.49}{n} 12y(12y/(12+2y))^{2/3} S_o^{1/2} \\
320 &= \frac{1.49}{0.014} 12y(12y/(12+2y))^{2/3} (0.005)^{1/2}
\end{aligned}
$$

Solving for y yields: $y = 2.45$ ft.

$$
\begin{aligned}
V &= Q/A \\
&= 320 \text{ ft}^3/\text{s} \ /(12 \text{ ft } \times 2.45 \text{ ft}) \\
&= 10.88 \text{ ft/s} \\
Fr &= V/\sqrt{gy} \\
&= 10.88/(32.2 \times 2.45)^{1/2} \\
Fr &= 2.11 \ (\underline{\underline{\text{supercritical}}})
\end{aligned}
$$

15.8 **Information and Assumptions**

provided in problem statement

Find

if flow subcritical or supercritical

Solution

First determine V:

$$\begin{aligned} V &= Q/A = 10 \text{ m}^3/\text{s } /((3 \times 1 \text{ m}^2) + 1^2 \text{ m}^2) = 2.50 \text{ m/s} \\ D &= A/T = 4 \text{ m}^2/5 \text{ m} = 0.80 \text{ m} \end{aligned}$$

Then

$$\begin{aligned} Fr &= V/(gD)^{0.5} = 2.50/(9.81 \times 0.80)^{0.50} = 0.89 \\ Fr &< 1 \end{aligned}$$

The flow is $\underline{\underline{\text{subcritical}}}$

15.9 **Information and Assumptions**

provided in problem statement

Find

the critical depth

Solution

For critical flow condition

$$V/\sqrt{gD} = 1 \text{ for critical flow condition or}$$

or

$$\begin{aligned}
(V/\sqrt{D}) &= \sqrt{g} \\
V &= Q/A = 20/(3y + y^2) \\
D &= A/T = (3y + y^2)/(3 + 2y) \\
(20/(3y + y^2))/((3y + y^2)/(3 + 2y))^{0.5} &= \sqrt{9.81} = 3.132
\end{aligned}$$

Solving for y yields; $\underline{\underline{y_{cr} = 1.4 \text{ m}}}$

15.10 **Information and Assumptions**

provided in problem statement

Find

alternate and sequent depths

Solution

For a rectangular channel

$$E = y + q^2/(2gy^2)$$

For this problem

$$q = Q/B = 18/6 = 3 \text{ m}^2/\text{s}$$

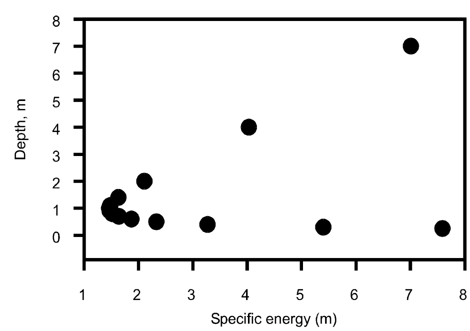
So

$$\begin{aligned} E &= y + 3^2/(2gy^2) \\ &= y + 0.4587/y^2 \end{aligned}$$

The calculated E versus y is shown below

y (m)	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.4	2.0	4.0	7.0
E (m)	7.59	5.4	3.27	2.33	1.87	1.64	1.52	1.47	1.46	1.48	1.63	2.11	4.03	7.01

The corresponding plot is



The alternate depth to $y = 0.30$ is $y = \underline{\underline{5.38}}$ m

Sequent depth:

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1 + 8F_1^2} - 1) \\ F_1 &= V/\sqrt{gy_1} = (3/0.3)/\sqrt{9.81 \times 0.30} = 5.83 \end{aligned}$$

Then

$$y_2 = (0.3/2)(\sqrt{1 + 8 \times 5.83^2} - 1) = \underline{\underline{2.33 \text{ m}}}$$

15.11 **Information and Assumptions**

provided in problem statement

Find

the discharge in the channel

Solution

$$d_{\text{brink}} \approx 0.71 y_c = 0.71(q^2/g)^{1/3}$$

Then for $d_{\text{brink}} = 0.35$ m and

$$\begin{aligned} q &= (0.35 \times g^{1/3}/0.71)^{3/2} \\ q &= 1.084 \text{ m}^3/\text{s} \end{aligned}$$

Then

$$Q = 4q = \underline{\underline{4.34}} \text{ m}^3\text{s}$$

15.12 **Information and Assumptions**

provided in problem statement

Find

the discharge in the channel

Solution

Solution like that for Problem 15.11:

$$\begin{aligned} q &= (1.20 \times 32.2^{1/3}/0.71)^{3/2} \\ q &= 12.47 \text{ m}^2/\text{s} \end{aligned}$$

Then

$$Q = 15 \times 12.47 = \underline{\underline{187 \text{ cfs}}}$$

15.13 **Information and Assumptions**

provided in problem statement

Find

depth at the brink

Solution

$$y_{\text{brink}} = 0.71 y_c$$

where

$$\begin{aligned} y_c &= \sqrt[3]{q^2/g} \\ q &= 500/15 = 33.33 \text{ ft}^2/\text{s} \\ y_{\text{brink}} &= 0.71 \sqrt[3]{33.33^2/32.2} = \underline{\underline{2.31 \text{ ft}}} \end{aligned}$$

15.14 **Information and Assumptions**

provided in problem statement

Find

discharge of water

Solution

$$H/(H+P) = (1.5/3.5) = 0.43$$

From Fig. 15.7 $C = 0.89$. Discharge is given by

$$\begin{aligned} Q &= 0.385\, CL\sqrt{2g}H^{1.5} \\ Q &= 0.385(0.89)(10\sqrt{64.4}(1.5) \\ Q &= \underline{\underline{50.5 \text{ cfs}}} \end{aligned}$$

15.15 **Information and Assumptions**

from Fig. 15.7 C=0.865
provided in problem statement

Find

the discharge

Solution

$$H/(H+P) = (0.6/(2.6)) = 0.23$$

The discharge is given by

$$\begin{aligned} Q &= 0.385(0.865)(5)\sqrt{2\times 9.81}(0.60)^{1.5} \\ Q &= \underline{\underline{3.43 \text{ m}^3/\text{s}}} \end{aligned}$$

15.16 **Information and Assumptions**

from Fig. 15.7 $C \approx 0.85$
provided in problem statement

Find

the water surface elevation

Solution

$$
\begin{aligned}
Q &= 0.385 C\, L\sqrt{2g} H^{3/2} \\
25 &= 0.385(0.85)(10)\sqrt{2 \times 9.81} H^{3/2} \\
(H)^{3/2} &= 1.725 \\
H &= 1.438,
\end{aligned} \tag{10}
$$

Water surface elevation = $\underline{\underline{101.4 \text{ m}}}$

15.17 **Information and Assumptions**

from Fig. 15.7 $C \approx 0.85$
provided in problem statement

Find

water surface elevation in upstream reservoir

Solution

$$\begin{aligned} Q &= 0.385C\,L\sqrt{2g}H^{3/2} \\ 1,200 &= 0.385(0.85)(40)\sqrt{64.4}H^{3/2} \\ H &= 5.07 \text{ ft} \end{aligned}$$

Water surface elevation = $\underline{\underline{305.1 \text{ ft}}}$

15.18 **Information and Assumptions**

provided in problem statement

Find

change in depth and maximum size of upstep

Solution

$$\begin{aligned} E_1 &= y_1 + V_1^2/2g = 3 + 3^2/(2 \times 9.81) = 3.46 \text{ m} \\ F_1 &= V_1/\sqrt{gy_1} = 3/\sqrt{9.81 \times 3} = 0.55 \text{ (subcritical)} \end{aligned}$$

Then

$$E_2 = E_1 - \Delta z_{\text{step}} = 3.46 - 0.30 = \underline{\underline{3.16 \text{ m}}}$$

$$\begin{aligned} y_2 + q^2/(2gy_2^2) &= 3.16 \text{ m} \\ y_2 + 9^2/(2gy_2^2) &= 3.16 \\ y_2 + 4.13/y_2^2 &= 3.16 \end{aligned}$$

Solving for y_2 yields $y_2 = 2.49$ m. Then

$$\Delta y = y_2 - y_1 = 2.49 - 3.00 = \underline{\underline{-0.51 \text{ m}}}$$

So water surface drops $\underline{\underline{0.21 \text{ m}}}$.
For a downward step

$$\begin{aligned} E_2 &= E_1 + \Delta z_{\text{step}} \\ &= 3.46 + 0.3 = \underline{\underline{3.76 \text{ m}}} \\ y_2 + 4.13/y_2^2 &= 3.76 \end{aligned}$$

Solving for y_2 gives $y_2 = 3.40$ m. Then

$$\Delta y = y_2 - y_1 = 3.40 - 3 = \underline{\underline{0.40 \text{ m}}}$$

Water surface elevation change = $\underline{\underline{+0.10 \text{ m}}}$
Max. upward step before altering upstream conditions:

$$\begin{aligned} y_c &= y_2 = \sqrt[3]{q^2/g} = \sqrt[3]{9^2/9.81} = 2.02 \\ E_1 &= \Delta z_{\text{step}} + E_2 \end{aligned}$$

where

$$E_2 = 1.5y_c = 1.5 \times 2.02 = 3.03 \text{ m}$$

Maximum size of step

$$z_{\text{step}} = E_1 - E_2 = 3.46 - 3.03 = \underline{\underline{0.43 \text{ m}}}$$

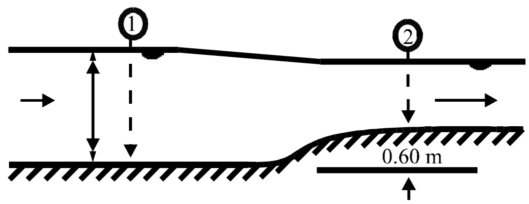
15.19 **Information and Assumptions**

provided in problem statement

Find

change in depth

Solution



$$
\begin{aligned}
E_2 &= E_1 - 0.60 \\
V_1 &= 2 \text{ m/s} \\
F_1 &= V_1/\sqrt{gy_1} = 2/\sqrt{9.81 \times 3} = 0.369
\end{aligned}
$$

Then

$$
\begin{aligned}
E_2 &= 3 + (2^2/(2 \times 9.81)) - 0.60 = 2.60 \text{ m} \\
y_2 + q^2/(2gy_2^2) &= 2.60
\end{aligned}
$$

where $q = 2 \times 3 = 6 \text{ m}^3/\text{s/m}$. Then

$$
\begin{aligned}
y_2 + 6^2/(2 \times 9.81 \times y_2^2) &= 2.60 \\
y_2 + 1.83/y_2^2 &= 2.60
\end{aligned}
$$

Solving, one gets $y_2 = 2.24$ m. Then

$$
\Delta y = y_2 - y_1 = 2.34 - 3.00 = \underline{\underline{-0.76 \text{ m}}}
$$

Water surface drops $\underline{\underline{0.16 \text{ m}}}$

For downward step of 15 cm we have

$$
\begin{aligned}
E_2 &= (3 + (2^2/(2 \times 9.81)) + 0.15 = 3.35 \text{ m} \\
y_2 + 6^2/(2 \times 9.81 \times y_2^2) &= 3.35 \\
y_2 + 1.83/y_2^2 &= 3.35
\end{aligned}
$$

Solving: $y_2 = 3.17$ m or

$$y_2 - y_1 = 3.17 - 3.00 = \underline{\underline{+0.17 \text{ m}}}$$

Water surface rises $\underline{\underline{0.02 \text{ m}}}$
The maximum upstep possible before affecting upstream water surface levels is for $y_2 = y_c$

$$y_c = \sqrt[3]{q^2/g} = 1.54 \text{ m}$$

Then

$$\begin{aligned}
E_1 &= \Delta z_{\text{step}} + E_{2,\text{crit}} \\
\Delta z_{\text{step}} &= E_1 - E_{2,\text{crit}} = 3.20 - (y_c + V_c^2/2g) = 3.20 - 1.5 \times 1.54 \\
\Delta z_{\text{step}} &= \underline{\underline{+0.89 \text{ m}}}
\end{aligned}$$

15.20 **Information and Assumptions**

provided in problem statement

Find

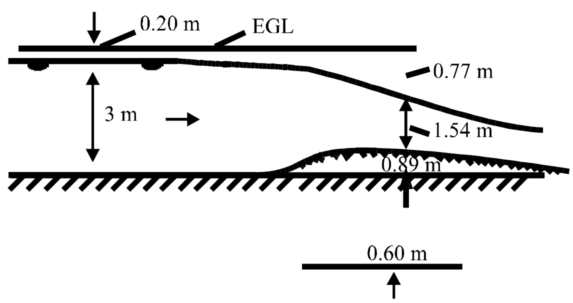
maximum value of Δz to permit unit flow rate of 6 m^2/s

Solution

$$y_c = (q^2/g)^{1/3} = (6^2/9.81)^{0.333} = 1.542 \text{ m}$$

where y_c is depth allowed over the hump for the given conditions.

$$\begin{aligned}
E_1 &= E_2 \\
V_1 &= q/y_1 = 6/3 = 2 \text{ m/s} \\
V_2 &= 6/1.542 = 3.891 \text{ m/s} \\
V_1^2/2g + y_1 &= V_2^2/2g + y_2 + \Delta z \\
2^2/2g + 3 &= (3.891^2/(2 \times 9.81)) + 1.542 + \Delta z \\
\Delta z &= 3.204 - 0.772 - 1.542 = \underline{\underline{0.89 \text{ m}}}
\end{aligned}$$



15.21 **Information and Assumptions**

provided in problem statement

Find

change in depth and water surface elevation and greatest contraction allowable

Solution

$$\begin{aligned} F_1 &= V_1/\sqrt{gy_1} = 3/\sqrt{9.81 \times 3} = 0.55 \text{ (subcritical)} \\ E_1 &= E_2 = y_1 + V_1^2/2g = 3 + 3^2/2 \times 9.81 = 3.46 \text{ m} \\ q_2 &= Q/B_2 = 27/2.6 = 10.4 \text{ m}^3/\text{s/m} \end{aligned}$$

Then

$$\begin{aligned} y_2 + q^2/(2gy_2^2) &= y_2 + (10.4)^2/(2 \times 9.81 \times y_2^2) = 3.46 \\ y_2 + 5.50/y_2^2 &= 3.46 \end{aligned}$$

Solving: $y_2 = \underline{\underline{2.71 \text{ m}}}$.

$$\Delta z_{\text{water surface}} = \Delta y = y_2 - y_1 = 2.71 - 3.00 = \underline{\underline{0.29 \text{ m}}}$$

Max. contraction without altering the upstream depth will occur with $y_2 = y_c$

$$E_2 = 1.5y_c = 3.45;\ y_c = 2.31 \text{ m}$$

Then

$$\begin{aligned} V_c^2/2g &= y_c/2 = 2.31/2 \text{ or } V_c = 4.76 \text{ m/s} \\ Q_1 &= Q_2 = 27 = B_2 y_c V_c \\ B_2 &= 27/(2.31 \times 4.76) = 2.46 \text{ m} \end{aligned}$$

The width for max. contraction = $\underline{\underline{2.46 \text{ m}}}$

15.22 **Information and Assumptions**

provided in problem statement

Find

"ship sqaut" of fully loaded supertanker

Solution

Write the energy equation from a section in the channel upstream of the ship to a section where the ship is located.

$$
\begin{aligned}
E_1 &= E_2 \\
V_1^2/2g + y_1 &= V_2^2/2g + y_2 \\
A_1 &= 35 \times 200 = 7,000 \text{ m}^2 \\
V_1 &= 5 \times 0.515 = 2.575 \text{ m/s} \\
2.575^2/(2 \times 9.81) + 35 &= (Q/A_2)^2/(2 \times 9.81) + y_2 \qquad (1) \\
\text{where } Q &= V_1 A_1 = 2.575 \times 7,000 \text{ m}^3/\text{s} \\
A_2 &= 200 \text{ m } \times y_2 - 29 \times 63
\end{aligned}
$$

where

$$
\begin{aligned}
Q &= V_1 A_1 = 2.575 \times 7,000 \text{ m}^3/\text{s} \qquad (2) \\
A_2 &= 200 \text{ m } \times y_2 - 29 \times 63 \qquad (3)
\end{aligned}
$$

Substituting Eq's (2) and (3) into Eq. (1) and solving for y_2 yields $y_2 = 34.70$ m. Therefore, the ship squat is $y_1 - y_2 = 35.0 - 34.7 =$ $\underline{\underline{0.30 \text{ m}}}$

15.23 **Information and Assumptions**

provided in problem statement

Find

determine depth downstream of roughness

Solution

Apply the momentum equation for a unit width

$$
\begin{aligned}
\sum F_x &= \sum \dot{m}_o V_o - \sum \dot{m}_i V_i \\
\gamma y_1^2/2 - \gamma y_2^2/2 - 2{,}000 &= -\rho V_1^2 y_1 + \rho V_2^2 y_2
\end{aligned}
$$

Let $V_1 = q/y_1$ and $V_2 = q/y_2$ and divide by γ

$$
\begin{aligned}
y_1^2/2 - y_2^2/2 - 2000/\gamma &= -q_1^2 y_1/(g y_1^2) + q_2^2 y_2/(g y_2^2) \\
1/2 - y_2^2/2 - 3.205 &= (+(20)^2/32.2)(-1 + 1/y_2)
\end{aligned}
$$

Solving for y_2 yields: $y_2 = \underline{\underline{1.43 \text{ ft}}}$

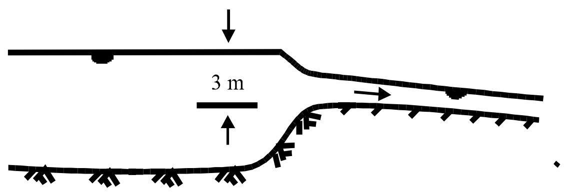
15.24 **Information and Assumptions**

provided in problem statement

Find

discharge

Solution



Assume negligible velocity in the reservoir and negligible energy loss. Then the channel entrance will act like a broad crested weir.

Thus

$$Q = 0.545\sqrt{g}LH^{3/2}$$

where $L = 4$ m and $H = 3$ m. Then

$$Q = 0.545\sqrt{9.81} \times 4 \times 3^{3/2} = \underline{\underline{35.5 \text{ m}^2/\text{s}}}$$

15.25 **Information and Assumptions**

provided in problem statement

Find

speed of the wave

Solution

$$V = \sqrt{gy} = \sqrt{32.2 \times 0.5} = 4.01 \text{ ft/s}$$

15.26 **Information and Assumptions**

provided in problem statement

Find

depth of water

Solution

$$
\begin{aligned}
V &= \sqrt{gy} \\
1.5 &= \sqrt{9.81y} \\
y &= \underline{\underline{0.23 \text{ m}}}
\end{aligned}
$$

15.27 As the waves travel into shallower water their speed is decreased ($V = \sqrt{gy}$); therefore, the wave lags that in deeper water. Thus, the wave crests tend to become parallel to the shoreline.

15.28 **Information and Assumptions**

provided in problem statement

Find

head lost, power dissipated and horizontal component of the force exerted by ramp

Solution

Let the upstream section (where $y = 3$ ft) be section 1 and the downstream section ($y = 2$ ft) be section 2. Then

$$V_1 = 18/3 = 6 \text{ ft/s}$$

and

$$\begin{aligned} V_2 &= 18/2 = 9 \text{ ft/s} \\ y_1 + V_1^2/2g + z_1 &= y_2 + V_2^2/2g + z_2 + h_L \\ 3 + 6^2/(2 \times 32.2) + 2 &= 2 + 9^2/(2 \times 32.2) + h_L \\ h_L &= \underline{\underline{2.30 \text{ ft}}} \end{aligned}$$

$$\begin{aligned} P &= Q\gamma h_L/550 \\ &= 18 \times 62.4 \times 2.3/550 = \underline{\underline{4.70 \text{ horsepower}}} \end{aligned}$$

Determine the force of ramp by writing the momentum equation between section 1 and 2. Let F_x be the force of the ramp on the water and assume x positive in the direction of flow. Then

$$\begin{aligned} \sum F_x &= \rho q(V_{2x} - V_{1x}) \\ \gamma y_1^2/2 - \gamma y_2^2/2 + F_x &= 1.94 \times 18(9 - 6) \\ (62.4/2)(3^3 - 2^2) + F_x &= 104.8 \\ F_x &= -51.2 \text{ lbf} \end{aligned}$$

The ramp exerts a force of $\underline{\underline{\text{51.2 lbs opposite to the direction of flow}}}$.

15.29 **Information and Assumptions**

provided in problem statement

Find

depth downstream of hydraulic jump

Solution

$$
\begin{aligned}
y_0 + q^2/(2gy_0^2) &= y_1 + q^2/(2gy_1^2) \\
5 + 2^2/(2(9.81)5^2) &= y_1 + 2^2/(2(9.81)y_1^2) \\
y_1 &= 0.206 \text{ m} \\
F_1 &= q/\sqrt{gy_1^3} = 2/\sqrt{9.81(0.206)^3} = 6.829 \\
y_2 &= (y_1/2)(\sqrt{1+8F_1^2} - 1) = (0.206/2)(\sqrt{1+8(6.829^2)} - 1 = \underline{\underline{1.89 \text{ m}}}
\end{aligned}
$$

15.30 **Information and Assumptions**

provided in problem statement

Find

if hydraulic jump can exist and depth downstream of jump

Solution

$$\begin{aligned} Fr &= V/\sqrt{gy} \\ V &= Q/(By) \\ V &= 3.60/(2 \times 0.30) \\ V &= 6 \text{ m/s} \\ Fr &= 6/\sqrt{gy} \\ &= 6/\sqrt{9.81 \times 0.30} = 3.5 > 1 \\ Fr &= 3.5 > 1 \end{aligned}$$

Therefore $\underline{\underline{\text{jump can form}}}$.

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1 + 8F_1^2} - 1) = (0.3/2)(\sqrt{1 + 8 \times 3.5^2} - 1) \\ &= \underline{\underline{1.34 \text{ m}}} \end{aligned}$$

15.31 **Information and Assumptions**

assume V_0 is negligible
provided in problem statement

Find

depth of flow on apron downstream of jump.

Solution

First develop the expression for y_1 and $V_{\text{theor.}}$.Write the energy equation from the upstream pool level to y_1.

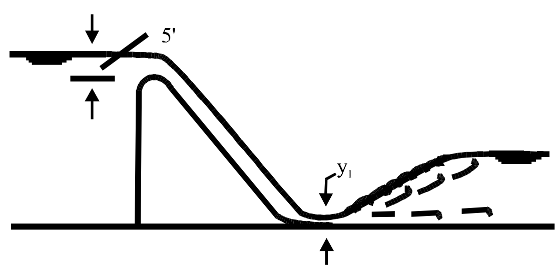
$$\begin{aligned} V_0^2/2g + z_0 &= V_1^2/2g + z_1 \\ 0 + 100 &= V_{\text{theor.}}^2/2g + y_1 \end{aligned} \tag{1}$$

But

$$V_{\text{theor}} = V_{\text{act}}/0.95 \tag{2}$$

and

$$V_{\text{act.}} = q/y_1 \tag{3}$$



Consider a unit width of spillway. Then

$$\begin{aligned} q &= Q/L = K\sqrt{2g}H^{1.5} \\ &= 0.5\sqrt{2g}(5^{1.5}) \\ q &= 44.86 \text{ cfs/ft} \end{aligned} \tag{4}$$

Solving Eqs. (1), (2), (3), and (4) yields

$$y_1 = 0.59 \text{ ft}$$

and

$$
\begin{aligned}
V_{\text{act.}} &= 76.03 \text{ ft/sec} \\
Fr_1 &= V/\sqrt{gy_1} = 76.03/\sqrt{(32.2)(0.59)} = 17.44
\end{aligned}
$$

Now solve for the depth of flow on the apron:

$$
\begin{aligned}
y_2 &= (y_1/2)((1+8Fr_1^2)^{0.5}-1) \\
&= (0.59/2)((1+8(17.44^2))^{0.5}-1) = \underline{\underline{14.3 \text{ ft}}}
\end{aligned}
$$

15.32 **Information and Assumptions**

provided in problem statement

Find

depth at y_1

Solution

$$y_2 = (y_1/2)((1 + 8Fr_1^2)^{0.5} - 1)$$

where

$$Fr_1^2 = V_1^2/(gy_1) = q^2/(gy_1^3)$$

Then

$$\begin{aligned} y_2 &= (y_1/2)((1 + 8q^2/(gy_1^3))^{0.5} - 1) \\ y_2 - y_1 &= (y_1/2)[((1 + 8q^2/(gy_1^3))^{0.5} - 1 - 2] \end{aligned}$$

However

$$\begin{aligned} y_2 - y_1 &= 14.0 \text{ ft (given)} \\ q &= 65 \text{ ft}^2/\text{s} \end{aligned}$$

Therefore

$$\begin{aligned} 14.0 \text{ ft} &= (y_1/2)[((1 + 8 \times 65^2/(gy_1^3))^{0.5} - 1) - 2] \\ y_1 &= \underline{\underline{1.08 \text{ ft}}} \end{aligned}$$

15.33 **Information and Assumptions**

provided in problem statement

Find

depth of flow downstream of jump

Solution

$$F_1 = V/\sqrt{gy} = 8/\sqrt{9.81(0.4)} = 4.04$$
$$y_2 = (0.4/2)(\sqrt{1 + 8(4.04)^2} - 1) = \underline{\underline{2.09 \text{ m}}}$$

15.34 **Information and Assumptions**

provided in problem statement

Find

depth of flow downstream of jump

Solution

Check Fr upstream to see if it is really supercritical flow

$$
\begin{aligned}
Fr &= V/(gD)^{0.5} \\
D &= A/T = (By + y^2)/(B + 2y) \\
D_{y=0.4} &= (5 \times 0.4 + 0.4^2)/(5 + 2 \times 0.4) = 0.372 \text{ m}
\end{aligned}
$$

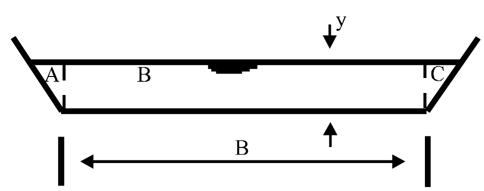
Then

$$
\begin{aligned}
Fr_1 &= 10 \text{ m/s}/((9.81 \text{ m/s}^2)(0.372))^{0.5} \\
Fr_1 &= 5.23
\end{aligned}
$$

so flow is supercritical and a jump will form. Applying the momentum equation (Eq. 15.23):

$$\bar{p}_1 A_1 + \rho Q V_1 = \bar{p}_2 A_2 + \rho Q V_2 \tag{1}$$

Evaluate $\bar{p}_1$ by considering the hydrostatic forces on the trapezoidal section divided into rectangular plus triangular areas as shown below:



Then

$$
\begin{aligned}
\bar{p}_1 A_1 &= \bar{p}_A A_A + \bar{p}_B A_B + \bar{p}_C A_C \\
&= (\gamma y_1/3)(y_1^2/2) + (\gamma y_1/2)\ By_1 + (\gamma y_1/3)(y_1^2/2) \\
&= \gamma(y_1^3/6) + \gamma B(y_1^2/2) + \gamma(y_1^3/6) \\
&= \gamma(y_1^3/3) + \gamma B(y_1^2/2) \\
\bar{p}_1 A_1 &= \gamma((y_1^3/3) + B(y_1^2/2))
\end{aligned}
$$

Also

$$\rho Q V_1 = \rho Q Q/A_1 = \rho Q^2/A_1$$

Equation (1) is then written as

$$\gamma((y_1^3/3)+(B(y_1^2/2))+\rho Q^2/A_1=\gamma((y_2^3/3)+B(y_2^2/2))+\rho Q^2/(By_2+y_2^2)$$

For $\gamma=9,810$ N/m^2, $B_3=5$ m, $y_1=40$ cm$=0.40$ m

$$\begin{aligned} Q &= V_1A_1=21.6\ \text{m}^3/\text{s} \\ A_1 &= 5\times 0.4+0.4^2=2.16\ \text{m}^2 \end{aligned}$$

Solving for y_2 yields: $y_2=\underline{\underline{2.45}}$ m

15.35 **Information and Assumptions**

provided in problem statement

Find

discharge per foot width of channel

Solution

$$y_2/y_1 = 12/0.5 = 0.5(1 + 8Fr_1^2)^{0.5} - 1)$$

Solving for Fr_1 yields $Fr_1 = 17.32$. Then

$$Fr_1 = V/(gy_1)^{1/2}$$

$$\begin{aligned} V &= Fr_1(gy_1)^{0.5} = 17.32(32.2 \times 0.5)^{0.5} \\ &= 69.50 \text{ ft/s} \end{aligned}$$

Finally

$$q = V_1 y_1 = (69.50ft/s)(0.50ft) = \underline{\underline{34.75}} \text{ ft}^2/\text{s}$$

15.36 **Information and Assumptions**

provided in problem statement

Find

critical depth and normal depth

Solution

$$
\begin{aligned}
y_c &= (q^2/g)^{1/3} \\
q &= 500/20 = 25 \ \text{ cfs/ft} \\
y_c &= (25^2/32.2)^{1/3} = \underline{\underline{2.69 \text{ ft}}}
\end{aligned}
$$

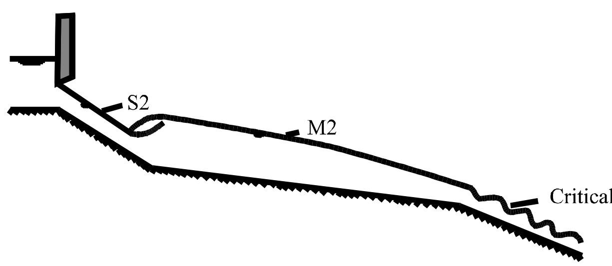
Solving for $y_{n,1}$yields $\underline{\underline{1.86 \text{ ft}}}$.

Thus one concludes that normal depth in $\underline{\underline{\text{reach 1 would be supercritical}}}$, in $\underline{\underline{\text{reach 2 subcritical}}}$ and in $\underline{\underline{\text{reach 3 critical}}}$. If reach 2 is long then the flow would be near normal depth in reach 2. Thus, the flow would probably go from supercritical flow in reach 1 to subcritical in reach 2. In going from sub to supercritical a hydraulic jump would form.

Determine jump height and location:

$$
\begin{aligned}
y_2 &= (y_1/2)((1+8Fr_1^2)^{0.5}-1) \\
Fr_1 &= V_1/(gy_1)^{0.5} = (25/1.86)/(32.2 \times 1.86)^{0.5} = 1.737 \\
y_2 &= (1.86/2)((1+8 \times 1.737^2)^{0.5}-1) = \underline{\underline{3.73 \text{ ft}}}
\end{aligned}
$$

Because y_2 is less than the normal depth in reach 2 the jump will probably occur in reach 1. The water surface profile could occur as shown below.



15.37 **Information and Assumptions**

provided in problem statement

Find

if hydraulic jump will form

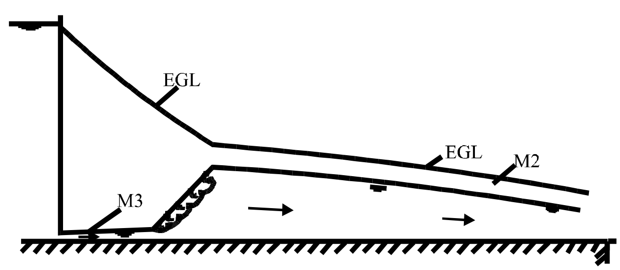
Solution

$$
\begin{aligned}
Fr_1 &= V_1/\sqrt{gy_1} = 10/\sqrt{9.81 \times 0.10} = 10.1 \qquad \text{(supercritical)} \\
V_2 &= q/y_2 = (0.10 \text{ m})\ (10 \text{ m/s})/(1.1 \text{ m}) = 0.91 \text{ m/s} \\
Fr_2 &= V_2/(gy_2)^{0.5} = 0.91/(9.81 \times 1.1)^{0.5} = 0.277
\end{aligned}
$$

A hydraulic jump will form because flow goes from supercritical to subcritical. Determine y_1 for a y_2 of 1.1 m:

$$
\begin{aligned}
y_1 &= (y_2/2)((1 + 8Fr_2^2)^{0.5} - 1) \\
&= (1.1/2)((1 + 8 \times .277^2)^{0.5} - 1) \\
&= 0.14 \text{ m}
\end{aligned}
$$

Therefore the jump will start at about the 29 m distance downstream of the sluice gate. Profile and energy grade line:



15.38 **Information and Assumptions**

provided in problem statement

Find

an estimate of shear stress on smooth bottom

Solution

$$\text{Re}_x = (10 \times 0.5)/(10^{-6}) = 5 \times 10^6$$

From Chapter 9 for $Re_x < 10^7$

$$\begin{aligned} c_f &= (0.058)/(\text{Re}_x^{0.2}) \\ &= (0.058)/(21.9) = 0.0026 \\ \tau_0 &= c_f \rho V^2/2 = (0.0026)(998)(10^2/2) \\ &= 130 \text{ N/m}^2 \end{aligned}$$

Therefore; the correct choice is d) $\underline{\underline{\tau > 40 \text{ N/m}^2}}$

15.39 **Information and Assumptions**

provided in problem statement

Find

horsepower lost in hydraulic jump

Solution

Assume negligible energy loss for flow under the sluice gate. Write the Bernoulli equation from a section upstream of the sluice gate to a section immediately downstream of the sluice gate.

$$\begin{aligned}
y_0 + V_0^2/2g &= y_1 + V_1^2/2g \\
65 + \text{neglig.} &= 1 + V_1^2/2g \\
V_1 &= \sqrt{64 \times 64.4} = 64.2 \text{ ft/s} \\
F_1 &= V_1/\sqrt{gy_1} = 64.2/\sqrt{32.2 \times 1} = 11.3
\end{aligned}$$

Now solve for the depth after the jump:

$$\begin{aligned}
y_2 &= (y_1/2)(\sqrt{1 + 8F_1^2} - 1) \\
&= (1/2(\sqrt{1 + 8 \times 11.3^2} - 1) = 15.5 \text{ ft} \\
h_L &= (y_2 - y_1)^3/(4y_1y_2) \\
&= (14.51)^3/(4 \times 1 \times 15.51) = \underline{\underline{49.2 \text{ ft}}} \\
P &= Q\gamma h_L/550 \\
&= (64.2 \times 1 \times 5) \times 62.4 \times 49.2/550 = \underline{\underline{1,793 \text{ horsepower}}}
\end{aligned}$$

15.40 Design problem:

For this experiment it is necessary to first produce supercritical flow in the flume and then force this flow to become subcritical. The supercritical flow could be produced by means of a sluice gate as shown in Prob. 15.39 and the jump could be forced by means of another sluice gate farther down the flume. Therefore, one needs to include in the design an upstream chamber that will include a sluice gate from which the high velocity flow will be discharged.

The relevant equation for the hydraulic jump is Eq. (15.28). Therefore, to verify this equation y_1, y_2 and V_1 will have to be measured or deduced by some other means. A fairly accurate measurement of y_2 can be made by means of a point gage or piezometer. The depth y_1 could also be measured in the same way; however, the degree of accuracy of this measurement will be less than for y_2 because y_1 is much smaller than y_2. Perhaps a more accurate measure of y_1 would be to get an accurate reading of the gate opening of the sluice gate and apply a coefficient of contraction to that reading to get y_1. The C_C for a sluice gate could be obtained from the literature.

The velocity, V_1, which will be needed to compute F_{r1} can probably be best calculated by Bernoulli's equation knowing the depth of flow in the chamber upstream of the sluice gate. Therefore, a measurement of that depth must be made.

Note that for use of V_1 and y_1 just downstream of the sluice gate, the hydraulic jump will have to start very close to the sluice gate because the depth will increase downstream due to the channel resistance. The jump location may be changed by operation of the downstream sluice gate. Other things that could or should be considered in the design:

A. Choose maximum design discharge. This will be no more than 5 cfs (see Prob. 13.77).

B. Choose reasonable size of chamber upstream of sluice gate. A 10 ft depth would be ample for a good experiment.

C. Choose width, height and length of flume.

D. Work out details of sluice gates and their controls.

15.41 **Information and Assumptions**

assume $n = 0.012$
provided in problem statement

Find

estimate of height of hydraulic jump

Solution

$$\begin{aligned} V &= (1/n)R^{2/3}S_0^{1/2} \\ R &= A/P = (0.4 \times 10)/(2 \times 0.4 + 10) = 0.370 \text{ m} \end{aligned}$$

Then

$$\begin{aligned} V &= (1/0.012)(0.370)^{2/3} \times (0.04)^{1/2} = 8.59 \text{ m/s} \\ F_1 &= V/\sqrt{gy_1} = 8.59/\sqrt{9.81 \times 0.40} = 4.34 \text{ (supercritical)} \\ y_2 &= (y_1/2)(\sqrt{1 + 8 \times F_1^2} - 1) = (0.40/2)(\sqrt{1 + 8 \times (4.34)^2} - 1) = \underline{\underline{2.26 \text{ m}}} \end{aligned}$$

15.42 **Information and Assumptions**

provided in problem statement

Find

estimate of shear force and F_s/F_H

Solution

Assume the shear stress will be the average of τ_{0_1}, uniform approaching the jump, and τ_{0_2}, uniform flow leaving the jump.

From Eq. 10.21

$$\tau_0 = f\rho V^2/8$$

where $f = f(\text{Re},\ k_s/4R)$

$$R_{e_1} = V_1(4R_1)/\nu \quad R_{e_2} = V_2 \times (4R_2)/\nu$$

From solution to Prob. 15.41

$$V_2 = V_1 \times 0.4/2.26 = 1.52 \text{ m/s}$$

$$\begin{array}{ll} R_{e_1} = 8.59 \times (4 \times 0.37)/10^{-6} & R_2 = A/P = (2.26 \times 10)/(2 \times 2.26 + 10) = 1.31 \text{ m} \\ R_{e_1} = 1.3 \times 10^7 & R_{e_2} = 1.52 \times (4 \times 1.56)/10^{-6} = 9.5 \times 10^6 \end{array}$$

Assume $k_s = 3 \times 10^{-3}$ m

$$\begin{array}{ll} k_s/4R_1 = 3 \times 10^{-3}/(4 \times 0.37) & k_s/4R_2 = 3 \times 10^{-3}/(4 \times 1.56) \\ k_s/4R_1 = 2 \times 10^{-3} & k_s/4R_2 = 4.8 \times 10^{-4} \end{array}$$

From Fig. 10-8, $f_1 = 0.024$ and $f_2 = 0.017$. Then

$$\begin{array}{ll} \tau_{0_1} = 0.024 \times 1,000 \times (6.87)^2/8 & \tau_{0_2} = 0.017 \times 1,000 \times (1.52)^2/8 \\ \tau_{0_1} = 142 \text{ N/m}^2 & \tau_{0_2} = 4.9 \text{ N/m}^2 \end{array}$$

$$\tau_{\text{avg}} = (142 + 4.9)/2 = 73 \text{ N/m}^2$$

Then

$$F_s = \tau_{\text{avg}} A_s = \tau_{\text{avg}} PL$$

where $L \approx 6y_2, P \approx B + (y_1 + y_2)$. Then

$$F_2 \approx 73(10 + (0.40 + 2.26))(6 \times 2.26) = 10,790 \text{ N}$$

$$F_H = (\gamma/2)(y_2^2 - y_1^2)B = (9,810/2)((2.26)^2 - (0.40)^2) \times 10 = 242,680 \text{ N}$$

Thus

$$F_s/F_H = 10,790/242,680 = \underline{\underline{0.044}}$$

Note: The above estimate probably gives an excessive amount of weight. to τ_{0_1} because τ_0 will not be linearly distributed. A better estimate might be to assume a linear distribution of velocity with an average f and integrate $\tau_0 dA$ from one end to the other.

15.43 **Information and Assumptions**

provided in problem statement

Find

type of water surface profile and shear stress on smooth bottom distance of 0.5 m downstream

Solution

$$q = 0.40 \times 10 = 4.0 \text{ m}^3/\text{s/m}$$

Then

$$y_c = \sqrt[3]{q^2/g} = \sqrt[3]{(4.0)^2/9.81} = 1.18m$$

Then we have $y < y_n < y_c$; therefore, the water surface profile will be an $\underline{\underline{S3}}$.

For the shear stress assume a boundary layer develops similar to a flat plate downstream of the plane of the sluice gate.

Then

$$\begin{aligned} \text{Re}_x &\approx V \times 0.5/\nu \\ \text{Re}_x &= 10 \times 0.5/10^{-6} = 5 \times 10^6 \\ c_f &= 0.058/\text{Re}_x^{1/5} = 0.00265 \text{ (from Ch. 9)} \end{aligned}$$

From Chapter 9 $c_f = 0.058/\text{Re}_x^{1/5} = 0.00265$. Then

$$\tau_0 = c_f \rho V_0^2/2 = 0.00265 \times 998 \times 10^2/2 = \underline{\underline{132 \text{ N/m}^2}}$$

15.44 **Information and Assumptions**

provided in problem statement

Find

water surface profile

Solution

$$
\begin{aligned}
y_n &= 2 \text{ ft} \\
y_c &= (q^2/g)^{1/3} = (10^2/32.2)^{1/3} = 1.46 \text{ ft.} \\
y &> y_n > y_c
\end{aligned}
$$

From Fig. 15-16 the profile isM1. Correct choice is $\underline{\underline{\text{c)}}}$.

15.45 The correct choice is $\underline{\underline{\text{d)}}}$.

15.46

$$F_1 = q/\sqrt{gy^3} = (5/3)/\sqrt{9.81(0.3)^3} = 3.24 > 1 \text{ (supercritical)}$$

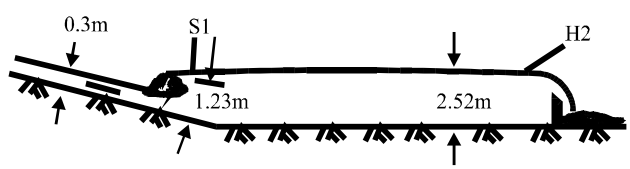
For flow over weir,

$$\begin{aligned} Q &= (0.40 + 0.05H/P)L\sqrt{2g}H^{3/2} \\ 5 &= (0.40 + 0.05H/1.6) \times 3\sqrt{2(9.81)}H^{3/2} \end{aligned}$$

Solving by iteration gives $H = 0.917$ m. Depth upstream of weir $= 0.917 + 1.6 = 2.52$ m

$$F_2 = (5/3)/\sqrt{9.81(2.52)^3} = 0.133 < 1 \text{ (subcritical)}$$

Therefore a hydraulic jump forms. $y_2 = (0.3/2)(\sqrt{1 + 8(3.24)^2} - 1) = 1.23$ m



15.47 **Information and Assumptions**

provided in problem statement

Find

water surface profile

Solution

The profile might be an M profile or an S profile depending upon whether the slope is mild or steep. However, if it is a steep slope the flow would be uniform right to the brink. Check to see if M or S slope. assume $n = 0.012$

$$\begin{aligned}
Q &= (1.49/n)AR^{0.667}S^{0.5} \\
AR^{2/3} &= Q/((1.49/n)(S^{0.5})); \\
&= 120/((1.49/0.012)(0.0001)^{0.5}) \\
(by)(by/(10+2y))^{.667} &= 96.6
\end{aligned}$$

With $b = 10$ ft we can solve for y to obtain $y = 5.2$ ft. Then

$$V = Q/A = 120/32 = 2.31 \text{ ft/s}$$

$$F = V/\sqrt{gy} = 2.31/(\sqrt{32.2 \times 5.2}) = 0.18 \text{ (subcritical)}$$

Therefore, the water surface profile will be an $\underline{\underline{\text{M2}}}$.

15.48 **Information and Assumptions**

provided in problem statement

Find

if hydraulic jump will occur and where it would be located.

Solution

First determine the depth upstream of the weir

$$Q = K\sqrt{2g}LH^{3/2}$$

where $K = 0.40 + 0.05H/P$. By trial and error (first assume K then solve for H, etc.) solve for H yield $H = 2.06$ ft. Then the velocity upstream of the weir will be

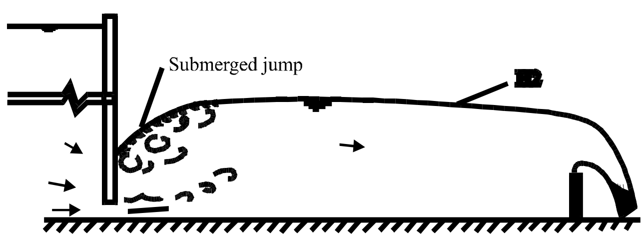
$$\begin{aligned} V &= Q/A = 108/(4.06 \times 10) = 2.66 \text{ ft/sec} \\ Fr &= V/\sqrt{gy} = 2.66/(32.2 \times (4.06))^{0.5} = 0.23 \text{ (subcritical)} \end{aligned}$$

The Froude number just downstream of the sluice gate will be determined:

$$\begin{aligned} V &= Q/A = 108/(10 \times 0.40) = 27 \text{ ft/sec} \\ Fr &= V/\sqrt{gy} = 27/\sqrt{32.2 \times 0.40} = 7.52 \text{ (supercritical)} \end{aligned}$$

Because the flow is supercritical just downstream of the sluice gate and subcritical upstream of the weir a jump will form someplace between these two sections.

Now determine the approximate location of the jump. Let y_2 = depth downstream of the jump and assume it is approximately equal to the depth upstream of the weir ($y \approx 4.06$ ft). By trial and error (utilizing Eq. (15.25)) it can be easily shown that a depth of 0.40 ft is required to produce the given y_2. Thus the jump will start immediately downstream of the sluice gate and it will be approximately 25 ft long. Actually, because of the channel resistance y_2 will be somewhat greater than $y_2 = 4.06$ ft; therefore, the jump may be submerged against the sluice gate and the water surface profile will probably appear as shown below.



15.49 **Information and Assumptions**

provided in problem statement

Find

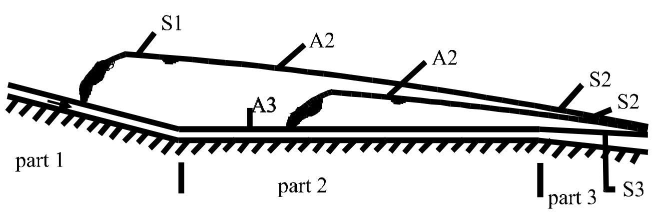
possible water surface profiles

Solution

$$y_c = \sqrt[3]{q^2/g} = \sqrt[3]{202/32.2} = 2.32ft.$$

Thus the slopes in parts 1 and 3 are steep.

If part 2 is very long, then a depth greater than critical will be forced in part 2 (the part with adverse slope). In that case a hydraulic jump will be formed and it may occur on part 2 or it may occur on part 1. These two possibilities are both shown below. The other possibility is for no jump to form on the adverse part. Also see this below:



15.50 **Information and Assumptions**

provided in problem statement

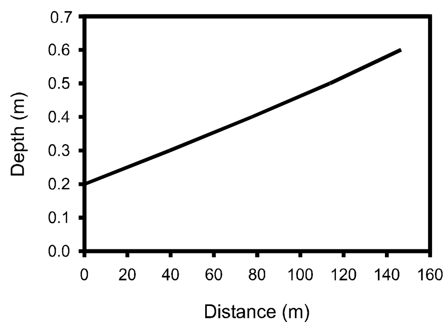
Find

water surface profile up to depth of 20 cm.

Solution

$$
\begin{aligned}
F_1 &= q/\sqrt{gy^3} = 3/\sqrt{9.81(0.2)^3} = 10.71 \\
F_2 &= 3/\sqrt{9.81(0.6)^3} = 2.06
\end{aligned}
$$

Therefore the profile is a continuous $H3$ profile.



y	$\bar{y}$	V	$\bar{V}$	E	ΔE	S_f	Δx	x
0.2		15		11.6678				0
	0.25		12.5		6.2710	0.1593	39.4	
0.3		10		5.3968				39.4
	0.35		8.75		2.1298	0.0557	38.2	
0.4		7.5		3.2670				77.6
	0.45		6.75		0.9321	0.0258	36.1	
0.5		6.0		2.3349				113.7
	0.55		5.5		0.4607	0.0140	32.9	
0.6		5.0		1.8742				146.6

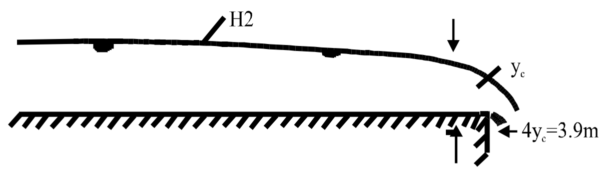
15.51 **Information and Assumptions**

provided in problem statement

Find

water depth 300 m upstream

Solution



$$
\begin{aligned}
q &= Q/B = 12/4 = 3 \text{ m}^3/\text{s/m} \\
y_c &= \sqrt[3]{q^2/g} = 0.972 \text{ m (This depth occurs near brink.)}
\end{aligned}
$$

Carry out a step solution for the profile upstream from the brink.

$$
\begin{aligned}
\text{Re} &\approx V \times 4R/\nu \approx 3 \times 1/10^{-6} \approx 3 \times 10^6 \\
k_s/4R &\approx 0.3 \times 10^{-3}/4 \approx 0.000075 \\
f &\approx 0.010
\end{aligned}
$$

See solution table below.

Solution Table for Problem 15.51

Section number upstream of y_c	Depth y,m	Velocity at section V,m/s	Mean Velocity in reach $(V_1+V_0)/2$	V^2	Hydraulic Radius R=A/P,m	Mean Hydraulic Radius $R_m=(R_1+R_2)/2$	$s_f=fV^2_{mean}/8gR_{mean}$	$\Delta x=((y_2+V^2_2/2g)-(y_1+V_1/2g))/S_f$	Distance upstream from brink x,m
1 at $y=y_c$	0.972	3.086			0.654				3.9m
			3.073	9.443		0.656	1.834×10^{-3}	0.1m	4.0m
2	0.980	3.060			0.658				
			3.045	9.272		0.660	1.790×10^{-3}	0.4m	4.4m
3	0.990	3.030			0.662				
			2.986	8.916		0.669	1.698×10^{-3}	1.7m	6.1m
4	1.020	2.941			0.675				
			2.886	8.327		0.684	1.551×10^{-3}	4.7m	10.9m
5	1.060	2.830			0.693				
			2.779	7.721		0.701	1.403×10^{-3}	7.7m	18.6m
6	1.100	2.727			0.710				
			2.613	6.828		0.730	1.192×10^{-3}	33.2m	51.8m
7	1.200	2.500			0.750				
			2.404	5.779		0.769	9.576×10^{-4}	55.3m	107.1m
8	1.300	2.308			0.788				
			2.225	4.951		0.806	7.83×10^{-4}	80.0m	187.1m
9	1.400	2.143			0.824				
			2.072	4.291		0.841	6.501×10^{-4}	107.4m	294.5m
10	1.500	2.000			0.857				

The depth 300 m upstream is approximately 1.51 m

15.52 Upstream of jump the profile will be an $\underline{\underline{H3}}$. Downstream of jump the profile will be an $\underline{\underline{H2}}$. The baffle blocks will cause the depth upstream of A to increase; therefore, the jump will move towards the sluice gate.

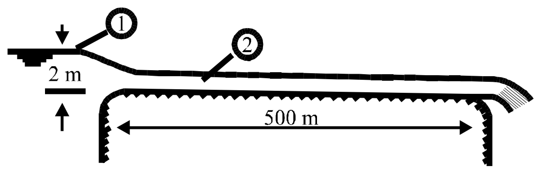
15.53 **Information and Assumptions**

provided in problem statement

Find

discharge in channel

Solution



The channel is steep; therefore, critical depth will occur just inside the channel entrance. Then write the energy equation from the reservoir, (1), to the entrance section (2). Assume $V_1 = 0$.

$$\begin{aligned}
y_1 + V_1^2/2g &= y_2 + V_2^2/2g \\
\text{Then } 2 &= y_2 + V_2^2/2g = y_c + 0.5y_c \\
\text{Solving for } y_c &: y_c = 2/1.5 = 1.33 \text{ m} \\
\text{Get } V_c &= V_2 : V_c^2/g = y_c = 1.33 \text{ or } V_c = 3.62 \text{ m/s} \\
\text{Then } Q &= V_c A_2 = 3.62 \times 1.33 \times 4 = 19.2 \text{ m}^3/\text{s}
\end{aligned}$$

Then

$$2 = y_2 + V_2^2/2g = y_c + 0.5y_c$$

Solving for y_c gives $y_c = 2/1.5 = 1.33$ m. Get

$$\begin{aligned}
V_c &= V_2 \\
V_c^2/g &= y_c = 1.33 \\
V_c &= 3.62 \text{ m/s}
\end{aligned}$$

Then

$$Q = V_c A_2 = 3.62 \times 1.33 \times 4 = \underline{\underline{19.2 \text{ m}^3/\text{s}}}$$

15.54 **Information and Assumptions**

provided in problem statement

Find

a) an estimate for the discharge and b) how the discharge would be estimated for a 100 m channel.

Solution

a) Assume uniform flow is established in the channel except near the downstream end. Then if the energy equation is written from the reservoir to a section near the upstream end of the channel, we have:

$$2.5 \approx V_n^2/2g + y_n \tag{1}$$

Also

$$\begin{aligned} V_n &= (1/n)R^{2/3}S^{1/2} \\ V_n^2/2g &= (1/n^2)R^{4/3}S/2g \end{aligned} \tag{2}$$

where

$$R = A/P = 3.5y_n/(2y_n + 3.5) \tag{3}$$

Then combining Eqs. (1), (2) and (3) we have

$$2.5 = ((1/n^2)((3.5y_n/(2y_n + 3.5))^{4/3}S/2g) + y_n \tag{4}$$

Assuming $n = 0.012$ and solving Eq. (4) for y_n yields: $y_n = 2.16$ m; also solving (2) yields $V_n = 2.58$ m/s. Then

$$Q = VA = 2.58 \times 3.5 \times 2.15 = \underline{\underline{19.5 \text{ m}^3/\text{s}}}$$

b) With only a 100 m-long channel, uniform flow will not become established in the channel; therefore, a trial-and-error type of solution is required. Critical depth will occur just upstream of the brink, so assume a value of y_c, then calculate Q and calculate the water surface profile back to the reservoir. Repeat the process for different values of y_c until a match between the reservoir water surface elevation and the computed profile is achieved.

15.55 **Information and Assumptions**

provided in problem statement

Find

the water surface profile

Solution

$$q = 10 \text{ m}^3/\text{s/m}$$
$$y_c = \sqrt[3]{q^2/g} = \sqrt[3]{10^2/9.81} = 2.17 \text{ m}$$

y	$\bar{y}$	V	$\bar{V}$	E	ΔE	$S_f \times 10^4$	Δx	x	elev.
52.17		0.1917		52.170				0	52.17
	51.08		0.1958		2.168	0.00287	-5,429		
50		0.20		50.002				5,430	52.17
	45		0.2222		9.999	0.00419	-25,024		
40		0.25		40.003				-30,450	52.18
	35		0.2857		9.997	0.00892	-25,048		
30		0.333		30.006				-55,550	52.22
	25		0.400		9.993	0.02447	-25,146		
20		0.50		20.013				-80,650	52.26
	15		0.6667		9.962	0.11326	-25,631		
10		1.00		10.051				-106,280	52.51
	9		1.1111		1.971	0.5244	-5,671		
8		1.25		8.080				-111,950	52.78
	7		1.4286		1.938	1.1145	-6,716		
6		1.667		6.142				-118,670	53.47

15.56 **Information and Assumptions**

assume $n = 0.015$, $K = 0.42$
provided in problem statement

Find

water surface profile from section 1 to section 2.

Solution

First, one has to determine whether the uniform flow in the channel is super or subcritical. Determine y_n and then see if for this y_n the Froude number is greater or less than unity.

$$\begin{aligned}
Q &= (1.49/n)AR^{2/3}S^{1/2} \\
12 &= (1.49/0.015) \times y \times y^{2/3} \times (0.04)^{1/2} \\
y_n &= 0.739 \text{ ft and } V = Q/y_n = 16.23 \text{ ft/s} \\
F &= V/\sqrt{gy_n} = 3.33
\end{aligned}$$

Solving for y_n gives $y_n = 0.739$ ft and

$$V = Q/y_n = 16.23 \text{ ft/s}$$

Therefore, uniform flow in the channel is supercritical and one can surmise that a hydraulic jump will occur upstream of the weir. One can check this by determining what the sequent depth is. If it is less than the weir height plus head on the weir height plus head on the weir then the jump will occur.

Now find sequent depth:

$$\begin{aligned}
y_2 &= (y_1/2)(\sqrt{1 + 8F_1^2} - 1) \\
&= (0.739/2)(\sqrt{1 + 8 \times 3.33^2} - 1) \\
y_2 &= \underline{\underline{3.13 \text{ ft}}}
\end{aligned}$$

and head on weir:

$$\begin{aligned}
Q &= K\sqrt{2g}LH^{3/2} \\
12 &= 0.42\sqrt{64.4} \times 1 \times H^{3/2} \\
H &= 2.33 \text{ ft} \\
H/P &= 2.33/3 = 0.78
\end{aligned}$$

so

$$K = 0.40 + 0.05 \times 0.78$$

A better estimate is

$$H = 2.26 \text{ ft} \quad K = 0.44$$

Then depth upstream of weir $= 3 + 2.26 = 5.56$ ft. Therefore, it is proved that a jump will occur. A rough estimate for the distance to where the jump will occur may be found by applying Eq. (15.35) with a single step computation. A more accurate calculation would include several steps.

The single-step calculation is given below:

$$\Delta x = ((y_1 - y_2) + (V_1^2 - V_2^2))/2g/(S_f - S_0)$$

where $y_1 = 3.13$ ft; $V_1 = q/y_1 = 12/3.13 = 3.83$ ft/s; $V_1^2 = 14.67$ ft^2/s^2 and $y_2 = 5.56$ ft; $V_2 = 2.16$ ft/s.

$$\begin{aligned} V_2^2 &= 4.67 \text{ ft}^2/\text{s}^2 \\ S_f &= fV_{\text{avg}}^2/(8gR_{\text{avg}}) \\ V_{\text{avg}} &= 3.00 \text{ ft/s} \\ R_{\text{avg}} &= 4.34 \text{ ft} \end{aligned}$$

Assume $k_s = 0.001$ ft so $k_s/4R = 0.00034$.

$$\text{Re} = V \times 4R/\nu = ((3.83 + 2.16)/2) \times 4 \times 4.34/(1.22 \times 10^{-5}) = 4.33 \times 10^6$$

Then

$$f = 0.015$$

and

$$\begin{aligned} S_f &= 0.015 \times 3.0^2/(8 \times 32.2 \times 4.34) = 0.000121 \\ \Delta x &= ((3.13 - 5.56) + (14.67 - 4.67)/(64.4))/(0.000121 - 0.04) = \underline{\underline{57.0 \text{ ft}}} \end{aligned}$$

Thus, the water surface profile is shown below: